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Lec 19 — 15/12

Part1 Cts

Ch3 ~~Youtube~~ — Fourier series

Dts

// Transform

Ch4 — Cts

Ch5 — Dts

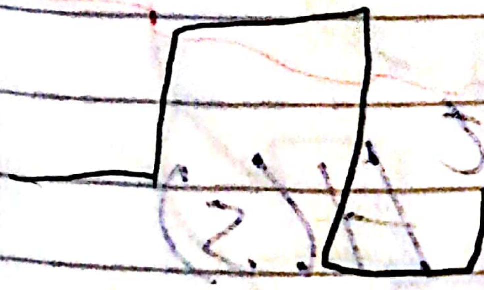
Aperiodic

e^{st} — Can be used to build a broad range of signal

$\boxed{LTI}$ → simple to compute

Input $\sum e$

$$e^{st}\left\{H(s)\right\} = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt$$



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Periodic CTS

Claim

$$x(t) = \sum_{K=-\infty}^{+\infty} a_k e^{jK\omega_0 t}$$

$$e^{j\Theta} = \cos\Theta + j\sin\Theta$$

$x(t) \longrightarrow$ Periodic

$$\boxed{x(t) = x(t+T)}$$

T :- Fundamental Period

$$\phi_K(t) = e^{jK\omega_0 t}$$: set of harmonical related Signal

$$\omega_0 = \frac{2\pi}{T}$$

$K = 0, \pm 1, \pm 2, \pm 3, \dots$

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$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

(1) multiply both sides by $e^{-jn\omega_0 t}$

$$x(t) \cdot e^{-jn\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}$$

(2) Integrat both sides from 0 to (T)

$$\int_0^T x(t) e^{-jn\omega_0 t} dt = \int_0^T \sum_{k=-\infty}^{+\infty} a_k e^{j(k-n)\omega_0 t} dt$$

$\frac{2\pi}{\omega_0}$

$$\int_0^T x(t) e^{-jn\omega_0 t} dt = \sum_{k=-\infty}^{+\infty} a_k \int_0^T e^{j(k-n)\omega_0 t} dt$$

Recall

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{j(k-n)\omega_0 t} = \cos((k-n)\omega_0 t) + j\sin((k-n)\omega_0 t)$$

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$$\cos[(k-n)\omega_0 t] + j\sin[(k-n)\omega_0 t]$$

$$\cos(\omega t + \phi) \qquad \sin(\omega t + \phi)$$

$$\omega = (k-n)\omega_0$$

$$T_0 \triangleq \frac{2\pi}{\omega} = \frac{2\pi}{(k-n)\omega_0}$$

$$= \frac{\cancel{2\pi} \cdot T}{(k-n)\cancel{2\pi}} = \boxed{\frac{T}{(k-n)}}$$

$$T_0 = \frac{T}{(k-n)} \quad ; \quad \begin{matrix} k \text{ integer} \\ n \text{ integer} \\ (k-n) \text{ integer} \end{matrix}$$

$$T = (k-n)T_0$$

$$= \text{integer} \cdot T_0$$

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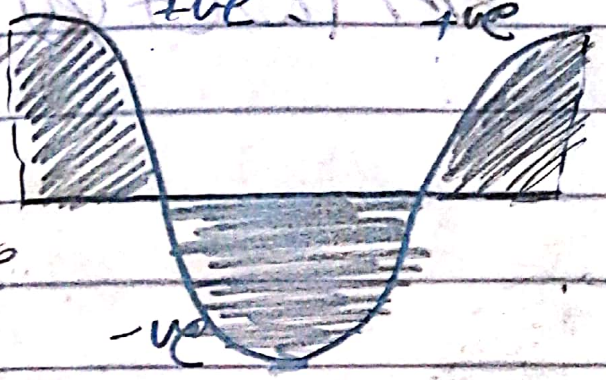
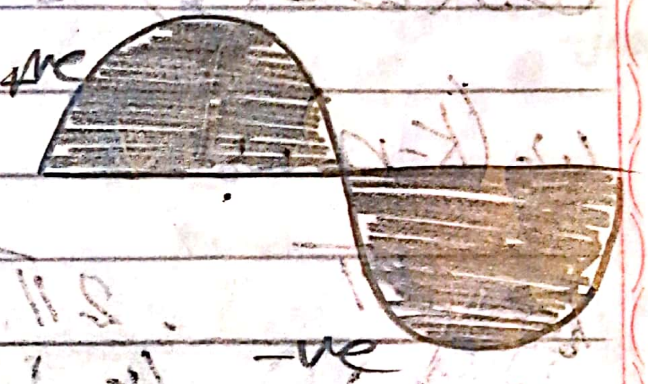
$$\int_0^T e^{j(k-n)\omega_0 t}\,dt = \int_0^T \cos(k-n)\omega_0 t + j\int_0^T \sin(k-n)\omega_0 t$$





