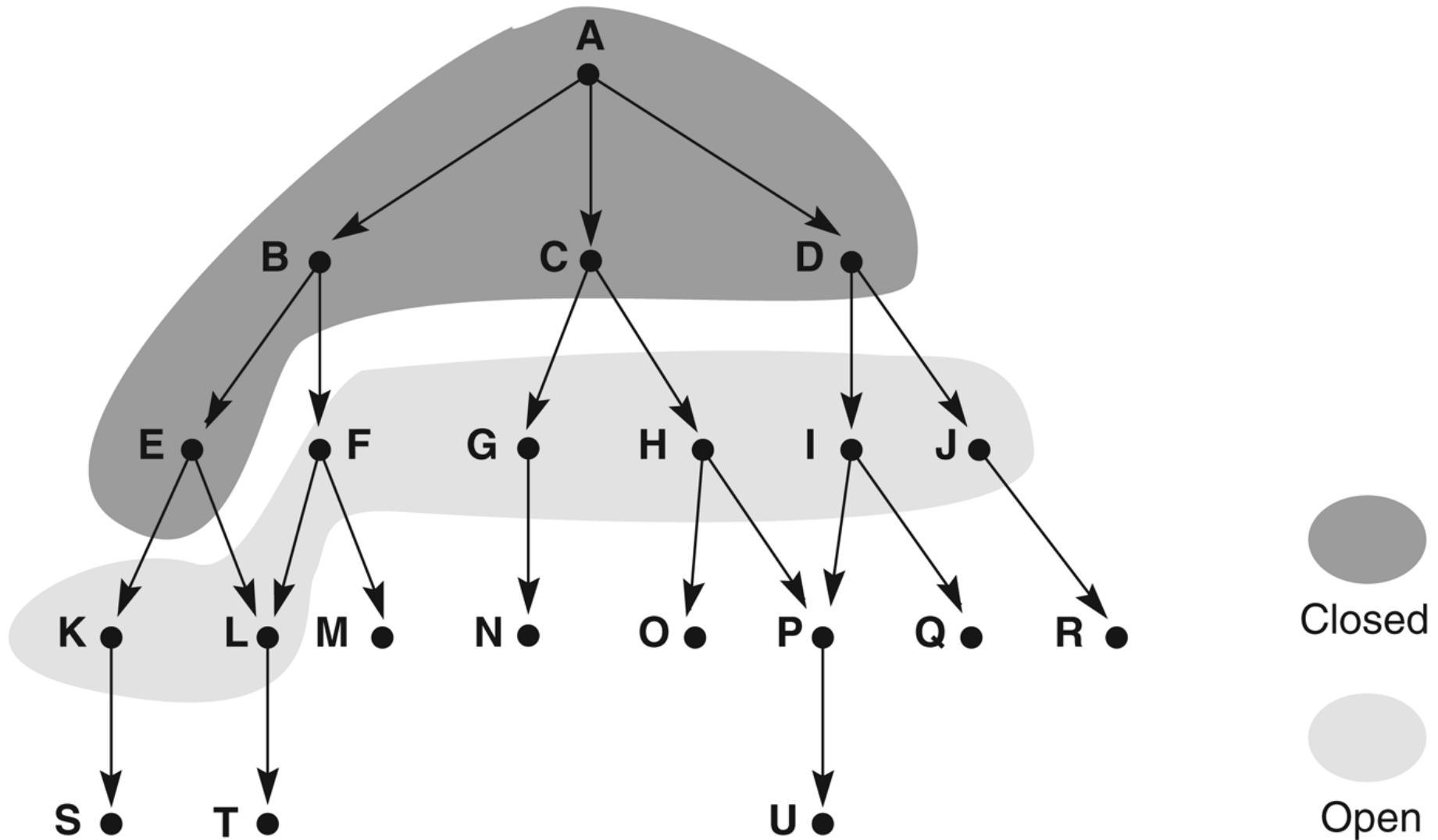


3

STRUCTURES AND STRATEGIES FOR STATE SPACE SEARCH

- | | | | |
|-----|-----------------------------------|-----|--|
| 3.0 | Introduction | 3.3 | Using the State Space to Represent Reasoning with the Predicate Calculus |
| 3.1 | Graph Theory | | |
| 3.2 | Strategies for State Space Search | 3.4 | Epilogue and References |
| | | 3.5 | Exercises |

Figure 3.14: Graph of Figure 3.13 at iteration 6 of breadth-first search. States on open and closed are highlighted.



Function depth_first_search algorithm

```

begin
  open := [Start];                                % initialize
  closed := [ ];
  while open ≠ [ ] do                             % states remain
    begin
      remove leftmost state from open, call it X;
      if X is a goal then return SUCCESS           % goal found
      else begin
        generate children of X;
        put X on closed;
        discard children of X if already on open or closed;
        put remaining children on left end of open % loop check
                                                    % stack
      end
    end;
  return FAIL                                     % no states left
end.

```

A trace of depth_first_search on the graph of Figure 3.13

1. **open = [A]; closed = []**
2. **open = [B,C,D]; closed = [A]**
3. **open = [E,F,C,D]; closed = [B,A]**
4. **open = [K,L,F,C,D]; closed = [E,B,A]**
5. **open = [S,L,F,C,D]; closed = [K,E,B,A]**
6. **open = [L,F,C,D]; closed = [S,K,E,B,A]**
7. **open = [T,F,C,D]; closed = [L,S,K,E,B,A]**
8. **open = [F,C,D]; closed = [T,L,S,K,E,B,A]**
9. **open = [M,C,D], as L is already on closed; closed = [F,T,L,S,K,E,B,A]**
10. **open = [C,D]; closed = [M,F,T,L,S,K,E,B,A]**
11. **open = [G,H,D]; closed = [C,M,F,T,L,S,K,E,B,A]**

Figure 3.15: Breadth-first search of the 8-puzzle, showing order in which states were removed from open.

