Name (... the fourth name): .... Degree:

ID: .... Midterm Exam allowed time: One Hour

Answer the following FOUR questions in the blank area (Four Pages)

[Q.1]

(a) The "System" is a tool that transform a signal to get another signal or process a signal to obtain a desired behavior or extracting a piece of information [1 degree]

The Fundamental Period is the smallest positive value for which a signal is repeated [1 degree]

$N = m\frac{2\pi}{w}$

(b) Consider the signal $x[n] = 1 + e^{j\frac{4\pi n}{7}} + e^{j\frac{4\pi n}{5}}$ is this signal periodic?
If it is periodic, what is its fundamental period? [3 degrees]

$x[n]$ can be re-written as

$x[n] = x_1[n] + x_2[n] + x_3[n]$

where $x_1[n] = 1$ = dc signal which is periodic with fundamental period $= \boxed{1 = N_1}$

and $x_2[n] = e^{j\frac{4\pi n}{7}}$ and $x_3[n] = e^{j\frac{4\pi n}{5}}$

$\therefore$ $x[n]$ will be peridic if it is a sum of periodic sub-signals.

For $x_2[n] = e^{j\frac{4\pi n}{7}}$:-

when compared with the general form of complex exponential signal e^{jwn}

$\therefore w_2 = \frac{4\pi}{7} \Rightarrow x_2[n]$ will be periodic if $\frac{w_2}{2\pi}$ is a rational number.

$\frac{w_2}{2\pi} = \frac{4\pi}{7} \times \frac{1}{2\pi} = \frac{2}{7}$ = rational number

$x_2[n]$ is periodic with fundamental period $= 7$ $\therefore \boxed{N_2 = 7}$

For $x_3[n] = e^{j\frac{4\pi n}{5}}$:-

when compared with the general form of complex exponential signal e^{jwn}

$\therefore w_3 = \frac{4\pi}{5} \Rightarrow x_3[n]$ will be periodic if $\frac{w_3}{2\pi}$ is a rational number

$\therefore \frac{w_3}{2\pi} = \frac{4\pi}{5} \times \frac{1}{2\pi} = \frac{1}{5}$ = rational number

$\therefore x_3[n]$ is periodic with fundamental period $\boxed{N_3 = 5}$

$\therefore x[n]$ = periodic signal + periodic signal + periodic signal

$\Rightarrow x[n]$ is periodic signal

The fundamental period of $x[n]$ will be

$N = LCM(N_1, N_2, N_3)$ (Least Common Multiplier)

$\therefore N = LCM(1, 7, 5) = \boxed{35}$

$\therefore$ the fundamental period of $x[n]$ is 35

[Q.2] If you have a system that has the following relationship between its input x[n] and its output y[n]

$$y[n] = n \cdot x[n]$$

[5 degrees]

Determine if this system is :

1- Memoryless ?   2- Causal ?   3- Stable ?   4- Time-Invariant ?   5- Linear ?

① $y[n] = n\, x[n]$

there are no time-shift, time-scaling, nor time-reverse, But this is NOT sufficient to prove that the system is memoryless, So we will check if there is a counter example, if any, through applying different values of (n) that cover all possibilities of an integer (n).

$n=0 \Rightarrow y[0] = 0 \cdot x[0] = 0$

$n=1 \Rightarrow y[1] = 1 \cdot x[1] = x[1]$

$n=-1 \Rightarrow y[-1] = (-1) \cdot x[-1] = -x[-1]$

$n=2\ (>1) \Rightarrow y[2] = 2 \cdot x[2] = 2\,x[2]$

$n=-2\ (<-1) \Rightarrow y[-2] = (-2) \cdot x[-2] = -2\,x[-2]$

we tried all possible values of (n) and we did not find a counter example, then it is not exist $\Rightarrow$ $\boxed{\text{the system is Memoryless}}$

② $\because$ the system is Memoryless as we explained in ① $\Rightarrow$ $\boxed{\text{the system is Causal}}$

③ let $\{|x[n]| \underset{\text{for all } n}{\leq} K\}$, K is +ve and $K < \infty$

then $|y[n]| = |n\, x[n]|$

$\because |A \cdot B| = |A| \cdot |B|$

$\therefore |y[n]| = |n| \cdot |x[n]| = K \cdot |n|$

as $n = \pm\infty \Rightarrow |y[\infty]| = K \cdot \infty = \infty$

$\therefore$ although $|x[\infty]| = K$ as assumed, the $|y[\infty]| = \infty$ = not bounded

$\therefore$ Bounded input Leads to non-bounded output $\Longrightarrow$ $\boxed{\text{the system is not stable}}$

