习题3.3：（A）5（3），6，10，（B）1，2，3，4，5，6，习题3.4：（A）1（1、3、5、7、9）

3.3 (A)

5. (3)

$$
\begin{aligned}
\int_{-1}^{1}|x|(x^2+\frac{\sin^3 x}{1+\cos x})\mathrm{d}x &= \int_{-1}^{1}|x|^3\mathrm{d}x+\int_{-1}^{1}\frac{|x|\sin^3 x}{1+\cos x}\mathrm{d}x \\
&= 2\int_{0}^{1}x^3\mathrm{d}x+0 \\
&= \frac{1}{2}x^4|_0^1 \\
&= \frac{1}{2}
\end{aligned}
$$

6.

令$x=t+T$

$$
\begin{aligned}
\int_{a}^{a+T}f(x)\mathrm{d}x &= \int_{a}^{0}f(x)\mathrm{d}x+\int_{0}^{T}f(x)\mathrm{d}x+\int_{T}^{a+T}f(x)\mathrm{d}x \\
&= \int_{a}^{0}f(x)\mathrm{d}x+\int_{0}^{T}f(x)\mathrm{d}x+\int_{T}^{a+T}f(t+T)\mathrm{d}(t+T) \\
&= \int_{a}^{0}f(x)\mathrm{d}x+\int_{0}^{T}f(x)\mathrm{d}x+\int_{0}^{a}f(t)\mathrm{d}t \\
&= \int_{a}^{0}f(x)\mathrm{d}x+\int_{0}^{T}f(x)\mathrm{d}x+\int_{0}^{a}f(x)\mathrm{d}x \\
&= \int_{0}^{T}f(x)\mathrm{d}x
\end{aligned}
$$

10.

(1)

令$x=\frac{\pi}{2}-t, x(\frac{\pi}{2})=0, x(0)=\frac{\pi}{2}$

$$
\begin{aligned}
\int_0^{\frac{\pi}{2}} f(\sin x)\mathrm{d}x &= \int_0^{\frac{\pi}{2}} f(\sin(\frac{\pi}{2}-t))\mathrm{d}(\frac{\pi}{2}-t) \\
&= -\int_{\frac{\pi}{2}}^0 f(\sin(\frac{\pi}{2}-t))\mathrm{d}t \\
&= \int_0^{\frac{\pi}{2}} f(\cos x)\mathrm{d}x
\end{aligned}
$$

(2)

令$x=a+(b-a)t, x(0)=a, x(1)=b$

$$
\begin{aligned}
\int_a^b f(x)\mathrm{d}x &= \int_a^b f[a+(b-a)t]\mathrm{d}[a+(b-a)t] \\
&= (b-a)\int_0^1 f[a+(b-a)t]\mathrm{d}t \\
&= (b-a)\int_0^1 f[a+(b-a)x]\mathrm{d}x
\end{aligned}
$$

(3)

令$x=1-t, x(1)=0, x(0)=1$

$$
\begin{aligned}
\int_0^1 x^m(1-x)^n\mathrm{d}x &= \int_0^1 (1-t)^m t^n \mathrm{d}(1-t) \\
&= -\int_1^0 t^n(1-t)^m\mathrm{d}t \\
&= \int_0^1 x^n(1-x)^m\mathrm{d}x
\end{aligned}
$$

(4)

当$a\geq 0$时

令$x=\sqrt{t}, x(0)=0, x(a^2)=a$

$$\begin{aligned}\int_0^a x^3 f(x^2)\mathrm{d}x &= \int_0^a (\sqrt{t})^3 f((\sqrt{t})^2)\mathrm{d}\sqrt{t} \\ &= \frac{1}{2}\int_0^{a^2} tf(t)\mathrm{d}t \\ &= \frac{1}{2}\int_0^{a^2} xf(x)\mathrm{d}x\end{aligned}$$

当$a < 0$时

令$x = -\sqrt{t}, x(0) = 0, x(a^2) = a$

$$\begin{aligned}\int_0^a x^3 f(x^2)\mathrm{d}x &= \int_0^a (-\sqrt{t})^3 f((-\sqrt{t})^2)\mathrm{d}(-\sqrt{t}) \\ &= \frac{1}{2}\int_0^{a^2} tf(t)\mathrm{d}t \\ &= \frac{1}{2}\int_0^{a^2} xf(x)\mathrm{d}x\end{aligned}$$

$\therefore$ 原式成立

3.3 (B)

1.