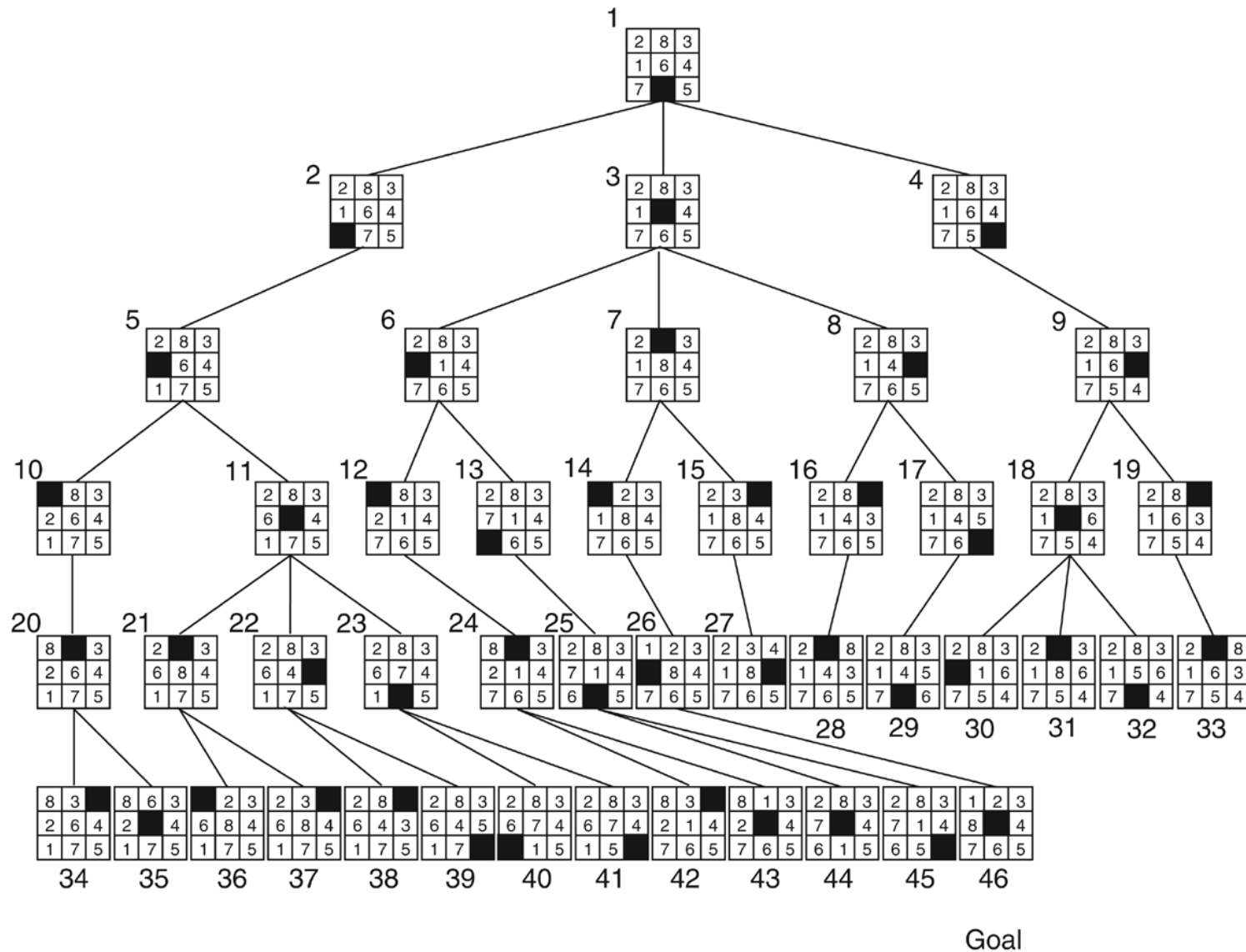


Figure 3.16: Graph of Figure 3.13 at iteration 6 of depth-first search. States on open and closed are highlighted.

