DATA COMPRESSION

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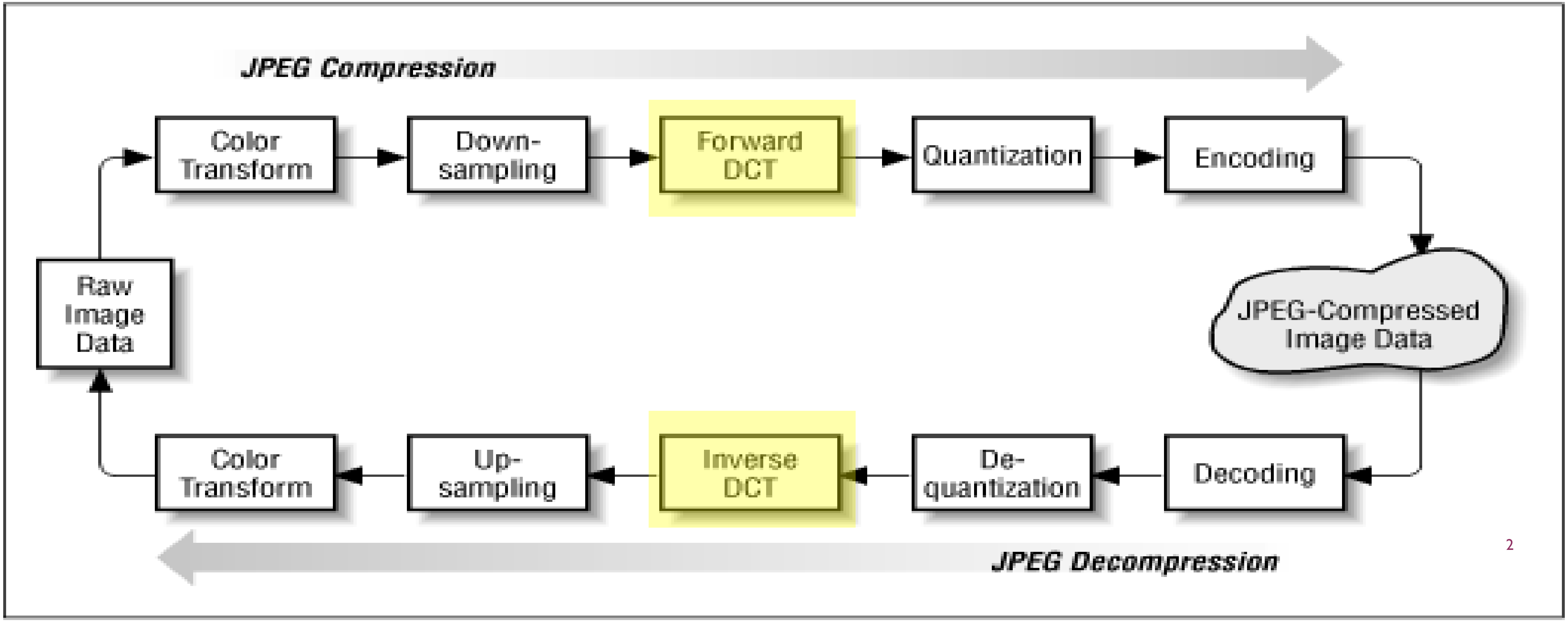
JPEG



JPEG Compression
Color Transform
Down-sampling
Forward DCT
Quantization
Encoding
Raw Image Data
JPEG-Compressed Image Data
Color Transform
Up-sampling
Inverse DCT
De-quantization
Decoding
JPEG Decompression

JPEG : JOINT PHOTOGRAPHIC EXPERTS GROUP

JPEG encoder Consists of:

1. Image/block preparation "Color Models"
2. DCT computation
3. Quantization
4. Entropy coding [vectoring, differential encoding, run-length encoding, Huffman encoding]
5. Frame building

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2. DCT computation
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From this point on, each color component is processed independently, so a "pixel" means a single value, even in a color image

2-JPEG: DISCRETE COSINE TRANSFORM (DCT)

