Signals and Systems

Lecture # 5

Basic Signals

Prepared by:

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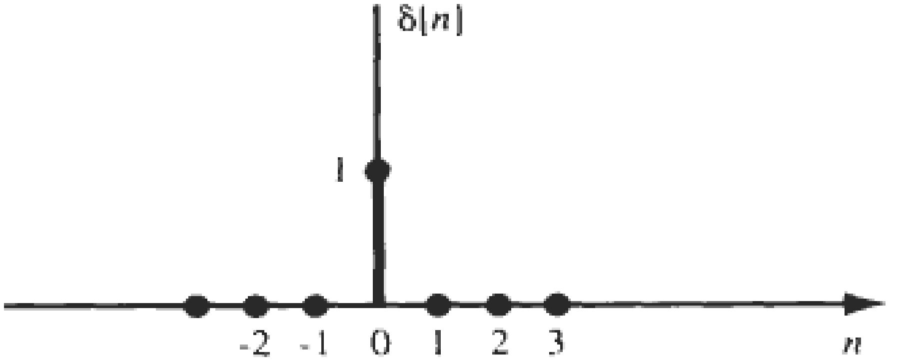
Topics of the lecture:

- **Discrete-Time Unit Impulse and Unit step Signals.**
- **Continuous-Time Unit Impulse and Unit step Signals.**

➢ Basic Signals

The discrete-time unit Impulse Signal:

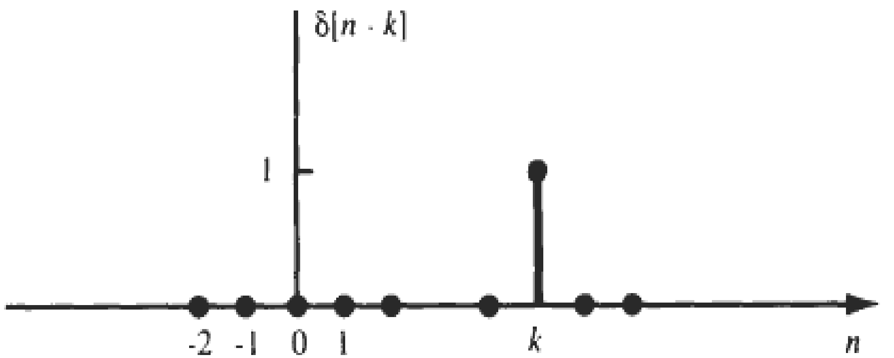
$$\delta[n] = \begin{cases} 1 & ; \quad n = 0 \\ 0 & ; \quad n \neq 0 \end{cases}$$



It is also called **unit sample** **signal**

The discrete-time shifted unit impulse Signal:

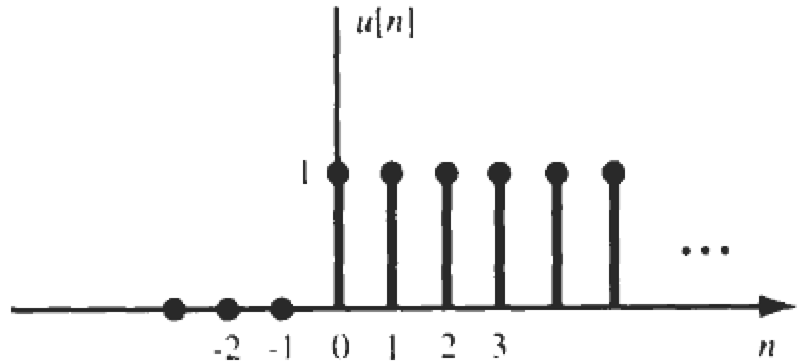
$$\delta[n-k] = \begin{cases} 1 & ; \quad n = k \\ 0 & ; \quad n \neq k \end{cases}$$



➢ Basic Signals

The discrete-time unit step Signal:

