Final (2012)
Question (1):- (a)

Any point (X,Y) has one of three cases depending on the function F(X,Y) provided that X is positive:

$$f(X,Y) = \begin{cases} 0 & (X,Y) \text{ on the line} \\ <0 & (X,Y) \text{ under the line} \\ >0 & (X,Y) \text{ above the line} \end{cases}$$

Initial Value of d: at (X=0, Y=R)

$d_{initial} = d(X,Y) = f(X+1, Y-\frac{1}{2}) = (X+1)^2 + (Y-\frac{1}{2})^2 - R^2$

$= (0+1)^2 + (R-\frac{1}{2})^2 - R^2$

$= 1 + R^2 - R + \frac{1}{4} - R^2 = \frac{5}{4} - R \simeq 1 - R$

change in d:

(1) if $d<0$ then: X++

$\Delta d = d(X+2, Y-\frac{1}{2}) - d(X,Y)$

$= [(X+2)^2 + (Y-\frac{1}{2})^2 - R^2] - [(X+1)^2 + (Y-\frac{1}{2})^2 - R^2]$

$= (X^2+4X+4) - (X^2-2X+1) = 2X+3$

2

if $d > 0$ then: $x{+}{+}$ $y{-}{-}$

$$\Delta d = d(x+2, y-\tfrac{3}{2}) - d(x,y)$$

$$= [(x+2)^2 + (y-\tfrac{3}{2})^2 - R^2] - [(x+1)^2 + (y-\tfrac{1}{2})^2 - R^2]$$

$$= [(x^2+4x+4) + (y^2-3y+\tfrac{9}{4})] + [(x^2+2x+1) + (y^2-y+1)]$$

$$= (2x+3) + (-2y+\tfrac{5}{4}) \simeq 2(x-y)+5$$

Algorithm:

Draw circle (x_c, y_c, R)

1. $x=0$
2. $y=R$
3. $d=1-R$
4. Draw points (x_c, y_c, x, y)
5. if ($d<0$)

while ($x<y$)

    d += 2x+3;
    x++;

else

    d += 2(x-y)+5;
    x++; y++;

6. Draw Point (x_c, y_c, x, y)

(b)

$\text{Slope} = \frac{15-9}{12-2} = \frac{6}{10} = 0.6 \quad \therefore \text{Slope} < 1$

Initial Value $= \Delta x - 2\Delta y = 10 - (2*6) = -2$

Change in $d<0$: $d_1 = 2\Delta x - 2\Delta y = 20 - 12 = 8$

Change in $d>0$: $d_2 = 2\Delta y = 2*6 = 12$

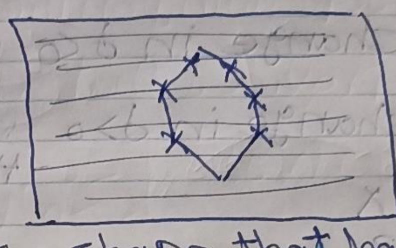
traces

x	y	Decision Variable
2	9	$d<0$ $d = d + d_1 = -2+8 = 6$
3	10	$d>0$ $d = d + d_2 = 6 + -12 = -6$
4	10	$d<0$ $d = d + d_1 = -6+8 = 2$
5	11	$d>0$ $d = d + d_2 = 2-12 = -10$
6	12	$d<0$ $d = d + d_1 = -10+8 = -2$
7	12	$d<0$ $d = d + d_1 = -2+8 = 6$
8	13	$d>0$ $d = d + d_2 = 6-12 = -6$
9	13	

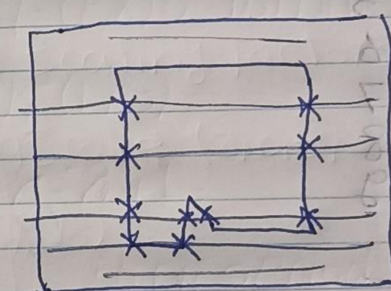
(d)

Question (2) :- (a)

convex shapes: is a shape that has at most 2 interesection with scan line



Non-convex shape: is a shape that has more than 2 intersection with scan line



Algorithm:

1. initialize the edge list array 'table' with edge information

2. start with y = the first index in table with non-empty edge list

3. let Active list = table [y]

4. while Active list is not empty
4.1. Sort Active list node in an ascending order with x

4.2. Draw horizontal lines between points represented by successive pairs of node in Active list

4.3. Increment y by 1

4.4. Delete from Active list those node with $y_{max} = y$

4.5. update x where y+ = min.V

4.6. Append to Active list the new node at table [y] if not

end while

Question (3) :- (b)

for(int theta=0 , theta ≤ 62.8 ; theta += $\frac{1}{20 * \frac{22}{7}}$)

int x= theta * cos(cos (theta));

int y= theta * sin (sin (theta));

setpixel (Round (x), Round (y), c)

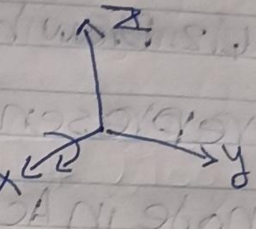
end for

Question (4):-

- Rotate an object about X axis with a rotation angle of 30° clockwise

∴ Rotation → from z to y

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(30) & \sin(30) & 0 \\ 0 & -\sin(30) & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- Scale 3D object with respect to a fixed point (3,2,8) where the scaling factors in x, y and z directions are (2, 0,5, 3

$$T = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Scale object on origin:-

$$S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0,5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. undo step 1:

