Pattern Recognition

Mini-project #5

Subject: Support Vector machines

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Contents

Abstract

Kernel method and Kernel machines are one of the main branches of pattern recognition recipes which are capable of addressing supervised problems (both linear and nonlinear). Probably, the most famous member of kernel machines family is SVM which is the main topic of this project.

In this mini-project we are trying to fit some SVM classifiers with various settings to retailMarketing.csv database and compare them based on classification accuracy. the effect of data normalization is the next topic we'll investigate and finally the effect of hyperparameters will be considered.

Support Vector Machine

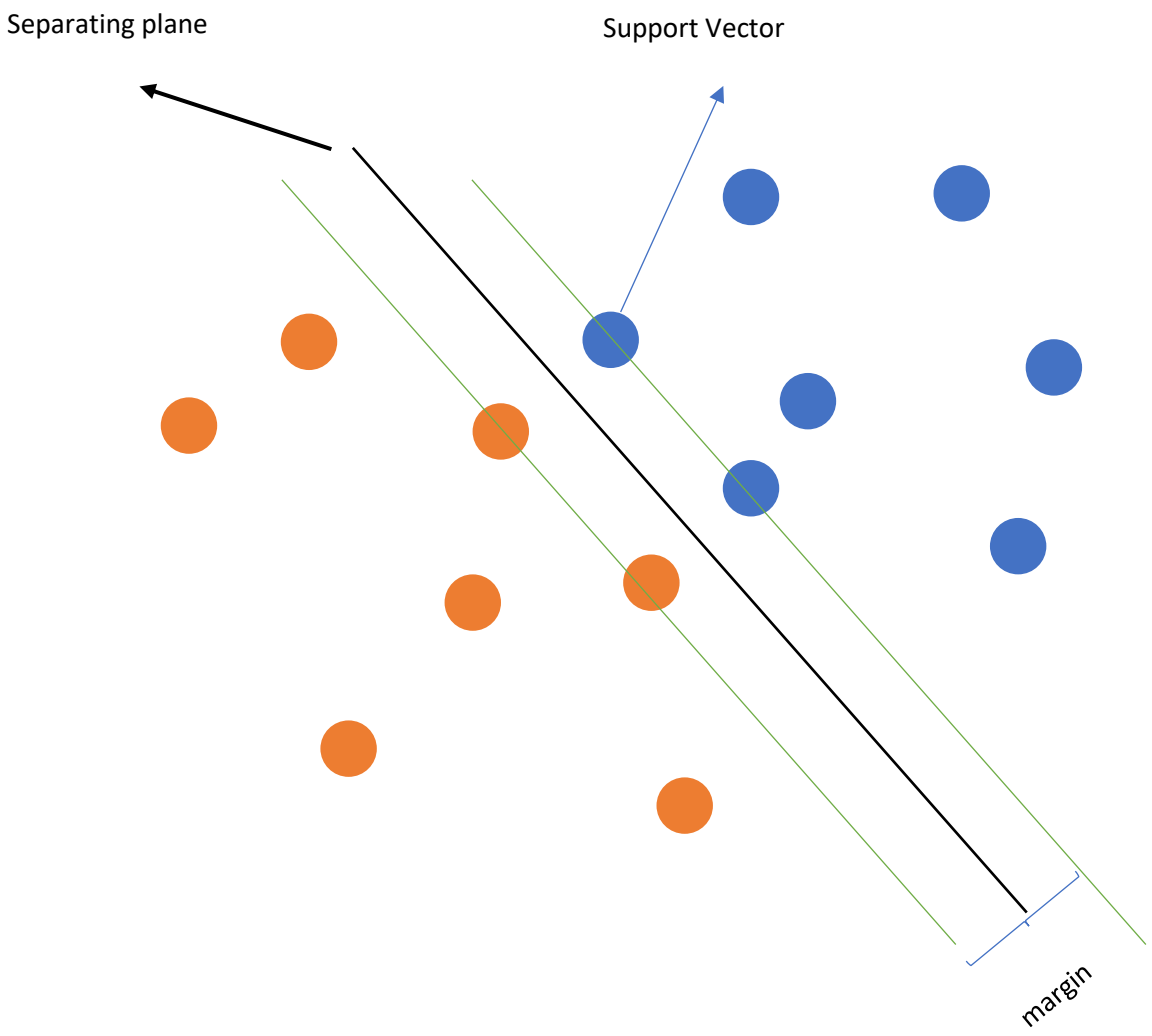
The main idea of the SVMs is to finding a hyperplane that best divides a dataset into two classes (hence, SVMs inherently used for binary classification) but, there may be infinitely many such hyperplanes!