1-D Discrete Cosine Transform

$$C(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left(\frac{(2x+1)u\pi}{2N} \right)$$

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } u = 0 \\ \sqrt{\frac{2}{N}} & \text{otherwise} \end{cases}.$$

$$\boxed{F(x)} \xrightarrow{\text{DCT}} \boxed{C(u)}$$

DISCRETE COSINE TRANSFORM (2D-DCT)

2-D Discrete Cosine Transform

the definition for alpha is the same as before

$$C(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left(\frac{(2x+1)u\pi}{2N}\right)\cos\left(\frac{(2y+1)v\pi}{2N}\right)$$

$$F(x,y) \xrightarrow{\text{DCT}} C(u,v)$$

INVERSE DCT

The DCT is invertible

Spatial samples can be recovered from the DCT coefficients

$$f(x) = \sum_{u=0}^{N-1} \alpha(u) C(u) \cos\left(\frac{(2x+1)u\pi}{2N}\right)$$

$$f(x,y) = \sum_{u=0}^{N-1} \sum_{v}^{N-1} \alpha(u)\alpha(v) C(u,v) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right)$$

$$\boxed{F(x,y)} \xleftarrow{\text{IDCT}} \boxed{C(u,v)}$$

SOME DCT PROPERTIES

- The DCT provides energy compaction

Low frequency coefficients have larger magnitude (typically)

High frequency coefficients have smaller magnitude (typically)

Most information is compacted into the lower frequency coefficients

(those coefficients at the 'upper-left')

- Compaction can be leveraged for compression

Use the DCT coefficients to store image data but discard a certain percentage of the high-frequency coefficients!

JPEG does this

DCT ON A 8X8 BLOCK

Y,Cb,Cr
dividing into 8x8 blocks

139	144	149	153	155	155	155	155
144	151	153	156	159	156	156	156
150	155	160	163	158	156	156	156
159	161	162	160	160	159	159	159
159	160	161	162	162	155	155	155
161	161	161	161	160	157	157	157
162	162	161	163	162	157	157	157
162	162	161	161	163	158	158	158

DC term

AC term

235.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	-0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

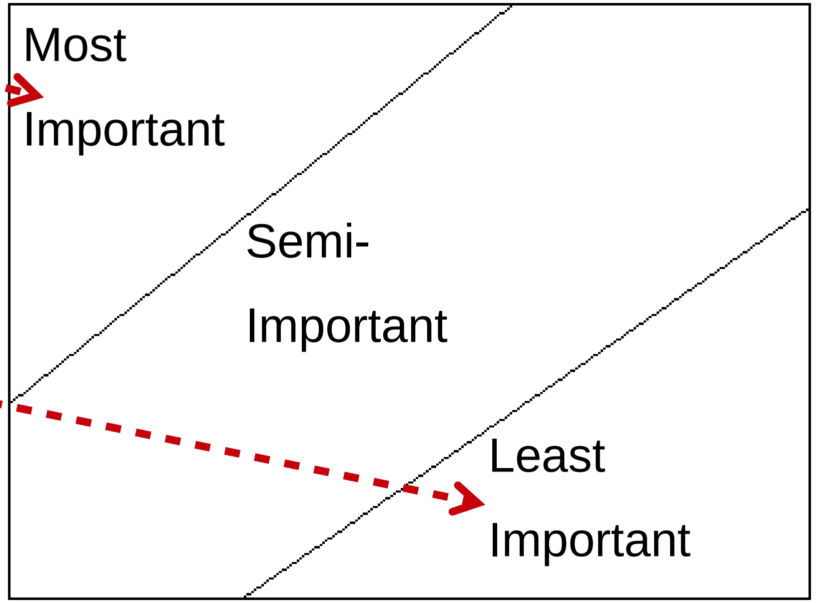
$$C(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left(\frac{(2x+1)u\pi}{2N}\right)\cos\left(\frac{(2y+1)v\pi}{2N}\right)$$

