概率统计第五次作业

201300035 方盛俊

4.1

18.

$\because$ 在 $[0,a]$ 上任意投掷一个质点

$\therefore F(0)=P(X\leqslant 0)=0, F(a)=P(X\leqslant a)=1$

$\because$ 质点落在 $[0,a]$ 中任意小区间内的概率与这个小区间的长度成正比例

$\therefore F(x)=\dfrac{x}{a}\cdot 1=\dfrac{x}{a}, 0\leqslant x<1$

$$\therefore F(x)=\begin{cases}0, & x<0\\ \dfrac{x}{a}, & 0\leqslant x<1\\ 1, & x>1\end{cases}$$

19.

(1) $P(X\leqslant 3)=F(3)=1-e^{-1.2}$

(2) $P(X>4)=1-F(4)=e^{-1.6}$

(3) $P(3<X\leqslant 4)=F(4)-F(3)=e^{-1.2}-e^{-1.6}$

(4) $P(X\leqslant 3 \text{ or } X>4)=1-e^{-1.2}+e^{-1.6}$

(5) $P(X=2.5)=0$

20.

(1)

$$P(X<2)=\lim_{x\to 2^-}F(x)=\ln 2$$

$$P(0<X\leqslant 3)=F(3)-F(0)=1$$

$$P(2<X<\frac{5}{2})=F(\frac{5}{2}-0)-F(2)=\ln\frac{5}{2}-\ln 2=\ln 5-2\ln 2$$

(2)

$$f(x)=\begin{cases}0, & x<1\\ \frac{1}{x}, & 1\leqslant x<e\\ 0, & x\geqslant e\end{cases}$$

21.

(1)

$$\because F(x)=\int_{-\infty}^{x}f(x)\mathrm{d}x=\int_{1}^{x}2(1-\frac{1}{x^2})\mathrm{d}x=2x+\frac{2}{x}-4$$

$$\therefore F(x)=\begin{cases}0, & x<0\\ 2x+\frac{2}{x}-4, & 1\leqslant x\leqslant 2\\ 1, & x>2\end{cases}$$

(2)

