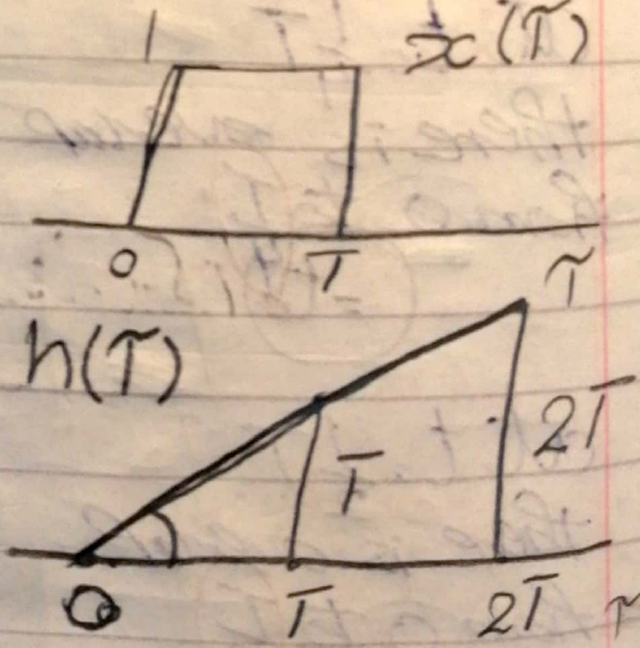
Lec. 11 10/11

$$x(t) = \begin{cases} 1 & 0 < t < T \\ 0 & \text{o.w.} \end{cases}$$

$$h(t) = \begin{cases} t & 0 < t < 2T \\ 0 & \text{o.w.} \end{cases}$$

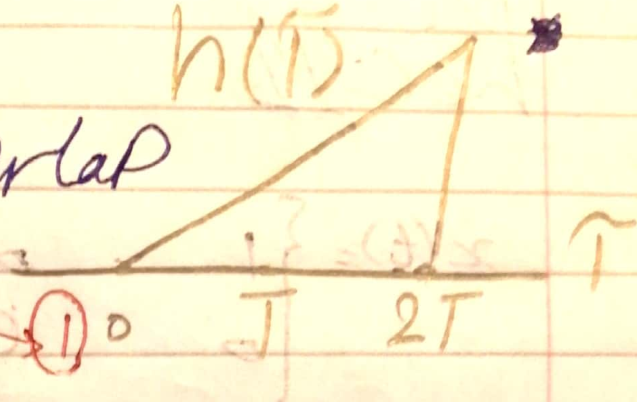
$$y(t) = x(t) * h(t) \,??$$



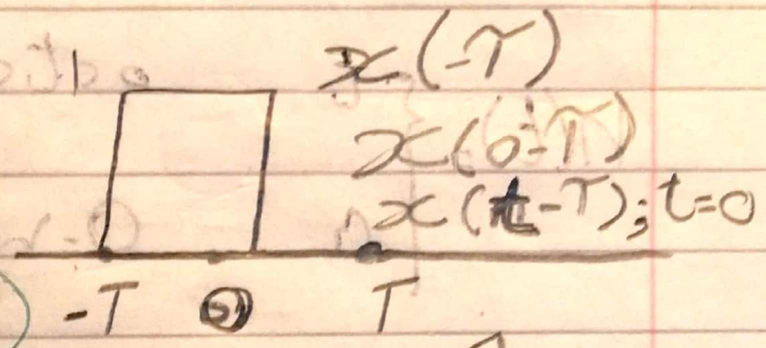
- for $t<0$

there is No overlap

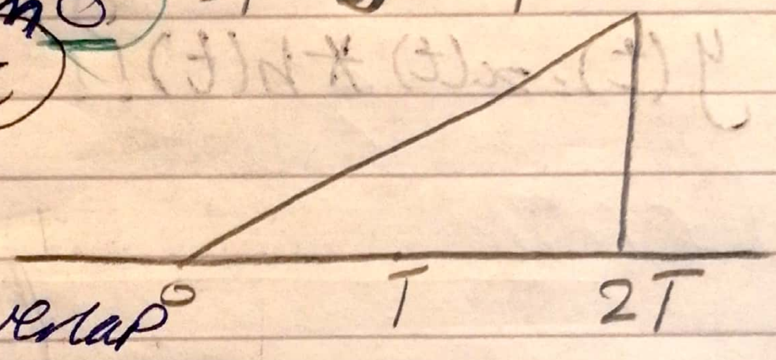
$\boxed{\therefore y(t)=0 \; ; \; t<0}$



- for $0<t<T$

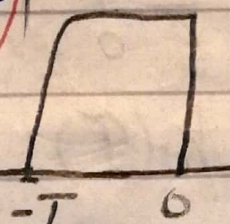


at $t=0$ there is overlap from 0 to 0 $=t$



at $t=\frac{1}{4}T$

there is overlap from 0 to $\frac{T}{4}$ $=t$

