Lattice Reduction Techniques To Attack RSA

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RSA ?

(e, N) is the **public key**, (d, N) is the **private key**.

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$$c = m^e \pmod{N}$$

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And to **decrypt** :

$$m = c^d \pmod{N}$$

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Use p and q to generate the pair **private key/public key** (d, e).


ATTACKS ?

On the implementation or on the mathematics.

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Model :

- Recover the plaintext $m^e = c \pmod{N}$
- Recover the private key d

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Relaxed model :

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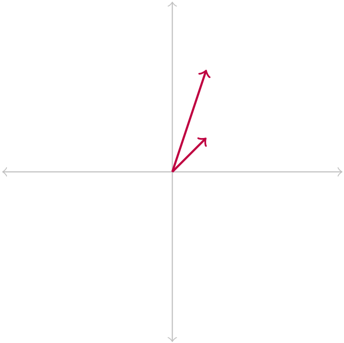
- Recover the private key d

Relaxed model :

- We know a part of the message
- We know an approximation of one of the prime
- The private exponent is too small

LATTICE ?

A bit like a **vector space**.



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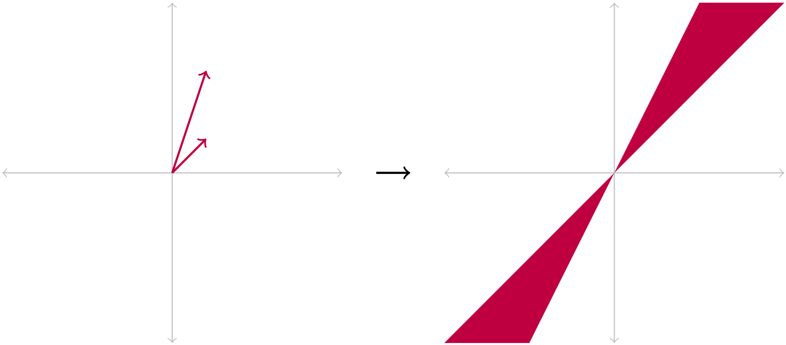
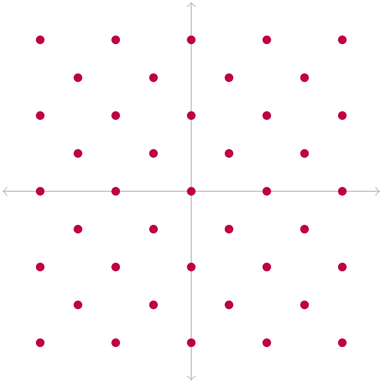
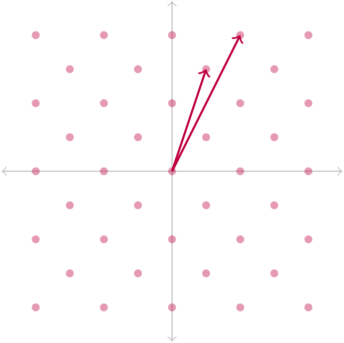




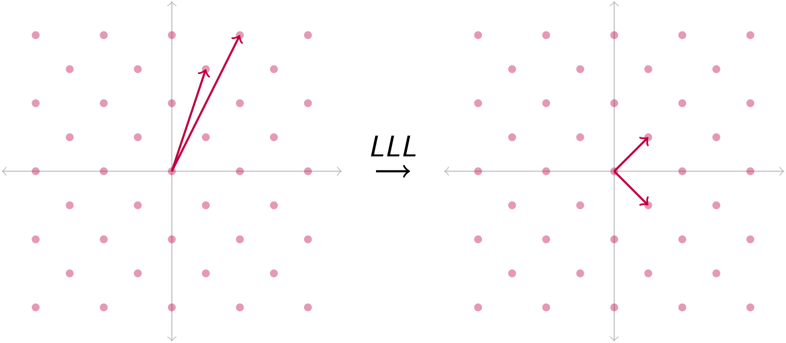
LLL, a lattice basis reduction algorithm

random basis



LLL, a lattice basis reduction algorithm

random basis **reduced basis**



$$B = \begin{pmatrix} \vec{b_1} \\ \vdots \\ \vec{b_n} \end{pmatrix} \xrightarrow{\quad\textbf{LLL}\quad} B' = \begin{pmatrix} \vec{b'_1} \\ \vdots \\ \vec{b'_n} \end{pmatrix}$$

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$$\|b'_1\| \leq \|b'_2\| \leq \ldots \leq \|b'_i\| \leq 2^{\frac{n(n-1)}{4(n+1-i)}} \cdot det(L)^{\frac{1}{n+1-i}}$$

COPPERSMITH ?

BONEH-DURFEE ?