Lec. 12 15/11

(LTI Systems) Properties

[1] Commutative:- $y[n] = x[n] * h[n]$

$$y[n] \triangleq \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k] \longrightarrow (1)$$

$m = n-k \Longrightarrow k = n-m$

at $k = -\infty \Longrightarrow m = n-(-\infty) = n+\infty = +\infty$

at $k = +\infty \Longrightarrow m = n-\infty = -\infty$

$$\therefore y[n] = \sum_{m=+\infty}^{-\infty} x[n-m]\,h[m]$$

$$= \sum_{m=-\infty}^{+\infty} h[m]\,x[n-m]$$

Rename:- $y[n] = \sum_{k=-\infty} h[k]\,x[n-k]$

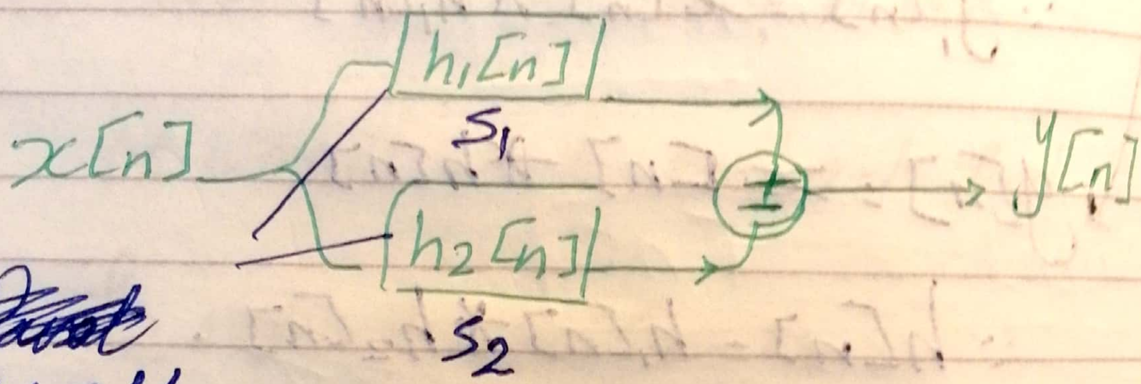
$= h[n] * x[n] \longrightarrow (2)$

$y[n]$

[2] Distributive:- توزيع

$$y[n] = x[n] * \{h_1[n] \pm h_2[n]\}$$

$$= \underbrace{x[n] * h_1[n]}_{S_1} \pm \underbrace{x[n] * h_2[n]}_{S_2}.$$



~~Parat~~
Parell

$$h[n] = h_1[n] \pm h_2[n]$$

$$x[n] \longrightarrow \boxed{h_1[n] \pm h_2[n]} \longrightarrow y[n]$$

[3] Associative:- لدمج

$$y[n] = \underbrace{x[n] * h_1[n]} * h_2[n]$$

$$y[n] = y_1[n] * h_2[n]$$

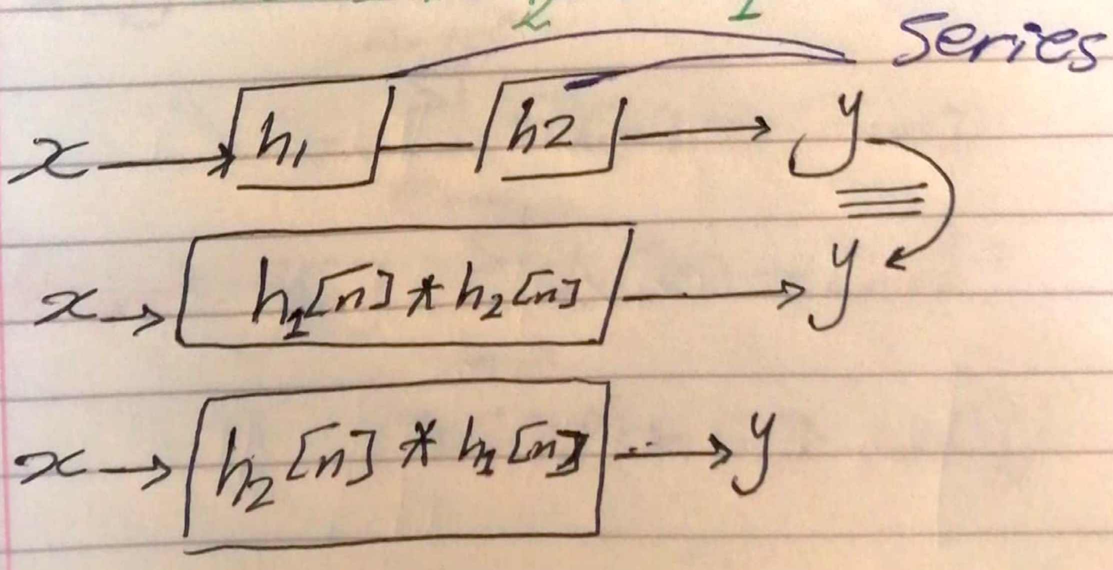
$$\because y_1[n] = x[n] * h_1[n]$$

$$y[n] = x[n] * h[n]$$

$$\because h[n] = h_1[n] * h_2[n] .$$

$$y[n] = x[n] * \underbrace{h_1[n] * h_2[n]}$$

$$= x[n] * h_2[n] * h_1[n] .$$



$$x \rightarrow \boxed{h_2[n]} \rightarrow \boxed{h_1[n]} \rightarrow y$$

[4] Memory less :- عديم الذاكرة

LTI

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, \underbrace{h[n-k]}$$

we need $h[n-k] = 0 \ \forall \ n \neq k$

re-write it to be $\boxed{n-k \neq 0}$

$\therefore$ The required condition is

$$h[n-k] = 0 \quad \forall \ n-k \neq 0$$

$$h[\text{عفريت}] = 0 \quad \forall \ \text{عفريت} \neq 0$$

$$\boxed{h[n] = 0 \quad \forall \ n \neq 0}$$

[5] Causality: LTI

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k].$$

