$$D_x y = 2x + y$$

$$y = \int 2x + 7dx = x^2 + 7x + C$$

$$D_x y + y = 7$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$D_x y = y(1 - y) = y - y^2$$

$$D_x y = \frac{dy}{dx} = y(1-y) \Leftrightarrow \frac{dy}{y(1-y)} = dx \Leftrightarrow \int \frac{dy}{y(1-y)} = \int dx \Leftrightarrow u = 1 - \frac{1}{y} \Rightarrow -\int \frac{1}{u} du = x + C \Leftrightarrow -\ln|u| = -\ln|1 - \frac{1}{y}| = x + C \Leftrightarrow \ln|1 - \frac{1}{y}| = -x + C \Leftrightarrow$$

$$1 - \frac{1}{y} \geq 0$$

$$1-\frac{1}{y}=e^{-x+C} \Leftrightarrow -1=e^{-x+C}y-y=y(e^{-x+C}-1) \Leftrightarrow y=\frac{-1}{e^{-x+C}-1}$$

$\mu(x)$

$$
\begin{aligned}
&D_x y + P(x)y = Q(x) \\
&\Leftrightarrow \mu(x)(D_x y + P(x)y) = D_x(\mu(x)y) \\
&\Leftrightarrow \mu(x)D_x y + \mu(x)P(x)y = \mu(x)D_x y + D_x\mu(x)y \\
&\Leftrightarrow \mu(x)P(x) = D_x\mu(x)y
\end{aligned}
$$

$$
P(x) = \frac{\mu_x(x)}{\mu(x)} = \frac{d\mu}{\mu} \Rightarrow \int P(x)dx = \int \frac{d\mu}{\mu} = \ln\mu \Rightarrow \mu(x) = e^{\int P(x)dx}
$$

I

$$x_0$$

$$y(x_0) = y_0$$

$$y(x)$$

$$y(x) = e^{-\int P(x)dx} \left(\int e^{\int P(x)dx} Q(x)dx + C \right)$$

$$\frac{dy}{dx} = f(x, y)$$

$$v = \alpha(x, y)$$

v

$D_x v$

$$D_x v + P(x) v = Q(x)$$

$$v = q(D_x y) \Rightarrow D_x v = w(D_x^2 y)$$

$$xD_x^2y + D_xy = Q(x)$$

$$v = D_x y \Rightarrow D_x v = D_x^2 y \Rightarrow x D_x v + v = Q(x)$$

$$yD_xy + (D_x^2y)^2 = 0$$

$$v = yD_xy \Rightarrow D_xv = (D_xy)^2 + yD_x^2y \Rightarrow D_xv = 0 \Rightarrow yD_xy = y\frac{dy}{dx} = v = c \Rightarrow \int y dy = \int c dx + C$$

$$D_x y = F\left(\frac{y}{x}\right)$$

$\Longrightarrow$

$$v = \frac{y}{x} \Rightarrow y = vx \Rightarrow D_x y = D_x(v)x + v$$

$$v`x + v = F(v) \Rightarrow \frac{D_x v}{F(v)-v} = \frac{1}{x}$$

$$f(x, y)dy = g(x, y)dx$$

$$y = ux \Rightarrow \frac{dx}{x} = h(u)du \Rightarrow$$

$$D_x y + P(x)y = Q(x)y^n$$

$$v = y^{1-n} \Rightarrow D_x v = (1-n)y^{-n} D_x y \Rightarrow D_x v + (1-n)P(x)v = (1-n)Q(x)$$

$$D_x y + P(x)y = Q(x)\frac{1}{y}$$

$$I(x,y) + J(x,y)D_x y = 0 \Leftrightarrow I(x,y)dx + J(x,y)dy = 0$$

$$I_y = J_x$$

$\exists \psi(x, y)$

$$\psi_x(x, y) = I(x, y)$$
$$\psi_y(x, y) = J(x, y)$$

$$\psi(x, y) = c$$

$$f(a) = b$$

$$x = a, y = b$$

ψ

c

x

t

$$D_x y = P(y)$$

$$D_x y = f(x)g(y)$$

$$D_x y = \frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)dx + C$$

$$aD_x^2y + bD_xy + cy = 0$$

a, b

$$a + b$$

$$y = e^{rx}$$

$$
\begin{aligned}
& a(r^2e^{rx}) + b(re^{rx}) + ce^{rx} = 0 \\
& \Leftrightarrow e^{rx}(ar^2 + br + c) = 0 \\
& \Leftrightarrow ar^2 + br + c = 0
\end{aligned}
$$

