Lec.6 25/10

(Basic Signals)

$e^{j\omega n}$ & $\cos(\omega n)$ & $\sin(\omega n)$

$\boxed{0 \longrightarrow 2\pi}$ $e^{j\omega n}$

$\boxed{-\pi \longrightarrow \pi}$ $e^{j\omega n}$ ω

$\omega + 2\pi$

$e^{j(\omega+2\pi)n}$

$e^{j\omega n} \cdot \underbrace{e^{j2\pi n}}_{=1}$

$\Rightarrow 2\pi n$ عدد صحيح

$\Rightarrow 2\pi t \neq 1$

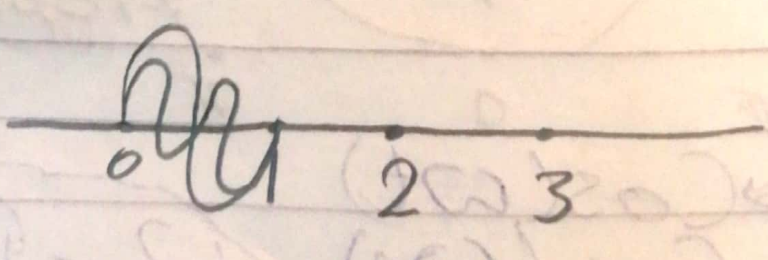
$\underbrace{\cos(2\pi n)}_{=1} + j\underbrace{\sin(2\pi n)}_{=0}$

$\boxed{\text{real values}}$

$e^{j\omega t} \cdot e^{j(\omega+2\pi)t}$

$e^{j\omega t} \cdot e^{j2\pi t}$

t ‖integer‖



$\cos(\omega n)$ ارسم في اليمين

$\omega = 0 \quad \cos(0)\ \forall n = 1$

$\omega = \frac{\pi}{8} \quad \cos(\frac{\pi}{8} n) \Longrightarrow$ 16 samples

$\omega = \frac{\pi}{4} \quad \cos(\frac{\pi}{4} n) \longrightarrow$ 8 Samples

$\omega = \frac{\pi}{2} \quad \cos(\frac{\pi}{2} n) \Longrightarrow$ 4 Samples.

$\omega = \pi \quad \cos(\pi n) \Longrightarrow$ 2 samples $\boxed{\text{freq. أعلى}}$

$\omega = \frac{3\pi}{2} \quad \cos(\frac{3\pi}{2} n) \Longrightarrow$ 4 Samples

$n=0 \Longrightarrow 1$

$n=1 \longrightarrow 0$

$n=2 \longrightarrow -1$

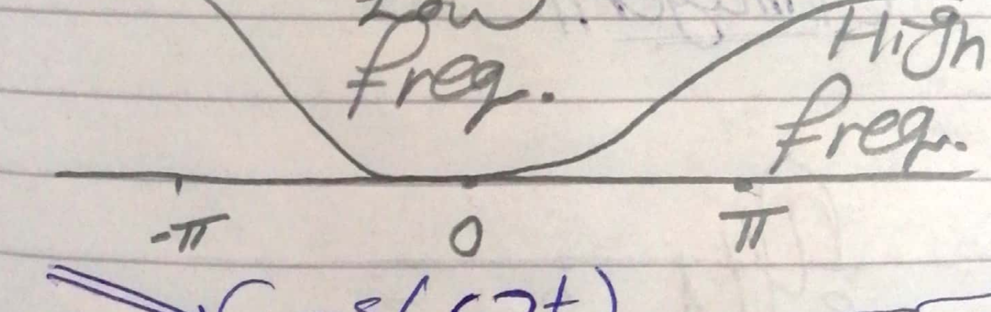
$n=3 \Longrightarrow 0$

$n=4 \longrightarrow 1$

$\omega = \frac{7\pi}{4} \quad \cos(\frac{7\pi}{4} n) \Longrightarrow$ 8 Samples.

