

Signals and Systems

Lecture # 1

Introduction and Complex Numbers Review

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Topics of the lecture:

- **Course Outlines.**
- **Introduction.**
- **Review of Complex Numbers.**
 - **Imaginary Number.**
 - **Complex Numbers Representations.**
 - **Complex Conjugate and its Properties.**
 - **Complex Number Magnitude and its Properties.**
 - **Complex Numbers Mathematical Operations**

➤ Course Outline.

- The Course will be divided into Two parts:
 - Part I : by Dr. Mohammed Refaey
 - Part II : by Dr. Sherif Abdo
- The Course Textbooks will be:
 - 1- "Signals and Systems", 2nd Edition , 1997
A.V.Oppenheim & A.S. Willsky (Prentice Hall)
 - 2- "Signals and Systems": 2nd Edition, 2011
Edward A. Lee & Pravin Varaiya
- The Course prerequisites (**include but not limited to**):
The Calculus (especially differentiation and Integration),
The Partial Fractions, The Complex Numbers,
Trigonometry, and Differential Equations ... etc
- The Course Total Degrees is 100:
 - 60 % Final Exam.
 - 20 % Midterm Exam.
 - 20 % Two Quizzes and Assignments.

➤ Introduction

Are you interested to study Signals and Systems?

Do not answer now!

Let us first know some applications of signals and Systems:

- 1- **Communications.** (e.g. the internet carrier signal, the mobile communications...etc)
- 2- **Aeronautics** . علوم الطيران (Study, design, and manufacturing of air flight-capable machines.)
- 3- **Astronautics** . الملاحة الفضائية (The science and technology of space flight.)
- 4- **Circuit Design.** (Design and test electrical circuits.)
- 5- **Acoustics** . الصوتيات (e.g. how to make the sound clear and effective to the audience)
- 6- **Seismology** . علم الزلازل (e.g. detecting earthquakes.)
- 7- **Biomedical Engineering** . الهندسة الطبية (e.g. design of medical imaging devices.)
- 8- **Energy Generation and Distribution Systems.** (e.g. Microwaves and Heaters.)
- 9- **Chemical Process Control.** (e.g. Adjusting the mix parameters according to sensors.)
- 10- **Speech Processing.** (e.g. Speaker Identification, Text to Speech, Speech Recognition.)
- 11- **Image Processing.** (e.g. Image restoration and enhancement...etc)

➤ Introduction

Are you involved? No

OK, let us **see** some examples:



