➢ Systems Properties

6- Linearity:

The system is said to be linear if :

$$x_1(t) \xrightarrow{S} y_1(t)$$

$$x_2(t) \xrightarrow{S} y_2(t)$$

And system has the following two properties:

1- Additive property: $$x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$$

2- Scaling/Homogeneity property:

$$ax_1(t) \xrightarrow{S} ay_1(t)$$

The two conditions can be meet together through satisfying the superposition property that:

$$ax_1(t) + bx_2(t) \xrightarrow{S} ay_1(t) + by_2(t)$$

i.e. a linear combination of inputs result in the same linear combination of outputs.

➢ Systems Properties

Linearity check algorithm:

1- let :

$$x_1(t) \xrightarrow{S} y_1(t)$$

$$x_2(t) \xrightarrow{S} y_2(t)$$

And let

$$x_3(t) = ax_1(t) + bx_2(t) \xrightarrow{S} y_3(t)$$

2- Get $y_3(t)$ in terms of $x_1(t)$ and $x_2(t)$ using the system equation. → I

3- Get the expression $ay_1(t) + b\,y_2(t)$ in terms of $x_1(t)$ and $x_2(t)$. → II

4- check if the result of I and II are equal?

- **If yes → linear system.**
- **If No → not linear system.**

➢ Systems Properties

<u>6- Linearity:</u>

Examples to be solved on the board:

1- $$y(t) = t\,x(t)$$

2- $$y(t) = x^2(t)$$

3- $$y[n] = Re\{x[n]\}$$

4- $$y[n] = 2x[n] + 3$$

➢ Systems Properties

6- Linearity:

1- $y(t) = t\,x(t)$

➢ Systems Properties

6- Linearity:

2- $$y(t) = x^2(t)$$

Systems Properties

6- Linearity:

$$y[n] = Re\left\{x[n]\right\}$$

➢ Systems Properties

6- Linearity:

$$y[n] = 2x[n] + 3$$

➢ Review on topics till now

Topics covered:

1- Complex Numbers.

2- Signals and Systems definitions and classifications.

3- Energy and Power.

4- Signals Transformations both for dependent and independent variables.

5- Even and Odd signals.

6- Periodic Signals.

7- Continuous-time Exponential Signals.

8- Discrete-time Exponential Signals.

9- The differences between Continuous-time Exponential and Discrete-time Exponential Signals.

10- Unit impulse and unit step signals.

11- System Properties.

Signals and Systems

Lectures # 8 & #9

Discrete-time LTI Systems

(Convolution Sum)

Prepared by:

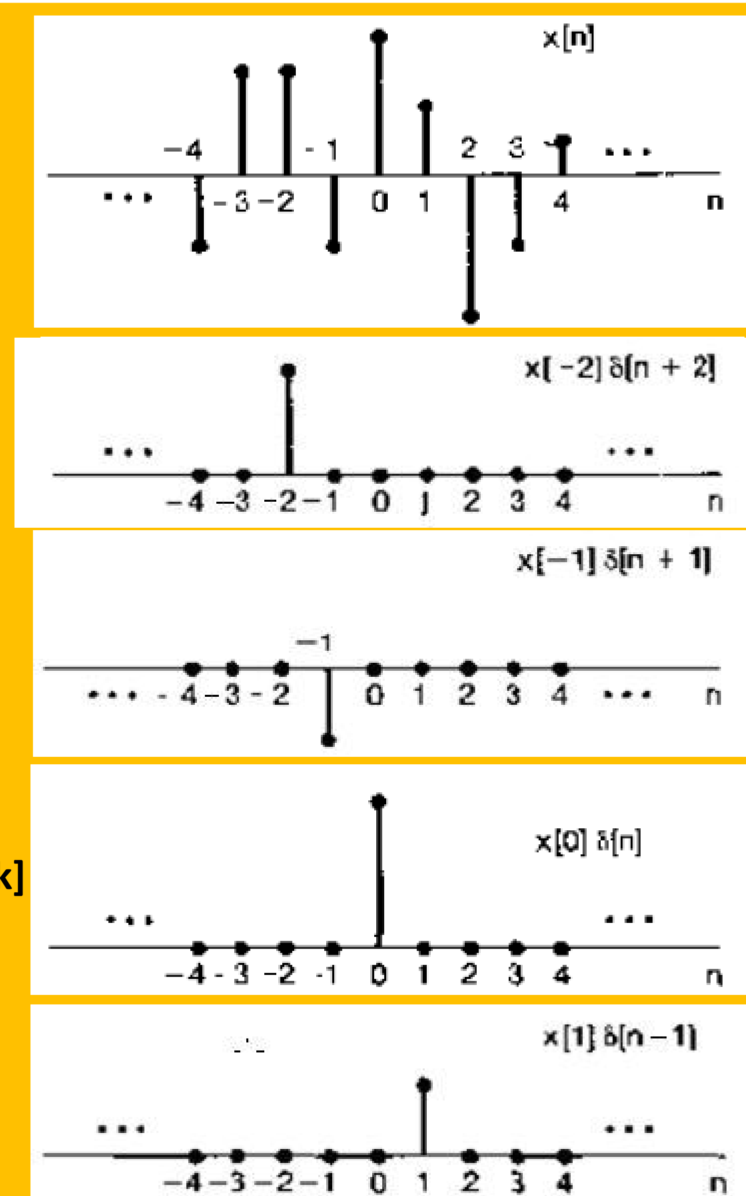
Dr. Mohammed Refaey

Topics of the lecture:

- **Convolution Sum Formula Derivation**
- **Convolution Sum Computation Algorithm**
- **Examples.**

➢ Convolution Sum Formula Derivation

If the original discrete-time signal x[n] is as in figure →



Recall: the unit impulse/sample signal

$$\delta[n]=\begin{cases}1 & ; \quad n=0\\ 0 & ; \quad n\neq 0\end{cases}$$

→ **x[n].δ[n+2] = x[-2].δ[n+2] = x[-2]**

→ **x[n].δ[n+1] = x[-1].δ[n+1] = x[-1]**

→ **x[n].δ[n] = x[0].δ[n] = x[0].δ[n-0] = x[0]**

→ **x[n].δ[n-1] = x[1].δ[n-1] = x[1]**

→ **And so on, then: x[k] = x[k]. δ[n-k] = x[n] . δ[n-k]**

$$\therefore x[n] = sum \quad of \quad all \quad samples$$

$$= \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Convolution Sum Formula Derivation

$$\therefore x[n] = sum \quad of \quad all \quad samples$$

$$\therefore x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$\because \qquad x[n] \xrightarrow{S} y[n]$$

$$let \qquad \delta[n] \xrightarrow{S} h[n]$$

$(as \;\; \delta \;\; is\ an\ impulse,\ h \;\; is\ called\ the\ impulse\ response)$

$$as \quad S \quad LTI \quad system:$$

$$\therefore \delta[n-k] \xrightarrow{S} h[n-k] \; (LTI \;\; \equiv \;\; same \;\; shift)$$

$$\because x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$\xrightarrow{S} \boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]} = x[n] * h[n]$$

Convolution Sum Formula