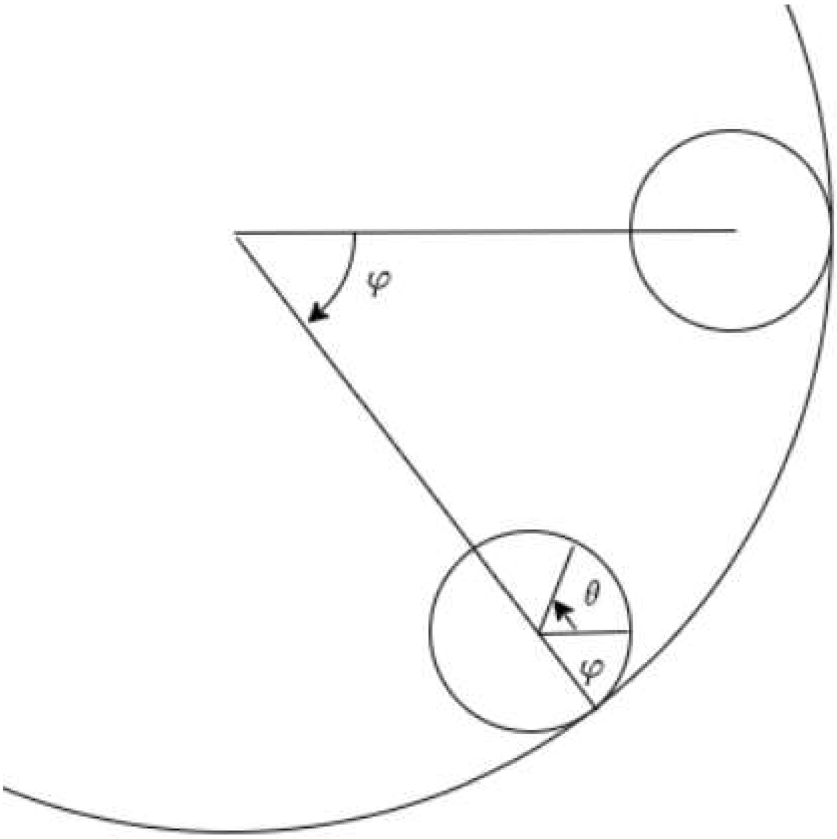
# 大学物理第六次作业

6.6, 7.1, 7.5, 7.8

## 6.6

### (a)




设小球质量为 $m$, 动能为 $E_k$, 转动动能为 $E_1$, 平动动能为 $E_2$, 小球半径为 $r$, 大球半径为 $R$.

由功能关系可知

$$E_k = mg(R - r)$$

$$E_k = E_1 + E_2$$

$$E_2 = \frac{1}{2}m[(R - r)\dot{\varphi}]^2$$

对转动角度分析可知:

$$\because r(\theta + \varphi) = R\varphi$$

$\therefore \theta = (\frac{R}{r} - 1)\varphi, \theta' = (\frac{R}{r} - 1)\varphi'$

小球的转动惯量 $I = \frac{2}{5}mr^2$

$\therefore E_1 = \frac{1}{2}(\frac{2}{5}mr^2)[(\frac{R}{r} - 1)\dot{\varphi}]^2 = \frac{1}{5}m[(R-r)\dot{\varphi}]^2$

$\therefore \frac{E_1}{E_2} = \frac{2}{5}$

$\therefore E_1 = \frac{2}{7}E_k = \frac{2}{7}mg(R-r)$

$E_2 = \frac{5}{7}E_k = \frac{5}{7}mg(R-r)$

### (b)

进行最低点进行受力分析:

$N - mg = m(R-r)\dot{\varphi}^2$

由 (a) 得

$E_1 = \frac{1}{5}m[(R-r)\dot{\varphi}]^2 = \frac{2}{7}mg(R-r)$

$\therefore \dot{\varphi}^2 = \frac{10g}{7(R-r)}$

$\therefore N = mg + m(R-r)\dot{\varphi}^2 = \frac{17}{7}mg$

## 7.1

### (a)

对于简谐运动有

$m\ddot{x} + kx = 0, \omega_0^2 = \frac{k}{m}$

解常微分方程可得

$x = A\cos(\omega_0 t + \varphi), \dot{x} = -\omega_0 A\sin(\omega_0 t + \varphi)$