Aeronautics example
صناعة

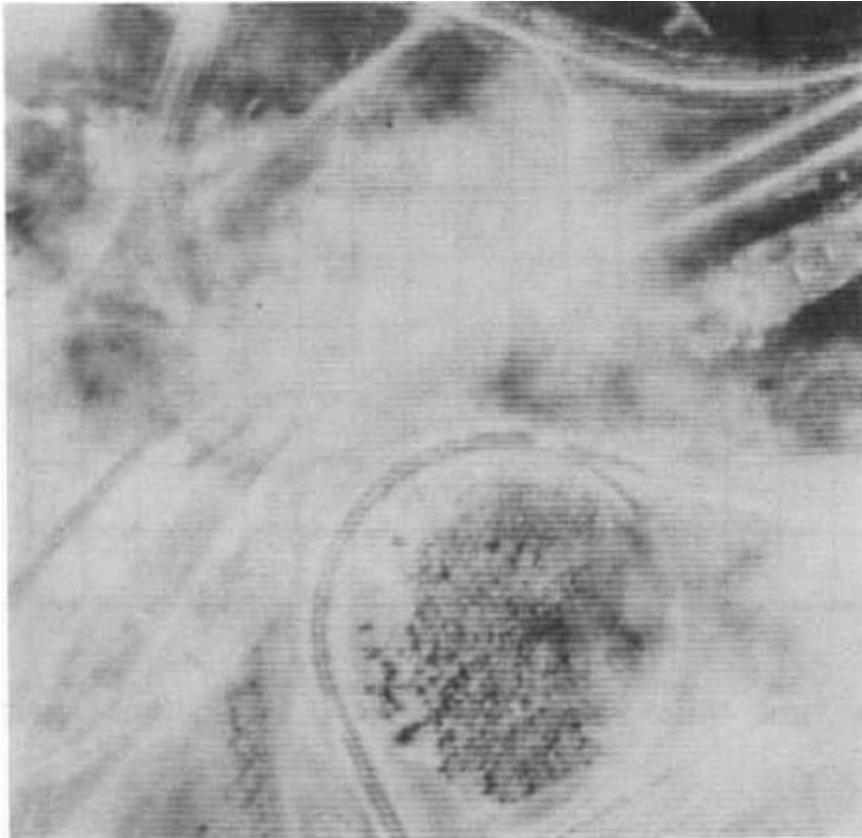


Astronautics example
قيادة

➤ Introduction

Are you involved? No

OK, let us **see** more examples:



Satellite image **Before Enhancement**



After Enhancement

➤ Introduction

Are you involved? No

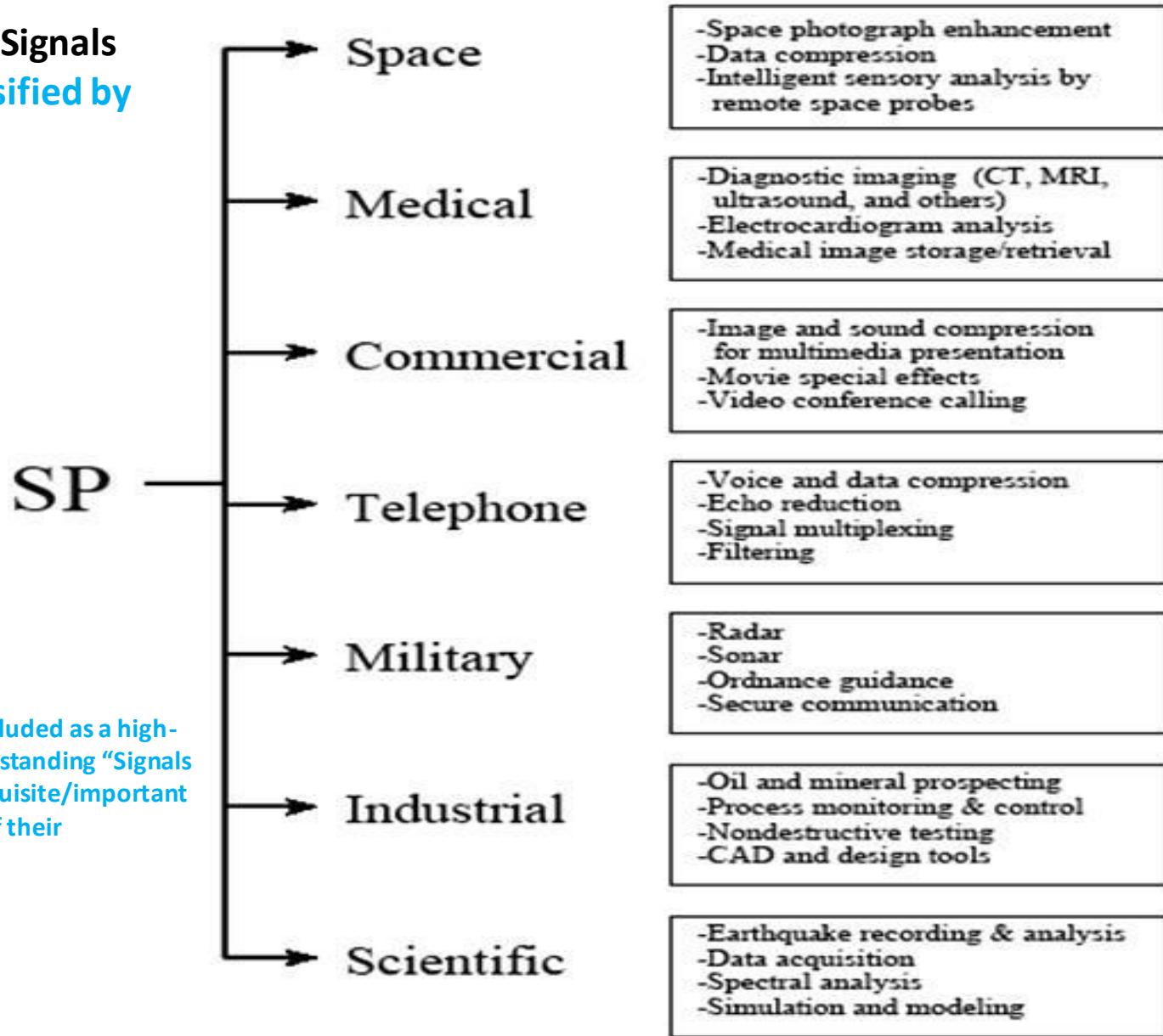
OK, let us **see** more examples:



