THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALL

UNIVERSITY OF LONDON

**CM1015**

**BSc EXAMINATION**

**COMPUTER SCIENCE**

**Numerical Mathematics**

Tuesday 3 September 2019 : 10.00 – 12.00

Time allowed: 2 hours

**DO NOT TURN OVER UNTIL TOLD TO BEGIN**

**INSTRUCTIONS TO CANDIDATES:**

This examination paper is in two parts: Part A and Part B. You should answer **ALL** of question 1 in Part A and **TWO** questions from Part B. Part A carries 40 marks, and each question from Part B carries 30 marks. If you answer more than **TWO** questions from **Part B** only your first **TWO** answers will be marked.

All answers must be written in the answer books, answers written on the question paper will not be marked. You may write notes in your answer book. Any notes or additional answers in the answer book(s) should be crossed out.

The marks for each part of a question are indicated at the end of the part in [.] brackets. There are 100 marks available on this paper.

Graph Paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

A handheld calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

**PART A**

Candidates should answer **ALL** of Question 1 in Part A.

**Question 1**

(a) Which of the following values are in the interval $[1, 15)$?

Select ALL that apply. [4]

a) 2

b) 15

c) 0

d) 1

(b) Find the $n^{th}$ term of the sequence pattern: $20, 17, 14, 8, 5 \cdots$, noting that the first term in the sequence has index $n = 1$.

Choose ONE option. [4]

a) $23 - 3n$

b) $23 - n - 3$

c) $3n + 23$

d) $3n - 23$

e) none of the other options is correct

(c) Calculate $\log_2 16$.

Select ALL correct statements. [4]

a) $\log_2 x = x^2$

b) $\log_2 16 = 4$

c) $\log_2 16 = 8$

d) $16 = 2^4$

e) $16 = 2^3$

f) $2 = 16^{-4}$

g) none of the other options is correct

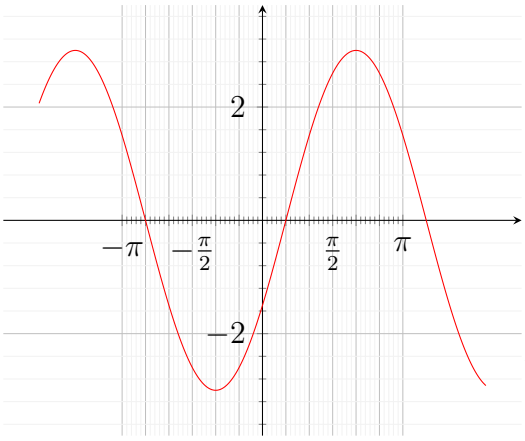
(d) The graph below is a transformation of one of the functions

$y = \sin(x)$,

$y = \cos(x)$ or

$y = \tan(x)$,

with $x$ in radians.




Determine the original function and the transformation. Select ALL true statements that describe the original function and a consistent transformation.

[4]

a) The graph shown is a transformation of the graph of $y = \cos(x)$

b) The graph shown is a transformation of the graph of $y = \tan(x)$

c) The transformation is a scaling in the y-direction by a factor of 3 followed by a translation in the x-direction by $-\frac{2\pi}{2}$

d) The transformation is a scaling in the y-direction by a factor of 3 followed by a translation in the x-direction by $-\frac{2\pi}{3}$

e) The transformation is a scaling in the y-direction by a factor of 2 followed by a translation in the x-direction by $-2\frac{\pi}{3}$

f) The function in the graph has a period of $\pi$

g) None of the other options is correct.

(e) A window cleaner needs to use a ladder to reach a window. The ladder must reach the bottom of the window, which is 3m above the ground. For safety reasons, the ladder must be between the angle of 60 and 75 degrees from the ground. What are the largest and smallest lengths the ladder can be?
Choose ONE option. [4]

a) 3.464101615 and 3.105828541

b) 0.803847577 and 1.732050808

c) 1.5 and 1.552914271

d) 3.212562 and 3.5542167

(f) Differentiate $y = x^3$ with respect to $x$.
Choose ONE option. [4]

a) $\frac{dy}{dx} = x^{3-1}$

b) $\frac{dy}{dx} = \frac{1}{3}x^2$

c) $\frac{dy}{dx} = \frac{1}{2}x^2$

d) $\frac{dy}{dx} = 3x^3$

e) None of the above options is correct.

