8.1 Introduction to trees: basic concepts

Notebook: Discrete Mathematics [CM1020]

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Cornell Notes

Topic:

8.1 Introduction to trees: basic concepts

Course: BSc Computer Science

Class: Discrete Mathematics-

Lecture

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Essential Question:

What is a tree?

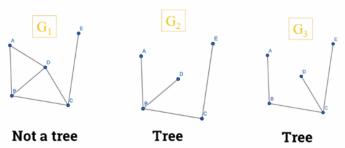
Questions/Cues:

- What is acyclic graph?
- What is a tree?
- What is a forest?
- What are some properties of trees?
- What is a rooted tree?
- What is a spanning tree?
- How we construct a spanning tree?
- What are non-isomorphic spanning trees?
- What is cost mean in terms of trees?
- What is a minimum-cost spanning tree?
- What are the two greedy algorithms for finding minimum-cost spanning trees?
- What is Kruskal's Algorithm?
- What is Prim's Algorithm?

Notes

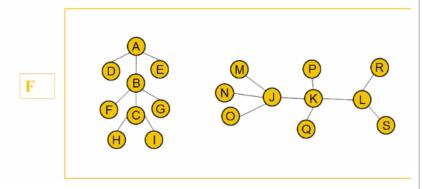
- Acyclic graph = A graph G is an acyclic graph if and only if G has no cycles. This includes no loops and no parallel edges.
- Tree = An undirected graph G is a tree if and only if it is connected and acyclic. This means there exists a path between any two vertices of G & G is cycle-free.

Hence, a tree can have neither loops nor multiple edges (parallel edges)



Definition of a forest

A disconnected graph containing no cycles is called a forest.

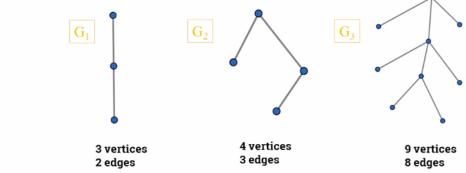


Theorem 1

An undirected graph is a **tree** if and only if there is **unique** simple path between any two of its vertices.

Theorem 2

A tree with n vertices has n-1 edges.



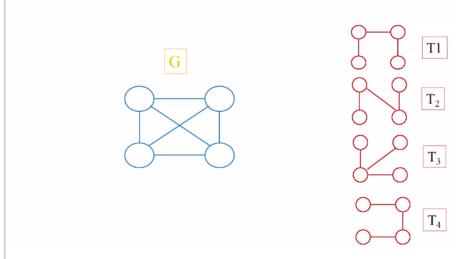
Rooted trees

A rooted tree is when one vertex has been designated as the root and every edge is directed away from the root.

Definition of spanning trees

A spanning tree of a graph G is a connected sub graph of G which contains all vertices of G, but with no cycles.

Example

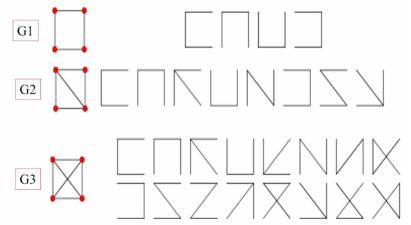


Constructing a spanning tree

To get a spanning tree of a graph G

- Keep all vertices of G
- 2. Break all the cycles but keep the tree connected.

Example of spanning trees



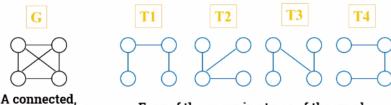
Non-isomorphic spanning trees

Two spanning trees are said isomorphic if there is a bijection preserving adjacency between the two trees.

***NOTE if we asked to draw all the spanning trees of a graph, we are only interested in drawing the non-isomorphic ones

Example

undirected graph



Four of the spanning trees of the graph

 T_1 , T_3 and T_4 are all isomorphic to each others

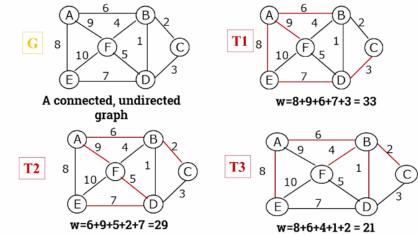
 T_1 , T_3 and T_4 are all non-isomorphic to T_2 .

Spanning trees cost

Suppose you have a connected undirected graph with a weight (or cost) associated with each edge.

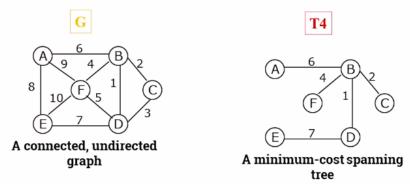
The cost of a spanning tree would be the sum of the costs of its edges.

The weight of spanning trees



Minimum-cost spanning trees

A minimum-cost spanning tree is a spanning tree that has the lowest weight (lowest cost).



Finding spanning trees

There are two basic algorithms for finding minimumcost spanning trees, and both are greedy algorithms:

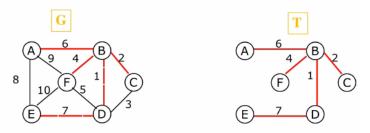
Kruskal's algorithm

Prim's algorithm

Kruskal's Algorithm

- 1. Start with the cheapest edge in the spanning tree.
- 2. Repeatedly add the cheapest edge that keeps the tree connected but free from any cycles.

Kruskal's algorithm



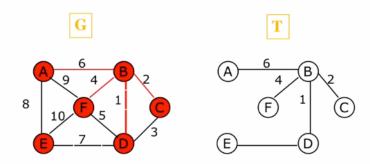
A connected, undirected graph

A minimum-cost spanning tree

Prim's Algorithm

- 1. Start with any one node in the spanning tree.
- 2. Repeatedly add the cheapest edge incident to that node and the node it leads to, for which the node is not already in the spanning tree.

Prim's algorithm



A connected, undirected graph

A minimum-cost spanning tree

Summary

In this week, we learned what tree is, what a forest is, what a rooted tree is? Alongside this, we explored some properties of trees, what a spanning tree and minimum-cost spanning trees are and the two greedy algorithms for finding minimum cost spanning trees; Kruskal's and Prim's Algorithm.