第四次作业

4.4

$$\therefore F = G\frac{m'm}{d^2} - G\frac{\frac{1}{8}m'm}{(d - \frac{1}{2}R)^2}$$

4.5

(a)

轨道半径
$$R=6371+640~(\mathrm{km})=7011~(\mathrm{km})\approx 7\times 10^6~(\mathrm{m})$$

地球质量
$$M=5.965 imes10^{24}~(ext{kg})pprox6 imes10^{24}~(ext{kg})$$

引力常数
$$G=6.67 imes 10^{-11}~(\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)$$

$$\because G\frac{Mm}{R^2} = m\frac{v^2}{R}$$

$$\therefore v = \sqrt{\frac{GM}{R}} = 7561.179046380834 \text{ (m/s)} = 7.561 \times 10^3 \text{ (m/s)}$$

(b)

$$\because G\frac{Mm}{R^2} = m\omega^2 R = m(\frac{2\pi}{T})^2 R$$

$$\therefore T = 2\pi \sqrt{rac{R^3}{GM}} = 2\pi imes \sqrt{rac{(7 imes 10^6)^3}{6.67 imes 10^{-11} imes 6 imes 10^{24}}} = 5816.856984931375 ext{ (s)} = 5.817 imes 10^3 ext{ (s)}$$

(c)

设新速度为 v', 新半径为 R, 新周期为T'\$

对于圆周运动的天体所拥有的动能和重力势能总能量

$$\because E = \frac{1}{2}mv^2 - G\frac{Mm}{R}$$

$$G\frac{Mm}{R^2} = m\frac{v^2}{R}$$

$$\therefore E = -\frac{GMm}{2R} = -\frac{1}{2}mv^2$$

$$\because E' = -rac{GMm}{2R'} = -rac{1}{2}mv'^2 = E - 1500 imes 1.4 imes 10^5 ext{J} = -6.5 imes 10^9 ext{J}$$

$$d = R' - 6.371 \times 10^6 \text{m} = 4 \times 10^5 \text{ (m)}$$

$$v' = \sqrt{\frac{-2E'}{m}} = 7.687 \times 10^3 \text{ (m/s)}$$

$$\because \frac{v'}{R'} = \frac{2\pi}{T'}$$

$$T' = \frac{2\pi R'}{v'} = 5.534 \times 10^3 \text{ (s)}$$

(d)

$$\therefore fs = 2\pi Rf = 1.4 \times 10^5 \text{J}$$

$$\therefore f = rac{1.4 imes 10^5 ext{J}}{2\pi R} = 3.18 imes 10^{-3} \; ext{N}$$

(e)

$$\therefore L = Rmv = -\frac{GMm}{2E} \cdot m\sqrt{\frac{-2E}{m}} = -\frac{GMm}{2}\sqrt{\frac{-2m}{E}}$$

:: E 在不断发生变化, 角动量不守恒

$$\therefore \frac{L - L'}{L} = \frac{\sqrt{-\frac{1}{E}} - \sqrt{-\frac{1}{E'}}}{\sqrt{-\frac{1}{E}}} = 1.6\%$$

4.7

(a)

$$\therefore E = rac{1}{2} m v^2 - G rac{m_{\oplus} m}{r}$$

$$Grac{m_\oplus m}{r^2}=mrac{v^2}{r}$$

$$\therefore E = -\frac{1}{2}mv^2 = -\frac{Gm_{\oplus}m}{2r}$$

$$\therefore E_A + E_B = -rac{Gm_\oplus m}{2r} - rac{Gm_\oplus m}{2r} = -rac{Gm_\oplus m}{r}$$

(b)

:: 碰撞前 A 和 B 的速度大小相等, 方向相反

... 发生非弹性碰撞后的碰撞碎片聚集在一起后, 速度为零, 即没有动能

$$\therefore E = -Grac{m_\oplus 2m}{R} = -2Grac{m_\oplus m}{R}$$

(c)

卫星碎片会进行加速度变大的加速直线运动, 即自由落体运动, 下落到地球.

4.12

(a)

对 a 点:

粒子的能量
$$E_a=rac{1}{2}mv_a^2-rac{k}{a}$$

$$\because v_a = \sqrt{rac{k}{2ma}}$$

$$\therefore E_a = \frac{k}{4a} - \frac{k}{a} = -\frac{3k}{4a}$$

对另一个极端点b点:

粒子的能量
$$E_b=rac{1}{2}mv_b^2-rac{k}{b}=E_a=-rac{3k}{4a}$$

由角动量守恒得 $L=av_a=bv_b$

联解可得
$$\frac{1}{2}m(\frac{av_a}{b})^2 - \frac{k}{b} = \frac{ak}{4b^2} - \frac{k}{b} = -\frac{3k}{4a}$$

$$\therefore a^2 - 4ab + 3b^2 = (a - 3b)(a - b) = 0$$

$$\therefore b = \frac{1}{3}a$$

(b)

$$\therefore v_b = rac{av_a}{b} = 3v_a = 3\sqrt{rac{k}{2ma}}$$

4.14

设约化质量为 $\mu=rac{m_1m_2}{m_1+m_2}$, 对于二维平面上:

$$\therefore -kx = \mu \ddot{x}$$
$$-ky = \mu \ddot{y}$$

解常微分方程可得:

$$\therefore x = A\cos(t\sqrt{rac{k}{\mu}} + arphi_1)$$

$$y=B\cos(t\sqrt{rac{k}{\mu}}+arphi_2)$$

:: 轨迹为椭圆

4.18

(a)

对不被撕裂且有最小密度的星球表面上一点:

$$\therefore G\frac{Mm}{R^2} = m\omega^2 R$$

$$M =
ho V = rac{4}{3}
ho\pi R^3$$

$$\therefore \rho = \frac{3\omega^2}{4\pi G}$$

对蟹状星云脉冲星:

$$\therefore \omega = 2\pi f = 60\pi \, (\mathrm{rad/s})$$

$$\therefore \rho = \frac{3 \times (60\pi)^2}{4 \times \pi \times 6.67 \times 10^{-11}} \text{ kg/m}^3 = 1.2717 \times 10^{14} \text{ kg/m}^3$$

(b)

$$\therefore M = \rho V = \frac{4}{3} \rho \pi R^3$$

$$\therefore R = \sqrt[3]{rac{3M}{4\pi
ho}} = (rac{3 imes2 imes10^{30}}{4 imes\pi imes1.2717 imes10^{14}})^{rac{1}{3}} \; ext{m} = 1.55 imes10^5 \; ext{m}$$