由物理量对时间平均公式 $\overline{P} = \frac{1}{T}\int_0^T P\mathrm{d}t$ , 势能公式 $\frac{1}{2}kx^2$, 动能公式 $\frac{1}{2}m\dot{x}^2$ 和 $\frac{1}{T} = \frac{\omega_0}{2\pi}$ 可得

$$\overline{E}_{pT}=\frac{1}{T}\int_0^T\frac{1}{2}kx^2\mathrm{d}t=\frac{1}{4\pi}kA^2\int_0^{2\pi}\cos^2(\omega_0t+\varphi)\mathrm{d}(\omega_0t)=\frac{1}{4}kA^2$$

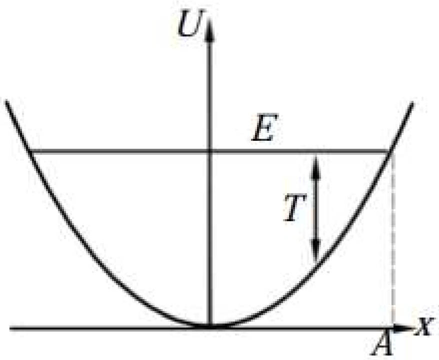
$$\overline{E}_{kT}=\frac{1}{T}\int_0^T\frac{1}{2}m\dot{x}^2\mathrm{d}t=\frac{1}{4\pi}m\omega_0^2A^2\int_0^{2\pi}\sin^2(\omega_0t+\varphi)\mathrm{d}(\omega_0t)=\frac{1}{4}kA^2$$

### (b)

$$\overline{E}_{px}=\frac{1}{A}\int_0^A\frac{1}{2}kx^2\mathrm{d}x=\frac{1}{6}kA^2$$

$$\begin{aligned}\overline{E}_{kx}&=\frac{1}{A}\int_0^A\frac{1}{2}m\dot{x}^2\mathrm{d}x\\&=\frac{1}{A}\int_0^A\frac{1}{2}m\dot{x}^3\mathrm{d}t\\&=\frac{1}{\omega_0A}\int_0^{\frac{\pi}{2}}\frac{1}{2}m(-\omega_0A\sin(\omega_0t+\varphi))^3\mathrm{d}(\omega_0t)\\&=-\frac{1}{2}m\omega_0^2A^2\int_0^{\frac{\pi}{2}}\sin^3(\omega_0t+\varphi)\mathrm{d}(\omega_0t)\\&=\frac{1}{3}kA^2\end{aligned}$$

### (c)




由该图我们可以看出, 动能所占的面积大于势能, 因而 $\overline{E}_{kx}>\overline{E}_{px}$.

## 7.5

设弹簧伸长量为 $X$, $x$ 为每个质点相对于原来位置的位移.

每个小质点的速度为 $\dot{x}=\frac{x}{X}\dot{X}$

弹簧动能为

$$E_k=\frac{1}{2}\int_0^X\dot{x}^2(\frac{m_s}{X}\mathrm{d}x)=\frac{m_s\dot{X}^2}{2X^3}\int_0^X x^2\mathrm{d}x=\frac{1}{6}m_s\dot{X}^2$$

由能量守恒得

$$-mgX-m_sg\frac{X}{2}+\frac{1}{2}kX^2+\frac{1}{2}m\dot{X}^2+\frac{1}{6}m_s\dot{X}^2=0$$

对时间求导得

$$-mg\dot{X}-\frac{1}{2}m_sg\dot{X}+kX\dot{X}+m\dot{X}\ddot{X}+\frac{1}{3}m_s\dot{X}\ddot{X}=0$$

当 $\dot{X}=0$ 时, 该式意义不大

当 $\dot{X}\neq 0$ 时,

$$\therefore -mg-\frac{1}{2}m_sg+kX+m\ddot{X}+\frac{1}{3}m_s\ddot{X}=0$$

$$\therefore (m+\frac{1}{3}m_s)\ddot{X}=-kX+mg+\frac{1}{2}m_sg$$

相当于是有恒定外力的简谐振动

$$\because \omega_0^2=\frac{k}{M}=\frac{k}{m+\frac{1}{3}m_s},\omega_0=\frac{2\pi}{T}$$

$$\therefore T=2\pi\sqrt{\frac{m+\frac{1}{3}m_s}{k}}$$

# 7.8

$\because$ 强迫阻尼振动一个周期后的动能和势能依然不变

$$\therefore \int F\dot{x}\mathrm{d}t-\int F_\gamma\dot{x}\mathrm{d}t=\Delta E_k+\Delta E_p=0$$

$$\therefore \overline{P}=\frac{1}{T}\int F\dot{x}\mathrm{d}t=\frac{1}{T}\int F_\gamma\dot{x}\mathrm{d}t$$

$\therefore$ 外力的平均功率等于阻尼力耗散的功率.