$$= \begin{cases} 0 & k \neq n \\ T & k = n \end{cases}$$

$$\int_0^T x(t)e^{-jn\omega_0 t}\,dt = \sum_{k=-\infty}^{+\infty} a_k \begin{cases} 0 & k \neq n \\ T & k = n \end{cases} = T a_n$$

$$a_n = \frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}\,dt$$

a_k

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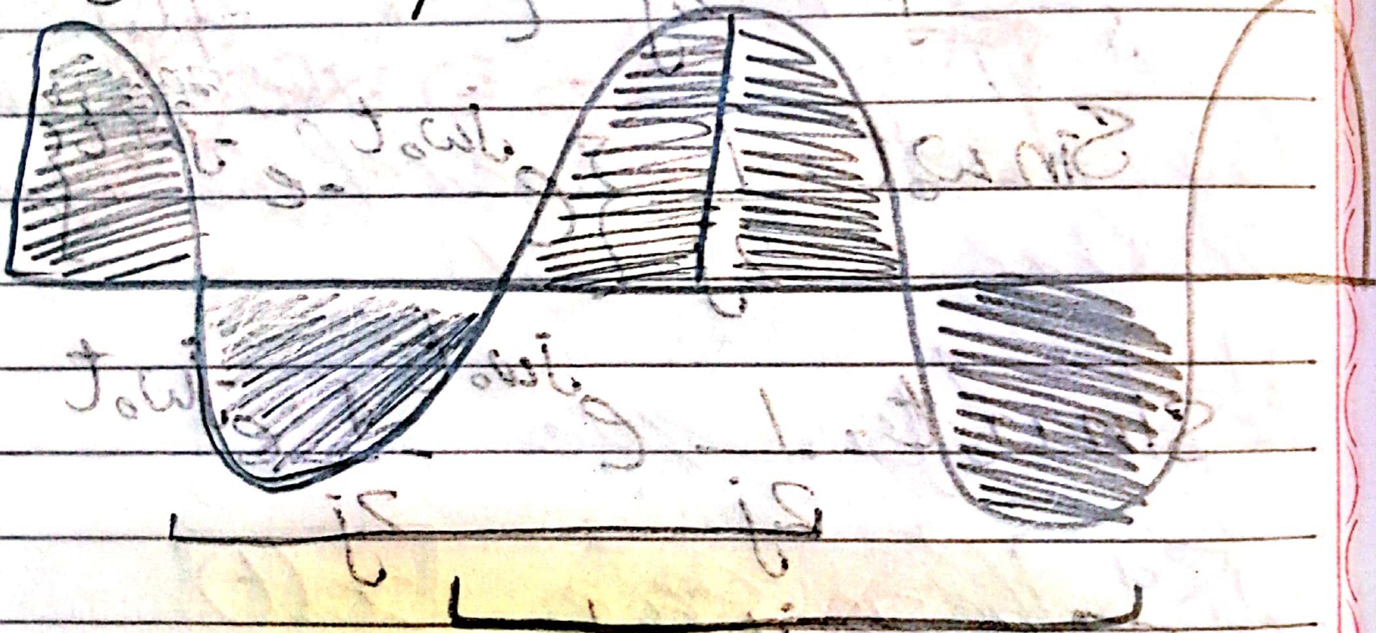
$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$