What is the best hyperplane?

In the literature of SVMs, the best hyperplane is the one that has farthest distance from data points of both classes. Intuitively, we need to consider only the nearest data points to such hyperplane. But we don't know the hyperplane nor the nearest points.

If we assume that the smallest distance of a data point to the separating plane be **m, then our goal is to maximize the m (m: margin)**

The b is called margin hence, SVMs are called maximum margin classifiers



Support Vector Machine (Mathematics)

Le the separating plane be (a and x' are in augmented form)

$$g(x) = W^T x + b$$

Because we assume that the smallest distance between a datapoint and the plane is **m** so:

$$\frac{|g(x_k)|}{||a||} \geq m \; for \; all \; data \; point \; x'_k$$

Also, we assumed we have two classes so, we may impose:

$$y_k = +1 \; if \; x_k \epsilon \omega_1$$

$$y_k = -1 \; if \; x_k \epsilon \omega_2$$

$$\frac{y_k g(x'_k)}{||a||} \geq b$$

For having one solution we impose: m||W|| = 1 so maximizing **m** equals minimizing ||W||, thus, our problem becomes:

$$\text{minimize}\{ ||W||^2 \} \quad w.r.t \quad y_k g(x_k) \geq 1$$

Using Lagrange multipliers with $\lambda_k > 0 \; we \; have$:

$$L(W, w_0, \lambda) = \frac{1}{2}||W||^2 - \Sigma_{k=1}^{N} \lambda_k y_k (W^T x_k + b - 1)$$

Integrating with respect to W and w_0 in order to find maxima:

$$\frac{\partial L(W, w_0, \lambda)}{\partial W} = 0 \rightarrow W = \Sigma_{k=1}^{N} \lambda_k y_k x_k$$

$$\frac{\partial L(W, w_0, \lambda)}{\partial b} = 0 \rightarrow 0 = \Sigma_{k=1}^{N} \lambda_k y_k$$

After inserting the above equations into L, we get:

$$L(\lambda) = \Sigma_{k=1}^{N} \lambda_k - \frac{1}{2} \Sigma_{i=1}^{N} \Sigma_{j=1}^{N} \lambda_i \lambda_j y_i y_j x_i^T x_j$$

The L, should be maximized with respect to λ subject to the constraints:

$$\Sigma_{k=1}^{N} \lambda_k y_k = 0 \quad and \quad \lambda_k \geq 0 \quad for \; k = 1, \ldots, N$$

Which can be solved by Quadratic programming methods.

Soft margin SVM

The method discussed in the previous page, would have no solution if the dataset would not be linearly separable!

Also, it tries to classifying all data points correctly (training error = 0!) but it doesn't mean that the generalizing error is low.

Both of the two above problems can be eliminated if we let the margin adopts some error (or margin be soft!)

Let g(x) be the separating hyperplane, our constraints become:

$$g(x_k) = W^T x_k + b \geq 1 - \varepsilon_k \qquad if\ y_k = +1$$

$$g(x_k) = W^T x_k + b \geq 1 + \varepsilon_k \qquad if\ y_k = -1$$

$$\varepsilon_i \geq 0\ for\ 1 \leq i \leq N$$

Following the same approach as previous page, we get:

$$Minimize \frac{1}{2}\left|\left|W\right|\right|^2 + C\Sigma_{k=1}^{N}\varepsilon_k \quad s.t\ \ y_k(W^T x_k + b) \geq 1 - \varepsilon_k \qquad \varepsilon_k \geq 0$$

Where C is a tradeoff parameter between error and the margin.

After derivation with respect to W and ε and substituting the result into the above equation finally, we have:

$$\max\{\Sigma_{k=1}^{N}\lambda_k - \frac{1}{2}\Sigma_{i=1,j=1}\lambda_i\lambda_j y_i y_j x_i^T x_j\} \qquad s.t\ \ 0 \leq\ \lambda_k \leq C,\ \ \Sigma_{i=1}^{N}\lambda_i y_i = 0$$

$$W\ is\ recovered\ as\ W = \Sigma_{i=1}^{N}\lambda_i y_i x_i$$

Which is same as before, except there is an upper bound C on λs. Again, QP solver can be used.

SVM for nonlinear data

Soft margin SVM always fits a hyperplane. But what if our decision boundary has more complex form? One idea is that to transforming the data points into a higher dimensional space and hope that the data points become linearly separable in the transformed space.

let $\phi(x)$ be the transformation function, then our dataset D in the transformed space becomes:

$$D = \{X_1, X_2, \ldots., X_N\} \rightarrow D_{New} = \{\phi(X_1), \phi(X_2), \ldots., \phi(X_N)\}$$

Now, we can try to fit a SVM into D_{New} .

