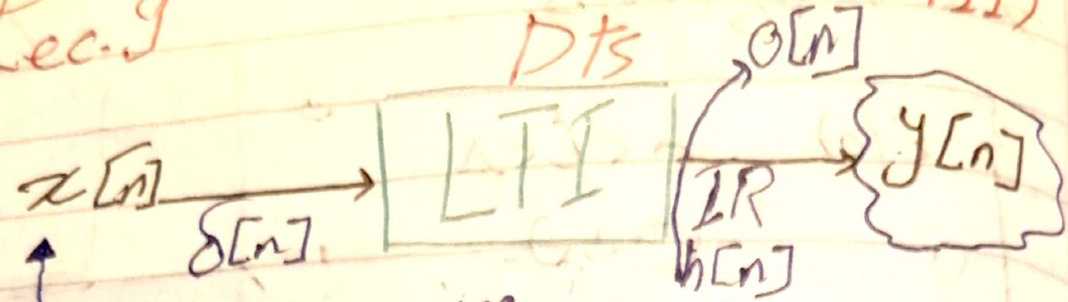
Lec. 9 (3/11)

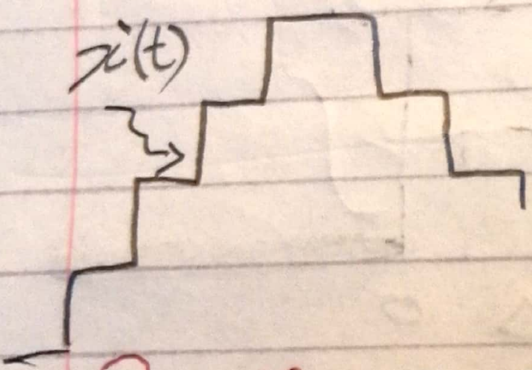
Dts

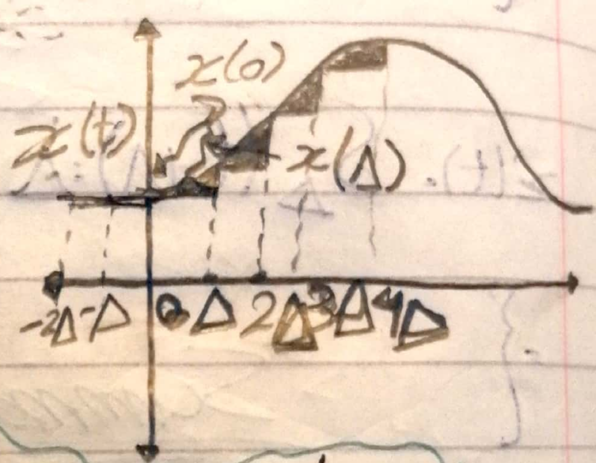


$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

Impulse Response

Cts

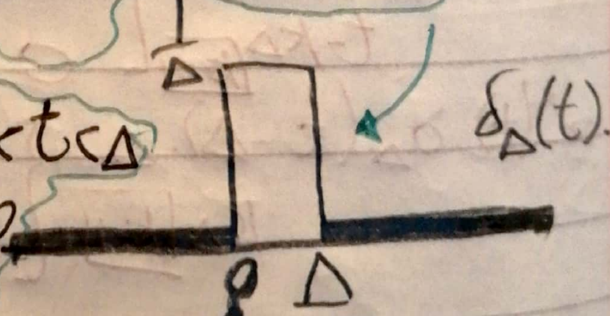


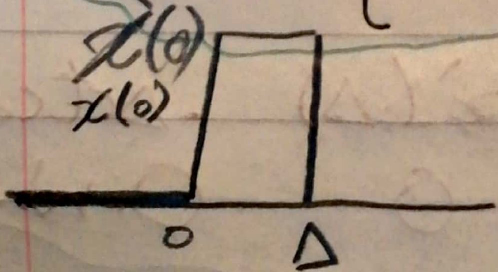


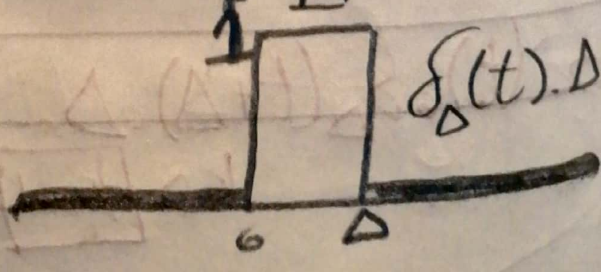
Recall

$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & 0<t<\Delta \\ 0 & o.w. \end{cases}$$