Recording Enhancement Process

➤ Introduction

**Applications of Signals
Processing classified by
field:**



Note: it may not be included as a high-end product but understanding "Signals and Systems" is prerequisite/important to development/use of their products/machines.

➤ Signals and systems Definition

Now let us know what is the signal? And what is the system?

The Signal:

It is a mean to convey information (or an abstraction of any measurable quantity) that is usually have some form of variations. The contained information point to the behavior or nature of some phenomena.

Mathematically:

The signal is a function of one or more independent variable that maps a domain, often time or space, into a range, often a physical measure such as air pressure or light intensity. **We usually called the independent variable time even if it is not actually time.**

The System:

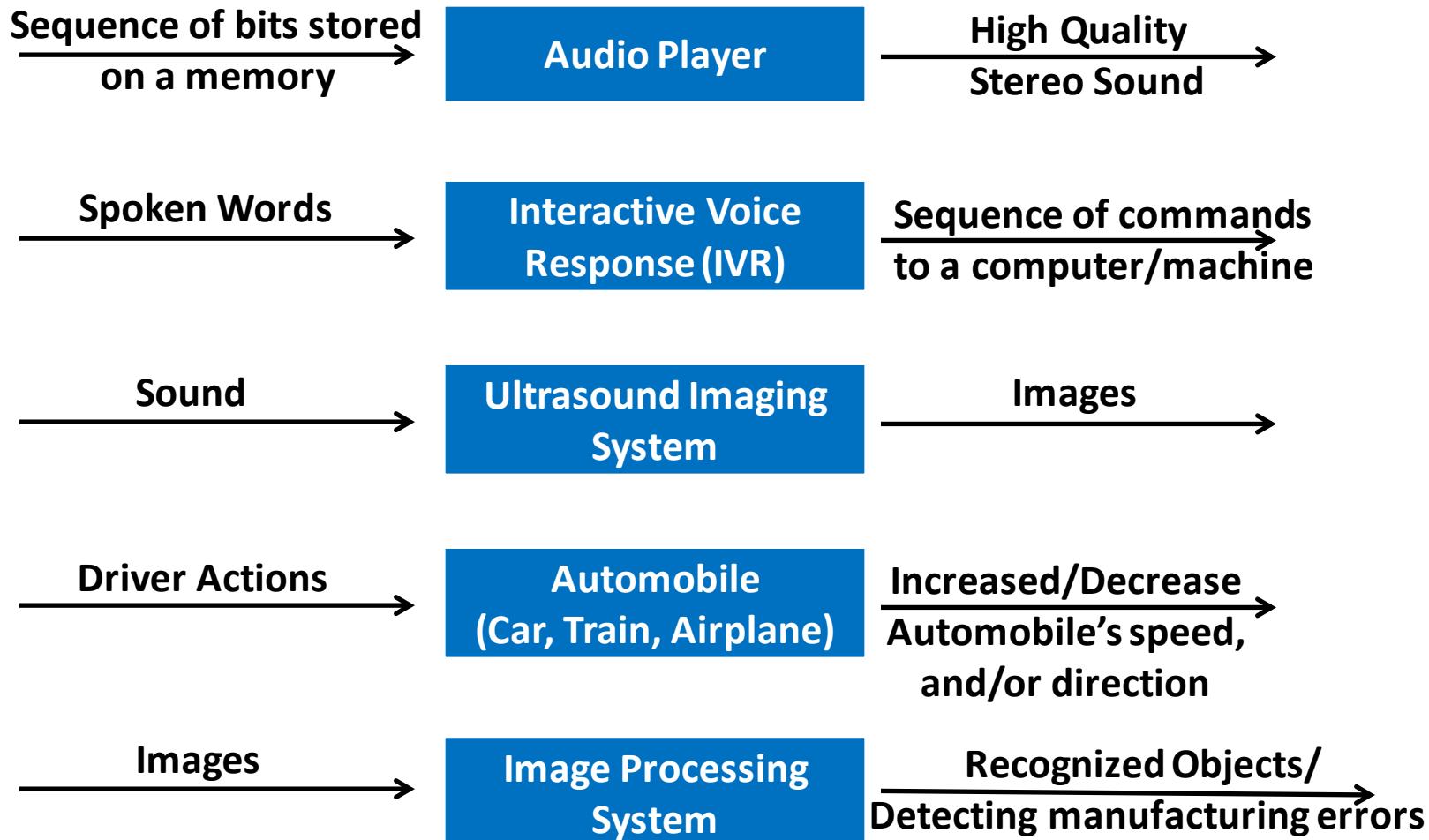
It is a tool that transform a signal to get another signal or process a signal to obtain a desired behavior or for extracting a piece of information.

