2.2 More about Functions-Reading

Notebook: Discrete Mathematics [CM1020]

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Cornell Notes

Topic:

2.2 More about functions-

Reading

Course: BSc Computer Science

Class: Discrete Mathematics-

Reading

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Essential Question:

What is function composition? Alongside this, what are the floor, ceiling and partial functions ?

Questions/Cues:

- What is function composition?
- What is the identify function in terms of function composition?
- What is the graph of a function?
- What are floor and ceiling functions?
- What is a partial function?

Notes

- Let $g: A \to B$ and $f: B \to C$, composition of f and g denoted for all $a \in A$ by $f \circ g$ is: $(f \circ g)(a) = f(g(a))$
 - In this case $f \circ g$ cannot be unless range of g is subset of domain f
 - composition of functions not commutative
- Identity function:

$$(f^{-1}o f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$$

 $(f o f^{-1})(b) = f(f^{-1}(b)) = f(a) = b$

o That is,
$$(f^{-1})^{-1} = f$$

Let f be a function from the set A to the set B. The graph of the function f is the set of ordered pairs $\{(a,b) \mid a \in A \text{ and } f(a) = b\}$.

The *floor function* assigns to the real number x the largest integer that is less than or equal to x. The value of the floor function at x is denoted by $\lfloor x \rfloor$. The *ceiling function* assigns to the real number x the smallest integer that is greater than or equal to x. The value of the ceiling function at x is denoted by $\lceil x \rceil$.

• Floor function often also called greatest integer function, denoted by [x]

TABLE 1 Useful Properties of the Floor and Ceiling Functions.

(n is an integer, x is a real number)

- (1a) |x| = n if and only if $n \le x < n + 1$
- (1b) $\lceil x \rceil = n$ if and only if $n 1 < x \le n$
- (1c) $\lfloor x \rfloor = n$ if and only if $x 1 < n \le x$
- (1d) $\lceil x \rceil = n$ if and only if $x \le n < x + 1$
- (2) $x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$
- (3a) $\lfloor -x \rfloor = -\lceil x \rceil$
- (3b) [-x] = -[x]
- $(4a) \quad \lfloor x + n \rfloor = \lfloor x \rfloor + n$
- (4b) $\lceil x + n \rceil = \lceil x \rceil + n$

A partial function f from a set A to a set B is an assignment to each element a in a subset of A, called the *domain of definition* of f, of a unique element b in B. The sets A and B are called the *domain* and *codomain* of f, respectively. We say that f is *undefined* for elements in A that are not in the domain of definition of f. When the domain of definition of f equals A, we say that f is a *total function*.

o We write $f:A\to B$, denoting f is partial function from A to B, same notation used for functions, context is different; determines whether f is a partial or total function

EXAMPLE 32 The function $f : \mathbb{Z} \to \mathbb{R}$ where $f(n) = \sqrt{n}$ is a partial function from \mathbb{Z} to \mathbb{R} where the domain of definition is the set of nonnegative integers. Note that f is undefined for negative integers.

Summary

In this week, we learned what function composition is and what is means for a function to be partial. Also, we looked at the floor and ceiling functions.