第一份

1.

解方程

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & x \\ 1 & x & 6 \end{vmatrix} = 1$$

$\therefore 12 + x + x - 2 - 6 - x^2 = 4 + 2x - x^2 = 1$

$\therefore x^2 - 2x - 3 = (x-3)(x+1) = 0$

$\therefore x = 3$或$x = -1$

2.

$\therefore D = 2 + 20 - 9 - 12 + 3 - 10 = -6$

$D_1 = 5 + 45 - 15 - 27 + 5 - 25 = -12$

$D_2 = 10 - 100 + 81 - 60 - 15 + 90 = 6$

$D_3 = 18 - 20 - 15 - 20 + 27 + 10 = 0$

$\therefore x_1 = \dfrac{D_1}{D} = 2, x_2 = \dfrac{D_2}{D} = -1, x_3 = \dfrac{D_3}{D} = 0$

第二份

1.

(1)

$\therefore \tau(542163) = 4 + 3 + 1 + 1 = 9$

(2)

$$
\begin{aligned}
&\tau(24\cdots(2n-2)(2n)(2n-1)(2n-3)\cdots 31)\\
=&1+2+\cdots+n+\cdots+2+1\\
=&n(n+1)-n\\
=&n^2
\end{aligned}
$$

2.

①当$n=2$时,

$\because \tau(x_1x_2)=I'$

$\therefore \tau(x_2x_1)=1-I'=\dfrac{n(n-1)}{2}-I'$成立

②假设$n=k$时,$\tau(x_1x_2\cdots x_k)=I'$

$\therefore \tau(x_kx_{k-1}\cdots x_1)=\dfrac{k(k-1)}{2}-I'$成立

当$n=k+1$时,

$$\therefore \tau(x_1x_2\cdots x_kx_{k+1})=\tau(x_1x_2\cdots x_k)+\sum_{i=1}^{k}\tau(x_ix_{k+1})=I=I'+I-I'$$

$$\therefore \sum_{i=1}^{k}\tau(x_ix_{k+1})=I-I'$$

$$\therefore \sum_{i=1}^{k}\tau(x_{k+1}x_i)=k-I+I'$$

$$
\begin{aligned}
\therefore \tau(x_{k+1}x_k\cdots x_1)&=\sum_{i=1}^{k}\tau(x_{k+1}x_i)+\tau(x_kx_{k-1}\cdots x_1)\\
&=k-I+I'+\frac{k(k-1)}{2}-I'\\
&=\frac{k(k+1)}{2}-I
\end{aligned}
$$

$\therefore$ 综上,$\tau(x_nx_{n-1}\cdots x_1)=\dfrac{n(n-1)}{2}-I$成立

第三份

1.

$\therefore D=(-1)^{\tau(23\cdots(n-1)n1)}1\times 2\times\cdots\times n=(-1)^{n-1}n!$

2.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

$\therefore$ 行列式中有$a_{23}a_{41}$的项是$(-1)^{\tau(2341)}a_{12}a_{23}a_{34}a_{41}$和$(-1)^{\tau(4321)}a_{14}a_{23}a_{32}a_{41}$

$\therefore$ 易知$-a_{12}a_{23}a_{34}a_{41}$符合要求

书上习题

1.(2)

$\therefore \tau(217986354)=18,$奇偶性为偶

3.

$12435\overset{(12)}{\to}21435\overset{(15)}{\to}25431\overset{(34)}{\to}25341$

5.

①当$n=2$时,

$\because \tau(x_1x_2)=K'$

$\therefore \tau(x_2x_1)=1-K'=\dfrac{n(n-1)}{2}-K'$成立

②假设$n=k$时,$\tau(x_1x_2\cdots x_k)=K'$

$\therefore \tau(x_kx_{k-1}\cdots x_1)=\dfrac{k(k-1)}{2}-K'$成立

当$n=k+1$时,

$$\therefore \tau(x_1x_2\cdots x_kx_{k+1}) = \tau(x_1x_2\cdots x_k) + \sum_{i=1}^{k}\tau(x_ix_{k+1}) = K = K' + K - K'$$

$$\therefore \sum_{i=1}^{k}\tau(x_ix_{k+1}) = K - K'$$

$$\therefore \sum_{i=1}^{k}\tau(x_{k+1}x_i) = k - K + K'$$

$$\begin{aligned}\therefore \tau(x_{k+1}x_k\cdots x_1) &= \sum_{i=1}^{k}\tau(x_{k+1}x_i) + \tau(x_kx_{k-1}\cdots x_1)\\ &= k - K + K' + \frac{k(k-1)}{2} - K'\\ &= \frac{k(k+1)}{2} - K\end{aligned}$$

$\therefore$ 综上, $\tau(x_nx_{n-1}\cdots x_1) = \dfrac{n(n-1)}{2} - K$成立

6.

