作业与练习

■ 习题1

5 . 2),6. 2),7~9,11.,13.,14.

5.2

```
            1       -1  |               -1/3    10/9    |
    1   -3  0       1   |       -3      -1      2       |
----------------------  |   --------------------------  |
1   -4  0   0       1   |   1   -3      0       1       |
1   -3  0   1           |   1   1/3     -2/3            |
--------------          |   --------------------------  |
    -1  0   -1      1   |       -10/3   2/3     1       |
    -1  3   0       -1  |       -10/3   -10/9   20/9    |
    ------------------  |   --------------------------  |
        -3  -1      2   |               16/9    -11/9   |
        -3  33/16
        --------------
            -49/16  2
            149/16  539/256
            ---------------
                    -27/256
```

所以最大公因式是1.

6.2

$q_1 = 2x$

$q_2 = -\frac{1}{3}x + \frac{1}{3}$

$q_3 = 6x + 9$

```
        2       -1      -5      4
  ----------------------------------
4       -2      -16     5       9
4       -2      -10     8
  ----------------------------------
                -6      -3      9

        -6      -3      9
  --------------------------
2       -1      -5      4
2       1       -3
  --------------------------
        -2      -2      4
        -2      -1      3
        ------------------
                -1      1

        -1      1
  -------------------
-6      -3      9
-6      6
  ------------------
        -9      9
        -9      9
        ----------
                0
```

$$
\begin{aligned}
x - 1 = -r_2 &= g(x) + q_2(x)f(x) - q_1(x)q_2(x)g(x) \\
&= q_2(x)f(x) + (1 - q_1(x)q_2(x))g(x)
\end{aligned}
$$

$\therefore u(x) = q_2(x) = -\dfrac{1}{3}x + \dfrac{1}{3}$

$\therefore v(x) = 1 - 2x(-\dfrac{1}{3}x + \dfrac{1}{3}) = \dfrac{2}{3}x^2 - \dfrac{2}{3}x + 1$

7.

```
1       0       t       u
--------------------------
1       1+t     2       2u
1       0       t       u
--------------------------
        1+t     2-t     u

        1+t             2-t         u
--------------------------------------
1       0               t           u
1       (2-t)/(1+t)     u/(1+t)
--------------------------------------
        -(2-t)/(1+t)    t-u/(1+t)   u
        1+t             2-t         u
        ------------------------------
        0               0           0
```

$$\begin{cases}-2+t=(1+t)^2\\ t-u=(2-t)(1+t)\end{cases}\Rightarrow\begin{cases}t^2+t+3=0\\ t^2-u-2=0\end{cases}\Rightarrow\begin{cases}t=\dfrac{-1\pm\sqrt{11}i}{2}\\ u=\dfrac{-9\pm\sqrt{11}i}{2}\end{cases}$$

$\therefore t=\dfrac{-1\pm\sqrt{11}i}{2}$

$\therefore u=\dfrac{-9\pm\sqrt{11}i}{2}$

8.

$\because d(x)$为$f(x)$和$g(x)$的一个组合

假设$h(x)$为$f(x)$和$g(x)$的任意一个公因式,$d(x)=u(x)f(x)+v(x)g(x)$

$\therefore h(x)|f(x),h(x)|g(x)$

$\therefore h(x)|u(x)f(x)+v(x)g(x)$

$\therefore h(x)|d(x)$

$\because d(x)|f(x),d(x)|g(x)$

$\therefore d(x)$能被$f(x)$和$g(x)$任意一个公因式整除,又是$f(x)$和$g(x)$的一个公因式

$\therefore d(x)$是$f(x)$和$g(x)$的一个最大公因式

9.

设$d(x)=(f(x),g(x)),c(x)=(f(x)h(x),g(x)h(x))$

$\therefore d(x)|f(x), d(x)|g(x)$

$\therefore d(x)h(x)|f(x)h(x), d(x)h(x)|g(x)h(x)$

$\therefore d(x)h(x)|c(x)$

$\because c(x)|f(x)h(x), c(x)|g(x)h(x)$

$\therefore c(x)|f(x), c(x)|g(x), c(x)|h(x)$

$\therefore c(x)|d(x)$

$\therefore c(x)|d(x)h(x)$

$\because c(x)|d(x)h(x)$且$d(x)h(x)|c(x)$且$h(x)$首项为1

$\therefore c(x) = d(x)h(x)$

$\therefore (f(x)h(x), g(x)h(x)) = (f(x), g(x))h(x)$

11.

$\because (f(x), g(x))|f(x), (f(x), g(x))|g(x)$

$\therefore$ 令$f(x) = (f(x), g(x))f'(x), g(x) = (f(x), g(x))g'(x)$

$\because u(x)f(x) + v(x)g(x) = (f(x), g(x))$

$\therefore u(x)(f(x), g(x))f'(x) + v(x)(f(x), g(x))g'(x) = (f(x), g(x))$

$\therefore f'(x)u(x) + g'(x)v(x) = 1$

$\therefore (u(x), v(x)) = 1$

13.

$\because (f_1(x), g_1(x)) = 1, (f_2(x), g_1(x)) = 1$

$\therefore u_1(x)f_1(x) + v_1(x)g_1(x) = 1, u_2(x)f_2(x) + v_2(x)g_1(x) = 1$

$$\begin{aligned}\therefore 1 &= 1 \times 1 \\ &= [u_1(x)f_1(x) + v_1(x)g_1(x)][u_2(x)f_2(x) + v_2(x)g_1(x)] \\ &= u_1(x)u_2(x)f_1(x)f_2(x) + [u_1(x)f_1(x)v_2(x) + v_1(x)u_2(x)f_2(x) + v_1(x)v_2(x)g_1(x)]g_1(x)\end{aligned}$$

$\therefore (f_1(x)f_2(x), g_1(x)) = 1$

$\because (f_i(x), g_1(x)) = 1 \qquad (i = 1, 2, \cdots, m)$

$\therefore$ 同理可得$(f_1(x)f_2(x)\cdots f_m(x), g_1(x))$

$\because (f_i(x), g_j(x)) = 1 \qquad (i = 1, 2, \cdots, m; \quad j = 1, 2, \cdots, n)$

$\therefore (f_1(x)f_2(x)\cdots f_m(x), g_j(x)) = 1 \qquad (j = 1, 2, \cdots, n)$

$\therefore$ 同理可得$(f_1(x)f_2(x)\cdots f_m(x), g_1(x)g_2(x)\cdots g_m(x)) = 1$

14.

$\because (f(x), g(x)) = 1$

$\therefore u(x)f(x) + v(x)g(x) = 1$

$\therefore 1 = u(x)f(x) + v(x)g(x) = [u(x) - v(x)]f(x) + v(x)[f(x) + g(x)]$

$\therefore (f(x), f(x) + g(x)) = 1$

$\therefore$ 同理可得$(g(x), f(x) + g(x)) = 1$

令$d(x) = f(x) + g(x)$,则

有$u_1(x)f(x) + v_1(x)d(x) = u_2(x)g(x) + v_2(x)d(x) = 1$

$$\begin{aligned}
\therefore 1 &= 1 \times 1 \\
&= [u_1(x)f(x) + v_1(x)d(x)][u_2(x)g(x) + v_2(x)d(x)] \\
&= u_1(x)u_2(x)f(x)g(x) + [u_1(x)v_2(x)f(x) + v_1(x)u_2(x)g(x) + v_1(x)v_2(x)d(x)]d(x)
\end{aligned}$$

$\therefore (f(x)g(x), f(x) + g(x)) = 1$