Signals and Systems

Lecture # 2

Signals Classification and Transformation

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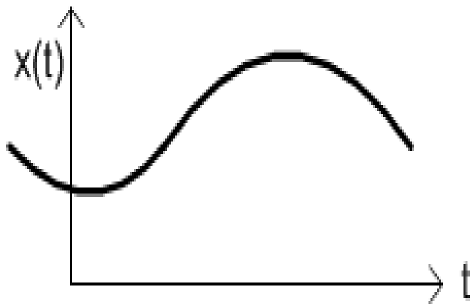
Topics of the lecture:

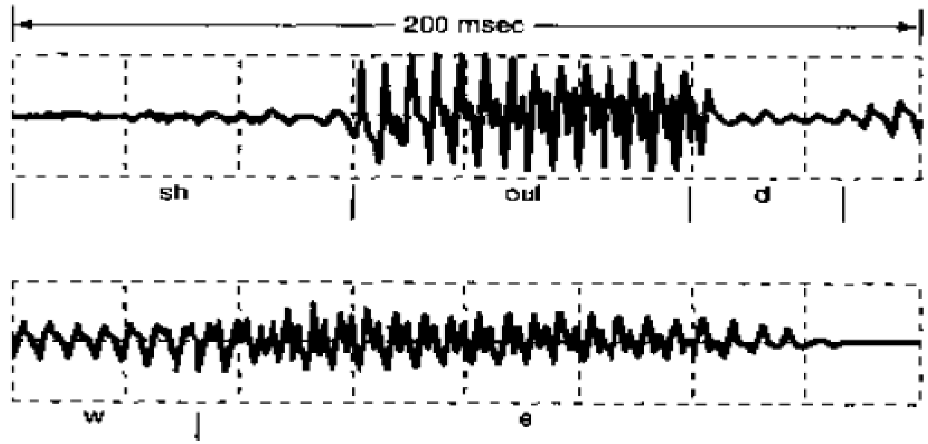
- **Signals Classification according to the Independent Variable.**
- **Signal’s Energy and Power.**
- **Signals Transformations of Independent Variable.**

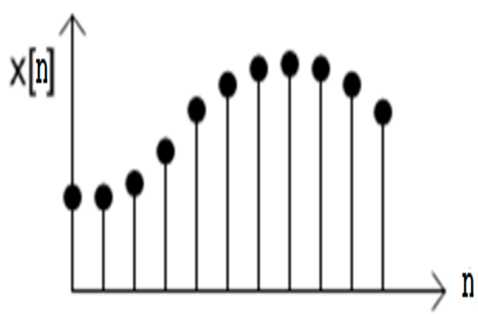
➢ Signal Classification according the independent variable

Signals are either *continuous in time* , that are defined at any time instant in its time domain (e.g. voltages and currents in electrical circuits or sound signals), OR, *discrete in time* , that are defined at integer time instants only in its time domain(e.g. closed stock market average , crime rate , or total population).

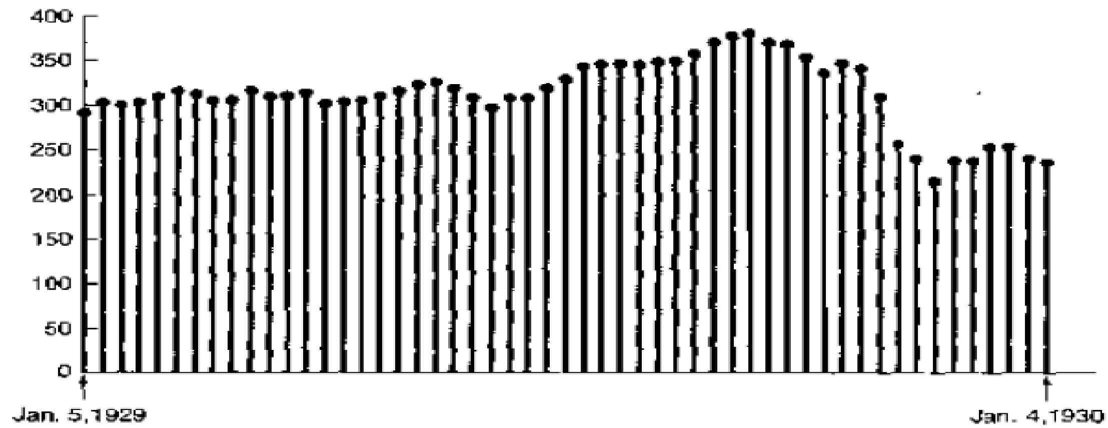


Continuous-Time





Discrete-Time



An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

➢ Signal Classification according the independent variable

	Continuous-Time Signals	Discrete-Time Signals
- **Domain:**	Its Domain is a continuous interval of Time $f(t)=x \quad , \quad t1 < t < t2$	Its Domain is a sequence of Time samples $f[n] = y \quad ,$ $n \in \{ n1 , n1+1 , n1+2 , \ldots, n2\}$
- **Range:**	Its range is a real valued e.g. $f(t)=1.43$	Its range is real valued e.g. $f[n]=9.32$
- **Symbol of IV:**	t	n
- **Function form:**	( )	[ ]
- **Examples:**	• Speech Signal • Voltage and Current	• Digital Images • Stock Market Index