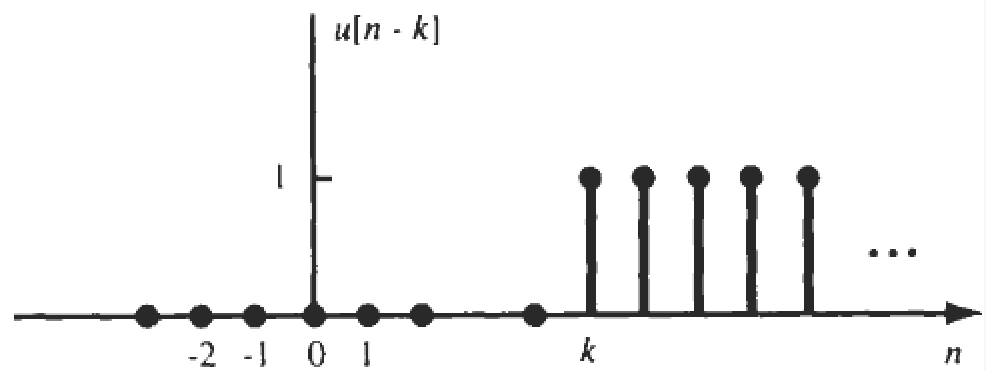
$$u[n]=\begin{cases}1 & ; \quad n\geq 0\\ 0 & ; \quad n<0\end{cases}$$



It is also called **unit sequence** **signal**

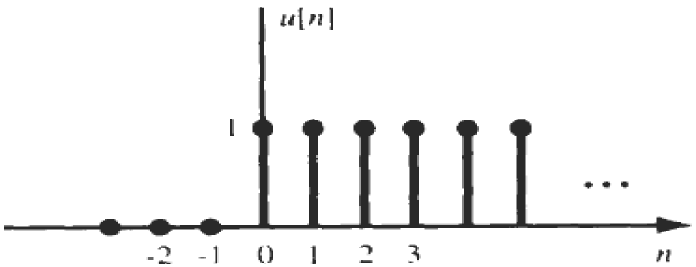
The discrete-time shifted unit step Signal:

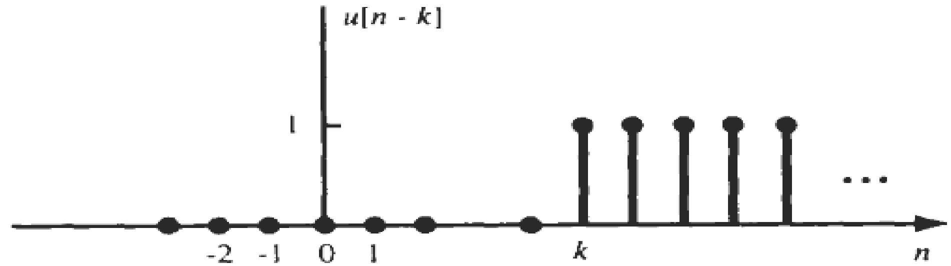
$$u[n-k]=\begin{cases}1 & ; \quad n\geq k\\ 0 & ; \quad n<k\end{cases}$$



➢ Basic Signals

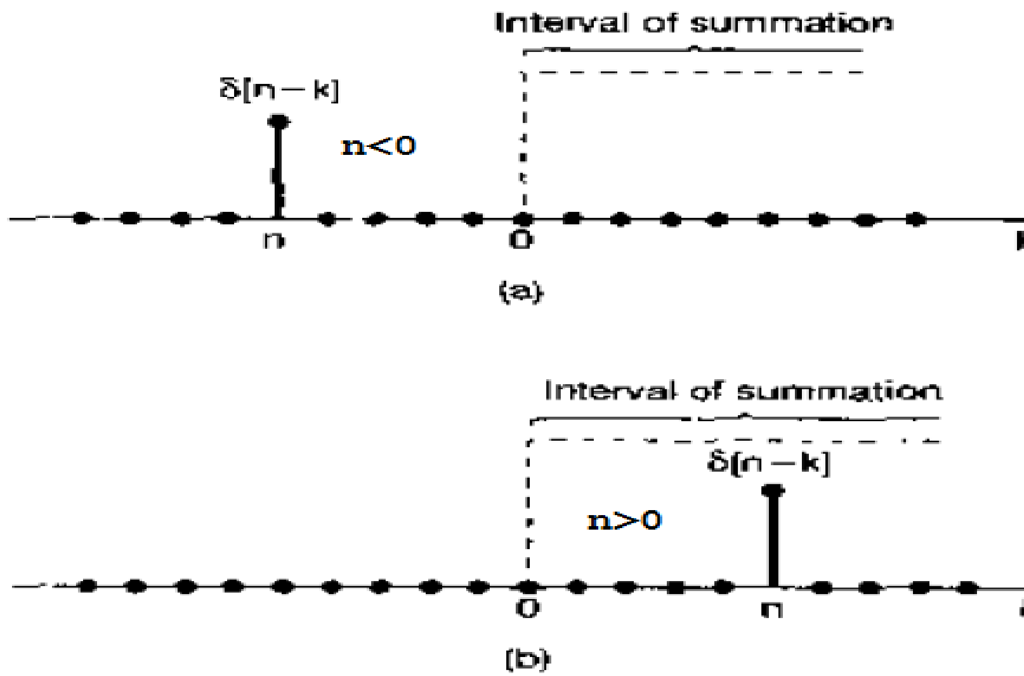
The relationship between discrete-time unit impulse and unit step signals:





