1.求一个次数尽可能低的实系数（复系数）多项式，使其以1,0,*i*, *i*, 1-*i*为根.

2.本章习题26.

24. 证明：如果$(x-1)|f(x^n)$，那么$(x^n-1)|f(x^n)$.

25. 证明：如果$(x^2+x+1)|f_1(x^3)+xf_2(x^3)$，那么

$$(x-1)|f_1(x),(x-1)|f_2(x).$$

26. 将多项式 x^n-1 在复数范围内和在实数范围内因式分解.

26.

24.

1.

$$\begin{aligned}\therefore f(x)&=x(x-1)(x-i)(x+i)(x-i)(x+i)(x-1+i)(x-1-i)\\&=x(x-1)(x^2+1)^2(x^2-2x+2)\end{aligned}$$

24.

$\because (x-1)|f(x^n)$

$\therefore f(0)=0$

$\therefore 0$是$f(x^n)=0$的根

$\therefore (x^n-1)|f(x)$

26.

在复数范围内分解$f(x)=x^n-1$

$\because 1=\cos 2k\pi+i\sin 2k\pi$

$\therefore x=1^{\frac{1}{n}}=(\cos 2k\pi+i\sin 2k\pi)^{\frac{1}{n}}=(e^{i2k\pi})^{\frac{1}{n}}=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$

$$\therefore f(x)=\prod_{k=0}^{n-1}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})]$$

在实数范围内分解$f(x)=x^n-1$

若n为奇数,

$$\therefore f(x)=\prod_{k=0}^{n-1}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})]$$

$$=(x-1)\prod_{k=1}^{\frac{n-1}{2}}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})]\prod_{k=\frac{n+1}{2}}^{n-1}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})]$$

$$=(x-1)\prod_{k=1}^{\frac{n-1}{2}}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})][x-(\cos\frac{2k\pi}{n}-i\sin\frac{2k\pi}{n})]$$

$$=(x-1)\prod_{k=1}^{\frac{n-1}{2}}(x^2-2\cos\frac{2k\pi}{n}+1)$$

若n为偶数,

$$\therefore f(x)=\prod_{k=0}^{n-1}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})]$$

$$=(x-1)(x+1)\prod_{k=1}^{\frac{n}{2}-1}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})]\prod_{k=\frac{n}{2}+1}^{n-1}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})]$$

$$=(x-1)(x+1)\prod_{k=1}^{\frac{n}{2}-1}[x-(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n})][x-(\cos\frac{2k\pi}{n}-i\sin\frac{2k\pi}{n})]$$

$$=(x-1)(x+1)\prod_{k=1}^{\frac{n}{2}-1}(x^2-2\cos\frac{2k\pi}{n}+1)$$