概率统计第十二次作业

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2.

(1)

由题目可知 $E(X_i) = 280, D(X_i) = 800^2$

对于 10000 个投保人, 由大数定理可知

$$\bar{X} = \frac{1}{10000}\sum_{i=1}^{10000} X_i \sim N(280, \frac{800^2}{10000} = 8^2)$$

因此

$$P(\sum_{i=1}^{10000} X_i > 2700000) = P(\bar{X} > 270) \approx 1 - \Phi(\frac{270-280}{8}) = \Phi(1.25) = 0.8944$$

(2)

由题目可知 $E(X_i) = 5, D(X_i) = 6$, 对于 50 张保单有

$$P(\sum_{i=1}^{50} X_i > 300) = P(\frac{1}{50}\sum_{i=1}^{n} X_i > 6) \approx 1 - \Phi(\frac{6-5}{\sqrt{\frac{6}{50}}}) = 1 - \Phi(\frac{300 - 50 \times 5}{\sqrt{50 \times 6}}) = 1 - \Phi(2.89) = 0.0019$$

7.

(1)

设第 i 只蛋糕售出价格为 X_i, 则有分布律

X_i	1	1.2	1.5

X_i	1	1.2	1.5
p_i	0.3	0.2	0.5

因此有

$$E(X_i) = 1 \times 0.3 + 1.2 \times 0.2 + 1.5 \times 0.5 = 1.29$$

$$D(X_i) = 1 \times 0.3 + 1.2^2 \times 0.2 + 1.5^2 \times 0.5 - 1.29^2 = 0.0489$$

由大数定律有

$$P(X \geqslant 400) \approx 1 - \Phi(\frac{400 - 300 \times 1.29}{\sqrt{300 \times 0.0489}}) = 1 - \Phi(3.39) = 0.0003$$

(2)

记随机变量 Y 为 300 只蛋糕中售价为 1.2 元蛋糕的只数,

于是有 $Y \sim b(300, 0.2)$, 由拉普拉斯定理可知

$$P(Y > 60) = 1 - \Phi(\frac{y - np}{\sqrt{np(1-p)}}) = 1 - \Phi(\frac{60 - 300 \times 0.2}{\sqrt{300 \times 0.2 \times (1 - 0.2)}}) = 1 - \Phi(0) = 0.5$$

9.

(1)

因为 $E(X_i) = 2.2, D(X_i) = 1.4^2$

因此 $\bar{X} \sim N(2.2, \frac{1.4^2}{50})$

$$P(\bar{X} < 2) \approx \Phi(\frac{2 - 2.2}{\sqrt{\frac{1.4^2}{50}}}) = \Phi(-1.03) = 0.1515$$

(2)

$$P(\sum_{i=1}^{52} X_i < 100) \approx \Phi(\frac{100 - 52 \times 2.2}{\sqrt{52 \times 1.4^2}}) = \Phi(-1.426) = 0.077$$

11.

(1)

由题意知 $E(\bar{X})=E(\bar{Y})=5, D(\bar{X})=D(\bar{Y})=\dfrac{0.3}{80}$

$P(4.9<\bar{X}<5.1)=\Phi(\dfrac{5.1-5}{\sqrt{\frac{0.3}{80}}})-\Phi(\dfrac{4.9-5}{\sqrt{\frac{0.3}{80}}})=2\Phi(1.63)-1=0.8968$

(2)

因为 $E(\bar{X}-\bar{Y})=E(\bar{X})-E(\bar{Y})=0, D(\bar{X}-\bar{Y})=D(\bar{X})+D(\bar{Y})=\dfrac{0.3}{40}$

$P(-0.1<\bar{X}<0.1)=\Phi(\dfrac{0.1}{\sqrt{\frac{0.3}{40}}})-\Phi(\dfrac{-0.1}{\sqrt{\frac{0.3}{40}}})=2\Phi(1.15)-1=0.7498$

12.

由题可知 $E(X_i)=1\times 0.6+2\times 0.3=1.2, D(X_i)=1^2\times 0.6+2^2\times 0.3-1.2^2=0.36$

因此要有 $0.95\leqslant P(\sum_{i=1}^{200}X_i\leqslant n)\approx\Phi(\dfrac{n-200\times 1.2}{\sqrt{200\times 0.36}})=\Phi(\dfrac{n-240}{\sqrt{72}})$

因为有 $0.95=\Phi(1.645)$

因此 $\dfrac{n-240}{\sqrt{72}}\geqslant 1.645$

解得 $n\geqslant 240+1.645\times\sqrt{72}=253.96$, 至少要有 254 个车位

13.

由题意可知 $Y=\bar{X}-\mu\sim N(0,\dfrac{400}{n})$

因此 $P(|\bar{X}-\mu|<1)=P(-1<Y<1)\approx\Phi(\dfrac{1}{20/\sqrt{n}})-\Phi(\dfrac{-1}{20/\sqrt{n}})=2\Phi(\dfrac{1}{20/\sqrt{n}})-1$

要 $P(|\bar{X}-\mu|<1)\geqslant 0.95$ 即 $2\Phi(\dfrac{1}{20/\sqrt{n}})-1\geqslant 0.95$ 即 $\Phi(\dfrac{1}{20/\sqrt{n}})\geqslant 0.975=\Phi(1.96)$

所以 $n \geqslant (20 \times 1.96)^2 = 1536.64$, 即 n 至少为 1537

14.

(1)

由中心极限定理可知

$P(X > 75) \approx 1 - \Phi(\frac{75 - 100 \times 0.8}{\sqrt{100 \times 0.8 \times (1 - 0.8)}}) = 1 - \Phi(-1.25) = \Phi(1.25) = 0.8944$

(2)

$P(X > 75) \approx 1 - \Phi(\frac{75 - 100 \times 0.7}{\sqrt{100 \times 0.7 \times (1 - 0.7)}}) = 1 - \Phi(1.09) = 0.1379$