令$x = \frac{\pi}{4} - \frac{t}{2}, x(\frac{\pi}{2}) = 0, x(-\frac{\pi}{2}) = \frac{\pi}{2}$

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin^m x \cdot \cos^m x\mathrm{d}x &= \frac{1}{2^m}\int_0^{\frac{\pi}{2}} \sin^m 2x\mathrm{d}x \\ &= \frac{1}{2^m}\int_0^{\frac{\pi}{2}} \sin^m(\frac{\pi}{2} - t)\mathrm{d}(\frac{\pi}{4} - \frac{t}{2}) \\ &= -\frac{1}{2^{m+1}}\int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \cos^m t\mathrm{d}t \\ &= \frac{1}{2^m}\int_0^{\frac{\pi}{2}} \cos^m x\mathrm{d}x\end{aligned}$$

2.

令$x = t + \frac{\pi}{4}, x(-\frac{\pi}{4}) = 0, x(n\pi - \frac{\pi}{4}) = n\pi$

令$t = u + n\pi, x(0) = n\pi, x(-\frac{\pi}{4}) = n\pi - \frac{\pi}{4}$

$$
\begin{aligned}
&\int_0^{n\pi} \sqrt{1-\sin 2x}\mathrm{d}x \\
&= \int_0^{n\pi} \sqrt{(\sin x - \cos x)^2}\mathrm{d}x \\
&= \int_0^{n\pi} \sqrt{2\sin^2(x-\frac{\pi}{4})}\mathrm{d}x \\
&= \int_0^{n\pi} \sqrt{2\sin^2 t}\mathrm{d}(t+\frac{\pi}{4}) \\
&= \int_{-\frac{\pi}{4}}^{n\pi-\frac{\pi}{4}} \sqrt{2\sin^2 t}\mathrm{d}t \\
&= \int_{-\frac{\pi}{4}}^{0} \sqrt{2\sin^2 t}\mathrm{d}t + \int_0^{n\pi} \sqrt{2\sin^2 t}\mathrm{d}t + \int_{n\pi}^{n\pi-\frac{\pi}{4}} \sqrt{2\sin^2 t}\mathrm{d}t \\
&= \int_{-\frac{\pi}{4}}^{0} \sqrt{2\sin^2 t}\mathrm{d}t + \int_0^{n\pi} \sqrt{2\sin^2 t}\mathrm{d}t + \int_{0}^{-\frac{\pi}{4}} \sqrt{2\sin^2 (u+n\pi)}\mathrm{d}u \\
&= \sqrt{2}\int_0^{n\pi} |\sin x|\mathrm{d}x \\
&= \sqrt{2}(\int_0^{\pi} \sin x\mathrm{d}x - \int_{\pi}^{2\pi} \sin x\mathrm{d}x + \cdots + (-1)^{n-1}\int_{(n-1)\pi}^{n\pi} \sin x\mathrm{d}x) \\
&= \sqrt{2}n\int_0^{\pi} \sin x\mathrm{d}x \\
&= \sqrt{2}n(-\cos x|_0^{\pi}) \\
&= 2\sqrt{2}n
\end{aligned}
$$

3.

令$x = \frac{\pi}{2} - x, x(\frac{11\pi}{2}) = -5\pi, x(-\frac{9\pi}{2}) = 5\pi$

$$
\begin{aligned}
&\int_0^{10\pi} \frac{\sin^3 x + \cos^3 x}{2\sin^2 x + \cos^4 x}\mathrm{d}x \\
=&\int_{-5\pi}^{5\pi} \frac{\sin^3 x}{2\sin^2 x + \cos^4 x}\mathrm{d}x + \int_{-5\pi}^{5\pi} \frac{\cos^3 x}{2\sin^2 x + \cos^4 x}\mathrm{d}x \\
=&\int_{-5\pi}^{5\pi} \frac{\cos^3 x}{2\sin^2 x + \cos^4 x}\mathrm{d}x \\
=&\int_{\frac{11\pi}{2}}^{-\frac{9\pi}{2}} \frac{\sin^3 x}{2\cos^2 x + \sin^4 x}\mathrm{d}(\frac{\pi}{2} - x) \\
=&\int_{-5\pi}^{5\pi} \frac{\sin^3 x}{2\sin^2 x + \cos^4 x}\mathrm{d}x \\
=&\ 0
\end{aligned}
$$

4.

