Lec-13

17/11

LaPlace Transform

عالم فرنسي

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DoD 1827 — 80

تفاضل
تكامل

(Transform)

- Provide us with a set of tools for system easier analysis
- Extension for FT to analyze more sys - even if it is Not stable
- Convert:-
  - Convolution
  - Time-shifts
  - Differentiations
  - Integrations

  + − / X

LTI

$x(t)$ I/P → [ ] → $y(t)$

I.R.

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

if $x(t) = e^{st} \Rightarrow x(t-\tau) = e^{s(t-\tau)}$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau) e^{st} \cdot e^{-s\tau}\, d\tau \qquad = e^{st} \cdot e^{-s\tau}$$

$$\therefore y(t) = e^{st} \underbrace{\int_{-\infty}^{+\infty} h(\tau) \cdot e^{-s\tau}\, d\tau}_{\text{Gain}}$$

e^{st} is an Eigen function for LTI system.

damming Variable

Eigen Value

$$\int_{-\infty}^{+\infty} h(t) e^{-st}\, dt$$

Pure Img. $s = j\omega$ $\qquad$ $s = $ Complex

$$\int_{-\infty}^{+\infty} h(t) e^{-j\omega t}\, dt \qquad \int_{-\infty}^{+\infty} h(t) e^{-st}\, dt$$

FT $\qquad$ LT

LT of any CTs Signal $x(t)$

$$\boxed{X(S) \triangleq \int_{-\infty}^{+\infty} x(t) e^{-st} dt}$$

Ex1 Compute LT for $x(t) = e^{-at} u(t)$

- $X(S) \triangleq \int_{-\infty}^{+\infty} x(t) e^{-st} dt$

$$= \int_{-\infty}^{+\infty} e^{-at} . u(t) . e^{-st} dt$$

$$\therefore X(S) = \int_{0}^{+\infty} e^{-at} . e^{-st} dt$$

$$= \int_{0}^{+\infty} e^{-(s+a)t} dt$$

$$= \frac{1}{-(s+a)} \left[e^{-(s+a)t} \Big|_{0}^{\infty} \right]$$

$$\therefore X(S) = \frac{1}{-(S+a)} \left[e^{-(s+a)(+\infty)} - \underbrace{e^{-(s+a)(0)}}_{=1} \right]$$

المسموحة ضوئيا بـ CamScanner

$$\therefore X(s) = \frac{-1}{s+a}\left[e^{-(s+a)(+\infty)} - 1\right]$$

To converge, we need $e^{-(s+a)(+\infty)} < \infty$

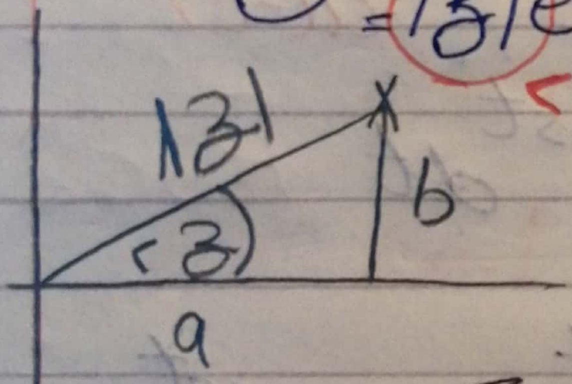
$$\therefore e^{\boxed{-(\sigma+j\omega+a)}(\infty)} < \infty$$

$$\left[e^{-(\sigma+a)} \cdot e^{-j\omega}\right]^{+\infty} < \infty$$

Recall: $z = a+jb$

$= |z| e^{j\angle z}$

$|z| < \infty$



$e^{-j\omega}$

مستحيل $\neq \infty$

$$\therefore \left[e^{-(\sigma+a)} \cdot e^{-j\omega}\right]^{+\infty} < \infty$$

$$e^{-(\sigma+a)\infty} < \infty$$

$$\boxed{\sigma + a > 0}$$

$$Re\{s\} > -a$$

$$X(s) = \frac{-1}{s+a}\left\{e^{-(s+a)\infty} - 1\right\}$$

$$X(s) = \frac{-1}{s+a}\left\{0-1\right\} ;\ \sigma > -a$$

$$\boxed{x(s) = \frac{1}{s+a}} ;\ Re\{s\} > -a$$

$$\rightarrow x(t) = e^{-at}.u(t)$$

Ex2: $x(t) = -e^{-at}.u(-t)$

$$X(s) \triangleq \int_{-\infty}^{+\infty} x(t) e^{-st}\, dt$$

$\boxed{-t > ve}$

$$= \int_{-\infty}^{+\infty} (-1) e^{-at}.u(-t).e^{-st}\, dt$$

$\boxed{0 \\ -ve}$

$$\therefore X(s) = -\int_{-\infty}^{0} e^{-at}.e^{-st}\, dt$$

$$= -\int_{-\infty}^{0} e^{-t(s+a)}\, dt$$

$$X(s) = \frac{\cancel{-}1}{\cancel{-}(s+a)}\left[e^{-(s+a)t} \Big|_{-\infty}^{0} \right]$$

$$\therefore X(s) = \frac{1}{s+a}\left[e^{-(s+a)(0)} - e^{-(s+a)(-\infty)} \right]$$

$$X(s) = \frac{1}{s+a}\left[1 - 0 \right] \; ; \; Re\{s\} + a < 0$$

$$= \boxed{\frac{1}{s+a}} \; ; \; Re\{s\} < -a$$

For Ex.1 and Ex.2 to compute the LT you must to mention $\boxed{\text{ROC}}$.

Region of Convergence

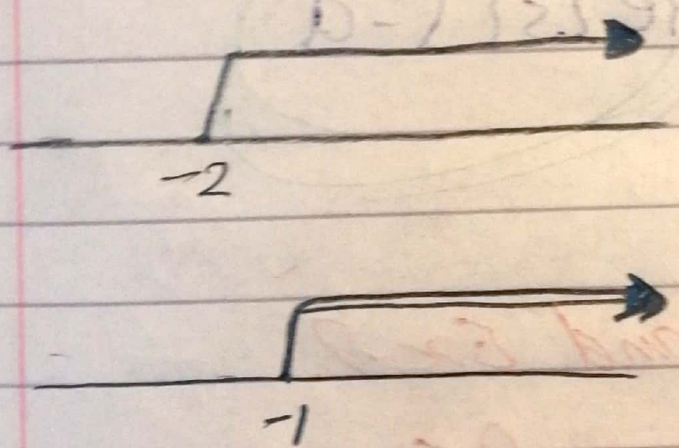
Ex $x(t) = 3e^{-2t}.u(t) - 2e^{-t}.u(t)$

$$X(s) = \boxed{3.\frac{1}{s+2}} - \boxed{2.\frac{1}{s+1}}$$

$Re\{s\} > -2$ $\qquad$ $Re\{s\} > -1$

$\Uparrow$ $\qquad$ $\Uparrow$

Roc1 $\qquad$ Roc2



$Roc = Roc1 \cap Roc2$

Ex1 Ex2 Ex3 Ex4

e^{-at} $\qquad$ e^{ja}

Roc $\cap$

$Re\{s\} > -Re\{a\}$

$< -Re\{a\}$

if a Complex

$X(s) = \frac{N(s)}{D(s)}$

= Rational Experssion