According to the previous discussions fitting a SVM leads to solving below problem:

$$\max\{\Sigma_{k=1}^{N}\lambda_k - \frac{1}{2}\Sigma_{i=1,j=1}\lambda_i\lambda_j y_i y_j x_i^T x_j\} \qquad s.t\ \ 0 \leq\ \lambda_k \leq C,\ \ \Sigma_{i=1}^{N}\lambda_i y_i = 0$$

$x_i^T x_j$ Is the inner product between two samples which in the case of D_{New} must be replaced with $\phi(x_i)^T\phi(x_j)$

But usually we don't know the appropriate $\phi(x)$.

Kernel trick

We don't need to knowing $\phi(x)$ explicitly, the only thing we need is the form of inner product of $\phi(x_i)^T\phi(x_j)$.

So, let define the kernel function as below:

$$K(x, x') = \phi(x)^T\phi(x')$$

Thus, instead of finding a vectorial function $\phi(x)$, we can define a scalar function K(x,x'). In this mini-project, various forms of kernel functions have been studied.

SVM for multiclass classification

As mentioned, SVMs intrinsically are designed to addressing binary classification problems.

For multiclass classification the SVMs can be applied after breaking the multiclass classification problem into multiple binary classification problem

There is two idea:

1) A binary classification per each pair of classes (One vs One), so if there are m classes, we need m(m-1)/2 SVMs then use majority voting for selecting the most probable class.
2) A binary classification per each class (One vs Rest), so if there are m classes, we need m SVMs then use some criteria to select the best one (for example each model should computes a score for its class, then select the class with highest score).

Preparing the dataset

Our problem is to fitting a SVM in order to predicting the age of customers based on their other recorded attributes in the retialMarking.csv dataset.

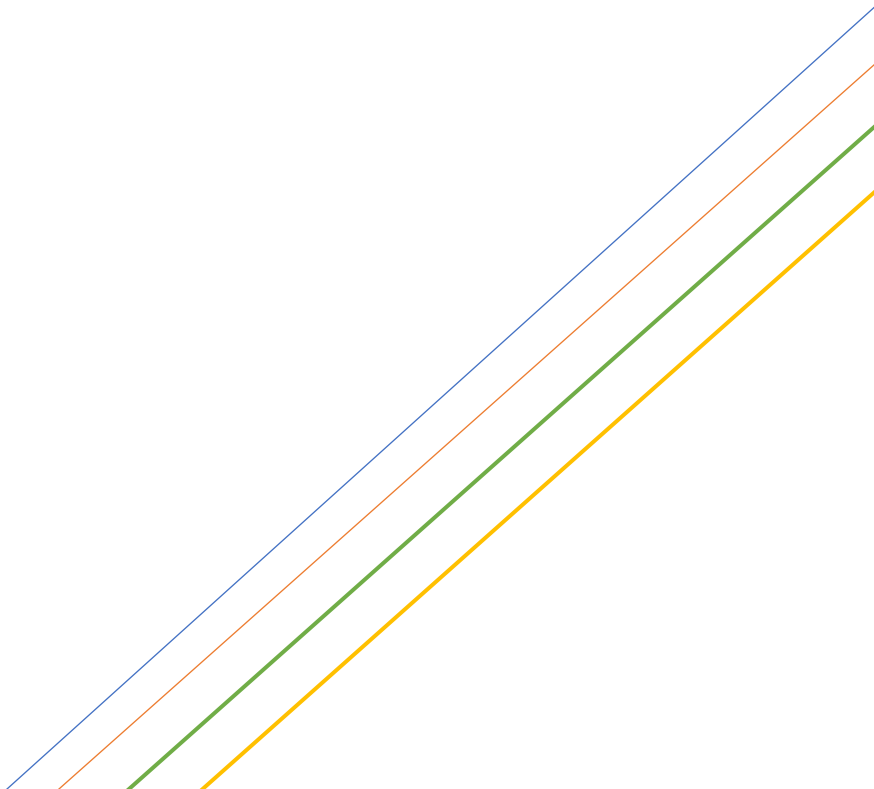
the dataset is prepared as following:

1) separating the age (our target) from other features (inputs)
2) the age can take three string values ('Young', 'Middle', 'Old) which are replaced with numbers (0,1,2) correspondingly.
3) any discrete feature which is stored as a binary vector is transformed into an appropriate number value equals to the index of element 1 in its binary vector representation.

