习题2.2（A）：9（3、4），10，11，13，14（3、5），18（3、5），19，（B）3，6，习题2.3：（A）3（3、7），5（3），习题2.4：（A）6，9，11

2.2 (A)

9.

(3)

$$
\begin{aligned}
f^{(50)}(x) &= \sum_{k=0}^{50} C_{50}^{k}(x^2)^{(k)}(\sin 2x)^{(50-k)} \\
&= C_{50}^{0}x^2(\sin 2x)^{(50)} + C_{50}^{1}(x^2)'(\sin 2x)^{(49)} + C_{50}^{2}(x^2)''(\sin 2x)^{(48)} \\
&= -2^{50}x^2\sin 2x + 100\times 2^{49}x(\sin 2x)^{(49)} + 2450\times 2^{48}\sin 2x
\end{aligned}
$$

(4)

$\because f(x)=\dfrac{1}{x^2-3x+2}=\dfrac{1}{(x-1)(x-2)}=\dfrac{1}{x-2}-\dfrac{1}{x-1}$

$\therefore f^{(n)}(x)=\dfrac{(-1)^n n!}{(x-2)^{n+1}}-\dfrac{(-1)^n n!}{(x-1)^{n+1}}$

10.

$\because f(x)=x(x-1)(x-2)\cdots(x-n)\quad (n\in\mathbb{N}^{+})$

$\therefore f'(0)=(-1)^n n!$

将$f(x)$展开得$f(x)=x^{n+1}+a_n x^n+a_{n-1}x^{n-1}+\cdots+a_0$

$\therefore f^{(n+1)}=1$

11.

当$n=1$时,

$$f'_-(0)=\lim_{x\to0^-}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0^-}\frac{3x^3+x^2(-x)}{x}=0$$
$$f'_+(0)=\lim_{x\to0^+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0^+}\frac{3x^3+x^2(x)}{x}=0$$

$\because f'_-(0)=f'_+(0)$

$\therefore f'(0)$存在$,f'(x)$存在$,f'(x)=9x^2+2x|x|$

当$n=2$时,

$$f''_-(0)=\lim_{x\to0^-}\frac{f'(x)-f(0)}{x-0}=\lim_{x\to0^-}\frac{9x^2+2x(-x)}{x}=0$$
$$f''_+(0)=\lim_{x\to0^+}\frac{f'(x)-f(0)}{x-0}=\lim_{x\to0^+}\frac{9x^2+2x(x)}{x}=0$$

$\because f''_-(0)=f''_+(0)$

$\therefore f''(0)$存在$,f''(x)$存在$,f''(x)=18x+2|x|$

当$n=3$时,

$$f'''_-(0)=\lim_{x\to0^-}\frac{f''(x)-f(0)}{x-0}=\lim_{x\to0^-}\frac{18x+2(-x)}{x}=16$$
$$f'''_+(0)=\lim_{x\to0^+}\frac{f''(x)-f(0)}{x-0}=\lim_{x\to0^+}\frac{18x+2x}{x}=20$$

$\because f''_-(0)\neq f''_+(0)$

$\therefore f'''(0)$不存在

$\therefore f^{(n)}(0)$存在的最高阶数n为2

13.

$$F'(x)=\lim_{t\to\infty}\{t^2[f'(x+\frac{\pi}{t})-f'(x)]\sin\frac{x}{t}+t[f(x+\frac{\pi}{t})-f(x)]\cos\frac{x}{t}\}$$

14.

(3)

$\because e^{x+y}+\cos(xy)=0$

$\therefore \mathrm{d}(e^{x+y}+\cos(xy))=0$

$\therefore \mathrm{d}e^{x+y}+\mathrm{d}\cos(xy)=0$

$\therefore e^{x+y}\mathrm{d}(x+y)-\sin(xy)\mathrm{d}(xy)=0$

$\therefore e^{x+y}\mathrm{d}x+e^{x+y}\mathrm{d}y-y\sin(xy)\mathrm{d}x+x\sin(xy)\mathrm{d}y=0$

$\therefore [e^{x+y}+x\sin(xy)]\mathrm{d}y=[y\sin(xy)-e^{x+y}]\mathrm{d}x$

$\therefore \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{y\sin(xy)-e^{x+y}}{x\sin(xy)+e^{x+y}}$

(5)