④ Let $x_1[n] \xrightarrow{S} y_1[n] = n \cdot x_1[n]$

and let $x_2[n] \xrightarrow{S} y_2[n] = n\, x_2[n]$

if $x_2[n] = x_1[n-n_0] \xrightarrow{S} y_2[n] = n \cdot x_1[n-$

$y_1[n-n_0] = (n-n_0) \cdot x_1[n-n_0] \neq y_2[n$

$\therefore$ $\boxed{\text{the system is Not time-invari}}$

⑤ let $x_1[n] \xrightarrow{S} y_1[n] = n\, x_1[n]$

and $x_2[n] \xrightarrow{S} y_2[n] = n\, x_2[n]$

and let $x_3[n] = a\,x_1[n] + b\,x_2[n] \xrightarrow{S}$

$y_3[n] = n\,x_3[n] = a\,n\,x_1[n] + b\,n\,x_2$

$= a\, y_1[n] + b\, y_2[$

$\therefore$ Linear combination of inputs lead the same Linear combination of ou

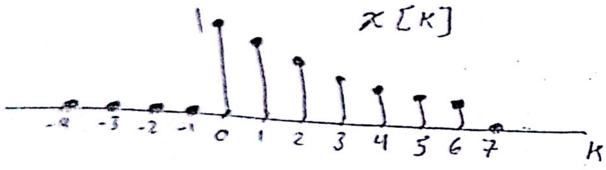
$\therefore$ $\boxed{\text{the system is Linear}}$

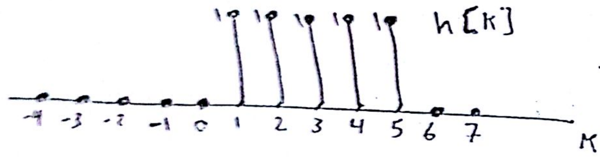
[Q.3] Suppose that $x[n] = \begin{cases} \alpha^n, & 0 \le n \le 6 \\ 0, & otherwise \end{cases}$; $0 < \alpha < 1$ and $h[n] = \begin{cases} 1, & 1 \le n \le 5 \\ 0, & otherwise \end{cases}$

(a) Compute and Plot $y[n] = x[n] * h[n]$? [4 degrees]

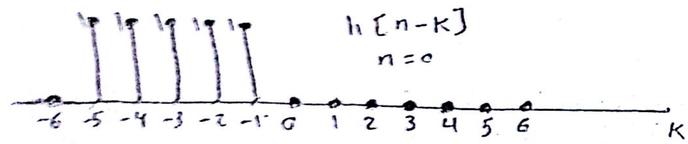
(b) If $h[n]$ is the impulse response of the system, is that system stable? [1 degree]

(a) - Re-name the independent variable to be (k) then





- Time-reverse h[k]



- For $n < 1$ there is no overlapping between $x[k]$ and $h[n-k]$

$\therefore$ $\boxed{y[n] = 0 \quad \text{for } n < 1}$ ⇒ I

- For $1 \le n \le 4$ there is a partial overlapping between $x[k]$ and $h[n-k]$

$\therefore y[n] = \sum_{k=L}^{U} x[k]\, h[n-k]$

at $n=1$ → overlapping from 0 to 0
at $n=2$ → " " 0 to 1
at $n=3$ → " " 0 to 2

$\therefore$ $\boxed{L = 0}$ and $\boxed{U = n-1}$

$\therefore y[n] = \sum_{k=0}^{n-1} \alpha^k \cdot 1 = \sum_{k=0}^{n-1} \alpha^k = \frac{\alpha^0[1-\alpha^n]}{1-\alpha}$

$\therefore$ $\boxed{y[n] = \frac{1-\alpha^n}{1-\alpha}, \quad 1 \le n \le 4}$ ⇒ II

- For $5 \le n \le 7$ there is a total overlapping between $x[k]$ and $h[n-k]$

$\therefore y[n] = \sum_{k=L}^{U} x[k]\, h[n-k]$

at $n=5$ → overlapping from 0 to 4
at $n=6$ → " " 1 to 5
at $n=7$ → " " 2 to 6

$\therefore$ $\boxed{L = n-5}$ and $\boxed{U = n-1}$

$\therefore y[n] = \sum_{k=n-5}^{n-1} \alpha^k \cdot 1 = \sum_{k=n-5}^{n-1} \alpha^k = \frac{\alpha^{n-5}[1-\alpha^5]}{1-\alpha}$

$\therefore$ $\boxed{y[n] = \frac{\alpha^{n-5} - \alpha^n}{1-\alpha} ; \quad 5 \le n \le 7}$ ⇒ III

- For $8 \le n \le 11$ there is another partial overlapping between $x[k]$ and $h[n-k]$

$y[n] = \sum_{k=L}^{U} x[k]\, h[n-k]$

at $n=8$ → overlapping from 3 to 6
at $n=9$ → " " 4 to 6
at $n=10$ → " " 5 to 6

$\therefore$ $\boxed{L = n-5}$ and $\boxed{U = 6}$

$\therefore y[n] = \sum_{k=n-5}^{6} \alpha^k \cdot 1 = \sum_{k=n-5}^{6} \alpha^k = \frac{\alpha^{n-5}[1-\alpha^{12-n}]}{1-\alpha}$