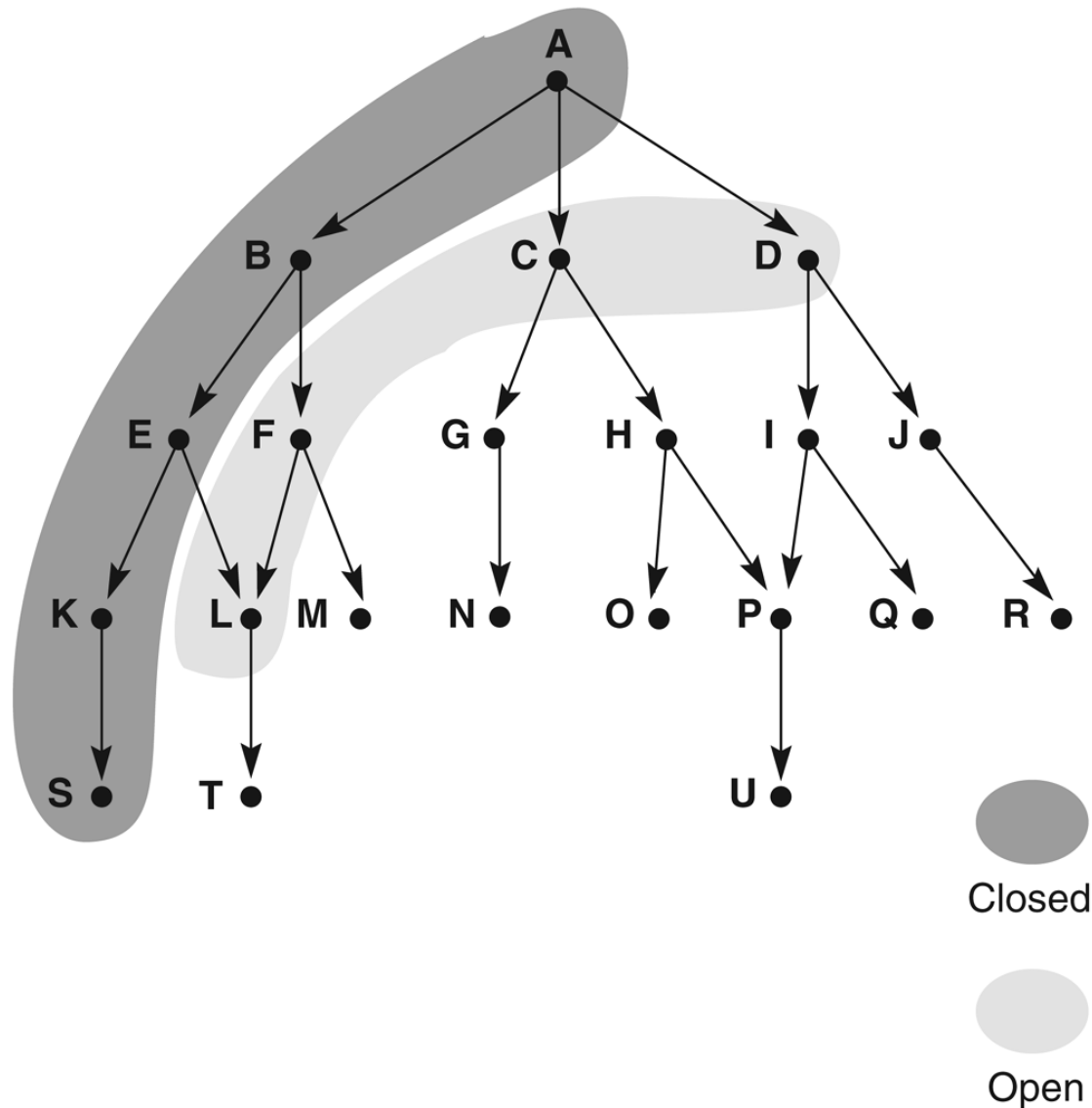
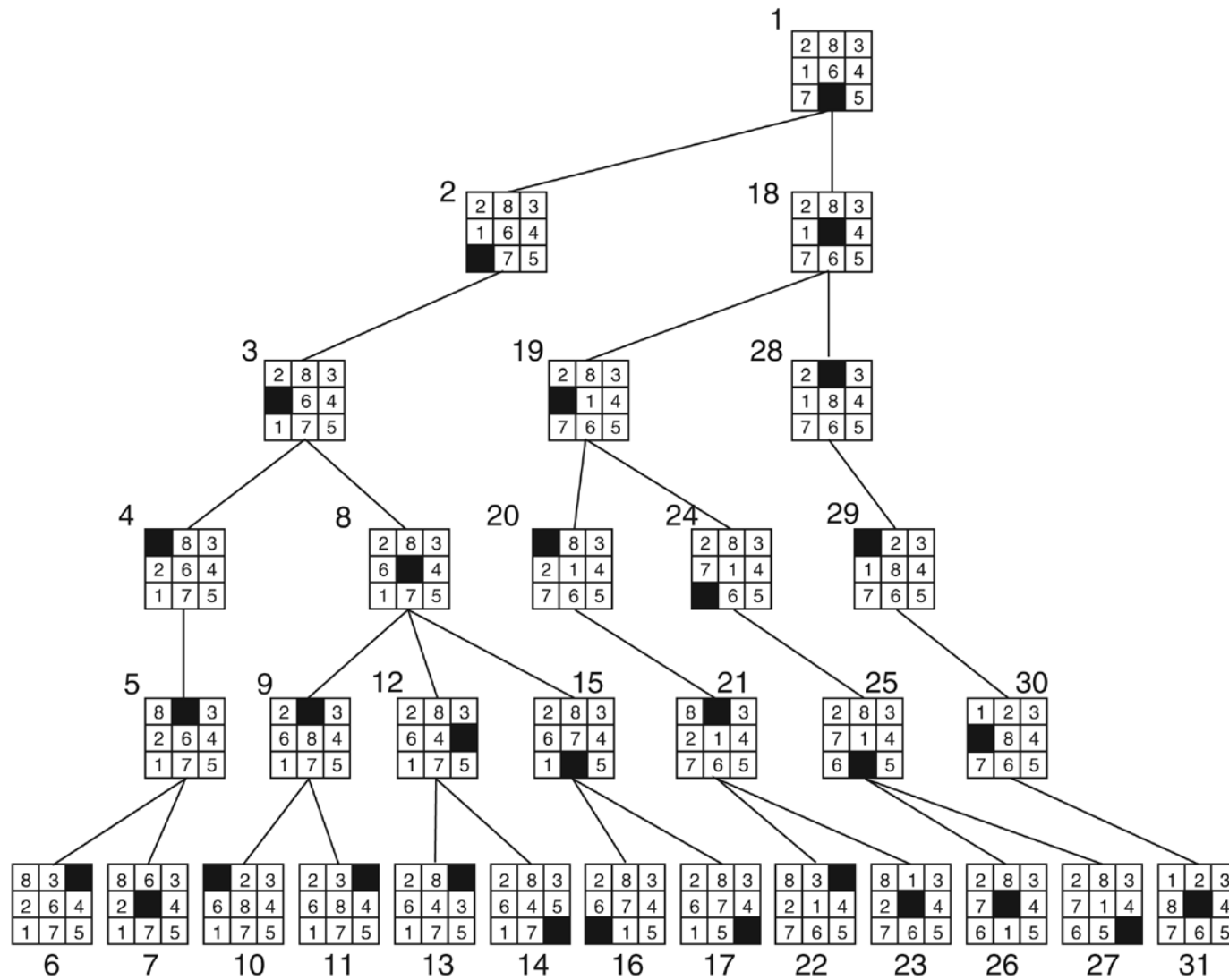


Figure 3.17: Depth-first search of the 8-puzzle with a depth bound of 5.



Goal

Figure 3.18: State space graph of a set of implications in the propositional calculus.