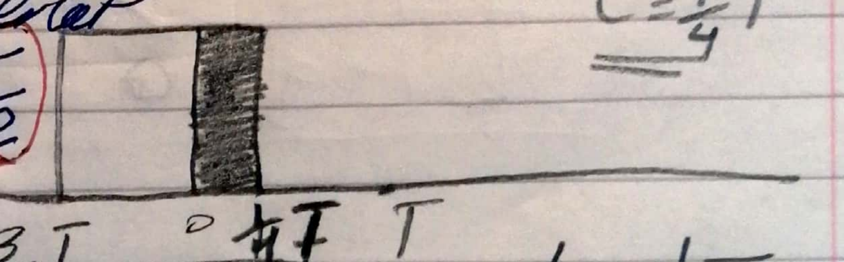


at $t=\frac{1}{2}T$

there is overlap from 0 to $\frac{T}{2}$ $=t$

$x(t-\tau)$

$t=\frac{1}{4}T$

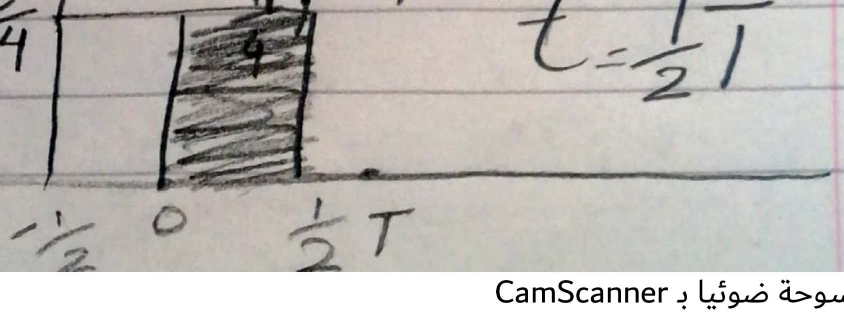


Low Limit

$\boxed{L=0}$

$\boxed{u=t}$

$t=\frac{1}{2}T$



بالتعويض

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{0}^{4} \tau \cdot 1 \, d\tau$$

$$= \int_{0}^{t} \tau \, d\tau = \left.\frac{\tau^2}{2}\right|_{0}^{t}$$

$$= \frac{t^2}{2} - 0 = \frac{t^2}{2}$$

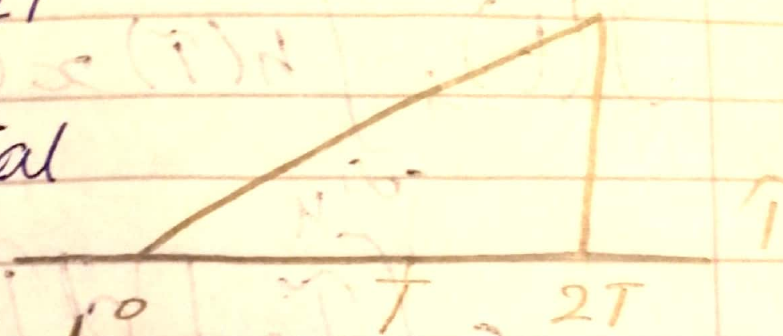
$$\boxed{\therefore y(t) = \frac{t^2}{2} \; ; \; 0 < t < T} \rightarrow (2)$$

- for $T \leq t \leq 2T$

there is a total overlap.