And finally, the dataset has been divided into 60% - 40% parts for training and testing respectively.

Problems

1) RBF and Polynomial Kernels
2) Linear Kernels
3) Kernels comparison
4) Effect of data normalization
5) Hyperparameter adjustment



RBF and Polynomial Kernels

The polynomial kernel is defined as below:

$$K(X,Y) = (X^T Y + c)^d$$

Where c and d are design parameters.

Radial Basis function Kernel is defined as below:

$$K(X,Y) = \exp\left(-\frac{||X-Y||^2}{2\sigma^2}\right)$$

Where σ is design parameter

Without any parameter modification,

The classification accuracy of SVM with RBF kernel is about 65%

The classification accuracy of SVM with polynomial kernel (degree = 3) is about 62%

Linear Kernel

Linear kernel means having no kernel, using the inner product of the input points directly without any further transformation.

After adjusting the parameter C,

The classification accuracy of SVM with Linear kernel is about 72%

Kernels comparison

Both RBF and polynomial kernels increase the dimensionality of feature space and then finding the best separating hyperplane in the transformed space. (the underlying feature space in the case of polynomial kernel is finite dimensional but for the case of RBF kernel is infinite dimensional!)

Generally, increasing the dimensionality of data points makes them sparser which is good for classification but, in the other hand, a higher dimensional space has more points ($R^2 \subset R^3 \subset R^4 \subset R^5$..) so there must be more sample points in a higher dimensional space to having a good representation of the underlying distribution of samples on that space and having low generalization error, alas, the number of samples usually cannot be increased Thus, the dimension of feature space must be taken carefully.

So, according to the above paragraph, although there is no general way to determining the best degree for a polynomial kernel but, polynomial kernels with higher degree usually leads to poor accuracy especially when the number of samples is small. in the case of this project, polynomial kernels with degree higher than 4 results slightly poorer accuracy.

RBF kernels are good choice when the features are continuous, but in this project, 6 out of 9 features are discrete.

Finally, the linear kernel (having no kernel) leads to a slightly better accuracy than other two kernels. This is because most of the features are discrete so, for example polynomial kernel generates intermixed features that are only a factor of other features thus, practically, no informative new feature is produced when the data points are transformed into a higher dimensional feature space hence, the accuracy is not improved sharply as dimension increased.

Effect of data normalization

features have been normalized as below:

$$x_{i(normalized)} = \frac{(x_i - x_{\min}\ )}{x_{\max}\ - x_{\min}} \quad for\ i = 1, \dots., N$$

Where x_{min} and x_{max} are minimum and maximum value of the feature.

(Note: Only continuous features (Salary and AmountSpent) have been normalized.)

	Before data normalization	After data normalization
Polynomial kernel (d = 3)	**0.04 s**	**0.016 s**
RBF kernel	**0.04 s**	**0.024 s**
Linear kernel	**71.0 s**	**0.014 s**

Recall that, a SVM problem equals a quadratic optimization problem as below:

$$Minimize \frac{1}{2}\left|\left|W\right|\right|^2 + C\Sigma_{k=1}^{N}\varepsilon_k \quad s.t\ \ y_k(W^T x_k + b) \geq 1 - \varepsilon_k \qquad \varepsilon_k \geq 0$$

Every $y_k(W^T x_k + b) \geq 1 - \varepsilon_k$ can thought as a hyperplane in W space and the intersection of all these planes construct the solution region (feasible region) which must be searched in order to find the optimal solution (using interior point, gradient methods or ….).

Normalizing the data points makes them more densely results in a narrower solution region to be searched so, the time consumed by the solver algorithms, generally, improved, after data normalization. The accuracy of classification may decrease due to the data point compaction

Hyperparameter adjustment

Usually, the optimal value of hyperparameters like C and γ and d (degree for polynomial kernel) is highly depending on the nature of the dataset thus, there is no analytical way to setting them properly. The only way is to searching for optimal values whether manually (trying different values) or systematically (using optimization programing).

For example, in this project our accuracy depending on the two hyperparameters parameters C and γ for both RBF and polynomial (d = 2 or 3, assuming higher degree kernels results in poor accuracy) kernels.

