

Assignment 4

201300035 方盛俊

Question 1. OWA and CWA

(1)

- (a) Album(Fantasy)
- (f) $\neg \text{StudioAlbum}(\text{2004_Incomparable_Concert}) \vee \neg \text{LiveAlbum}(\text{2004_Incomparable_Concert})$
- (h) $\exists x. \text{hasFriend}(\text{Jay_Chou}, x)$
- (i) $\exists x. (\text{hasFriend}(\text{Jay_Chou}, x) \wedge \exists y. (\text{dancesWith}(x, y) \wedge \text{Song}(y)))$
- (k) $\text{hasFriend}(\text{Vincent_Fang}, \text{Jay_Chou})$

(2)

- (a) Album(Fantasy): No
- (b) StudioAlbum(The_Eight_Dimensions): Yes
- (c) LiveAlbum(Common_Jasmin_Orange): No
- (d) $\neg \text{LiveAlbum}(\text{Common_Jasmin_Orange})$: Yes
- (e) $\neg \text{EP}(\text{Secret})$: Yes
- (f) $\neg \text{StudioAlbum} \sqcup \neg \text{LiveAlbum}(\text{2004_Incomparable_Concert})$: Yes
- (g) $\neg \text{StudioAlbum} \sqcup \neg \text{LiveAlbum}(\text{Eason_Chan})$: Yes
- (h) $\exists \text{hasFriend}. T(\text{Jay_Chou})$: Yes
- (i) $\exists \text{hasFriend}. \exists \text{dancesWith}. \text{Song}(\text{Jay_Chou})$: Yes
- (j) $\exists \text{hasFriend}. \text{Composer}(\text{Jay_Chou})$: No
- (k) $\exists \text{hasFriend}. \{\text{Jay_Chou}\}(\text{Vincent_Fang})$: No
- (l) DebutAlbum(2004_Incomparable_Concert): No
- (m) Song(Rewind): No
- (n) Singer(Jay_Chou): Yes
- (o) Singer(Jolin_Tsai): No
- (p) Lyricist(Jay_Chou): No
- (q) Composer(Jay_Chou): Yes
- (r) Composer(Ta-yu_Lo): No
- (s) Police(Jay_Chou): No
- (t) Police(Jolin_Tsai): No

- (u) \neg Singer-SongWriter \sqcup \neg Police(Vincent_Fang): Yes
- (v) \neg Singer-SongWriter \sqcup \neg Police(Ta-yu_Lo): yes
- (w) Singer-SongWriter(Jay_Chou): No
- (x) Singer-SongWriter(Jolin_Tsai): No
- (y) \neg SongWriter(Vincent_Fang): Yes
- (z) \neg Dancer(Will_Liu): Yes

(3)

- (a) Album(Fantasy): Don't know
- (b) StudioAlbum(The_Eight_Dimensions): Yes
- (c) LiveAlbum(Common_Jasmin_Orange): Don't know
- (d) \neg LiveAlbum(Common_Jasmin_Orange): Don't know
- (e) \neg EP(Secret): Don't know
- (f) \neg StudioAlbum \sqcup \neg LiveAlbum(2004_Incomparable_Concert): Don't know
- (g) \neg StudioAlbum \sqcup \neg LiveAlbum(Eason_Chan): Don't know
- (h) \exists hasFriend.T(Jay_Chou): Yes
- (i) \exists hasFriend. \exists dancesWith.Song(Jay_Chou): Yes
- (j) \exists hasFriend.Composer(Jay_Chou): Don't know
- (k) \exists hasFriend.{Jay_Chou}(Vincent_Fang): Don't know
- (l) DebutAlbum(2004_Incomparable_Concert): Don't know
- (m) Song(Rewind): Don't know
- (n) Singer(Jay_Chou): Yes
- (o) Singer(Jolin_Tsai): Don't know
- (p) Lyricist(Jay_Chou): Don't know
- (q) Composer(Jay_Chou): Yes
- (r) Composer(Ta-yu_Lo): Don't know
- (s) Police(Jay_Chou): Don't know
- (t) Police(Jolin_Tsai): Don't know
- (u) \neg Singer-SongWriter \sqcup \neg Police(Vincent_Fang): Don't know
- (v) \neg Singer-SongWriter \sqcup \neg Police(Ta-yu_Lo): Don't know
- (w) Singer-SongWriter(Jay_Chou): Don't know
- (x) Singer-SongWriter(Jolin_Tsai): Don't know
- (y) \neg SongWriter(Vincent_Fang): Don't know
- (z) \neg Dancer(Will_Liu): Don't know

