机器人学导论第二次作业

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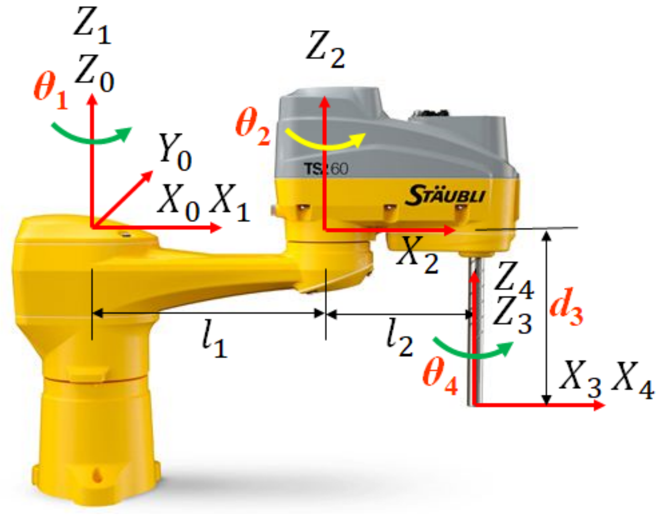
问题:

SCARA教学机器人 (四个自由度), 机器人的末端装置即为连杆4的坐标系, 根据给出的坐标系关系, 建立个连杆坐标的 D-H 参数表, 求解运动学正逆解方程.



解答:

首先是由图建立 D-H 参数表:

连杆 i	θ_i	α_{i-1}	a_{i-1}	d_i
1	$\theta_1(0^\circ)$	0	0	0
2	$\theta_2(0^\circ)$	0	l_1	0
3	0	0	l_2	d_3
4	$\theta_4(0^\circ)$	0	0	0

由连杆变换公式

$$ {}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

可得:

$\theta_3 = 0, \alpha_{i-1} = 0, a_{i-1} = 0, d_i = 0$

$$ {}^0T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^3T_4 = \begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & 0 \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

进行运动学正解 ${}^0T_4 = {}^0T_1(\theta_1){}^1T_2(\theta_2){}^2T_3(\theta_3){}^3T_4(\theta_4)$ 可得最后结果:

$$
{}^0T_4 = \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_4) & -\sin(\theta_1+\theta_2+\theta_4) & 0 & l_1\cos(\theta_1)+l_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2+\theta_4) & \cos(\theta_1+\theta_2+\theta_4) & 0 & l_1\sin(\theta_1)+l_2\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

对 ${}^0T_4 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^0T_1^1T_2^2T_3^3T_4$ 进行运动学逆解有

${}^0T_1^{-1}(\theta_1){}^0T_4 = {}^1T_2(\theta_2){}^2T_3(\theta_3){}^3T_4(\theta_4)$

即有

$$
\begin{bmatrix} n_x\cos(\theta_1)+n_y\sin(\theta_1) & o_x\cos(\theta_1)+o_y\sin(\theta_1) & a_x\cos(\theta_1)+a_y\sin(\theta_1) & p_x\cos(\theta_1)+p_y\sin(\theta_1) \\ -n_x\sin(\theta_1)+n_y\cos(\theta_1) & -o_x\sin(\theta_1)+o_y\cos(\theta_1) & -a_x\sin(\theta_1)+a_y\cos(\theta_1) & -p_x\sin(\theta_1)+p_y\cos(\theta_1) \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_2+\theta_4) & -\sin(\theta_2+\theta_4) & 0 & l_1+l_2\cos(\theta_2) \\ \sin(\theta_2+\theta_4) & \cos(\theta_2+\theta_4) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

有 $\cos(\theta_2) = \dfrac{p_x\cos(\theta_1)+p_y\sin(\theta_1)-l_1}{l_2}$ 和 $\sin(\theta_2) = \dfrac{-p_x\sin(\theta_1)+p_y\cos(\theta_1)}{l_2}$

