Assignment 1

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⋆ **This assignment, due on 25th March, contributes to 10% of the total mark of the course.**

Question 1. Basic Understanding of KR and Ontologies
In Philosophy, *ontology* is the study of existence and being as such, and of the fundamental classes and relationships of existing things. In Computer Science and Artificial Intelligence, *ontology* is a formal description of knowledge about a domain of interest based on a fixed vocabulary of terms. Explain in 5-7 sentences as to why a logic-based ontology can be used as a "computational" KR model?

Question 2. Expressivity & Computability
Make up a natural language sentence that is unable to be modelled in the formal languages you learnt in the lecture, say in Description Logics or First-Order Logic. Following this example, there may come naturally a belief that a logic-based KR language should be designed as expressive as possible in any circumstances to capture as much domain knowledge as possible. Say in 5-7 sentences your opinion?

Question 3. Manchester Syntax
(1) Given the following ontology definition:

Class: LivestockOwner

SubClassOf: Person that hasLivestock some Cow and hasLivestock only (Cow or Sheep)

Class: Cow

SubClassOf: Animal

Class: Dog

SubClassOf: Animal

Individual: Tom

Types: LivestockOwner

Facts: hasLivestock Timo

Individual: Timo

Types: not Sheep

- What additional information do we know about Timo?

(2) Given the following ontology definition:

Class: LivestockOwner

SubClassOf: Person that hasLivestock some Cow and hasLivestock some Sheep

Class: Cow

SubClassOf: Animal

Class: Sheep

SubClassOf: Animal

Individual: Tom

Types: LivestockOwner

Facts: hasLivestock Timo hasLivestock Fido

Individual: Timo

Types: Cow

Individual: Fido

- Is Fido a Dog?

(3) Given the following ontology definition:

Class: LivestockOwner

SubClassOf: Person that hasLivestock some Cow

- Is owning a Cow enough to recognize a Person as a LivestockOwner?

(4) Given the following ontology definition:

Class: LivestockOwner

SubClassOf: Person that hasLivestock some Cat and hasLivestock only Cat

- How many Cows does a LivestockOwner have as Livestocks?

Question 4. ALC Extensions & FOL

Consider the following sentences:

- Every Chinese couple have at most 3 children.
- ML is a course taught by SFM who is a professor working at NJU.
- NJU is a university whose members are a school or a department.
- NJU has at least 30,000 students.
- All members of AI School are undergraduates, graduates, or teachers.
- The domain of the relation "citizenOf" consists of countries.

(1) Translate these sentences into one or multiple SHOIQ inclusions. State which concept names, role names, and nominals are used.
(2) Translate the LAST TWO inclusions into equivalent first-order logic.

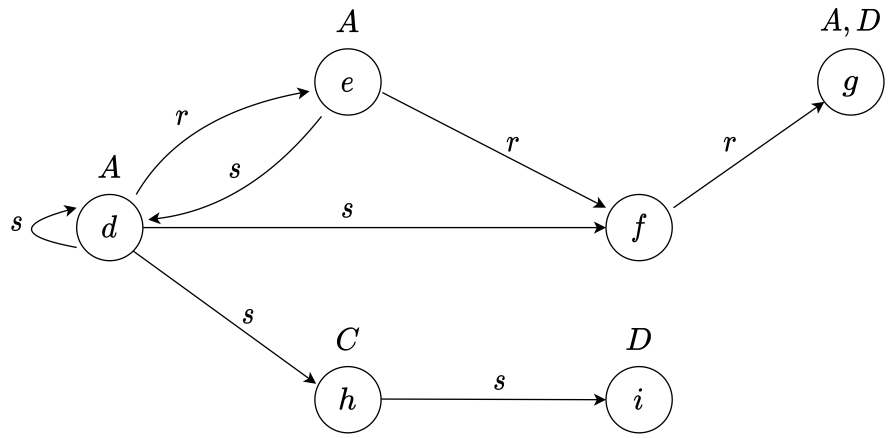
Question 5. DL Semantics

Prove or disprove the following in regard of description logics:

- There is an ontology that has no model at all.
- There is an ontology that has only finite models.
- Every ontology has either no model or infinite many models.
- A satisfiable class must always have a non-empty interpretation.
- An unsatisfiable class may have a non-empty interpretation in some model.
- An unsatisfiable class will be a subclass of any other class.

