（A）1（1、4），5，10（4），11（2），12（3、6、10），13（4、5），17（2、4），习题1.4（A）2，4，7（3、6），（B）2（1）

1.3(A)

1.

(1)

$\forall \varepsilon > 0, \exists M,$使得当$x \in (-\infty, M),$有$|f(x) - a| < \varepsilon$

(4)

$\forall M > 0, \exists \delta,$使得当$0 < |x - x_0| < \delta,$有$f(x) > M$

5.

(1)

不正确.

令$f(x) = -1, a = 1$

则有$\lim\limits_{x \to x_0} |f(x)| = |a| = 1$成立,

但是$\lim\limits_{x \to x_0} f(x) = a = 1$不成立.

(2)

正确.

$\because \lim\limits_{x \to x_0} f(x) = a$

$\therefore \lim\limits_{x \to x_0} f^2(x) = \lim\limits_{x \to x_0} f(x) \cdot \lim\limits_{x \to x_0} f(x) = a^2$

(3)

当n是非零实数时,

正确.

$\because \lim\limits_{n \to \infty} f(\frac{1}{n}) = a$

$\therefore \lim\limits_{x \to 0^+} f(x) = a$且$\lim\limits_{x \to 0^-} f(x) = a$

$\therefore \lim\limits_{x \to 0^+} f(x) = a$

当n是正整数时,

不正确.

假设$f(x) = D(x) = \begin{cases} 1, & x \in \mathbf{Q} \\ 0, & x \in \mathbf{R} \backslash \mathbf{Q} \end{cases}$

$\because n \in \mathbf{N}^+$

$\therefore \frac{1}{n} \in \mathbf{Q}^+$

$\therefore \lim\limits_{n \to \infty} f(\frac{1}{n}) = 1 \neq \lim\limits_{x \to 0^+} f(x) = 0$

(4)

正确.

设$\lim\limits_{x \to x_0} f(x) = A, \lim\limits_{x \to x_0} [f(x) + g(x)] = B$

$\therefore \forall \varepsilon_1 > 0, \exists \delta,$当$0 < |x - x_0| < \delta$时,使得$|f(x) - A| < \varepsilon_1$

$\therefore \forall \varepsilon_2 > 0, \exists \delta,$当$0 < |x - x_0| < \delta$时,使得$|f(x) + g(x) - B| < \varepsilon_2$

$\therefore |g(x) - (B - A)| - |f(x) - A| \leq |f(x) + g(x) - B| < \varepsilon_2$

$\therefore |g(x) - (B - A)| < |f(x) - A| + \varepsilon_2 < \varepsilon_1 + \varepsilon_2$

(5)

正确.

设 $\lim\limits_{x\to x_0} f(x)=A, \lim\limits_{x\to x_0} f(x)g(x)=B$

$$\begin{aligned}\lim_{x\to x_0} g(x) &= \lim_{x\to x_0}\frac{f(x)g(x)}{g(x)}\\ &= \frac{\lim\limits_{x\to x_0} f(x)g(x)}{\lim\limits_{x\to x_0} g(x)}\\ &= \frac{B}{A}\end{aligned}$$

(6)

不正确.

令$x_0=0,\ f(x)=\begin{cases}x^2 & (x\neq 0)\\ 1 & (x=0)\end{cases}$

满足$f(x)$在x_0的某领域内$f(x)>0,$

但是 $\lim\limits_{x\to x_0^-} f(x)=a=0.$

$\therefore a>0$不一定成立.

10.(4)

证明$\lim\limits_{x\to 1}\dfrac{x^2}{x+1}=\dfrac{1}{2}$

证明:

$\therefore$ 要证$\forall\varepsilon>0,\exists\delta,$当$0<|x-1|<\delta$时,使得$|f(x)-\dfrac{1}{2}|<\varepsilon$

$$\begin{aligned}\therefore |\frac{x^2}{x+1}-\frac{1}{2}| &= |\frac{2x^2-x-1}{2x+2}|\\ &= |\frac{(2x+1)(x-1)}{2x+2}|\\ &< |\frac{(2x+2)(x-1)}{2x+2}|\\ &= |x-1|\\ &< \varepsilon\end{aligned}$$

$\therefore$ 取$\delta=\varepsilon$,则有$|f(x)-\frac{1}{2}|<\varepsilon$

$\therefore \lim\limits_{x\to 1}\frac{x^2}{x+1}=\frac{1}{2}$

11.(2)