$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$



Let $m = n - k$ **OR** $k = n - m$

$$\therefore \quad u[n] = \sum_{k=+\infty}^{0} \delta[n-k]$$

$$\therefore \quad u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

➢ Basic Signals

The unit impulse sequence can be used to sample the value of a signal at $n = 0$. In particular, since $\delta[n]$ is nonzero (and equal to 1) only for $n = 0$, it follows that

$$x[n]\delta[n] = x[0]\delta[n] = x[0]$$

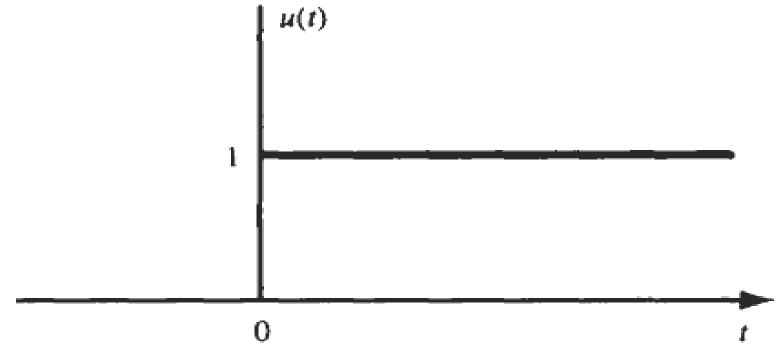
More generally, if we consider a unit impulse $\delta[n - n_0]$ at $n = n_0$, then

$$x[n]\delta[n - n_0] = x[n_0]\delta[n - n_0] = x[n_0]$$

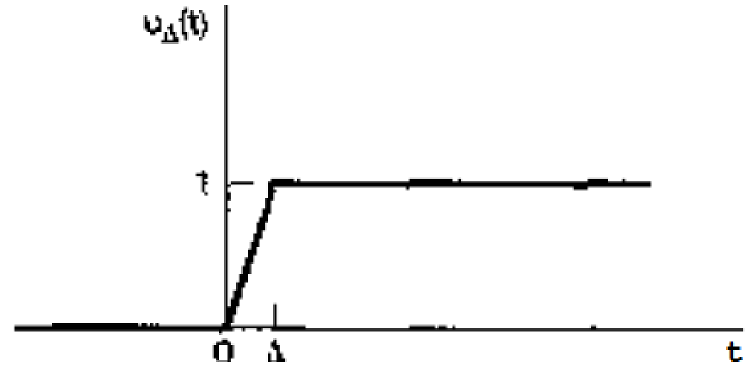
➢ Basic Signals

<u>The continuous-time unit step Signal:</u>

$$u(t) = \begin{cases} 1 & ; \quad t > 0 \\ 0 & ; \quad t < 0 \end{cases}$$



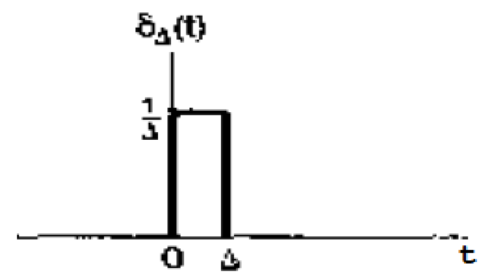
As the there is no such sudden change in real practical application. So an approximation of the ideal case is usually is what is happen.



Continuous approximation to the unit step, $u_\Delta(t)$.

Similarly to the discrete-time case the unit continuous-time impulse signal is the differentiation of the unit step signal

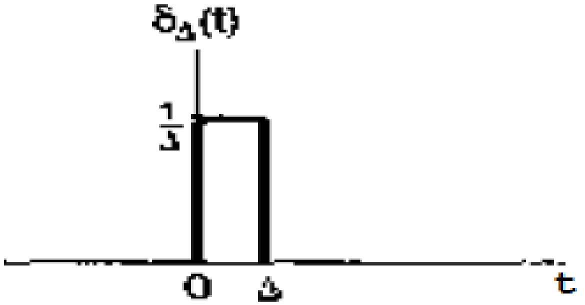
$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)$$



Derivative of $u_\Delta(t)$.

