01 A. Sets

Notebook:	Discrete Mathematics [CM1020]		
Created:	2019-10-07 2:31 PM	**Updated:**	2019-10-08 4:39 PM
Author:	SUKHJIT MANN		
Tags:	Cardinality, Intersection, Powerset, Set Builder, Set Difference, Set Theory, Sets, Subset, ...		

Cornell Notes	**Topic:** 01 A. Sets	Course: BSc Computer Science Class: Discrete Mathematics-Lecture Date: October 7, 2019

Essential Question:

What are sets (how can we write them and "measure" them and perform operations on them?) and what are "elements" within a set, to that what are sets within sets?

Questions/Cues:

- What is Set Theory?
- Who introduced the concept of sets in math?
- What other branches of Math has Set Theory helped to establish?
- What is a set?
- What is standard set notation?
- What is the symbol for denoting an element of a set?
- What is meant by the cardinality of a set?
- What is a subset of a set?
- What are the special sets?
- What are the different ways to represent a set?
- Can sets be elements of another set?
- What is a powerset of a set?
- What is the cardinality of a powerset?
- What is the union of sets?
- What is the intersection of two sets?
- What is set difference?
- What is symmetric difference?

Notes

1. Set Theory, mathematics that deal with properties of well-defined collection of objects
2. Sets introduced by George Cantor, German Mathematician.
3. Basis of other fields like counting theory, relations, graph theory and finite state machines.
4. Set is well-defined **collection** of an **kind** of **objects**.
    - **Unordered** collection of **unique** objects.

    - No duplicate elements allowed
5. Set defined as ie. **E = {2,4,6,8}**; curly braces to show elements of set and capital letter to denote Set.
6. $\in$ is "element of" set symbol
7. $\emptyset$ = empty set
    - $\emptyset \subseteq \emptyset$
    - $\emptyset \subseteq$ **A**
8. Given set S, Cardinality of S, # of elements contained in S. |S| = cardinality of S.
9. A only subset of B if and only if all ele A also ele B, written as A$\subseteq$ B
    - Any set is a subset of itself A$\subseteq$ A
    - {{x}} = element of a set
10. Special Sets:

- N = natural numbers {1,2,3,4}
- Z = integers {-3,-2,-1,0,1,2,3}
- Q = rational numbers (form a/b, a and b $\in$ Z, b $\neq$ 0)
- N $\subseteq$ Z $\subseteq$ Q $\subseteq$ R

Description method = word explanation of set ie. S1 = set of all vowels in the eng alphabet
Listing method = listing all elements of set.
Set builder notation = ie. Even = {2n | n $\in$ Z}
| = such that

- elements of a set can be sets themselves ie. {1,2,3,4} $\subseteq$ A but {1,2,3,4} $\in$ B
    - A = {1,2,3,4,5,6,7,8,9} B = { {12,3,4}, {5,6}, {7,8,9} }
        - {1,2,3,4} $\subseteq$ A but {1,2,3,4} $\in$ B
- Given S, powerset of S, **P(S)** = set with all subsets of S inside
- Given S, | **P(S)** | = $2^{|S|}$
        - $P(\emptyset) = \{\emptyset\}$
        - $\emptyset \subseteq P(\emptyset)$
        - $\{\emptyset\} \subseteq P(\emptyset)$
        - $P(P(\emptyset)) = \{\emptyset, \{\emptyset\}\}$
        - Given A, if | A | = n, find |P(P(P(A)))|
            - $|P(A)| = 2^{|A|} = 2^n$

              $|P(P(A))| = 2^{|P(A)|} = 2^{2^n}$

              $|P(P(P(A)))| = 2^{|P(P(A))|} = 2^{2^{2^n}}$
- Given sets A and B, union of A and B, $A \cup B$, contains all elem. in **EITHER** A **or** B. $A \cup B = \{x \mid x \in A \, or \, x \in B\}$
- Given sets A and B, intersection of A and B, $A \cap B$, contains all elem. in **BOTH** A **and** B. $A \cap B = \{x \mid x \in A \, and \, x \in B\}$
- Given sets A and B, set difference $A - B$, contains the elem. that in A **but not in** B. $A - B = \{x \mid x \in A \, and \, x \notin B\}$
- Given sets A and B, symmetric difference $A \oplus B$, contains the elem. in A **or in** B **but not in** BOTH. $A \oplus B = \{x \mid (x \in A \, or \, x \in B \, and \, x \notin A \cap B\}$

A	B	AUB	A∩B	A - B	A⊕B
0	0	0	0	0	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	1	0	0

Summary

In this week, we've defined what set theory is and what sets are. Alongside this, we've looked at what makes up a set, how to count the elements of a set and most importantly how to denote a set in various way such set builder notation. Furthermore, we've looked at how sets can be nested other sets, special sets used in mathematics, powersets or set of all subsets within a set and what kinds of operation we can perform on sets.