Subject :

Date / /201

موضوع الدرس :

التاريخ : / / ٢٠١

Lec. 20

16/12

$x(t)$ Periodic with Period T

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$

Subject:

Date: / /201

+ve

Ex 3:-

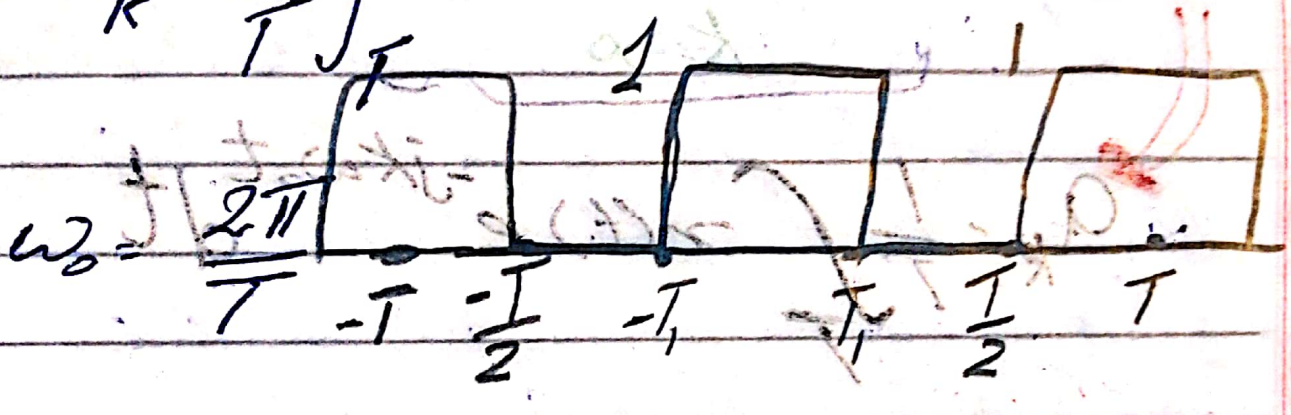
Periodic $x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < \frac{T}{2} \end{cases}$ with Period $= T$

$|t| < T_1$, $-t < T_1$, $t > -T_1$

$a_k = ??$ $(-T_1 > t > -\frac{T}{2})$ $(T_1 < t < \frac{T}{2})$ +ve

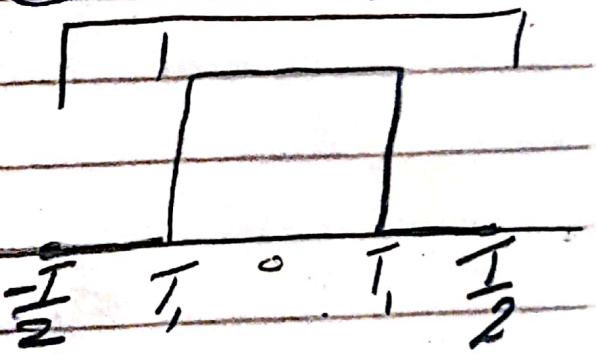
-ve

$$\rightarrow a_k = \frac{1}{T} \int_{T} x(t)\, e^{-jk\omega_0 t}\, dt$$



Period $-\frac{T}{2}$ $\frac{T}{2}$

$$a_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-jk\omega_0 t}\, dt$$



موضوع الدرس :

Subject :

التاريخ : / / ٢٠١

Date : / / 201

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Big|_{-T_1}^{T_1}\right]$$

$$= \frac{-1}{jk\omega_0 T}\left[e^{-jk\omega_0 T_1} - e^{+jk\omega_0(+T_1)}\right]$$

$$a_k = \frac{1}{jk\omega_0 T}\left[e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}\right]$$

$\theta = k\omega_0 T_1$

$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$

$\omega_0 = \frac{2\pi}{T}$ ، $T = \frac{2\pi}{\omega_0}$

$$= \frac{2j\sin(k\omega_0 T_1)}{jk\omega_0 \frac{2\pi}{\omega_0}}$$

$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$$

OR

$$= \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T}$$

Subject :

Date : / /201

موضوع الدرس :

التاريخ : / /

$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} \quad ; \quad a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T}$$