$q \rightarrow p$

$r \rightarrow p$

$v \rightarrow q$

$s \rightarrow r$

$t \rightarrow r$

$s \rightarrow u$

s

t

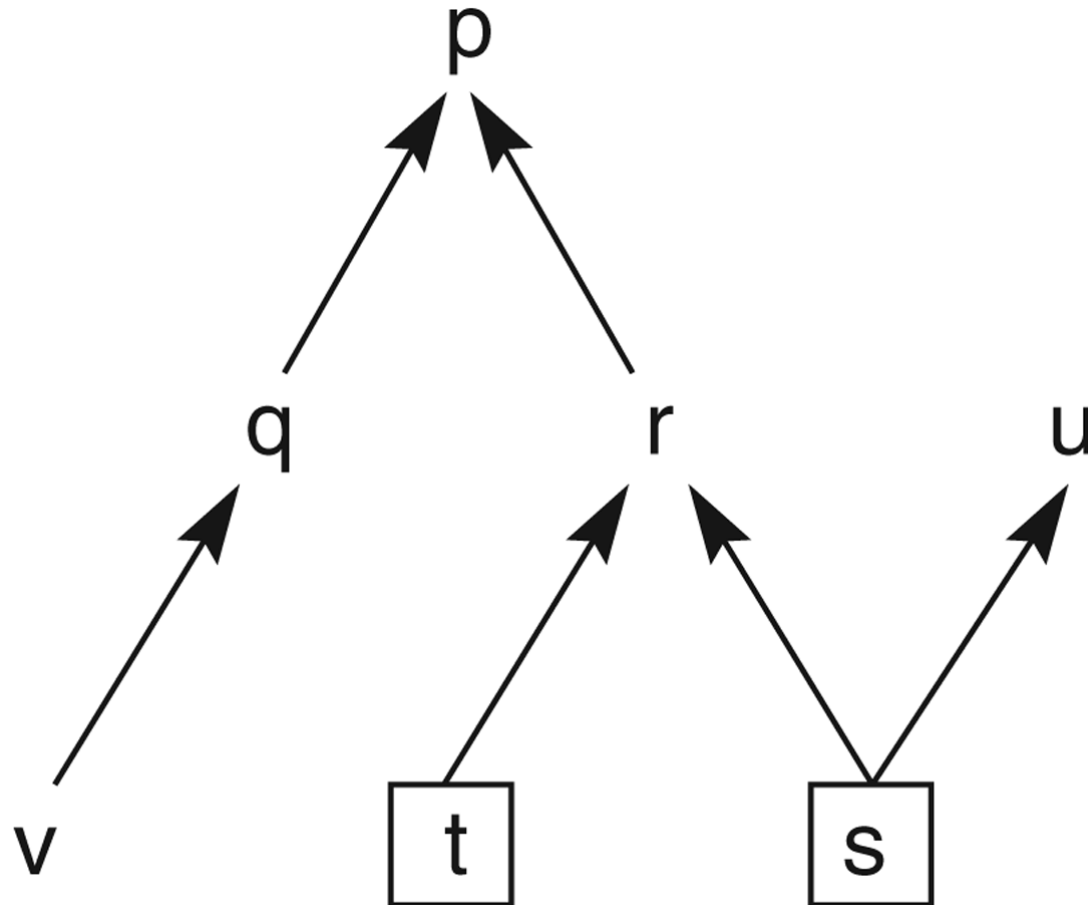
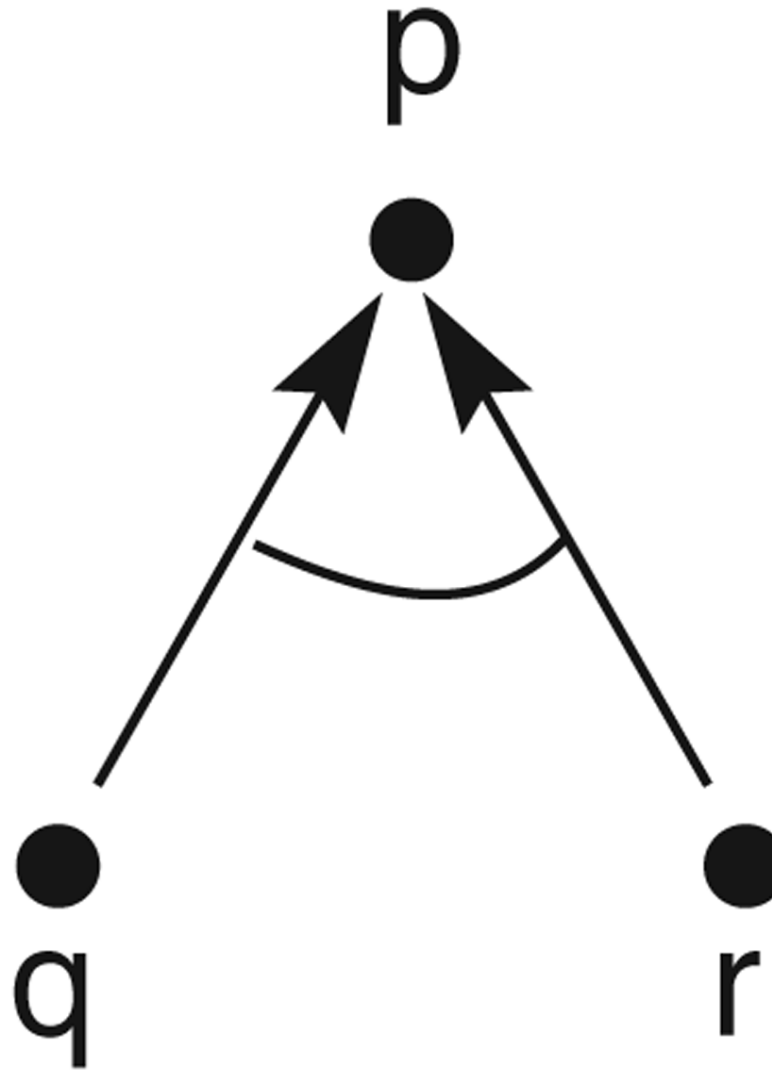


Figure 3.19: And/or graph of the expression $q \sqcup r \not\sqsubseteq p$.



DEFINITION

HYPERGRAPH

A hypergraph consists of:

N, a set of nodes.

H, a set of hyperarcs defined by ordered pairs in which the first element of the pair is a single node from **N** and the second element is a subset of **N**.

An ordinary graph is a special case of hypergraph in which all the sets of descendant nodes have a cardinality of 1.

Figure 3.20: And/or graph of the expression $q \Delta r \not\subseteq p$.

