

2.1 Introduction to Functions

Notebook: Discrete Mathematics [CM1020]

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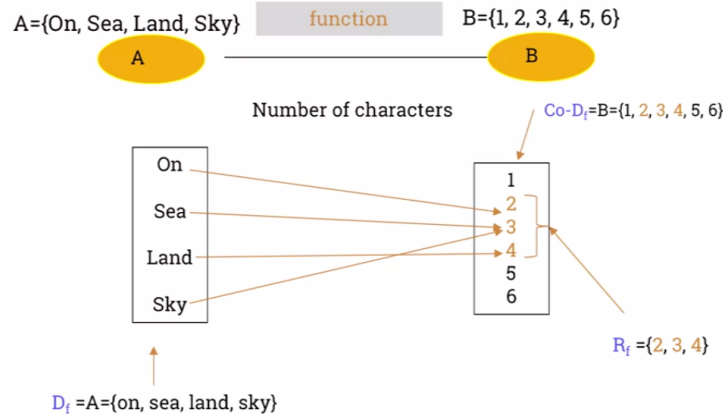
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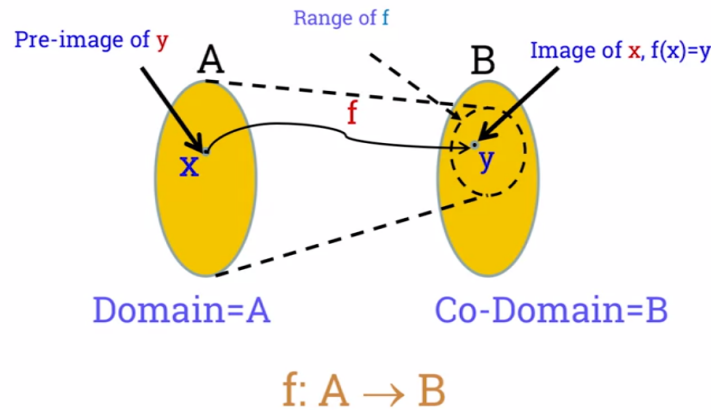
Tags: Domain, Exponential, Injective, Linear, Quadratic, Range, Surjective

Cornell Notes	Topic: 2.1 Introduction to functions	Course: BSc Computer Science
		Class: Discrete Mathematics- Lecture
		Date: October 21, 2019
Essential Question:		
What is a function and what are its properties and how do we graph it?		
Questions/Cues:		
<ul style="list-style-type: none">• What is a function?• What is the formal definition of a function?• What is the domain, co-domain and range of a function?• What is a linear function?• What is a Quadratic function?• What is an Exponential function?• What is an Injective function?• What is an Surjective function?		
Notes		
<ul style="list-style-type: none">• Function = A rule that relates how one quantity depends on other quantities, a relation between set of inputs and set of outputs, in which each input maps to exactly one output<ul style="list-style-type: none">◦ function maps an element of set to an element in another set• Formal Def of Function = function f from set A to set B is an assignment of exactly 1 element of B to each element of A<ul style="list-style-type: none">◦ $f : A \rightarrow B$◦ $x \in A : x \rightarrow f(x) = y (y \in B)$• $x \in A \rightarrow f(x) = y \in B ; f : A \rightarrow B$<ol style="list-style-type: none">1. A is set of inputs called domain of f, $D_f = A$2. B is set of containing outputs called co-domain of f, $co-D_f = B$3. set of all outputs called range of f, R_f4. Y called image of x, x called pre-image of y, $f(x) = y$		

Domain, co-domain and range



Visualization



- Linear function = of the form $f(x) = ax + b$
 - straight line function
 - passes through point $(0, b)$
 - a is gradient or slope
 - if a is positive, increasing function, $x_1 \leq x_2$ then $f(x_1) \leq f(x_2)$
 - if a is negative, then decreasing function, $x_1 \leq x_2$ then $f(x_1) \geq f(x_2)$
- Quadratic function = of the form $f(x) = ax^2 + bx + c$
 - where a, b, c are number with $a \neq 0$
- Exponential function = of the form $f(x) = b^x$ where $b > 0$ and $b \neq 1$
 - variable b called the base of function
 - if $b > 1$, then exponential growth
 - if $0 < b < 1$, then exponential decay
 - $(0, 1)$ is common point for both graphs and x -axis is the horizontal asymptote for both graphs
 - Domain is set of all real numbers and range is > 0

$$b^x b^y = b^{x+y}$$

$$b^{-x} = \frac{1}{b^x}$$

$$\frac{b^x}{b^y} = b^{x-y}$$

$$(b^x)^y = b^{xy}$$

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

- Injective function (one to one) = any two different inputs lead to two different outputs
 - *for all $a, b \in A$, if $a \neq b$ then $f(a) \neq f(b)$*
 - Equivalently, *for all $a, b \in A$, if $f(a) = f(b)$ then $a = b$*
- Surjective function (onto) = every element of the co-domain of f , B has at least one pre-image in domain of f , A
 - *for all $y \in B$ there exists $x \in A$ such that $y = f(x)$*
 - Equivalently, $D_f = R_f$

Summary

In this week, we learned what is function is, with what the domain, co-domain, and range of a function represent. Also we explored the equations and graphs of three common function, Linear, Quadratic and Exponential. Lastly, we looked at the injective and surjective functions.