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Lec-22

27/12

1- $x(t)$ is a real signal

2- $x(t)$ is Periodic with Period $\boxed{T=4}$ and it has Fourier series Coefficients a_k.

3- $a_k = 0$ for $|k| > 1$

4- The Signal with Fourier Series Coefficients $b_k = e^{-jk(\frac{\pi}{2})} \cdot a_{-k}$ is odd.

5- $\frac{1}{4}\int_{4} |x(t)|^2 \, dt = \frac{1}{2}$

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$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}$$

Fact #3 a_{-1}, a_0, a_1

$$\boxed{x(t) = a_{-1} e^{-j\omega_0 t} + a_0 + a_1 e^{j\omega_0 t}}$$

Fact #2

$$T=4 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$x(t) = a_0 + a_{-1} e^{-j(\frac{\pi}{2})t} + a_1 e^{j(\frac{\pi}{2})t}$$

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Fact#1 ← $x(t)$ is real

← $a_{-k} = a_k^*$

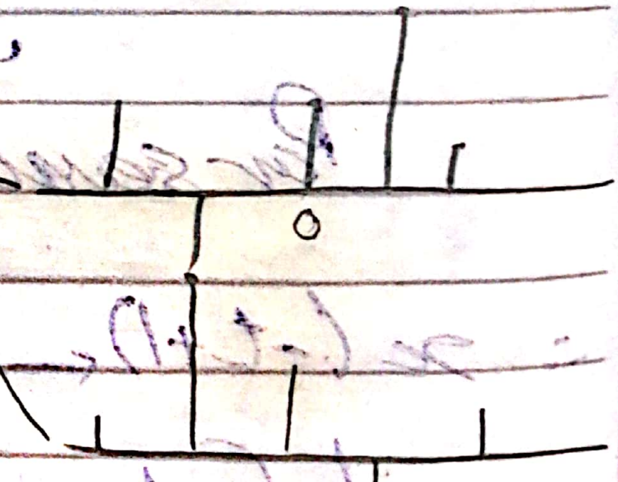
∴ $a_{-1} = a_1^*$

$$\boxed{\therefore x(t) = a_0 + a_1 e^{j(\frac{\pi}{2})t} + a_1^* e^{-j(\frac{\pi}{2})t}}$$

Fact #4

$x(t) \xleftrightarrow{FS} a_k$

$x(-t) \xleftrightarrow{FS} a_{-k}$



$y_1(t) = x(-t)$ → a_{-k}

$y(t) = y_1(t-1)$ → b_k

$= \underline{x(-t+1)}$

$b_k = e^{-jk(\frac{\pi}{2})} \cdot a_{-k}$

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$$x(t) \longrightarrow a_k$$

$$x(t-t_0) \longrightarrow e^{-jk\omega_0 t_0} \cdot a_k$$

$$\boxed{b_{-1} = -b_1^*} \longrightarrow \text{odd}$$

$$b_0 = \boxed{0 = a_0}$$

Fact #5:

$$\frac{1}{4}\underbrace{\int_4 |x(t)|^2 dt}_{} = \frac{1}{2}$$

Parseval's relation

$$\therefore x(-t+1) \longleftrightarrow b_k$$

$$\frac{1}{4}\int_4 |x(-t+1)|^2 dt = \sum_{k=-\infty}^{+\infty} |b_k|^2$$

$$b_k = e^{-jk\left(\frac{\pi}{2}\right)} \cdot a_{-k}$$

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$$|b_{-1}|^2 + |b_0|^2 + |b_1|^2 = \frac{1}{2}$$

$$|-b_1|^2 + 0 + |b_1|^2 = \frac{1}{2}$$

$$|b_1|^2 + |b_1|^2 = \frac{1}{2}$$

$$\boxed{\frac{1}{2}\times} \; 2|b_1|^2 = \frac{1}{2}$$

$$|b_1|^2 = \frac{1}{4} \Longrightarrow |b_1| = \frac{1}{2} \begin{cases} b_1 = \frac{1}{2}j \\ b_1 = -\frac{1}{2}j \end{cases}$$

$$b_k = e^{-jk\frac{\pi}{2}} a_{-k}$$

$$b_{-1} = e^{j\frac{\pi}{2}} a_1 \qquad a_1 = b_{-1} e^{-j\frac{\pi}{2}}$$

$$\boxed{b_{-1} = -b_1}$$

$$a_1 = -b_1 \, e^{-j\frac{\pi}{2}}$$

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Case 1: if $b_1 = \frac{1}{2}j$ $\quad e^{j\theta} = \cos\theta + j\sin\theta$

$= -j\sin(\frac{\pi}{2})$

$a_1 = -e^{-j\frac{\pi}{2}} \cdot b_1$ $\quad = -j$

$= jb_1$

$\therefore a_1 = \frac{1}{2}j \cdot -j = \boxed{-\frac{1}{2}}$

$x(t) = -\frac{1}{2}e^{j\frac{\pi}{2}t} + \left(-\frac{1}{2}e^{-j\frac{\pi}{2}t}\right)$

$= -\frac{1}{2}\left\{e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t}\right\}$