(4)

- (a) $\text{answer}(F1(x), D_{\text{music}}) = \{ \text{Jay_Chou}, \text{Eason_Chan} \}$
- (b) $\text{answer}(F2(x), D_{\text{music}}) = \{ \text{Will_Liu}, \text{Black_Cat}, \text{The_Eight_Dimensions}, \text{Secret}, \text{Together}, \text{Ta-yu_Lo}, \text{Jolin_Tsai}, \text{Vincent_Fang}, \text{Hidden_Track}, \text{Herbalist_Manual}, \text{Jay},$

Common_Jasmin_Orange, Initial_D, Elimination, Fantasy, Pearl_of_the_Orient, 2004_Incomparable_Concert, Rewind }

- (c) $\text{answer}(F3(x, y), D_{\text{music}}) = \{ (\text{Jay_Chou}, \text{Vincent_Fang}), (\text{Jay_Chou}, \text{Will_Liu}) \}$
- (d) $\text{answer}(F4(x), D_{\text{music}}) = \{ \text{Vincent_Fang} \}$

(5)

- (a) $\text{cetanswer}(F1(x), D_{\text{music}}) = \{ \text{Jay_Chou}, \text{Eason_Chan} \}$
- (b) $\text{cetanswer}(F2(x), D_{\text{music}}) = \emptyset$
- (c) $\text{cetanswer}(F3(x, y), D_{\text{music}}) = \{ (\text{Jay_Chou}, \text{Vincent_Fang}), (\text{Jay_Chou}, \text{Will_Liu}) \}$
- (d) $\text{cetanswer}(F4(x), D_{\text{music}}) = \emptyset$

Question 2. Querying with TBox

(1)

The certain answers in the context of D_{music} is same with Question 2. (3).

The certain answers in the context of $(\mathcal{T}, D_{\text{music}})$:

- (a) $\text{Album}(\text{Fantasy})$: Yes
- (b) $\text{StudioAlbum}(\text{The_Eight_Dimensions})$: Yes
- (c) $\text{LiveAlbum}(\text{Common_Jasmin_Orange})$: No
- (d) $\neg \text{LiveAlbum}(\text{Common_Jasmin_Orange})$: Yes
- (e) $\neg \text{EP}(\text{Secret})$: Don't know
- (f) $\neg \text{StudioAlbum} \sqcup \neg \text{LiveAlbum}(\text{2004_Incomparable_Concert})$: Yes
- (g) $\neg \text{StudioAlbum} \sqcup \neg \text{LiveAlbum}(\text{Eason_Chan})$: Yes
- (h) $\exists \text{hasFriend.T}(\text{Jay_Chou})$: Yes
- (i) $\exists \text{hasFriend}.\exists \text{dancesWith.Song}(\text{Jay_Chou})$: Yes
- (j) $\exists \text{hasFriend.Composer}(\text{Jay_Chou})$: Don't know
- (k) $\exists \text{hasFriend}.\{\text{Jay_Chou}\}(\text{Vincent_Fang})$: Don't know
- (l) $\text{DebutAlbum}(\text{2004_Incomparable_Concert})$: No
- (m) $\text{Song}(\text{Rewind})$: Yes
- (n) $\text{Singer}(\text{Jay_Chou})$: Yes
- (o) $\text{Singer}(\text{Jolin_Tsai})$: Don't know
- (p) $\text{Lyricist}(\text{Jay_Chou})$: Yes
- (q) $\text{Composer}(\text{Jay_Chou})$: Yes
- (r) $\text{Composer}(\text{Ta-yu_Lo})$: Don't know
- (s) $\text{Police}(\text{Jay_Chou})$: No
- (t) $\text{Police}(\text{Jolin_Tsai})$: Don't know
- (u) $\neg \text{Singer-SongWriter} \sqcup \neg \text{Police}(\text{Vincent_Fang})$: Don't know

- (v) \neg Singer-SongWriter \sqcup \neg Police(Ta-yu_Lo): Don't know
- (w) Singer-SongWriter(Jay_Chou): Don't know
- (x) Singer-SongWriter(Jolin_Tsai): Don't know
- (y) \neg SongWriter(Vincent_Fang): Don't know
- (z) \neg Dancer(Will_Liu): Don't know