$\because y=\sin(x+y)$

$\therefore \mathrm{d}y=\mathrm{d}\sin(x+y)$

$\therefore [1-\cos(x+y)]\mathrm{d}y=\cos(x+y)\mathrm{d}x$

$\therefore [1-\cos(x+y)]\mathrm{d}^2y+2\sin(x+y)\mathrm{d}x\mathrm{d}y+\sin(x+y)\mathrm{d}y^2+\sin(x+y)\mathrm{d}x^2=0$

$\therefore [1-\cos(x+y)]\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\sin(x+y)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\sin(x+y)\dfrac{\mathrm{d}y^2}{\mathrm{d}x^2}+\sin(x+y)=0$

$\therefore [1-\cos(x+y)]\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\sin(x+y)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\sin(x+y)\dfrac{\mathrm{d}y^2}{\mathrm{d}x^2}+\sin(x+y)=0$

$\because \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\cos(x+y)}{1-\cos(x+y)}$

$\therefore [1-\cos(x+y)]\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\sin(x+y)\dfrac{\cos(x+y)}{1-\cos(x+y)}+\sin(x+y)[\dfrac{\cos(x+y)}{1-\cos(x+y)}]^2+\sin(x+y)=0$

$\therefore \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{\sin(x+y)}{[1-\cos(x+y)]^3}$

18.

(3)

当$a=0$时,

$\therefore x=0, y=0$

$$\therefore \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=2}=\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}=0$$

当$a\neq 0$时,

$$\therefore \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=2}=\frac{\left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{t=2}}{\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=2}}=\frac{\left.\frac{2a(1+t^2)-4at^2}{(1+t^2)^2}\right|_{t=2}}{\left.\frac{6at(1+t^2)-6at^3}{(1+t^2)^2}\right|_{t=2}}=\frac{10a-16a}{60a-48a}=-\frac{1}{2}$$

(5)

$$\therefore \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\frac{\mathrm{d}^2y}{\mathrm{d}t^2}}{\frac{\mathrm{d}x}{\mathrm{d}t}\cdot\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{(tf(t)-f(t))''}{f'(t)\cdot f'(t)}=\frac{2f'(t)+(t-1)f''(t)}{[f'(t)]^2}$$

19.

$\because \Gamma: r=r(\theta), x=r(\theta)\cos\theta, x=r(\theta)\sin\theta$

$$\therefore k=\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}=\frac{r'(\theta)\sin\theta+r(\theta)\cos\theta}{r'(\theta)\cos\theta-r(\theta)\sin\theta}$$

对于心型线$r(\theta)=a(1-\cos\theta), r'(\theta)=a\sin\theta$

$$\begin{aligned}\therefore k&=\frac{a\sin^2\theta+a(1-\cos\theta)\cos\theta}{a\sin\theta\cos\theta-a(1-\cos\theta)\sin\theta}\\&=\frac{a\sin^2\theta-a\cos^2\theta+a\cos\theta}{2a\sin\theta\cos\theta-a\sin\theta}\\&=\frac{-a\cos 2\theta+a\cos\theta}{a\sin 2\theta-a\sin\theta}\end{aligned}$$

2.2 (B)

3.

$$\therefore f'_-(0)=\lim_{x\to0^-}\frac{\frac{1}{x}(1-\cos ax)-0}{x-0}=\lim_{x\to0^-}\frac{1}{x^2}\cdot\frac{1}{2}(ax)^2=\frac{a^2}{2}$$

$$f'_+(0)=\lim_{x\to0^+}\frac{\frac{1}{x}\ln(b+x^2)-0}{x-0}=\lim_{x\to0^+}\frac{1}{x^2}\ln(b+x^2)$$

$\therefore$ 只有$b=1$时, $f'_+(0)$才存在, 且$f'_+(0)=1$

$$\therefore f'_-(0)=f'_+(0)=\frac{a^2}{2}=1$$

$$\therefore a=\sqrt{2}$$

$$\therefore f'(x)=\begin{cases}\dfrac{\sqrt{2}x\sin\sqrt{2}x+\cos\sqrt{2}x-1}{x^2}, & x<0\\ 1, & x=0\\ \dfrac{2}{x^2+1}-\dfrac{1}{x^2}\ln(x^2+1), & x>0\end{cases}$$

6.