(g) What is the number 247 in base 10 converted to binary?
Choose ONE option. [4]

a) 010100111

b) 111100010

c) 11110111

d) 11101111

(h) Calculate the inverse of the matrix:

$$\begin{pmatrix} 9 & 4 \\ 2 & 2 \end{pmatrix}$$

Choose ONE option. [4]

a) 0.1

b)
$$\begin{pmatrix} 2 & 2 \\ 4 & 9 \end{pmatrix}$$

c)
$$\begin{pmatrix} 0.2 & -0.4 \\ -0.2 & 0.9 \end{pmatrix}$$

d)
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(i) You have a standard six sided dice with the values 1 through 6 on the sides. How many ways are there to get a total of 7 with 2 dice rolls?

Choose ONE option. [4]

a) 6

b) 3

c) 7

d) $\frac{7}{12}$

(j) What is the probability of being dealt only 1 queen from a standard, 52 card deck containing 4 queens if you are dealt 3 cards and the cards are not returned to the deck after being dealt? (to 1 decimal place).

Choose ONE option. [4]

a) 6.8%

b) 20.4%

c) 13%

d) 19.3%

**PART B**

Candidates should answer any **TWO** questions from Part B.

**Question 2**

Consider the points $A, B$ and $C$ with coordinates $(2, 5)$, $(1, -1)$ and $(-2, 3)$ respectively. Consider the triangle $[ABC]$.

(a) Using graph paper represent the triangle in the coordinate space. [3]

(b) Consider the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

i. Write the coordinates of the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$. [3]

ii. Find the lengths $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$. Show all your calculations clearly stating any results, shortcuts or techniques used. [3]

iii. Is the triangle $[ABC]$ right-angled? Show your reasoning and calculations. [2]

iv. Compute the dot product between $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

v. Use the dot product, or otherwise, to compute the angle between $AB$ and $AC$ in degrees and in radians. Show all your calculations clearly stating any results, shortcuts or techniques used. [5]

(c) Convert the vector $(6.3, \frac{\pi}{3}$ rad$)$ in polar coordinates to cartesian coordinates. [3]

(d) You are given vectors as follows:

$$u = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} \text{ and } v = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}.$$

Compute $u \times v$ the cross product (vector product) of vectors $u$ and $v$. [3]

(e) Write the following in the form $a^b$, simplifying as necessary. If more than one solution is possible, choose the one where $a$ is a positive integer and as small as possible. [3]

i. $\frac{3^{-2}}{3^3} \cdot 27$

ii. $\frac{\sqrt{3}}{3}$

iii. $(2^{-6} \cdot 16)^{-2}$

(f) Compute the following, showing your working. [3]

i. $\log_{16}\left(\frac{1}{4}\right)$

ii. $\log_2(256)$

iii. $\log_2(-1)$

**Question 3**

(a) For the function $f(x) = 3\sin(x + \frac{\pi}{2})$, with $x$ in radians, state the: [3]

i. period

ii. amplitude

iii. phase when $x = 2$

(b) Prepare a table of values to plot the graph of $f(x) = 3\sin(x + \frac{\pi}{2})$, with $x$ in radians between $-\pi$ rad and $\pi$ rad. Your table should contain FIVE values for $x$. [3]

(c) Using graph paper, plot the graph of $f(x) = 3\sin(x + \frac{\pi}{2})$, with $x$ in radians between $-\pi$ rad and $\pi$ rad, adding comments if necessary. [3]

(d) Given $y = \frac{1}{x^3}$, compute $\frac{dy}{dx}$. [2]

(e) Given $y = log(3x)$, compute $\frac{dy}{dx}$. [2]

(f) Given $y = \cos(x^2)$, with $x$ in radians, compute $\frac{dy}{dx}$. [3]

(g) Without sketching the graph of $y = \cos(x^3)$, with $x$ in radians, find the maxima and minima of the function. [5]

(h) Consider a particle moving in a straight line with constant acceleration. The acceleration might be positive or negative.