$\omega = \frac{15\pi}{8} \quad \cos(\frac{15\pi}{8} n) \longrightarrow$ 16 Samples

$\omega = 2\pi \quad \cos(2\pi n) \longrightarrow \forall n = 1$



$\Longrightarrow \cos(\omega t)$

$\cos(\omega n)$

$\frac{\omega}{2\pi}$ = Rational Number

Rational Number $= \frac{\text{عدد صحيح}}{\text{عدد صحيح}}$

$\cos(\omega n + \phi)$

$\cos\left(\frac{33\pi}{5} n + \phi\right)$

$\frac{5}{1}$

$\cos(\text{عشري}\, n + \phi)$

$\omega =$ عشري

$\frac{\omega}{2\pi} = RN = \frac{K}{N} = \frac{4}{24} = \frac{1}{6}$ → Fundimantal Period

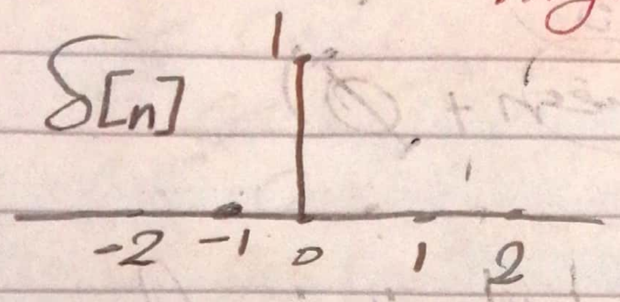
$T_1 \quad + \quad T_2$

$$\boxed{L T_1 = K T_2}$$

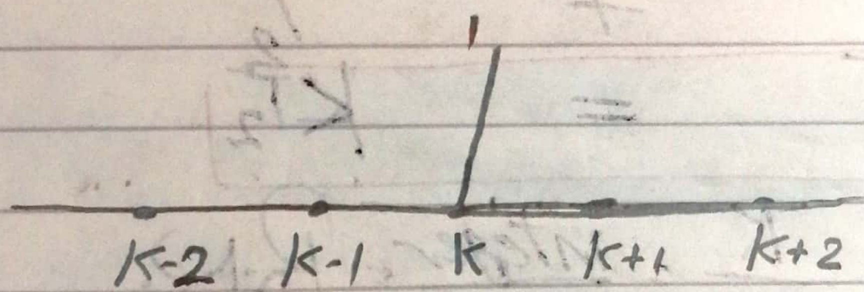
$$\frac{T_1}{T_2} = \frac{K}{L} = \frac{\text{integer}}{\text{integer}} = R.N.$$

Unit Sample
Unit import

$$\underbrace{\delta[n]}_{\text{Argument}} = \begin{cases} 1 & \text{Argument} = 0,\ n = 0 \\ 0 & \text{Otherwise} \end{cases}$$



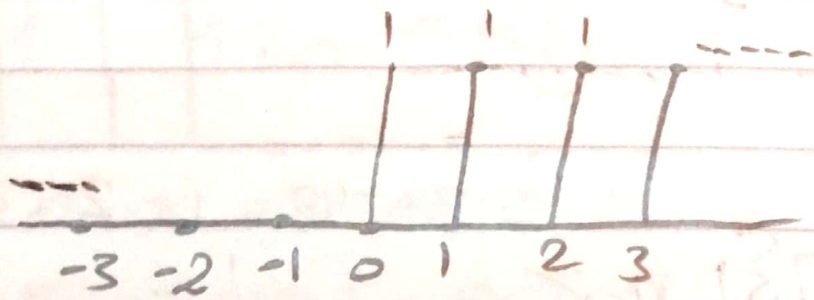
$$\delta[\underbrace{n-k}] = \begin{cases} 1 & n-k=0,\ \boxed{n=k} \\ 0 & \boxed{n \neq k}\ \text{otherwise} \end{cases}$$