$$r = r_1, r_2$$

$$y = Ae^{rx} + Be^{rx}$$

$\forall A, B$

$$r_1 = r_2$$

$$y = Ae^{rx} + Bxe^{rx}$$

$$D_x^2 y + p(x) D_x y + q(x) y = g(x)$$

$$L[y] = D_x^2 y + p(x) D_x y + q(x) y$$

$$L[y] = g(x)$$

$$y = (c_1 y_1 + c_2 y_2 = y_h) + y_p$$

y_h

$$L[y] = 0$$

y_p

$$g(x)$$

$P_n(x)$

$$t^s Q_n(x)$$

$$P_n(x)e^{\alpha x}$$

$$t^s Q_n(x) e^{\alpha x}$$

$$P_n(x)e^{\alpha x}(\sin \beta t + \cos \beta t)$$

$$t^s e^{\alpha x}(Q_n(x)\sin\beta x + R_n(x)\cos\beta x)$$

S

≥ 0

$P_n(x), Q_n(x), R_n(x)$

$$L[y_p] = g(t)$$

$$y_h = c_1 y_1 + c_2 y_2$$

u_1, u_2

$$D_x(u_1)y_1 + D_x(u_2)y_2 = 0$$

$$D_x(u_1)D_x(y_1) + D_x(u_2)D_x(y_2) = g(x)$$

$$w(f, g)$$

$$w(f, g) := \begin{vmatrix} f & g \\ D_x f & D_x g \end{vmatrix}$$

$$w(f, g) \equiv 0$$

$$w(f,g)(x)=0, \forall x$$

f, g

$$w(f,g) \not\equiv 0$$

$\exists x$

$$w(f,g)(x) \neq 0$$

$$D_x u_1 = \frac{-y_2 g}{w(y_1,y_2)}$$

$$D_x u_2 = \frac{y_1 g}{w(y_1, y_2)}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$
\begin{aligned}
y_p &= -y_1(x)\int_{x_0}^{x} \frac{y_2(s)g(s)}{w(y_1,y_2)(s)}ds + y_2(x)\int_{x_0}^{x} \frac{y_1(s)g(s)}{w(y_1,y_2)(s)}ds \\
&= \int_{x_0}^{x} \frac{y_2(x)y_1(s) - y_1(x)y_2(s)}{w(y_1,y_2)(s)} g(s)ds = 0
\end{aligned}
$$

y_1, y_2

N

n

$$a_0 D_x^n y + a_1 D_x^{n-1} y + \cdots + a_{n-1} D_x y + a_n y = 0$$

$$y = e^{rx} \Rightarrow a_0 r^n + a_1 r^{n-1} + \cdots + a_{n-1} r + a_n = 0$$

$$D_t x_i = \sum_{j=1}^{n} (P_{ij}(t) x_j) + g_i(t)$$

$1 \leq i \leq n$

$\Leftrightarrow$

$$D_t\vec{x} = \left[P_{ij}(t)\right]_{ij}\vec{x} = \begin{bmatrix} g_1(t) \\ \vdots \\ g_n(t) \end{bmatrix}$$

$$g_i(t) = 0, \forall t$$

$$\{\vec{e_i}\}$$

$\Re^n$

$\vec{x_i}$

$$\vec{x_i}(t_0) = \vec{e_i}$$

$$\{\vec{x_i}\}$$

$$D_x\vec{x} = A\vec{x}$$

$\lambda_1, \cdots, \lambda_n$

$$\vec{v_1}, \cdots, \vec{v_n}$$

$$\vec{x} = \sum_{i=1}^{n} c_i e^{\lambda_i} \vec{v_i}$$

$$\vec{x} = \vec{x_1} + \vec{x_2}$$

$$\vec{x_1} = c_1 \vec{v_1} e^{\lambda_1 t}$$

$$\vec{x_2} = \vec{w}te^t \Rightarrow D_t\vec{x_2} = \vec{w}(e^t + te^t)$$

$\vec{w}$

$$aD_x^2 x + bx = 0 \Rightarrow$$

$$x_1 = x, x_2 = D_x x \Rightarrow D_x x_2 = \frac{-b x_1}{a}, D_x x_1 = x_2$$

$$y_1 = x_1,$$

$$y_2 = D_x x_1,$$

$$y_3 = x_2,$$

$$y_4 = D_x x_2$$

$$D_t X = AX$$

$$\forall X \in M_{n\times n}(S)$$

$$\vec{x} = c_1\vec{x_1} + c_2\vec{x_2} + \cdots + c_n\vec{x_n}$$

$$\vec{x_i} = \vec{v_i} e^{\lambda_i t}$$

$$D_t\vec{x} = A\vec{x}$$

$$X = \Phi(t) = \begin{bmatrix} \vec{x_1} & \vec{x_2} & \cdots & \vec{x_n} \end{bmatrix}$$

$$\vec{x}(0) = \vec{x_0}$$

$$\Phi(t)\vec{c} = \vec{x_0} \Leftrightarrow \vec{c} = \Phi^{-1}(t)\vec{x}$$

$$D_t X = AX,$$

$$\vec{x}(t) = \Phi(t)\Phi^{-1}(0)\vec{x_0}$$

$$X = e^{At}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!} \Rightarrow e^{At} = \sum_{n=0}^{\infty} \frac{A^n t^n}{n!}$$

$$AB = BA$$

$$e^{A+B} = e^A e^B; (e^A)^{-1} = e^{-A}; e^{0 \in M_{n \times n}(S)} = I$$

$$A = diag(a_1, a_2, \cdots, a_n)$$

$$e^A = diag(e^{a_1}, e^{a_2}, \cdots, e^{a_n})$$

$$A = SDS^{-1}$$

D

$$e^A = Se^D S^{-1}$$

A

$$A^n = 0$$

$$e^A = \sum_{i=0}^{n} \frac{A^i}{i!}$$

$$\left(= \sum_{n=0}^{\infty} \frac{A^n}{n!} \right)$$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^2 \neq 0$$

$$A^3 = 0 \Rightarrow e^A = \sum_{i=0}^{2} \frac{A^i}{i!} = (A^0 = I) + A + \frac{1}{2}A^2$$

$$C = A + B$$

$$e^{Ct} = e^{At} e^{Bt}$$

$$A = nI$$

$$AB = nIB = nB = Bn = BnI = BA$$

$$D_t\vec{x} = a\vec{x},$$

$$\vec{x}(t) = e^{At}\vec{x_0}$$

$$(= \Phi(t)\Phi^{-1}(0)\vec{x_0} \Rightarrow e^{At} = \Phi(t)\Phi^{-1}(0)\vec{x_0})$$