Lec.15 1/12

$$x(t) = \delta(t)$$

$$X(s) \triangleq \int_{-\infty}^{+\infty} x(t)\, e^{-st}\, dt$$

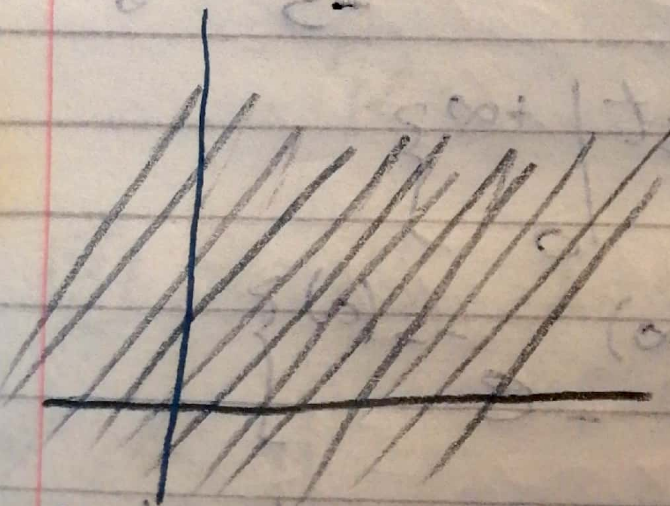
$$\therefore X(s) = \int_{-\infty}^{+\infty} \delta(t) \boxed{e^{-st}} dt = 1$$

$t = 0$

Zero

$e^{0} = 1$

$$\delta(t) \xrightarrow{\mathcal{L}T} 1$$



ROC entire s-Plane

$$x(t) = \boxed{u(t)}$$

$$X(s) \triangleq \int_{-\infty}^{+\infty} x(t)\, e^{-st}\, dt$$

حيث

$$u(\;) = \begin{cases} 1 & \boxed{(\;) > 0} \\ 0 & \boxed{o.w} \end{cases}$$

argument

كل ما بين القوسين

$$= \int_{-\infty}^{+\infty} \boxed{u(t)}\, e^{-st}\, dt$$

Basic signal
(limited integration)

$$= \int_{0}^{+\infty} e^{-st}\, dt = \frac{e^{-st}}{-s}\Big|_{0}^{+\infty}$$

$$X(s) = \frac{1}{-s}\left\{ e^{-st}\Big|_{0}^{+\infty} \right\}$$

$s = \sigma + j\omega$
$= 0$

$$= \frac{1}{-s}\left\{ e^{-s(+\infty)} - e^{-s(0)} \right\}$$

$$= \frac{-1}{s}\left\{ 0 - 1 \right\} = \frac{-1}{s} \cdot (-1)$$

$$\boxed{\therefore X(s) = \frac{1}{s} \;;\; Re\{s\} > 0}$$

$$\int x(t) e^{-st} dt$$

$s = \sigma + j\omega$

0

$s = \sigma + j\omega$

$\boxed{\begin{matrix} 0 - j\infty \\ 0 + j\infty \end{matrix}}$

$$\int x(t)\, e^{-\sigma t} \cdot e^{-j\omega t} dt$$

ROC Properties

[1] ROC ≡ strips Parallel to $j\omega$-axis

[2] for $X(s) \equiv \dfrac{N(s)}{D(s)}$

$N(s) \xrightarrow{=0}$ Zeros

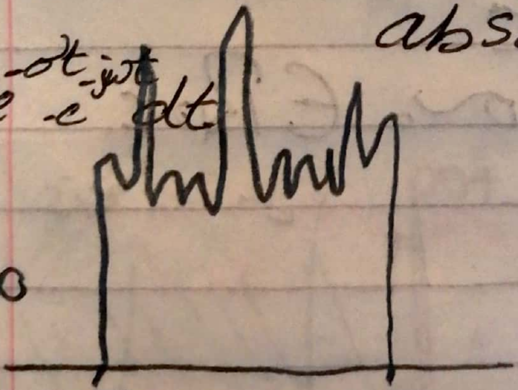
$D(s) \xrightarrow{=0}$ Poles

∴ ROC ∉ any Pole.