Unit step signal $u[n]$

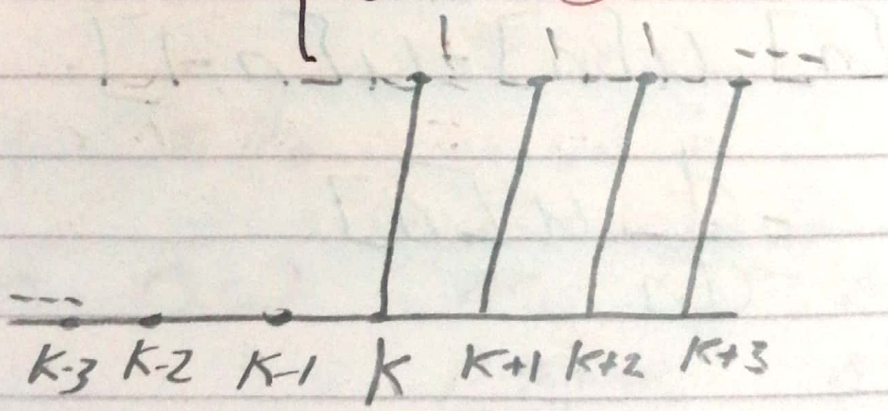
Argument ≥ 0

$$u[\underbrace{n}_{\text{Argument}}] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

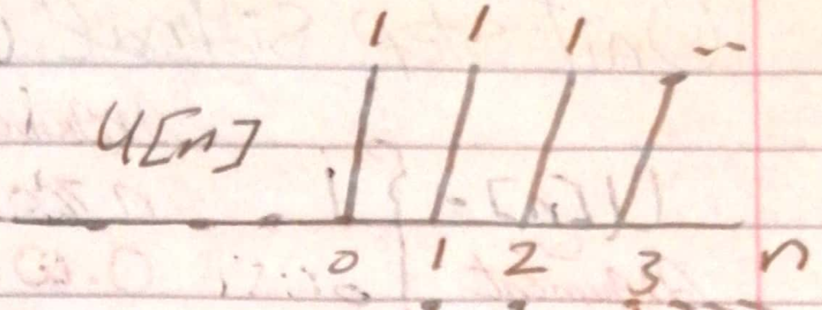


$n-k \geq 0$

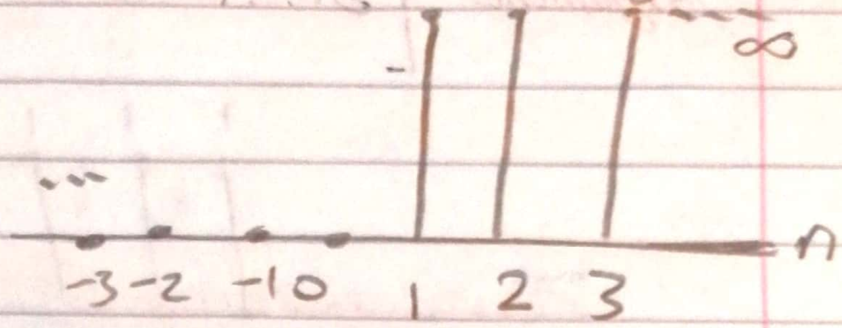
$$u[\underbrace{n-k}] = \begin{cases} 1 & n \geq k \\ 0 & \text{o.w.} \end{cases}$$



