UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics – Sample Exam Paper

Now

Time allowed: 2 hours

DO NOT TURN OVER UNTIL TOLD TO BEGIN

INSTRUCTIONS TO CANDIDATES:

This examination paper is in two parts: Part A and Part B. You should answer **ALL** of question 1 in Part A and **TWO** questions from Part B. Part A carries 40 marks, and each question from Part B carries 30 marks. If you answer more than **TWO** questions from **Part B** only your first **TWO** answers will be marked.

All answers must be written in the answer books; answers written on the question paper will not be marked. You may write notes in your answer book. Any notes or additional answers in the answer book(s) should be crossed out.

The marks for each part of a question are indicated at the end of the part in [.] brackets. There are 100 marks available on this paper.

Calculators are not permitted in this examination.

A handheld calculator may not be used when answering questions on this paper.

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PART A

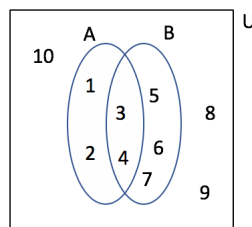Candidates should answer **ALL** of Question 1 in Part A.

Question 1

(a) Let A be a set with 5 elements. What is the number of subsets which can be formed from A?

Choose ONE option. [4]

    i. 10

    ii. 25

    iii. 32

    iv. 64

(b) Given the following Venn diagram representing two sets A and B, subsets of the universal set U:
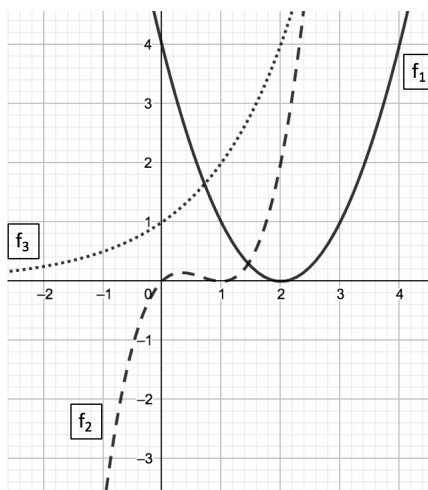

Which one of the following sets represents $(\overline{A \cap B})$?

Choose ONE option. [4]

    i. $\{1, 2, 5, 6, 7, 8, 9, 10\}$

    ii. $\{3, 4, 8, 9, 10\}$

    iii. $\{8, 9, 10\}$

    iv. none of the other options is correct

(c) The following graph shows the curves of three functions, f_1, f_2 and f_3:

Which of these functions is/are not invertible?

Choose ONE option. [4]

  i. f_3

 ii. f_1 and f_3

iii. f_2 and f_3

iv. f_1 and f_2

(d) Let p and q be the following propositions concerning a positive integer n where p means 'n has is less than or equal to 7 ' and q means 'n cannot be represented using 3 binary binary digits' Which one of the following logical expressions is equivalent to a correct formalisation of the sentence below?

' if n is less than or equal to 7 then n can be represented using 3 binary digits'

Choose ONE option. [4]

   i. $p \wedge \neg q$

  ii. $p \vee \neg q$

  iii. $p \rightarrow \neg q$

  iv. $p \rightarrow q$

(e) Given the statement $S(x)$: ' $x^2 + 1 = 5$', select the right statement from the following.
Choose ONE option. [4]

   i. S can be expressed using propositional logic

  ii. S is not a proposition, as its truth value is a function depending on x

  iii. The truth value of $S(2)$ is False

  iv. The truth value of $S(3)$ is True

(f) The number of ways k objects that can be selected from n objects where the ordering of the outputs is not important can be calculated using which one of the following formulations?
Choose ONE option. [4]

   i. $\frac{n!}{k!(n-k)!}$

  ii. $\frac{n!}{(n-k)!}$

  iii. $\frac{n!}{k!}$

  iv. $n(n-k)$

(g) Which one of the following degree sequences cannot represent a simple graph? [4]

    i. $5, 3, 3, 2, 2$

    ii. $4, 2, 2, 2, 2$

    iii. $2, 2, 2, 2, 2$

    iv. $4, 3, 3, 2, 2$

(h) Which one the following is a correct definition of a Hamiltonian path? [4]

    i. A Hamiltonian path in a graph G is a path that uses each edge in G precisely once

    ii. A Hamiltonian path in graph G is a path that visits each vertex in G exactly once

    iii. A Hamiltonian path path is a trail in which neither vertices nor edges are repeated

    iv. A Hamiltonian path is a walk in which no edge is repeated

(i) Let $S = \{1, 2, 3\}$ and $\mathcal{R}$ be a relation on elements in S with

$\mathcal{R} = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 1), (3, 3)\}$

Which one of the following statements is correct about the relation $\mathcal{R}$?