[3] $x(t)$ finite-domain.
absolutely integral.

$$\int_{-\infty}^{+\infty} x(t) e^{-\sigma t} \cdot e^{-j\omega t} dt$$

$-\infty$



$+\infty$

$$\int_{-\infty}^{+\infty} |\quad| \, dt < \infty$$

Complex

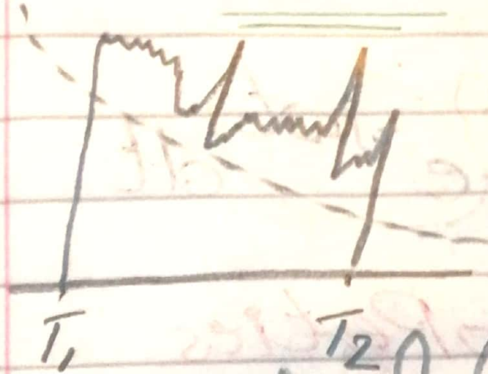
T_1 T_2

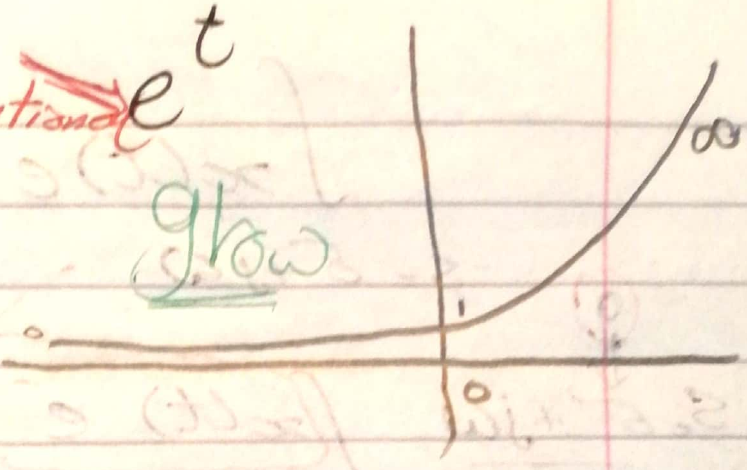
$|x(t)| < \infty \ \forall t$

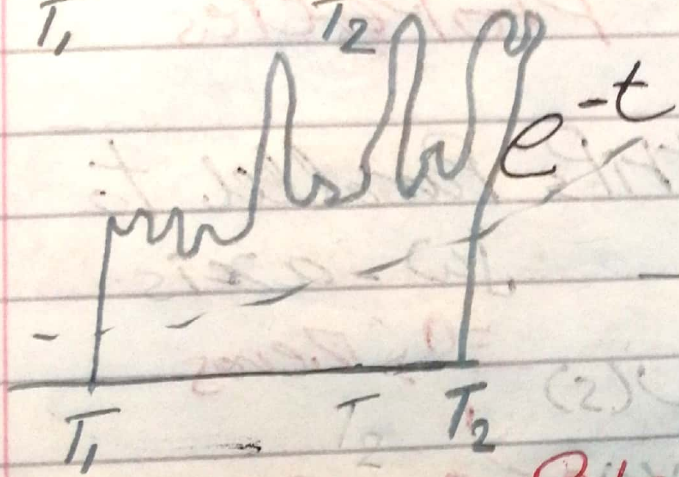
$x(t) = e^{-\sigma t}$

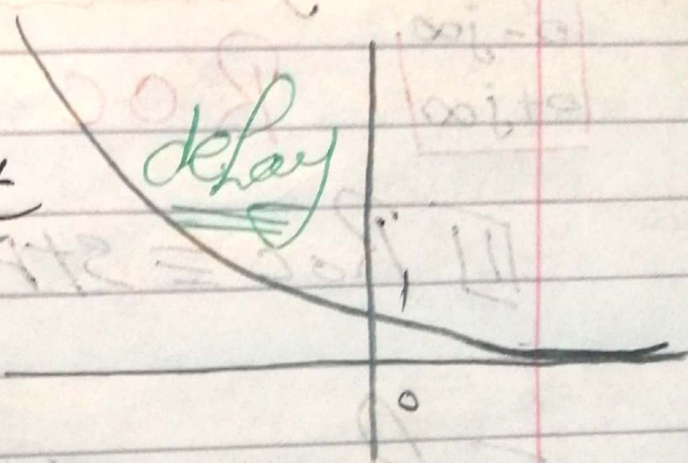
$\boxed{4}$ Real exponential e^{t}

$x(t) = e^{-\sigma t}$









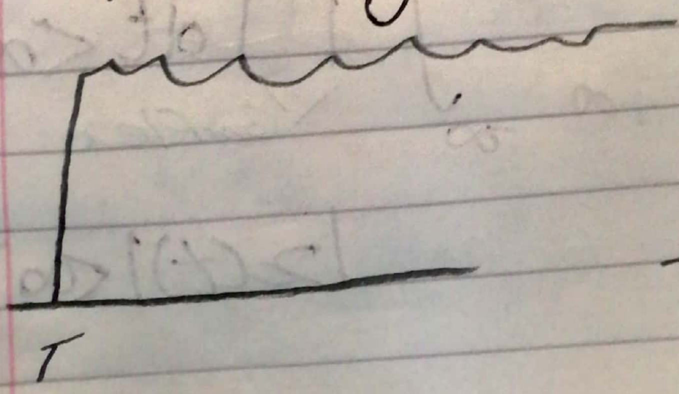
Pole at ∞

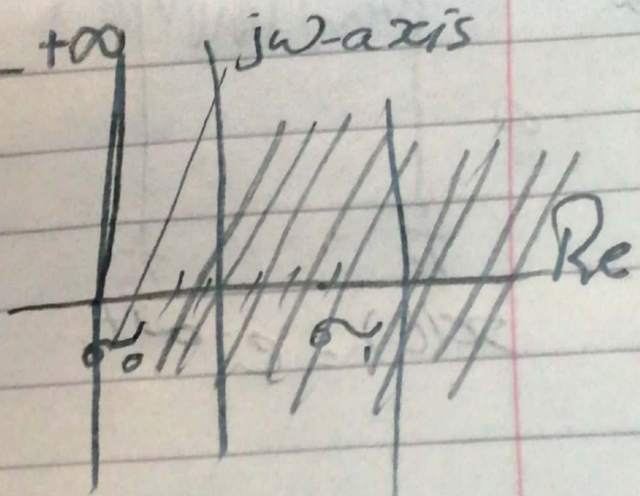
Zero at 0

$\boxed{4}$ $x(t)$ right-sided

if $\sigma_0 \in ROC$

then any $\sigma_1 > \sigma_0 \in ROC$





$$X(s) \triangleq \int_{-\infty}^{+\infty} \underbrace{x(t) e^{-\sigma t}}\, \underbrace{e^{-j\omega t}}\, dt$$