令$x = n\pi - t, x(n\pi) = 0, x(0) = n\pi$

$$
\begin{aligned}
\int_0^{n\pi} x|\sin x|\mathrm{d}x &= \int_0^{n\pi} (n\pi - t)|\sin(n\pi - t)|\mathrm{d}(n\pi - t) \\
&= \int_0^{n\pi} (n\pi - t)|\sin(t)|\mathrm{d}t \\
&= n\pi \int_0^{n\pi} |\sin(x)|\mathrm{d}x - \int_0^{n\pi} x|\sin(x)|\mathrm{d}x
\end{aligned}
$$

$$
\begin{aligned}
\int_0^{n\pi} x|\sin x|\mathrm{d}x &= \frac{1}{2}n\pi \int_0^{n\pi} |\sin(x)|\mathrm{d}x \\
&= \frac{1}{2}n^2\pi \int_0^{\pi} \sin(x)\mathrm{d}x \\
&= n^2\pi
\end{aligned}
$$

5.

$$
\begin{aligned}
&\int_{\frac{1}{2}}^{2}(1+x-\frac{1}{x})e^{x+\frac{1}{x}}\mathrm{d}x \\
=&\int_{\frac{1}{2}}^{2}e^{x+\frac{1}{x}}\mathrm{d}x+\int_{\frac{1}{2}}^{2}x\mathrm{d}e^{x+\frac{1}{x}} \\
=&\int_{\frac{1}{2}}^{2}e^{x+\frac{1}{x}}\mathrm{d}x+xe^{x+\frac{1}{x}}|_{\frac{1}{2}}^{2}-\int_{\frac{1}{2}}^{2}e^{x+\frac{1}{x}}\mathrm{d}x \\
=&\frac{3}{2}e^{\frac{5}{2}}
\end{aligned}
$$

6.

$$
\begin{aligned}
\int\frac{xe^x}{(1+x)^2}x &= \int\frac{1}{1+x}\mathrm{d}e^x-\int\frac{1}{(1+x)^2}\mathrm{d}e^x \\
&= \frac{e^x}{1+x}-\int e^x\mathrm{d}\frac{1}{1+x}-\int\frac{1}{(1+x)^2}\mathrm{d}e^x \\
&= \frac{e^x}{1+x}+\int\frac{e^x}{(1+x)^2}\mathrm{d}x-\int\frac{1}{(1+x)^2}\mathrm{d}e^x \\
&= \frac{e^x}{1+x}
\end{aligned}
$$

3.4 (A)

1.

(1)

$$
\begin{aligned}
\therefore S &= \int_0^1 9-x^2-x^2\mathrm{d}x \\
&= \int_0^1 9-2x^2\mathrm{d}x \\
&= (9x-\frac{2}{3}x^3)|_0^1 \\
&= \frac{25}{3}
\end{aligned}
$$

(3)