(2)

All assertions α :

- Song(Herbalist_Manual)
- Song(Elimination)
- Singer(Jay_Chou)
- Lyricist(Vincent_Fang)

Question 3. Computing $\mathcal{I}_{\mathcal{T}, \mathcal{A}}$ in \mathcal{EL}

(1)

The initial assignment (with obvious abbreviations) is given by

- $S(d_{\text{Guitarist}}) = \{ \text{Guitarist} \}$
- $S(d_{\text{Bassist}}) = \{ \text{Bassist} \}$
- $S(d_{\text{Drummer}}) = \{ \text{Drummer} \}$
- $S(d_{\text{RockBand}}) = \{ \text{RockBand} \}$
- $S(d_{\text{Manager}}) = \{ \text{Manager} \}$
- $S(d_{\text{Employee}}) = \{ \text{Employee} \}$
- $S(\text{John_Lennon}) = \{ \text{Guitarist} \}$
- $S(\text{Paul_McCartney}) = \{ \text{Bassist} \}$
- $S(\text{Ringo_Starr}) = \{ \text{Drummer} \}$
- $S(\text{Beatles}) = \{ \text{RockBand} \}$
- $S(\text{Brian_Epstein}) = \emptyset$
- $R(\text{managed_by}) = \{ (\text{Beatles}, \text{Brian_Epstein}) \}$
- $R(\text{plays_for}) = \emptyset$

Update S using (simpleR):

- $S(d_{\text{Manager}}) = \{ \text{Manager}, \text{Employee} \}$

Update R using (rightR):

- $R(\text{plays_for}) = \{ (d_{\text{Guitarist}}, d_{\text{RockBand}}), (\text{John_Lennon}, d_{\text{RockBand}}), (d_{\text{Bassist}}, d_{\text{RockBand}}), (\text{Paul_McCartney}, d_{\text{RockBand}}), (d_{\text{Drummer}}, d_{\text{RockBand}}), (\text{Ringo_Starr}, d_{\text{RockBand}}) \}$
- $R(\text{managed_by}) = \{ (\text{Beatles}, \text{Brian_Epstein}), (d_{\text{RockBand}}, d_{\text{Manager}}), (\text{Beatles}, d_{\text{Manager}}), (d_{\text{Manager}}, d_{\text{Manager}}) \}$

So we have:

- $\Delta_{\mathcal{T}, \mathcal{A}}^{\mathcal{I}} = \{ d_{\text{Guitarist}}, d_{\text{Bassist}}, d_{\text{Drummer}}, d_{\text{RockBand}}, d_{\text{Manager}}, d_{\text{Employee}}, \text{John_Lennon}, \text{Paul_McCartney}, \text{Ringo_Starr}, \text{Beatles}, \text{Brian_Epstein} \}$
- $\text{Guitarist}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ d_{\text{Guitarist}}, \text{Guitarist} \}$
- $\text{Bassist}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ d_{\text{Bassist}}, \text{Paul_McCartney} \}$
- $\text{Drummer}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ d_{\text{Drummer}}, \text{Ringo_Starr} \}$
- $\text{RockBand}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ d_{\text{RockBand}}, \text{Beatles} \}$
- $\text{Manager}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ d_{\text{Manager}} \}$
- $\text{Employee}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ d_{\text{Employee}}, d_{\text{Manager}} \}$
- $\text{plays_for}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ (d_{\text{Guitarist}}, d_{\text{RockBand}}), (\text{John_Lennon}, d_{\text{RockBand}}), (d_{\text{Bassist}}, d_{\text{RockBand}}), (\text{Paul_McCartney}, d_{\text{RockBand}}), (d_{\text{Drummer}}, d_{\text{RockBand}}), (\text{Ringo_Starr}, d_{\text{RockBand}}) \}$
- $\text{managed_by}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}} = \{ (\text{Beatles}, \text{Brian_Epstein}), (d_{\text{RockBand}}, d_{\text{Manager}}), (\text{Beatles}, d_{\text{Manager}}), (d_{\text{Manager}}, d_{\text{Manager}}) \}$

(2)

- $\exists \text{plays_for}.\text{RockBand}(\text{John_Lennon})$:

Because $(\text{John_Lennon}, d_{\text{RockBand}}) \in \text{plays_for}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}}$, $\mathcal{I}_{\mathcal{T}, \mathcal{A}}$ gives the answer "Yes".