Mathematically:

A system is a function that maps signals from its domain—its input signals—into a signal in its range—its output signals. The domain and the range are both sets of signals; we call a set of signals a signal space. Thus, systems are functions whose domains and ranges are signal spaces.

➤ Signals and systems Definition

Examples:



➤ Motivations

What are the types of problems that signals and systems techniques try to answer?

- 1- *System Characterization* in detail to **understand** how it will respond to different inputs. (e.g. aircraft/ electrical circuit)
- 2- *System design* to **react** to inputs in a specific way. This usually involves a signal enhancement or restoration. (e.g. air traffic control tower in the airport to avoid the loud noise around when communicating with a pilot)
- 3- *Extracting specific pieces of information.*
(e.g. electrocardiogram estimation of heart rates)
- 4- *Design of Signals with particular properties.*
(e.g. the carrier signal in long distance communications)
- 5- *Modification and control* the characteristics **of a given system**.
(e.g. chemical process **control** through sensors)

➤ Review of Complex Numbers.

Imaginary Number:

An Imaginary Number, when squared, gives a negative result

imaginary  negative

• Unit Imaginary Number:

Each numbering system should have a unit to can be counted !!!

The "unit" Imaginary Number (the equivalent of 1 for Real Numbers) is $\sqrt{-1}$ (the square root of negative one). In mathematics we use (*i*) (for imaginary) but in electronics use (*j*) (because (*i*) already means current, and the next letter after (*i*) is (*j*)). We will use (*j*) in this course!

It is “*imaginary*” and useful ?!

- imaginary numbers give us the ability to find **solutions** (roots) for *quadratic equations* like : $x^2 + 1 = 0$
- using imaginary numbers and real numbers together makes it a lot **easier** to do the **calculations** in many applications as they *encode the magnitude and phase together in a compact simpler form*.
- the imaginary numbers are *not imaginary, they are exist* and *fill a gap in math*. For example: *imaginary x imaginary = real* → i.e. exist !

➤ Review of Complex Numbers.

- Dealing with imaginaries gives you the ability to work with the square root of negatives. BUT, in the same time **you lose something !**

$$\therefore j = \sqrt{-1}$$

$$\text{and as } (A^a)^b = (A^b)^a$$

$$\text{then: } j^2 = (\sqrt{-1})^2 = \sqrt{(-1)^2} = \sqrt{1} = \pm 1$$

HERE this is not true , as you should always make $j^2 = -1$ not +1

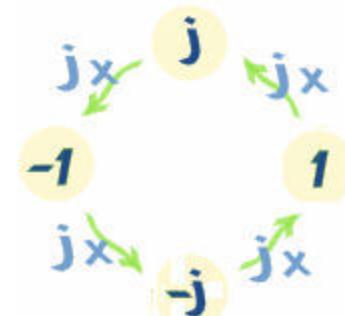
- Interesting Property:

$$\underline{\text{Note}}: \quad j = j^5 = j^9 = \sqrt{-1} \quad ; \quad j^2 = j^6 = j^{10} = -1$$

$$j^3 = j^7 = j^{11} = -j \quad ; \quad j^4 = j^8 = j^{12} = 1$$

in general, $j^m = j^{4n} \cdot j^k = j^k$, where m,n, and k are integers

$$\text{for example : } j^{99} = j^{96+3} = j^{4 \times 24} \cdot j^3 = j^3 = -j$$



➤ Review of Complex Numbers.