$k \neq 0$

$$Ta_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0}$$

if $k = 0$

$$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} x(t)\,dt = \frac{2\sin(\omega T_1)}{k\omega_0}\Big|_{\omega = k\omega_0}$$

$$= \frac{2T_1}{T}$$

$$Ta_k = \frac{2\sin(\omega T_1)}{\omega}\Big|_{\omega = k\omega_0}$$

$\frac{1}{\pi} = \frac{1}{3.14}$

Subject:
Date: / /201

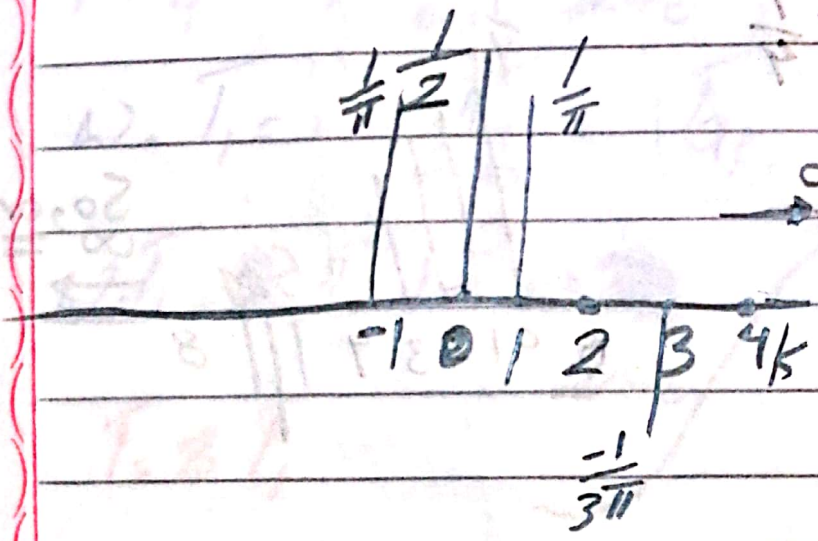
موضوع الدرس:
التاريخ: / /

$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$$

$T = 4T_1$

$$a_0 = \frac{2T_1}{T}$$

$$= \frac{2T_1}{4T_1} = \frac{1}{2}$$



$$T_1 = \frac{T}{4}$$

$$k\omega_0 T_1 = k\omega_0 \frac{1}{4} \cdot \frac{2\pi}{\omega_0}$$

$$= k \cdot \frac{1}{2} \cdot \pi$$

$$k\omega_0 T_1 = k \cdot \frac{\pi}{2}$$

one cycle

4-Point

$$\frac{\sin(k \cdot \frac{\pi}{2})}{k\pi}$$

Subject:

Date: / / 201

$T = 8T_1$

$a_0 = \frac{2}{T} = \frac{2T_1}{8T_1} = \frac{1}{4}$

$T_1 = \frac{T}{8} = \frac{1}{48} \cdot \frac{2\pi}{\omega_0}$

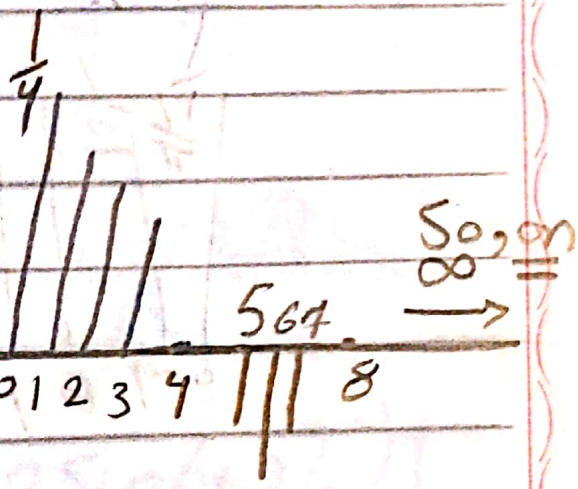
$= \frac{\pi}{4\omega_0}$

$k\omega_0 T_1 = k \cdot \omega_0 \cdot \frac{\pi}{4\omega_0}$

$= k\frac{\pi}{4} \equiv$ one cycle

$= 8$-Point

$\frac{\sin(k \cdot \frac{\pi}{4})}{k\pi}$



$\frac{\sin(k\omega_0 T_1)}{k\pi}$

موضوع الدرس:

Subject:

التاريخ: / / ٢٠١

Date: / /201

$T = 4T_1$

$T = 16T_1$

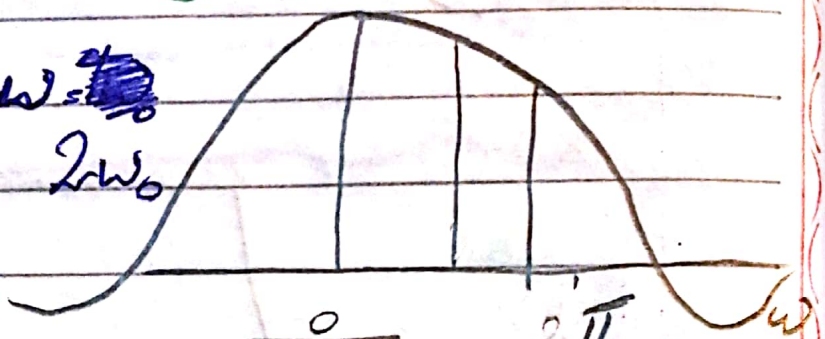
$\omega T_1 =$

$T_1 = \frac{T}{4} = \frac{1}{4}\frac{2\pi}{\omega_0} = \frac{\pi}{2\omega_0}$

$\omega \cdot T_1 = \omega \cdot \frac{\pi}{2\omega_0}$

T_{a_k}

$\omega \cdot T_1 = \pi$ if $\omega = 2\omega_0$



$T = 8T_1$

T_{a_k}

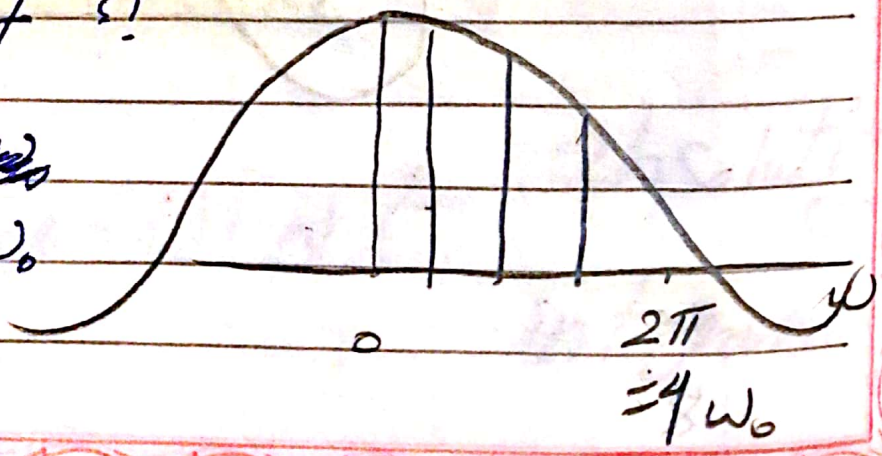
$T_1 = \frac{1}{8}T = \frac{1}{4}\frac{1}{8}\cdot\frac{2\pi}{\omega_0}$

$= \frac{\pi}{4\omega_0}$

$\omega \cdot T_1 = \omega \cdot \frac{\pi}{4\omega_0}$

$\omega \cdot T_1 = \pi$ if ??

$\omega = 4\omega_0$



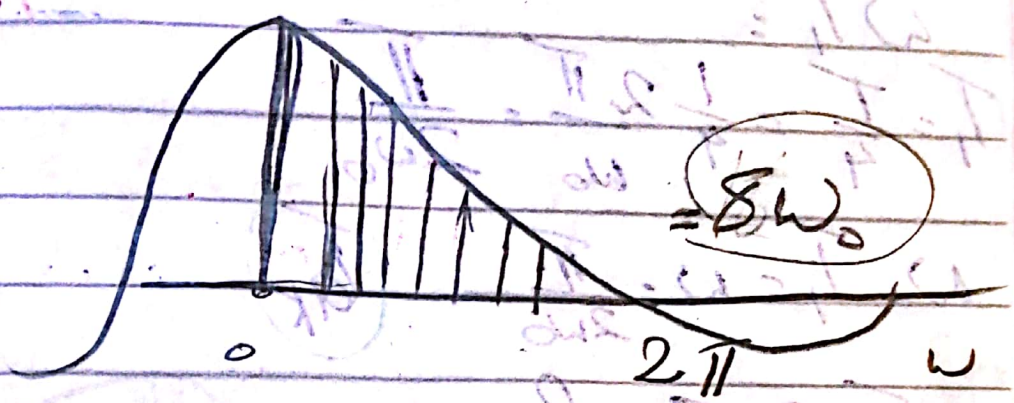
Subject :

