EXAM 2013

Question 3

a-

the general form

of affine transformation in 2D spaces is given by the following equation:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

The transform parameters are thus given by a matrix A=$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and a vector b= $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. It is usually preferable to have the parameters in a single matrix for easy manipulation. For this purpose, homogeneous coordinate systems are used for representation of affine transforms.

b-

Proof:

$$x = R\cos\theta$$
$$y = R\sin\theta$$
$$x' = R\cos(\varphi + \theta)$$
$$= R\cos\varphi\cos\theta - R\sin\varphi\sin\theta$$
$$= x\cos\theta - y\sin\theta$$
$$y' = R\sin(\varphi + \theta)$$
$$= R\cos\varphi\sin\theta + R\sin\varphi\cos\theta$$
$$= x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} cos\Theta & -sin\Theta & 0 \\ sin\Theta & cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

this if it counter clockwise so when you need clock wise we will change the Θ will be - Θ

it will be $$\begin{bmatrix} cos\Theta & sin\Theta & 0 \\ -sin\Theta & cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c

1-

$$\begin{bmatrix} 1 & 0 & -12 \\ 0 & 1 & -48 \\ 0 & 0 & 1 \end{bmatrix}$$

2-

first find the center of the rectangle if you draw it by co-ordinates it will be easier

x=(79+30)/2=54.5

y=(100+50)/2=75   the center =(54.5,75)

we need to rotate this rectangle about its center by angle Θ and we cant do this directly so

To rotate an object about a general center (a, b):

1. Translate the object such that (a, b) becomes the origin using the translation matrix:

$$\begin{bmatrix} 1 & 0 & -54.5 \\ 0 & 1 & -75 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Rotate the object about the origin using the rotation matrix:

$$\begin{bmatrix} cos\Theta & -sin\Theta & 0 \\ sin\Theta & cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Undo step 1; i.e. translate the origin to (a, b)

$$\begin{bmatrix} 1 & 0 & 54.5 \\ 0 & 1 & 75 \\ 0 & 0 & 1 \end{bmatrix}$$

rotation(-54.5,75, Θ)= $\begin{bmatrix} 1 & 0 & 54.5 \\ 0 & 1 & 75 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\Theta & -sin\Theta & 0 \\ sin\Theta & cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -54.5 \\ 0 & 1 & -75 \\ 0 & 0 & 1 \end{bmatrix}$

3-

To scale an object uniformly about its center (a, b) with a scaling factor of 0.5:

1- Translate (a, b) to O using

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2- Scale about O

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3- Undo step 1

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So the composite scaling is given by:

$$T^{-1}\mathrm{ST} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5- answer like this . if you understand this example you will answer the one in exam walah a3lm ana sarecht 3la el net

Example. Let W be the plane generated by the vectors $(1,1,1)$ and $(1,0,1)$. Find the orthogonal projection $P : \mathbb{R}^3 \longrightarrow W$.

Solution. We notice first that $((1,1,1),(1,0,1)) = 2 \neq 0$, so this is not an orthogonal basis. Using Gram-Schmidt we get:
$v_1 = (1,1,1)$
$v_2 = (1,0,1) - \frac{2}{3}(1,1,1) = (\frac{1}{3}, -\frac{2}{3}), \frac{1}{3} = \frac{1}{3}(1,-2,1)$.

To avoid fractions, we can use $(1,-2,1)$ instead of $\frac{1}{3}(1,-2,1)$. Thus the orthogonal projection is:

$$\begin{aligned} P(x,y,z) &= \frac{x+y+z}{3}(1,1,1) + \frac{x-2y+z}{6}(1,-2,1) \\ &= \Big(\frac{2x+2y+2z}{6} + \frac{x-2y+z}{6}, \\ &\quad \frac{2x+2y+2z}{6} - 2\frac{x-2y+z}{6}, \\ &\quad \frac{2x+2y+2z}{6} + \frac{x-2y+z}{6}\Big) \\ &= \left(\frac{x+z}{2}, y, \frac{x+z}{2}\right). \end{aligned}$$

Question 4

a

alaah a3lm

b

1- Compute the basis vectors of the camera coordinate system to get the rotation matrix:
w=VPN = (1, 1, 1)

+ - + kol el nas btnssaha 3lshan keda byst3gebo men el - in J

u = $VPN \times VUP = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = i - j + 0k = (1, -1, 0)$

$$v = u \times w = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (-1)i - (1)j + (2)k = (-1, -1, 2)$$

check if V.U = 0 and V.W = 0 then

$$u = \frac{(1, -1, 0)}{\sqrt{1^2 + (-1)^2 + (0)^2}} = \frac{(1, -1,\ 0)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$$

$$v = \frac{(-1, -1, 2)}{\sqrt{(-1)^2 + (-1)^2 + (2)^2}} = \frac{(-1, -1, 2)}{\sqrt{6}} = \left(\frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

$$w = \frac{(1, 1, 1)}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{(1,1,1)}{\sqrt{3}} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Rotate u,v,w

So, $R = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2- Translate COP to O

$T = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3- Camera view matrix =R*T