IMPORTANCE OF DCT COEFFICIENTS

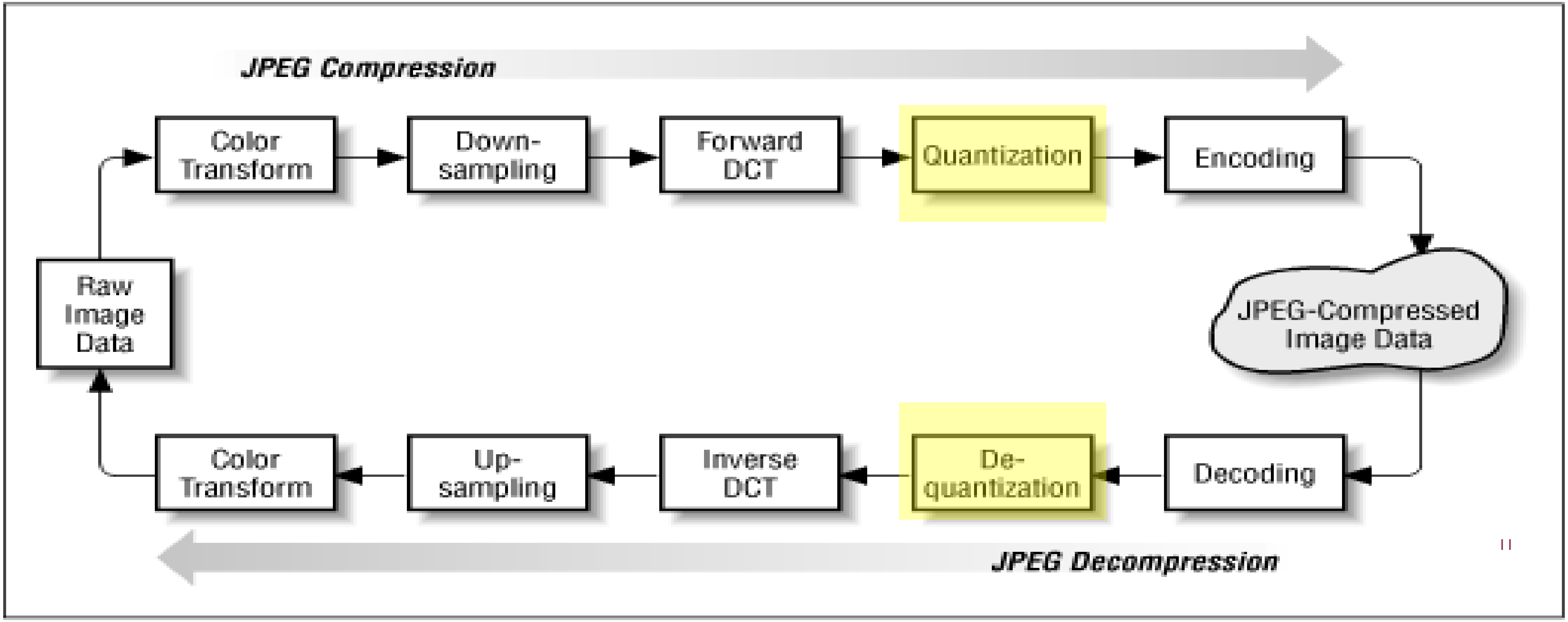
- Using the DCT, the entries will be organized based on the human visual system.

- **The most important values** to
- our eyes will be placed in the
- upper left corner of the matrix.

- **The least important values**
- will be mostly in the lower
- right corner of the matrix.



JPEG



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3-JPEG: QUANTIZATION

- After DCT transform we have fraction, +ve and –ve numbers, which mean more bits is needed to store each block, we have to quantize DCT blocks But any change in any number of DCT will propagate to overall image ,so HOW TO QUANTIZE??
- We formulate quntizer for each block with 64 different steps , but HOW TO DETERMINE STEP SIZE??

