习题 5.3 (A)

1.

(2)

$$rac{\partial z}{\partial x} = rac{1}{\sqrt{1-rac{x^2}{x^2+y^2}}} \cdot rac{\sqrt{x^2+y^2}-x \cdot rac{x}{\sqrt{x^2+y^2}}}{x^2+y^2} = rac{1}{|y|} - rac{x^2}{|y|(x^2+y^2)}$$

$$rac{\partial z}{\partial y} = rac{1}{\sqrt{1 - rac{x^2}{x^2 + y^2}}} \cdot rac{-x \cdot rac{y}{\sqrt{x^2 + y^2}}}{x^2 + y^2} = -rac{xy}{|y|(x^2 + y^2)}$$

(6)

$$rac{\partial u}{\partial x} = rac{zx^{z-1}}{u^z}$$

$$rac{\partial u}{\partial y} = -rac{zx^z}{y^{z-1}}$$

$$\frac{\partial u}{\partial z} = (\frac{x}{y})^z \ln \frac{x}{y}$$

(9)

$$u = xze^{\sin(yz)}$$

$$\frac{\partial u}{\partial x} = ze^{\sin(yz)}$$

$$\frac{\partial u}{\partial y} = xz^2 e^{\sin(yz)} \cos(yz)$$

$$rac{\partial u}{\partial z} = x e^{\sin(yz)} + xyz e^{\sin(yz)}\cos(yz)$$

2. (2)

$$egin{split} f_y(\pi,rac{\pi}{4}) &= rac{2\sin(x-2y)\cos(x+y) + \sin(x+y)\cos(x-2y)}{\cos^2(x+y)} \ &= rac{2\sinrac{\pi}{2}\cosrac{5\pi}{4} + \sinrac{5\pi}{4}\cosrac{\pi}{2}}{\cos^2rac{5\pi}{4}} \ &= -2\sqrt{2} \end{split}$$

4.

(1)

$$\therefore f_x(0,0) = \lim_{x o 0} rac{x\sinrac{1}{x^2+0^2}-0}{x} = \sinrac{1}{x^2}$$
, $f_x(0,0)$ 不存在.

$$\therefore f_y(0,0) = \lim_{y o 0} rac{0 imes \sinrac{1}{0^2+y^2}-0}{y} = 0$$
, $f_y(0,0)$ 存在.

(2)

$$f_x^+(0,0) = \lim_{x o 0^+} rac{|x|g(x,0)-0}{x} = g(0,0) \ f_x^-(0,0) = \lim_{x o 0^-} rac{|x|g(x,0)-0}{x} = -g(0,0)$$

$$egin{aligned} \therefore f_y^+(0,0) &= \lim_{y o 0^+} rac{|y|g(0,y) - 0}{y} = g(0,0) \ f_y^-(0,0) &= \lim_{y o 0^+} rac{|y|g(0,y) - 0}{y} = -g(0,0) \end{aligned}$$

若偏导数 $f_x(0,0), f_y(0,0)$ 存在, 那么 g(0,0) = -g(0,0)

即 g(0,0)=0 时两个偏导数均存在.

假设 f 在 (0,0) 可微

$$\therefore \Delta f = f_x(0,0)\Delta x + f_y(0,0)\Delta y + o(\rho)$$

$$\therefore \Delta f = f(0 + \Delta x, 0 + \Delta y) - f(0, 0) = |\Delta x - \Delta y| g(\Delta x, \Delta y)$$

$$\therefore \lim_{(\Delta x, \Delta y) o (0,0)} rac{\Delta f}{
ho} = rac{|\Delta x - \Delta y| g(\Delta x, \Delta y)}{\sqrt{\Delta x^2 + \Delta y^2}} = 0$$

$$\therefore g(0,0)=0$$
 时 f 在 $(0,0)$ 处可微

6.

(1)

设
$$e_l = (\cos \theta, \sin \theta)$$

$$egin{aligned} \therefore rac{\partial f(0,0)}{\partial oldsymbol{l}} &= \lim_{t o 0} rac{f(t\cos heta,t\sin heta) - f(0,0)}{t} \ &= \lim_{t o 0} rac{(t^2\cos heta\sin heta)^{rac{1}{3}}}{t} \ &= \lim_{t o 0} rac{(rac{1}{2}\sin2 heta)^{rac{1}{3}}}{t^{rac{1}{3}}} \end{aligned}$$

当
$$heta=rac{k\pi}{2}$$
,即沿着两个坐标轴正负方向时,有 $rac{\partial f(0,0)}{\partial m{l}}=0$

即沿着两个坐标轴正负方向存在方向导数.