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

composite scaling matrix

$$T^{-1} \cdot ST = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- transform figure 1 into figure 2:-

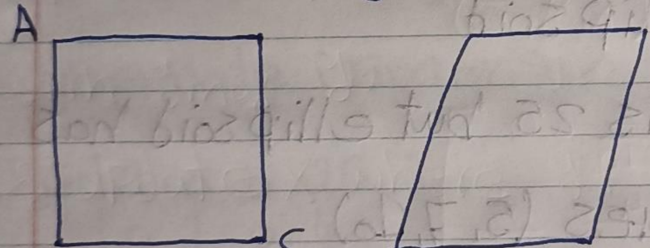


figure 1 figure 2

$$T = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

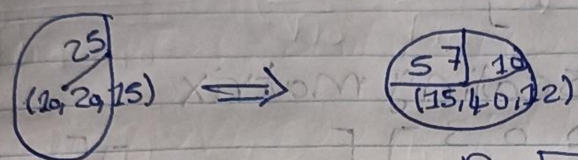
$x' = x + \alpha y$
$y' = y + Bx$

$x\cancel{y}' = x + \alpha y$

$y' = y$

Map the sphere $(x-10)^2+(y-20)^2+(z-15)^2 = 625$ the ellipsoid:

$$\frac{(x-15)^2}{25}+\frac{(y-40)^2}{44}+\frac{(z-12)^2}{100}=1$$



$R = \sqrt{625} = 25$ $\qquad$ $R = \sqrt{25}\ \sqrt{44}\ \sqrt{100}$

- if we want to map shape into another there is 2 difference:-

① center of ellipsoid

② R in sphere is 25 but ellipsoid has 3 different values (5, 7, 10)

① translate sphere to origin

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② we need to convert R which (25) to (A, B, C) which are (5, 7, 10) so we need to Apply two scaling one to convert (25) to 1 and other to Resrate 1 to (5, 7, 10):

$$S_1 = \begin{bmatrix} \frac{1}{25} & 0 & 0 & 0 \\ 0 & \frac{1}{25} & 0 & 0 \\ 0 & 0 & \frac{1}{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ translate from origin to ellipsoid center

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

find Matrix $= T_2 \, S_2 \, S_1 \, T_1$ Xinput

$$= \begin{bmatrix} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{25} & 0 & 0 & 0 \\ 0 & \frac{1}{25} & 0 & 0 \\ 0 & 0 & \frac{1}{35} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- find the orthogonal projection of an object the plane passing through the points $(1,-3,0)$, $(2,4,1)$, $(0,1,0)$

$$\hat{u} = \frac{(1,-3,0)}{\sqrt{1^2+3^2+0^2}} = \left(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}, 0\right)$$

$$\hat{v} = \frac{(2,4,1)}{\sqrt{2^2+4^2+1^2}} = \left(\frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}, \frac{1}{\sqrt{21}}\right)$$

$$\hat{w} = \frac{(0,1,0)}{\sqrt{0^2+1^2+0^2}} = (0,1,0)$$

1 convert to $o(x,y,z)$

$$C = \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} & 0 \\ \frac{2}{\sqrt{21}} & \frac{4}{\sqrt{21}} & \frac{1}{\sqrt{21}} \\ 0 & 1 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C^{-1} to convert from $o(x,y,z)$ to $o(u,v,w)$

$$C^{-1} = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{2}{\sqrt{21}} & 0 \\ \frac{-3}{\sqrt{10}} & \frac{4}{\sqrt{21}} & \frac{1}{\sqrt{21}} \\ 0 & 1 & 0 \end{bmatrix}$$

Composite $= C^{-1} \cdot P \cdot C$

Find orthogonal projection of point (6, 2, 3) on plane passing through points (1, 3, 5), (2, 4, 1) and (0, 1, 0)

1. construct 3 vectors

$u \rightarrow (1, 3, 5)$

$v \rightarrow (2, 4, 1)$

$w \rightarrow (0, 1, 0)$

2. Normalize vectors:

$$\hat{u} = \frac{(1, 3, 5)}{\sqrt{1^2+3^2+5^2}} = \left(\frac{1}{\sqrt{35}}, \frac{-3}{\sqrt{35}}, \frac{5}{\sqrt{35}}\right)$$

$$\hat{v} = \frac{(2, 4, 1)}{\sqrt{2^2+4^2+1^2}} = \left(\frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}, \frac{1}{\sqrt{21}}\right)$$

$$\hat{w} = \frac{(0, 1, 0)}{\sqrt{0^2+1^2+0^2}} = (0, 1, 0)$$

3. convert to $o(x, y, z)$

$$C = \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{35}} & \frac{3}{\sqrt{35}} & \frac{5}{\sqrt{35}} \\ \frac{2}{\sqrt{21}} & \frac{4}{\sqrt{21}} & \frac{1}{\sqrt{21}} \\ 0 & 1 & 0 \end{bmatrix}$$

4- return to x, y, z origin

$$P = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5. convert to o(u, v, w)

$$C^{-1} = [\hat{u} | \hat{v} | \hat{w}] = \begin{bmatrix} \frac{1}{\sqrt{35}} & \frac{2}{\sqrt{21}} & 0 \\ \frac{-3}{\sqrt{35}} & \frac{4}{\sqrt{21}} & 1 \\ \frac{5}{\sqrt{35}} & 1 & 0 \end{bmatrix}$$

composite $= C^{-1} . P . C$

Final (2017):-
Question (1):- (b)