<u>The continuous-time unit impulse Signal:</u>

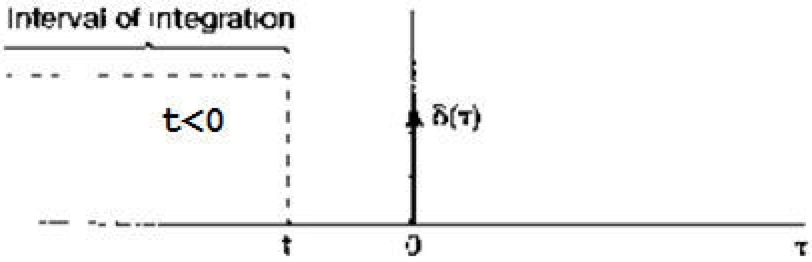
$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)$$



Derivative of $u_\Delta(t)$.

Note that $\delta_\Delta(t)$ is a short pulse of duration Δ and unit area. As the Δ becomes smaller the $\delta_\Delta(t)$ becomes narrower and higher <u>maintaining</u> the unit area. As the Δ goes to zero the $\delta_\Delta(t)$ goes to ∞

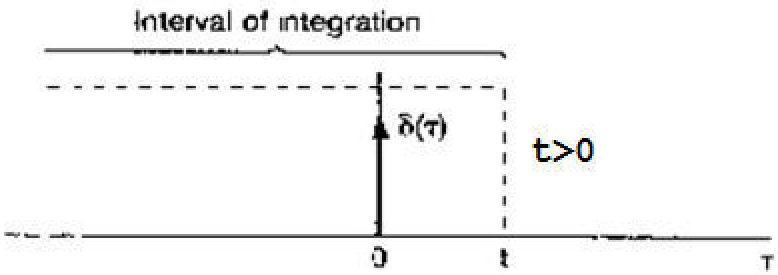
$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$



(a)

Then u(t) can be thought as a running integral of $\delta(t)$

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$



Basic Signals

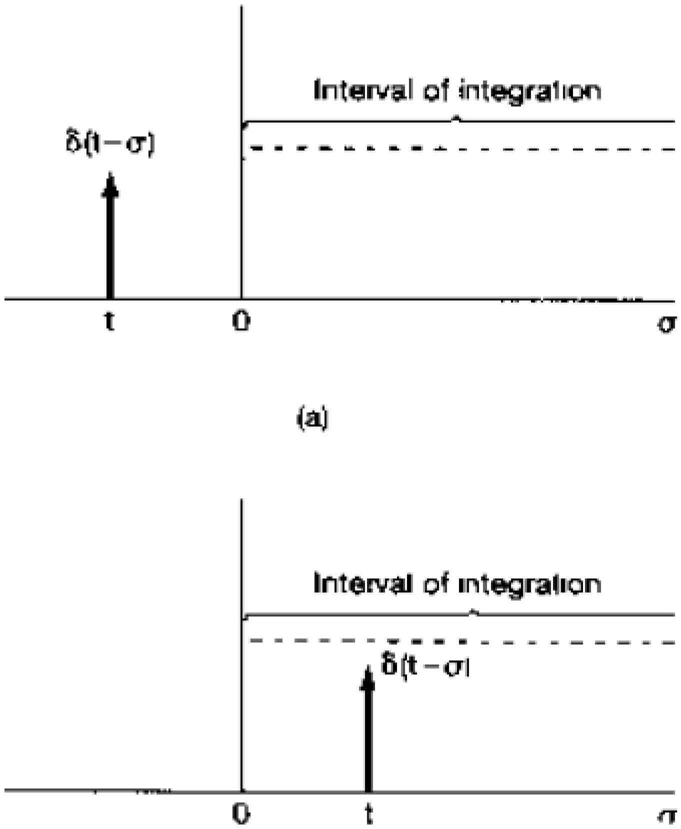
The relationship between continuous-time unit impulse and unit step signals:

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

Let $\sigma = t - \tau$ **OR** $\tau = t - \sigma$

$$u(t) = \int_{\infty}^{0} \delta(t-\sigma)(-d\sigma)$$

$$u(t) = \int_{0}^{\infty} \delta(t-\sigma)\, d\sigma$$



➢ Basic Signals

$$x_1(t) = x(t)\delta_\Delta(t).$$

In Figure (a) we have depicted the two time functions $x(t)$ and $\delta_\Delta(t)$, and in Figure (b) we see an enlarged view of the nonzero portion of their product. By construction, $x_1(t)$ is zero outside the interval $0 \leq t \leq \Delta$. For Δ sufficiently small so that $x(t)$ is approximately constant over this interval,