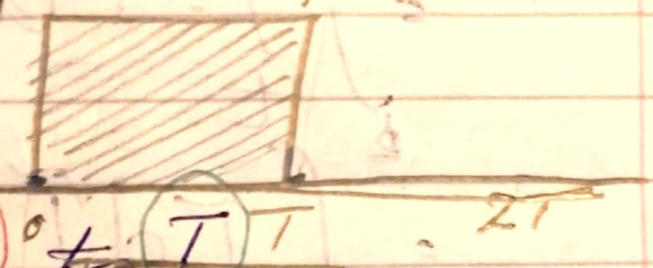
- for $T \le t \le 2T$

there is a total overlap



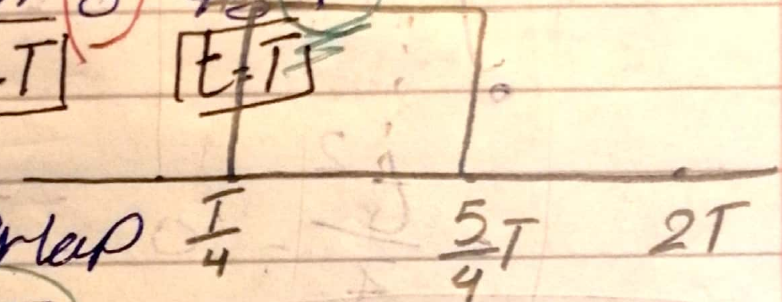
at $t = T$ there is overlap from 0 to T

$\boxed{t-T}$ $\boxed{t=T}$



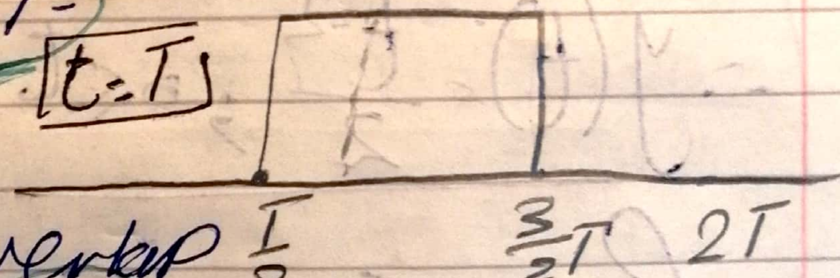
at $t = 1\frac{1}{4}T$ there is overlap from $\frac{T}{4}$ to $\frac{5}{4}T$

$\boxed{t-T}$ $\boxed{t=T}$



at $t = 1\frac{1}{2}T$ there is overlap from $\frac{T}{2}$ to $\frac{3}{2}T$

$\boxed{t-T}$ $\boxed{t=T}$



$\boxed{L = t - T}$ $\boxed{U = t}$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{t-T}^{t} \tau \cdot 1 \, d\tau =$$

$$y(t) = \frac{\tau^2}{2} \Big|_{t-T}^{t}$$

$$= \frac{1}{2}\left[t^2 - (t-T)^2\right] = \frac{1}{2}\left[t^2 - (t^2 - 2tT + T^2)\right]$$

$$= \frac{1}{2}\left[\cancel{t^2} - \cancel{t^2} + 2tT - T^2\right]$$

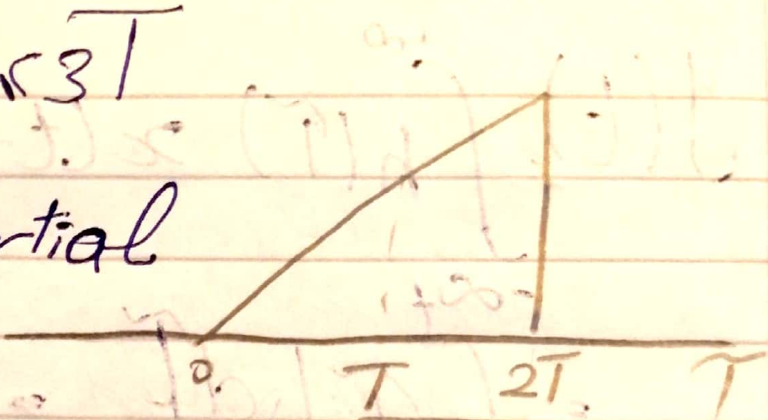
$$= \frac{1}{2}\left(2tT - T^2\right) = \frac{1}{2} T (2t - T)$$

$$= tT - \frac{T^2}{2}$$

$$\boxed{\therefore y(t) = tT - \frac{T}{2} \; ; \; T < t \leq 2T} \rightarrow 3$$