当$x<1$时, $n\to\infty, e^{n(x-1)}\to0$

$$\therefore f(x)=ax+b$$

当$x=1$时, $n\to\infty, e^{n(x-1)}=1$

$$\therefore f(x)=\frac{x^2+ax+b}{2}$$

当$x>1$时, $n\to\infty, e^{n(x-1)}=\infty$

$$\therefore f(x)=x^2$$

$$\therefore f(x)=\begin{cases}ax+b, & x<1\\ \dfrac{x^2+ax+b}{2}, & x=1\\ x^2, & x>1\end{cases}$$

对于连续性：

$$\because f(1+0)=1$$

$$\therefore f(1-0)=a+b=1, f(1)=\frac{a+b+1}{2}=1$$

$$\therefore a+b=1$$

对于可导性：

$$\therefore f'_+(1)=\lim_{\Delta x\to 0^+}\frac{(1+\Delta x)^2-1}{\Delta x}=\lim_{\Delta x\to 0^+}\frac{\Delta x^2+2\Delta x}{\Delta x}=2$$

$$\therefore f'_-(1)=\lim_{\Delta x\to 0^-}\frac{a(1+\Delta x)+b-1}{\Delta x}=2$$

$$\therefore a=2,b=-1$$

$$\therefore f(x)=\begin{cases}2x-1, & x<1\\ \dfrac{x^2+2x-1}{2}, & x=1\\ x^2, & x>1\end{cases}$$

$$\therefore f(x)=\begin{cases}2, & x\le 1\\ 2x, & x>1\end{cases}$$

2.3 (A)

3.

(3)

$$\begin{aligned}\mathrm{d}y&=\mathrm{d}[e^{-x}\cos(3-x)]\\&=\cos(3-x)\mathrm{d}e^{-x}+e^{-x}\mathrm{d}\cos(3-x)\\&=[\sin(3-x)-\cos(3-x)]e^{-x}\mathrm{d}x\end{aligned}$$

(7)

$$\because y=\sqrt[3]{\frac{1-x}{1+x}}$$

$$\therefore \ln y=\frac{1}{3}\ln(1-x)-\frac{1}{3}\ln(1+x)$$

$$\therefore \frac{y'}{y}=\frac{1}{3x-3}-\frac{1}{3x+3}$$

$$\therefore \mathrm{d}y=y'\mathrm{d}x=(\frac{1}{3x-3}-\frac{1}{3x+3})\sqrt[3]{\frac{1-x}{1+x}}\mathrm{d}x$$

5.(3)

$\because f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

取$x_0 = 25$

$\therefore \sqrt{25.4} \approx \sqrt{25} + \dfrac{(25.4 - 25)}{2\sqrt{25}} = 5 + \dfrac{0.4}{10} = 5.04$

2.4 (A)

6.

$\because f$为奇函数

$\therefore f(a) = -f(-a)$

当$a > 0$时,

$\therefore$ 由拉格朗日中值定理知

$$\exists \xi \in (-a, a), f'(\xi) = \frac{f(a) - f(-a)}{a - (-a)} = \frac{f(a)}{a}$$

当$a < 0$时, 同理可知也成立

$\therefore \forall a \neq 0$, 必定存在$\xi$在$-a$和$a$之间, 使得$f'(\xi) = \dfrac{f(a)}{a}$

9.

令$F(x) = \arcsin x + \arccos x$

假设$F(x_0) \neq \dfrac{\pi}{2}$

$\because F(-1) = \dfrac{\pi}{2}$

$\therefore$ 由$Lagrange$定理可知$\exists \xi \in (-1, x_0), F'(\xi) = \dfrac{F(x_0) - F(-1)}{\xi + 1} \neq 0$

$$\because F'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

$\therefore$ 产生矛盾

$$\therefore F(x) = \frac{\pi}{2}$$

11.

令$F(x) = e^x f(x), x_1$和x_2是$f(x)$的两个零点

$\therefore F(x_1) = F(x_2) = e^x f(x_1) = e^x f(x_2) = 0$

$\because f(x)$可微

$\therefore \exists \xi \in (x_1, x_2), F'(\xi) = e^\xi f(\xi) + e^\xi f'(\xi) = 0$

$\therefore \exists \xi \in (x_1, x_2), f(\xi) + f'(\xi) = 0$