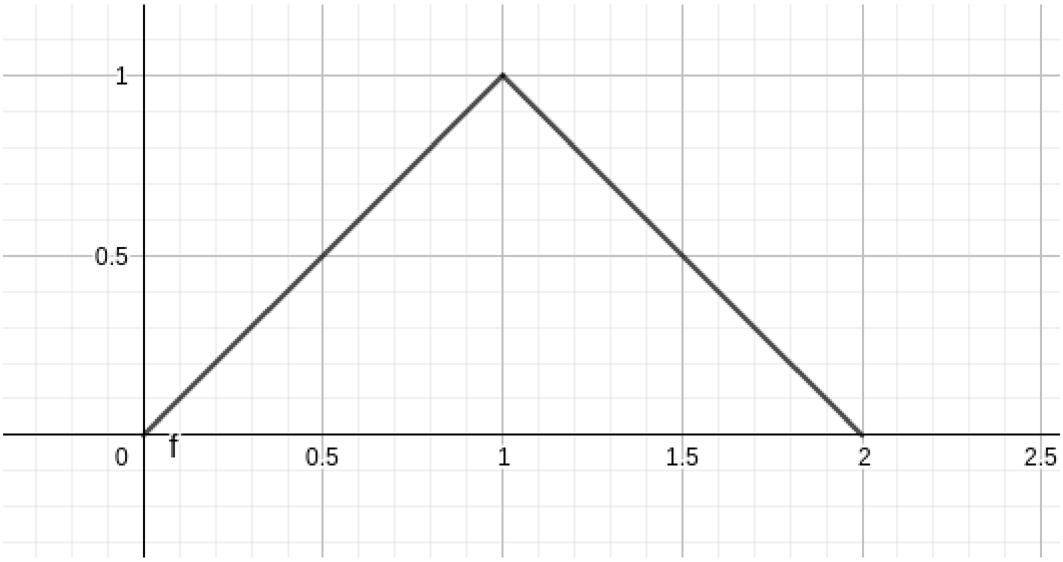
当 $0\leqslant x<1$ 时,

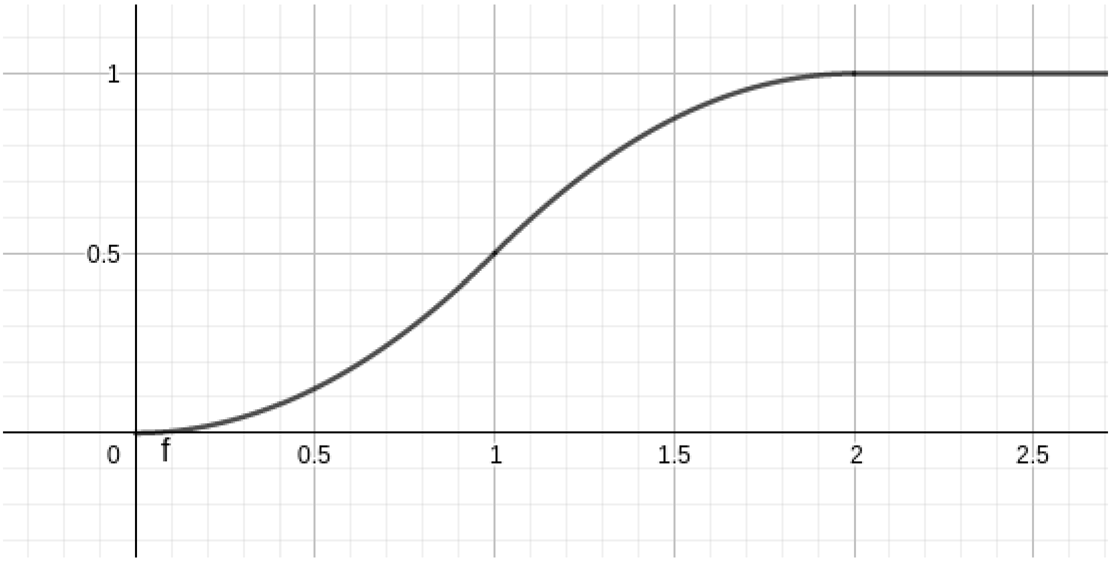
$$F(x)=0+\int_{0}^{x}x\mathrm{d}x=\frac{x^2}{2}$$

当 $1\leqslant x<2$ 时,

$$F(x)=\frac{1}{2}+\int_{1}^{x}(2-x)\mathrm{d}x=-\frac{x^2}{2}+2x-1$$

$$\therefore F(x)=\begin{cases}0, & x<0\\ \frac{x^2}{2}, & 0\leqslant x<1\\ -\frac{x^2}{2}+2x-1, & 1\leqslant x<2\\ 1, & x\geqslant 2\end{cases}$$





23.

$$\because f(x)=\frac{1000}{x^2},x>1000$$

$$\therefore F(x)=\int_{1000}^{x}\frac{1000}{x^2}\mathrm{d}x=\frac{x-1000}{x},x>1000$$

$$\therefore P(X\leqslant 1500)=F(1500)=\frac{1500-1000}{1500}=\frac{1}{3}$$

$$\therefore p=1-(\binom{5}{1}\cdot\frac{1}{3}\cdot(\frac{2}{3})^4+\binom{5}{0}(\frac{2}{3})^5)=\frac{131}{243}$$

24.

$$\because f(x)=\frac{1}{5}e^{-\frac{x}{5}},x>0$$

$$\therefore F(x)=\int_0^x \frac{1}{5}e^{-\frac{x}{5}}\,\mathrm{d}x=1-e^{-\frac{x}{5}}$$

$$\therefore P(X>10)=1-F(10)=e^{-2}$$

$$\therefore Y\sim B(5,e^{-2})$$

$$\therefore P(Y=k)=\binom{5}{k}e^{-2k}(1-e^{-2})^{5-k}$$

$$\therefore P(Y\geqslant 1)=1-P(Y=0)=1-(1-e^{-2})^5$$

25.