[4]

    i. $\mathcal{R}$ is reflexive, is symmetric and is transitive

    ii. $\mathcal{R}$ is NOT reflexive, is symmetric and is transitive

    iii. $\mathcal{R}$ is reflexive, is symmetric and is NOT transitive

    iv. $\mathcal{R}$ is NOT reflexive, NOT symmetric and NOT transitive

(j) Let $S = \{a, b, c\}$ and $\mathcal{R}$ be a relation on elements in S with

$$\mathcal{R} = \{(c, b), (a, a), (b, c)\}$$

Which one of the following statements is correct about the relation $\mathcal{R}$?

[4]

    i. $\mathcal{R}$ is NOT reflexive, is NOT symmetric, and is NOT transitive.

    ii. $\mathcal{R}$ is NOT reflexive, is NOT symmetric, and is transitive.

    iii. $\mathcal{R}$ is NOT reflexive, is symmetric, and is NOT transitive.

    iv. $\mathcal{R}$ is reflexive, is symmetric, and is transitive.

PART B

Candidates should answer any **TWO** questions from Part B.

Question 2   Set theory, Propositional and Predicate logic & Combinatorics

(a)   i. Rewrite the following three sets using the listing method:
- $A = \{n^2 + (-1)^n : n \in \mathbb{Z} \text{ and } 0 \leq n < 5\}$
- $B = \{n + \frac{1}{n} : n \in \mathbb{Z}^+ \text{ and } n < 6\}$
- $C = \{(-2)^{-n} : n \in \mathbb{Z}^+ \text{ and } n < 5\}$

[3]

ii. Given the following Venn diagram representing three sets A, B and C intersecting in the most general way. Three binary digits are used to refer to each one of the 8 region in this diagram. In terms of A, B and C, Write the set representing the area comprising the regions 011, 101 and 111. The answer must be written in its simplest
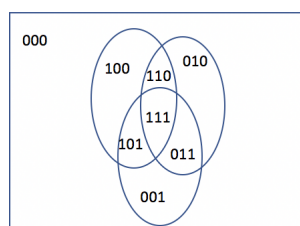
[3]

iii. Given three sets A, B and C. Using set identities, prove that the expression $(A - B) - (B - C)$ is equivalent to $A - B$. [4]

(b)   i. Let p and q be two propositions. Assume that p is false and q is is true. Determine the truth value of each of the following: $p \rightarrow q$; $\neg p \vee q$; $q \oplus \neg p$ and $\neg q \rightarrow p$.

[4]

ii. Let p, q and r denote the following statements:

$p :$ 'I finish my home work'

$q :$ 'I will go to the gym'

$r :$ ' it's raining '

1. Write the following the following statements into their corresponding symbolic forms.
- if i finish my homework and it is raining then i will go to the gym.

- i will go to the gym or finish my homework but not both.

[4]

2. Write in words the contrapositive of the the following statement: 'if i finish my homework and it is raining then i will go to the gym.' [2]

(c) Let A be the set of all students, $p(x)$ be the proposition: 'x is enrolled on the Discrete Mathematics module' and $S(x)$ is the set of all students in the same year as x' where x is an element of A. Use rules of inference with quantifiers to formalise the three following statements:

i. None of the students is enrolled in Discrete Mathematics module. [2]

ii. Not every student is enrolled in Discrete Matheamtics module.

[2]

iii. There exists a student, none of whose classmates play football.
"Classmates of a student x" means 'students in the same class as x'

[2]

(d) Suppose that a sales person has to visit eight different cities including London. They must begin their trip in London, but can visit the other seven cities in any order they wish.

How many possible orders can the sales person use when visiting these cities?