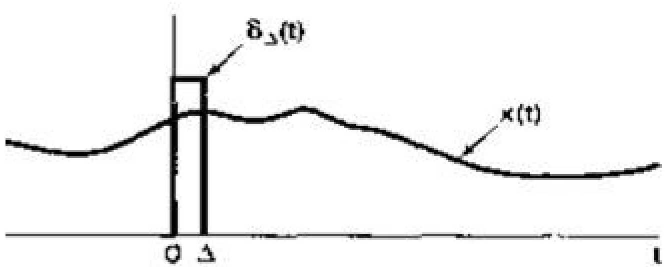
$$x(t)\delta_\Delta(t) \approx x(0)\delta_\Delta(t).$$

Since $\delta(t)$ is the limit as $\Delta \to 0$ of $\delta_\Delta(t)$, it follows that

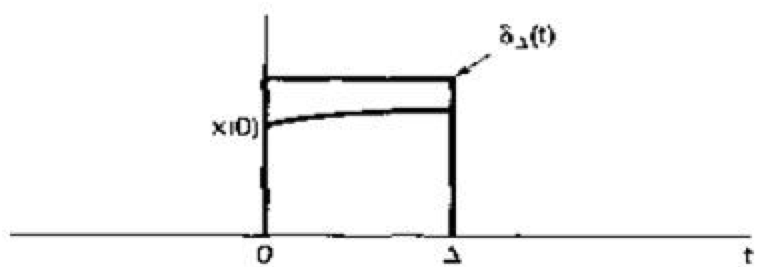
$$x(t)\delta(t) = x(0)\delta(t).$$

By the same argument, we have an analogous expression for an impulse concentrated at an arbitrary point, say, t_0. That is,

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0).$$



(a)



(b)

Signals and Systems

Lecture # 6

Systems Properties

Prepared by:

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Topics of the lecture:

- Systems Interconnections.
- Systems Properties.
  1. Memoryless
  2. Invertibility
  3. Causality
  4. Stability

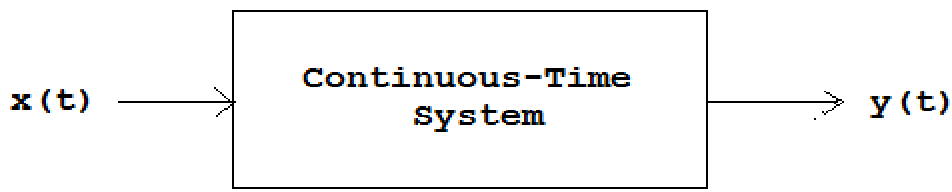
➢ Systems

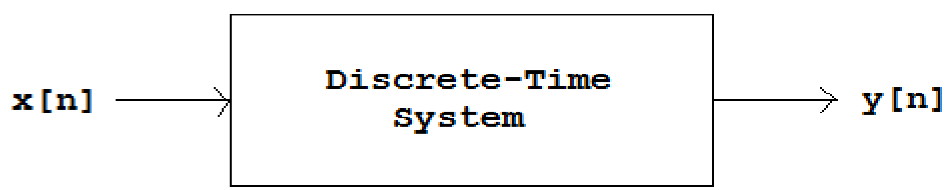
<u>Recall:</u>

The system is the tool or the process through which the input signal is used to get another output signal or make the system to act in a certain behavior.

A system may consists of physical components (Hardware Realization) or may consists of an algorithm that compute the output signal from the input signal (Software Realization).

Physical systems in the broadest sense are an interconnection of components, devices, or subsystems.