235.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	-0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

3-JPEG: QUANTIZATION

- Each of the *64 positions* of the DCT output block has its *own quantization* coefficient, with the higher-order terms being quantized more heavily than the low-order terms (that is, the *higher-order terms have larger quantization coefficients*).
- *Separate* quantization tables are employed for *luminance and chrominance* data, with the chrominance data being quantized more heavily than the luminance data. This allows JPEG to exploit further the eye's differing sensitivity to luminance and chrominance.

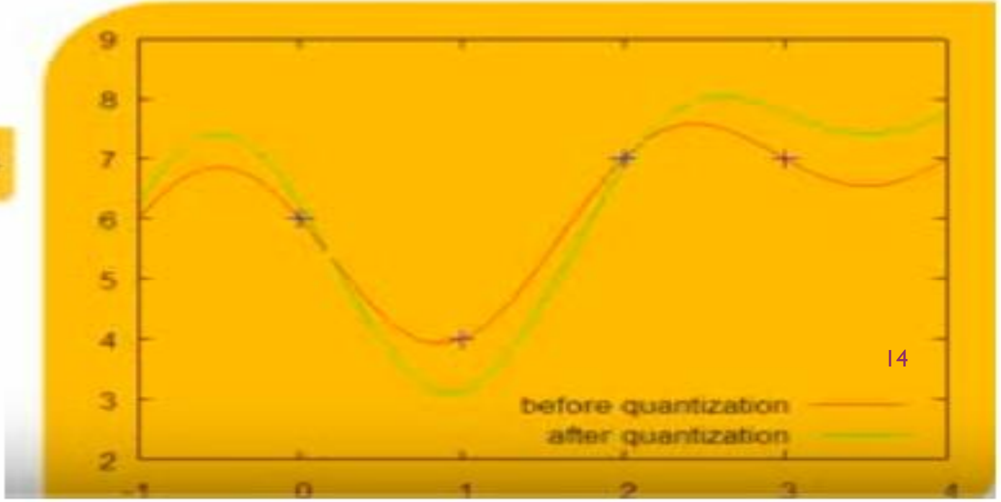
Example: Uniform quantization with $\Delta = 2$ of x = {6.00, -1.47, 1.00, 1.69}

$$q = \left\lfloor \frac{x}{\Delta} + 0.5 \right\rfloor \Rightarrow q = \{3, -1, 1, 1\}$$

$$\hat{x} = q \cdot \Delta \Rightarrow \hat{x} = \{6, -2, 2, 2\}$$

Quantization error:

$$MSE = \frac{1}{4}\left((6-6)^2 + ((-2)-(-1.47))^2 + (2-1)^2 + (2-1.69)^2\right) = 0.34$$



QUANTIZED DCT COEFFICIENTS

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Quantization Matrix

235.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	-0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