$\Delta y = 14 - 10 = 4 \qquad \Delta x = 6 - 12 = -6$

$|5| \leq |-6| \quad \therefore$ slope $< 1 \quad \therefore$ negative

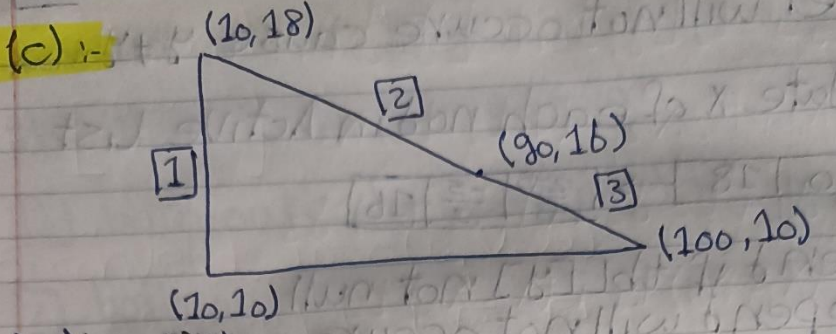
$d = -\Delta x - 2\Delta y = -(-6) - 2(4) = -2$

$ch_1 = -\Delta x - \Delta y = 6 - 4 = 2$

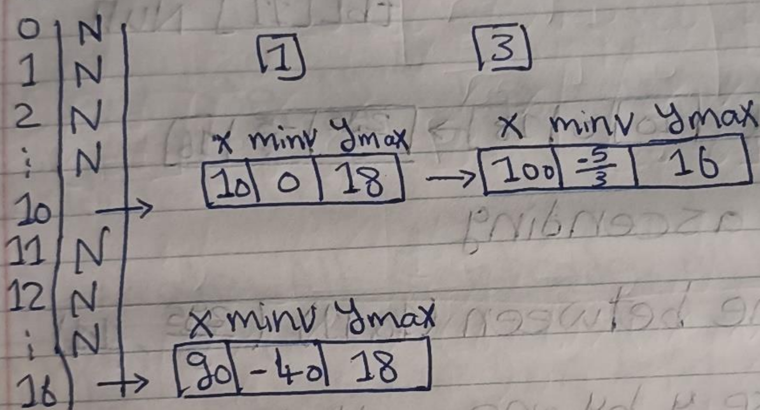
$ch_2 = -2\Delta y = -2(4) = -8$

x	y	decision Variable		
6	14	-2	$\therefore d \leq 0$	$\therefore d += ch_1$
7	13	0	$d \leq 0$	$d += ch_1$
8	12	2	$d > 0$	$d += ch_2$
9	12	-6	$d \leq 0$	$d += ch_1$
10	11	-4	$d \leq 0$	$d += ch_1$
11	10	-2	$d \leq 0$	$d += ch_1$
12	9	0		

(c) :-



Active list



start with y=0

1. while (tbl[y] == Null) y++;
2. Active = tbl[y] ∴ Active = tbl [10]
3. while (Active != Null)

∴ y = 10

① sort x ascending ∴ [10 | 0 | 18] → [100 | -5/3 | 16]

② Draw Line between (10, 10) (100, 10)

③ Increase y by one ∴ y = 11

④ delet will not occure cause $y \neq y_{max}$

⑤ update x of each nod in Active List

| 10 | 0 | 18 | → | $\frac{295}{3}$ | $\frac{-5}{3}$ | 16 |

⑥ Append if tbL[y] not null

∴ Append will not occure cause tbL[11] Null

∴ $y = 11$

Active → | 10 | 0 | 18 | → | $\frac{295}{3}$ | $\frac{-5}{3}$ | 16 |

① sort X ascending

② Draw Line between (10, 11) ($\frac{295}{3}$, 11)

③ Increase y by one ∴ $y = 12$

④ delet will not occure cause $y \neq y_{max}$

⑤ update x of each node in Active List

| 10 | 0 | 18 | → | $\frac{290}{3}$ | $\frac{-5}{3}$ | 16 |

⑥ Append if tbL[y] not Null

∴ Append will not occure cause

tbL[12] Null

Question (2) :- (a)

the line will accepted if

$\boxed{1}$ both outcode not empty

$\boxed{2}$ both out code not have common side

condition of $\boxed{1}$

if (out code$_1$. All != 0 && out code$_2$. All != 0)

condition of $\boxed{2}$

if(outcode$_1$.All & outcode$_2$ All)) != 0

(b) $\boxed{1}$ $t_2 + t_1 \leq 1 \Rightarrow t_2 \leq 1 - t_1$

for(t_1 = 0,0 , $t_1 \leq 1$, t_1 += $\frac{1}{max(P_1, P_2, P_3)}$)

for(t_2=0,0, $t_2 \leq 1 - t_1$, t_2+= $\frac{1}{max(P_1, P_2, P_3)}$

Point P = $t_1 * P_1 + t_2 * P_2 + (1 - t_1 - t_2) * P_3$

~~Draw Point~~

Drawpixel (P.x , P.y, color)

endfor

endfor

$\boxed{2}$ $\begin{bmatrix} x \\ y \end{bmatrix} = t_1 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + t_2 \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + (1-t_1-t_2) \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$