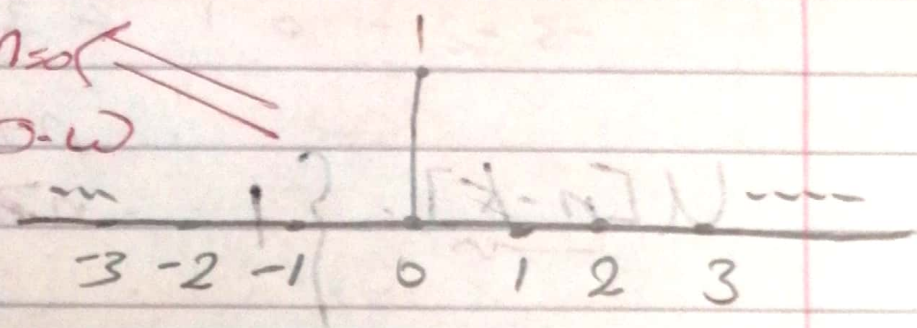
$u[n]$



$u[n-1]$



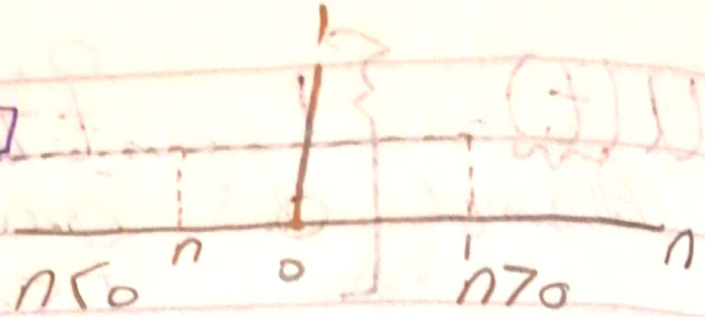
$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & o.w \end{cases}$$



$$\delta[n] = u[n] - u[n-1].$$

$$= \frac{d}{dn} u[n]$$

Running Sum $= \sum_{m=-\infty}^{n} \delta[m]$



$$= \begin{cases} 0 & n<0 \\ 1 & n \geq 0 \end{cases} = u[n]$$

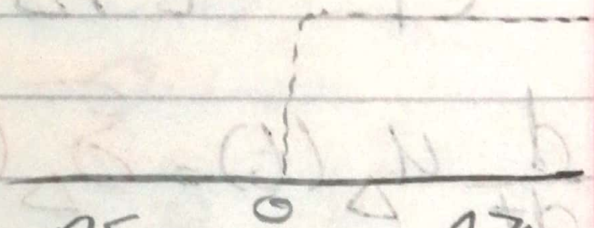
$K = n - m \implies m = n - K$

at $m = -\infty \implies K = n - (-\infty) = n + \infty = \boxed{+\infty}$

at $m = n \implies K = n - n = 0$

$$\sum_{k=+\infty}^{0} \delta[n-k] = \sum_{k=0}^{+\infty} \delta[n-k].$$

$$= \begin{cases} 1 & n \geq 0 \\ 0 & \text{o.w.} \end{cases}$$



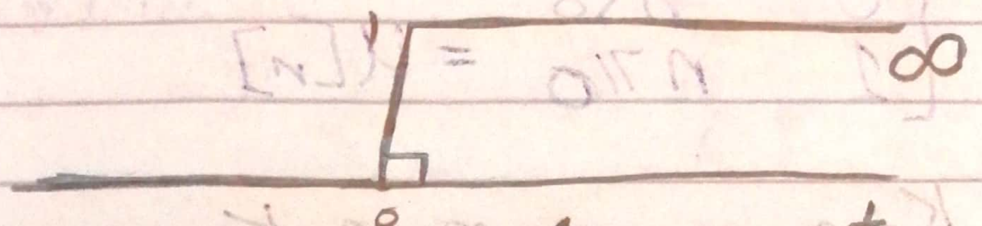
$n<0$ $n \geq 0$

$n-K = -ve$ $n-K =$

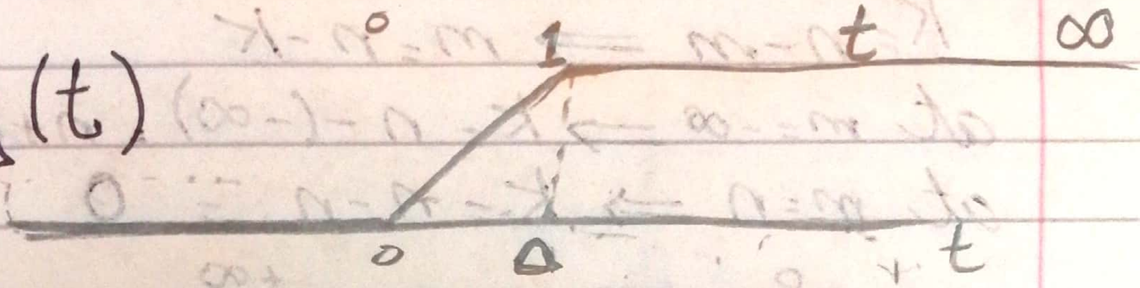
$\delta[-ve] = 0$ $\sum \delta[0] = 1$

$$U(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{o.w.} \quad t < 0 \end{cases}$$