if $\sigma_0 \in \text{RoC} \Longrightarrow X(s) < \infty$

$$\therefore \int_{-\infty}^{+\infty} |x(t)|\, e^{-\sigma_0 t}\, dt < \infty$$

if

$\sigma_1 > \sigma_0 \qquad (\sigma_1 - \sigma_0) \equiv +\underline{ve}$

$\sigma_1 = \sigma_0 + (\sigma_1 - \sigma_0)$

$$\int_{-\infty}^{+\infty} |x(t)|\, e^{-\sigma_1 t}\, dt$$

$$= \int_{-\infty}^{+\infty} \underbrace{|x(t)|\, e^{-\sigma_0 t}}\, e^{-(\sigma_1 - \sigma_0)t}\, dt$$

$\sigma_1 - \sigma_0 = +ve$

as (t) increases $-ve \; +ve = -ve$

$-(\sigma_1 - \sigma_0)t \downarrow$

$$< e^{-(\sigma_1 - \sigma_0)T} \int_{-\infty}^{+\infty} |x(t)|\, e^{-\sigma_0 t}\, dt$$

$< \infty \qquad < \infty$

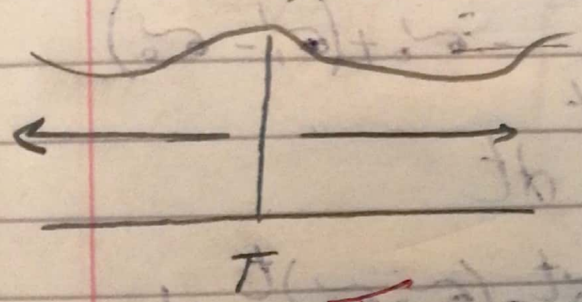
[5] $x(t)$ Left-sided

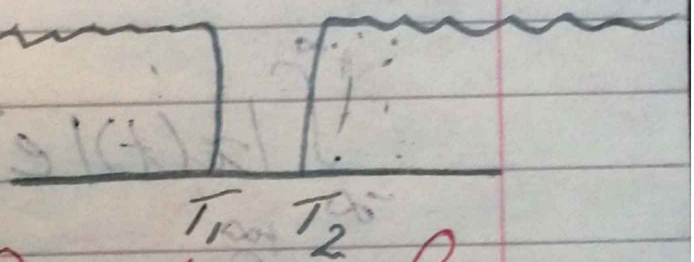
$-(\sigma_1-\sigma_0)t$ $\sigma_0 \in ROC$

