第十次作业

12.8, 12.9, 12.13, 13.6, 13.12, 14.2

12.8

$$\because 60\ {}^\circ\mathrm{C} = (273+60)\ \mathrm{K} = 333\ \mathrm{K}$$

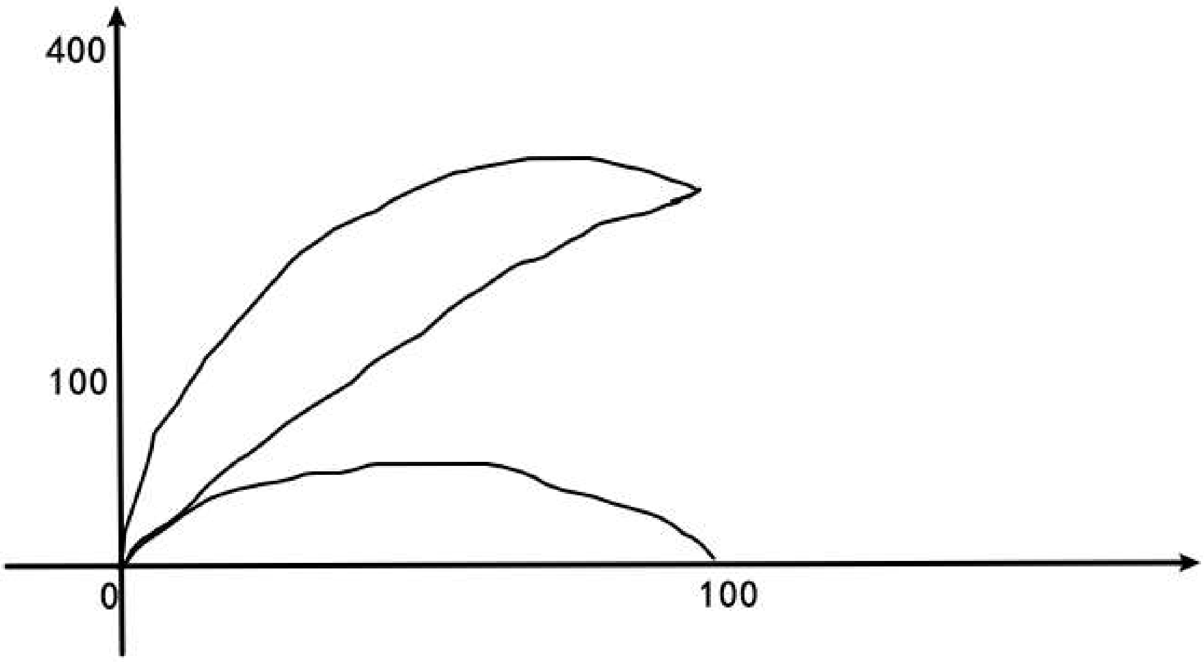
$$\because \Delta Q_b = \frac{1}{2} \times 20 \times 10^{-3} \times 400^2\ \mathrm{J} = mc(T_f - 333\ \mathrm{K})$$

$$\therefore T_f = 333\ \mathrm{K} + \frac{\frac{1}{2} \times 20 \times 10^{-3} \times 400^2}{20 \times 10^{-3} \times 400}\ \mathrm{K} = 533\ \mathrm{K}$$

$$\begin{aligned}\therefore \Delta S &= \left(\int_{333}^{T_f} + \int_{T_f}^{288}\right) \frac{mc\mathrm{d}T}{T} + \frac{\Delta Q_s}{T_s} \\ &= 20 \times 10^{-3} \times 400 \times \ln\frac{288}{333}\ \mathrm{J/K} \\ &+ \frac{\frac{1}{2} \times 20 \times 10^{-3} \times 400^2 + 20 \times 10^{-3} \times 400 \times 45}{273+15}\ \mathrm{J/K} \\ &= 5.65\ \mathrm{J/K}\end{aligned}$$

12.9

(a)



(b)

$$
\begin{aligned}
\frac{S}{k_B} &= \ln C_{1500}^{n} = \ln \frac{1500!}{(1500-n)!n!} \\
&= \ln \frac{1500 \cdot 1499 \cdot \cdots \cdot (1500-n+1)}{n \cdot (n-1) \cdot \cdots \cdot 1} \\
&= \ln \frac{1500}{n} + \ln \frac{1499}{n-1} + \cdots + \ln \frac{1500-n+1}{1}
\end{aligned}
$$

(3)

$$\because \frac{x}{1500} = \frac{100-x}{100}$$

$$\therefore x = \frac{1500}{16} = 93.75$$

12.13

将 T, p 看作独立变量.