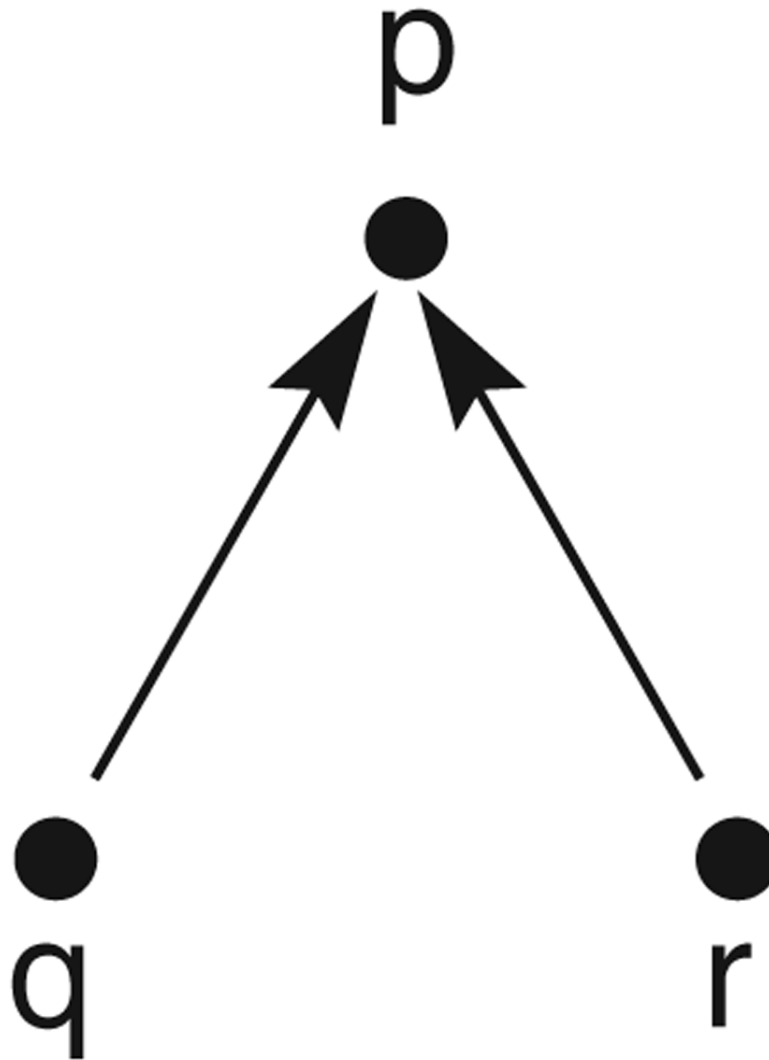


Figure 3.21: And/or graph of a set of propositional calculus expressions.

a
b
c
a \wedge **b** \rightarrow **d**
a \wedge **c** \rightarrow **e**
b \wedge **d** \rightarrow **f**
f \rightarrow **g**

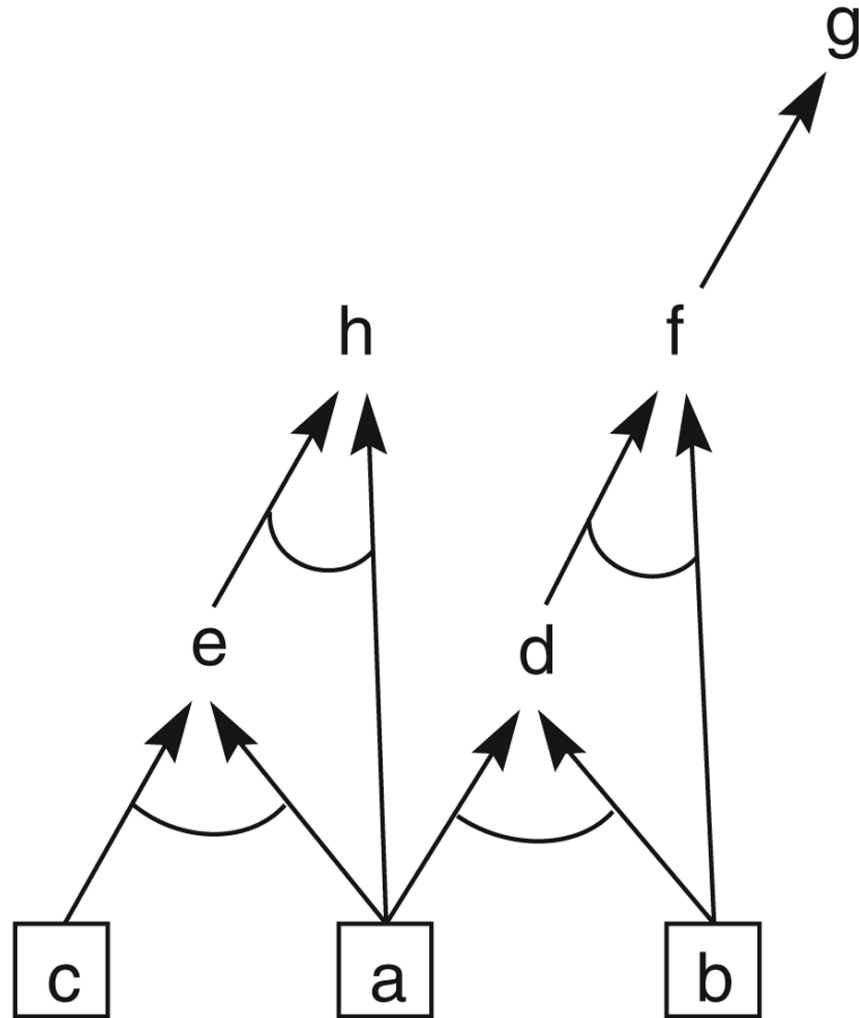
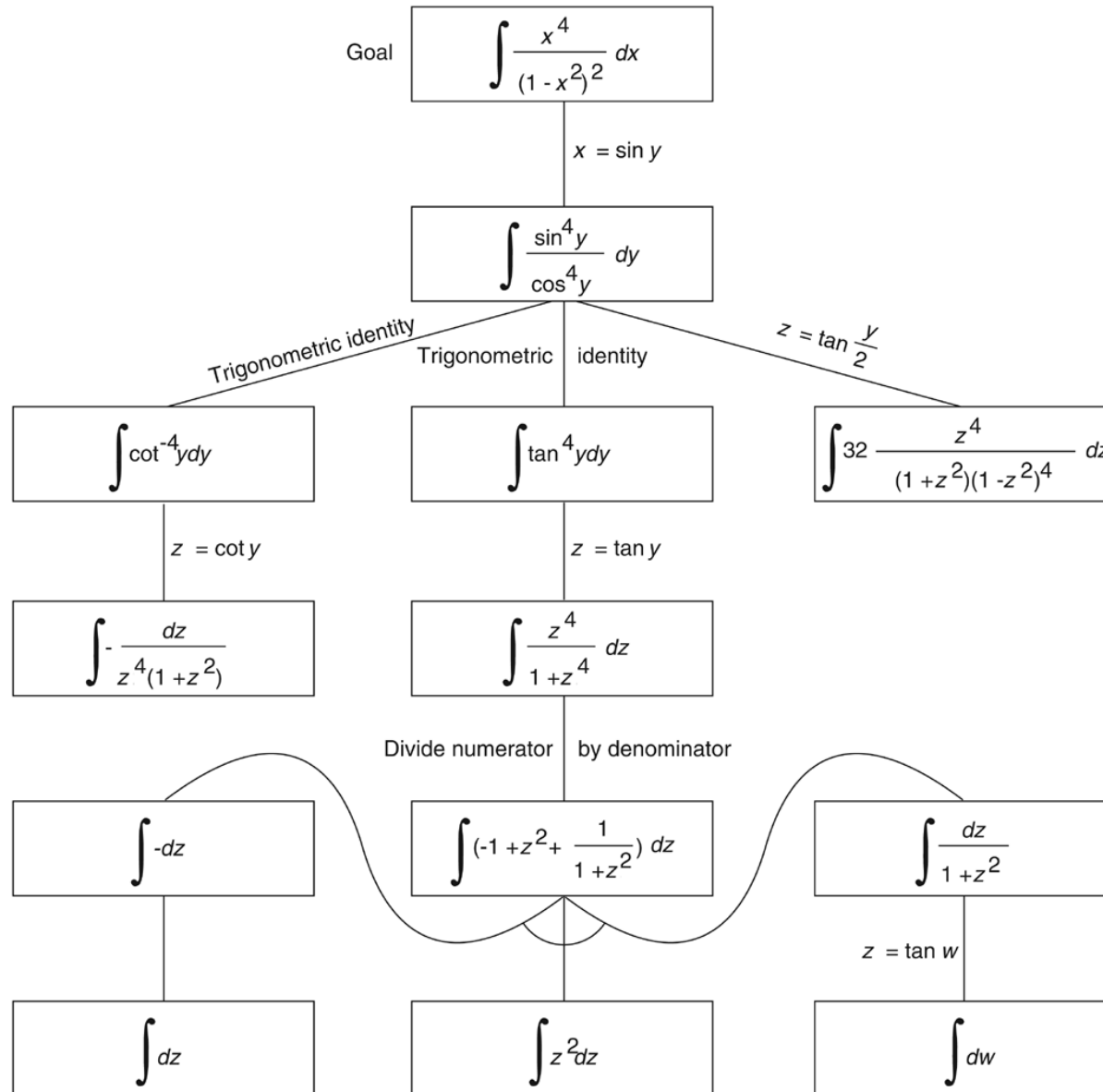


