

# Data Mining: Concepts and Techniques

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## — Chapter 6 —

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# Chapter 6. Classification and Prediction

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- What is classification? What is prediction? 
- Issues regarding classification and prediction
- Classification by decision tree induction
- Bayesian classification
- Rule-based classification
- Classification by back propagation
- Support Vector Machines (SVM)
- Associative classification
- Lazy learners (or learning from your neighbors)
- Other classification methods
- Prediction
- Accuracy and error measures
- Ensemble methods
- Model selection
- Summary

# Classification vs. Prediction

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- **Classification**

- predicts categorical class labels (discrete or nominal)
- classifies data (constructs a model) based on the training set and the values (**class labels**) in a classifying attribute and uses it in classifying new data

- **Prediction**

- models continuous-valued functions, i.e., predicts unknown or missing values

- Typical applications

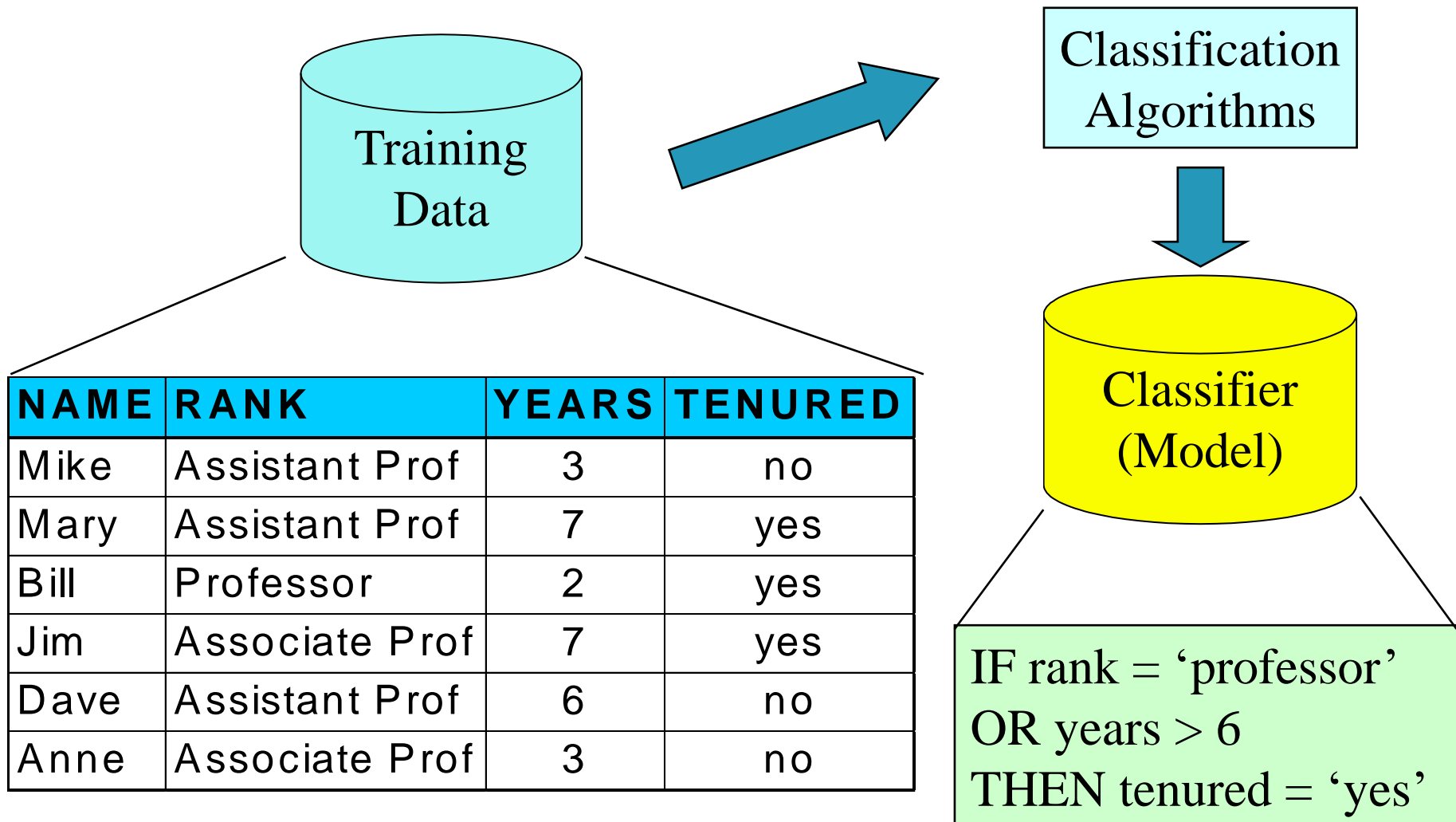
- Credit approval
- Target marketing
- Medical diagnosis
- Fraud detection

