21.

28.

29.

30.

1.判断 $A=\begin{bmatrix}2 & 1 & -5\\ 3 & 2 & 4\\ 1 & 0 & 3\end{bmatrix}$是否可逆?若可逆则求出$A^{-1}$.

2.用逆矩阵解方程组 $\begin{cases}2x_1-x_2-x_3=4\\ 3x_1+4x_2-2x_3=11\\ 3x_1-2x_2+4x_3=11\end{cases}$.

3.解矩阵方程:

$$\begin{bmatrix}1 & 4\\ -1 & 2\end{bmatrix}X\begin{bmatrix}2 & 0\\ -1 & 1\end{bmatrix}=\begin{bmatrix}3 & 1\\ 0 & -1\end{bmatrix}.$$

$$\boxed{X=\begin{bmatrix}1 & 1\\ \frac{1}{4} & 0\end{bmatrix}}$$

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1.

$$\because A=\begin{bmatrix}2 & 1 & -5\\ 3 & 2 & 4\\ 1 & 0 & 3\end{bmatrix}$$

$\therefore |A|=17,$矩阵A可逆

$$\because [A\quad E]\to\begin{bmatrix}2 & 1 & -5 & 1 & 0 & 0\\ 3 & 2 & 4 & 0 & 1 & 0\\ 1 & 0 & 3 & 0 & 0 & 1\end{bmatrix}\to\begin{bmatrix}1 & 0 & 3 & 0 & 0 & 1\\ 0 & 1 & -11 & 1 & 0 & -2\\ 0 & 0 & 17 & -2 & 1 & 1\end{bmatrix}$$

$$\to\begin{bmatrix}1 & 0 & 0 & \frac{15}{17} & -\frac{5}{17} & \frac{12}{17}\\ 0 & 1 & 0 & -\frac{5}{17} & \frac{11}{17} & -\frac{23}{17}\\ 0 & 0 & 1 & -\frac{2}{17} & \frac{1}{17} & \frac{1}{17}\end{bmatrix}$$

$$\therefore A^{-1}=\begin{bmatrix}\frac{15}{17} & -\frac{5}{17} & \frac{12}{17}\\ -\frac{5}{17} & \frac{11}{17} & -\frac{23}{17}\\ -\frac{2}{17} & \frac{1}{17} & \frac{1}{17}\end{bmatrix}$$

2.

令$A=\begin{pmatrix}2&-1&-1\\3&4&-2\\3&-2&4\end{pmatrix},x=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix},B=\begin{pmatrix}4\\11\\11\end{pmatrix}$

$\therefore$ 原式可以写成$Ax=B$,其中$|A|=60\neq 0$,即A可逆

$\therefore x=A^{-1}B$

$$\because (A\quad B)\rightarrow\begin{pmatrix}2&-1&-1&4\\3&4&-2&11\\3&-2&4&11\end{pmatrix}\rightarrow\begin{pmatrix}1&-\frac{1}{2}&-\frac{1}{2}&2\\0&1&-11&-10\\0&0&1&1\end{pmatrix}$$

$$\rightarrow\begin{pmatrix}1&0&0&3\\0&1&0&1\\0&0&1&1\end{pmatrix}$$

$\therefore x=\begin{pmatrix}3\\1\\1\end{pmatrix}$,即$x_1=3,x_2=1,x_3=1$

3.

$\because$ 令$P=\begin{pmatrix}1&4\\-1&2\end{pmatrix},Q=\begin{pmatrix}2&0\\-1&1\end{pmatrix}$

$\therefore |P|=6,|Q|=2,P,Q$均可逆

$\therefore X=P^{-1}\begin{pmatrix}3&1\\0&-1\end{pmatrix}Q^{-1}$

$\because P^{*}=\begin{pmatrix}2&-4\\1&1\end{pmatrix},Q^{*}=\begin{pmatrix}1&0\\1&2\end{pmatrix}$

$$\therefore X=\begin{pmatrix}\frac{1}{3}&-\frac{2}{3}\\\frac{1}{6}&\frac{1}{6}\end{pmatrix}\begin{pmatrix}3&1\\0&-1\end{pmatrix}\begin{pmatrix}\frac{1}{2}&0\\\frac{1}{2}&1\end{pmatrix}=\begin{pmatrix}1&1\\\frac{1}{2}&0\end{pmatrix}\begin{pmatrix}\frac{1}{2}&0\\\frac{1}{2}&1\end{pmatrix}=\begin{pmatrix}1&1\\\frac{1}{4}&0\end{pmatrix}$$

21.

$$\because\begin{pmatrix}O&A\\C&O\end{pmatrix}\begin{pmatrix}O&C^{-1}\\A^{-1}&O\end{pmatrix}=\begin{pmatrix}AA^{-1}&O\\O&CC^{-1}\end{pmatrix}=E$$

$$\therefore X^{-1} = \begin{pmatrix} O & C^{-1} \\ A^{-1} & O \end{pmatrix}$$

28.