Question 6. Interpretation as Graph

Consider the following graph G:



and the following set of concepts S_C:

- $\neg A$
- $\exists r.(A \sqcup B)$
- $\exists s.\exists s.\neg A$
- $\neg A \sqcap \neg B$
- $\forall r.(A \sqcup B)$
- $\leq 1 s.\top$

First, we read the graph G as an interpretation, where the vertices are elements of the domain and the labels of vertices and edges result from the interpretation function.

- List for each of the concepts in S_C which elements of the domain they have as instances.

Question 7. DL Semantics

(1) Consider the interpretation $\mathcal{I}$ defined by:

$$\Delta^{\mathcal{I}} = \{a, b, c, d, e\}$$

$$P^{\mathcal{I}} = \{a, b, d\}$$

$$Q^{\mathcal{I}} = \{d, e\}$$

$$r^{\mathcal{I}} = \{(a, b), (a, d), (d, e)\}$$

Determine the following sets:

- $(Q \sqcap \geq 2r.P)^{\mathcal{I}}$
- $(\forall r.Q)^{\mathcal{I}}$
- $(\neg \exists r.Q)^{\mathcal{I}}$
- $(\forall r.\top \sqcap \exists r^{-}.P)^{\mathcal{I}}$
- $(\exists r^{-}.\bot)^{\mathcal{I}}$

(2) Consider the interpretation $\mathcal{I}$ defined by:

$$\Delta^{\mathcal{I}} = \{1, 2, 3, 4, 5, 6\}$$

$$A^{\mathcal{I}} = \{1, 2\}$$

$$B^{\mathcal{I}} = \{3, 4, 5, 6\}$$

$$r^{\mathcal{I}} = \{(1, 3), (1, 5), (2, 6)\}$$

Determine the following sets:

- $A \sqcap B$
- $\exists r.B$
- $\exists r.(A \sqcap B)$
- $\top$
- $A \sqcap \exists r.B$

Which of the following are true?

- $\mathcal{I} \models A \equiv \exists r.B$
- $\mathcal{I} \models A \sqcap B \sqsubseteq \top$
- $\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$
- $\mathcal{I} \models \top \sqsubseteq B$
- $\mathcal{I} \models B \sqsubseteq \exists r.A$

Question 8. DL Semantics
Which of the following statements hold:

- if $C \sqsubseteq D$ holds, then $\exists r.C \sqsubseteq \exists r.D$ holds.
- $\exists r.C$ is equivalent to $\leq 1 r.\top$.
- $\leq 0 r.\top$ is equivalent to $\forall r.\bot$.
- $\forall r.(A \sqcup B)$ is equivalent to $(\forall r.A) \sqcup (\forall r.B)$.
- $\exists r.(A \sqcup B)$ is equivalent to $(\exists r.A) \sqcup (\exists r.B)$.

Justify your answers.

Question 9. DL Semantics
Let $\mathcal{T} = \{\mathsf{Parent} \sqsubseteq \exists \mathsf{hasChild}.\mathsf{Person}, \mathsf{Mother} \sqsubseteq \mathsf{Parent}\}$. Show that $\mathcal{T} \not\models \mathsf{Parent} \sqsubseteq \mathsf{Mother}$ by giving an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \not\models \mathsf{Parent} \sqsubseteq \mathsf{Mother}$.

Question 10. DL Semantics
Let $\mathcal{T}$ be an $\mathcal{ALC}$ TBox, which is a finite set of concept inclusions. Let X and Y be complex $\mathcal{ALC}$ concepts (note that a complex concept can also be an atomic concept). Show that:

- $X \sqsubseteq_{\mathcal{T}} Y$ if and only if $X \sqcap \neg Y$ is not satisfiable with respect to $\mathcal{T}$.
- X is satisfiable with respect to $\mathcal{T}$ if and only if $X \not\sqsubseteq \bot$.

Question 11. Bisimulation
Interpretations of ALC can be represented as graphs, with edges labelled by roles and nodes labelled by sets of concept names. More precisely, in such a graph:

> each node corresponds to an element in the domain of the interpretation and it is labelled with all the concept names to which this element belongs in the interpretation;
>
> an edge with label r between two nodes says that the correspondsing two elements of the interpretation are related by the role r.