构造数列$\{a_n\}, a_n=2\pi n$

$\therefore$ 对于$\{f(a_n)\}, a_n\to+\infty$时,
$f(a_n)=2\pi n(1+\sin(2\pi n))=2\pi n\to+\infty$

构造数列$\{b_n\}, b_n=2\pi n-\frac{\pi}{2}$

$\therefore$ 对于$\{f(b_n)\}, b_n\to+\infty$时,
$f(b_n)=(2\pi n-\frac{\pi}{2})(1+\sin(2\pi n-\frac{\pi}{2}))=0\to 0$

由$Heine$定理可知$\lim\limits_{x\to+\infty}x(1+\sin x)$极限不存在.

12.

(3)

$$\begin{aligned}\lim_{x\to+\infty}\sqrt{x}(\sqrt{a+x}-\sqrt{x})&=\lim_{x\to+\infty}\frac{a\sqrt{x}}{\sqrt{a+x}+\sqrt{x}}\\&=\frac{a}{\lim\limits_{x\to+\infty}\sqrt{\frac{a}{x}+1}+1}\\&=\frac{a}{2}\end{aligned}$$

(6)

$$
\begin{aligned}
\lim_{x\to1}(\frac{2}{1-x^2}-\frac{3}{1-x^3}) &= \lim_{x\to1}(\frac{2}{(1-x)(1+x)}-\frac{3}{(1-x)(1+x+x^2)}) \\
&= \lim_{x\to1}(\frac{2(1+x+x^2)}{(1-x)(1+x)(1+x+x^2)}-\frac{3(1+x)}{(1-x)(1+x)(1+x+x^2)}) \\
&= \lim_{x\to1}\frac{(2x+1)(x-1)}{(1-x)(1+x)(1+x+x^2)} \\
&= \lim_{x\to1}\frac{-2x-1}{(1+x)(1+x+x^2)} \\
&= -\frac{1}{2}
\end{aligned}
$$

(10)

$n \in \mathbf{N}_+$

$$
\begin{aligned}
\lim_{x\to+\infty}\frac{\sqrt[n]{1+x}-1}{x} &= \lim_{x\to+\infty}\frac{\sqrt[n]{1+x}}{x}-\lim_{x\to+\infty}\frac{1}{x} \\
&= \lim_{x\to+\infty}\frac{\sqrt[n]{1+x}}{x}
\end{aligned}
$$

当$n=1$时,

$$
\lim_{x\to+\infty}\frac{\sqrt[n]{1+x}}{x}=\lim_{x\to+\infty}\frac{1+x}{x}=1
$$

当$n>1$时,

$$
\begin{aligned}
\lim_{x\to+\infty}\frac{\sqrt[n]{1+x}}{x} &= \lim_{x\to+\infty}\sqrt[n]{\frac{1+x}{x^n}} \\
&= \sqrt[n]{\lim_{x\to+\infty}\frac{1+x}{x^n}} \\
&= \sqrt[n]{\lim_{x\to+\infty}\frac{1}{x^n}+\lim_{x\to+\infty}\frac{1}{x^{n-1}}} \\
&= 0
\end{aligned}
$$

13.

(4)

$$
\begin{aligned}
\lim_{x\to1}(1-x)\tan\frac{\pi x}{2} &= \lim_{x\to1}\frac{2}{\pi}(\frac{\pi}{2}-\frac{\pi x}{2})\tan\frac{\pi x}{2} \\
&= \lim_{x\to1}\frac{2}{\pi}(\frac{\pi}{2}-\frac{\pi x}{2})\cdot\frac{\cos(\frac{\pi}{2}-\frac{\pi x}{2})}{\sin(\frac{\pi}{2}-\frac{\pi x}{2})} \\
&= \lim_{x\to1}\frac{2}{\pi}\cos(\frac{\pi}{2}-\frac{\pi x}{2}) \\
&= \frac{2}{\pi}
\end{aligned}
$$

(5)

$$
\begin{aligned}
\lim_{x\to\infty}(1-\frac{2}{x})^{3x} &= \lim_{x\to\infty}[(1-\frac{1}{\frac{x}{2}})^{\frac{x}{2}}]^6 \\
&= [\lim_{x\to\infty}(1-\frac{1}{\frac{x}{2}})^{\frac{x}{2}}]^6 \\
&= e^6
\end{aligned}
$$

17.