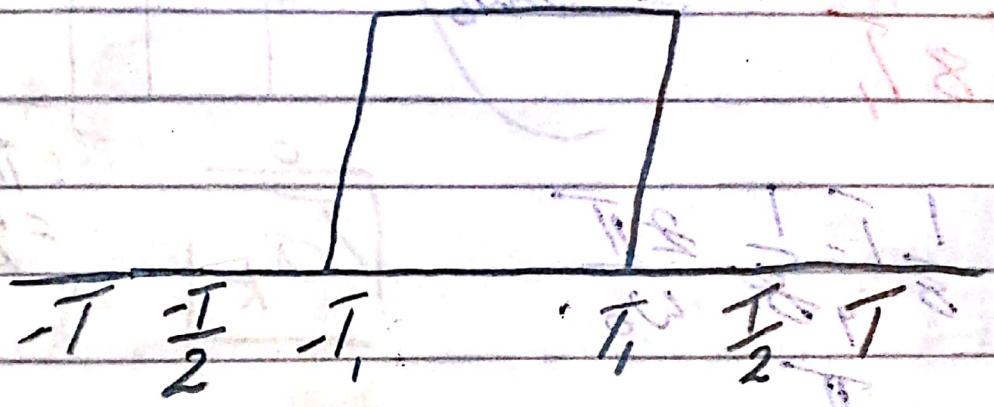
Date : / /201

موضوع البحث :

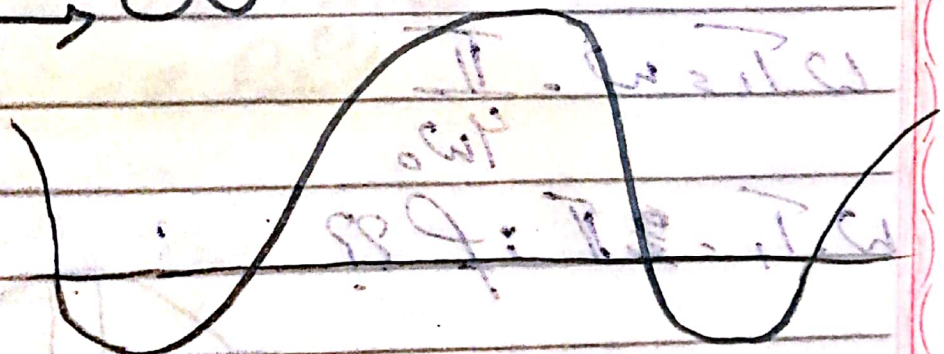
التاريخ : / /٢٠١

$T = 16 T_1$





$T \to \infty$



Subject :

Date : / / 201

موضوع الدرس :

التاريخ : / /

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\, dt$$

$$|x(t)| e^{j\angle x}$$

$$z = |z| e^{j\angle z}$$

$$x = \frac{1}{\tan\left(\frac{\pi}{2}\right)}\Big|_{t=0}^{\infty}$$

$|x(t)| < \infty \quad \forall t$

Convergance of FS

$|x(t)| < \infty \quad \forall t$

$\int_T |x(t)|\, dt$ absolutly integrable

Subject :

Date : / /

موضوع الدرس :