The complex number:

is made up of **both** real and imaginary components and its usually represented as:

$$Z = a + jb$$

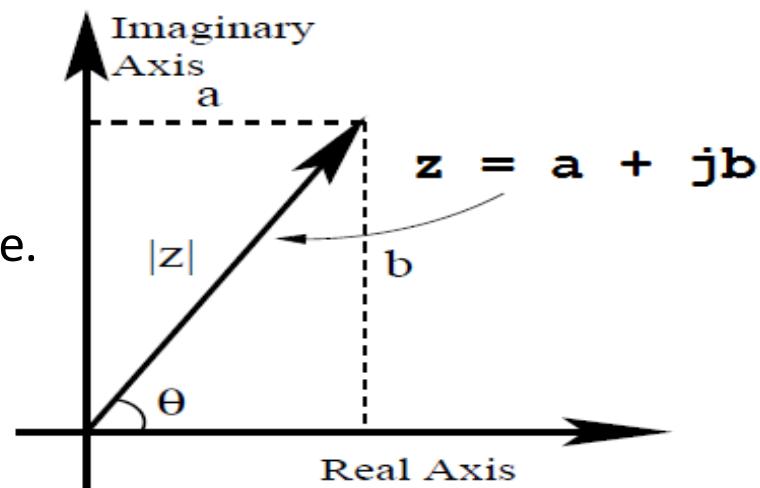
Where **a** is called the real part of the complex number **Z**, or $a = \text{Re}\{Z\}$,

b is called the imaginary part of the complex number **Z**, or $b = \text{Im}\{Z\}$,

and (**j**) is the square root of (-1), i.e. $j = \sqrt{-1}$

The Complex Plane:

- A complex number can be **visualized** in a two-dimensional number plane, known as an **Argand diagram**, or the complex plane as shown in Figure.
- It is conventional to represent a complex number as a **vector** in the complex plane, usually called a **phasor**.



If $Z_1 = a + jb$ and $Z_2 = c + jd$, then $Z_1 = Z_2$ iff $a = c$ and $b = d$

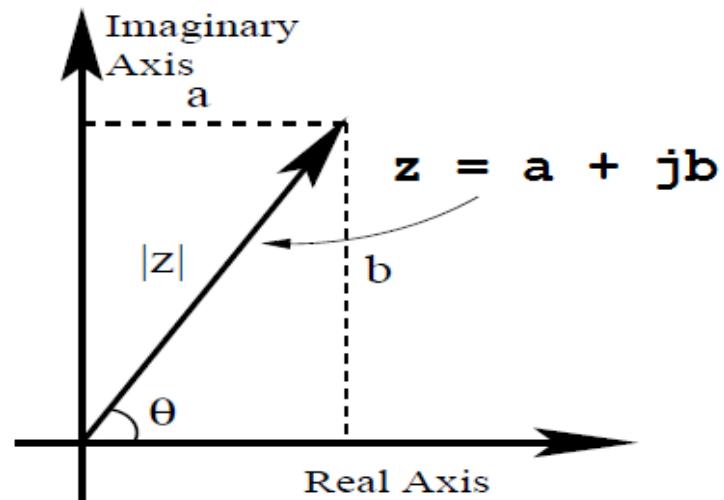
➤ Review of Complex Numbers.

The Complex Magnitude :

From the Figure, it can be easily seen (using the Pythagorean theorem) that the **magnitude**, or **length**, of the vector representing the complex number is:

$$(|z|)^2 = a^2 + b^2$$

$$|z| = \sqrt{a^2 + b^2}$$



It is *also* called **absolute value** or **modulus**.