And $(\mathcal{T}, \mathcal{A})$ gives the certain answer "Yes".

- $\exists \text{managed_by}.\text{Manager}(\text{Paul_McCartney})$

There is no $x \in \text{Manager}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}}$ and $(\text{Paul_McCartney}, x) \in \text{managed_by}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}}$ so $\mathcal{I}_{\mathcal{T}, \mathcal{A}}$ doesn't give the answer "Yes".

And $(\mathcal{T}, \mathcal{A})$ doesn't give the certain answer "Yes".

- $\exists \text{plays_for}.\exists \text{managed_by}.\text{Manager}(\text{Ringo_Starr})$

Because $(\text{Ringo_Starr}, d_{\text{RockBand}}) \in \text{plays_for}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}}$ and $(d_{\text{RockBand}}, d_{\text{Manager}}) \in \text{managed_by}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}}$, $\mathcal{I}_{\mathcal{T}, \mathcal{A}}$ gives the answer "Yes".

And $(\mathcal{T}, \mathcal{A})$ gives the certain answer "Yes".

(3)

- $F(x, y) = \exists z.(\text{plays_for}(x, z) \wedge \text{plays_for}(y, z))$

$\text{answer}(F(x, y), \mathcal{I}_{\mathcal{T}, \mathcal{A}}) = \{ (d_{\text{Guitarist}}, d_{\text{Guitarist}}), (d_{\text{Guitarist}}, \text{John_Lennon}), (d_{\text{Guitarist}}, d_{\text{Bassist}}), (d_{\text{Guitarist}}, \text{Paul_McCartney}), (d_{\text{Guitarist}}, d_{\text{Drummer}}), (d_{\text{Guitarist}}, \text{Ringo_Starr}), (\text{John_Lennon}, d_{\text{Guitarist}}), (\text{John_Lennon}, \text{John_Lennon}), (\text{John_Lennon}, d_{\text{Bassist}}), (\text{John_Lennon}, \text{Paul_McCartney}), (\text{John_Lennon}, d_{\text{Drummer}}), (\text{John_Lennon}, \text{Ringo_Starr}), (d_{\text{Bassist}}, d_{\text{Guitarist}}), (d_{\text{Bassist}}, \text{John_Lennon}), (d_{\text{Bassist}}, d_{\text{Bassist}}), (d_{\text{Bassist}}, \text{Paul_McCartney}), (d_{\text{Bassist}}, d_{\text{Drummer}}), (d_{\text{Bassist}}, \text{Ringo_Starr}), (\text{Paul_McCartney}, d_{\text{Guitarist}}), (\text{Paul_McCartney}, \text{John_Lennon}), (\text{Paul_McCartney}, d_{\text{Bassist}}), (\text{Paul_McCartney}, \text{Paul_McCartney}), (\text{Paul_McCartney}, d_{\text{Drummer}}), (\text{Paul_McCartney}, \text{Ringo_Starr}), (d_{\text{Drummer}}, d_{\text{Guitarist}}), (d_{\text{Drummer}}, \text{John_Lennon}), (d_{\text{Drummer}}, d_{\text{Bassist}}), (d_{\text{Drummer}}, \text{Paul_McCartney}), (d_{\text{Drummer}}, d_{\text{Drummer}}), (d_{\text{Drummer}}, \text{Ringo_Starr}), (\text{Ringo_Starr}, d_{\text{Guitarist}}), (\text{Ringo_Starr}, \text{John_Lennon}), (\text{Ringo_Starr}, d_{\text{Bassist}}), (\text{Ringo_Starr}, \text{Paul_McCartney}), (\text{Ringo_Starr}, d_{\text{Drummer}}), (\text{Ringo_Starr}, \text{Ringo_Starr}) \}$

$\text{cetanswer}(F(x, y), (\mathcal{T}, \mathcal{A})) = \{ (\text{John_Lennon}, \text{John_Lennon}), (\text{Paul_McCartney}, \text{Paul_McCartney}), (\text{Ringo_Starr}, \text{Ringo_Starr}) \}$

- $F = \exists x.\text{managed_by}(x, x)$

Because $(d_{\text{Manager}}, d_{\text{Manager}}) \in \text{managed_by}^{\mathcal{I}_{\mathcal{T}, \mathcal{A}}}$, $\mathcal{I}_{\mathcal{T}, \mathcal{A}}$ gives the answer "Yes".

