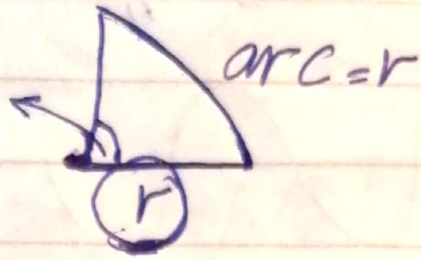
Lec3 : (Exponentials and Sinusoidal Signals Relationship)

التردد

f = frequency $= \frac{cyc}{sec}$ 1 rad

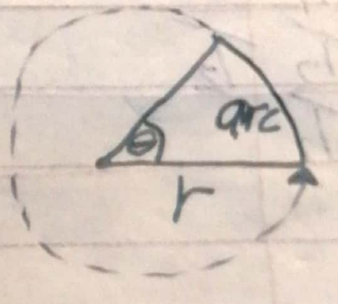


ω = Angular frequency $= \frac{rad}{sec}$

$$f = \frac{cyc}{sec} \times \frac{rad}{rad} = \frac{cyc}{rad} \cdot \boxed{\frac{rad}{sec}} \rightarrow \boxed{\omega}$$

$$f = \frac{cyc}{rad} \cdot \omega = \frac{1}{2\pi} \cdot \omega \qquad \boxed{\omega = 2\pi f}$$

$$\frac{\theta}{arc} = \frac{2\pi}{Circumference}$$



$$= \frac{1}{r} = \frac{2\pi}{Circumference}$$

محيط الدائرة

Circumference $= 2\pi r$.

$$\pi = \frac{\text{Circumference}}{\text{diameter}} \left(\frac{\text{محيط الدائرة}}{\text{القطر}} \right)$$

$$= 3.14159......$$

$$\frac{22}{7} \neq \pi \neq 3.14$$

$$e^{jwt} = \cos(wt) + j\sin(wt).$$

$$wt = 0 \Longrightarrow e^{j0} = \cos 0 + \cancel{j\sin 0} = 1$$

$$wt = \frac{\pi}{2} \Longrightarrow e^{j\frac{\pi}{2}} = \cancel{\cos\frac{\pi}{2}} + j\sin\frac{\pi}{2} = j$$

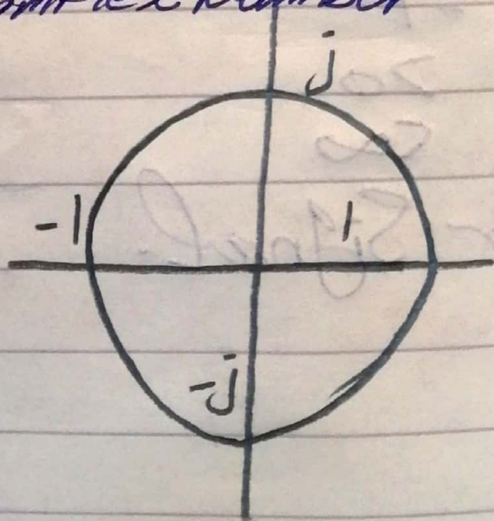
$$wt = \pi \Longrightarrow e^{j\pi} = \cos\pi + \cancel{j\sin\pi} = -1$$

$$wt = \frac{3\pi}{2} \Longrightarrow e^{j\frac{3\pi}{2}} = \cos\frac{3\pi}{2} + j\sin\frac{3\pi}{2} = -j$$