The Phase Angle of a complex number:

It is the angle of the phasor of the complex number **with the positive Real-Axis**:

$$\theta = \tan^{-1} \left(\frac{b}{a} \right)$$

It is usually called the **Argument** of the complex number. $\theta = \text{Arg } \{Z\} = \angle Z$

$$0 \leq \theta < 2\pi$$

➤ Review of Complex Numbers.

Polar Form:

If you have $Z = a + j b$ (Rectangular Form)

from the graph :

$$\sin(\theta) = \frac{b}{|Z|} \Rightarrow b = |Z| \sin(\theta)$$

$$\text{and } \cos(\theta) = \frac{a}{|Z|} \Rightarrow a = |Z| \cos(\theta)$$

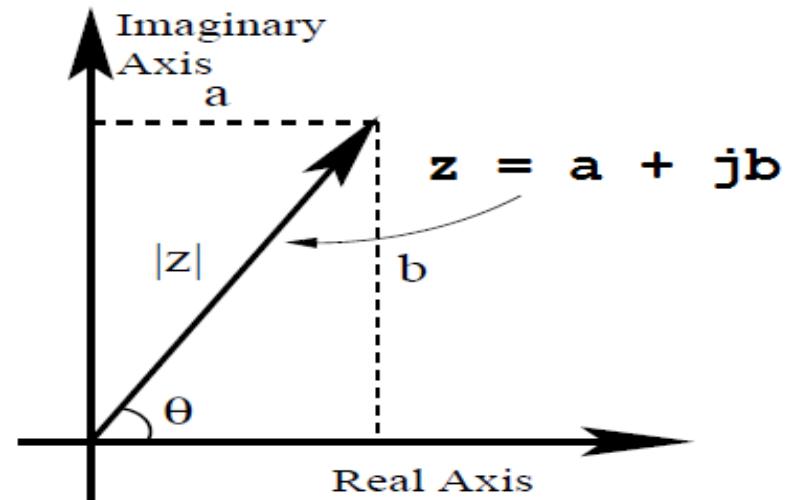
Then Z can be rewritten as :

$$\begin{aligned} Z &= |Z| \cos(\theta) + j |Z| \sin(\theta) \\ &= |Z| (\cos(\theta) + j \sin(\theta)) \end{aligned}$$

From Euler's Formula : $e^{j\theta} = \cos(\theta) + j \sin(\theta)$

$$\text{Then } Z = |Z| e^{j\theta} = |Z| e^{j\angle Z}, \text{ if } r = |Z| \Rightarrow \underline{\underline{Z = r e^{j\theta}}}$$

which is called the polar form of the complex number (Z)



➤ Review of Complex Numbers.

Addition and Subtraction:

If you have $Z_1 = a + j b$ and $Z_2 = c + j d$

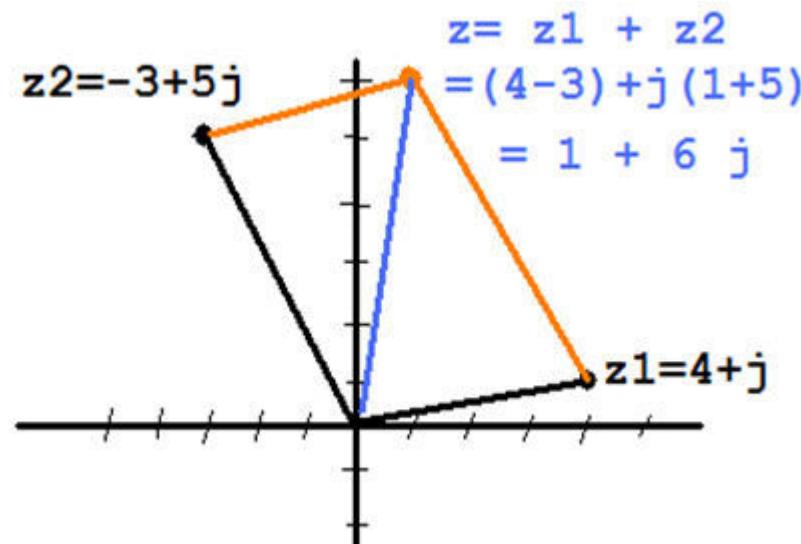
Then: $Z_1 + Z_2 = a + j b + c + j d = (a + c) + j (b + d)$

i.e. add the **real** part **to the real** part and the **imaginary** part **to the imaginary** part.