$\because \tau(431265) = 6, \tau(452316) = 8$

$\therefore$ 两项的符号都为正

8.(3)

设该行列式为D

$\therefore D = (-1)^{\tau((n-1)(n-2)\cdots 21n)}1\times 2\times\cdots\times n = (-1)^{\frac{(n-1)(n-2)}{2}}n!$

10.

由行列式定义可知, 每一项都是取自不同行、不同列4个元素的乘积

对于含x^4项：

$\because$ 每一项都是取自不同行不同列的4个元素乘积,
且行列式里每一个元素要么是0次要么是1次的

$\therefore$ 取出来的四个元素必定都是1次的

$\therefore$ 只能取$a_{11}, a_{22}, a_{33}, a_{44}$组成的项

$\therefore$ 包含x^4的项的系数是2

对于含x^3项：

$\therefore$ 取出来的四个元素有且仅有三个1次项

$\therefore$ 只能取$a_{12}, a_{21}, a_{33}, a_{44}$组成的项

$\therefore$ 包含x^3的项的系数是-1

13.

(2)

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = 3xy(x+y) - (x+y)^3 - x^3 - y^3$$
$$= 3x^2y + 3xy^2 - x^3 - 3x^2y - 3xy^2 - y^3 - x^3 - y^3$$
$$= -2x^3 - 2y^3$$

(4)

$$D = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix} \xlongequal[r_4-4r_1]{r_2-2r_1, r_3-3r_1} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -7 \\ 0 & -2 & -8 & -10 \\ 0 & -7 & -10 & -13 \end{vmatrix}$$
$$\xlongequal[r_4-7r_2]{r_3-2r_2} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -7 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & 4 & 36 \end{vmatrix} \xlongequal{r_4+r_2} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -7 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 40 \end{vmatrix}$$

$\therefore D = 1 \times (-1) \times (-4) \times 40 = 160$

(5)

$$D = \begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} \xlongequal[r_3-r_4]{r_1-r_4, r_2-r_4} \begin{vmatrix} x & 0 & 0 & y \\ 0 & -x & 0 & y \\ 0 & 0 & y & y \\ 1 & 1 & 1 & 1-y \end{vmatrix}$$
$$\xlongequal{T} \begin{vmatrix} x & 0 & 0 & 1 \\ 0 & -x & 0 & 1 \\ 0 & 0 & y & 1 \\ y & y & y & 1-y \end{vmatrix} \xlongequal{r_4+(y-1)r_3} \begin{vmatrix} x & 0 & 0 & 1 \\ 0 & -x & 0 & 1 \\ 0 & 0 & y & 1 \\ y & y & y^2 & 0 \end{vmatrix}$$

$\therefore D = x^2y^2 - xy^2 + xy^2 = x^2y^2$

(6)

$$D = \begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 & (a+3)^2 \\ b^2 & (b+1)^2 & (b+2)^2 & (b+3)^2 \\ c^2 & (c+1)^2 & (c+2)^2 & (c+3)^2 \\ d^2 & (d+1)^2 & (d+2)^2 & (d+3)^2 \end{vmatrix} \xlongequal{T} \begin{vmatrix} a^2 & b^2 & c^2 & d^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 & (d+1)^2 \\ (a+2)^2 & (b+2)^2 & (c+2)^2 & (d+2)^2 \\ (a+3)^2 & (b+3)^2 & (c+3)^2 & (d+3)^2 \end{vmatrix}$$

$$\xlongequal[r_4-r_1]{r_2-r_1,r_3-r_1} \begin{vmatrix} a^2 & b^2 & c^2 & d^2 \\ 2a+1 & 2b+1 & 2c+1 & 2d+1 \\ 4a+4 & 4b+4 & 4c+4 & 4d+4 \\ 6a+9 & 6b+9 & 6c+9 & 6d+9 \end{vmatrix} \xlongequal[r_4-3r_2]{r_3-2r_2} \begin{vmatrix} a^2 & b^2 & c^2 & d^2 \\ 2a+1 & 2b+1 & 2c+1 & 2d+1 \\ 2 & 2 & 2 & 2 \\ 6 & 6 & 6 & 6 \end{vmatrix}$$

$\therefore D = 0$