第九次作业

10.4，10.5，11.2，11.5，11.10

10.4

$\because h=(L-R\varphi-R\tan\theta)\cos\theta$

$\therefore \frac{\mathrm{d}L}{\mathrm{d}T}-\frac{\mathrm{d}R}{\mathrm{d}T}\left(\frac{\pi}{2}-\theta+\tan\theta\right)$

$\therefore \frac{\mathrm{d}L}{\mathrm{d}T}\equiv L\alpha_{\text{steel}},\frac{\mathrm{d}R}{\mathrm{d}T}\equiv R\alpha_{\text{Al}}$

$\therefore L\alpha_{\text{steel}}-R\alpha_{\text{Al}}\left(\frac{\pi}{2}-\theta+\tan\theta\right)=0$

$\therefore R=\frac{2L}{\pi-2\theta+2\theta}\frac{\alpha_{\text{steel}}}{\alpha_{\text{Al}}}\approx 0.63\ \mathrm{m}$

10.5

$\because a^2x\rho_{\text{Hg}}g=a^3\rho_{\text{Al}}g,x=a\frac{\rho_{\text{Al}}}{\rho_{\text{Hg}}}=3.97\ \mathrm{cm}$

同时我们有

$\ln x=\ln a+\ln\rho_{\text{Al}}-\ln\rho_{\text{Hg}}$

$\frac{\mathrm{d}\ln x}{\mathrm{d}T}=\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}T}+\frac{1}{\rho_{\text{Al}}}\frac{\mathrm{d}\rho_{\text{Al}}}{\mathrm{d}T}-\frac{1}{\rho_{\text{Hg}}}\frac{\mathrm{d}\rho_{\text{Hg}}}{\mathrm{d}T}$

$\therefore \frac{1}{\rho_{\text{Al}}}\frac{\mathrm{d}\rho_{\text{Al}}}{\mathrm{d}T}=\frac{V}{m}\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{m}{V}\right)=V\left(-\frac{1}{V^2}\right)\frac{\mathrm{d}V}{\mathrm{d}T}=-\frac{1}{V}\frac{\mathrm{d}v}{\mathrm{d}t}=-\beta$

$\therefore \int\mathrm{d}\ln x=\int\mathrm{d}T(\alpha_{\text{Al}}-\beta_{\text{Al}}+\beta_{\text{Hg}})=(\alpha_{\text{Al}}-\beta_{\text{Al}}+\beta_{\text{Hg}})\int\mathrm{d}T$

$\therefore \ln\frac{x}{x_0}(\alpha_{\text{Al}}-\beta_{\text{Al}}+\beta_{\text{Hg}})\Delta T=6.7\times10^{-3}$

$\therefore \Delta x=x_0(e^{6.7\times10^{-3}}-1)=x_0\times6.7\times10^{-3}=0.27\ \mathrm{mm}$

11.2

$$\because p=\frac{AT^3}{V}, U=BT^n\ln\left(\frac{V}{V_0}\right)+f(T)$$

由书中 P.213 页可得到方程

$$\left(\frac{\partial U}{\partial V}\right)_T=T\left(\frac{\partial p}{\partial T}\right)_V-p$$

则可得

$$\frac{BT^n}{V}=\frac{T\cdot 3AT^2}{V}-\frac{AT^3}{V}$$

解得

$$n=3, B=2A$$

11.5

$$\because p_1=p_0+\frac{mg}{A}$$

$\therefore pV^\gamma$ 为常量

$$\therefore \frac{\mathrm{d}p}{p}=-\gamma\frac{\mathrm{d}V}{V}=-\gamma\frac{Ax}{V_0}$$

$$\therefore f=\mathrm{d}p\cdot A=-\gamma\frac{pA^2}{V_0}x$$

$$\therefore \omega^2=\frac{\gamma A^2 p}{mV_0}, v=\frac{\omega}{2\pi}$$

11.10

$$\eta=1-\left(\frac{V_1}{v_2}\right)^{\gamma-1}=1-\left(\frac{V_1}{v_2}\right)^{\frac{2}{5}}$$