则有 $(p_x\cos(\theta_1)+p_y\sin(\theta_1)-l_1)^2+(-p_x\sin(\theta_1)+p_y\cos(\theta_1))^2 = l_2^2$

则有 $2l_1p_x\cos(\theta_1)+2l_1p_y\sin(\theta_1) = l_1^2-l_2^2+p_x^2+p_y^2$

解得

$$
\theta_1 = 2\operatorname{atan}\left(\frac{2l_1p_y-\sqrt{-l_1^4+2l_1^2l_2^2+2l_1^2p_x^2+2l_1^2p_y^2-l_2^4+2l_2^2p_x^2+2l_2^2p_y^2-p_x^4-2p_x^2p_y^2-p_y^4}}{l_1^2+2l_1p_x-l_2^2+p_x^2+p_y^2}\right)
$$

或

$$
\theta_1 = 2\operatorname{atan}\left(\frac{2l_1p_y+\sqrt{-l_1^4+2l_1^2l_2^2+2l_1^2p_x^2+2l_1^2p_y^2-l_2^4+2l_2^2p_x^2+2l_2^2p_y^2-p_x^4-2p_x^2p_y^2-p_y^4}}{l_1^2+2l_1p_x-l_2^2+p_x^2+p_y^2}\right)
$$

且有 $\theta_2 = \operatorname{atan2}(-p_x\sin(\theta_1)+p_y\cos(\theta_1), p_x\cos(\theta_1)+p_y\sin(\theta_1)-l_1)$

且可以求出 $d_3 = p_z$

再使用

${}^1T_2^{-1}(\theta_2){}^0T_1^{-1}(\theta_1){}^0T_4 = {}^2T_3(\theta_3){}^3T_4(\theta_3)$

即有

$$
\begin{bmatrix} n_x\cos(\theta_1+\theta_2)+n_y\sin(\theta_1+\theta_2) & o_x\cos(\theta_1+\theta_2)+o_y\sin(\theta_1+\theta_2) & a_x\cos(\theta_1+\theta_2)+a_y\sin(\theta_1+\theta_2) & -l_1\cos(\theta \\ -n_x\sin(\theta_1+\theta_2)+n_y\cos(\theta_1+\theta_2) & -o_x\sin(\theta_1+\theta_2)+o_y\cos(\theta_1+\theta_2) & -a_x\sin(\theta_1+\theta_2)+a_y\cos(\theta_1+\theta_2) & l_1\sin(\theta_2 \\ n_z & o_z & a_z & \\ 0 & 0 & 0 & \end{bmatrix}
\begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & l_2 \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

${}^3T_3^{-1}(\theta_3){}^1T_2^{-1}(\theta_2){}^0T_1^{-1}(\theta_1){}^0T_4 = {}^3T_4(\theta_3)$

$$
\begin{bmatrix}
n_x \cos(\theta_1+\theta_2) + n_y \sin(\theta_1+\theta_2) & o_x \cos(\theta_1+\theta_2) + o_y \sin(\theta_1+\theta_2) & a_x \cos(\theta_1+\theta_2) + a_y \sin(\theta_1+\theta_2) & -l_1 \cos(\theta \\
-n_x \sin(\theta_1+\theta_2) + n_y \cos(\theta_1+\theta_2) & -o_x \sin(\theta_1+\theta_2) + o_y \cos(\theta_1+\theta_2) & -a_x \sin(\theta_1+\theta_2) + a_y \cos(\theta_1+\theta_2) & l_1 \sin(\\
n_z & o_z & a_z & \\
0 & 0 & 0 &
\end{bmatrix}
$$

$$
\begin{bmatrix}
\cos(\theta_4) & -\sin(\theta_4) & 0 & 0 \\
\sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

最后的 $\theta_4 = \operatorname{atan2}(-n_x \sin(\theta_1+\theta_2) + n_y \cos(\theta_1+\theta_2), n_x \cos(\theta_1+\theta_2) + n_y \sin(\theta_1+\theta_2))$