$\therefore$ $\boxed{y[n] = \frac{\alpha^{n-5} - \alpha^7}{1-\alpha}, \quad 8 \le n \le 11}$ ⇒ IV

- For $n > 11$ there is no overlapping between $x[k]$ and $h[n-k]$

$\therefore$ $\boxed{y[n] = 0 \quad \text{for } n > 11}$ → V

From I, II, III, IV, and V

$$\therefore y[n] = \begin{cases} 0 & ; \quad n < 1 \\ \left\{\frac{1-\alpha^n}{1-\alpha}\right\} & ; \quad 1 \le n \le 4 \\ \left\{\frac{\alpha^{n-5}-\alpha^n}{1-\alpha}\right\} & ; \quad 5 \le n \le 7 \\ \left\{\frac{\alpha^{n-5}-\alpha^7}{1-\alpha}\right\} & ; \quad 8 \le n \le 11 \\ 0 & ; \quad n > 11 \end{cases}$$

(b) The system is stable if $\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$

$\therefore \sum_{n=-\infty}^{+\infty} |h[n]| = 5 < \infty$

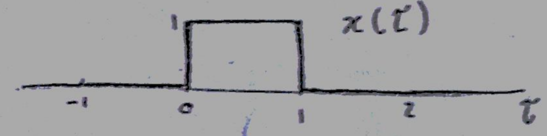
$\therefore$ $\boxed{\text{the system is stable}}$

[Q:4] Suppose that $x(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$ if $h(t) = x(t/2)$

Compute and sketch $y(t) = x(t) * h(t)$? [5 degrees]

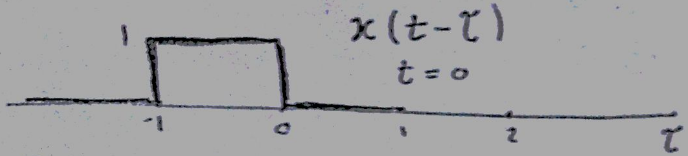
- Let the independent variable to be (τ)

then



$\Rightarrow$

- Time-reverse $x(\tau)$



- For $t<0$ there is no overlapping between $h(\tau)$ and $x(t-\tau)$

$\boxed{\therefore y(t) = 0 \quad \text{For } t<0} \Rightarrow$ I

- For $0<t<1$ there is a partial overlapping between $h(\tau)$ and $x(t-\tau)$

$\therefore y(t) = \int_L^U h(\tau)\, x(t-\tau)\, d\tau$

at $t=0 \rightarrow$ overlapping From 0 to 0
at $t=\frac{1}{4} \rightarrow$ " " 0 to $\frac{1}{4}$
at $t=\frac{1}{2} \rightarrow$ " " 0 to $\frac{1}{2}$

$\therefore \boxed{L=0}$ and $\boxed{U=t}$

$\therefore y(t) = \int_0^t 1.1\, d\tau = \int_0^t d\tau = \tau \Big|_0^t = t$

$\boxed{\therefore y(t) = t, \quad 0<t<1} \Rightarrow$ II

- For $1<t<2$ there is a total overlapping between $h(\tau)$ and $x(t-\tau)$

$y(t) = \int_L^U h(\tau)\, x(t-\tau)\, d\tau$

at $t=1 \longrightarrow$ overlapping From 0 to 1
at $t=\frac{3}{2} \longrightarrow$ " " $\frac{1}{2}$ to $\frac{3}{2}$
at $t=2 \longrightarrow$ " " 1 to 2

$\therefore \boxed{L=t-1}$ and $\boxed{U=t}$

$\therefore y(t) = \int_{t-1}^t 1.1\, d\tau = \int_{t-1}^t d\tau = \tau \Big|_{t-1}^t$

$\therefore y(t) = t-(t-1) = 1$

$\boxed{\therefore y(t) = 1, \quad 1<t<2} \Rightarrow$ III

- For $2<t<3$ there is another partial overlapping between $h(\tau)$ and $x(t-\tau)$

$y(t) = \int_L^U h(\tau)\, x(t-\tau)\, d\tau$

at $t=2 \rightarrow$ overlapping From 1 to 2
at $t=\frac{5}{2} \rightarrow$ " " $\frac{3}{2}$ to 2
at $t=3 \rightarrow$ " " 2 to 2

$\therefore \boxed{L=t-1}$ and $\boxed{U=2}$

$\therefore y(t) = \int_{t-1}^2 1.1\, d\tau = \tau \Big|_{t-1}^2 = 2-t+1$

$\boxed{\therefore y(t) = 3-t, \quad 2<t<3} \Rightarrow$ IV

- For $t>3$ there is no overlapping between $h(\tau)$ and $x(t-\tau)$

$\boxed{\therefore y(t) = 0, \quad t>3} \Rightarrow$ V

From I, II, III, IV, and V

$$\therefore y(t) = \begin{cases} 0 & ; t<0 \\ t & ; 0<t<1 \\ 1 & ; 1<t<2 \\ 3-t & ; 2<t<3 \\ 0 & ; t>3 \end{cases}$$

then,