

Page 4

In math form, old A3C objective could be written as  $J_\theta = \mathbb{E}_t[\nabla_\theta \log \pi_\theta(a_t|s_t)A_t]$ . The new objective proposed by the PPO is  $J_\theta = \mathbb{E}_t[\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)}A_t]$ . But if we just start to blindly maximize this value, it will lead to very large update to the policy weights. To limit the update, the clipped objective is used. If we write the ratio between the new and the old policy as  $r_t(\theta) = \frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)}$ , the clipped objective could be written as this

$$J_\theta^{clip} = \mathbb{E}_t[\min(r_t(\theta)A_t, \text{clip}(r_t(\theta), 1 - \epsilon, 1 + \epsilon)A_t)]$$

This objective limits the ratio between the old and the new policy to be in the interval  $[1 - \epsilon, 1 + \epsilon]$ , so, by varying  $\epsilon$  we can limit the size of the update.

Another difference from the A3C method, is the way we estimate the advantage. In the A3C paper, the advantage obtained from the finite-horizon estimation of T steps in the form:

$$A_t = -V(s_t) + r_t + \gamma r_{t+1} + \dots + \gamma^{T-t+1}r_{T-1} + \gamma^{T-t}V(s_T)$$

In the PPO paper, the authors used more general estimation in the form of

$$A_t = \sigma_t + (\gamma\lambda)\sigma_{t+1} + (\gamma\lambda)^2\sigma_{t+2} + \dots + (\gamma\lambda)^{T-t+1}\sigma_{T-1}$$

where  $\sigma_t = r_t + \gamma V(s_{t+1}) - V(s_t)$ . The original A3C estimation, is a special case of the proposed method with  $\lambda = 1$ .

Page 9

As the first step, TRPO method defines the discounted visitation frequencies of the state:  $\rho_\pi(s) = P(s_0 = s) + \gamma P(s_1 = s) + \gamma^2 P(s_2 = s) + \dots$ . In this equation,  $P(s_i = s)$  equals to the sampled probability of state s to be met at position i of the sampled trajectories. Then, TRPO defines the optimisation objective as  $L_\pi(\tilde{\pi}) = \eta(\pi) + \sum_s \rho_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s, a)$ , where  $\eta(\pi) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r(s_t)]$  is the expected discounted reward of the policy and  $\tilde{\pi} = \arg \max_a A_\pi(s, a)$  defines the deterministic policy. To address the issue with large policy updates, TRPO defines the additional constraint on the policy update, expressed as a maximum Kullback-Leibler divergence between the old and the new policies, which could be written as  $\bar{D}_{KL}^{\rho_{\theta_{old}}}(\theta_{old}, \theta) \leq \delta$ .