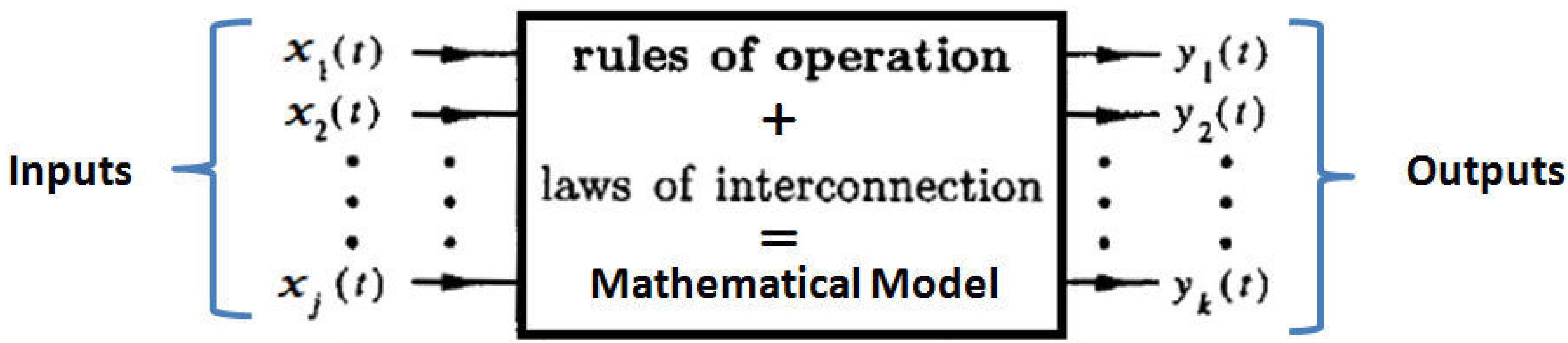




➢ Systems

A system is characterized by its **inputs**, its **outputs** (or **responses**), and the **rules of operation** (or **laws**) adequate to describe its behavior. For example, in electrical systems, the laws of operation are the familiar voltage-current relationships for the resistors, capacitors, inductors, transformers, transistors, and so on, as well as the laws of interconnection (i.e., Kirchhoff's laws). Using these laws, we derive mathematical equations relating the outputs to the inputs. These equations then represent a **mathematical model** of the system. Thus a system is characterized by its inputs, its outputs, and its mathematical model.

A system can be conveniently illustrated by a "black box" with one set of accessible terminals where the input variables $x_1(t), x_2(t), \ldots, x_j(t)$ are applied and another set of accessible terminals where the output variables $y_1(t), y_2(t), \ldots, y_k(t)$ are observed. Note that the direction of the arrows for the variables in Fig. is always from cause to effect.



➢ Systems Interconnections

Many real systems are built as interconnection of *subsystems*.

Example:

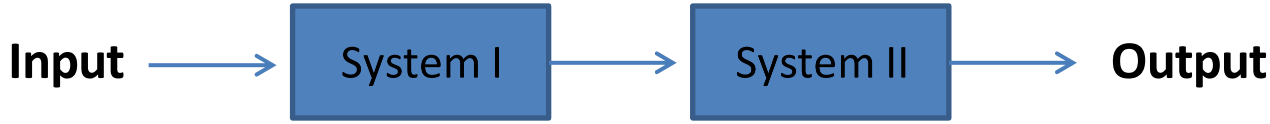
The Audio System Ξ **Receiver** **+** **Player** **+** **Amplifier** **+** **Speaker**

Viewing (through *block diagram*) a system as an interconnection of *subsystems* has the advantage of:

1- Facilitate the understanding of existing systems by understanding the function of each subsystem and how it is connected to other components.

