

Homework 5: Commitment Schemes

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1. The Setup phase of the KZG polynomial commitment scheme involves computing commitments to powers of a secret evaluation point τ . This is called the “trusted setup” and is often generated in a multi-party computation known as the “Powers of Tau” ceremony. One day, you find the value of τ on a slip of paper. How can you use it to make a fake KZG opening proof?
2. Construct a **vector commitment scheme** from the KZG polynomial commitment scheme. (Hint: For a vector $m = (m_1, \dots, m_q)$, is there an “interpolation polynomial” $I(X)$ such that $I(i) = m[i]$?)

Fun fact: *The Verkle tree [1] is a Merkle tree that uses a **vector commitment** instead of a hash function. Using the KZG vector commitment scheme, can you see why a Verkle tree is more efficient?*

3. The KZG polynomial commitment scheme makes an opening proof π for the relation $p(x) = y$. Can you extend the scheme to produce a multiproof π , that convinces us of $p(x_i) = y_i$ for a list of points and evaluations (x_i, y_i) ? (Hint: assume that you have an interpolation polynomial $I(X)$ such that $I(x_i) = y_i$.)

References

- [1] J. Kuszmaul. Verkle trees. <https://math.mit.edu/research/highschool/primes/materials/2018/Kuszmaul.pdf>, 2019.