Lec-8. (1/11)

Linearity:

Super Position Property

A- Additive Property

$x_1 \rightarrow y_1$

$x_2 \rightarrow y_2$

$x_1 + x_2 \rightarrow y_1 + y_2$

B- Scaling / Homogenity property

$x_1 \rightarrow y_1$

$a\,x_1 \rightarrow a\,y_1$

$x_1 \rightarrow y_1$

$x_2 \rightarrow y_2$

$a x_1 \pm b x_2 \rightarrow a y_1 \pm b y_2$

$y = f(x)$ sys. equ.

output Input

Let $x_1 \xrightarrow{S} y_1$

and let $x_2 \xrightarrow{S} y_2$

and let $x_3 = a x_1 + b x_2 \xrightarrow{S} a y_3$ $\cancel{+ a y_2}$

$y_3 \overset{??}{=} a y_1 \pm b y_2$

Ex 1 $\boxed{y(t) = t\, x(t)}$ sys. equ.

Linear??

Let $x_1(t) \xrightarrow{S} y_1(t) = t\, x_1(t) \rightarrow (1)$

$x_2(t) \xrightarrow{S} y_2(t) = t\, x_2(t) \rightarrow (2)$

and let $x_3(t) \longrightarrow y_3(t) = t\, \underbrace{x_3(t)} \rightarrow (3)$

$= \underbrace{a x_1(t) + b x_2(t)}$

$y_3(t) = t\,(a x_1(t) + b x_2(t))$

$= a t\, x_1(t) + b t\, x_2(t)$

from (1) and (2)

$\therefore y_3(t) = a y_1 + b y_2$

$\therefore$ The is Linear system.

Ex 2 $y[n] = Re\{x[n]\}$

Let $x_1[n] \xrightarrow{\text{ك}} y_1[n] = Re\{x_1[n]\} \to ①$

Let $x_2[n] \xrightarrow{\text{ك}} y_2[n] = Re\{x_2[n]\} \to ②$

Let $x_3[n] = a x_1[n] + b x_2[n] \longrightarrow y_3[n] =$

$= Re\{\underbrace{x_3[n]}\} ③$

$\therefore y_3[n] = Re\{a x_1[n] + b x_2[n]\}$

a & b are both real

a & b = j

Linear

Non Linear

Recall

$z = m + jk$

$Re\{z\} = m$

$j z = jm - k$

$= \{-k + jm\}$

$Real\{j z\} = \boxed{-k} = -Im\{z\}.$

Case I - a & b are both real.

$y_3[n] = Re\{a x_1[n]\} + Re\{b x_2[n]\}$

$= a Re\{x_1[n]\} + b Re\{x_2[n]\}$

$= a y_1 + b y_2$

$\therefore$ The system is Linear.

Case III :- $a = j$ or $b = j$
& b is real & a is real.

$$y_3[n] = Re\{a x_1[n] + b x_2[n]\}.$$

$$= Re\{\underbrace{\overset{\swarrow j}{a} x_1[n]}\} + Re\{b x_2[n]\}$$

$$= -Im\{x_1[n]\} + b Re\{x_2[n]\}.$$

$$\neq a\, y_1[n] + b\, y_2[n].$$

∴ The system is Non-Linear.

Not Linear.

Chapter 2

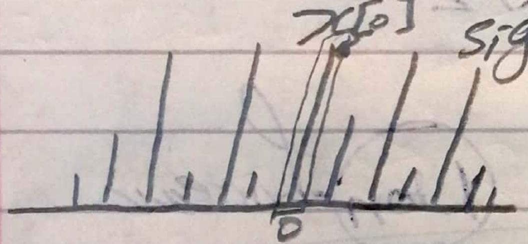
LTI

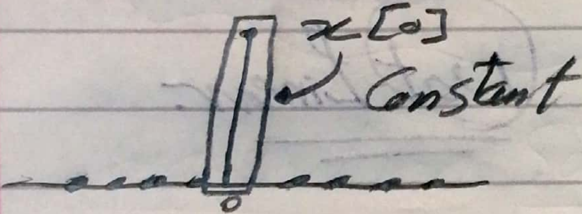
Linear

Time-Invariant

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise.} \end{cases}$$

$$x[n] \cdot \delta[n] = x[0]\,\delta[n] = x[0].$$



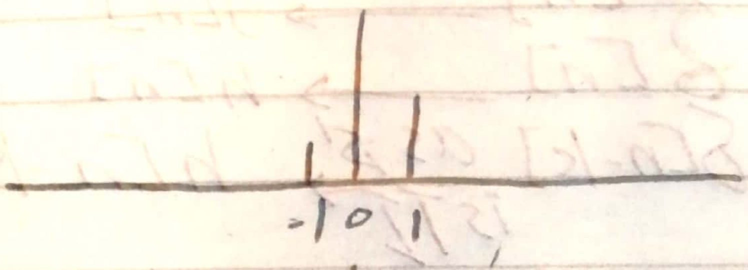


Discreat time system (LTI)

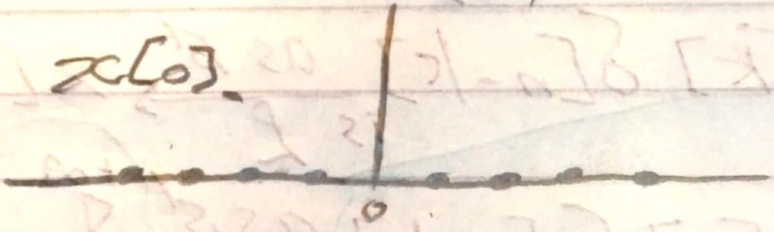
$$x[n]\,\delta[n] = x[0]\,\delta[n] = x[0]$$

$$x[n]\,\delta[n-1] = x[1]\,\delta[n-1] = x[1].$$

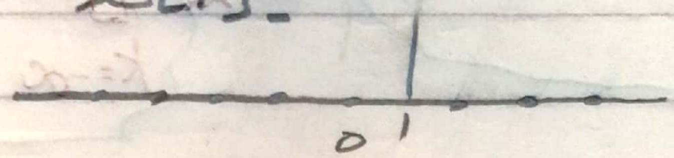
$$x[n]\,\delta[n+1] = x[-1]\,\delta[n+1] = x[-1].$$



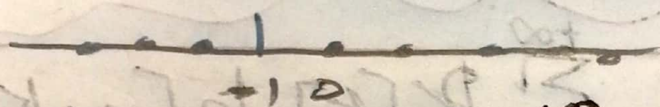
$x[0]$.



$x[1]$.



$x[-1]$.



$$x[n]\,\delta[n-k] = x[k]. \quad \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k] = \boxed{x[n]}$$

impulse $x \longrightarrow y$

$\delta[n] \longrightarrow$ impulse Response $h[n]$

$$\delta[n-k] \xrightarrow[TI]{as\ S} h[n-k].$$

$$\sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k] \xrightarrow[L]{as\ S} \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k].$$

$$x[n] \longrightarrow y[n]$$

$$\delta[n] \longrightarrow h[n]$$

$$\delta[n-k] \xrightarrow[\text{is TI}]{\text{as } S} h[n-k]$$

$$x[k]\,\delta[n-k] \xrightarrow[\text{is L}]{\text{as } S} x[k]\,h[n-k].$$

$$\sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k] \xrightarrow[\text{is L}]{\text{as } S} \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k].$$

$$\therefore\ x[n] \xrightarrow{S} y[n].$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k].$$

Convolution Sum formula.

$$y[n] = 5x[n]+3$$

$$h[n] = 5\delta[n]+3.$$