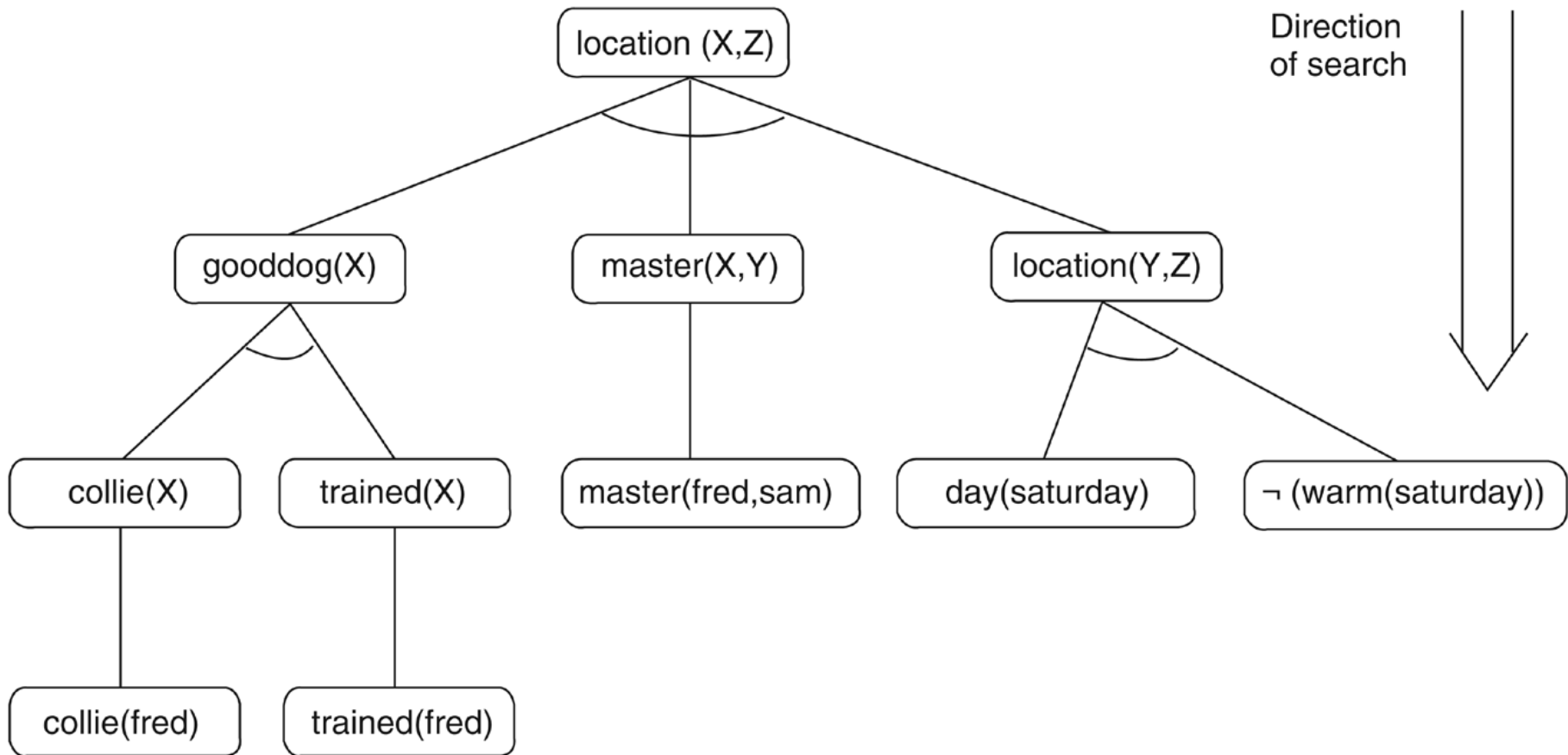
Figure 3.22: And/or graph of part of the state space for integrating a function, from Nilsson (1971).



The facts and rules of this example are given as English sentences followed by their predicate calculus equivalents:

1. Fred is a collie.
collie(fred).
2. Sam is Fred's master.
master(fred,sam).
3. The day is Saturday.
day(saturday).
4. It is cold on Saturday.
 \neg (**warm(saturday)**).
5. Fred is trained.
trained(fred).
6. Spaniels are good dogs and so are trained collies.
 $\forall X[\text{spaniel}(X) \vee (\text{collie}(X) \wedge \text{trained}(X)) \rightarrow \text{gooddog}(X)]$
7. If a dog is a good dog and has a master then he will be with his master.
 $\forall (X,Y,Z) [\text{gooddog}(X) \wedge \text{master}(X,Y) \wedge \text{location}(Y,Z) \rightarrow \text{location}(X,Z)]$
8. If it is Saturday and warm, then Sam is at the park.
 $(\text{day}(\text{saturday}) \wedge \text{warm}(\text{saturday})) \rightarrow \text{location}(\text{sam},\text{park}).$
9. If it is Saturday and not warm, then Sam is at the museum.
 $(\text{day}(\text{saturday}) \wedge \neg (\text{warm}(\text{saturday}))) \rightarrow \text{location}(\text{sam},\text{museum}).$

Figure 3.23: The solution subgraph showing that fred is at the museum.



