习题 p.100
15(1),16(4),17(1)、(3),18(1)、(2)、(4),

15.(1)

$$
\begin{array}{cccc}
A_{11}=-6 & A_{12}=0 & A_{13}=0 & A_{14}=0 \\
A_{21}=-12 & A_{22}=6 & A_{23}=0 & A_{24}=0 \\
A_{31}=15 & A_{32}=-6 & A_{33}=-3 & A_{34}=0 \\
A_{41}=7 & A_{42}=0 & A_{43}=1 & A_{44}=-2
\end{array}
$$

16.(4)

$$
\begin{vmatrix}
1 & \frac{1}{2} & 0 & 1 & -1 \\
2 & 0 & -1 & 1 & 2 \\
3 & 2 & 1 & \frac{1}{2} & 0 \\
1 & -1 & 0 & 1 & 2 \\
2 & 1 & 3 & 0 & \frac{1}{2}
\end{vmatrix}
=
\begin{vmatrix}
1 & \frac{1}{2} & 0 & 1 & -1 \\
0 & -1 & -1 & -1 & 4 \\
0 & \frac{1}{2} & 1 & -\frac{5}{2} & 3 \\
0 & -\frac{3}{2} & 0 & 0 & 3 \\
0 & 0 & 3 & -2 & \frac{5}{2}
\end{vmatrix}
$$

$$
=\begin{vmatrix}
-1 & -1 & -1 & 4 \\
\frac{1}{2} & 1 & -\frac{5}{2} & 3 \\
-\frac{3}{2} & 0 & 0 & 3 \\
0 & 3 & -2 & \frac{5}{2}
\end{vmatrix}
=\begin{vmatrix}
-1 & -1 & -1 & 4 \\
0 & \frac{1}{2} & -3 & 5 \\
0 & \frac{3}{2} & \frac{3}{2} & -3 \\
0 & 3 & -2 & \frac{5}{2}
\end{vmatrix}
=-\begin{vmatrix}
\frac{1}{2} & -3 & 5 \\
\frac{3}{2} & \frac{3}{2} & -3 \\
3 & -2 & \frac{5}{2}
\end{vmatrix}
$$

$$
=-\begin{vmatrix}
\frac{1}{2} & -3 & 5 \\
0 & \frac{21}{2} & -18 \\
0 & 16 & -\frac{55}{2}
\end{vmatrix}
=\frac{21\times 55}{8}-8\times 18=\frac{3}{8}
$$

17.

(1)

$$
\begin{aligned}
&\begin{vmatrix}
x & y & 0 & \cdots & 0 & 0 \\
0 & x & y & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & \cdots & x & y \\
y & 0 & 0 & \cdots & 0 & x
\end{vmatrix} \\
=&x\begin{vmatrix}
x & y & \cdots & 0 & 0 \\
\vdots & \vdots & & \vdots & \vdots \\
0 & 0 & \cdots & x & y \\
0 & 0 & \cdots & 0 & x
\end{vmatrix} + (-1)^{n+1}y\begin{vmatrix}
y & 0 & \cdots & 0 & 0 \\
x & y & \cdots & 0 & 0 \\
\vdots & \vdots & & \vdots & \vdots \\
0 & 0 & \cdots & x & y
\end{vmatrix} \\
=&x^n + (-1)^{n+1}y^n
\end{aligned}
$$

(3)

$$
\begin{aligned}
&\begin{vmatrix}
x_1 - m & x_2 & \cdots & x_n \\
x_1 & x_2 - m & \cdots & x_n \\
\vdots & \vdots & & \vdots \\
x_1 & x_2 & \cdots & x_n - m
\end{vmatrix} = \begin{vmatrix}
\sum\limits_{i=1}^{n} x_i - m & x_2 & \cdots & x_n \\
\sum\limits_{i=1}^{n} x_i - m & x_2 - m & \cdots & x_n \\
\vdots & \vdots & & \vdots \\
\sum\limits_{i=1}^{n} x_i - m & x_2 & \cdots & x_n - m
\end{vmatrix} \\
=&(\sum_{i=1}^{n} x_i - m)\begin{vmatrix}
1 & x_2 & \cdots & x_n \\
1 & x_2 - m & \cdots & x_n \\
\vdots & \vdots & & \vdots \\
1 & x_2 & \cdots & x_n - m
\end{vmatrix} = (\sum_{i=1}^{n} x_i - m)\begin{vmatrix}
1 & 0 & \cdots & 0 \\
1 & -m & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
1 & 0 & \cdots & -m
\end{vmatrix} \\
=&(-m)^{n-1}(\sum_{i=1}^{n} x_i - m)
\end{aligned}
$$

18.

