17.

求t值使$f(x) = x^3 - 3x^2 + tx - 1$有重根.

解:

$\therefore f'(x) = 3x^2 - 6x + t$

```
            3           -6          t
---------------------------------------------
1           -3          t           -1
1           -2          t/3
---------------------------------------------
            -1          2t/3        -1
            -1          2           -t/3
            ---------------------------------
                        2(t/3-1)      t/3-1
```

当$t = 3$时, $(f(x), f'(x)) = \frac{1}{3} f'(x) = (x-1)^2$

当$t \neq 3$时,

```
            2(t/3-1)    t/3-1
---------------------------------
3           -6          t
3           3/2
---------------------------------
            -15/2       t
            -15/2       -15/4
            ---------------------
                        t+15/4=0
```

$$\therefore t = -\frac{15}{4}$$

20.

$$\therefore f(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$$

$$\therefore f'(x)=1+x+\frac{x^2}{2!}+\cdots+\frac{x^{n-1}}{(n-1)!}$$

假设$f(x)$有重根a,则$f(a)=0,f'(x)=0$

$$\therefore f(a)=1+a+\frac{a^2}{2!}+\cdots+\frac{a^n}{n!}=0,$$
$$f'(a)=1+a+\frac{a^2}{2!}+\cdots+\frac{a^{n-1}}{(n-1)!}=0$$

$$\therefore f(a)-f'(x)=\frac{a^n}{n!}=0$$

$\therefore a=0$

$\therefore f(a)=f(0)=1\neq 0$,产生矛盾

$\therefore f(x)$没有重根.

22.

证明：x_0是$f(x)$的k重根的充分必要条件是$f(x_0)=f'(x_0)=\cdots=f^{(k-1)}(x_0)=0$,而$f^{(k)}(x_0)\neq 0$.

$\Rightarrow$:

$\because x_0$是$f(x)$的k重根

$\therefore (x-x_0)$是$f(x)$的k重因式

$\therefore (x-x_0)$是$f'(x)$的$k-1$重因式

$\cdots$

$\therefore (x-x_0)$是$f^{(k-1)}(x)$的1重因式

$\therefore (x-x_0)$不是$f^{(k)}(x)$的因式

$\therefore f(x_0)=f'(x_0)=\cdots=f^{(k-1)}(x_0)=0,f^{(k)}(x_0)\neq 0$

$\Leftarrow$:

若$f(x)=0$,则$f^{(k)}(x_0)=0$与题目$f^{(k)}(x_0)\neq 0$矛盾,
后续同理不再重复叙述函数为0的情况.

不妨设$f(x)=(x-x_0)q_1(x)+r(x)$

$\therefore f(x_0)=r(x)=0$

$\therefore f'(x)=(x-x_0)q_1'(x)+q_1(x)$

$\therefore f'(x_0)=q_1(x)=0$

$\therefore q_1(x)=(x-x_0)q_2(x), f(x)=(x-x_0)^2q_2(x)$

$\dots$

$\therefore$ 同理可得$q_{k-1}=(x-x_0)q_k(x), f(x)=(x-x_0)^kq_k(x)$

$\therefore f(x)=(x-x_0)^kq_k(x)$

$\therefore x_0$是$f(x)$的k重根.