The particle has moved for $15s$ and measurements indicate that the initial velocity was $4m/s$ and the velocity at time equals $15s$ was $3m/s$.

i. Calculate the acceleration of the particle. [2]

ii. Give the equation of the velocity of the particle as a function of time. [2]

iii. Describe the movement of the particle for the first ten seconds. [2]

iv. Comment on what will happen to the particle as the time increases beyond the first fifteen seconds. [1]

v. Give the equation of motion that describes the displacement of the particle as a function of time. [2]

**Question 4**

In a drawer you have socks of THREE colours: $2$ red socks, $7$ green socks and $4$ blue socks.

(a) You choose a sock at random. What is the probability that it is blue? [2]

(b) You replace the sock and then take THREE socks out at random

i. What is the probability that they are all blue? [2]

ii. What is the probability that they are all the same colour? [2]

iii. What is the probability that they are THREE different colours? [2]

iv. In how many ways can you choose at least ONE green sock? [2]

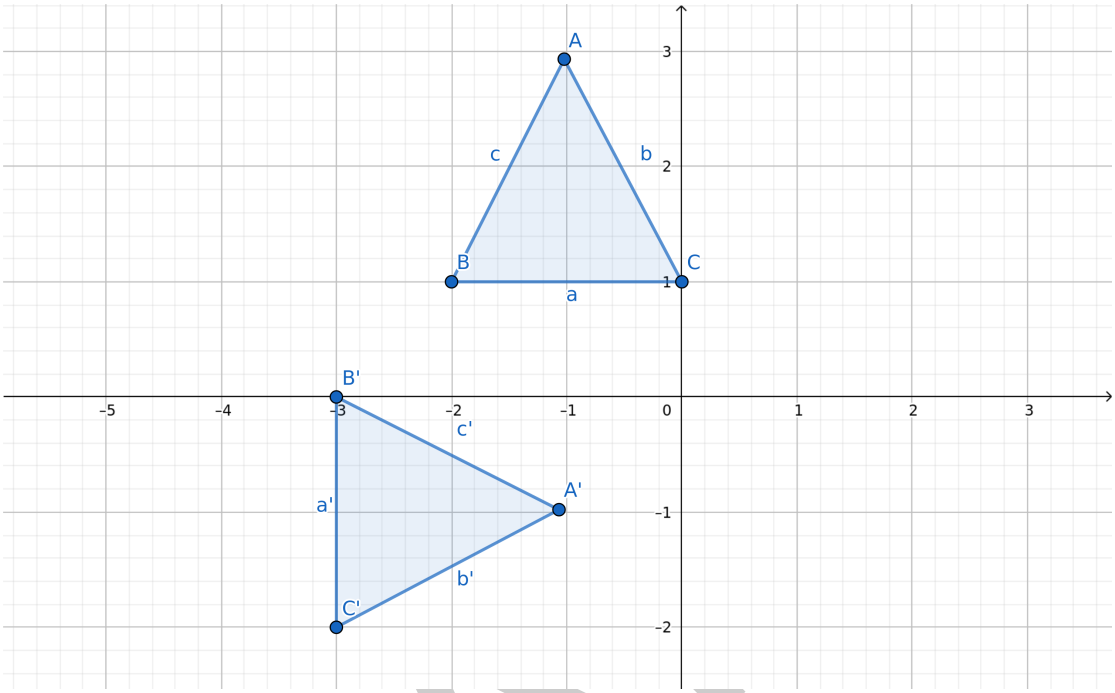
(c) A triangle has sides of length $15$ and $3$ and one internal angle of $55°$.

i. Draw ALL the feasible triangles that satisfy these requirements. [3]

ii. Choose ONE of the triangles, stating clearly which one you have chosen. Solve your chosen triangle completely showing the calculations you needed to reach the answers, and stating clearly any shortcuts, results or techniques you used. [5]

iii. Are there any feasible triangles with sides $4, 5$ and $10$? Explain your answer. [2]

(d) Consider the transformation $S$ that maps the triangle $[ABC]$ to the triangle $[A'B'C']$ with the vertices $A$, $B$ and $C$ mapped to $A'$, $B'$ and $D'$ respectively:




i. Describe the geometrical transformation $S$. [3]

ii. Give the matrix that describes the transformation $S$. [3]

iii. Consider the transformation $T$ defined as the translation by the vector:

$$\begin{pmatrix} -4 \\ 3 \end{pmatrix}.$$

Use homogeneous coordinates to give the matrix that describes the transformation $T$. [2]

iv. Give the matrix that describes the transformation: $S$ followed by $T$. [2]

END OF PAPER

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