第四次作业

4.4

$$\therefore F = G\frac{m'm}{d^2} - G\frac{\frac{1}{8}m'm}{(d-\frac{1}{2}R)^2}$$

4.5

(a)

轨道半径 $R = 6371 + 640\ (\mathrm{km}) = 7011\ (\mathrm{km}) \approx 7 \times 10^6\ (\mathrm{m})$

地球质量 $M = 5.965 \times 10^{24}\ (\mathrm{kg}) \approx 6 \times 10^{24}\ (\mathrm{kg})$

引力常数 $G = 6.67 \times 10^{-11}\ (\mathrm{N \cdot m^2/kg^2})$

$$\because G\frac{Mm}{R^2} = m\frac{v^2}{R}$$

$$\therefore v = \sqrt{\frac{GM}{R}} = 7561.179046380834\ (\mathrm{m/s}) = 7.561 \times 10^3\ (\mathrm{m/s})$$

(b)

$$\because G\frac{Mm}{R^2} = m\omega^2 R = m(\frac{2\pi}{T})^2 R$$

$\therefore T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi \times \sqrt{\frac{(7\times10^6)^3}{6.67\times10^{-11}\times 6\times 10^{24}}} = 5816.856984931375\ (\mathrm{s}) = 5.817 \times 10^3\ (\mathrm{s})$

(c)

设新速度为 v', 新半径为 R, 新周期为T'$

对于圆周运动的天体所拥有的动能和重力势能总能量

$$\because E = \frac{1}{2}mv^2 - G\frac{Mm}{R}$$

$$G\frac{Mm}{R^2} = m\frac{v^2}{R}$$

$$\therefore E=-\frac{GMm}{2R}=-\frac{1}{2}mv^2$$

$$\because E'=-\frac{GMm}{2R'}=-\frac{1}{2}mv'^2=E-1500\times1.4\times10^5\mathrm{J}=-6.5\times10^9\mathrm{J}$$

$$\therefore d=R'-6.371\times10^6\mathrm{m}=4\times10^5\ (\mathrm{m})$$

$$\therefore v'=\sqrt{\frac{-2E'}{m}}=7.687\times10^3\ (\mathrm{m/s})$$

$$\because \frac{v'}{R'}=\frac{2\pi}{T'}$$

$$\therefore T'=\frac{2\pi R'}{v'}=5.534\times10^3\ (\mathrm{s})$$

(d)

$$\because fs=2\pi Rf=1.4\times10^5\mathrm{J}$$

$$\therefore f=\frac{1.4\times10^5\mathrm{J}}{2\pi R}=3.18\times10^{-3}\ \mathrm{N}$$

(e)

$$\therefore L=Rmv=-\frac{GMm}{2E}\cdot m\sqrt{\frac{-2E}{m}}=-\frac{GMm}{2}\sqrt{\frac{-2m}{E}}$$

$\because E$ 在不断发生变化, 角动量不守恒

$$\therefore \frac{L-L'}{L}=\frac{\sqrt{-\frac{1}{E}}-\sqrt{-\frac{1}{E'}}}{\sqrt{-\frac{1}{E}}}=1.6\%$$

4.7

(a)

$$\because E=\frac{1}{2}mv^2-G\frac{m_\oplus m}{r}$$

$$G\frac{m_\oplus m}{r^2}=m\frac{v^2}{r}$$

$$\therefore E=-\frac{1}{2}mv^2=-\frac{Gm_\oplus m}{2r}$$

$$\therefore E_A + E_B = -\frac{Gm_{\oplus}m}{2r} - \frac{Gm_{\oplus}m}{2r} = -\frac{Gm_{\oplus}m}{r}$$

(b)

$\because$ 碰撞前 A 和 B 的速度大小相等, 方向相反

$\therefore$ 发生非弹性碰撞后的碰撞碎片聚集在一起后, 速度为零, 即没有动能

$$\therefore E = -G\frac{m_{\oplus}2m}{R} = -2G\frac{m_{\oplus}m}{R}$$

(c)

卫星碎片会进行加速度变大的加速直线运动, 即自由落体运动, 下落到地球.

4.12

(a)

对 a 点:

粒子的能量 $E_a = \frac{1}{2}mv_a^2 - \frac{k}{a}$

$\because v_a = \sqrt{\frac{k}{2ma}}$

$\therefore E_a = \frac{k}{4a} - \frac{k}{a} = -\frac{3k}{4a}$

对另一个极端点 b 点:

粒子的能量 $E_b = \frac{1}{2}mv_b^2 - \frac{k}{b} = E_a = -\frac{3k}{4a}$

由角动量守恒得 $L = av_a = bv_b$

联解可得 $\frac{1}{2}m(\frac{av_a}{b})^2 - \frac{k}{b} = \frac{ak}{4b^2} - \frac{k}{b} = -\frac{3k}{4a}$

$\therefore a^2 - 4ab + 3b^2 = (a-3b)(a-b) = 0$

$\therefore b = \frac{1}{3}a$

(b)

$$\therefore v_b = \frac{a v_a}{b} = 3 v_a = 3\sqrt{\frac{k}{2ma}}$$

4.14

设约化质量为 $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$, 对于二维平面上:

$$\therefore -kx = \mu \ddot{x}$$

$$-ky = \mu \ddot{y}$$

解常微分方程可得:

$$\therefore x = A\cos(t\sqrt{\frac{k}{\mu}} + \varphi_1)$$

$$y = B\cos(t\sqrt{\frac{k}{\mu}} + \varphi_2)$$

∴ 轨迹为椭圆

4.18

(a)

对不被撕裂且有最小密度的星球表面上一点:

$$\because G\frac{Mm}{R^2} = m\omega^2 R$$

$$M = \rho V = \frac{4}{3}\rho\pi R^3$$

$$\therefore \rho = \frac{3\omega^2}{4\pi G}$$

对蟹状星云脉冲星:

$$\because \omega = 2\pi f = 60\pi \ (\mathrm{rad/s})$$

$$\therefore \rho = \frac{3 \times (60\pi)^2}{4 \times \pi \times 6.67 \times 10^{-11}} \ \mathrm{kg/m^3} = 1.2717 \times 10^{14} \ \mathrm{kg/m^3}$$

(b)

$$\because M = \rho V = \frac{4}{3}\rho\pi R^3$$

$$\therefore R = \sqrt[3]{\frac{3M}{4\pi\rho}} = (\frac{3 \times 2 \times 10^{30}}{4 \times \pi \times 1.2717 \times 10^{14}})^{\frac{1}{3}}\ \mathrm{m} = 1.55 \times 10^5\ \mathrm{m}$$