2.2 More about Functions-Reading

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Cornell Notes	**Topic:** 2.2 More about functions-Reading	Course: BSc Computer Science Class: Discrete Mathematics-Reading Date: October 29, 2019

Essential Question:

What is function composition? Alongside this, what are the floor, ceiling and partial functions ?

Questions/Cues:

- What is function composition?
- What is the identify function in terms of function composition?
- What is the graph of a function?
- What are floor and ceiling functions?
- What is a partial function?

Notes

- Let $g: A \to B$ and $f: B \to C$, composition of f and g denoted for all $a \in A$ by $f\ o\ g$ is: $(f\ o\ g)(a) = f(g(a))$
  - In this case $f\ o\ g$ cannot be unless range of g is subset of domain f
  - composition of functions not commutative
- Identity function :

$$(f^{-1} o\, f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$$
$$(f\, o\, f^{-1})(b) = f(f^{-1}(b)) = f(a) = b$$

  - That is, $(f^{-1})^{-1} = f$

Let f be a function from the set A to the set B. The *graph* of the function f is the set of ordered pairs $\{(a, b) \mid a \in A \text{ and } f(a) = b\}$.

The *floor function* assigns to the real number x the largest integer that is less than or equal to x. The value of the floor function at x is denoted by $\lfloor x \rfloor$. The *ceiling function* assigns to the real number x the smallest integer that is greater than or equal to x. The value of the ceiling function at x is denoted by $\lceil x \rceil$.

- Floor function often also called greatest integer function, denoted by [x]

TABLE 1 Useful Properties of the Floor and Ceiling Functions.
(n is an integer, x is a real number)

(1a)	$\lfloor x \rfloor = n$ if and only if $n \le x < n+1$
(1b)	$\lceil x \rceil = n$ if and only if $n-1 < x \le n$
(1c)	$\lfloor x \rfloor = n$ if and only if $x-1 < n \le x$
(1d)	$\lceil x \rceil = n$ if and only if $x \le n < x+1$
(2)	$x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$
(3a)	$\lfloor -x \rfloor = -\lceil x \rceil$
(3b)	$\lceil -x \rceil = -\lfloor x \rfloor$
(4a)	$\lfloor x+n \rfloor = \lfloor x \rfloor + n$
(4b)	$\lceil x+n \rceil = \lceil x \rceil + n$

A *partial function* f from a set A to a set B is an assignment to each element a in a subset of A, called the *domain of definition* of f, of a unique element b in B. The sets A and B are called the *domain* and *codomain* of f, respectively. We say that f is *undefined* for elements in A that are not in the domain of definition of f. When the domain of definition of f equals A, we say that f is a *total function.*

- We write $f: A \to B$, denoting f is partial function from A to B, same notation used for functions, context is different; determines whether f is a partial or total function

EXAMPLE 32 The function $f : \mathbf{Z} \to \mathbf{R}$ where $f(n) = \sqrt{n}$ is a partial function from $\mathbf{Z}$ to $\mathbf{R}$ where the domain of definition is the set of nonnegative integers. Note that f is undefined for negative integers. ◀

Summary

In this week, we learned what function composition is and what is means for a function to be partial. Also, we looked at the floor and ceiling functions.