So, the classification accuracy is as below:

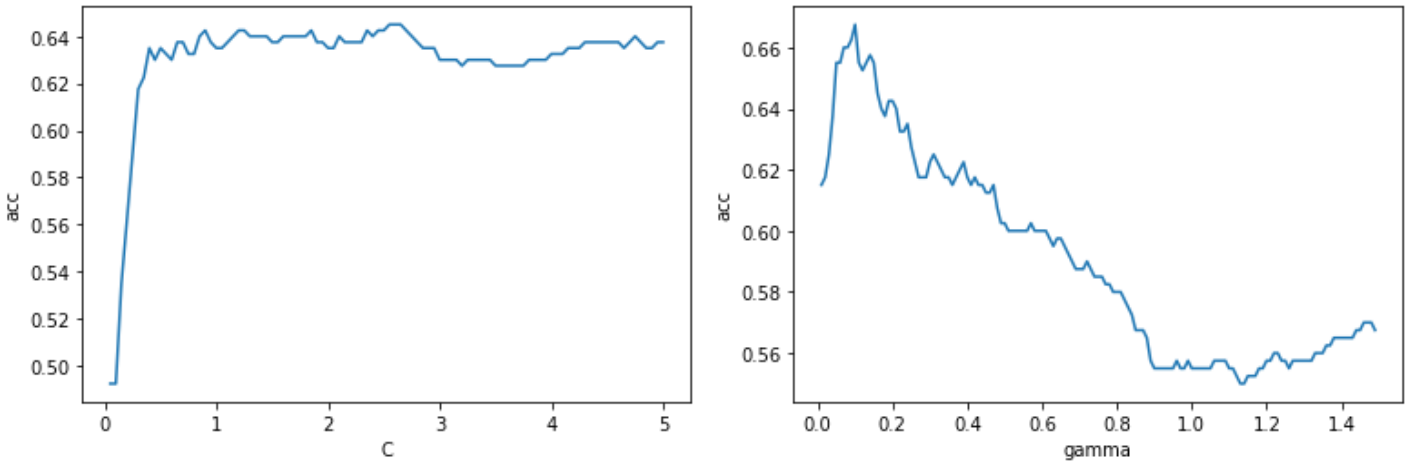
$$acc = f(C, \gamma)$$

We must find the global maxima of acc (if any) which needs to probing on the grid domain of (C, γ).

in this mini-project, I have fixed one parameter and find the optimal value for the other, generally this approach cannot find the global maxima (even local maxima!).

below table lists, accuracy per various parameters setting for a SVM with RBF kernel.

C	γ	acc
1	0.001	62%
0.8	0.01	49%
0.8	0.09	64%
4.85	0.2	69%
0.7	0.00002	48.75%



Left: Accuracy of a SVM with RBF kernel versus parameters C (while γ kept constant).

Right: Accuracy of a SVM with RBF kernel versus parameters γ (while C kept constant).

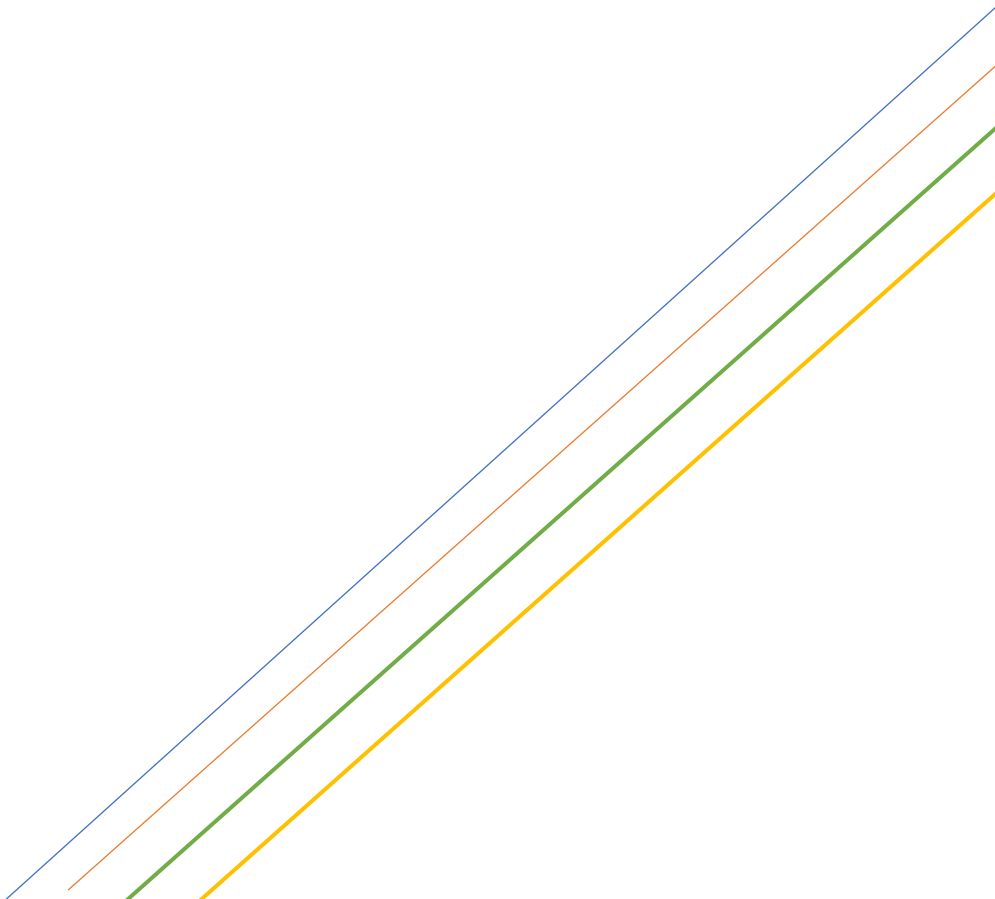
Using linear SVM, the overall accuracy of about 70% - 75% was achieved which is the best accuracy I've reached.

