1.求多项式 $f(x)=x^5+x^4-6x^3-14x^2-11x-3$ 的有理根.

2.证明下列多项式在有理数域上不可约.

(1) $f(x)=5x^4-6x^3+12x+6$

(2) $f(x)=x^6+x^3+1$

27. 求下列多项式的有理根：

1) $x^3-6x^2+15x-14$；

2) $4x^4-7x^2-5x-1$；

3) $x^5+x^4-6x^3-14x^2-11x-3$.

28. 判断下列多项式在有理数域上是否可约？

1) x^2+1；

2) $x^4-8x^3+12x^2+2$；

3) x^6+x^3+1；

4) x^p+px+1，p 为奇素数；

5) $x^4+4kx+1$，k 为整数.

27.(2), 28.(5)

1.

$\therefore$ 由题目可知x只可能是$\pm1,\pm3$

$\therefore f(-1)=-1+1+6-14+11-3=0$

$\therefore f(1)=1+1-6-14-11-3=-30\neq0$

```
1       1       -6      -14     -11     -3
        -1      0       6       8       3
-------------------------------------------
1       0       -6      -8      -3      0
        3       9       9       3
-----------------------------------
1       3       3       1       0
        -3      0       -9
---------------------------
1       0       3       -8
```

$\therefore f(x)=(x+1)(x-3)(x^3+3x^2+3x+1)=(x+1)^4(x-3)$

$\therefore f(x)$的有理根为-1和3

2.

(1)

令素数$p=2$,则$2\nmid 5,\ 2|-6,12,6,\ 2^2\ \nmid 6$,
由$Eisenstain$判别法可知

$f(x)=5x^4-6x^3+12x+6$在有理数域上不可约

(2)

$\because f(x)=x^6+x^3+1$

$\therefore f(x)$的有理根只能为± 1

$\because f(1)=3\neq 0, f(-1)=1\neq 0$

$\therefore f(x)$在有理数域上无有理根,即无一次因式

当$(x^2+ax+1)|f(x)$时,

```
                 1       -a          a^2-1   2a-a^3-1        1
                                     1       a               1
 ---------------------------------------------------------------
 1       0       0       1           0       0               1
 1       a       1
 ---------------------------
         -a      -1      1
         -a      -a^2    -a
         ------------------------------
                 a^2-1   a+1         0
                 a^2-1   a^3-a       a^2-1
                 ---------------------------------
                         2a-a^3+1    1-a^2       0
                         2a-a^3+1    2a^2-a^4+a  2a-a^3+1
                         ------------------------------------
                                     a^4-3a^2-a+1 a^3-2a-1   1
                                     1             a         1
```

$\therefore a^4-3a^2+a+1=1, a^3-2a-1=a$

$\therefore a(a^3-3a+1)=0, a^3-3a-1=0$

$\therefore$易知无解,所以此情况不成立.

当$(x^3+ax^2+bx+1)|f(x)$时,

```
                        1           -a          a^2-b            1
                        1           a           b                1
---------------------------------------------------------------------
1       0       0       1           0           0                1
1       a       b       1
----------------------------------------
        -a      -b      0           0
        -a      -a^2    -ab         -a
        ------------------------------------------
                a^2-b   ab          a           0
                a^2-b   a^3-ab      a^2b-b^2    a^2-b
                ---------------------------------------------------
                        2ab-a^3     b^2-a^2b+a  b-a^2            1
                        1           a           b                1
```

$$\therefore \begin{cases} 2ab-a^3-1=0 \\ b^2-a^2b=0 \\ -a^2=0 \end{cases} \Rightarrow \begin{cases} -1=0 \\ b=0 \\ a=0 \end{cases}$$

$\therefore$ 产生矛盾,此情况也不成立

$\therefore$ 综上$f(x)$在有理数域上不可约

27.(2)

令$f(x)=4x^4-7x^2-5x-1$

$\therefore$ 易知有理根只可能是$\pm 1,\pm\frac{1}{2},\pm\frac{1}{4}$

$\therefore f(1)=4-7-5-1=-9\neq 0, f(-1)=4-7+5-1=1\neq 0$

```
4       0       -7      -5      -1
        -2      1       3       1
------------------------------------
4       -2      -6      -2      0
        -2      2       -2
----------------------------
4       -4      -4      0
        -2      3
--------------------
4       -6      -1
```

经检验$\frac{1}{2}$和$\pm\frac{1}{4}$均不符合

$\therefore f(x)$的有理根为$-\frac{1}{2},-\frac{1}{2}$

28.(5)

令$x=t+1$,令$f(t)=(t+1)^4+4k(t+1)+1$

$\therefore f(t)=t^4+4t^3+6t^2+(4+4k)t+4k+2$

$\therefore$ 令$p=2$,则$p\nmid 1, p|4,6,(4+4k),(4k+2), p^2\nmid 4k+2$

$\therefore$ 由$Eisenstain$判别法可知$f(t)$有理数域上不可约

$\therefore x^4+4kx+1$在有理数域上也不可约