Transformation

types

1. Constant transformation
2. linear transformation
3. Affline transformation

note: the composition of an affline transformation gives another affline transformation.

Homogeneus space: Hypothetical space exceeds the physical space by one degree of freedom

Basic affline transformation:

① Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{glLoadIdentity()}$$

② Translation matrix

$\bar{x} = x + a$
$\bar{y} = y + b$
$\bar{z} = z + c$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogenus space

→ glTranslated (a, b, c);

③ scaling (around origine(0,0,0))

$$\bar{x} = \alpha x \qquad \bar{y} = \beta y \qquad \bar{z} = \gamma z$$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ glScaled(α,β,γ);

④ Rotation (around z)

$$\bar{x} = x\cos\theta - y\sin\theta$$
$$\bar{y} = x\sin\theta + y\cos\theta$$
$$\bar{z} = z$$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ glRotated(θ, 0, 0, 1)

note: the inverse of any rotation matrix is its transpose

⑤ shearing (perpendicular to z)

$$\bar{x} = x + \alpha z$$
$$\bar{y} = y + \beta z$$
$$\bar{z} = z$$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \alpha & 0 \\ 0 & 1 & \beta & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

coordinates system:. system consists of a point (origine) and 3 independent vectors

orthogonal basis: $v_1 \perp v_2$ $\qquad \|v_1\| = \|v_2\| = \|v_3\| = 1$

$v_2 \perp v_3$

$v_1 \perp v_3$

① from any system to world system

$$P = \begin{bmatrix} v_1 & v_2 & v_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \alpha \\ 1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} R & t \\ \hline 000 & 1 \end{array}\right]$$

② from the world system to another

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} v_1^T \\ v_2^T \\ v_3^T \end{bmatrix} P$$

$$= \left[\begin{array}{c|c} R^T & -R^T t \\ \hline 000 & 1 \end{array}\right]$$

- Camera Paramters
① eye position (cop)
② Central point (TP)
③ viewers up direction (up)

→ glLookat( );

- Derivation of Camera View Matrix

$w = Vpn = Tp - Cop$ $\qquad \hat{w} = w / \|w\|$

$u = w \times Vup$ $\qquad \hat{u} = u / \|u\|$

$v = u \times w$ $\qquad \hat{v} = v / \|v\|$

① translation

$$= \begin{bmatrix} 1 & 0 & 0 & -copx \\ 0 & 1 & 0 & -copy \\ 0 & 0 & 1 & -copz \end{bmatrix} \text{then} \rightarrow$$

② Rotation

$$= \begin{bmatrix} \hat{u}^T & 0 \\ \hat{v}^T & 0 \\ \hat{w}^T & 0 \\ 000 & 1 \end{bmatrix}$$

Newpoint = RT * old point

projection

① Orthogonal

$\bar{x} = x$

$\bar{y} = y$

$\bar{z} = f$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & F \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② perspective

$\bar{x} = xF/Z$

$\bar{y} = yF/Z$

$\bar{z} = F$

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & F & 0 & 0 \\ 0 & 0 & F & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- standard device : (normalized space) cube with the dimensions 2x2x2 and has the origin in the Center.

- Field of view → Standard device cube

① parallel projection

transformation = Scaling + translation

$x = \text{left}$ $\quad x = \text{right}$

$y = \text{Bottom}$ $\quad y = \text{top}$

$Z = \text{near}$ $\quad Z = \text{far}$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ \bar{w} \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & d \\ 0 & b & 0 & e \\ 0 & 0 & c & f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ \bar{w} \end{bmatrix} = \begin{bmatrix} 2/r-l & 0 & 0 & -(r+l)/r-l \\ 0 & 2/T-B & 0 & -(T+B)/T-B \\ 0 & 0 & 2/F-N & -(F+N)/F-N \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ glOrtho(l, R, T, B, F, N)

② Frustum projection

$x = left$ $\quad$ $x = right$
$z = near$ $\quad$ $z = near$

$y = top$ $\quad$ $y = Bottom$
$z = near$ $\quad$ $z = near$

$z = far$

transformation:
scaling + translation + shearing + projection

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ \bar{w} \end{bmatrix} = \begin{bmatrix} a & 0 & g & 0 \\ 0 & b & h & 0 \\ 0 & 0 & c & f \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\downarrow$

Convert from Homogenous to 3D

$$\bar{\bar{x}} = a/z + g$$
$$\bar{\bar{y}} = b/z + h$$
$$\bar{\bar{z}} = f/z + c$$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \\ \bar{w} \end{bmatrix} = \begin{bmatrix} 2n/r-l & 0 & -(r+l)/r-l & 0 \\ 0 & 2n/T-B & -(t+B)/t-B & 0 \\ 0 & 0 & -(f+n)/f-n & -2fn/f-n \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

View port mapping

(x_0, y_0, w, h)
scaling + translation

$$\begin{bmatrix} a & 0 & c \\ 0 & b & d \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} w/2 & 0 & x_0 + w/2 \\ 0 & -h/2 & y_0 + h/2 \\ 0 & 0 & 1 \end{bmatrix}$$

Subject موضوع الدرس Date

Hidden face removal

```
bool visible (cop, A, B, c)
{
    v1 = B - A;
    v2 = C - B;
    N = v1 x v2;

    v = A - cop;
    return (v.N < 0);
}
```