$\sigma_1 < \sigma_0 \in ROC$

$$\int_{-\infty}^{T} < e^{-(\sigma_1-\sigma_0)T}$$

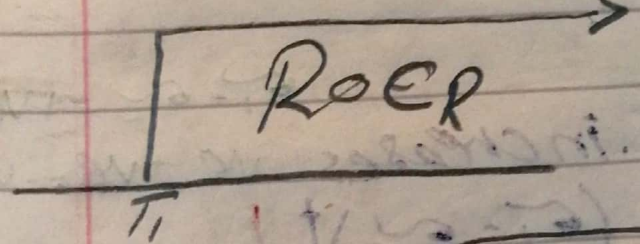
two-sided

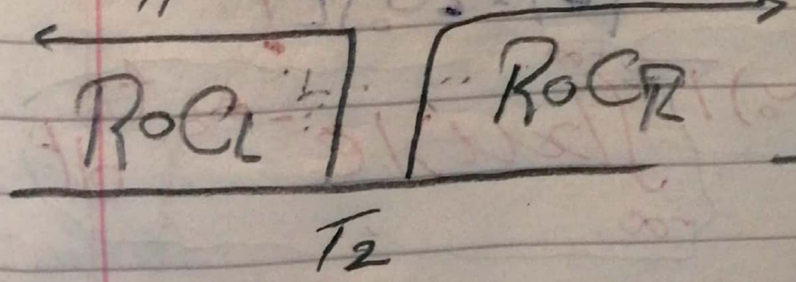


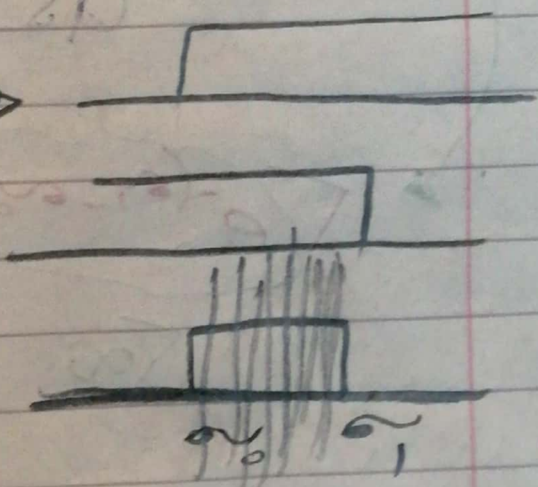


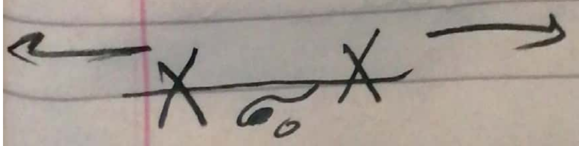
Two ≡ Right + left

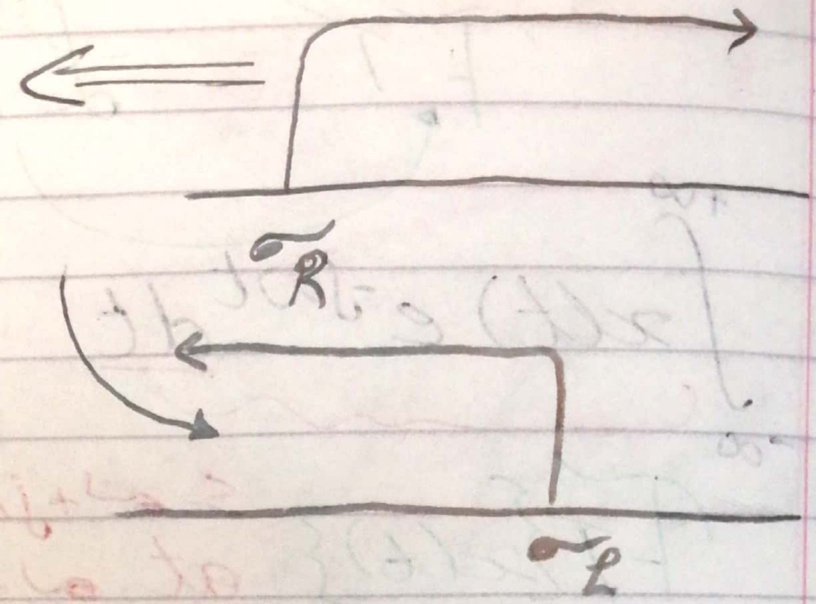
ROC_R ROC_L

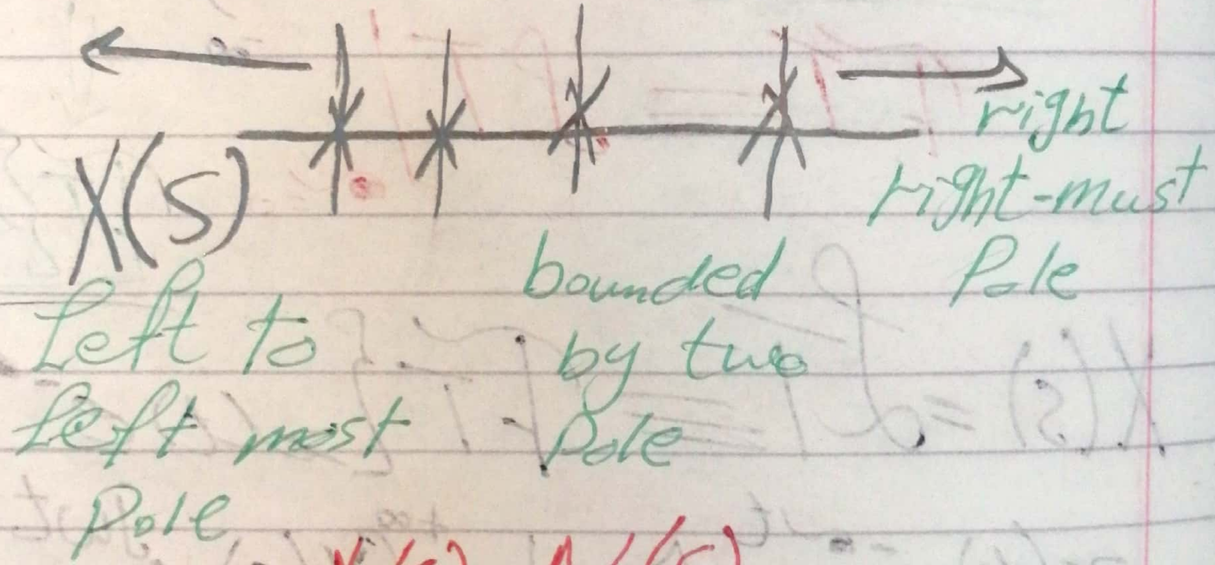




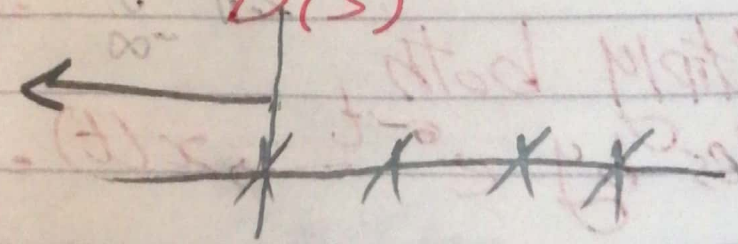





σ_R
σ_L


X(S)
right
right-must
Pole
bounded
by two
Pole
Left to
Left most
Pole

$$X(s) = \frac{N(s)}{D(s)}$$



$$FT \qquad \mathcal{L}T\Big|_{\boxed{\sigma=0}} \quad s=j\omega$$

$$\underbrace{\int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt} \qquad \underbrace{\int_{-\infty}^{+\infty} x(t)\, e^{-st}\, dt}$$

$$FT\{x(t)\} \quad s=\sigma+j\omega \text{ at } \sigma=0 \qquad \int_{-\infty}^{+\infty} x(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt$$

$$\Downarrow$$