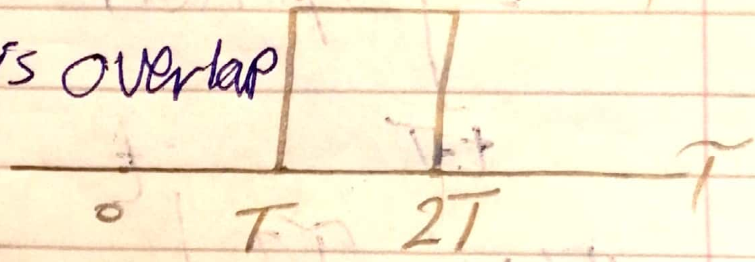
for $2T<t<3T$

there is Partial overlap



at $t=2T$ there is overlap from T to $2T$

$\boxed{t-T}$



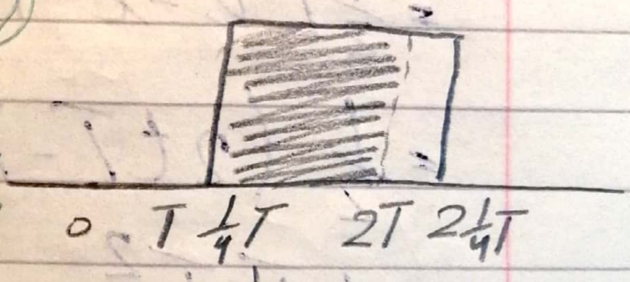
at $t=2\frac{1}{4}T$ there is overlap from $1\frac{1}{4}T$ to $2T$

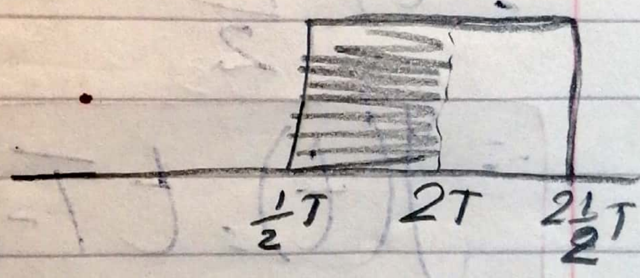
$\boxed{t-T}$

at $t=2\frac{1}{2}T$ there is overlap from $1\frac{1}{2}T$ to $2T$

$\boxed{t-T}$

$\boxed{L=t-T}$ $\boxed{U=2T}$





$$\therefore y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{t-T}^{2T} \tau \cdot 1 \, d\tau$$

$$\therefore y(t) = \frac{\tau^2}{2} \Big|_{t-T}^{2T}$$

$$= \frac{1}{2}\left[4T^2 - t^2 + 2tT - T^2\right]$$

$$= \frac{1}{2}\left[3T^2 - t^2 + 2tT\right]$$

$$= \frac{3}{2}T^2 + tT - \frac{t^2}{2}$$

$$\boxed{\therefore y(t) = \frac{3}{2}T + tT - \frac{t^2}{2}} \rightarrow (4)$$

- for ~~t>3t~~ $t > 3T$

there is no overlap

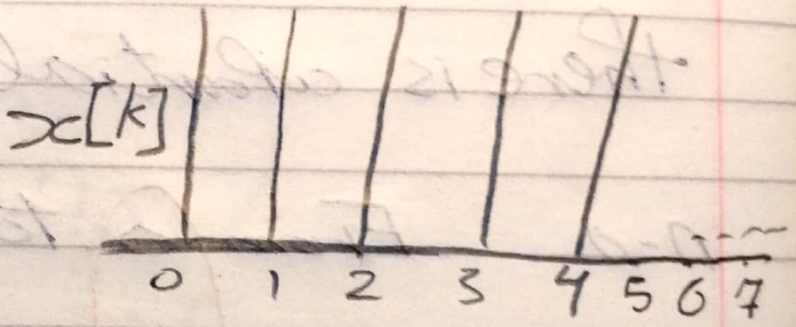
$$\boxed{\therefore y(t) = 0 \, ; \, t > 3T} \Rightarrow (5)$$