Argument



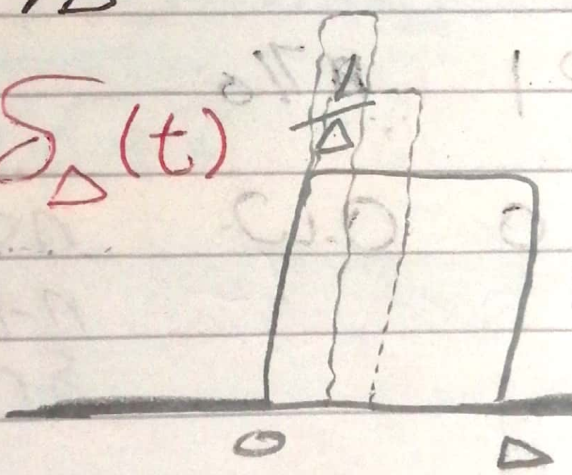
$U_\Delta(t)$



$$= \begin{cases} 0 & t < 0 \\ \frac{t}{\Delta} & 0 < t < \Delta \\ 1 & t \geq \Delta \end{cases}$$

$$\frac{d}{dt} u_\Delta(t) = \delta_\Delta(t)$$

as $\Delta \downarrow$ $\frac{1}{\Delta} \uparrow$



$\frac{1}{\Delta} \cdot \Delta = 1$

Area = 1

$$\lim_{\Delta \to 0} \quad \frac{1}{\Delta} \longrightarrow \infty$$

as $\Delta \rightarrow 0$

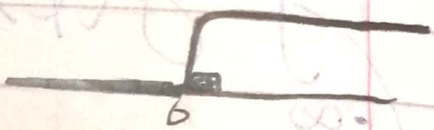
$$\lim_{\Delta\to 0} \delta_\Delta(t) = \delta(t)$$

Area = 1

$$\delta(t) = \begin{cases} 1 & \boxed{\text{Concentrated at } 0} \\ 0 & \text{O.W.} \end{cases}$$

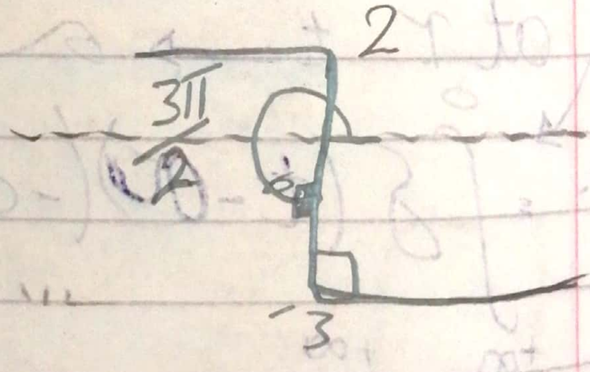
$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)$$