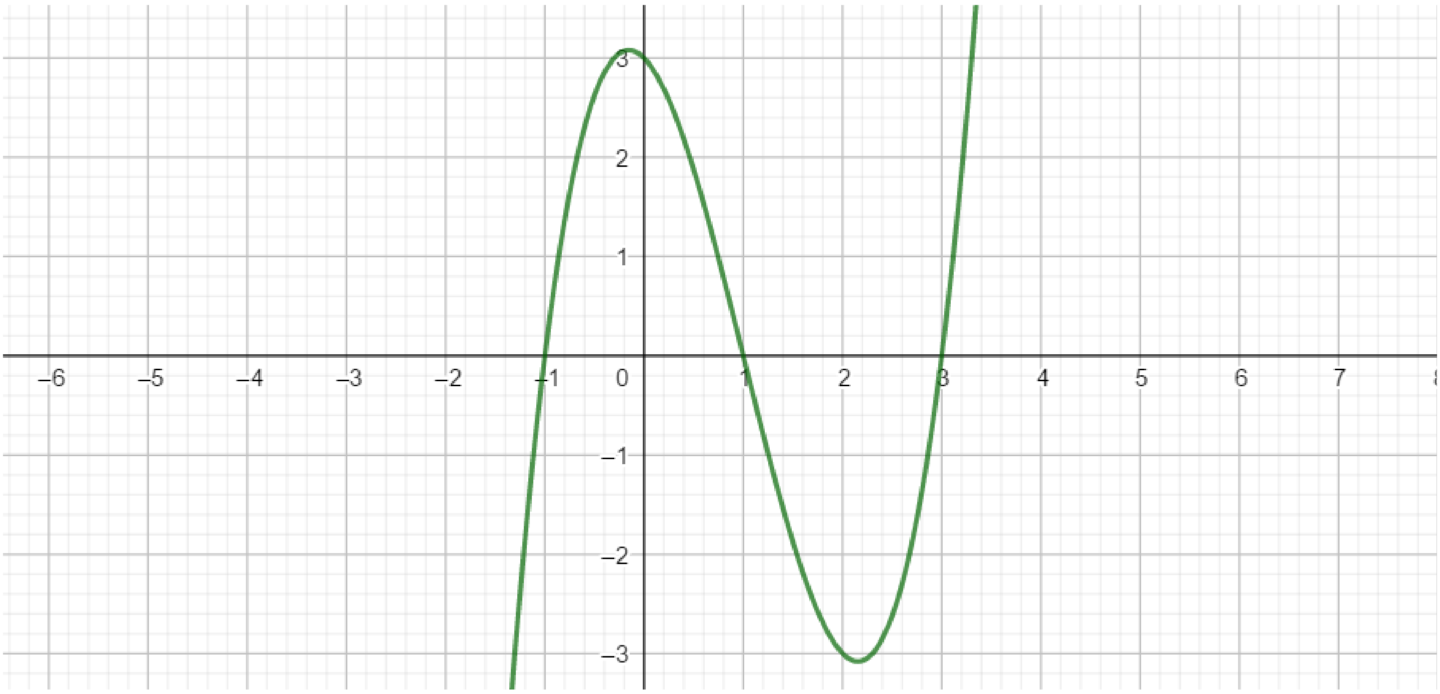
$\therefore y=(\sqrt{a}-\sqrt{x})^2=x-2\sqrt{ax}+a, 0\leq x\leq a$

$$\begin{aligned}\therefore S&=\int_0^a(x-2\sqrt{a}\sqrt{x}+a)\mathrm{d}x\\&=(\frac{1}{2}x^2-\frac{4}{3}\sqrt{a}x^{\frac{3}{2}}+ax)|_0^a\\&=\frac{1}{6}a^2\end{aligned}$$

(5)

令$f(x)=x(x-1)(x-2)-3(x-1)=(x-1)(x^2-2x-3)=(x-1)(x+1)(x-3)=x^3-3x^2-x+3$

$\therefore$ 对于$f(x)=0$解得$x_1=-1, x_2=1, x_3=3,$
且$f(x)$在$(-1,1)$大于0, 在$(1,3)$小于0



$$\begin{aligned}\therefore S&=\int_{-1}^1 f(x)\mathrm{d}x+\int_1^3-f(x)\mathrm{d}x\\&=\int_{-1}^1(x^3-3x^2-x+3)\mathrm{d}x+\int_1^3(-x^3+3x^2+x-3)\mathrm{d}x\\&=(\frac{1}{4}x^4-x^3-\frac{1}{2}x^2+3x)|_{-1}^1-(\frac{1}{4}x^4-x^3-\frac{1}{2}x^2+3x)|_1^3\\&=8\end{aligned}$$

(7)

令$\theta = \frac{1}{2}t, \theta(0) = 0, \theta(\pi) = \frac{\pi}{2}$

$$
\begin{aligned}
\therefore S &= 4\int_0^{\frac{\pi}{2}} \frac{1}{2}\rho^2(\theta)\mathrm{d}\theta \\
&= 4\int_0^{\frac{\pi}{2}} \sin 2\theta \mathrm{d}2\theta \\
&= 4\int_0^{\pi} \sin t \mathrm{d}t \\
&= -4\cos x|_0^{\pi} \\
&= 8
\end{aligned}
$$

(9)

$$
\begin{aligned}
\therefore S &= \int_0^{2\pi} y(t)\mathrm{d}x(t) \\
&= \int_0^{2\pi} a(1-\cos t)\mathrm{d}a(t-\sin t) \\
&= a^2\int_0^{2\pi} (1-\cos t)^2\mathrm{d}t \\
&= a^2\int_0^{2\pi} (1-2\cos t+\cos^2 t)\mathrm{d}t \\
&= a^2(x-2\sin x)|_0^{2\pi} + a^2\int_0^{2\pi} \frac{1+\cos 2t}{2}\mathrm{d}t \\
&= 2\pi a^2 + a^2\int_0^{2\pi} \frac{1+\cos 2t}{2}\mathrm{d}t \\
&= 3\pi a^2
\end{aligned}
$$