(2)

$$
\begin{aligned}
\lim_{x\to1}(2-x)^{\sec\frac{\pi x}{2}} &= \lim_{x\to1}(2-x)^{\frac{\frac{\pi x}{2}}{\sin\frac{\pi x}{2}}\cdot\frac{1}{\frac{\pi x}{2}}} \\
&= \lim_{x\to1}(2-x)^{\frac{2}{\pi x}} \\
&= 1
\end{aligned}
$$

(4)

$$
\begin{aligned}
\lim_{x\to0}\left(\frac{\pi+e^{\frac{1}{x}}}{1+e^{\frac{4}{x}}}+\arctan\frac{1}{x}\right) &= \lim_{x\to0}\left(\frac{\frac{\pi}{e^{\frac{1}{x}}}+1}{\frac{1}{e^{\frac{1}{x}}}+e^{\frac{3}{x}}}+\arctan\frac{1}{x}\right) \\
&= \lim_{x\to0}\left(\frac{\frac{\pi}{e^{\frac{1}{x}}}+1}{\frac{1}{e^{\frac{1}{x}}}+e^{\frac{3}{x}}}+\arctan\frac{1}{x}\right) \\
&= \lim_{x\to0}\left(0+\arctan\frac{1}{x}\right) \\
&= \infty
\end{aligned}
$$

1.4(A)

2.

(1) 错误; 无穷小量是个变量, 并未绝对值很小的常数.

(2)

无穷小量就是数0错误, 因为无穷小是个变量, 可能大于零也可能小于0;

数0是无穷小量正确, 当自变量趋向于任何数时, 数0都等于0.

(3) 正确; 无穷大量的绝对值一定大于任意给定的正数.

(4) 错误; 例如 $n \to \infty, a_n = [1+(-1)^n]n$ 是无界变量, 但不是无穷大量.

(5) 错误; 例如无穷大量与0的乘积还是0, 而0是有界量.

(6) 错误; 例如 $n \to \infty, n$个$\frac{1}{n}$的和是1.

4.

(1) x的同阶无穷小, 阶数为1.

(2) x的低阶无穷小, 阶数为$\frac{1}{2}$.

(3) x的等价无穷小, 阶数为1.

(4) x的高阶无穷小, 阶数为$\frac{5}{3}$.

(5) x的高阶无穷小, 阶数为2.

7.

(3)

$$
\begin{aligned}
\lim_{x\to 0}\frac{\sqrt{1+\sin^2 x}-1}{x\tan x} &= \lim_{x\to 0}\frac{\frac{1}{2}\sin^2 x}{x\tan x} \\
&= \lim_{x\to 0}\frac{\frac{1}{2}x^2}{x^2} \\
&= \frac{1}{2}
\end{aligned}
$$

(6)

$$
\begin{aligned}
\lim_{x\to 0^-}\frac{(1-\sqrt{\cos x})\tan x}{(1-\cos x)^{\frac{3}{2}}} &= \lim_{x\to 0^-}\frac{(1-\sqrt{\cos x})x}{(\frac{1}{2}x^2)^{\frac{3}{2}}} \\
&= \lim_{x\to 0^-}\frac{1-\cos x}{\frac{\sqrt{2}}{4}x^2(1+\sqrt{\cos x})} \\
&= \lim_{x\to 0^-}\frac{\sqrt{2}}{1+\sqrt{\cos x}} \\
&= \frac{\sqrt{2}}{2}
\end{aligned}
$$

1.4(B)

2.(1)

$\because \lim\limits_{0\to+\infty}(\sqrt{x^2-x+1}-ax+b)=0$

$$
\begin{aligned}
\therefore \lim_{0\to+\infty}\frac{\sqrt{x^2-x+1}}{ax-b} &= \lim_{0\to+\infty}\sqrt{\frac{x^2-x+1}{ax^2-2abx+b^2}} \\
&= \lim_{0\to+\infty}\sqrt{\frac{1-\frac{1}{x}+\frac{1}{x^2}}{a-\frac{2ab}{x}+\frac{b^2}{x^2}}} \\
&= \frac{1}{a} \\
&= 1
\end{aligned}
$$

$\therefore a=1, b$可取任意值.