当
$$heta=rac{k\pi}{2}$$
 时, $\lim_{t o 0}rac{(rac{1}{2}\sin 2 heta)^{rac{1}{3}}}{t^{rac{1}{3}}}$ 极限不存在,不存在方向导数.

 $\therefore f(x,y)$ 在点 (0,0) 只沿着两个坐标轴的正负方向存在方向导数.

(2)

$$\Leftrightarrow (x,y) = (t\cos\theta, t\sin\theta)$$

$$\lim_{(x,y) o (0,0)} f(x,y) - f(0,0) = f(t\cos heta,t\sin heta) = (rac{1}{2}t^2\sin2 heta)^rac{1}{3} = 0$$

 $\therefore f(x,y)$ 在点 (0,0) 连续.

13. (2)

令
$$f(x,y)=x^y, (x_0,y_0)=(1,1), \Delta x=-0.03, \Delta y=0.05$$
,则

$$f_x(1,1) = yx^{y-1}|_{(1,1)} = 1$$
, $f_y(1,1) = x^y \ln x|_{(1,1)} = 0$

$$\therefore 0.97^{1.05} pprox f(1,1) + f_x(1,1) \Delta x + f_y(1,1) \Delta y = 1 - 0.03 = 0.97$$

18.

$$: l = (3-1, -2, 2-1) = (2, -2, 1)$$

$$\therefore \boldsymbol{e}_l = (\tfrac{2}{3}, -\tfrac{2}{3}, \tfrac{1}{3})$$

$$egin{aligned} dots & rac{\partial u}{\partial x} = rac{\sqrt{y^2 + z^2}}{(x + \sqrt{y^2 + z^2})\sqrt{y^2 + z^2}}, \ & rac{\partial u}{\partial y} = rac{y}{(x + \sqrt{y^2 + z^2})\sqrt{y^2 + z^2}}, \ & rac{\partial u}{\partial y} = rac{z}{(x + \sqrt{y^2 + z^2})\sqrt{y^2 + z^2}}, \end{aligned}$$

$$\therefore rac{\partial u}{\partial oldsymbol{l}} = rac{2\sqrt{y^2+z^2}-2y+z}{3(x+\sqrt{y^2+z^2})\sqrt{y^2+z^2}}$$

22.

$$\because \frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{\partial r}{\partial z} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\therefore
abla r = (rac{x}{\sqrt{x^2 + y^2 + z^2}}, rac{y}{\sqrt{x^2 + y^2 + z^2}}, rac{z}{\sqrt{x^2 + y^2 + z^2}})$$

$$\because \frac{\partial \frac{1}{r}}{\partial x} = -\frac{x}{(x^2 + y^2 + x^2)^{\frac{3}{2}}}, \frac{\partial \frac{1}{r}}{\partial y} = -\frac{y}{(x^2 + y^2 + x^2)^{\frac{3}{2}}}, \frac{\partial \frac{1}{r}}{\partial z} = -\frac{z}{(x^2 + y^2 + x^2)^{\frac{3}{2}}}$$

$$\therefore \nabla \frac{1}{r} = (-\frac{x}{(x^2 + y^2 + x^2)^{\frac{3}{2}}}, -\frac{y}{(x^2 + y^2 + x^2)^{\frac{3}{2}}}, -\frac{z}{(x^2 + y^2 + x^2)^{\frac{3}{2}}})$$

23. (1)

$$rac{\partial z}{\partial x} = f_1 + 2f_2 + 2xf_3$$

$$\frac{\partial z}{\partial y} = f_2 - 3yf_3$$

24. (4)

$$T = \frac{1}{2a\sqrt{\pi t}} \exp(-\frac{(x-a)^2}{4a^2t})$$

$$\therefore \frac{\partial T}{\partial t} = -\frac{1}{4at\sqrt{\pi t}} \exp(-\frac{(x-a)^2}{4a^2t}) + \frac{(x-a)^2}{8a^3t^2\sqrt{\pi t}} \exp(-\frac{(x-a)^2}{4a^2t})$$

$$\therefore a^{2} \frac{\partial^{2} T}{\partial x^{2}} = a^{2} \frac{\partial}{\partial x} \left(-\frac{2(x-a)}{8a^{3}t\sqrt{\pi t}} \exp\left(-\frac{(x-a)^{2}}{4a^{2}t} \right) \right)$$

$$= -\frac{1}{4at\sqrt{\pi t}} \exp\left(-\frac{(x-a)^{2}}{4a^{2}t} \right) + \frac{(x-a)^{2}}{8a^{3}t^{2}\sqrt{\pi t}} \exp\left(-\frac{(x-a)^{2}}{4a^{2}t} \right)$$

$$\therefore \frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2}$$