Substitutions = {fred/X, sam/Y, museum/Z}

Five rules for a simple subset of English grammar are:

1. **A sentence is a noun phrase followed by a verb phrase.**
sentence \leftrightarrow np vp
2. **A noun phrase is a noun.**
np \leftrightarrow n
3. **A noun phrase is an article followed by a noun.**
np \leftrightarrow art n
4. **A verb phrase is a verb.**
vp \leftrightarrow v
5. **A verb phrase is a verb followed by a noun phrase.**
vp \leftrightarrow v np
6. **art \leftrightarrow a**
7. **art \leftrightarrow the**
(“a” and “the” are articles)
8. **n \leftrightarrow man**
9. **n \leftrightarrow dog**
(“man” and “dog” are nouns)
10. **v \leftrightarrow likes**
11. **v \leftrightarrow bites**
(“likes” and “bites” are verbs)

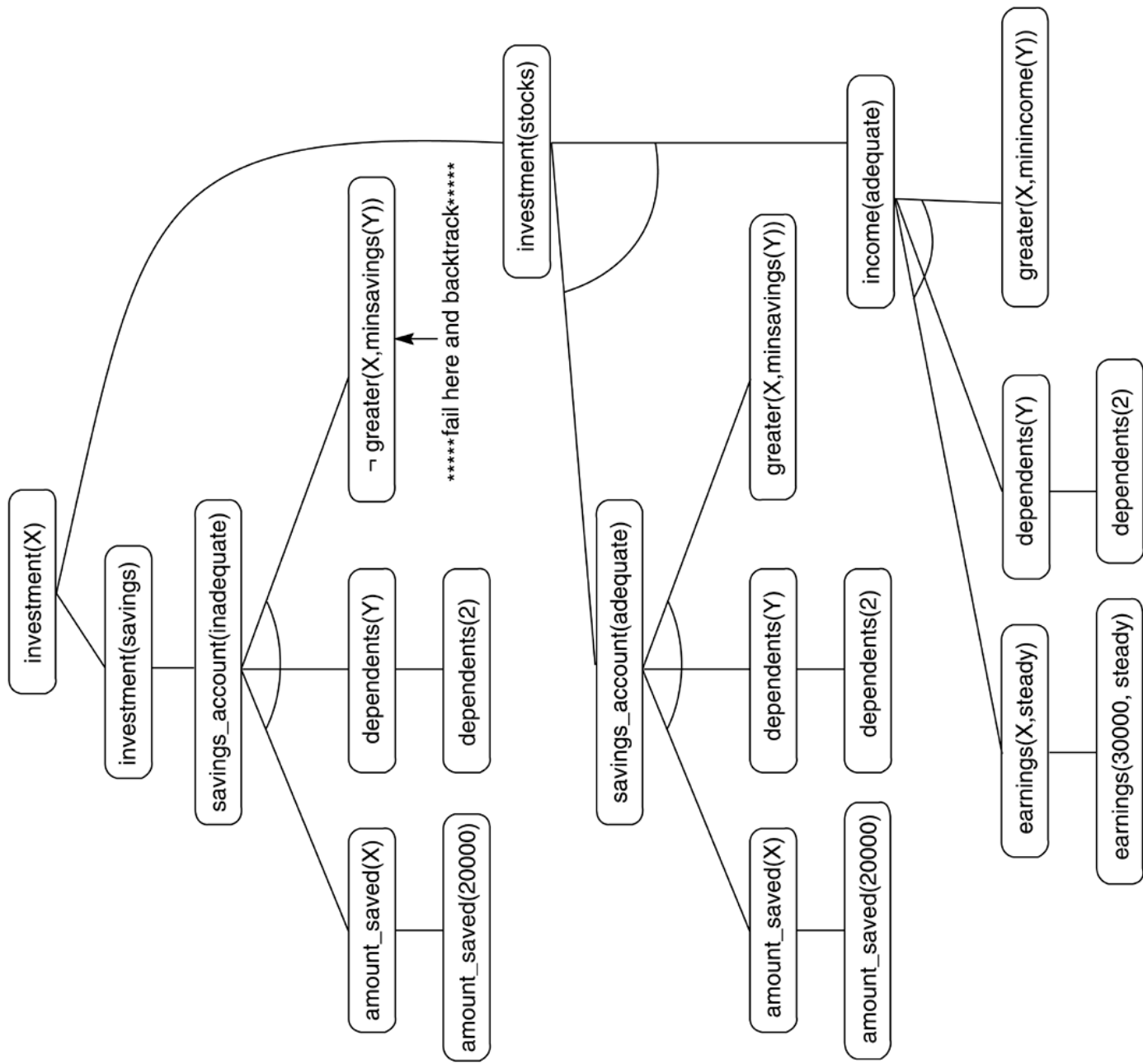
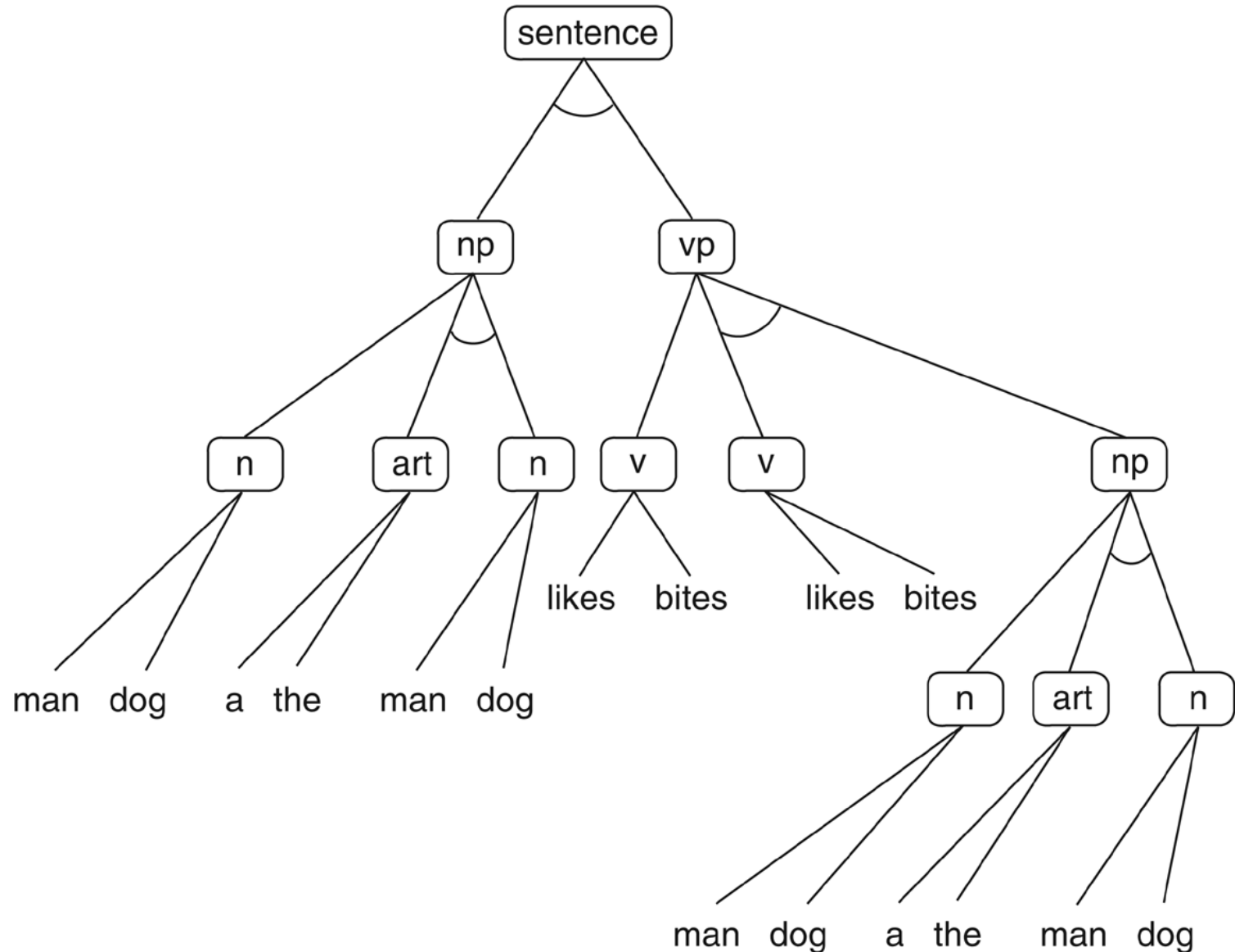