# Classification—A Two-Step Process

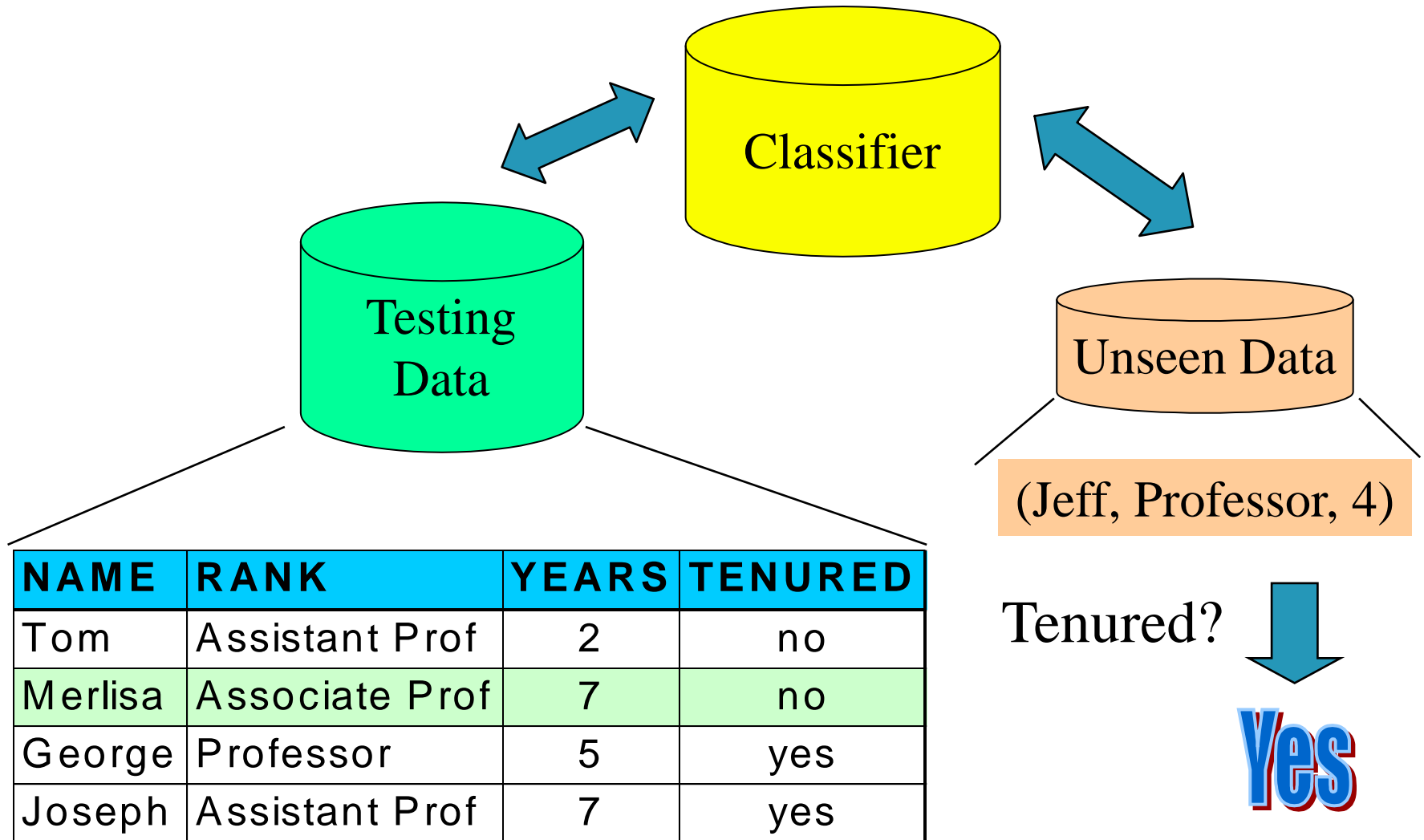
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- **Model construction**: describing a set of predetermined classes
  - Each tuple/sample is assumed to belong to a predefined class, as determined by the **class label attribute**
  - The set of tuples used for model construction is **training set**
  - The model is represented as classification rules, decision trees, or mathematical formulae
- **Model usage**: for classifying future or unknown objects
  - **Estimate accuracy** of the model
    - The known label of test sample is compared with the classified result from the model
    - Accuracy rate is the percentage of test set samples that are correctly classified by the model
    - Test set is independent of training set, otherwise over-fitting will occur
  - If the accuracy is acceptable, use the model to **classify data** tuples whose class labels are not known