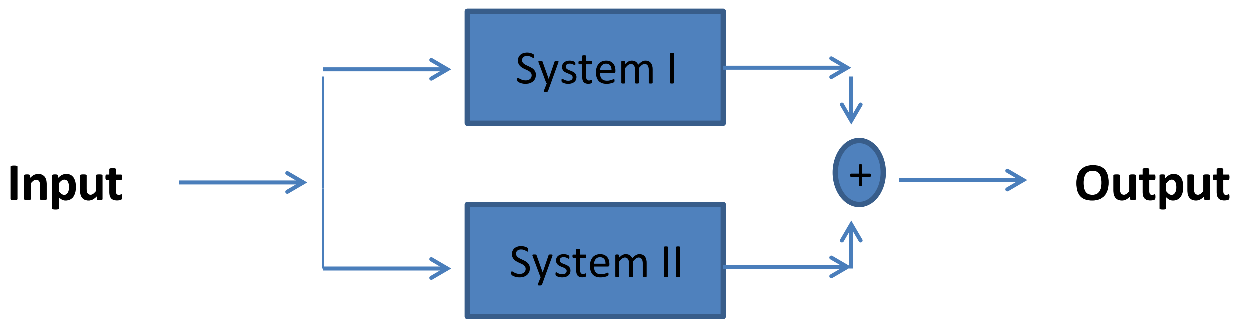
2- Help us to build complex systems

➢ Systems Interconnections



Series (cascade) Interconnection

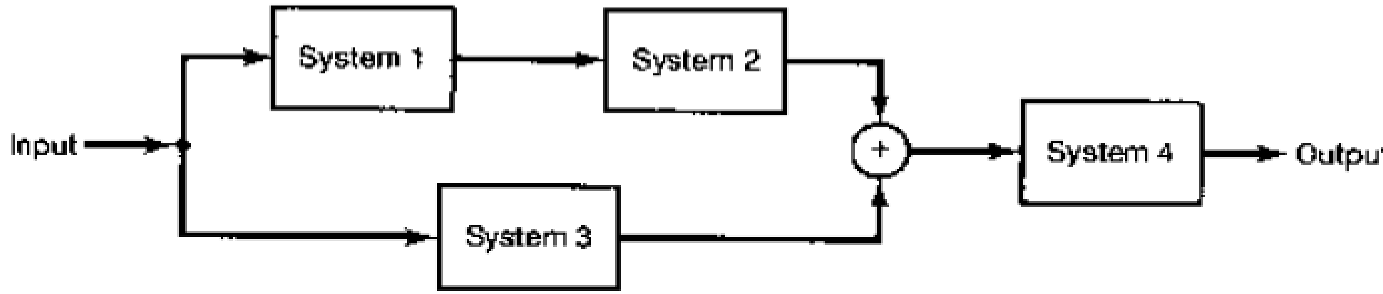
Example: **Radio Receiver** → **Amplifier**