And : $Z_1 - Z_2 = (a - c) + j (b - d)$

i.e. subtract the **real** part **from the real** part and the **imaginary** part **from the imaginary** part

Complex numbers
addition is similar to
vectors sum.



➤ Review of Complex Numbers.

Multiplication:

If you have $Z_1 = a + j b$ and $Z_2 = c + j d$

Then: $Z_1 \cdot Z_2 = (a + j b) \cdot (c + j d)$

$$\begin{aligned} &= a(c + j d) + j b(c + j d) = ac + ja d + jb c - bd \\ &= ac - bd + j(ad + bc) \end{aligned}$$

The **commutative** and **distributive** properties hold for the **product** of complex numbers.

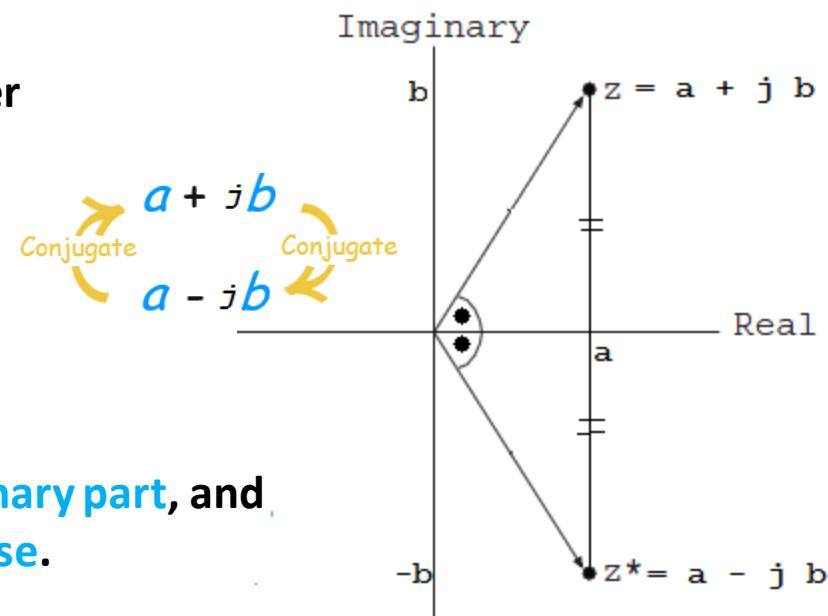
Complex Conjugation:

The complex conjugate of a complex number

$$Z = a + j b,$$

is denoted Z^* , and is defined by:

$$Z^* = a - j b$$



The complex conjugate Z^* has:

- the **same real part** but **opposite imaginary part**, and
- the **same magnitude** but **opposite phase**.

➤ Review of Complex Numbers.

Complex Conjugation Properties:

$$➤ (Z^*)^* = Z$$

$$➤ (Z_1 + Z_2)^* = Z_1^* + Z_2^*$$

$$➤ (Z_1 \cdot Z_2)^* = Z_1^* \cdot Z_2^*$$

$$➤ \text{if } Z_2 \neq 0, \quad \left(\frac{Z_1}{Z_2} \right)^* = \frac{Z_1^*}{Z_2^*}$$

$$➤ (Z^n)^* = (Z^*)^n$$

➤ if Z is real, then $Z = Z^*$

➤ Review of Complex Numbers.

Complex Conjugation Properties:

prove that: $(Z^n)^* = (Z^*)^n$

let $Z = r(\cos(\theta) + j \sin(\theta))$

then by DeMoivre's Theorem:

$$\begin{aligned} Z^n &= [r(\cos(\theta) + j \sin(\theta))]^n = r^n (\cos(n\theta) + j \sin(n\theta)) \\ \therefore (Z^n)^* &= (r^n (\cos(n\theta) + j \sin(n\theta)))^* = r^n (\cos(n\theta) - j \sin(n\theta)) \\ &= [r(\cos(\theta) - j \sin(\theta))]^n \\ &= (Z^*)^n \end{aligned}$$

➤ Review of Complex Numbers.