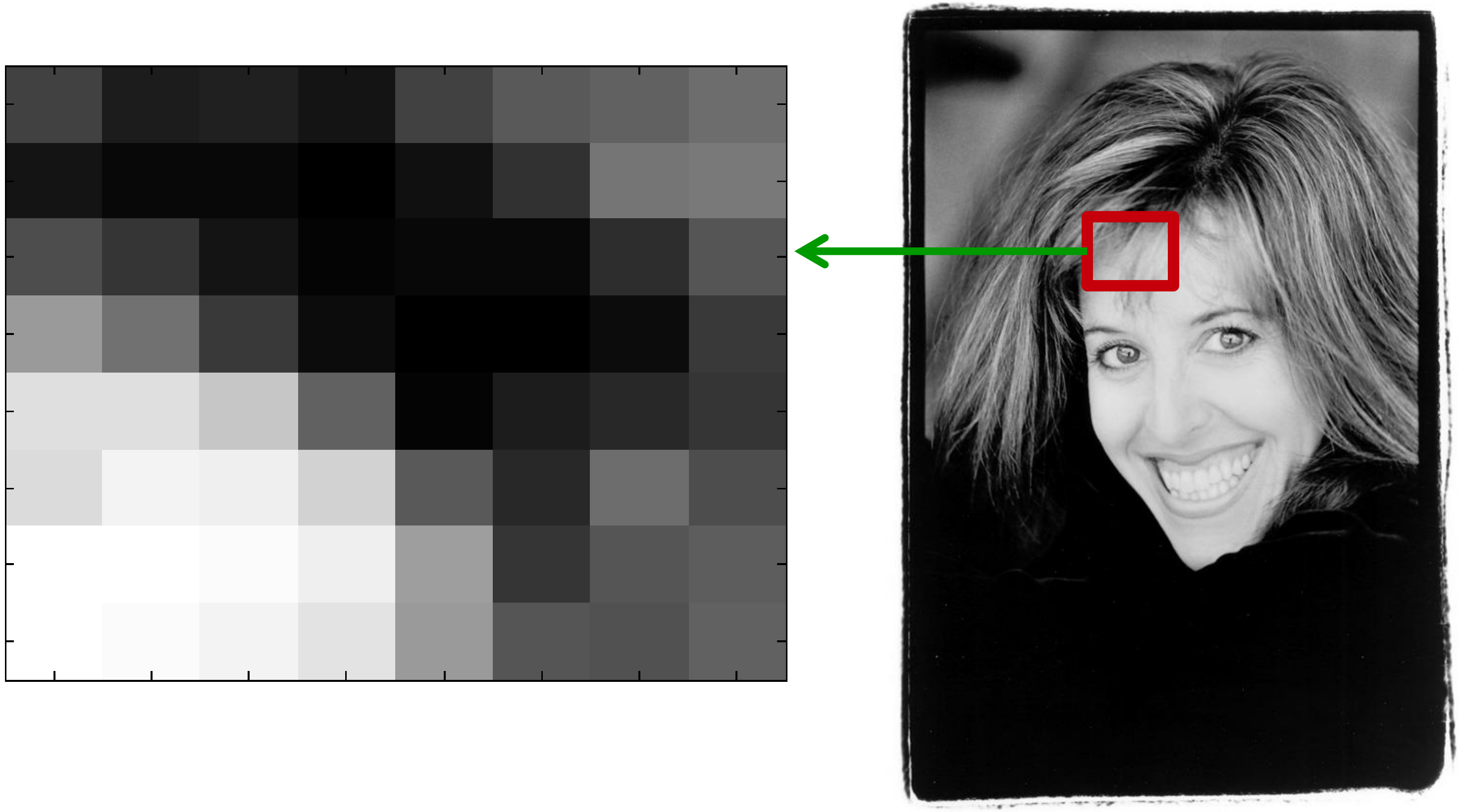
DCT Block

15	0	-1	0	0	0	0	0
-2	-1	0	0	0	0	0	0
-1	-1	0	0	0	0	0	0
-1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Quantizatied DCT B