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Ex 1:- $x(t) = \sin \omega_0 t$

$$x(t) = a_1 H(s_1) e^{s_1 t} + a_2 H(s_2) e^{s_2 t}$$

$$\sin \theta = \frac{1}{2j} \left\{ e^{j\theta} - e^{-j\theta} \right\}$$

$$\sin \omega_0 t = \frac{1}{2j} \left\{ e^{j\omega_0 t} - e^{-j\omega_0 t} \right\}$$

$$\sin \omega_0 t = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

$$\sum_k a_k e^{jk\omega_0 t}$$

$$a_1 = \frac{1}{2j}$$

$$a_{-1} = \frac{-1}{2j}$$

$$a_k = \begin{cases} \frac{1}{2j} & k = 1 \\ \frac{-1}{2j} & k = -1 \\ 0 & \text{otherwise} \end{cases}$$

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$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\frac{1}{\cancel{2}}\left(e^{j\theta} + e^{-j\theta}\right) = \frac{\cancel{2}\cos\theta}{\cancel{2}}$$

$$\frac{\cancel{2}j\sin\theta}{\cancel{2}} = \frac{1}{2}\left\{e^{j\theta} - e^{-j\theta}\right\}$$

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Ex2 : $x(t)$:

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

dc
Const
average value of the signal.

Furier series coefficient
spectral coefficient

تحليل Analysis eque. $\boxed{a_k}$

تركيب Synthesis eque. $\boxed{x(t)}$

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$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos(2\omega_0 t + \frac{\pi}{4})$

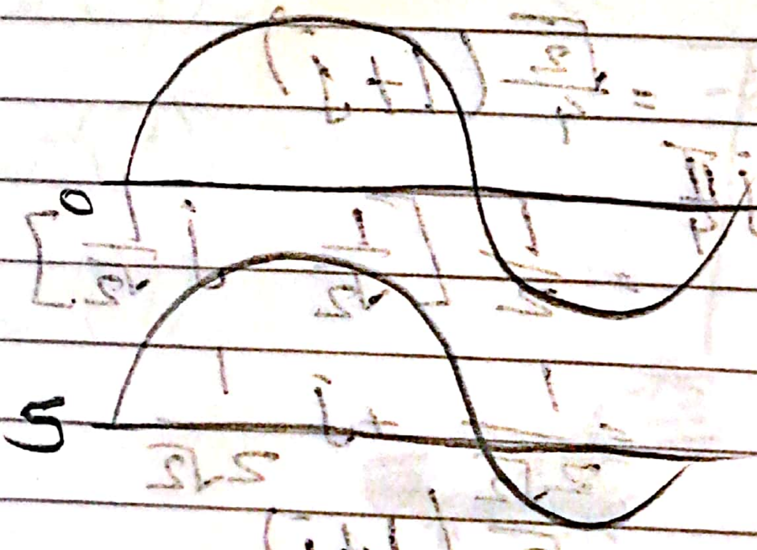
(DC-Component)

$\rightarrow 1 + \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) +$

$\cancel{2}\left[\frac{1}{\cancel{2}}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\right] +$

$\frac{1}{2}\left[e^{j(2\omega_0 t)} \cdot e^{j\frac{\pi}{4}} + e^{-j2\omega_0 t} \cdot e^{-j\frac{\pi}{4}}\right]$

$5 + \sin(2)$



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$$a_0 = 1$$

$$a_1 = 1 + \frac{1}{2j} = 1 - \frac{1}{2}j$$

$$a_{-1} = 1 - \frac{1}{2j} = 1 + \frac{1}{2}j$$

$$a_2 = \frac{1}{2} \cdot e^{j\frac{\pi}{4}} = \frac{1}{2}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}(1+j)$$

$$a_{-2} = \frac{1}{2} \cdot e^{-j\frac{\pi}{4}} = \frac{1}{2}\left[\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}(1+j)$$

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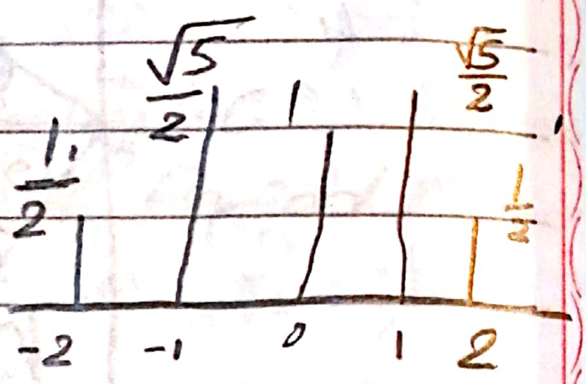
$a_0 = 1 \qquad |a_0| = 1$

