思考

1.

证明: 设 A 合同于 B, 则 A 可逆当且仅当 B 可逆, 此时 A^{-1} 合同于 B^{-1}.

$\because A$ 合同于 B

$\therefore B = C'AC, C$ 可逆, 即 $|C| \neq 0$

$\therefore |B| = |C'AC| = |C|^2|A|$

$\therefore |B| = 0$ 当且仅当 $|A| = 0$

$\therefore A$ 可逆当且仅当 B 可逆

$\because B^{-1} = (C'AC)^{-1} = C^{-1}A^{-1}(C')^{-1} = C^{-1}A^{-1}(C^{-1})'$

$\therefore A^{-1}$ 合同于 B^{-1}

2.

证明: 设 A 合同于 B, C 合同于 D, 则 $\begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}$ 合同于 $\begin{pmatrix} B & 0 \\ 0 & D \end{pmatrix}$.

$\because B = E'AE, D = F'CF$

$\therefore \begin{pmatrix} B & 0 \\ 0 & D \end{pmatrix} = \begin{pmatrix} E'AE & 0 \\ 0 & F'CF \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix}' \begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix} \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix}$

练习

1.

$$\begin{pmatrix} 0 & -2 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

2.

$$\begin{pmatrix} 1 & \frac{5}{2} & 6 \\ \frac{5}{2} & 4 & 7 \\ 6 & 7 & 5 \end{pmatrix}$$

3.

$$\begin{pmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{2} \\ \frac{1}{2} & 1 & \cdots & \frac{1}{2} \\ \vdots & \vdots & & \vdots \\ \frac{1}{2} & \frac{1}{2} & \cdots & 1 \end{pmatrix}$$

4.

$$\begin{aligned} \because \sum_{i=1}^{n}(x_i - \overline{x})^2 &= \sum_{i=1}^{n}(x_i^2 - 2x_i\overline{x} + \overline{x}^2) \\ &= \sum_{i=1}^{n} x_i^2 - 2\overline{x}\sum_{i=1}^{n} x_i + n\overline{x}^2 \\ &= \sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2 \\ &= \sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i^2 + 2\sum_{i<j} x_i x_j) \\ &= \frac{n-1}{n}\sum_{i=1}^{n} x_i^2 - \frac{2}{n}\sum_{i<j} x_i x_j \end{aligned}$$

$$\begin{pmatrix} \frac{n-1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & \frac{n-1}{n} & \cdots & -\frac{1}{n} \\ \vdots & \vdots & & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & \frac{n-1}{n} \end{pmatrix}$$