EXAMPLE: DCT WITH QUANTIZATION

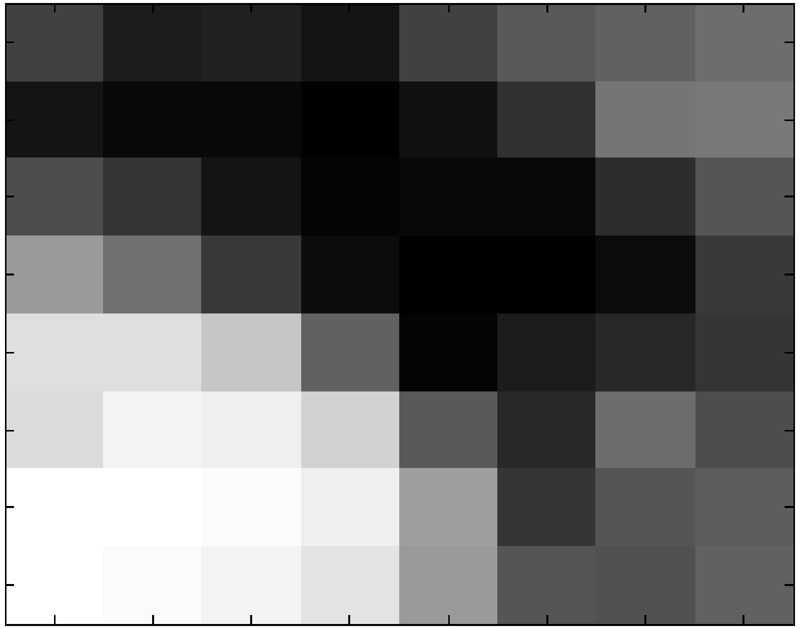
- 8 x 8 Pixels Original Image



EXAMPLE: DCT WITH QUANTIZATION

- **Gray-Scale Example:** Value Range 0 (black) --- 255 (white)

  - 63 33 36 28 63 81 86 98
  - 27 18 17 11 22 48 104 108
  - 72 52 28 15 17 16 47 77
  - 132 100 56 19 10 9 21 55
  - 187 186 166 88 13 34 43 51
  - 184 203 199 177 82 44 97 73
  - 211 214 208 198 134 52 78 83
  - 211 210 203 191 133 79 74 86



X

EXAMPLE: DCT WITH QUANTIZATION

63	33	36	28	63	81	86	98
27	18	17	11	22	48	104	108
72	52	28	15	17	16	47	77
132	100	56	19	10	9	21	55
187	186	166	88	13	34	43	51
184	203	199	177	82	44	97	73
211	214	208	198	134	52	78	83
211	210	203	191	133	79	74	86

DCT →

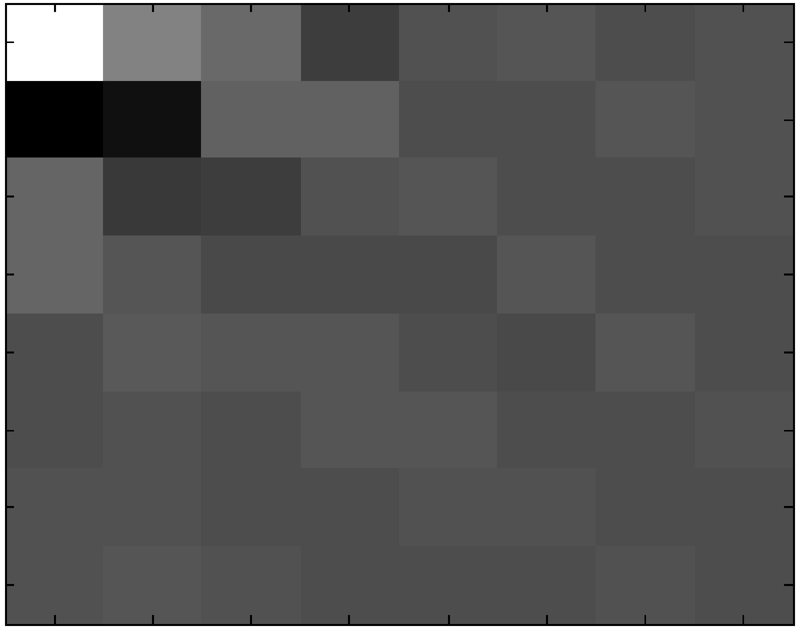
-304	210	104	-69	10	20	-12	7
-327	-260	67	70	-10	-15	21	8
93	-84	-66	16	24	-2	-5	9
89	33	-19	-20	-26	21	-3	0
-9	42	18	27	-7	-17	29	-7
-5	15	-10	17	32	-15	-4	7
10	3	-12	-1	2	3	-2	-3
12	30	0	-3	-3	-6	12	-1

EXAMPLE: DCT WITH QUANTIZATION

- 2D-DCT of matrix

Numbers are coefficients of polynomial

- -304 210 104 -69 10 20 -12 7
- -327 -260 67 70 -10 -15 21 8
- 93 -84 -66 16 24 -2 -5 9
- 89 33 -19 -20 -26 21 -3 0
- -9 42 18 27 -7 -17 29
- -5 15 -10 17 32 -15 -4 7
- 10 3 -12 -1 2 3 -2 -3
- 12 30 0 -3 -3 -6 12 -1