The Complex Magnitude Properties:

- $|Z| = 0$ iff $Z = 0$
- $|Z| = |Z^*|$
- $|Z_1 \cdot Z_2| = |Z_1| \cdot |Z_2|$
- if $Z \neq 0$, then $\left| \frac{1}{z} \right| = \frac{1}{|z|}$
- $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$
- $Z \cdot Z^* = |Z|^2$

➤ Review of Complex Numbers.

Division:

If you have $Z_1 = a + jb$ and $Z_2 = c + jd$

Then:

$$\frac{Z_1}{Z_2} = \frac{a + jb}{c + jd}$$

Is usually rewritten by rationalizing the denominator to make it simpler.

i.e. can be represented as a ratio.

$$\begin{aligned}\frac{Z_1}{Z_2} &= \frac{a + jb}{c + jd} \times \frac{c - jd}{c - jd} \\ &= \frac{(ac + bd) + j(bc - ad)}{c^2 + d^2}\end{aligned}$$

Example :

Express $\frac{5+5j}{1+3j}$ in the form of $(a + jb)$

We multiply the numerator and denominator by the conjugate of the denominator ($1 - 3j$).

$$\begin{aligned}\left(\frac{5+5j}{1+3j} \right) \times \left(\frac{1-3j}{1-3j} \right) &= \frac{5-15j+5j+15}{1-3j+3j+9} \\ &= \frac{20-10j}{10} \\ &= \frac{20}{10} - \frac{10}{10} j = 2 - j\end{aligned}$$

➤ Review of Complex Numbers.

Examples to be solved on the board:

1 – show that the $\text{Arg}\{Z^*\} = -\text{Arg}\{Z\}$?

2 – Get the value of $e^{-1+j\frac{\pi}{6}}$?

3 – Simplify $\frac{-20 + \sqrt{-75}}{5}$?

4 – Show that when multiplying two complex numbers,
actually we multiply their magnitudes and
add their phase angles ?

5 – Proof that : $\cos^2(\theta) + \sin^2(\theta) = 1$

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

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$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

➤ Review of Complex Numbers.

Assignment:

1- Proof the complex conjugate properties?

2- Proof the complex magnitude properties?

3 – Proof that : $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

➤ Review of Complex Numbers.

4- Consider a series AC electrical circuit with two resistors and a capacitor. The output complex voltage

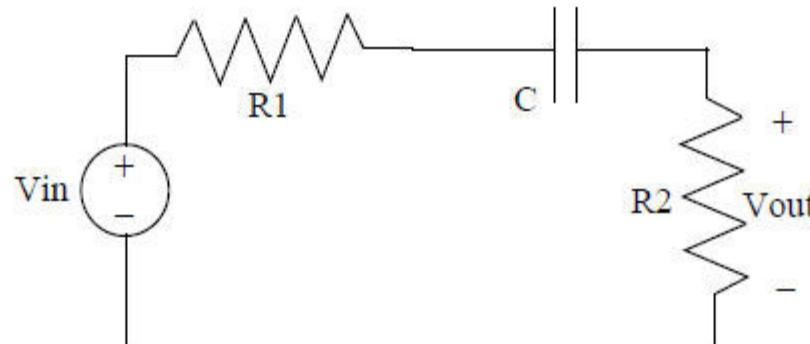


Figure : A simple AC circuit.

is related to the input complex voltage by the voltage divider law

$$\hat{V}_{out} = \frac{R_2}{R_1 + R_2 - i/(\omega C)} \hat{V}_{in}$$

If $R_1 = 100\Omega$, $R_2 = 200\Omega$, $C = 50\mu F$, and $\omega = 2\pi(60)$ cycles/s, and $\hat{V}_{in} = 100V$, then what is the

- a) magnitude of the output voltage,
- b) phase of the output voltage.
- c) plot $|\hat{V}_{out}/\hat{V}_{in}|$ as a function of different ω 's.