(1)

$$
\begin{vmatrix}
a_0 & 1 & 1 & \cdots & 1 \\
1 & a_1 & 0 & \cdots & 0 \\
1 & 0 & a_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
1 & 0 & 0 & \cdots & a_n
\end{vmatrix}
=
\begin{vmatrix}
a_0 - \sum_{i=1}^{n} \frac{1}{a_i} & 1 & 1 & \cdots & 1 \\
0 & a_1 & 0 & \cdots & 0 \\
0 & 0 & a_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & a_n
\end{vmatrix}
$$

$$
= a_1 a_2 \cdots a_n (a_0 - \sum_{i=1}^{n} \frac{1}{a_i})
$$

(2)

$$
\begin{vmatrix}
x & 0 & 0 & \cdots & 0 & a_0 \\
-1 & x & 0 & \cdots & 0 & a_1 \\
0 & -1 & x & \cdots & 0 & a_2 \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & \cdots & x & a_{n-2} \\
0 & 0 & 0 & \cdots & -1 & x + a_{n-1}
\end{vmatrix}
$$

$$
= (-1)^{n+1} a_0
\begin{vmatrix}
-1 & x & 0 & \cdots & 0 \\
0 & -1 & x & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & x \\
0 & 0 & 0 & \cdots & -1
\end{vmatrix}
+ (-1)^{n+2} a_1
\begin{vmatrix}
x & 0 & 0 & \cdots & 0 \\
0 & -1 & x & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & x \\
0 & 0 & 0 & \cdots & -1
\end{vmatrix}
$$

$$
+ \cdots + (-1)^{2n-1} a_{n-2}
\begin{vmatrix}
x & 0 & 0 & \cdots & 0 \\
-1 & x & 0 & \cdots & 0 \\
0 & -1 & x & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & -1
\end{vmatrix}
+ (-1)^{2n} (x + a_{n-1})
\begin{vmatrix}
x & 0 & 0 & \cdots & 0 \\
-1 & x & 0 & \cdots & 0 \\
0 & -1 & x & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & x
\end{vmatrix}
$$

$$
= (-1)^{n+1} (-1)^{n-1} a_0 + (-1)^{2n} a_1 x + \cdots + (-1)^{2n} a_{n-2} x^{n-2} + (-1)^{2n} (x + a_{n-1}) x^{n-1}
$$

$$
= x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0
$$

(4)

①当$n = 2$时,

$$
\begin{vmatrix}
\cos\alpha & 1 \\
1 & 2\cos\alpha
\end{vmatrix}
= 2\cos^2\alpha - 1 = \cos 2\alpha \text{成立}
$$

$$
\begin{vmatrix}
\cos\alpha & 1 & 0 \\
1 & 2\cos\alpha & 1 \\
0 & 1 & 2\cos\alpha
\end{vmatrix}
= 2\cos\alpha\cos 2\alpha - \cos\alpha = \cos\alpha\cos 2\alpha - \sin\alpha\sin 2\alpha = \cos 3\alpha \text{成立}
$$

②假设当$n-1,n-2$时成立,

对于n的情况：

$$\begin{vmatrix} \cos\alpha & 1 & 0 & \cdots & 0 & 0 \\ 1 & 2\cos\alpha & 1 & \cdots & 0 & 0 \\ 0 & 1 & 2\cos\alpha & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2\cos\alpha & 1 \\ 0 & 0 & 0 & \cdots & 1 & 2\cos\alpha \end{vmatrix}_n$$

$$=2\cos\alpha\cos[(n-1)\alpha]-\begin{vmatrix} \cos\alpha & 1 & 0 & \cdots & 0 & 0 \\ 1 & 2\cos\alpha & 1 & \cdots & 0 & 0 \\ 0 & 1 & 2\cos\alpha & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2\cos\alpha & 0 \\ 0 & 0 & 0 & \cdots & 1 & 1 \end{vmatrix}_{n-1}$$

$$=2\cos\alpha\cos[(n-1)\alpha]-\begin{vmatrix} \cos\alpha & 1 & 0 & \cdots & 0 \\ 1 & 2\cos\alpha & 1 & \cdots & 0 \\ 0 & 1 & 2\cos\alpha & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 2\cos\alpha \end{vmatrix}_{n-2}$$

$$=2\cos\alpha\cos[(n-1)\alpha]-\cos[(n-2)\alpha]$$
$$=2\cos\alpha\cos[(n-1)\alpha]-\cos[(n-1)\alpha-\alpha]$$
$$=2\cos\alpha\cos[(n-1)\alpha]-\cos\alpha\cos[(n-1)\alpha]-\sin\alpha\sin[(n-1)\alpha]$$
$$=\cos\alpha\cos[(n-1)\alpha]-\sin\alpha\sin[(n-1)\alpha]$$
$$=\cos n\alpha$$

1.

$$D_n=\begin{vmatrix} a & 0 & \cdots & 0 & 1 \\ 0 & a & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & a & 0 \\ 1 & 0 & \cdots & 0 & a \end{vmatrix}=a\begin{vmatrix} a & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & a & 0 \\ 0 & \cdots & 0 & a \end{vmatrix}+(-1)^{n+1}\begin{vmatrix} 0 & \cdots & 0 & 1 \\ a & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & a & 0 \end{vmatrix}=a^n+a^{n-2}$$

2.

$$
\begin{aligned}
D_{n+1} =& \begin{vmatrix} -a_1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ a_1 & -a_2 & 0 & \cdots & 0 & 0 & 1 \\ 0 & a_2 & -a_3 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{n-1} & -a_n & 1 \\ 0 & 0 & 0 & \cdots & 0 & a_n & 1 \end{vmatrix} \\
=& \begin{vmatrix} -a_1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & -a_2 & 0 & \cdots & 0 & 0 & 2 \\ 0 & 0 & -a_3 & \cdots & 0 & 0 & 3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -a_n & n \\ 0 & 0 & 0 & \cdots & 0 & 0 & n+1 \end{vmatrix} \\
=& (n+1)\prod_{i=1}^{n}(-a_i)
\end{aligned}
$$