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{7}{\sqrt{3}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii - dont know

iii -

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{7}{\sqrt{3}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{10}{\sqrt{6}} \\ \frac{5}{\sqrt{3}} \\ 0 \end{pmatrix}$$

do this for the other two points

it will be $\begin{pmatrix} \frac{-5}{\sqrt{2}} \\ \frac{-5}{\sqrt{6}} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{6}} \\ 0 \\ 0 \end{pmatrix}$ fe 7aga ana mesh mot2aked menha en ay point

benzwadlha 0 fe a5er el matrix $\begin{pmatrix} 0 \\ 0 \\ 5 \\ 0 \end{pmatrix}$ w7na bndrb wala bnzwadlha 1 $\begin{pmatrix} 0 \\ 0 \\ 5 \\ 1 \end{pmatrix}$ m3 el 3lm an lw b 1 ht5tlf wana shayfha ht2sr fe eldarb wde el eldoc 7l beha ama lw b 0 mesh ht2sr walaho a3lm

iV - dont know

2012 Question 4

1- if it counter clockwise

$$\begin{bmatrix} cos30 & -sin30 & 0 & 0 \\ sin30 & cos30 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

if it clockwise

$$\begin{bmatrix} cos30 & sin30 & 0 & 0 \\ -sin30 & cos30 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2- solved before

3- el so2al da ertgal btre2a 3'abya :D

we will make sheering

$$\begin{bmatrix} 1 & \propto & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ 29 \\ 1 \end{pmatrix} = \begin{pmatrix} 52 \\ 29 \\ 1 \end{pmatrix}$$

wallah a3lm :D

5- solved before

Question 5

a-- allah a3lm

b- solved before ma3ada rakm iii - i dont know it

2011

question 3

b

i- solved before

ii-solved before

iii-shear with x-axis

$$\begin{bmatrix}1 & 2 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{pmatrix}3\\4\\1\end{pmatrix}=\begin{pmatrix}11\\4\\1\end{pmatrix}$$

-shear with y-axis

$$\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{pmatrix}3\\4\\1\end{pmatrix}=\begin{pmatrix}3\\10\\1\end{pmatrix}$$

iv- de b2a sba7 el faty :D

it seem that this object rotate about its center then translate,so first time you need to compute the center of this object bec of this object is square or rectangle so the its easy to compute

x=(22+6)/2=14 y=(29+13)/2=21

second we need to rotate by drawing and imagination you can see that the $\Theta = 45$

rotation and we have the rotation clock wise so it will be

rotation(14,21, 45)= $\begin{bmatrix}1 & 0 & 14\\0 & 1 & 21\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}cos45 & sin45 & 0\\-sin45 & cos45 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -14\\0 & 1 & -21\\0 & 0 & 1\end{bmatrix}$

we see that the center (14,21) seem not to be (30,21) so it translate so to know a and b

a=30-14=16

b=21-21=0

T= $\begin{bmatrix}1 & 0 & 16\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

wallah a3ly w a3lm :D

c- solved before

Question 4

b-Note, since we are given a single point and for easy computation, go step by step instead of computing the overall matrix, as follows:

1- Translate COP to O
   So the point becomes: (5, 3, 4)-(-2, -1, 1)=(7, 4, 3)
2- Compute the basis vectors of the camera coordinate system to get the rotation matrix:
   CP is Focused point then VPN=CP-COP
   VPN=(0,0,0) - (-2,-1,1)=(2,1,-1)

$$VPN \times VUP = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 3 & 0 & 4 \end{vmatrix} = 4i - 11j + 3k = (4, -11, 3)$$

$$u = \frac{(4,-11,3)}{\sqrt{4^2 + (-11)^2 + (3)^2}} = \frac{(4,-11,3)}{\sqrt{146}} = \left(\frac{4}{\sqrt{146}}, \frac{-11}{\sqrt{146}}, \frac{3}{\sqrt{146}}\right)$$

$$w = \frac{(2,1,-1)}{\sqrt{2^2 + 1^2 + (-1)^2}} = \frac{(2,1,-1)}{\sqrt{6}} = \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$$

$$v = u \times w = \begin{vmatrix} i & j & k \\ \frac{4}{\sqrt{146}} & \frac{-11}{\sqrt{146}} & \frac{3}{\sqrt{146}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \end{vmatrix} = \frac{4}{\sqrt{219}} i - \frac{-5}{\sqrt{219}} j + \frac{13}{\sqrt{219}} k$$

So, $$R = \begin{pmatrix} \frac{4}{\sqrt{146}} & \frac{-11}{\sqrt{146}} & \frac{3}{\sqrt{146}} \\ \frac{4}{\sqrt{219}} & \frac{5}{\sqrt{219}} & \frac{13}{\sqrt{219}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \end{pmatrix}$$

3. Apply R to the resulting point of step 1:

$$\begin{pmatrix} \frac{4}{\sqrt{146}} & \frac{-11}{\sqrt{146}} & \frac{3}{\sqrt{146}} \\ \frac{4}{\sqrt{219}} & \frac{5}{\sqrt{219}} & \frac{13}{\sqrt{219}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 7 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{-7}{\sqrt{146}} \\ \frac{87}{\sqrt{219}} \\ \frac{15}{\sqrt{6}} \end{pmatrix}$$

Parallel projection is accomplished by dropping the z-component so it is $(\frac{-7}{\sqrt{146}}, \frac{87}{\sqrt{219}})$

For perspective projection, as we know, is the mapping:

$$(x, y, z) \rightarrow \left(x\frac{D}{z}, y\frac{D}{z}\right)$$

So, the perspective projection is:

$$\left(\frac{-7}{\sqrt{146}} * \frac{4}{\frac{15}{\sqrt{6}}}, \frac{87}{\sqrt{219}} \cdot \frac{4}{\frac{15}{\sqrt{6}}})\right) = (-0.378, 3.84)$$

2010
Question 1
a- solved before
b-solved before without iii

mesh 3arfha a3ta2ed hn23od natsha2leb fe el

Question 4
solved before