$$\because S = f(T, p)$$

$$\therefore \mathrm{d}S = \left(\frac{\partial S}{\partial T}\right)_p \mathrm{d}T + \left(\frac{\partial S}{\partial p}\right)_T \mathrm{d}p$$

$$\therefore T\mathrm{d}S = T\left(\frac{\partial S}{\partial T}\right)_p \mathrm{d}T + T\left(\frac{\partial S}{\partial p}\right)_T \mathrm{d}p$$

我们知道 $T\left(\frac{\partial S}{\partial T}\right)_P = C_p$ 且 $\frac{\partial S}{\partial P}_T = -\left(\frac{\partial V}{\partial T}\right)_P$

$\therefore T\mathrm{d}S = C_p\mathrm{d}T - T\left(\frac{\partial V}{\partial T}\right)_p \mathrm{d}p = C_p\mathrm{d}T - TV\alpha\mathrm{d}p$

$\because \mathrm{d}S = \frac{\mathrm{d}Q}{T} = \frac{1}{T}\left[\left(\frac{\partial U}{\partial T}\right)_V \mathrm{d}T + \left(\left(\frac{\partial U}{\partial V}\right)_T + P\right)\mathrm{d}V\right]$

且 $\left(\frac{\partial}{\partial V}\right)_T \left(\frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V\right) = \left(\frac{\partial}{\partial T}\right)_V \left(\left(\frac{\partial U}{\partial V}_T + P\right)\frac{1}{T}\right)$

$\therefore \frac{1}{T}\left(\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right)_T = \frac{1}{T}\left(\left(\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}_T\right)_V + \left(\frac{\partial P}{\partial T}\right)_V\right) - \left(\left(\frac{\partial U}{\partial V}\right)_T + P\right)\right)\frac{1}{T^2}$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

$\therefore T\mathrm{d}S = C_V\mathrm{d}T + T\left(\frac{\partial P}{\partial T}\right)_V \mathrm{d}V = C_V\mathrm{d}T + T\frac{\alpha}{\kappa}\mathrm{d}V$

13.6

$$\begin{aligned}
N &= \int_{v_m}^{\infty} n_0 \left(\frac{1}{\pi v_m^2}\right) \exp\left(-\frac{v^2}{v_m^2}\right) 4\pi v^2 \mathrm{d}v \\
&= \frac{4n_0}{\sqrt{\pi}} \int_1^{\infty} e^{-x^2} x^2 \mathrm{d}x \\
&= \frac{2n_0}{\sqrt{\pi}} \left(e^{-1} + \int_1^{\infty} e^{-x^2}\mathrm{d}x\right) \\
&= \frac{2n_0}{\sqrt{\pi}} \left(e^{-1} + \frac{\sqrt{\pi}}{2} - \int_0^1 e^{-x^2}\mathrm{d}x\right) \\
&= n_0\left(1 + \frac{2}{e\sqrt{\pi}} - erf(1)\right) \\
&= n_0\left(1 + \frac{2}{e\sqrt{\pi}} - 0.8427\right) \\
&= 0.57241 n_0
\end{aligned}$$

13.12

$\because 4\pi r^2 \sigma T_b^4 = 4\pi R_\odot^2 \sigma T_\odot^4 \frac{\pi r^2}{4\pi D^2}$

$$\therefore T_b = T_\odot \sqrt[4]{\frac{R_\odot^2}{4D^2}} = T_\odot \sqrt{\frac{\theta}{2}} = 5700\ \mathrm{K}\sqrt{\frac{0.50\times\pi}{2\times 180}} = 266\ \mathrm{K}$$

14.2

$$\because p = \frac{nRT}{V-nb} - a\frac{n^2}{V^2}$$

$$\therefore \left(\frac{\partial p}{\partial V}\right)_T = -\frac{nRT}{(V-nb)^2} + 2a\frac{n^2}{V^2}\cdot\frac{1}{V} = 0$$

$$\therefore \left(\frac{\partial^2 p}{\partial V^2}\right)_T = 2\frac{nRT}{(V-nb)^3} - 6a\frac{n^2}{V^2}\cdot\frac{1}{V^2} = 0$$

$$\therefore \frac{nRT}{(V-nb)^2} = 2a\frac{n^2}{V^2}\cdot\frac{1}{V}$$

$$\frac{nRT}{(V-nb)^3} = 3a\frac{n^2}{V^2}\cdot\frac{1}{V^2}$$

$$\therefore V - nb = \frac{2}{3}V, V_C = 3nb$$

$$\therefore nRT_C = 4n^2b^2\cdot 2a\frac{n^2}{27n^3b^3}$$

$$\therefore RT_C = \frac{8a}{27b}$$

$$\therefore p_C = \frac{n\frac{8a}{27b}}{2nb} - a\frac{1}{9b^2} = \frac{a}{27b^2}$$