$$wt = 2\pi \Longrightarrow e^{j2\pi} = \cos 2\pi + \cancel{j\sin 2\pi} = 1$$

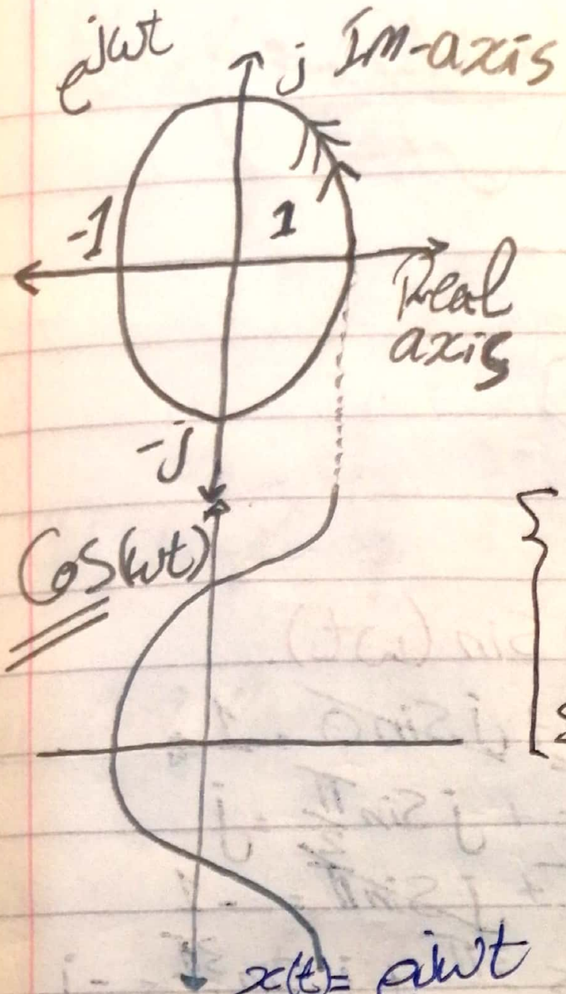
$\therefore e^{jwt}$ is a Periodic.

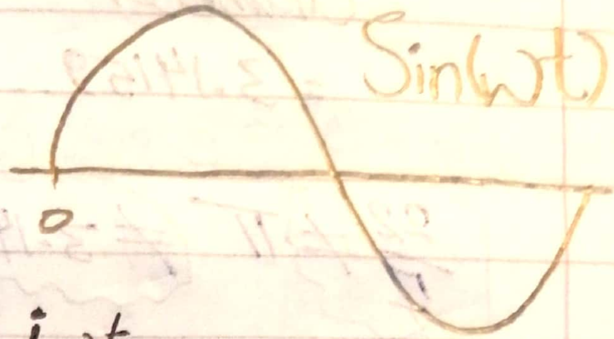
$\hookrightarrow$ is a Complex Number



$|| e^{j\measuredangle}$

wt





$$\left.\begin{matrix} e^{j\omega t} \\ \cos(\omega t+\phi) \\ \sin(\omega t+\phi) \end{matrix}\right\}$$ Periodic

$\underline{x(t)} = e^{j\omega t}$

$$P_\infty = \lim_{T\to\infty} \frac{1}{\cancel{2T}} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_\infty = 1 \qquad = t \Big|_{-T}^{T}$$

> 0

$< \infty$

$$= T - (-T) = \cancel{2T}$$

Power Signal.

$$\int_{-\pi}^{\pi} \cos^2 \omega t \, dt$$

$$\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$= \cos^2\alpha - \{1 - \cos^2\alpha\}$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\cos(2\alpha)$$

$$\int_{-\pi}^{\pi} \left\{\frac{1}{2} + \frac{1}{2}\underbrace{\cos(2\alpha)}\right\} dt.$$

$e^{j\omega t}$

(one signal)

one freq (ω)

$e^{j\omega t} = e^{j\omega(t+T)}$

$e^{j\omega t} = e^{j\omega t} \cdot e^{j\omega T}$

$e^{j\omega T} = 1$

$= \cos(\omega T) + j\sin(\omega T)$

$\omega T = K 2\pi$; K is integer

(set of signals)

~~More~~ than one freq (ω)

$\omega T = K 2\pi$

$T = K \frac{2\pi}{\omega}$

Smallest $\rightarrow K = \pm 1$

Smallest +ve $\rightarrow K = 1$

$$\boxed{T_0 = \frac{2\pi}{|\omega|}}$$

$e^{j\omega t} \longrightarrow$ set of Signals.