$$\because X\sim U(0,5)$$

求 $4x^2+4Kx+K+2=0$ 有实根的概率, 即 $\Delta=(4K)^2-4\times 4(K+2)\geqslant 0$

$$\therefore P(X\leqslant -1\ \text{ and }\ X>2)=P(2<X\leqslant 5)=\int_2^5\frac{1}{5-0}\mathrm{d}x=\frac{3}{5}$$

18.

$$\therefore E(X)=\int_{-\infty}^{+\infty}f(x)\mathrm{d}x=\int_0^{+\infty}x\cdot\frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}\,\mathrm{d}x=\sqrt{\iint_S\frac{x^2y^2}{\sigma^4}e^{-\frac{x^2+y^2}{2\sigma^2}}\,\mathrm{d}x\mathrm{d}y}=\frac{\sqrt{2\pi}\sigma}{2}$$

$$\therefore E(X^2)=\int_0^{+\infty}x^2\cdot\frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}\,\mathrm{d}x=2\sigma^2$$

$$\therefore D(X)=E(X^2)-E(X)^2=(2-\frac{\pi}{2})\sigma^2$$

4.2

设 X 为长方形的宽的随机变量, Y 为长方形周长的随机变量.

$$\because X\sim U(0,2),Y=2X+\frac{20}{X}$$

$$\therefore E(X)=\int_0^2 x\cdot\frac{1}{2-0}\mathrm{d}x=1,E(\frac{1}{X})=\int_0^2\frac{1}{x}\cdot\frac{1}{2-0}=\infty$$

$$\therefore E(X^2)=\int_0^2 x^2\cdot\frac{1}{2-0}\mathrm{d}x=\frac{4}{3},E(\frac{1}{X^2})=\int_0^2\frac{1}{x^2}\cdot\frac{1}{2-0}=\infty$$

即 $E(\frac{1}{X})$ 和 $E(\frac{1}{X^2})$ 期望不存在.

$$\therefore E(Y)=2E(X)+20E(\frac{1}{X})=2+20E(\frac{1}{X})$$

$D(Y)=E(Y^2)-E(Y)^2=E(4X^2+80+\frac{400}{X^2})-4E(X)^2-80E(X)E(\frac{1}{X})-400E(\frac{1}{X})^2=\frac{244}{3}+400E(\frac{1}{X^2})-80E(\frac{1}{X})-400E(\frac{1}{X})^2$

4.3

$$\therefore 1=\int_{-\infty}^{+\infty}f(x)=\int_0^{+\infty}Ae^{-x}\mathrm{d}x=A$$

$$\therefore E(Y)=\int_0^{+\infty}e^{-2x}\cdot e^{-x}\mathrm{d}x=\int_0^{+\infty}e^{-3x}\mathrm{d}x=\frac{1}{3}$$

4.4

$$\because \int_{-\infty}^{+\infty}e^{-\frac{(t-\mu)^2}{2\sigma^2}}\mathrm{d}t=\int_{-\infty}^{+\infty}e^{-\frac{x^2}{2\sigma^2}}\mathrm{d}x$$

$\therefore \int_{-\infty}^{+\infty}e^{-\frac{x^2}{2\sigma^2}}\mathrm{d}x\int_{-\infty}^{+\infty}e^{-\frac{y^2}{2\sigma^2}}\mathrm{d}y=\iint_S e^{-\frac{x^2+y^2}{2\sigma^2}}\mathrm{d}x\mathrm{d}y=\int_0^{2\pi}\mathrm{d}\theta\int_0^{+\infty}e^{-\frac{\rho^2}{2\sigma^2}}\cdot\rho\mathrm{d}\rho=2\pi\cdot\int_0^{+\infty}\frac{1}{2}e^{-\frac{t}{2\sigma^2}}\mathrm{d}t=2\pi\sigma^2$

$$\therefore \int_{-\infty}^{+\infty}e^{-\frac{(t-\mu)^2}{2\sigma^2}}\mathrm{d}t=\sqrt{2\pi}\sigma$$