But $(\mathcal{T}, \mathcal{A})$ doesn't give the certain answer "Yes".

Question 4. Conjunctive queries over database and interpretation

(1)

The finite first-order interpretation $\mathcal{I}_{\mathcal{D}}$ corresponding to \mathcal{D} .

- $\text{ID}^{\mathcal{I}_{\mathcal{D}}} = \{ 2001, 2002, 2003, 2004, 30000160, 30000170, 30000180 \}$
- $\text{Name}^{\mathcal{I}_{\mathcal{D}}} = \{ \text{Jay_Chou}, \text{Jolin_Tsai}, \text{Stefanie_Sun}, \text{Ta-yu_Lo} \}$
- $\text{StudentID}^{\mathcal{I}_{\mathcal{D}}} = \{ 2001, 2002, 2003, 2004 \}$
- $\text{Since}^{\mathcal{I}_{\mathcal{D}}} = \{ 2020, 2021, 2020 \}$
- $\text{CourseID}^{\mathcal{I}_{\mathcal{D}}} = \{ 30000160, 30000170, 30000180 \}$
- $\text{Title}^{\mathcal{I}_{\mathcal{D}}} = \{ \text{KR\&P}, \text{PR\&CV}, \text{NLP} \}$
- $\text{Person}^{\mathcal{I}_{\mathcal{D}}} = \{ (2001, \text{Jay_Chou}), (2002, \text{Jolin_Tsai}), (2003, \text{Stefanie_Sun}), (2004, \text{Ta-yu_Lo}) \}$

- Enrollment $^{\mathcal{I}_{\mathcal{D}}} = \{ (2002, 2020), (2003, 2021), (2004, 2020) \}$
- Attendance $^{\mathcal{I}_{\mathcal{D}}} = \{ (2001, 30000160), (2002, 30000160), (2002, 30000170), (2003, 30000180) \}$
- Course $^{\mathcal{I}_{\mathcal{D}}} = \{ (30000160, \text{KR\&P}), (30000180, \text{PR\&CV}), (30000170, \text{NLP}) \}$

(2)

First-order queries f_Q :

- $F_a(x, y) = \text{Person}(x, y)$
- $F_b(x) = \exists y. \exists z. (\text{Person}(y, x) \wedge \text{Attendance}(y, z) \wedge \text{Course}(z, \text{KR\&P}))$
- $F_c(x) = \exists y. \exists z. (\text{Person}(y, x) \wedge \text{Enrollment}(y, z) \wedge \forall c. \neg \text{Attendance}(y, c))$

$F_a(x, y)$ and $F_b(x)$ are conjunctive queries but $F_c(x)$ is not conjunctive query, because there is a universal quantification $\forall c$ in $F_c(x)$.

(3)

Answer Q in the context of \mathcal{D} :

- $\text{answer}(Q_a, \mathcal{D}) = \{ (2001, \text{Jay_Chou}), (2002, \text{Jolin_Tsai}), (2003, \text{Stefanie_Sun}), (2004, \text{Ta-yu_Lo}) \}$
- $\text{answer}(Q_b, \mathcal{D}) = \{ \text{Jay_Chou}, \text{Jolin_Tsai} \}$
- $\text{answer}(Q_c, \mathcal{D}) = \{ \text{Ta-yu_Lo} \}$

Answer f_Q in the context of $\mathcal{I}_{\mathcal{D}}$:

- $\text{answer}(F_a, \mathcal{I}_{\mathcal{D}}) = \{ (2001, \text{Jay_Chou}), (2002, \text{Jolin_Tsai}), (2003, \text{Stefanie_Sun}), (2004, \text{Ta-yu_Lo}) \}$
- $\text{answer}(F_b, \mathcal{I}_{\mathcal{D}}) = \{ \text{Jay_Chou}, \text{Jolin_Tsai} \}$
- $\text{answer}(F_c, \mathcal{I}_{\mathcal{D}}) = \{ \text{Ta-yu_Lo} \}$

Question 5. Certain answers in different contexts

(1)

- $\text{cetanswer}(r(x, y) \wedge Y(y), \mathcal{A}) = \emptyset$
- $\text{cetanswer}(\exists y(r(x, y) \wedge Y(y)), \mathcal{A}) = \emptyset$
- $\text{cetanswer}(\exists x, y(r(x, y) \wedge r(y, x)), \mathcal{A}) = \text{"Yes"}$
- $\text{cetanswer}(\exists z, w(r(x, y) \wedge r(y, z) \wedge r(z, x) \wedge r(z, w) \wedge W(w)), \mathcal{A}) = \emptyset$