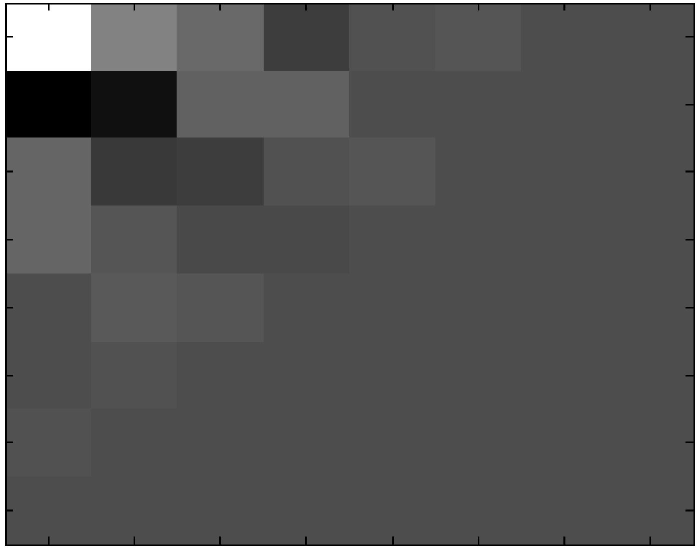


Y

EXAMPLE: DCT WITH QUANTIZATION

- Cut the least significant components (High Frequency Components) after quantization
- “Assume quantization step=1”

- -304 210 104 -69 10 20 -12 0
- -327 -260 67 70 -10 -15 0 0
- 93 -84 -66 16 24 0 0 0
- 89 33 -19 -20 0 0 0 0
- -9 42 18 0 0 0 0 0
- -5 15 0 0 0 0 0 0
- 10 0 0 0 0 0 0 0
- 0 0 0 0 0 0 0 0



EXAMPLE: DCT WITH QUANTIZATION

-304	**210**	**104**	**-69**	**10**	**20**	**-12**	**0**
-327	**-260**	**67**	**70**	**-10**	**-15**	**0**	**0**
93	**-84**	**-66**	**16**	**24**	**0**	**0**	**0**
89	**33**	**-19**	**-20**	**0**	**0**	**0**	**0**
-9	**42**	**18**	**0**	**0**	**0**	**0**	**0**
-5	**15**	**0**	**0**	**0**	**0**	**0**	**0**
10	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**

IDCT $\Rightarrow$

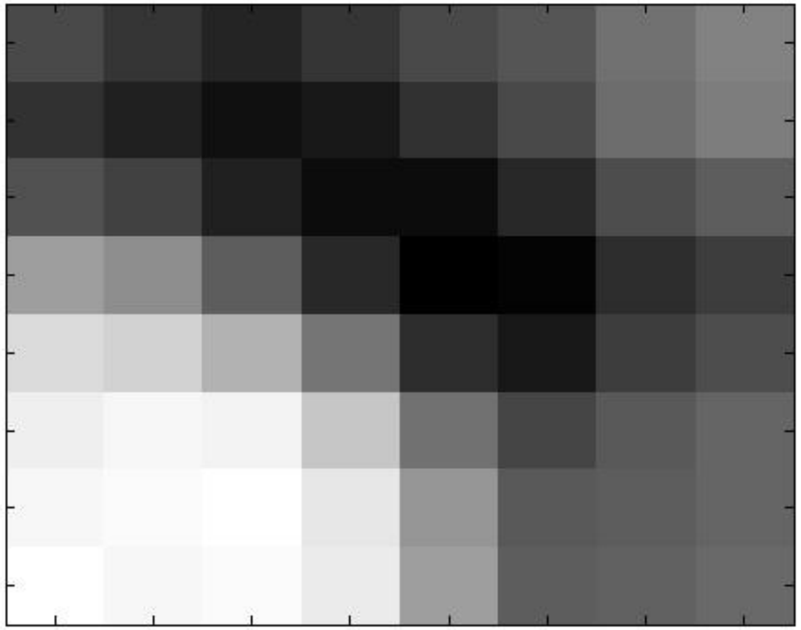
55	41	27	39	56	69	92	106
35	22	7	16	35	59	88	101
65	49	21	5	6	28	62	73
130	114	75	28	-7	-1	33	46
180	175	148	95	33	16	45	59
200	206	203	165	92	55	71	82
205	207	214	193	121	70	75	83
214	205	209	196	129	75	78	85