# Process (1): Model Construction



# Process (2): Using the Model in Prediction



# Supervised vs. Unsupervised Learning


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- Supervised learning (classification)
  - Supervision: The training data (observations, measurements, etc.) are accompanied by labels indicating the class of the observations
  - New data is classified based on the training set
- Unsupervised learning (clustering)
  - The class labels of training data is unknown
  - Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data



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# Issues: Data Preparation

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- Data cleaning
  - Preprocess data in order to reduce noise and handle missing values
- Relevance analysis (feature selection)
  - Remove the irrelevant or redundant attributes
- Data transformation
  - Generalize and/or normalize data


# Issues: Evaluating Classification Methods

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- Accuracy
  - classifier accuracy: predicting class label
  - predictor accuracy: guessing value of predicted attributes
- Speed
  - time to construct the model (training time)
  - time to use the model (classification/prediction time)
- Robustness: handling noise and missing values
- Scalability: efficiency in disk-resident databases
- Interpretability
  - understanding and insight provided by the model
- Other measures, e.g., goodness of rules, such as decision tree size or compactness of classification rules

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# Decision Tree Induction: Training Dataset

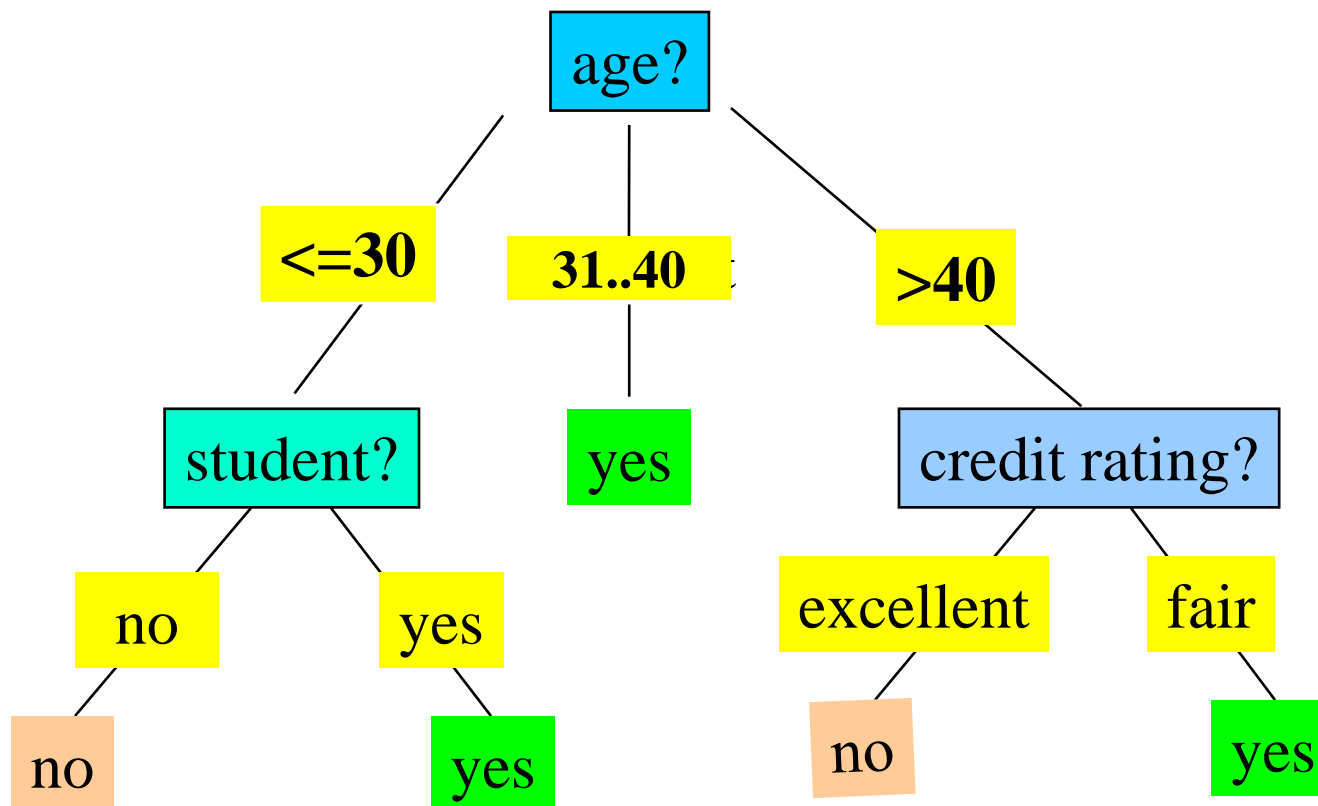
This follows an example of Quinlan's ID3 (Playing Tennis)

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no



# Output: A Decision Tree for "*buys\_computer*"

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# Algorithm for Decision Tree Induction

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- Basic algorithm (a greedy algorithm)
  - Tree is constructed in a **top-down recursive divide-and-conquer manner**
