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15.

$$
\begin{aligned}
n\sum_{i=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2&=(n-1)\sum_{i=1}^{n}x_i^2-2\sum_{1\le i<j\le n}^{n}x_ix_j\\
&=\sum_{1\le i<j\le n}(x_i-x_j)^2\\
&\ge 0
\end{aligned}
$$

对任意一组数 $(c_1,c_2,\cdots,c_n)$ 带入原式都有 $\sum\limits_{1\le i<j\le n}(c_i-c_j)^2$

$\therefore n\sum\limits_{i=1}^{n}x_i^2-(\sum\limits_{i=1}^{n}x_i)^2$ 半正定

17.

$Ax=0\Rightarrow A^TAx=0\Rightarrow x^TA^TAx=0$

$x^TA^TAx=0\Rightarrow (Ax)^TAx=0\Rightarrow Ax=0\Rightarrow A^TAx=0$

$\therefore Ax=0$ 与 $A^TAx=0$ 有相同的解, 根据其基础解系的性质可知

$\therefore n-\mathrm{rank}(A)=n-\mathrm{rank}(A^TA)$

$\therefore \mathrm{rank}(A)=\mathrm{rank}(A^TA)$