Appendix

Codes for Preparing dataset

Code for Implementing OVO SVM

Codes for Problems



Required libraries

Following libraries are used:

1) NumPy: for matrix manipulation
2) Pandas: for reading csv
3) Matplotlib: plotting
4) Time: for determining consumption time
5) Sklearn: for SVM classification

Loading and preparing data

Function load_dataset() receives one Boolean parameter which indicates whether data should be normalized or not.

```
def load_dataset(normalized = False):
  dataset = pd.read_csv("drive/MyDrive/retailMarketing.csv")
  N_samples = 1000
  #--------------------Normalization---------------
  if(normalized):
    AS_min = dataset.AmountSpent.min()
    AS_max = dataset.AmountSpent.max()
    dataset.AmountSpent = (dataset.AmountSpent - AS_min)/(AS_max - AS_min)
    sal_min = dataset.Salary.min();
    sal_max = dataset.Salary.max()
    dataset.Salary = (dataset.Salary - sal_min) / (sal_max - sal_min)
  #------------------------------------------------
  #aux associate-table for mapping values of age(our target) to numbers
  ages = {"Young": 0, "Middle": 1, "Old": 2}
  prepared_dataset = np.zeros((1000,10))
##Preapare samples---------------------------------------
  c = np.zeros((10,1))  #holds the current sample
  for i in range(N_samples):
    c[0] = dataset.AmountSpent[i]
    #catalogs_6 -> 0, catalogs_12 -> 1, catalogs_18 -> 2, catalogs_24 -> 3
    c[1] = np.argmax([dataset.Catalogs_6[i],dataset.Catalogs_12[i],dataset.Catalogs_18[i],data
set.Catalogs_24[i]])
    c[2] = dataset.Children[i]
    #Female -> 0, Male -> 1
    c[3] = np.argmax([dataset.Gender_Female[i],dataset.Gender_Male[i]])
    #Low -> 0, Medium -> 1, High -> 2
    c[4] = np.argmax([dataset.History_Low[i],dataset.History_Medium[i],dataset.History_High[i]
])
    #Close -> 0, Far -> 1
    c[5] = np.argmax([dataset.Location_Close[i],dataset.Location_Far[i]])
    #Single -> 0, Married -> 1
    c[6] = np.argmax([dataset.Married_Single[i],dataset.Married_Married[i]])
    #Own -> 0, Rent -> 1
    c[7] = np.argmax([dataset.OwnHome_Own[i],dataset.OwnHome_Rent[i]])
    c[8] = dataset.Salary[i]
    #Young -> 0, Middle -> 1, Old -> 2
    c[9] = ages[dataset.Age[i]] #target!
    prepared_dataset[i,:] = c.reshape(1,10) #put the sample into dataset
```

Function load_dataset() continue…

After preparation, dataset is shuffled and divided into 60% - 40% parts for training and testing respectively.

```
  #-------------------shuffling------------------------
  np.random.shuffle(prepared_dataset)
  #----------------------------------------------------

  #-------------------splitting------------------------
  #60% for training, 40% for testing

  x_train = prepared_dataset[:600,:8]
  y_train = prepared_dataset[:600,9]

  x_test = prepared_dataset[600:1000,:8]
  y_test = prepared_dataset[600:1000,9]

  return x_train,y_train,x_test,y_test
  #----------------------------------------------------

  #----------------------------END OF Load Dataset-----------------------------
```

Implementing One vs One for multiclass classification

In this paradigm, there is a SVM per each pair of classes, we have three classes (Young, Middle, Old) so, we need 3 SVM. Each SVM should be trained on its own dataset which contains samples of its two target classes only.