$\acute{x} = t_1 x_1 + t_2 x_2 + (1-t_1-t_2) \cdot x_3$

$\acute{y} = t_1 y_1 + t_2 y_2 + (1-t_1-t_2) y_3$

$\acute{x} - x_3 = (x_1 - x_3) t_1 + (x_2 - x_3) t_2$

$\acute{y} - y_3 = (y_1 - y_3) t_1 + (y_2 - y_3) t_2$

matrix form $= \begin{bmatrix} \acute{x} - x_3 \\ \acute{y} - y_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$

$\therefore \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \dfrac{1}{(x_1 - x_3) * (y_2 - y_3) - (x_2 - x_3) * (y_1 - y_3)}$

$* \begin{bmatrix} y_2 - y_3 & -(x_2 - x_3) \\ -(y_1 - y_3) & x_1 - x_3 \end{bmatrix}$

if : $t_1, t_2 \leq 1 \Rightarrow$ inside

$t_1, t_2 > 1 \Rightarrow$ outside

Algorithm:

for($t_1 = 0.0$, $t_1 \leq 1$, $t_1 += \frac{1}{max(P_1, P_2, P_3)}$)

for($t_2 = 0.0$, $t_2 \leq 1 - t_1$, $t_2 += \frac{1}{max(P_1, P_2, P_3)}$)

if ($t_1 + t_2 > 1$

continue

else

Point $P = t_1 . P_1 + t_2 P_2 + (1 - t_1 - t_2) P_3$

if ($P.X \leq P_1.X$ && $P.X \leq P_2.X$ && $P.X \leq P_3.X$

&& $P.y \leq P_1.y$ && $P.y \leq P_2.y$ && $P.y \leq P_3.y$)

the pixel is inside

(c) Draw Elipse (X_c, Y_c, A, B, color):
X=0 y=B

Draw 4 Point (X_c, Y_c, X, Y, Color)

while ($X B^2 < Y A^2$):

X++

$y = (Round(sqrt(B^2 + A^2 - \frac{X^2}{A^2}))$

Draw 4 point (X_c, Y_c, X, Y, Color)

endwhile

y=0 X=A

Draw 4 Point (X_c, Y_c, X, Y, color)

while ($X B^2 > Y A^2$):

y++

$X = (Round(sqrt(B^2 + A^2 - \frac{y^2}{B^2}))$

Draw 4 point (X_c, Y_c, X, Y, color)

endwhile

end

Draw 4 Point (X_c, Y_c, X, Y, Color):

set pixel ($X+X_c$, $Y+Y_c$, color)

set pixel ($X-X_c$, $Y-Y_c$, color)

drawline ($X+X_c$, $Y+Y_c$, $X-X_c$, $Y-Y_c$, color)

setpixel ($X+X_c$, $Y-Y_c$, color)

set pixel ($X-X_c$, $Y+Y_c$, color)

drawline ($X+X_c$, $Y-Y_c$, $X-X_c$, $Y+Y_c$, color)

Question (3) :- (a)
the homogeueous corrdinate system:
Is related to the standard 2D system through the relation H and S with H mapping from standard to homogeuous and S from homogeuoues to standard:

$$H: (X, Y) \rightarrow (X, Y, 1)$$

$$S: (X, Y, W) \rightarrow \left(\frac{X}{W}, \frac{Y}{W}\right)$$

Importance of homogeueous coordinate system:

It used for representation of affine

transformation and helpfull with operation of X and + of matrices so it can simplify the operation

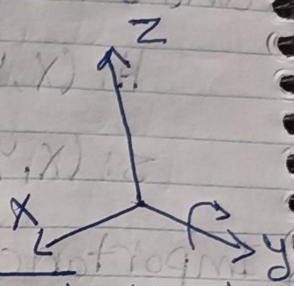
Affine transforma in homogeueous coordinate system:

$$T\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(b) ∴ rotate from x to z

$$R_y = \begin{bmatrix} \cos(80) & 0 & -\sin(60) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(60) & 0 & \cos(60) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∴ Rotatef(-θ, 0, 1, 0)



(c) 1 translate the object such that (10,12) become the origin using the translation matrix:

$$T = \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{bmatrix}$$

translatef(-10, -12, 1)

2 Rotate the object about the origin using the rotation matrix:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Rotatef(30, 0, 0, 1)

3. undo step 1: translate the origin to (10, 12)

$$T^{-1} = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix}$$ translate (10, 12, 1)

Composite matrix:

$$T^{-1}.R.T = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{bmatrix}$$

(d) shearing in X-direction:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$X = 5$
$y = 12$
$\alpha = 4$
$B = 4$

shearing in Y-direction

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ B & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

final (2018)

Question (1):- (b)

*Recursive Algorithm:

flood fill (X, y, Bc, Fc)

1. ColoRREF C = Getpixel (X, y);
2. if (C == Bc || C == Fc) return;
3. setpixel (X, y, Fc);
4. Floodfill (X+1, y, Bc, Fc);
5. Floodfill (X, y+1, Bc, Fc);
6. floodfill (X-1, y, Bc, Fc);
7. Floodfill (X, y-1, Bc, Fc);

*Non-Recursive Algorithm:

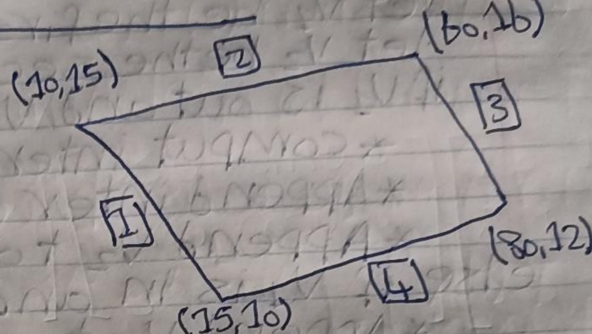
#include <queue>

floodfill (X, y, Bc, Fc);

1. struct point:
   1.1 X, y;
   1.2 point (X=0, y=0) : X(x) y(y) {};
2. queue <point> qu;
3. qu.push (point (x, y));
4. while (!qu.empty()):
   4.1. point P = qu.top ();
   4.2 qu.pop ();
   4.3 ColoRREF C = Getpixel (P.X, P.y);
   4.4 if (C == Bc || C == Fc) continue;
   4.5 setpixel (P.X, P.y, Fc);
   4.6 qu.push (point (X+1, y));
   4.7 qu.push (point (X, y+1));

4.8 qu.push (point (x-1, y));
4.9 qu.push (point (x, y-1));
5. end while
end

(c)



Active list

0	N
1	N
2	N
3	N
⋮	
10	→ [1] x: 15, minv: -1, ymax: 15 → [4] x: 80, minv: $\frac{65}{2}$, ymax: 12
11	N
12	→ 80 | -5 | 16 [3]
13	N
14	N
15	→ 60 | 50 | 16 [2]

Question (2) :- (a)

Idea:
for each edge in the polygon:
let V_1 be the first vertex
let V_2 be the second vertex
if V_1 is out and V_2 is in then:
* comput intersection as inter
* Append inter to outlist$_1$
* Append V_2 to out list$_1$
else if V_1 is in and V_2 ~~out~~ is in then:
* Append V_2 to out list$_1$
else if V_1 is in and V_2 is out then:
* compute intersection as inter
* Append inter to out list$_1$

(b) Draw circle (X_c, Y_c, R):
1. X=0 ;
2. y=R;
3. d=1-R;
4. d_1= 3;
5. d_2=5-2*R;
6. Draw Point $(X_c, Y_c, X, Y, Color)$;
7. while (X<y):
   7.1 if (d<0):
   7.1.1 d += d_1;
   7.1.2 d_2 += 2;
   7.1.3 d_1 += 2;

7.2 else :
  7.2.1 $d+=d_2$;
  7.2.2 $d_2+=4$;
  7.2.3 $d_1+=2$;
  7.2.4 y--;
7.3 Drawpoint (xc, yc, x, y, color);
end

Drawpoint (xc, yc, x, y) :
1. setpixel (xc+x, yc+y, color);
2. Drawline (xc, yc, xc+x, yc+y, color);
3. set pixel (xc+y, yc+x), color);
4. Drawline (xc, yc, xc+y, yc+x, color);
5. setpixel (xc-x, yc+y, color);
6. Drawpoint (xc, yc, xc-x, yc+y, color);
7. setpixel (xc-x, yc-y, color);
8. Drawline (xc, yc, xc-x, yc-y, color);
9. setpixel (xc+x, yc-y, color);
10. Drawline (xc, yc, xc+x, yc-y, color);
11. setpixel (xc-y, yc+x, color);
12. Drawline (xc, yc, xc-y, yc+x), color);
13. set pixel (xc+y, yc-x, color);
14. Drawline (xc, yc, xc+y, yc-x, color);
15. set pixel (xc-y, yc-x, color);
16. Drawline (xc, yc, xc-y, yc-x, color);
end

(e)

the same question in 2017

Q2(b) Page 17, 18

Question (3):- (a)

General form of Affine transformation in homogeueous space:

$$T\left(\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(b) Translate (10,12) to origin

$$T = \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{bmatrix}$$ gl translate f (-10, -12, 1);

* Rotate about origin with angle 30° anti clock

$$R = \begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

gl Rotate f (30, 0, 0, 1);

undo step 1:

$$T^{-1} = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix}$$ glTranslate (10, 12, 1);

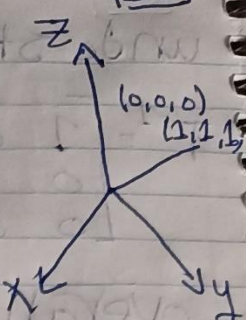
∴ overall matrix =

$$T^{-1}.R.T = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 10 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 16 - 5\sqrt{3} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 7 - 6\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

(c)



$\boxed{1}$ translate line to origin so the line will be parell with z axes:

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

gp translate (-1,-1,0)

$\boxed{2}$ clockwise → from x to y and $-\theta$

Rotate the line about z clock wise

$$R = \begin{bmatrix} \cos(60) & -\sin(60) & 0 & 0 \\ \sin(60) & \cos(60) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

undo step 1:

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ compsite matrix $= T^{-1} . R . T$

(d) shearing in x-direction:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

shearing in y-direction:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where $x=5$ $y=12$, $\alpha=\beta=4$

(e) 1. Construct the vectors:

$u \rightarrow (0.6, 0.8)$ $\quad v \rightarrow (-0.8, 0.6)$

2. Normalize the vectors:

$$\hat{u} = \frac{(0.6, 0.8)}{\sqrt{(0.6)^2+(0.8)^2}} = (0.6, 0.8)$$

$$\hat{v} = \frac{(-0.8, 0.6)}{\sqrt{(-0.8)^2+(0.6)^2}} = (-0.8, 0.6)$$

3. Translate to origin:

$$T_1 = \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -25 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Rotat $(\hat{u}, \hat{v}) \rightarrow (x, y)$

$$R = \begin{bmatrix} 0.6 & 0.8 & 0 \\ -0.8 & 0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. undo step 3

$$T^{-1} = \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 25 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Matrix $= T^{-1} . R . T$

Final (2019)

Question (1):- (a)

Bresenham Algorithm (X_1, y_1, X_2, y_2)

1. $X = X_1$
2. $y = y_{12}$
3. $dx = X_2 - X_1$
4. $dy = y_2 - y_1$
5. setpixel(x, y)
6. $d = dx - 2dy$
7. $d_1 = -2 * dy$
8. $d_2 = 2dx - 2dy$
9. while ($X < X_2$):
   - 9.1 if ($d > 0$):
     - X++ y++;
   - 9.2 else:
     - X++;
     - $d += d_2$
   - 9.3. setpixel (X, y);

   end while

end

DDA Algorithm	Bresenham Algo
* less efficient	* more efficient
* calculation speed less	* calculation speed faster
* costlier	* cheaper
* less accuracy	* more accuracy
* more complex	* simple complex
* use Multiplication and division	* use only substraction and addition

(b) $(6,10) \rightarrow (12,18)$ slope $= 1.33 > 1$

$x_1 = 6$ $x_2 = 12$ $y_1 = 10$ $y_2 = 18$

$\Delta x = 6$ $\Delta y = 8$

$d_{initial} = 2\Delta x - \Delta y = 12 - 8 = 4$

$d_1 = 2\Delta x - 2\Delta y = 12 - 16 = -4$

$d_2 = 2\Delta x = 12$

traces

~~X~~ X	y	deision		
6	10	$d \geq 0$	$d += d_1$	$d = 0$
7	11	$d \geq 0$	$d += d_1$	$d = 4$
8	12	$d < 0$	$d += d_2$	$d = 8$
~~9~~ 8	13	$d \geq 0$	$d += d_1$	$d = 4$
~~10~~ 9	14	$d \geq 0$	$d += d_1$	$d = 0$
~~11~~ 10	15	$d \geq 0$	$d += d_1$	$d = 4$
~~12~~ 11	16	$d < 0$	$d += d_2$	$d = 8$
11	17	$d \geq 0$	$d += d_1$	$d = 4$
12	18			

Question (2) :- (b)

use parametric equation of line:

$$x(t) = x_1 + (x_2 - x_1)\,t$$
$$y(t) = y_1 + (y_2 - y_1)\,t$$

blug parametric equation into ellipse:

$$\frac{(x_1 + (x_2 - x_1)t)^2}{a^2} + \frac{(y_1 + (y_2 - y_1)t)^2}{b^2} = 1$$

$$\left[\frac{1}{a^2}\left(x_1^2 + 2x_1(x_2 - x_1)t + (x_2 - x_1)^2 t^2\right)\right] + \left[\frac{1}{b^2}\left(y_1^2 + 2y_1(y_2 - y_1)t + (y_2 - y_1)^2 t^2\right)\right] = 1$$

using form of quadratic equation:

$$t^2\left[\frac{(x_2 - x_1)^2}{a^2} + \frac{(y_2 - y_1)^2}{b^2}\right] + t\left[\frac{2x(x_2 - x_1)}{a^2} + \frac{2y(y_2 - y_1)}{b^2}\right] + \left[\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1\right] = 0$$

$$\Rightarrow At^2 + Bt - C = 0 \qquad t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$\therefore d = B^2 - 4AC \rightarrow$ determent

if $d < 0$ the line and ellipse does not interesct

if $d = 0$ the line and ellipse tangeat

if $d > 0$ the line and ellipse interesct

زي اللي قبله بس لو قاطع circle وعين المعادلة

$(x-x_c)^2+(y-y_c)^2=R^2$

use parametric equation of line

$$x(t)=x_1+(x_2-x_1)t$$
$$y(t)=y_1+(y_2-y_1)t$$

blug parametric equation into circle:

$$(x_1+(x_2-x_1)t)^2+(y_1+(y_2-y_1)t)^2=R^2$$

$$[x_1^2+2x_1(x_2-x_1)t+(x_2-x_1)^2t^2]+$$

$$[y_1^2+2y_1(y_2-y_1)t+(y_2-y_1)^2t^2]=R^2$$

using form of qudratic equation:

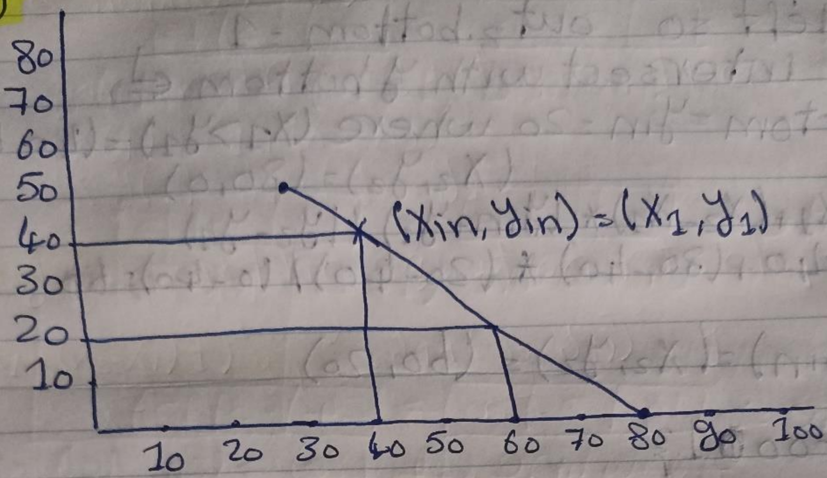
$$t^2[(x_2-x_1)^2+(y_2-y_1)^2]+t[2x(x_2-x_1)+2y(y_2-y_1)]$$
$$+x_1^2+y_1^2-R^2=0$$