[4]

Question 3   Boolean Algebra, Functions & Recursion

(a) Given the following logical circuit with three inputs a, b and c:
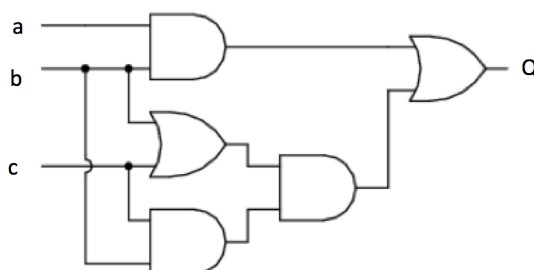
   i. Identify the logical gates used in this circuit.   [2]

   ii. What is the the logical expression of the output of this circuit?   [4]

   iii. Simplify the logical expression in (ii). Explain your answer.   [4]

   iv. Draw the resulting simplified circuit.   [2]

(b) i. Name two properties a function has to satisfy to be an invertible function.   [2]

   ii. Show that the function $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = 3x - 5$ is a bijection.   [2]

   iii. Consider the following function invertible $f : \mathbb{R} \to \mathbb{R}^+$ with $f(x) = 2^{x+1}$.
     Find its inverse function, f^{-1}.   [2]

   iv. . Plot the curve of f and f^{-1} in the graph.   [4]

(c) i. Consider the recursive relation:
     $f_n = n * f_{n-1}$ for all integer $n \geq 1$ with $f_1 = 1$.
     What are the values of f_2 and f_3?   [2]

   ii. Let $S_n = \sum_{i=1}^{i=n} (4i - 1)$ for all $n \in \mathbb{Z}^+$.

    1. Find S_1 and S_2.

                     [1]

    2. Prove by induction that $S_n = 2n^2 + n$ for all $n \in \mathbb{Z}^+$.

                     [5]

Question 4   Graph Theory, Trees & Relations

(a) i. Give a definition of a simple graph.   [2]

   ii. Is it possible to draw a simple graph with a degree sequence, 4, 3, 3, 2? If yes draw the graph and if no, explain why.   [2]

  iii. Draw the two graphs with adjacency lists

- $a_1 : a_2, a_5$
- $a_2 : a_1, a_3, a_4, a_5$
- $a_3 : a_2, a_4, a_5$
- $a_4 : a_2, a_3, a_5$
- $a_5 : a_1, a_2, a_3, a_4$

and

- $b_1 : b_2, b_3, b_4, b_5$
- $b_2 : b_1, b_5$
- $b_3 : b_1, b_4, b_5$
- $b_4 : b_1, b_3, b_5$
- $b_5 : b_1, b_2, b_3, b_4$

                                                                                  [2]

   1.   Write down the degree sequence for each graph above.   [2]

    2.   Are these graphs isomorphic? If so, show the correspondence between them.   [2]

(b) i. What is the number of vertices in a tree with n edges?   [2]

  ii. Explain how to find the minimum spanning tree in a weighted graph using Prim's algorithm.   [2]

  iii. A television company are setting up a new cable television network. Their system has a base station in the local business park (P) and 6 hubs (A - F) which then serve the local areas. The company wants to find the cheapest way to connect all hubs. The following graph shows the cost of each connection (in the unit $£10000$). Which cables should they lay and how much will it cost them.   [6]
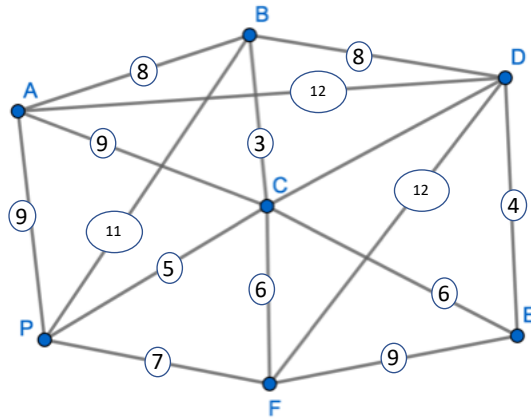
(c) Let S be the set of integers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let $\mathcal{R}$ be a relation defined on S by the following condition such that,
for all $x, y \in S$, xRy if $x \mod 3 = y \mod 3$.

    i. Draw the digraph of $\mathcal{R}$.   [2]

   ii. Say with reason whether or not $\mathcal{R}$ is

- reflexive;
- symmetric;
- anti-symmetric;
- transitive.

In the cases where the given property does not hold provide a counter example to justify this.   [4]

  iii. is $\mathcal{R}$ a partial order? Explain your answer.   [1]

  iv. is $\mathcal{R}$ an equivalence relation? If the answer is yes, write down the equivalence classes for this relation and if the answer is no, explain why.   [3]

END OF PAPER

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