Per each test samples we get three predictions corresponding to our three SVMs. We choose the most occurred class between these three predictions as final result of OVO SMV.

Function ovo_prepare_dataset() receives two parameters X and Y corresponding to inputs and targets.

Each SVM needs an input list and corresponding target list so, we need six list for our three SVMs.

So, ovo_prepare_dataset() defines six list and then initiates a loop through all samples in X and append the samples into defined lists based on their targets.

```
def ovo_prepare_dataset(x,y):
  #create an array of 6 lists as follows:
  #dataset[0],dataset[1] : x_train,y_train for young_middle
  #dataset[2],dataset[3] : x_train,y_train for young_old
  #dataset[4],dataset[5] : x_train_y_Train for middle_old
  #target: 0 -> young, 1 -> middle, 2 -> old
  dataset = [list() for i in range(6)] #array of 6 lists
```

```
N_samples = x.shape[0]
  for i in range(N_samples):
    c = y[i]
    if(c == 0): #young
      dataset[0].append(x[i]) #young_middle
      dataset[1].append(c)
      dataset[2].append(x[i])#young_old
      dataset[3].append(c)
    elif(c == 1): #middle
      dataset[0].append(x[i]) #young_middle
      dataset[1].append(c)
      dataset[4].append(x[i]) #middle_old
      dataset[5].append(c)
    elif(c == 2): #old
      dataset[2].append(x[i])#young_old
      dataset[3].append(c)
      dataset[4].append(x[i])#middle_old
      dataset[5].append(c)

  return dataset
```

Function ovo_SVC() receives an array of six list created using ovo_prepare_dataset() function along with test samples

Then defines three SVM each for a pair of classes and returns the prediction results of these three SVMs

```
#------------------------------------------------------------------------
def ovo_SVC(training_set,test_set):
  #young_middle
  y_pred_0 = svm.SVC(kernel = 'rbf').fit(training_set[0],training_set[1]).predict(test_set)
  #young_old
  y_pred_1 = svm.SVC(kernel = 'rbf').fit(training_set[2],training_set[3]).predict(test_set)
  #middle_old
  y_pred_2 = svm.SVC(kernel = 'rbf').fit(training_set[4],training_set[5]).predict(test_set)
  return y_pred_0,y_pred_1,y_pred_2
#------------------------------------------------------------------------
```

Function eval_ovo_SVC receives the prediction lists returned by ovo_prepare_dataset() function along true targets (y_test)

Then choose the best predict according to majority voting and finally computes the accuracy.

```
#----------------------Evaluating ovo SVC-------------------------
def eval_ovo_SVC(y_pred_0,y_pred_1,y_pred_2,y_test,N_test_samples):
  y_pred = np.zeros((N_test_samples,1))
  for i in range(N_test_samples):
      temp = np.zeros((3,1))
      temp[int(y_pred_0[i])] +=1
      temp[int(y_pred_1[i])] +=1
      temp[int(y_pred_2[i])] +=1
      y_pred[i] = np.argmax(temp)
  acc = accuracy_score(y_pred,y_test)
  return acc
 #------------------------------------------------------------------
```

Script for problem #1

```
#------------------------------PROBLEM #1-------------------------------------
x_train,y_train,x_test,y_test = load_dataset(normalized=False)
#---------------RBF kernel--------------------------
t0 = time.clock()
y_pred = svm.SVC(kernel='rbf').fit(x_train,y_train).predict(x_test)
print("Kernel: RBF,C = 1.0 ->","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")
t1 = time.clock() - t0
print("(( elapsed time for RBF kernel,without data normalization: ",t1,"))")

#----------------------Polynomial kernel-------------
t0 = time.clock()
y_pred = svm.SVC(kernel='poly').fit(x_train,y_train).predict(x_test)
print("Kernel: Poly,C = 1.0,degree = 3 ->","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")
t1 = time.clock() - t0
print("(( elapsed time for RBF kernel,without data normalization: ",t1,"))")
#-----------------------------------------------------

#-----------ovo SVM for multiclass classification-----
#implemented manually
x_train,y_train,x_test,y_test = load_dataset(False)
train_data = ovo_prepare_dataset(x_train,y_train)
preds = ovo_SVC(train_data,x_test)
acc = eval_ovo_SVC(*preds,y_test,400) * 100
print("\none vs one SVM Implemented using 3 distinct SVM\nAccuracy: ",acc,'%')
print('\n')

#-------------------------------------------------------------------------------
```