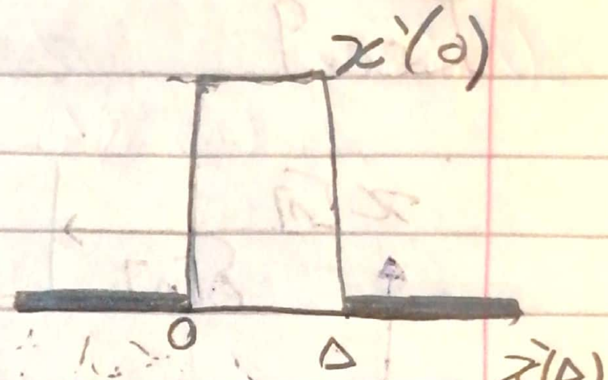
$$\delta_\Delta(t)\cdot\Delta = \begin{cases} 1 & 0<t<\Delta \\ 0 & o.w. \end{cases}$$





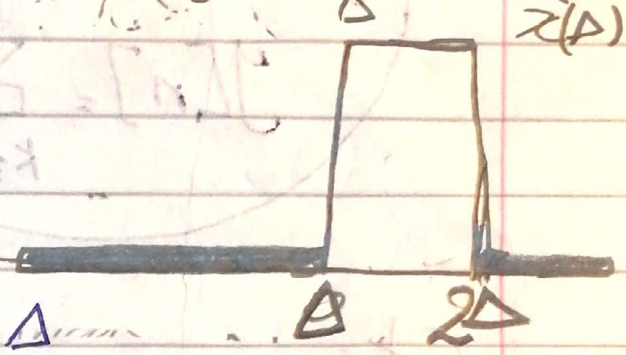


$\hat{x}(t) \cdot \delta_\Delta(t) \cdot \Delta$



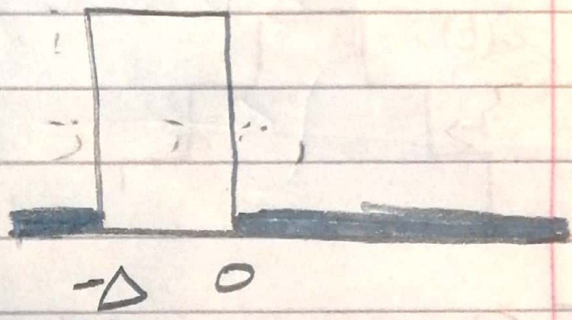
$$= \begin{cases} \hat{x}(0) & 0<t<\Delta \\ 0 & \text{O.W.} \end{cases}$$

$\hat{x}(t) \cdot \delta_\Delta(t-\Delta) \cdot \Delta$



$$= \begin{cases} \hat{x}(\Delta) & \Delta<t<2\Delta \\ 0 & \text{O.W.} \end{cases}$$