(2)

- $\text{cetanswer}(r(x, y) \wedge Y(y), \mathcal{A}) = \{ (\text{Jay_Chou}, \text{Jolin_Tsai}), (\text{Jolin_Tsai}, \text{Stefanie_Sun}), (\text{Stefanie_Sun}, \text{Jay_Chou}), (\text{Jolin_Tsai}, \text{Jolin_Tsai}), (\text{Stefanie_Sun}, \text{Stefanie_Sun}) \}$
- $\text{cetanswer}(\exists y(r(x, y) \wedge Y(y)), \mathcal{A}) = \{ \text{Jay_Chou}, \text{Jolin_Tsai}, \text{Stefanie_Sun} \}$
- $\text{cetanswer}(\exists x, y(r(x, y) \wedge r(y, x)), \mathcal{A}) = \text{"Yes"}$
- $\text{cetanswer}(\exists z, w(r(x, y) \wedge r(y, z) \wedge r(z, x) \wedge r(z, w) \wedge W(w)), \mathcal{A}) = \{ (\text{Jay_Chou}, \text{Jolin_Tsai}), (\text{Stefanie_Sun}, \text{Jay_Chou}), (\text{Jolin_Tsai}, \text{Jolin_Tsai}), (\text{Jolin_Tsai}, \text{Stefanie_Sun}) \}$

Question 6 (with 1 bonus mark). Simpleness of ABox

This doesn't affect the lower bound of the data complexity results.

Lower bound:

- \mathcal{ALC} : coNP-hard
- \mathcal{EL} : P-hard
- DL-Lite: AC^0

Because questions like non-3-colorability and path system accessibility can be still reduced into \mathcal{ALC} and \mathcal{EL} CQ-entailment problem by the way same with simple ABoxes.

This doesn't affect the upper bound of the data complexity results, too.

Upper bound:

- \mathcal{ALC} : coNP-complete, because we can still use tableau algorithm to solve the problem.
- \mathcal{EL} : P-complete, because we can reduce it to Datalog query entailment with PTime data complexity.
- DL-Lite: AC^0 , because we can reduce it to entailment of their FO-rewriting q_T .

So this doesn't affect the data complexity result.

Question 7 (with 1 bonus mark). k-colorability

It is possible to show that the problem of conjunctive query entailment (CQ-entailment) in ALC is coNP-hard w.r.t. data complexity using a reduction from non-k-colorability in graphs.

We can consider the following \mathcal{ALC} TBox and Boolean CQ:

$$\mathcal{T} = \{ \top \sqsubseteq \bigsqcup_{C \in \text{colors}} C, C \sqcap \exists r.C \sqsubseteq D \text{ for } C \in \text{colors} \}$$

$$q = \exists x.D(x)$$

And we translated the input graph $G = (V, E)$ into the ABox:

$$\mathcal{A} = \{(u, v) : r|\{u, v\} \in E\}$$

Then it is straightforward to prove that $(\mathcal{T}, \mathcal{A}) \models q$ if and only if \mathcal{A} is not k -colourable.

Because 3-colorability is NP-hard, thus k -colorability is NP-hard also. So we can know that the query entailment problem for conjunctive queries is coNP-hard w.r.t data complexity in \mathcal{ALC} .

But if we let k is fixed, it depends on the value of k whether the query entailment problem for conjunctive queries is coNP-hard w.r.t data complexity in \mathcal{ALC} .

For example, let $k = 1$ and then a graph is 1-colorable if and only if it is totally disconnected, that is all its vertices are isolated. It can be identified in PTime. So it can not prove the proposition.

Let $k = 2$ and then a graph is 2-colorable if and only if its vertices having same color can be taken as disjoint sets. Thus, A graph is 2-colorable if and only if it is bipartite, which can be also solved in PTime. So it can not prove the proposition.

Let $k = 3$ and then 3-colorability is NP-hard and it can prove that the query entailment problem for conjunctive queries is coNP-hard w.r.t data complexity in \mathcal{ALC} .

So if k is fixed, it depends on the value of k whether the query entailment problem for conjunctive queries is coNP-hard w.r.t data complexity in \mathcal{ALC} .