第十一次作业

15.9, 15.17, 17.1, 17.9, 18.1, 19.4, 20.4

15.9

对于无限薄的的 $\mathrm{d}x'$ 形成的电场:

$$E=\frac{1}{4\pi\varepsilon_0}2\pi(-eax')\mathrm{d}x'$$

因此

$$\begin{aligned}E(x)&=\frac{1}{4\pi\varepsilon_0}2\pi\int_{-\frac{x_m}{2}}^{x}(-eax')\mathrm{d}x'-\frac{1}{4\pi\varepsilon_0}2\pi\int_{x}^{\frac{x_m}{2}}(-eax')\mathrm{d}x'\\&=-\frac{ea}{2\varepsilon_0}\left[\frac{x^2}{2}-\frac{x_m^2}{8}-\frac{x_m^2}{8}+\frac{x^2}{2}\right]\\&=\frac{ea}{2\varepsilon_0}\left[\frac{x_m^2}{4}-x^2\right]\end{aligned}$$

在薄片之外, $E=0$

15.17

由两块金属板电势相等可知:

$$\frac{d}{3}\cdot E_1=\frac{2d}{3}\cdot E_2$$

对塑料薄片上的一小块面积分析, 由高斯定理可知:

$$\frac{q}{\varepsilon_0}=\frac{\sigma\mathrm{d}S}{\varepsilon_0}=(E_1+E_2)\mathrm{d}S$$

联解可得:

$$E_1=\frac{2\sigma}{3\varepsilon_0},E_2=\frac{\sigma}{3\varepsilon_0}$$

17.1

由公式 $\boldsymbol{B}=\frac{\mu_0}{4\pi}\oint\frac{I\mathrm{d}\boldsymbol{l}\times\boldsymbol{r}}{r^3}$ 可知

$$\therefore B=\frac{\mu_0}{4\pi}\left(\frac{I\cdot\pi R_2}{r^2}-\frac{I\cdot\pi R_1}{r^2}\right)=\frac{\mu_0 I}{4r^2}(R_2-R_1)$$

方向垂直纸面向外.

17.9

已知 $2\ \mathrm{keV}\ll 0.511\ \mathrm{MeV}, V_{\parallel}=V_0\cos 89^\circ, V_{\perp}=V_0\cos 1^\circ$

(a)

因为 $V_{\parallel}$ 是平动, 而 $V_{\perp}$ 是转动, 所以正电子的轨迹是螺旋线, 轴线沿 $\vec{B}$ 方向.

(b)

$$\because qV_{\perp}B=m\frac{V_{\perp}^2}{r}$$

$$\omega=\frac{V_{\perp}}{r}=\frac{qB}{m}$$

$$\therefore T=\frac{2\pi}{\omega}=3.57\times 10^{-10}\ \mathrm{s}$$

$$p=V_{\parallel}T=0.17\ \mathrm{mm}$$

$$r=1.5\times 10^{-3}\ \mathrm{m}$$

18.1

$$\therefore B(r)=\frac{\mu_0}{4\pi}\cdot\frac{2I}{r}$$

$$\therefore \frac{\mathrm{d}\Phi}{\mathrm{d}t}(r)=B\frac{\mathrm{d}A}{\mathrm{d}t}=B\cdot\mathrm{d}r\cdot v$$

$$\begin{aligned}\therefore \varepsilon&=-\int_a^b B(r)\cdot v\cdot\mathrm{d}r\\&=-\frac{\mu_0}{4\pi}2Iv\ln\frac{b}{a}\\&=-10^{-7}\times 2\times 100\times 5.0\times\ln 20\ \mathrm{V}\\&=-3.0\times 10^{-4}\ \mathrm{V}\end{aligned}$$

19.4

$$\because I = Q\frac{\omega}{2\pi}$$

$$\therefore \mu = I\pi r^2 = \frac{1}{2}Q\omega r^2$$

20.4

$$E_\rho = \frac{\lambda}{2\pi\varepsilon_0\rho}$$

(a)

$$\therefore q = C\varepsilon = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{R_2}{R_1}\right)}\varepsilon$$

$$\because \varepsilon_0 E_\rho 2\pi\rho l = q$$

$$\therefore E_\rho = \frac{1}{\rho}\cdot\frac{\varepsilon}{\ln\left(\frac{Rr_2}{R_1}\right)}$$

$$\because B_\varphi 2\pi\rho = \mu_0 I = \mu_0\frac{\varepsilon}{R}$$

$$\therefore B_\varphi = \frac{\mu_0}{4\pi}\cdot\frac{2\varepsilon}{\rho R}$$

(b)

$$\therefore S = \frac{\varepsilon^2}{2\pi R\ln\left(\frac{R_2}{R_1}\right)}\frac{1}{\rho^2}k$$

(c)

$$\therefore P = \frac{\varepsilon^2}{2\pi R\ln\left(\frac{R_2}{R_1}\right)}\int_{R_1}^{R2}\frac{1}{\rho^2}\cdot 2\pi\rho\,\mathrm{d}r = \frac{\varepsilon^2}{R}$$

所以是合理的.