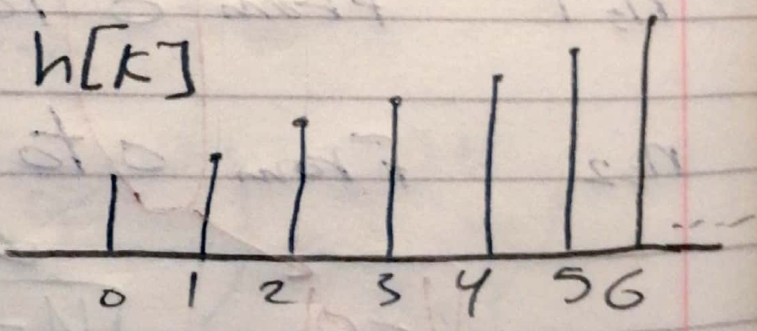
$$
\therefore y(t) \begin{cases} 0 & ; \; t<0 \\ \frac{t^2}{2} & ; \; 0 \leq t < T \\ tT - \frac{t^2}{2} & ; \; T \leq t \leq 2T \\ \frac{3T^2}{2} + tT - \frac{t^2}{2} & ; \; 2T < t \leq 3T \\ 0 & ; \; t > 3T \end{cases}
$$

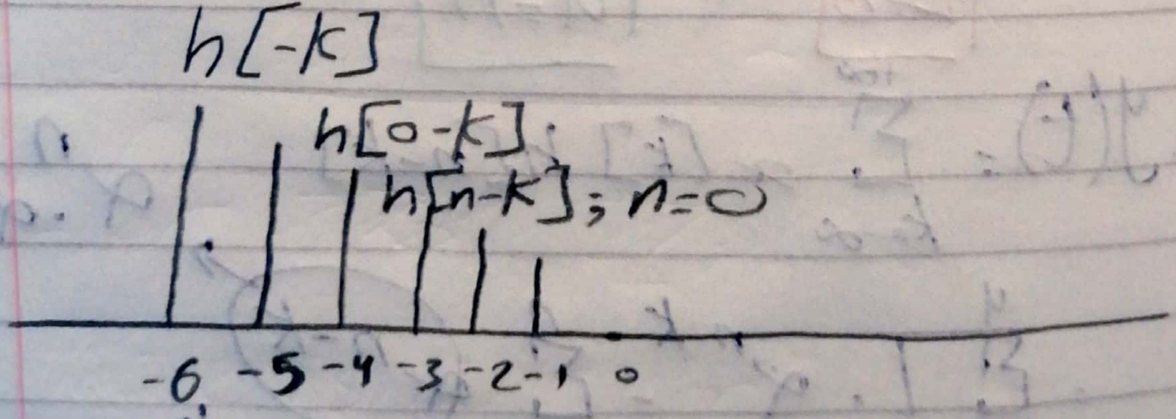
Q2 : $x[n] = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{O.W.} \end{cases}$

$h[n] = \begin{cases} \alpha^n & 0 \le n \le 6 \\ 0 & \text{O.W.} \end{cases}$

$\boxed{\alpha > 1}$







- for $n<0$

there is No overlap

$\boxed{\therefore y[n] = 0 \; ; \; n<0}$ → ①

- for $0 \le n < 4$

there is a partial overlap.

$n=0$ From 0 to 0 $(=n)$

$n=1$ From 0 to 1 $(=n)$

$n=2$ From 0 to 2 $(=n)$

$\boxed{L=0}$ $\boxed{U=n}$

$$\therefore y(t) = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$$\alpha^{n} \cdot \alpha^{-k}$$

$$= \sum_{k=L}^{U} 1 \cdot \alpha^{n-k} = \sum_{k=0}^{n} \alpha^{n-k}$$

$$= \alpha^{n} \sum_{k=0}^{n} \alpha^{-k} = \alpha^{n} \sum_{k=0}^{n} \left(\frac{1}{\alpha}\right)^{k}$$

Recall $$\sum_{k=a}^{b} r^k = r^a \cdot \frac{1-r^{b-a+1}}{1-r}$$

$$\alpha^n \sum_{k=0}^{n} \left(\frac{1}{\alpha}\right)^k = \alpha^n \left[\underbrace{\left(\frac{1}{\alpha}\right)^0}_{1} \cdot \frac{1-\left(\frac{1}{\alpha}\right)^{n+1}}{1-\left(\frac{1}{\alpha}\right)} \right]$$

$$= \alpha^n \left(\frac{1-\left(\frac{1}{\alpha}\right)^{n+1}}{1-\left(\frac{1}{\alpha}\right)} \right)$$

$$= \alpha^n \left(\frac{1-\alpha^{-n-1}}{1-\alpha^{-1}} \right) \cdot \frac{\alpha}{\alpha}$$

$$= \alpha^n \left(\frac{\alpha-\alpha^{-n}}{\alpha-1} \right) = \frac{\alpha^{n+1}-1}{\alpha-1}$$