Figure 3.24: And/or graph searched by the financial advisor.

Figure 3.25: And/or graph for the grammar of Example 3.3.6. Some of the nodes (np, art, etc.) have been written more than once to simplify drawing the graph.



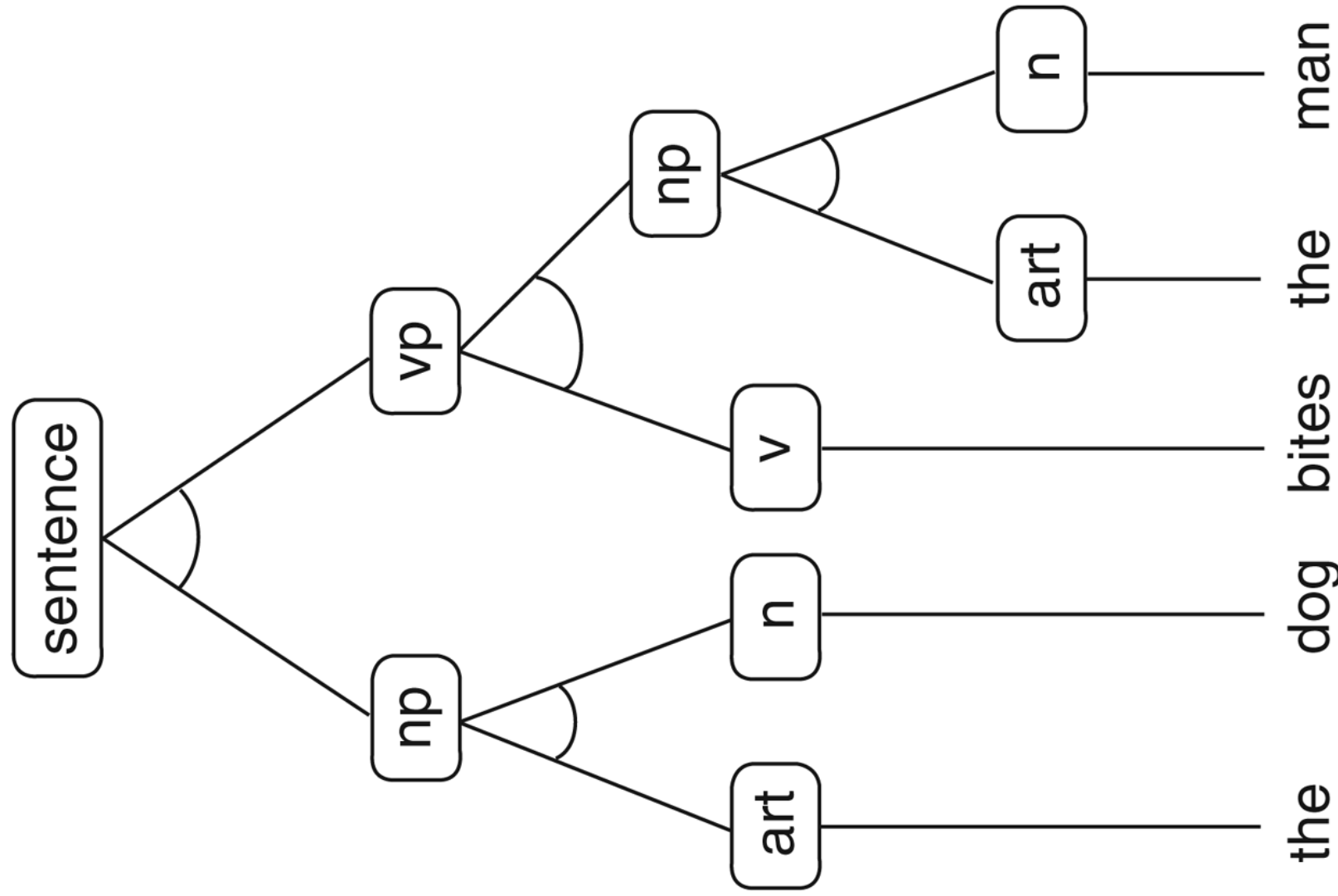


Figure 3.26: Parse tree for the sentence “The dog bites the man.” Note that this is a subtree of the graph of Figure 3.25.

Figure 3.27: A graph to be searched.