$\hat{x}(t) \cdot \delta_\Delta(t+\Delta) \cdot \Delta$



$= \{$

Summary

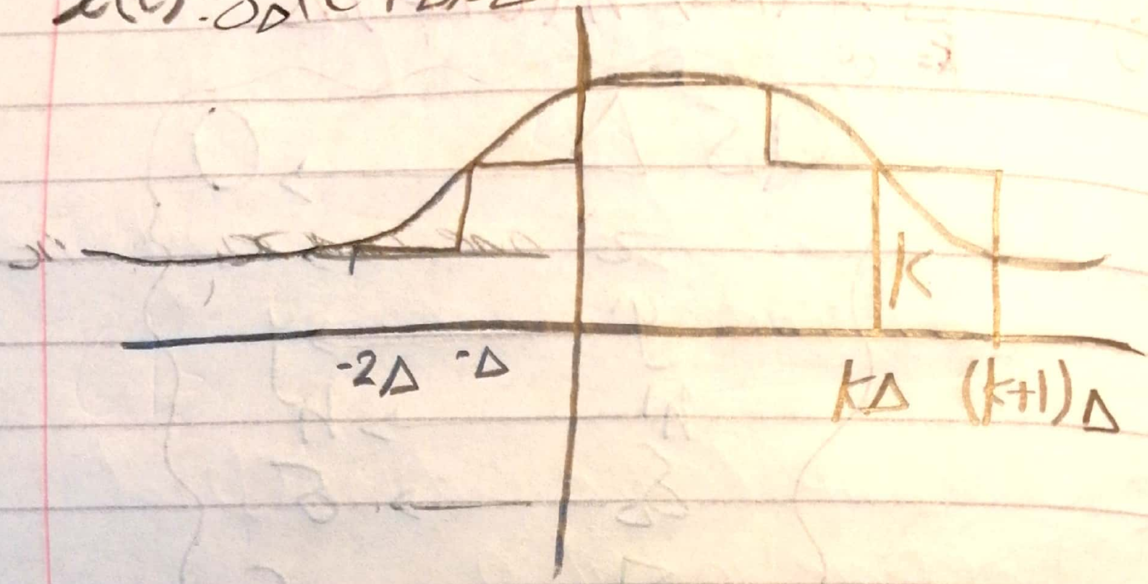
$$\hat{x}(t) \cdot \delta_\Delta(\underbrace{t}_{t=k\Delta\ \boxed{k=0}}) \cdot \Delta = \begin{cases} x(0) & 0<t<\Delta \\ 0 & \text{O.W.} \end{cases}$$

$$\hat{x}(t) \cdot \delta_\Delta(\underbrace{t-\Delta}_{k\Delta\ \boxed{k=1}}) \cdot \Delta = \begin{cases} \hat{x}(\Delta) & \Delta<t<2\Delta \\ 0 & \text{O.W.} \end{cases}$$

$$\hat{x}(t) \cdot \delta_\Delta(\underbrace{t+\Delta}_{k\Delta\ \boxed{k=-1}}) \cdot \Delta = \begin{cases} \hat{x}(-\Delta) & -\Delta<t<0 \\ 0 & \text{O.W.} \end{cases}$$

$$x'(t).\delta_\Delta(t-k\Delta).\Delta = \begin{cases} x'(k\Delta) & k\Delta < t < (k+1)\Delta \\ 0 & o.w. \end{cases}$$

$x'(t).\delta_\Delta(t-k\Delta).\Delta$



$$-\infty < k < +\infty$$

$$\sum_{k=-\infty}^{+\infty} x'(t)\delta_\Delta(t-k\Delta).\Delta = x'(t)$$

LTI

I/P $x \longrightarrow y$ O/P

$x' \longrightarrow y'$

$\delta \longrightarrow h$

$\delta' \longrightarrow h'$

$\delta_\Delta(t-k\Delta) \xrightarrow{TI} h'(t-k\Delta)$

$$\sum_{k=-\infty}^{+\infty} x'(k\Delta) \cdot \delta_\Delta(t-k\Delta)\Delta = \longrightarrow \sum_{k=-\infty}^{+\infty} x'(k\Delta) \cdot h'(t-k\Delta) \cdot \Delta$$

$$= y'(t) = \sum_{k=-\infty}^{+\infty} x'(k\Delta) \cdot h'(t-k\Delta) \cdot \Delta$$

as $\Delta \longrightarrow 0$

$x' \longrightarrow x$

$y' \longrightarrow y$

$h' \longrightarrow h$

$\delta_\Delta \longrightarrow \delta$

$\sum \longrightarrow \int$

$$y'(t) = \sum_{k=-\infty}^{+\infty} x'(k\Delta) h'(t-k\Delta) \cdot \Delta$$

$\boxed{\text{as } \Delta \to 0}$

$$\lim_{\Delta \to 0} y'(t) = \lim_{\Delta \to 0} \sum$$

$$y(t) = \int_{-\infty}^{+\infty} x(k\Delta)\, h(t-k\Delta)\Delta$$

$$y(t) = \int_{-\infty}^{+\infty} x(k\Delta)\, h(t-k\Delta)\, \Delta$$

$k\Delta = \tau \qquad \Delta dk = d\tau$

$dk = 1$ as k is the Counter of the summation.

$$\boxed{y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] \Longrightarrow$$ Convolution Sum formula.