EXAMPLE: DCT WITH QUANTIZATION

- Apply Inverse DCT in the Image

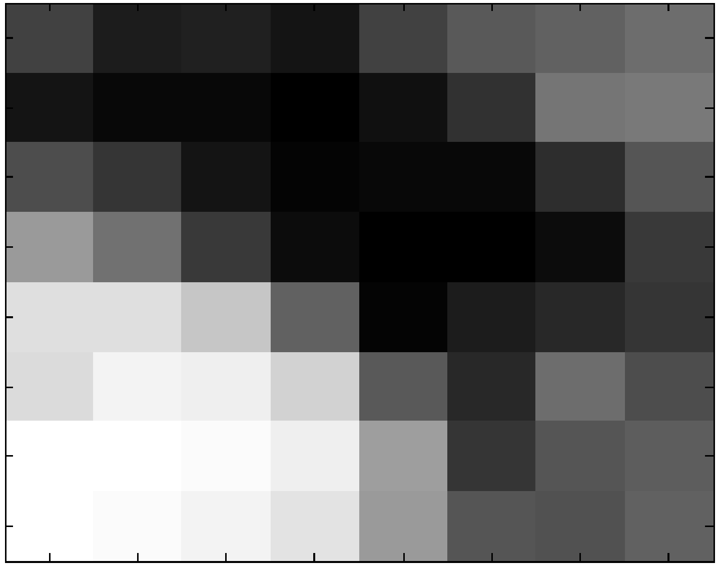
- 55 41 27 39 56 69 92 106
- 35 22 7 16 35 59 88 101
- 65 49 21 5 6 28 62 73
- 130 114 75 28 -7 -1 33 46
- 180 175 148 95 33 16 45 59
- 200 206 203 165 92 55 71 82
- 205 207 214 193 121 70 75 83
- 214 205 209 196 129 75 78 85

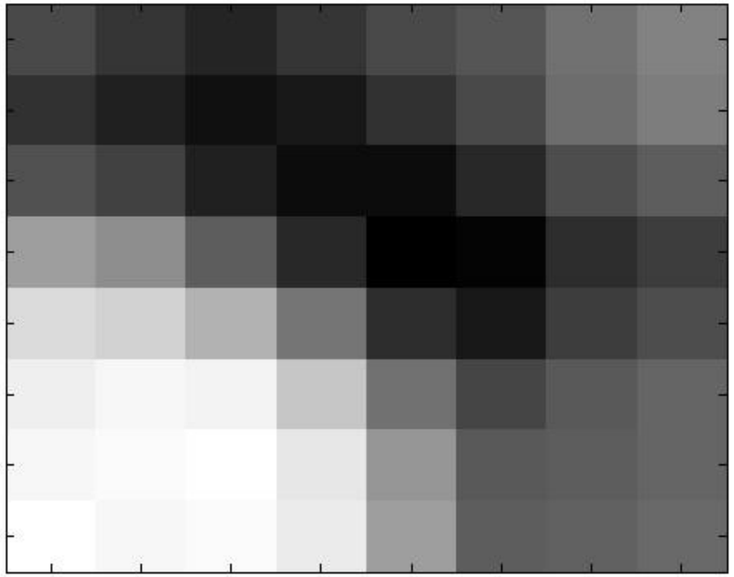
IDCT



EXAMPLE: DCT WITH QUANTIZATION

- Original Compressed-Decompressed





EXAMPLE: DCT WITH QUANTIZATION

- Original Compressed-Decompressed