$$\delta(t-t_0) = \frac{d}{dt} u(t-t_0)$$

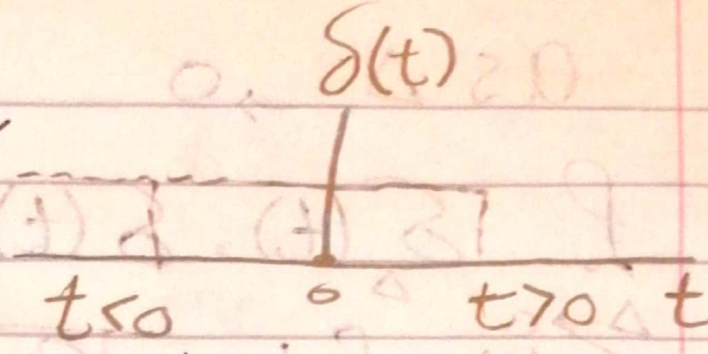


$$= \begin{cases} 1 & t = 0 \\ 0 & \text{O.W.} \end{cases}$$

$$-5\,\delta(t-t_0)$$



$$\int_{-\infty}^{t} \delta(\tau)\, d\tau$$



$= \text{Area under the curve}$

$$\begin{cases} 0 & t<0 \\ 1 & t\geq 0 \end{cases} = u(t)$$

$$\int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$y = f(x)\ 5-3x$

$\boxed{\frac{dy}{dx}} = \boxed{dy = dx}$ F3

$d\sigma = -d\tau$

$\tau = t - \sigma \Longrightarrow \sigma = t - \tau$ as Constant

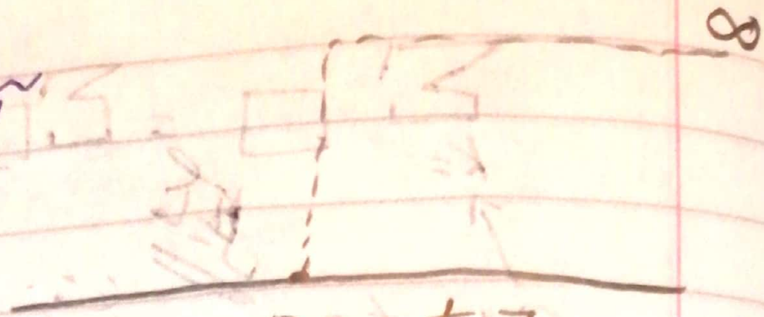
at $\tau = -\infty \Longrightarrow \sigma = t-(-\infty) = t+\infty = +\infty$

at $\tau = t \Longrightarrow \sigma = t - t = 0$

$$\therefore = \int_{+\infty}^{0} \delta(t-\sigma)(-d\sigma) = \int_{0}^{+\infty} \delta(t-\sigma)\, d\sigma$$

$$= \int_{0}^{+\infty} \delta(t-\tau)\, d\tau$$

$$\int_{0}^{+\infty} \delta(t-\tau)\,d\tau$$



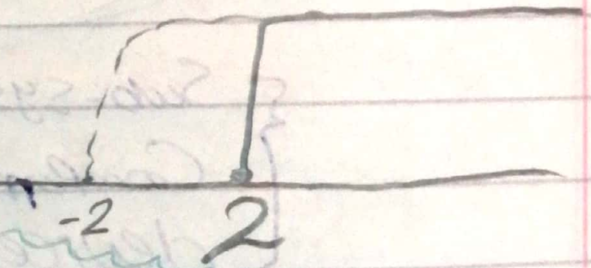
$$\begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases} = u(t)$$

$t<0$	$t \geq 0$
$t-\tau$	$t-\tau=0$
$\delta(t-\tau)=0$	$\delta(t-\tau)=1$

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$$

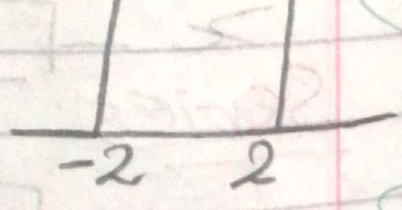
$$\delta(\tau-2)\ \cancel{d\tau} = u(t-2)$$

$$\delta(\tau+2)$$



$$\int_{-\infty}^{t} \delta(t+2) - \delta(t-2)\,dt$$

$$\rightarrow u(t+2) - u(t-2)$$

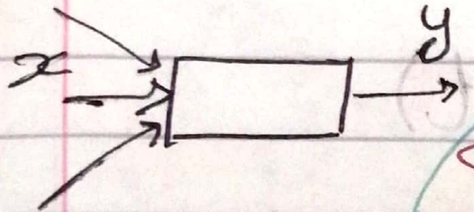


افكار السنة الي فاتت

$\sum_{k=} \square = \sum_{l=} \square$

Counter
dummy variable

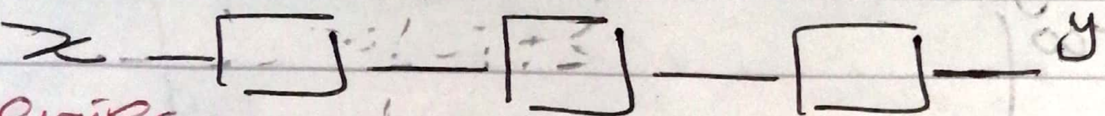
* Systems *



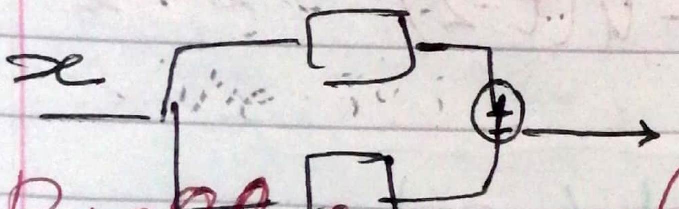
S/W
(software)