- **In some books, if the domain and range are discrete the signal is called digital signal**

Signal's Energy and Power

- As too many signals are related to ***physical quantities capturing energy and power***, it is useful to define and measure the signal's energy and power.

$$in\ electrical\ circuit,\ the\ power\ consumed:$$

$$P(t) = V(t).I(t) = \frac{1}{R}V^2(t), \qquad V\ is\ the\ voltage$$

$$in\ autombile,\ the\ power\ consumed\ through\ friction:$$

$$P(t) = b.V^2(t), \qquad V\ is\ the\ automobile\ speed$$

- In the above two examples, though they are different they have **something in common**, that is the ***power is*** a ***constant*** (that **could be ignored** for analysis purposes) ***times a square of the system variable***.

Signal's Energy and Power

Energy: **is the capacity for doing work. You must have energy to accomplish work - it is like the "currency" for performing work.**

Power: **is the rate of doing work or the rate of using energy.**

- For most of this class we will use a broad definition of power and energy that applies to any signal $x(t)$ or $x[n]$
- **Instantaneous signal power**

$$P(t) = |x(t)|^2 \qquad\qquad P[n] = |x[n]|^2$$

- **Signal energy**

$$E(t_0, t_1) = \int_{t_0}^{t_1} |x(t)|^2 \, \mathrm{d}t \qquad\qquad E(n_0, n_1) = \sum_{n=n_0}^{n_1} |x[n]|^2$$

- **Average signal power**

$$P(t_0, t_1) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} |x(t)|^2 \, \mathrm{d}t$$

$$P(n_0, n_1) = \frac{1}{n_1 - n_0 + 1} \sum_{n=n_0}^{n_1} |x[n]|^2$$

|.| means magnitude of possibly complex valued x(t) or x[n]

➢ Signal's Energy and Power

Usually, the limits are taken over an infinite time interval to get the total Energy:

$$E_\infty \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt = \int_{-\infty}^{+\infty} |x(t)|^2\, dt,$$

in discrete time → $E_\infty \triangleq \lim_{N\to\infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2$

and total Average Power: $P_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$

The division may lead to undefined limits

in discrete time → $P_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$

➢ Signal's Energy and Power

We will encounter many types of signals :

- Some have infinite average power, energy, or both
- A signal is called an **energy signal** if $E_\infty < \infty$
- A signal is called a **power signal** if $0 < P_\infty < \infty$
- A signal can be an energy signal, a power signal, or neither type
- A signal can not be both an energy signal and a power signal

Signal Energy & Power Tips

- There are a few rules that can help you determine whether a signal has finite energy and average power
- Signals with finite energy have zero average power: $E_\infty < \infty \Rightarrow P_\infty = 0$
- Signals of finite duration and amplitude have finite energy: $x(t) \leq K$ for $|t| > c$, $K < \infty \Rightarrow E_\infty < \infty$
- Signals with finite average power have infinite energy: $P_\infty > 0 \Rightarrow E_\infty = \infty$

Signal's Energy and Power

Determine whether the energy and average power of each of the following signals is finite.

a- $$x(t) = \begin{cases} 8 & |t| < 5 \\ 0 & \text{otherwise} \end{cases}$$

b- $$x[n] = j$$

c- $$x[n] = A\cos(\omega n + \phi)$$

d- $$x(t) = \begin{cases} e^{at} & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

e- $$x[n] = e^{j\omega n}$$

➢ Signal’s Energy and Power

a-

$$E_\infty = \int_{-\infty}^{+\infty} |x(\sigma)|^2 d\sigma = \int_{-5}^{+5} |8|^2 d\sigma = \int_{-5}^{+5} 64 d\sigma = 640$$

$$P_\infty = 0 \quad as\ E_\infty\ is\ finite$$

b-

$$E_\infty = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |j|^2 = \sum_{n=-\infty}^{+\infty} 1^2 = \infty$$

$$P_\infty = \lim_{N->\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 = \lim_{N->\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |j|^2$$

$$= \lim_{N->\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} 1^2 = \lim_{N->\infty} \frac{1}{2N+1} (2N+1)$$

$$= \lim_{N->\infty} 1 = 1$$

➢ Signal's Energy and Power

a-

$$x(t) = \begin{cases} 8 & |t| < 5 \\ 0 & \text{otherwise} \end{cases}$$

➢ Signal's Energy and Power

b- $x[n] = j$

➢ Signal's Energy and Power

C− $$x[n] = A\cos(\omega n + \phi)$$

➢ Signal's Energy and Power

d−

$$x(t) = \begin{cases} e^{at} & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

➢ Signal's Energy and Power

e-

$$x[n] = e^{j\omega n}$$