$\omega T = K 2\pi$

$\omega = k \frac{2\pi}{T}$; $K = \cancel{0}, \pm 1, \pm 2, \ldots$

(if $K = 1$ fundamental freq.) $\omega_0 = \frac{2\pi}{T_0}$ if $K \neq 1$

$\omega_k = K \frac{2\pi}{T_k}$

$\omega_k = k \omega_0$

$T_{0K} = \frac{2\pi}{\omega_k} = \frac{2\pi}{k\omega_0}$

ω_k

$T_{0K} = \frac{1}{K} \frac{2\pi}{\omega_0} = \frac{T_0}{K}$

$\downarrow T_{0K} = \frac{T_0}{|K|\uparrow}$

$e^{j\omega t}$

$\omega = \frac{2\pi}{T}$

$\omega \propto \frac{1}{T}$

$\underbrace{\phi_k(t) = e^{jk\omega_0}}_{\text{Set of harmonically relation signal.}}$

$\left(e^{j\omega t}\right)$ for diff. $\omega \Rightarrow$ different signal.

It's always Periodic $\Leftarrow$ Periodic signal

Power Signal

$\left(e^{j\omega n}\right)$

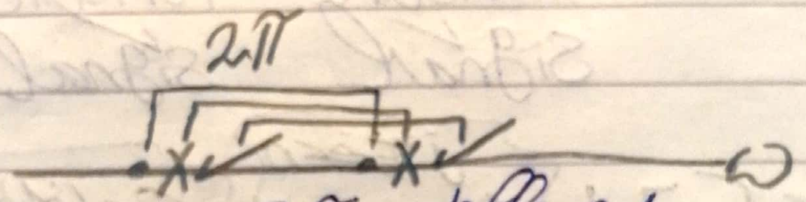
Power Signal

freq. $= \omega$ $\quad$ freq. $= \omega + 2\pi$

$e^{j\omega n}$ $\quad$ $e^{j(\omega+2\pi)n}$

$= e^{j\omega n} \cdot e^{j2\pi n}$

$= e^{j\omega n} \cdot 1$



2π different frequencies.

0 $\quad$ 2π

−π $\quad$ π

$x[n] = e^{j\omega n} \overset{??}{=} x[n+N]$

$e^{j\omega n} \overset{??}{=} e^{j\omega(n+N)}$

$e^{j\omega n} \overset{??}{=} e^{j\omega n} \cdot e^{j\omega N}$

$1 \overset{??}{=} e^{j\omega N}$ $\quad$ $\omega N = K 2\pi$

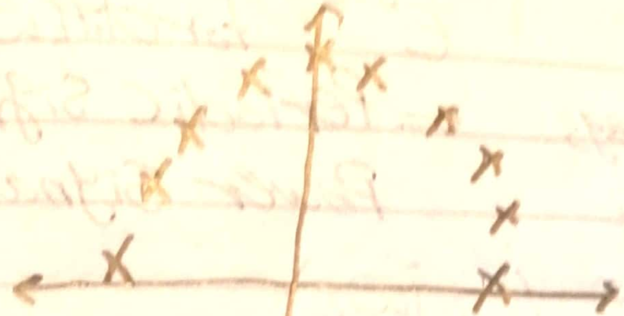
$e^{j\,\text{صفر}\,n}$

$\because \omega =$ صفر

$\dfrac{\omega}{2\pi} = \dfrac{K}{N} = \dfrac{\text{integer}}{\text{integer}} =$ Rational Numbers.

$e^{j\omega n}$

$e^{j\omega t}$



$x(t) = 1 + e^{j\,[illegible]\,t} + \cos(8t) \ldots$

Periodic signal   Periodic signal.

$x[n] = 1 + e^{j\,[illegible]\,n} + \cos([illegible]\, n) \ldots$

$$\frac{\omega}{2\pi} = \frac{K}{N} = \frac{\text{integer}}{\text{integer}} = \frac{2}{6} = \text{Simplest form}$$

$$= \frac{1}{3}$$

3 → fundumental Period