$$\Rightarrow At^2+Bt-c=0 \qquad t=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$d=B^2-4AC \Rightarrow$ determent

if $d<r$ then line cuts circle in two point
if $d=r$ then line and circle are tangent
if $d>r$ then line not cut circle in any point

(c)



$out_1 = \{0010\}$ $out_2 = \{0101\}$

$\boxed{1}$ $out_2.left = 0$ $out_1.bottom = 0$ $out_1.top = 1$

interesect with top side ⇐

$\therefore y_{top} = 40$ $y_{in} = y_{top} = 40$ ~~where (x_1, y_1)~~

where $(x_1, y_1) = (30, 50)$ $(x_2, y_2) = (80, 0)$

$x_{in} = x_1 + (x_2 - x_1) * (y_{in} - y_1) / (y_2 - y_1)$

$= 30 + (80 + 30) * (40 - 50) / (0 - 50) = 40$

$\therefore (x_{in}, y_{in}) = (x_1, y_1) = (40, 40)$

$out_1 = \{0000\}$ # trivial empty

$out_2 = \{0101\}$

$out_2.left = 0$ $out_2.bottom = 1$

intersect with y bottom

$\therefore y_{bottom} = y_{in} = 20$ where $(x_1, y_1) = (40, 40)$

$(x_2, y_2) = (80, 0)$

$x_{in} = x_1 + (x_2 - x_1) * (y_{in} - y_1) / (y_2 - y_1)$

$= 40 + (80 - 40) * (20 - 40) / (0 - 40) = 60$

$(x_{in}, y_{in}) = (x_2, y_2) = (60, 20)$

$out_2 = \{0001\}$

$out_2.left = 0$ $out_2.bottom = 0$ $out_2.top = 0$

$out_2.right = 1$

$\therefore$ interesect with x right

$\therefore x_{right} = x_{in} = 40$ where $(x_1, y_1) = (40, 40)$

$(x_2, y_2) = (60, 20)$

$y_{in} = y_1 + (x_{in} - x_1) * (y_2 - y_1) / (x_2 - x_1)$

$= 40 + (40 - 40) * (20 - 40) / (60 - 40) = 40$

$\therefore (x_{in}, y_{in}) = (x_2, y_2) = (40, 40)$

$out_2 = \{0000\}$

$(out_1.All == 0$ && $out_2.All == 0)$

Draw point $(40, 40)$

Question (3):-

نفس السؤال في 2012 سؤال (4) صفحة 8

7, 8, 9, 10, 11, 12, 13

final (2015)

Question (2):- (c)

(30, 10) (15, 15) $= \frac{15-30}{15-10} = -3$

(15, 15) (30, 20) $= \frac{30-15}{20-15} = 3$

(30, 20) (10, 10) $= \frac{10-30}{10-20} = 2$

(10, 10) (30, 10) X have some y

1 N
2 N
3 N
: N
10 → [30 | -3 | 15] → [10 | 2 | 20]
11
12
13
14
15 → [15 | 3 | 20]

x	minv	ymax
30	-3	15

x	minv	ymax
10	2	20

x	minv	ymax
15	3	20

$y=10$
sort ⇒ $\boxed{10 \mid 2 \mid 20} \rightarrow \boxed{30 \mid -3 \mid 15}$
Drawline $(10,10) \rightarrow (30,10)$

$y=11$
update ⇒ $\boxed{12 \mid 2 \mid 20} \rightarrow \boxed{27 \mid -3 \mid 15}$
Drawline $(12,11) \rightarrow (27,15)$

$y=12$
update ⇒ $\boxed{14 \mid 2 \mid 20} \rightarrow \boxed{24 \mid -3 \mid 15}$
Drawline $(14,12) \rightarrow (24,12)$

$y=13$
update ⇒ $\boxed{16 \mid 2 \mid 20} \rightarrow \boxed{21 \mid -3 \mid 15}$
Drawline $(16,13) \rightarrow (21,13)$

$y=14$
update ⇒ $\boxed{18 \mid 2 \mid 20} \rightarrow \boxed{18 \mid -3 \mid 15}$
Drawline $(18,14) \rightarrow (18,14)$

$y=15$
~~update~~ delete ⇒ $\boxed{18 \mid -3 \mid 15}$
update ⇒ $\boxed{20 \mid 2 \mid 20}$
Drawline $(15,15) \rightarrow (20,15)$

end

Question (3):-

1 compute center of figure (1)

center $\frac{10+18}{2} = 14$ $\quad \frac{21+15}{2} = 18$

2 Translate shape to origin

$$T = \begin{bmatrix} 1 & 0 & -14 \\ 0 & 1 & -18 \\ 0 & 0 & 1 \end{bmatrix}$$

3 Rotate figure (2) at origin anti-clockwise:

$$R = \begin{bmatrix} \cos(45) & -\sin(45) & 0 \\ \sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4 compute center of figure (2)