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau \Longrightarrow$$ Convolution integral formula.

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$-k+n$

① Renaming of I.V.

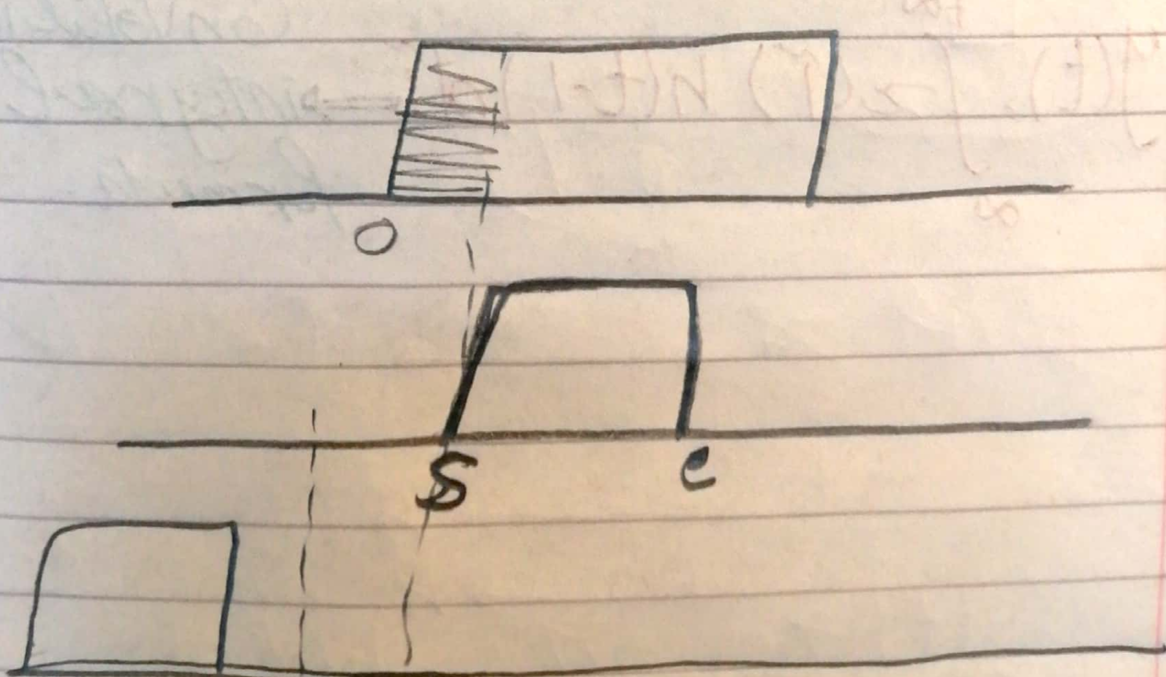
$x[n] \_ x[k]$

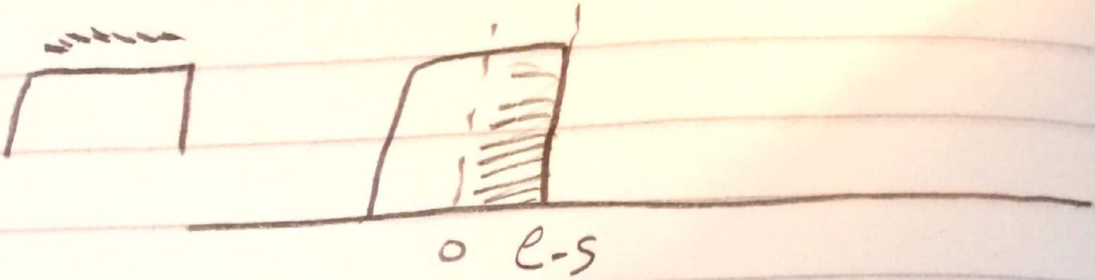
$h[n] \_ h[k]$

② Time-Reverse for one and only one of the Signals

③ To Compute the output (y) at any time instant n

- Time-shift by n
- Multiplication.
- Summation.