$= -\frac{1}{2} \cdot 2\cos(\frac{\pi}{2}t)$

$= -\cos(\frac{\pi}{2}t)$

Case 2: $x(t) = \cos(\frac{\pi}{2}t)$

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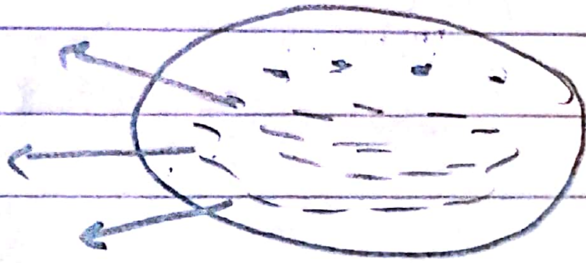
$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad e^{j\omega t}$$

$\omega = k\omega_0$

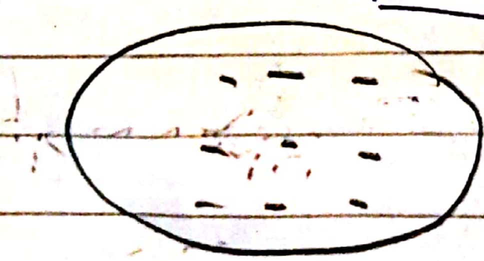


$$\sum a_k e^{jk\omega_0 n} \qquad e^{j\omega n}$$

$\omega \qquad \omega + 2\pi$

$\boxed{e^{j\omega n}}$

$$e^{j(\omega+2\pi)n}$$
$$= e^{j\omega n} \cdot \cancel{e^{j2\pi n}}$$
$$= \boxed{e^{j\omega n}} \qquad = 1$$



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$$\phi_k[n] = e^{jk\omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right)n}$$

$$\phi_{k+N}[n] = e^{j[k+N]\omega_0 n} = e^{jk\omega_0 n} \cdot e^{jN\omega_0 n}$$

$$\underline{\phi_k[n]} = e^{jk\left(\frac{2\pi}{N}\right)n} \cdot \underbrace{e^{jN\left(\frac{2\pi}{N}\right)n}}_{=1}$$

$$x[n] = \sum a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n}$$

$$x[n] = e^{-jH\left(\frac{2\pi}{N}\right)n} \cdot \sum_{k=\langle N\rangle} a_k e^{jk\left(\frac{\pi}{2}\right)n} \cdot e^{-jH\left(\frac{\pi}{N}\right)n}$$

$\phi_0 \quad \phi_1 \quad \phi_N \quad \phi_{N+1}$

ω

$0 \qquad 2\pi$

$-\pi \qquad \pi$

$k\omega_0$

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$$x[n] \cdot e^{-jr\left(\frac{2\pi}{N}\right)n} = \sum_{k=\langle N\rangle} a_k e^{j(k-r)\left(\frac{2\pi}{N}\right)n}$$

$$\sum_{n=\langle N\rangle} x[n] e^{-jr\left(\frac{2\pi}{N}\right)n}$$

$$= \sum_{n=\langle N\rangle} \sum_{k=\langle N\rangle} a_k e^{j(k-r)\left(\frac{2\pi}{N}\right)n}$$

$$= \sum_{k=\langle N\rangle} a_k \cdot \underbrace{\sum_{n=\langle N\rangle} e^{j(k-r)\left(\frac{2\pi}{N}\right)n}}$$

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$$\sum_{n=\langle N\rangle} e^{j(k-r)\left(\frac{2\pi}{N}\right)n} \cdot \quad \sum_{k=a}^{b} r^k = r^a \frac{1-r^{b-a+1}}{1-r} \quad r\neq 1$$

$$= \sum_{n=N_1}^{N-1+N_1} e^{j(k-r)\left(\frac{2\pi}{N}\right)n}$$

$$= \begin{cases} e^{j(k-r)\left(\frac{2\pi}{N}\right)N_1} \dfrac{1-\cancel{e^{j(k-r)\left(\frac{2\pi}{N}\right)(N_2-N_1+1)}}}{1-e^{j(k-r)\left(\frac{2\pi}{N}\right)}} & \\ 0 & k\neq r \end{cases}$$

1

$$j(k-r)2\pi$$

$$\sum_{n=\langle N\rangle} e^{j(k-r)\left(\frac{2\pi}{N}\right)n} = \begin{cases} 0 & \text{if } k\neq r \\ N & \text{if } k=r,\ r+PN \end{cases}$$

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if $k = r + PN$

$\therefore P$ is integer

$$\sum_{\langle N \rangle} e^{j(\cancel{k} r - PN - \cancel{r})\left(\frac{2\pi}{N}\right)n}$$

$P \, 2\pi n$

$$\sum_{n=\langle N \rangle} x[n] e^{-jr\left(\frac{2\pi}{N}\right)n} = \sum_{k=\langle N \rangle} a_k \begin{cases} N & \text{if } k = r \\ & k = r + PN \\ 0 & k \neq r \end{cases}$$

$$a_k = a_{k+PN}$$

$= N \, a_k$

$= N \, a_{k+PN}$

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$$a_r = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jr\left(\frac{2\pi}{N}\right)n}$$

$$a_{\text{صحيح}} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-j\,\text{صحيح}\left(\frac{2\pi}{N}\right)n}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$$x[n] = \sum_{k=\langle N \rangle} a_k\, e^{jk\left(\frac{2\pi}{N}\right)n}$$

synthesis equ.

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$$

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$e^{j\omega n}$

$\frac{\omega}{2\pi} = RN$ $N = 13$

$= \frac{M}{N}$

$a_0 = a_{13} = a_{-13} = a_{PN}$

P is integer

$a_1 = a_{14} = a_{-12}$

a_{N-1}

time domain → a_k → $k\omega$

$x(t)$ FS

$x[n]$ FT a_k