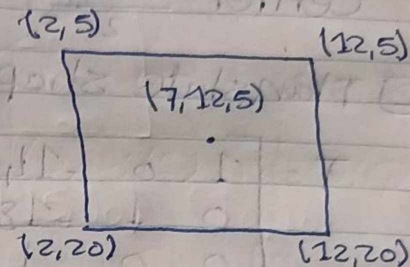
center $= \frac{30+30}{2} = 30$ $\quad \frac{23+13}{2} = 18$

5 translate figure (2) to (30,18)

$$T^{-1} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 18 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Matrix $= T^{-1} . R . T$

Map A rectangle left = 2 top = 5 width = 10 height = 15 to squre centered at (8,2) with lenght = 1



$\boxed{1}$ calculate center of rectangle

$$center = \frac{12+2}{2} = 7$$

$$\frac{20+5}{2} = 12{,}5$$

$\boxed{2}$ translate Rectangle to origin:

$$T = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & -12{,}5 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{3}$ scale width to 1 lenght to 1

$$S_1 = \begin{bmatrix} \frac{1}{10} & 0 & 0 \\ 0 & \frac{1}{15} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{4}$ translate from origin to squre center

$$T_2 = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

composite Matrix

$$= T_2 . S_1 . T$$

write an efficent algorithm to fill an ellipse using the following parametric equations:

$$X = A\cos\theta$$
$$y = B\sin\theta$$

```
Draw Elipse(Xc, yc, R, r2, color):
    dtheta = 1.0/R
    for(theta=0.0, theta<6.28, theta+=dtheta)
        X=Xc+r2 * cos(theta)
        y=yc+R * sin(theta)
        setpixel (Round(X), Round(y), color)
        Draw line (Xc, yc, Round(X), Round(y), color)
    end for
end
```

trace the convex polygon filling algorithm compute the edge table values for a polygon of vertices (15,10), (80,12), (60,15), (10,16)

Initial table:

$$\begin{array}{c} \begin{array}{cc} X_{left} & X_{right} \end{array} \\ \begin{bmatrix} \infty & -\infty \\ \infty & -\infty \\ \infty & -\infty \end{bmatrix} \end{array}$$

$\boxed{1}$ (10,16) (15,10) swap ⇒ (15,10) (10,16)

$\therefore y=10 \quad X=15 \quad \min V = \frac{-5}{6}$

$y=10 \ [15 \quad 15] \rightarrow y{+}{+} \quad X{+}{=}\min V$

$\therefore y=11 \quad X=\frac{85}{6} \quad \min V = \frac{-5}{6}$

$y=11 \ \left[\frac{85}{6} \quad \frac{85}{6}\right] \rightarrow y{+}{+} \quad X{+}{=}\min V$

$\therefore y=12 \quad X=\frac{40}{3} \quad \min V = \frac{-5}{6}$

$y=12 \ \left[\frac{40}{3} \quad \frac{40}{3}\right] \rightarrow y{+}{+} \quad X{+}{=}\min V$

$y=13 \quad X=\frac{25}{2} \quad \min V = \frac{-5}{6}$

$y=13 \ \left[\frac{25}{2} \quad \frac{25}{2}\right] \rightarrow y{+}{+} \quad X{+}{=}\min V$

$y=14 \quad X=\frac{35}{3} \quad \min V = \frac{-5}{6}$

$y=14 \ \left[\frac{35}{3} \quad \frac{35}{3}\right] \rightarrow y{+}{+} \quad X{+}{=}\min V$

Edg 2 table $(point_2(10,16), point_3(60,15))$:

$y=15 \quad X=65 \quad minV=50$

$y=15 \; [60 \quad 60] \rightarrow y{+}{+} \quad X{+}{=}minV$

Edg 2 Table $(point_3(60,15), point_4(80,12))$:

$y=12 \quad X=80 \quad minV=\frac{-20}{3}$

$y=12 \; [\frac{40}{3} \quad 80] \rightarrow y{+}{+} \quad X{+}{=}minV$

$y=13 \quad X=\frac{220}{3}$

$y=13 \; [\frac{25}{2} \quad \frac{220}{3}] \rightarrow y{+}{+} \quad X{+}{=}minV$

$y=14 \quad X=\frac{200}{3}$

$y=14 \; [\frac{35}{3} \quad \frac{200}{3}] \rightarrow y{+}{+} \quad X{+}{=}minV$

$y=15 \quad X=60$

Edg 2 Table $(point_4(80,12), point_1(15,10))$:

$y=10 \quad X=15 \quad minV=\frac{65}{2}$

$y=10 \; [15 \quad 32,5] \rightarrow y{+}{+}$

$y=11 \quad X=\frac{95}{2}$

$y=11 \; [\frac{85}{6} \quad \frac{95}{2}] \rightarrow y{+}{+} \quad X=80$

Edg Table:

$$
\begin{array}{c}
y \\
10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15
\end{array}
\begin{bmatrix}
15 & 32.5 \\
\frac{85}{6} & \frac{95}{2} \\
\frac{40}{3} & 80 \\
\frac{25}{2} & \frac{220}{3} \\
\frac{35}{3} & \frac{200}{3} \\
60 & 60
\end{bmatrix}
$$