طبيعية

$h[n-k] = 0 \quad \forall\ k > n$

$\boxed{n-k < 0}$

طبيعية

$$\boxed{h[n] = 0 \quad \forall\ n < 0}$$

[6] Stability: LTI

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k].$$

طبيعية طبيعية

if $|x[n]| \leq \mathbb{K}\ ;\ \mathbb{K} < \infty\ ;\ \forall n$

$\therefore\ |x[n-k]| \leq \mathbb{K}.$

we need $|y[n]| < \infty$

$$\therefore |y[n]| = \left| \sum_{k=-\infty}^{+\infty} h[k] \cdot x[n-k] \right|$$

$\because |A+B| \le |A| + |B|$

$$\therefore |y[n]| \le \sum_{k=-\infty}^{+\infty} |h[k]x[n-k]|$$

$\because |A \cdot B| = |A| \cdot |B|$

$$\therefore |y[n]| \le \sum_{k=-\infty}^{+\infty} |h[k]| \, |x[n-k]| \quad \mathbb{K}$$

$$\le \sum_{k=-\infty}^{+\infty} |h[k]| \cdot \mathbb{K}$$

$$\therefore |y[n]| \le \mathbb{K} \cdot \sum_{k=-\infty}^{+\infty} |h[k]|$$

$\therefore$ the required Condition for LTI Sys. to be stable is

$$\boxed{\sum_{n=-\infty}^{+\infty} |h[n]| < \infty}$$

$\boxed{7}$ Invertibity:-

$$x \rightarrow \underset{h_1}{\boxed{S}} \xrightarrow{\;y\;} \underset{h_1^{-1}}{\boxed{S^{-1}}} \rightarrow x$$

$$x * \underbrace{\boxed{h_1 * h_1^{-1}}}_{\delta[n]} = x$$

$$\underset{h}{y[n]} = \underset{\delta}{x}[n-n_0]$$

$$\therefore h_1[n] = \delta[n-n_0]$$

$$\boxed{\omega[n] = y[n+n_0]} = x[n+\cancel{n_0}-\cancel{n_0}]$$

$$= x[n]$$

$$h_1^{-1} = \delta[n+n_0]$$

$$h_1[n] * h_1^{-1}[n]$$

$$\delta[n-n_0] \qquad \delta[n+n_0]$$

$$= \sum_{k=-\infty}^{+\infty} h_1[k]\, h_1^{-1}[n-k]$$

$$= \sum_{k=-\infty}^{+\infty} \underbrace{\delta[k-n_0]}_{\boxed{k=n_0}}\, h_1^{-1}[n-k+n_0]$$

$$\delta[n-\cancel{n_0}+\cancel{n_0}]$$

$$= 0+0+0+0+\delta[n]+0+0+0+\cdots$$

$$\boxed{\therefore h_1[n] * h_1^{-1}[n] = \delta[n]}$$