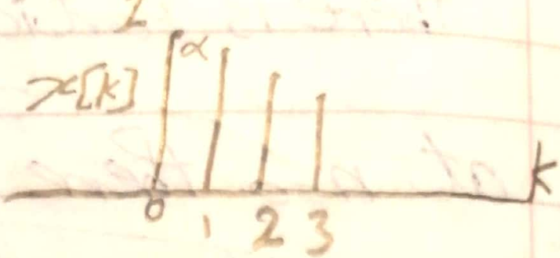
Q1: $x[n] = \alpha^n u[n]$; $0 < \alpha < 1$.

$h[n] = u[n]$.

- $y[n] = x[n] * h[n]$

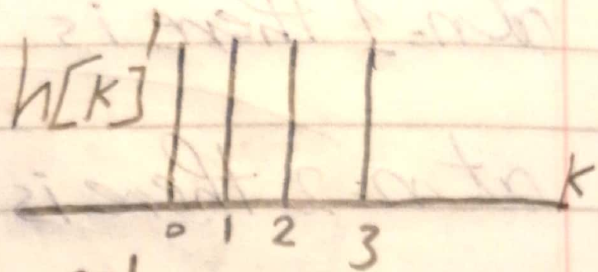
  Convolution.



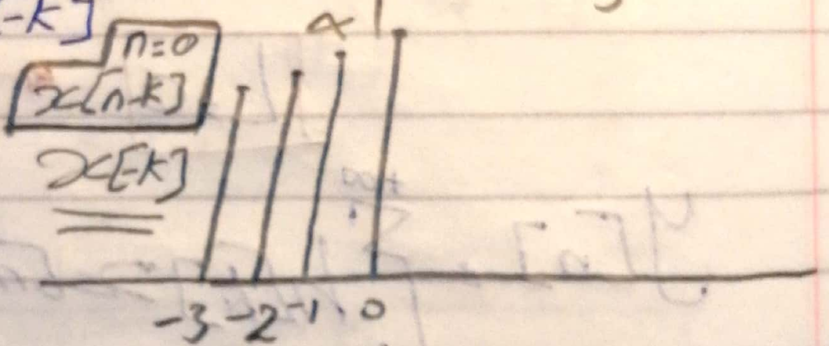
$\boxed{0}$ Draw

$\boxed{1}$ Renaming

$\boxed{2}$ Time-Reverse for $x[k]$



- get $x[-k]$



- for $n < 0$

there is No overlap between $h[k]$ and $x[n-k]$

$\boxed{y[n] = 0 \; ; \; n < 0}$ → (1)

- for $n \geq 0$

there is Partial overlap

at $n=0$ there is overlap From 0 to $\boxed{0=n}$

at $n=1$ there is Overlap From 0 to $\boxed{1=n}$

at $n=2$ there is over lap From 0 to $\boxed{2=n}$

$\boxed{L=0}$ $\boxed{u=n}$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]$$

$$= \sum_{k=L}^{u} h[k]\, x[n-k].$$

$$= \sum_{k=0}^{n} u[k]\, \alpha^{n-k}\, u[n-k]$$

for $n \geq 0$ $\;=1 \qquad =1$

$$\therefore y[n] = \sum_{k=0}^{n} \alpha^{n-k} = \alpha^{n} \cdot \sum_{k=0}^{n} \alpha^{-k}$$

$\alpha^{n} \cdot \alpha^{-k}$

$$\therefore y[n] = \alpha^n \sum_{k=0}^{n} \left(\frac{1}{\alpha}\right)^k = \alpha^n \left[\left(\frac{1}{\alpha}\right)^0 \cdot \frac{1-\left(\frac{1}{\alpha}\right)^{n-0+1}}{1-\frac{1}{\alpha}} \right]$$

$$= \alpha^n \left[\frac{1-\alpha^{-n-1}}{1-\alpha^{-1}} \right]$$

Recall

$$\sum_{k=a}^{b} r^k = r^a \cdot \frac{1-r^{b-a+1}}{1-r}$$

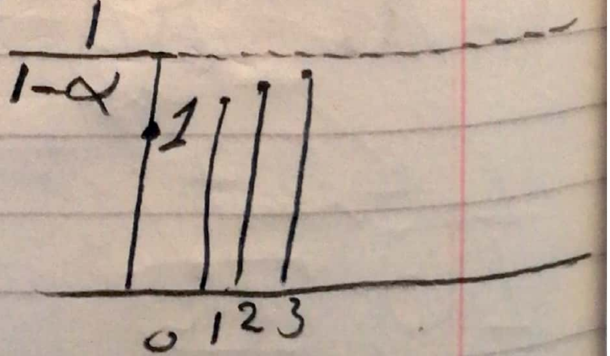
$$= \frac{\alpha^n - \alpha^{-1}}{1-\alpha^{-1}} \cdot \frac{\alpha}{\alpha} = \frac{\alpha^{n+1}-1}{\alpha-1}$$

$$= \frac{\cancel{-}(1-\alpha^{n+1})}{\cancel{-}(1-\alpha)}$$

for $n \geq 0$

$$\boxed{y[n] = \frac{1-\alpha^{n+1}}{1-\alpha} \; ; \; n \geq 0} \longrightarrow (2)$$

$$\therefore y[n] = \begin{cases} 0 & ; n < 0 \\ \dfrac{1-\alpha^{n+1}}{1-\alpha} & ; n \geq 0 \end{cases}$$



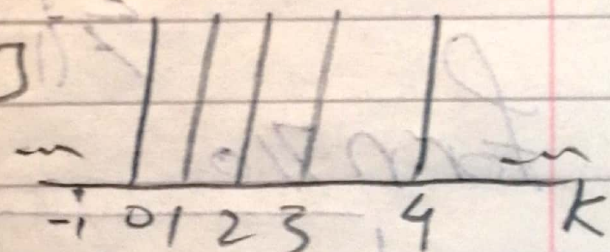
Q2

$$x[n] = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & O.W. \end{cases}$$

$$h[n] = \begin{cases} \alpha^n & 0 \le n \le 6 \\ 0 & O.W. \end{cases}$$

$\alpha > 1$

$x[k]$



$h[n-k]$