第二章

3.

T2: 空元素

X	X+1	$X\cdot 0$
0	1	0
1	1	0

$$\therefore X + 1 = 1, X \cdot 0 = 0$$

T3: 同一律

X	X + X	$X\cdot X$
0	0	0
1	1	1

$$\therefore X + X = X, X \cdot X = X$$

T4: 还原律

X	\overline{X}	$\overline{\overline{X}}$
0	1	0
1	0	1

$$\therefore \overline{\overline{X}} = X$$

T5: 互补律

X	$X + \overline{X}$	$X\cdot \overline{X}$
0	1	0
1	1	0

$$\therefore X + \overline{X} = 1, X \cdot \overline{X} = 0$$

5.

未处理好优先级问题.

$$X+Y\cdot Z$$
 的反应该是 $\overline{X}\cdot (\overline{Y}+\overline{Z})$

6.(1)

$$\begin{split} F &= W \cdot X \cdot Y \cdot Z \cdot (\overline{W} \cdot X \cdot Y \cdot Z + W \cdot \overline{X} \cdot Y \cdot Z + W \cdot X \cdot \overline{Y} \cdot Z + W \cdot X \cdot Y \cdot \overline{Z}) \\ &= (W \cdot \overline{W}) \cdot X \cdot Y \cdot Z \cdot X \cdot Y \cdot Z + W \cdot (X \cdot \overline{X}) \cdot Y \cdot Z \cdot W \cdot Y \cdot Z \\ &+ W \cdot X \cdot (Y \cdot \overline{Y}) \cdot Z \cdot W \cdot X \cdot Z + W \cdot X \cdot Y \cdot W \cdot X \cdot Y \cdot (Z \cdot \overline{Z}) \\ &= 0 \end{split}$$

7.

(5)

$$F = \overline{W \cdot X} \cdot \overline{\overline{Y} + \overline{Z}}$$

w	Х	Y	Z	$\overline{W\cdot X}$	$\overline{\overline{Y}} + \overline{Z}$	F
0	0	0	0	1	0	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	1	1
0	1	0	0	1	0	0
0	1	0	1	1	0	0
0	1	1	0	1	0	0
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	1	0	0

W	X	Y	Z	$\overline{W\cdot X}$	$\overline{\overline{Y}+\overline{Z}}$	F
1	0	1	1	1	1	1
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	0	1	0

(8)

$$F = \overline{\overline{\overline{A} + B} + \overline{C}} + D$$

A	В	С	D	$\overline{A+B}$	$\overline{\overline{A+B}}+\overline{\overline{C}}$	F
0	0	0	0	1	0	1
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	0	0
0	1	1	0	0	1	0
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	0
1	0	1	0	0	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	1
1	1	0	1	0	0	0
1	1	1	0	0	1	0
1	1	1	1	0	1	0

8.

(1)

积之和表达式: $F(A,B,C)=\overline{A}\cdot B\cdot \overline{C}+A\cdot \overline{B}\cdot \overline{C}+A\cdot B\cdot \overline{C}+A\cdot B\cdot C$

和之积表达式: $F(A,B,C) = (A+B+C)\cdot (A+B+\overline{C})\cdot (A+\overline{B}+\overline{C})\cdot (\overline{A}+B+\overline{C})$

(2)

积之和表达式: $F(W,X,Y) = \overline{W} \cdot X \cdot \overline{Y} + W \cdot X \cdot \overline{Y} + W \cdot X \cdot Y$

和之积表达式: $F(W,X,Y)=(W+X+Y)\cdot(W+X+\overline{Y})\cdot(W+\overline{X}+\overline{Y})\cdot(\overline{W}+X+Y)\cdot(\overline{W}+X+\overline{Y})$

(4)

积之和表达式: $F = \overline{V} + W \cdot \overline{X}$

和之积表达式: $F = \overline{V} + W \cdot \overline{X} = (\overline{V} + W) \cdot (\overline{V} + \overline{X})$

12.

(1)

对于 2 输入与非门 $nand = \overline{x \cdot y}$:

$$\operatorname{not}(x) = \operatorname{nand}(x, 1) = \overline{x \cdot 1} = \overline{x}$$

$$\operatorname{and}(x,y) = \operatorname{not}(\operatorname{nand}(x,y)) = \overline{\overline{x \cdot y} \cdot 1} = x \cdot y$$

$$\operatorname{or}(x,y) = \operatorname{nand}(\operatorname{not}(x),\operatorname{not}(y)) = \overline{\overline{x\cdot 1}\cdot \overline{y\cdot 1}} = x+y$$

∴ 2 输入与门, 2 输入或门以及反向器都能由 2 输入与非门表示

:: 2 输入与非门能构成逻辑门的完全集

(2)

对于 2 输入异或门 $xor = x \cdot \overline{y} + \overline{x} \cdot y$:

异或可以看作是模 2 加法.

任意一个仅由 x, y, 0, 1 和异或运算组成的 2 输入逻辑函数,都可以表示成若干个 x, y, 0, 1 的模 2 加法,即

$$F = m_1 x + m_2 y + m_3 \cdot 1 + m_4 \cdot 0 = m_1 x + m_2 y + m_3$$
 (mod 2)

若异或运算能构成逻辑门的完全集, 那么一定能表示出与门:

x	Y	F
0	0	0
0	1	0
1	0	0
1	1	1

带入可得

$$egin{cases} m_3 = 0 \ m_2 + m_3 = 0 \ m_1 + m_3 = 0 \ m_1 + m_2 = 1 \end{cases}$$
 (mod 2)

四个条件不可能同时满足,产生矛盾

∴ 2 输入异或门不能作为逻辑门的完全集

13.

(2)

WX \ YZ	00	01	11	10		WX \ YZ	00	01	11	10
00		1				00			1	
01	1	1	1	1	→	01	1	1	1	1
11			1	1		11			1	1
10		1				10		1		

$$\therefore F = \overline{W} \cdot X + \overline{X} \cdot \overline{Y} \cdot Z + W \cdot X \cdot Y$$

$$\therefore F = \overline{\overline{W} \cdot X + \overline{X} \cdot \overline{Y} \cdot Z + W \cdot X \cdot Y} = \overline{W \cdot \overline{X} \cdot \overline{X} \cdot \overline{X} \cdot \overline{Y} \cdot Z} \cdot \overline{W \cdot X \cdot Y}$$

(5)

ABICD	00	01	11	10		AB \ CD	00	01	11	10
00	1	1	1	1		00	1	1	1	1
01			1		→	01			1	
11	1			1		11	1)	(1
10	1	1	1	1		10	1	1	1	1

$$\therefore F = \overline{B} + \overline{A} \cdot C \cdot D + A \cdot B \cdot \overline{D}$$

$$\therefore F = \overline{B \cdot \overline{\overline{A} \cdot C \cdot D} \cdot \overline{A \cdot B \cdot \overline{D}}}$$