Script for problem #2

```
#------------------------------PROBLEM #2-------------------------------------
t0 = time.clock()
y_pred = svm.SVC(C = 0.5,kernel='linear').fit(x_train,y_train).predict(x_test)
print("Kernel: Linear,C = 1.0 ->","Acc: ",accuracy_score(y_pred,y_test)*100,"%")
t1 = time.clock() - t0
print("(( elapsed time for Linear kernel,without data normalization: ",t1,"))")
#-------------------------------------------------------------------------------
```

Script for problem #3

```
#------------------------------PROBLEM #3-------------------------------------
x_train,y_train,x_test,y_test = load_dataset(normalized=True)
#----------------RBF kernel---------------------------
t0 = time.clock()
y_pred = svm.SVC(kernel='rbf').fit(x_train,y_train).predict(x_test)
print("Kernel: RBF,C = 1.0 ->","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")
t1 = time.clock() - t0
print("(( elapsed time for RBF kernel,with data normalization: ",t1,"))")

#----------------------Polynomial kernel-------------
t0 = time.clock()
y_pred = svm.SVC(kernel='poly').fit(x_train,y_train).predict(x_test)
print("Kernel: Poly,C = 1.0,degree = 3 ->","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")
t1 = time.clock() - t0
print("(( elapsed time for RBF kernel,with data normalization: ",t1,"))")
#-----------------------------------------------------

#----------------------Linear kernel------------------
t0 = time.clock()
y_pred = svm.SVC(C = 1,kernel='linear').fit(x_train,y_train).predict(x_test)
print("Kernel: Linear,C = 1.0 ->","Acc: ",accuracy_score(y_pred,y_test)*100,"%")
t1 = time.clock() - t0
print("(( elapsed time for Linear kernel,with data normalization: ",t1,"))")
#-----------------------------------------------------
```

Script for problem #4

```
#------------------------------PROBLEM #4-------------------------------------
x_train,y_train,x_test,y_test = load_dataset(normalized=True)
y_pred = svm.SVC(C = 1,kernel='rbf').fit(x_train,y_train).predict(x_test)
print("Kernel: RBF,C = 1.0,gamma = 0.001 ->","Acc: ",accuracy_score(y_pred,y_test)*100,"%")

y_pred = svm.SVC(C = 0.8,gamma = 0.01,kernel='rbf').fit(x_train,y_train).predict(x_test)
print("Kernel: RBF,C = 0.8,gamma = 0.01 ->","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")

#parameters adjusted based on results of problem 5 (!)
y_pred = svm.SVC(C = 0.8,gamma = 0.09,kernel='rbf').fit(x_train,y_train).predict(x_test)
print("Kernel: RBF,C = 0.8,gamma = 0.09 ->","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")

#parameters adjusted based on results of problem 5 (!)
y_pred = svm.SVC(C = 4.849999999999991,gamma = 0.2,kernel='rbf').fit(x_train,y_train).predict(
x_test)
print("Kernel: RBF,C = 4.849999999999991,gamma = 0.05 -
>","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")

y_pred = svm.SVC(C = 0.7,gamma = 0.00002,kernel='rbf').fit(x_train,y_train).predict(x_test)
print("Kernel: RBF,C = 4.1,gamma = 0.1 ->","Acc: ",accuracy_score(y_pred,y_test) * 100,"%")
#-----------------------------------------------------------------------------
```

Script for problem #5

```
#-----------------------------PROBLEM #5 -------------------------------------
x_vals = list()
y_vals = list()
c = 0.05
step = 0.05
while(c <= 5):
  y_pred = svm.SVC(C = c,kernel = 'rbf').fit(x_train,y_train).predict(x_test)
  acc = accuracy_score(y_test,y_pred)
  x_vals.append(c)
  y_vals.append(acc)
  c += step
plt.plot(x_vals,y_vals)
plt.xlabel("C")
plt.ylabel("acc")
plt.show()

c_i = np.argmax(np.array(y_vals))
best_c = x_vals[c_i]
g = 0.01
step = 0.01
x_vals.clear()
y_vals.clear()
while(g <= 1.5):
  y_pred = svm.SVC(C = best_c,kernel = 'rbf',gamma = g).fit(x_train,y_train).predict(x_test)
  acc = accuracy_score(y_test,y_pred)
  x_vals.append(g)
  y_vals.append(acc)
  g += step

plt.plot(x_vals,y_vals)
plt.xlabel("gamma")
plt.ylabel("acc")
plt.show()
#------------------------------------------------------------------------------
```