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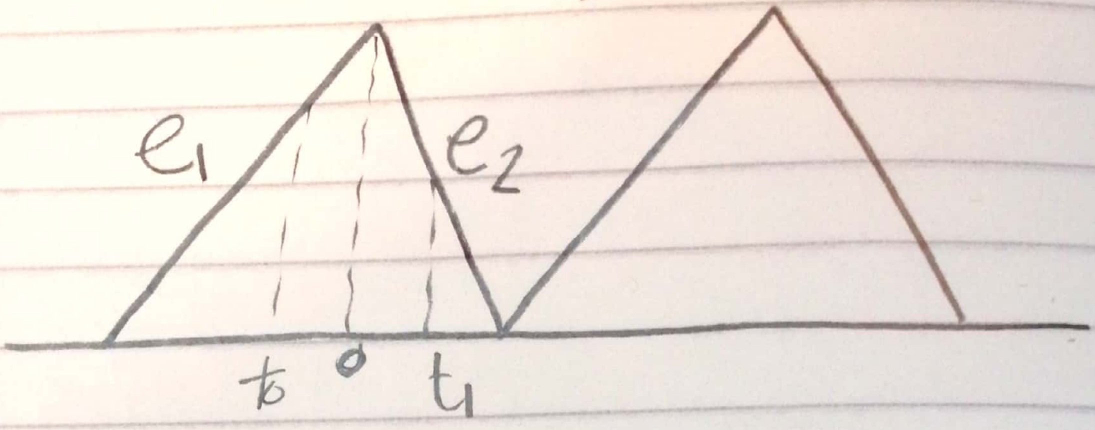
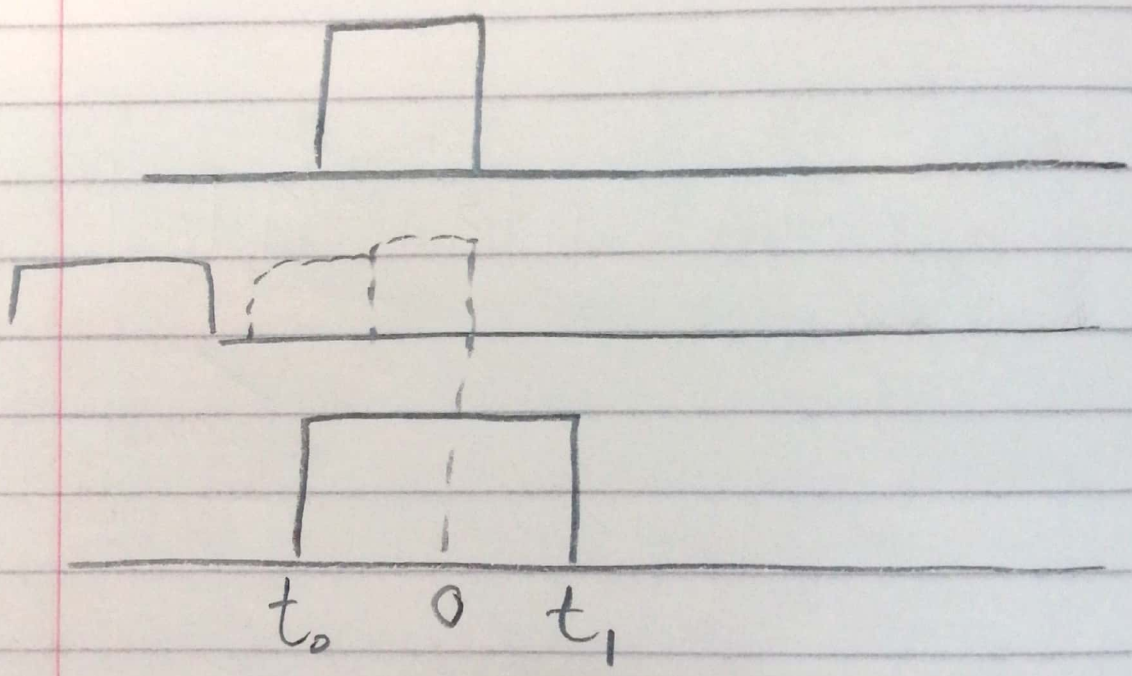

e1
e2
t0
0
t1


t0
0
t1