Definition 1 (Bisimulation) *Let $\mathcal{I}_1$ and $\mathcal{I}_2$ be interpretations. The relation $\bigotimes \subseteq \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_2}$ is a* bisimulation *between $\mathcal{I}_1$ and $\mathcal{I}_2$ if:*

(i) $d_1 \bigotimes d_2$ *implies* $d_1 \in A^{\mathcal{I}_1}$ iff $d_2 \in A^{\mathcal{I}_2}$, *for any* $d_1 \in \Delta^{\mathcal{I}_1}$, $d_2 \in \Delta^{\mathcal{I}_2}$, *and A any concept name;*

(ii) $d_1 \bigotimes d_2$ *and* $(d_1, d_1') \in r^{\mathcal{I}_1}$ *implies the existence of* $d_2' \in \Delta^{\mathcal{I}_2}$ *such that* $d_1' \bigotimes d_2'$ and $(d_2, d_2') \in r^{\mathcal{I}_2}$, *for any* $d_1, d_1' \in \Delta^{\mathcal{I}_1}$, $d_2 \in \Delta^{\mathcal{I}_2}$, *and r any role name;*

(iii) $d_1 \bigotimes d_2$ *and* $(d_2, d_2') \in r^{\mathcal{I}_2}$ *implies the existence of* $d_1' \in \Delta^{\mathcal{I}_1}$ *such that* $d_1' \bigotimes d_2'$ and $(d_1, d_1') \in r^{\mathcal{I}_1}$, *for any* $d_1 \in \Delta^{\mathcal{I}_1}$, $d_2, d_2' \in \Delta^{\mathcal{I}_2}$, *and r any role name;*

Given $d_1 \in \Delta^{\mathcal{I}_1}$ and $d_2 \in \Delta^{\mathcal{I}_2}$, we define $(\mathcal{I}_1, d_1) \sim (\mathcal{I}_2, d_2)$ if there is a bisimulation $\bigotimes$ between $\mathcal{I}_1$ and $\mathcal{I}_2$ such that $d_1 \bigotimes d_2$, and say that $d_1 \in \mathcal{I}_1$ is bisimilar *to $d_2 \in \mathcal{I}_2$.*

Intuitively, d_1 and d_2 are bisimilar if (i) they belong to the same concept name and (ii) for each role name r, they have bisimilar r-successors.[1].

[1] An element q is called an r-successor of an element p if the two elements are related via the relation r such that $r(p, q)$

- Show that: if $(\mathcal{I}_1, d_1) \sim (\mathcal{I}_2, d_2)$, then the following holds for all $\mathcal{ALC}$ concepts C: $d_1 \in C^{\mathcal{I}_1}$ if and only if $d_2 \in C^{\mathcal{I}_2}$ (complete the proof in the lecture slides).
- Show that: $\mathcal{ALCQ}$ is more expressive than $\mathcal{ALC}$, i.e., there is an $\mathcal{ALCQ}$ concept C such that $C \not\equiv D$ holds for all $\mathcal{ALC}$ concepts D.
- Show that: $\mathcal{S}$ is more expressive than $\mathcal{ALC}$.

Question 12. Make the acquaintance of Protégé
Choose from the File menu "Open from URL" to load the pizza ontology into Protégé `https://protege.stanford.edu/ontologies/pizza/pizza.owl`. Activate the view "Ontology metric", which is accessible from the menu "Window" → "Views" → "Ontology view" → "Ontology metrics".

- What is the axiom count and what is the logical axiom count? Why does it differ? (To find out details of the pizza ontology, use the tabs for "Entities", "Classes", and "'Object properties').
- Find axioms in the pizza ontology that respectively
  - use nominals,
  - use negations,
  - declare a sub-property of an object property, and
  - declare an inverse property.
- Start the reasoner from the menu "Reasoner". In the tab "Classes" in the view for "Class hierarchy" the class "Ice Cream" is shown in red. Why?
- What are the inferred superclasses for the classes "CajunSpiceTopping" and "SloppyGiuseppe"?
- Make the object property "hasIngredient" functional. Synchronize the reasoner (in the "Reasoner" menu). How do the reasoning results change?

Question 13. Develop your first ontology with Protégé
Your task in this question is to develop a Beijing 2022 ontology using Protégé, where you will use a Beijing 2022 Olympics Winter Games Schedule (a PDF file) alongside the information at `https://olympics.com/en/beijing-2022/sports/` as the basis for your ontology's content. In the first instance you should model the schedule. There are several sub-tasks you may have to do in developing your ontology:

- Produce some competency questions that will guide the ontology's content, design and evaluation; what should the ontology be "competent" to answer?
- Identify the relevant "terms" (different types of sports) from the schedule, and you may want to categorize these terms somehow (produce a hierarchy of sports items).
- Users may want to know where and when their favorite winter sports take place. In this case, what additional information should be included in your ontology? Should they be included as part of the ontology content, or as meta-information, for example, as comments? Also, users may want to know the profile of the medalists of some sports, what will you do in this case to fulfill such requirements? You do not need to answer these questions; instead, reflect your inclination in your ontology.