التاريخ : / /

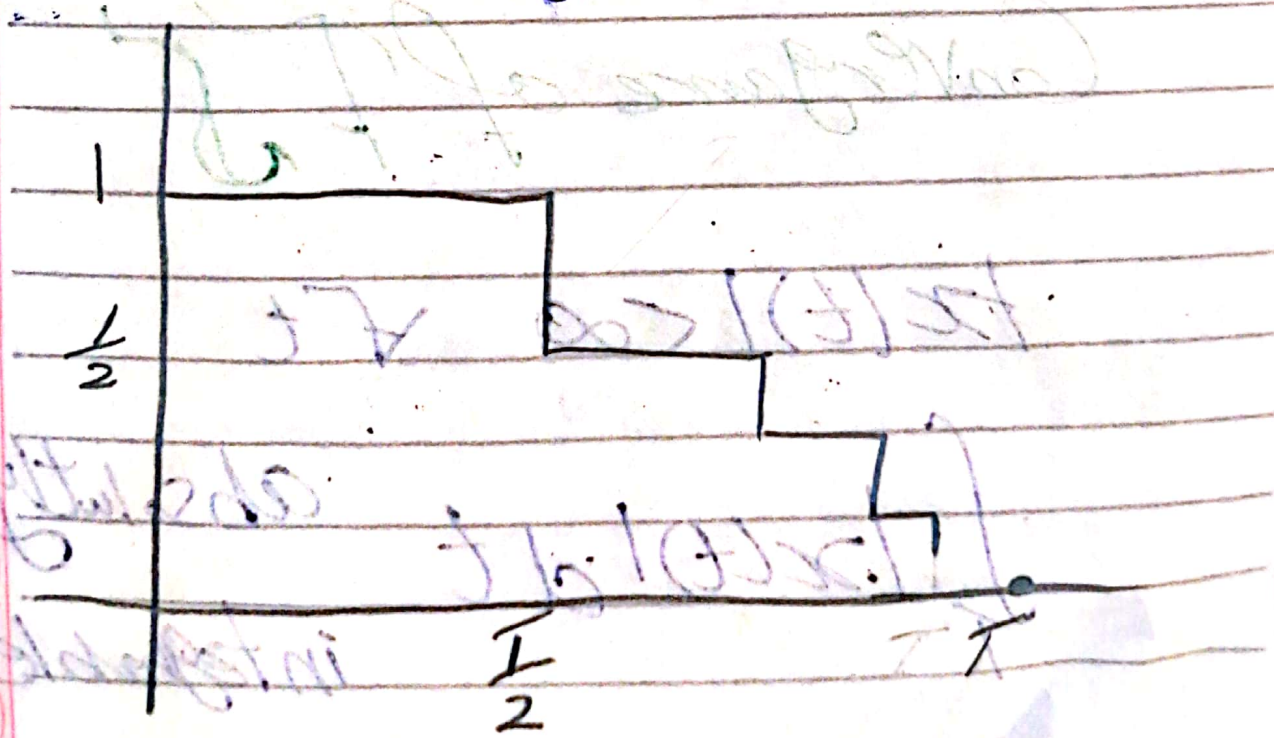
Dirichlet

1. $\int |f| < \infty$

Counter example

2. finite number of Local Maximam and Local minimam $\sin\left(\frac{2\pi}{t}\right)$

3. finite number of discontinuity



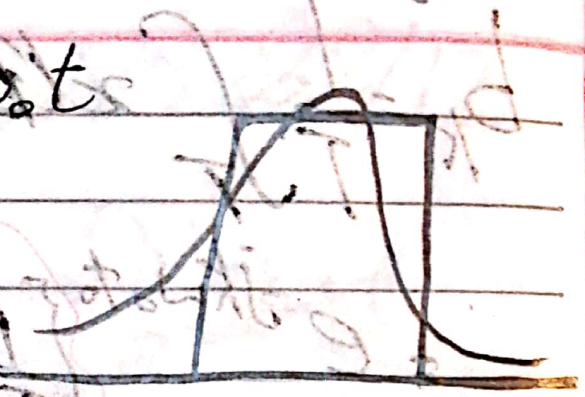
Subject :

موضوع الدرس :

Date :

التاريخ :

$$x(t) = \sum_{k=-N}^{+N} a_k e^{jk\omega_0 t}$$



F.S Properties

Time-shift

$$x(t) \xleftrightarrow{FS} a_k$$

$$y(t) = x(t-t_0) \xleftrightarrow{FS} b_k$$

$$b_k = \frac{1}{T}\int_T y(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_T x(t-t_0)\, e^{-jk\omega_0 t}\, dt$$

Let $\tilde{T} = t - t_0 \Longrightarrow t = \tilde{T} + t_0 \Longrightarrow dt = d\tilde{T}$

$$b_k = \frac{1}{T}\int_T x(\tilde{T})\, e^{-jk\omega_0(\tilde{T}+t_0)}\, d\tilde{T}$$

Subject :

Date : / /201

$$b_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, e^{-jk\omega_0 t_0}\, dt$$

$$= e^{-jk\omega_0 t_0}\left\{\frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt\right\}$$

$$\therefore b_k = e^{-jk\omega_0 t_0}\cdot a_k$$

$$= e^{-jk\omega_0 t_0}\,|a_k|\, e^{j\angle a_k}$$