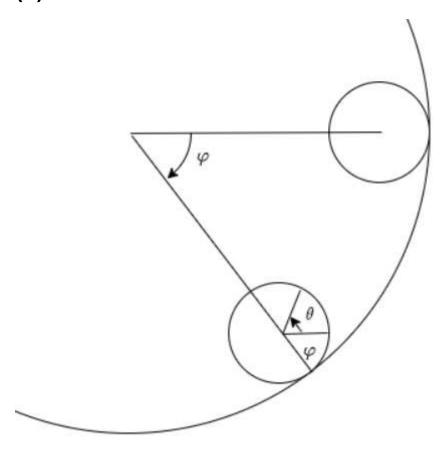
# 大学物理第六次作业

6.6, 7.1, 7.5, 7.8

6.6

(a)



设小球质量为 m, 动能为  $E_k$ , 转动动能为  $E_1$ , 平动动能为  $E_2$ , 小球半径为 r, 大球半径为 R.

由功能关系可知

$$E_k = mg(R-r)$$

$$E_k = E_1 + E_2$$

$$E_2=rac{1}{2}m[(R-r)\dot{arphi}]^2$$

对转动角度分析可知:

$$\because r(\theta + \varphi) = R\varphi$$

$$\therefore heta = (rac{R}{r} - 1)arphi, heta' = (rac{R}{r} - 1)arphi'$$

小球的转动惯量  $I=rac{2}{5}mr^2$ 

$$\therefore E_1 = \frac{1}{2} (\frac{2}{5} m r^2) [(\frac{R}{r} - 1)\dot{\varphi}]^2 = \frac{1}{5} m [(R - r)\dot{\varphi}]^2$$

$$\therefore \frac{E_1}{E_2} = \frac{2}{5}$$

$$\therefore E_1 = rac{2}{7}E_k = rac{2}{7}mg(R-r)$$

$$E_2=rac{5}{7}E_k=rac{5}{7}mg(R-r)$$

### (b)

进行最低点进行受力分析:

$$N-mg=m(R-r)\dot{arphi}^2$$

由 (a) 得

$$E_1 = rac{1}{5} m [(R-r) \dot{arphi}]^2 = rac{2}{7} m g (R-r)$$

$$\therefore \dot{\varphi}^2 = \frac{10g}{7(R-r)}$$

$$\therefore N = mg + m(R-r)\dot{arphi}^2 = rac{17}{7}mg$$

# 7.1

## (a)

对于简谐运动有

$$m\ddot{x}+kx=0, \omega_0^2=rac{k}{m}$$

解常微分方程可得

$$x=A\cos(\omega_0 t+arphi), \dot{x}=-\omega_0 A\sin(\omega_0 t+arphi)$$

由物理量对时间平均公式  $\overline{P}=rac{1}{T}\int_0^T P\mathrm{d}t$ ,势能公式  $rac{1}{2}kx^2$ ,动能公式  $rac{1}{2}m\dot{x}^2$  和  $rac{1}{T}=rac{\omega_0}{2\pi}$  可得

$$\overline{E}_{pT}=rac{1}{T}\int_0^Trac{1}{2}kx^2\mathrm{d}t=rac{1}{4\pi}kA^2\int_0^{2\pi}\cos^2(\omega_0t+arphi)\mathrm{d}(\omega_0t)=rac{1}{4}kA^2$$

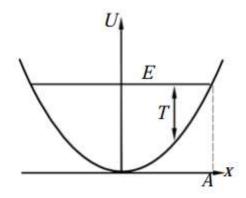
$$\overline{E}_{kT}=rac{1}{T}\int_0^Trac{1}{2}m\dot{x}^2\mathrm{d}t=rac{1}{4\pi}m\omega_0^2A^2\int_0^{2\pi}\sin^2(\omega_0t+arphi)\mathrm{d}(\omega_0t)=rac{1}{4}kA^2$$

(b)

$$egin{align} \overline{E}_{px} &= rac{1}{A} \int_0^A rac{1}{2} k x^2 \mathrm{d}x = rac{1}{6} k A^2 \ \overline{E}_{kx} &= rac{1}{A} \int_0^A rac{1}{2} m \dot{x}^2 \mathrm{d}x \ &= rac{1}{A} \int_0^A rac{1}{2} m \dot{x}^3 \mathrm{d}t \ &= \frac{1}{A} \int_0^A \frac{1}{2} m \dot{x}^3 \mathrm{d}t$$

$$egin{aligned} &=rac{1}{\omega_0A}\int_0^{rac{\pi}{2}}rac{1}{2}m(-\omega_0A\sin(\omega_0t+arphi))^3\mathrm{d}(\omega_0t)\ &=-rac{1}{2}m\omega_0^2A^2\int_0^{rac{\pi}{2}}\sin^3(\omega_0t+arphi)\mathrm{d}(\omega_0t)\ &=rac{1}{3}kA^2 \end{aligned}$$

(c)



由该图我们可以看出,动能所占的面积大于势能,因而  $\overline{E}_{kx} > \overline{E}_{px}$  .

### 7.5

设弹簧伸长量为 X, x 为每个质点相对于原来位置的位移.

每个小质点的速度为  $\dot{x}=rac{x}{X}\dot{X}$ 

弹簧动能为

$$E_k = rac{1}{2} \int_0^X \dot{x}^2 (rac{m_s}{X} \mathrm{d}x) = rac{m_s \dot{X}^2}{2 X^3} \int_0^X x^2 \mathrm{d}x = rac{1}{6} m_s \dot{X}^2$$

由能量守恒得

$$-mgX-m_sgrac{X}{2}+rac{1}{2}kX^2+rac{1}{2}m\dot{X}^2+rac{1}{6}m_s\dot{X}^2=0$$

对时间求导得

$$-mg\dot{X}-rac{1}{2}m_sg\dot{X}+kX\dot{X}+m\dot{X}\ddot{X}+rac{1}{3}m_s\dot{X}\ddot{X}=0$$

当  $\dot{X}=0$  时,该式意义不大

当 $\dot{X} \neq 0$ 时,

$$\therefore -mg-rac{1}{2}m_sg+kX+m\ddot{X}+rac{1}{3}m_s\ddot{X}=0$$

$$\therefore (m+rac{1}{3}m_s)\ddot{X} = -kX + mg + rac{1}{2}m_sg$$

相当于是有恒定外力的简谐振动

$$\because \omega_0^2 = rac{k}{M} = rac{k}{m + rac{1}{2}m_s}, \omega_0 = rac{2\pi}{T}$$

$$\therefore T = 2\pi \sqrt{rac{m + rac{1}{3}m_s}{k}}$$

#### 7.8

::强迫阻尼振动一个周期后的动能和势能依然不变

$$\therefore \int F \dot{x} \mathrm{d}t - \int F_{\gamma} \dot{x} \mathrm{d}t = \Delta E_k + \Delta E_p = 0$$

$$\therefore \overline{P} = \frac{1}{T} \int F \dot{x} dt = \frac{1}{T} \int F_{\gamma} \dot{x} dt$$

:. 外力的平均功率等于阻尼力耗散的功率.