H/W
(Hardware)

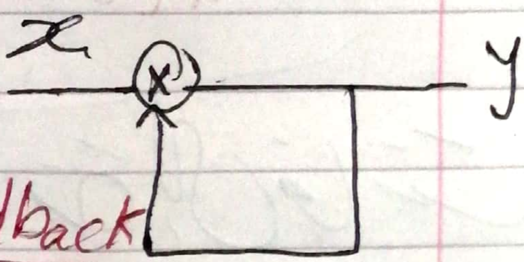
- Sub-systems
- Components
- devices



Series



Parallal



feedback

[1] Memoryless :-

الـ output $y(t)$ عند أي لحظة زمنية يعتمد فقط ع الـ input في نفس اللحظة الزمنية ,

$t = t_0$ ——— $x(t)$

$t = t_0$

System function

equ. $y = f(x)$

$$y(t) = 3 - 5\,x(t)$$

output ... input

Counter example

7 values

<-1

$-1<$, 0 , >0 , <1 , >1

$-1 \qquad 0 \qquad 1$

$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{\pi}{4}$

$j, -j$

Time delay
Time advanced
Time reverse

Not Memoryless

Variable Gain

$$y[n] = \cos[n+3] \cdot x[n]$$

$n=0$ $\quad y[0] = \cos[3] \cdot x[0]$

$y[1] = \cos[4] \cdot x[1]$

$y[-1] = \cos[2] \cdot x[-1]$

$y[2] = \cos[5] \cdot x[2]$

$y[-2] = \cos[1] \cdot x[-2]$

$y[\frac{1}{2}] = \cos[\frac{3}{2}] \cdot x[\frac{1}{2}]$

$y[-\frac{1}{2}] = \cos[\frac{5}{2}] \cdot x[-\frac{1}{2}]$.

$$y[n] = \sum_{m=-\infty}^{n} x[m]$$

$= \dots + x[n-1] + x[n]$

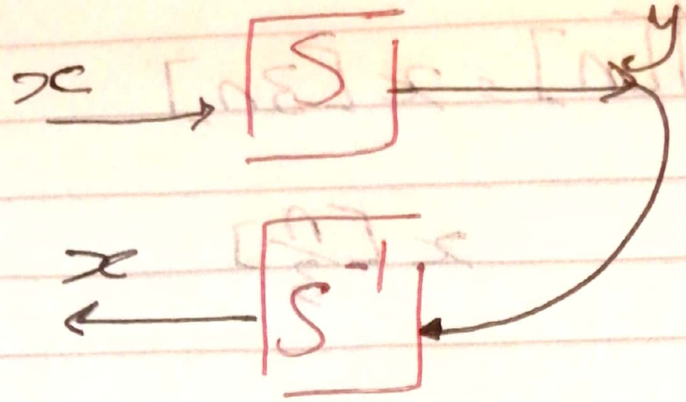
$$y[n] = x[-n] + 3$$

$y[0] = x[0] + 3$

$y[1] = x[-1] + 3$

$\therefore$ The system

2 Invertibility



$\Leftarrow y(t) = 2x(t)$

for $x_1 \neq x_2 \Longrightarrow y_1 \neq y_2$

Invertible

$\omega(t) = \frac{1}{2} y(t) = \frac{1}{\cancel{2}} \cdot \cancel{2} x(t) = x(t)$

$y[n] = (x[n])^2$

$\begin{cases} 4 \leftarrow 2 \\ 4 \leftarrow -2 \end{cases}$ Non-Invertible

$y(t) = t$