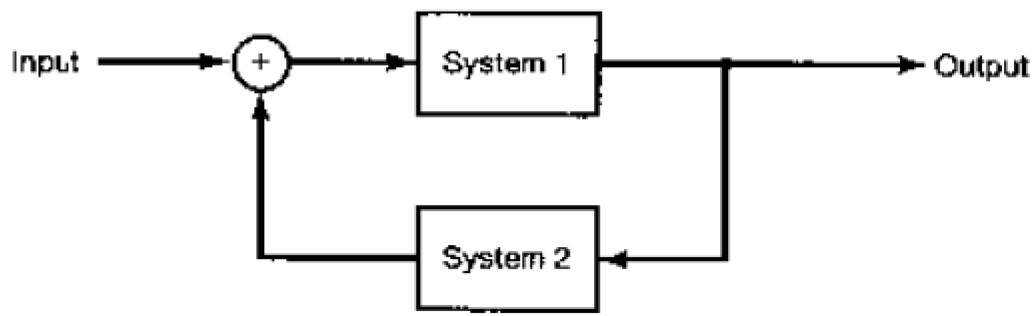
Parallel Interconnection

Example: **Mic#1**, **Mic#2** } **one Audio Stream**

➢ Systems Interconnections



Series-Parallel Interconnection



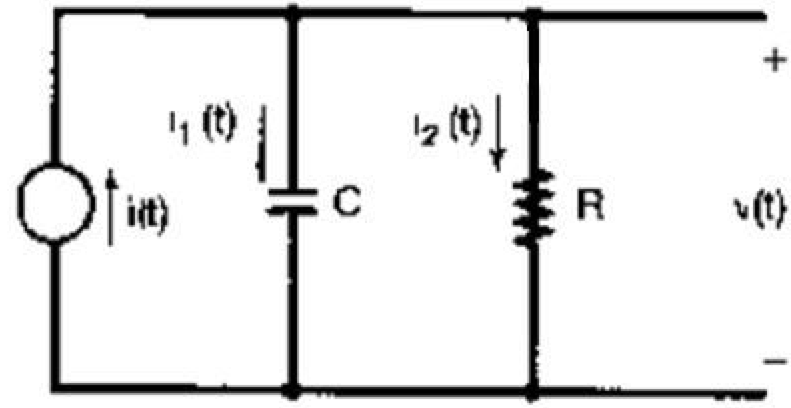
Feedback Interconnection

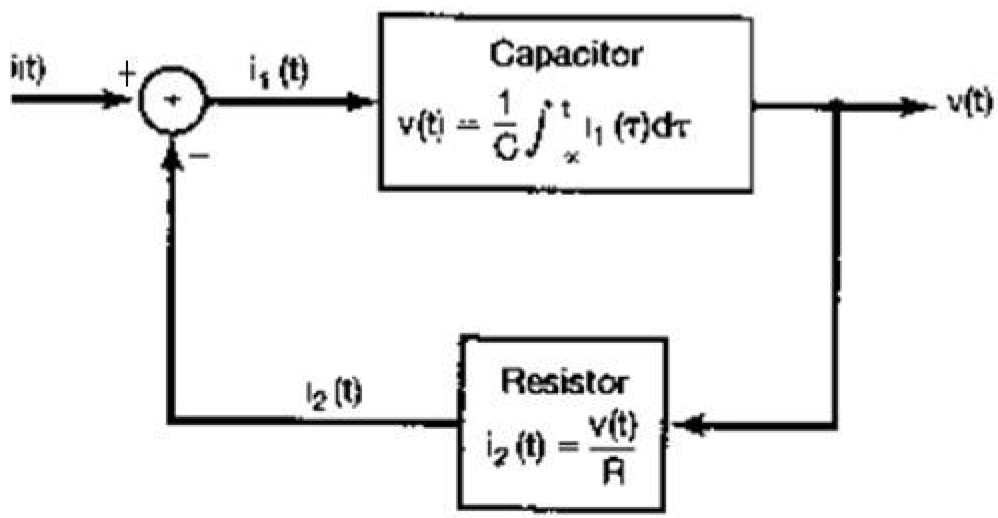
Examples:

-Fluid flow control in a car **- Autopilot and sensors in airplane**

➢ Systems Interconnections

Real Example:





Systems Properties

1- Memoryless:

The system is said to be *Memoryless* if its output at any time instant depends only on the input at the same time instant.

i.e.

$$y(t)\,\Big|_{t=t_o} \xleftarrow{\;depends\,only\,on\;} x(t)\,\Big|_{t=t_o}$$

i.e.

$CT:$

$$x(t) \xrightarrow{\;S\;} y(t)$$

$$y(t) = S\big(x(t)\big) = function \quad in\ x(t)$$

$DT:$

$$x[n] \xrightarrow{\;S\;} y[n]$$

$$y[n] = S\big(x[n]\big) = function \quad in\ x[n]$$

Examples:

1-

$$y[n] = \left(2x[n] + x^2[n]\right)^2$$

This system is *Memoryless* as the output at any time n_0 is a function of the input at that time instant only and there is no need for memory.

2- The voltage (v) across the resistor (R) depends on the electrical current passing through it at the same time instant (i):

$$v(t) = R.i(t)$$

This system is also *Memoryless*

➢ Systems Properties

1- Memoryless:

An example of a discrete-time system with memory is an *accumulator* or *summer*

$$y[n] = \sum_{k=-\infty}^{n} x[k],$$

and a second example is a *delay*

$$y[n] = x[n-1].$$

A capacitor is an example of a continuous-time system with memory, since if the input is taken to be the current and the output is the voltage, then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau,$$

where C is the capacitance.

Roughly speaking, the concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times other than the current time.

In many physical systems, memory is directly associated with the storage of energy. For example, the capacitor stores energy by accumulating electrical charge.

➢ Systems Properties

1- Memoryless:

A useful strategy to check the memory property of a system is to use the counter example technique. To find this counter example, if any, check for different values representing or covering the whole real number scale.
You can use the 7 values:

($t<-1$), ($t=-1$), ($-1<t<0$), ($t=0$), ($0<t<1$), ($t=1$), ($t>1$)

And do not miss a complex values if applicable like (j), (-j)
Also take care of (π), ($\pi/4$), ($\pi/2$), ($3\pi/4$), ($3\pi/2$) if the system consider angles.

Check- $y(t) = x(t)\cos(t+3)$ **is it *Memoryless?***

Note I*: *Memoryless system does not contain time-shift, time-reverse, nor time scaling for the input signal *{may exist in the multiplying gain}*. ***So, check if any one of them exist first, if exist then the system is not memoryless.***

***Note II*:** **do not be fooled by mathematical symbols like summation and integration as they need memory.**

➢ Systems Properties

2- Invertibility:

The system is said to be invertible if distinct values of the input lead to distinct values of the output.

It has a reverse system, when cascaded together with its origin system gives us the identity system. i.e. the output of the two systems connected in series is the input itself $x(t) \rightarrow \boxed{H} \rightarrow y(t) \rightarrow \boxed{H^{-1}} \rightarrow x(t)$

Examples of invertible systems:

1- y (t) = 2 x (t) **where the inverse is** **w (t) = ½ y (t)**

2- $y[n] = \sum_{k=-\infty}^{n} x[k]$ **where the inverse is** **w[n] = y[n] - y[n-1]**

Examples of noninvertible systems:

1- y (t) = 0

2- y[n] = $x^2[n]$

- The strategy of trying to find { ***two inputs have the same output*** **} could be used to verify that a system is {** ***not*** **} invertible.**

Invertibility is important in applications like data encoding for secure systems