(1)

$$\begin{aligned}
(A \quad E) &\to \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\to \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 & -1 & 1 & 0 & 0 \\ 0 & 0 & -2 & -2 & -1 & 0 & 1 & 0 \\ 0 & -2 & -2 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \\
&\to \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 & -1 & 1 & 0 & 0 \\ 0 & 0 & -2 & -2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 & 1 & -1 & -1 & 1 \end{pmatrix} \\
&\to \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \\
&\to \begin{pmatrix} 1 & 1 & 1 & 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \\
&\to \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix}
\end{aligned}$$

$$\therefore A^{-1} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

(2)

令$A=\begin{pmatrix} B & B \\ B & -B \end{pmatrix}$,其中$B=\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $|B|=-2\neq 0$

$\therefore$ 存在$B^{-1}=\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

$$\therefore \begin{pmatrix} E & O \\ -\frac{1}{2}E & E \end{pmatrix}\begin{pmatrix} E & E \\ O & E \end{pmatrix}\begin{pmatrix} B & B \\ B & -B \end{pmatrix}=\begin{pmatrix} 2B & O \\ 0 & -B \end{pmatrix}$$

$$\therefore \begin{pmatrix} B & B \\ B & -B \end{pmatrix}^{-1}\begin{pmatrix} E & E \\ O & E \end{pmatrix}^{-1}\begin{pmatrix} E & O \\ -\frac{1}{2}E & E \end{pmatrix}^{-1}=\begin{pmatrix} 2B & O \\ 0 & -B \end{pmatrix}^{-1}$$

$$\begin{aligned}\therefore A^{-1}=\begin{pmatrix} B & B \\ B & -B \end{pmatrix}^{-1}&=\begin{pmatrix} 2B & O \\ 0 & -B \end{pmatrix}^{-1}\begin{pmatrix} E & O \\ -\frac{1}{2}E & E \end{pmatrix}\begin{pmatrix} E & E \\ O & E \end{pmatrix}\\ &=\begin{pmatrix} \frac{1}{2}B^{-1} & O \\ O & -B^{-1} \end{pmatrix}\begin{pmatrix} E & O \\ -\frac{1}{2}E & E \end{pmatrix}\begin{pmatrix} E & E \\ O & E \end{pmatrix}\\ &=\begin{pmatrix} \frac{1}{2}B^{-1} & \frac{1}{2}B^{-1} \\ \frac{1}{2}B^{-1} & -\frac{1}{2}B^{-1} \end{pmatrix}\end{aligned}$$

$$\therefore A^{-1}=\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

29.

$$\because \begin{pmatrix} E_m & O \\ -A & E_n \end{pmatrix}\begin{pmatrix} E_m & B \\ A & E_n \end{pmatrix}=\begin{pmatrix} E_m & B \\ O & E_n-AB \end{pmatrix}$$

$$\begin{vmatrix} E_m & O \\ -A & E_n \end{vmatrix}=|E_m||E_n|=1$$

$$\begin{aligned}\therefore \begin{vmatrix} E_m & B \\ A & E_n \end{vmatrix}&=\begin{vmatrix} E_m & O \\ -A & E_n \end{vmatrix}\begin{vmatrix} E_m & B \\ A & E_n \end{vmatrix}\\ &=\begin{vmatrix} E_m & B \\ O & E_n-AB \end{vmatrix}\\ &=|E_m||E_n-AB|\\ &=|E_n-AB|\end{aligned}$$

$$\because \begin{pmatrix} E_m & -B \\ O & E_n \end{pmatrix}\begin{pmatrix} E_m & B \\ A & E_n \end{pmatrix}=\begin{pmatrix} E_m-BA & B \\ O & E_n \end{pmatrix}$$

$$\begin{vmatrix} E_m & -B \\ O & E_n \end{vmatrix} = |E_m||E_n| = 1$$

$$\begin{aligned} \therefore \begin{vmatrix} E_m & B \\ A & E_n \end{vmatrix} &= \begin{vmatrix} E_m & -B \\ O & E_n \end{vmatrix} \begin{vmatrix} E_m & B \\ A & E_n \end{vmatrix} \\ &= \begin{vmatrix} E_m - BA & B \\ O & E_n \end{vmatrix} \\ &= |E_n - BA||E_m| \\ &= |E_n - BA| \end{aligned}$$

$$\therefore \begin{vmatrix} E_m & B \\ A & E_n \end{vmatrix} = |E_n - AB| = |E_n - BA|$$

30.

$$\because \begin{pmatrix} \lambda E_m & O \\ -A & E_n \end{pmatrix} \begin{pmatrix} E_m & B \\ A & \lambda E_n \end{pmatrix} = \begin{pmatrix} \lambda E_m & \lambda B \\ O & \lambda E_n - AB \end{pmatrix}$$

$$\begin{vmatrix} \lambda E_m & O \\ -A & E_n \end{vmatrix} = |\lambda E_m||E_n| = \lambda^m$$

$$\because \begin{pmatrix} \lambda E_m & -B \\ O & E_n \end{pmatrix} \begin{pmatrix} E_m & B \\ A & \lambda E_n \end{pmatrix} = \begin{pmatrix} \lambda E_m - BA & O \\ A & \lambda E_n \end{pmatrix}$$

$$\begin{vmatrix} \lambda E_m & -B \\ O & E_n \end{vmatrix} = |\lambda E_m||E_n| = \lambda^m$$

$$\therefore \begin{vmatrix} \lambda E_m & O \\ -A & E_n \end{vmatrix} \begin{vmatrix} E_m & B \\ A & \lambda E_n \end{vmatrix} = \begin{vmatrix} \lambda E_m & -B \\ O & E_n \end{vmatrix} \begin{vmatrix} E_m & B \\ A & \lambda E_n \end{vmatrix}$$

$$\therefore \begin{vmatrix} \lambda E_m & O \\ O & \lambda E_n - AB \end{vmatrix} = \begin{vmatrix} \lambda E_m - BA & O \\ O & \lambda E_n \end{vmatrix}$$

$$\therefore \lambda^m |\lambda E_n - AB| = \lambda^n |\lambda E_m - BA|$$

$$\therefore |\lambda E_n - AB| = \lambda^{n-m} |\lambda E_m - BA|$$