$a_1 = 1 - \frac{1}{2}j \qquad |a_1| = \frac{\sqrt{5}}{2}$

$a_{-1} = 1 + \frac{1}{2}j \qquad |a_{-1}| = \frac{\sqrt{5}}{2}$



$a_2 = \frac{\sqrt{2}}{4}(1+j) \qquad |a_2| = \frac{1}{2}$

$a_{-2} = \frac{\sqrt{2}}{4}(1-j) \qquad |a_{-2}| = \frac{1}{2}$

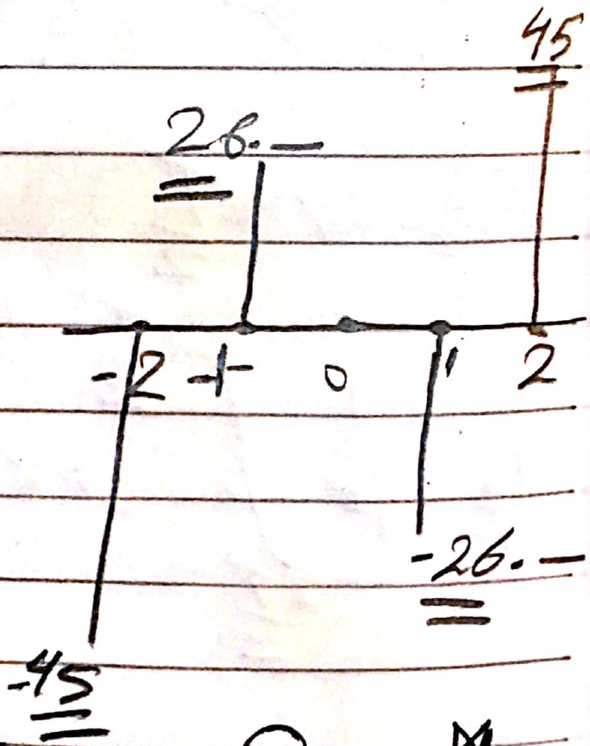
$a_k = 0, \; k \neq (0, 1, 2, -1, -2)$

$\tan^{-1}\left(\frac{-\frac{1}{2}}{1}\right) =$

$\tan^{-1}\left(\frac{\frac{1}{2}}{1}\right) =$

$\tan^{-1}(1) =$

$\tan^{-1}(-1)$



[redacted]

$a_k = a_k^*$

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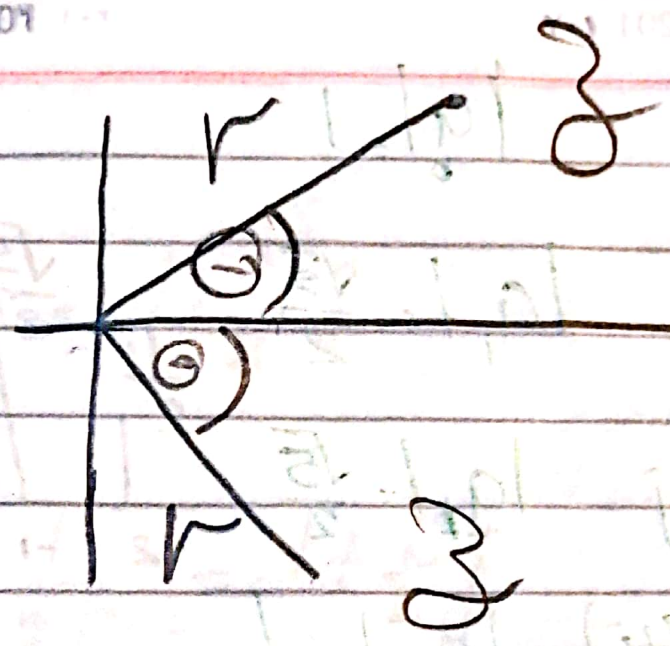

ق٢ = ق٤ + π

(ق١، ق٢، ق٣)

ق٤ = ق٢