$$FT \equiv \mathcal{L}T\Big|_{\sigma=0} \qquad FT\{x(t)\, e^{-\sigma t}\}$$

$$X(s) = \mathcal{L}T \equiv FT\{x(t)\, e^{-\sigma t}\}$$

$$x(t)\, e^{-\sigma t} \triangleq \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\underbrace{s}_{=\sigma+j\omega})\, e^{j\omega t}\, d\omega$$

Multiply both Sides by $e^{\sigma t}$ $\longrightarrow$ $x(t) = \dfrac{1}{2\pi} \displaystyle\int X(s)\, e^{j\omega t}\, e^{\sigma t}\, d\omega$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(s)\, e^{(\sigma+j\omega t)}\, d\omega$$

$$\therefore X(t) = \frac{1}{2\pi \boxed{j}} \int_{-\infty}^{+\infty} X(s)\, e^{st}\, \cancel{d\omega}\ ds$$

$s = \sigma + j\omega$

$ds = j\,d\omega$

$d\omega = \frac{ds}{j}$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} x(s) e^{st}\, dt$$

$$X(s) = \sum_{i=1}^{m} \frac{A_i}{s + a_i}$$

Ex 1   Ex 2

$$X(s) = \frac{N(s)}{D(s)}$$

Proper. fraction

D.O N < D.O D

Digrey of Numonator   Digrey of denomonator