  - At start, all the training examples are at the root
  - Attributes are categorical (if continuous-valued, they are discretized in advance)
  - Examples are partitioned recursively based on selected attributes
  - Test attributes are selected on the basis of a heuristic or statistical measure (e.g., **information gain**)
- Conditions for stopping partitioning
  - All samples for a given node belong to the same class
  - There are no remaining attributes for further partitioning – **majority voting** is employed for classifying the leaf
  - There are no samples left

# Attribute Selection Measure: Information Gain (ID3/C4.5)

- Select the attribute with the highest information gain
- Let  $p_i$  be the probability that an arbitrary tuple in  $D$  belongs to class  $C_i$ , estimated by  $|C_{i,D}|/|D|$
- **Expected information** (entropy) needed to classify a tuple in  $D$ :

$$Info(D) = -\sum_{i=1}^m p_i \log_2(p_i)$$

- **Information** needed (after using  $A$  to split  $D$  into  $v$  partitions) to classify  $D$ :

$$Info_A(D) = \sum_{j=1}^v \frac{|D_j|}{|D|} \times I(D_j)$$

- **Information gained** by branching on attribute  $A$

$$Gain(A) = Info(D) - Info_A(D)$$

# Attribute Selection: Information Gain

- Class P: buys\_computer = "yes"
- Class N: buys\_computer = "no"

$$Info(D) = I(9,5) = -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right) = 0.940$$

$$Info_{age}(D) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) = 0.694$$

age	p <sub>i</sub>	n <sub>i</sub>	I(p <sub>i</sub> , n <sub>i</sub> )
<=30	2	3	0.971
31...40	4	0	0
>40	3	2	0.971

$\frac{5}{14} I(2,3)$  means "age <=30" has 5 out of 14 samples, with 2 yes'es and 3 no's. Hence

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Similarly,

$$Gain(income) = 0.029$$

$$Gain(student) = 0.151$$

$$Gain(credit\_rating) = 0.048$$

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
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<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

# Computing Information-Gain for Continuous-Value Attributes

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- Let attribute A be a continuous-valued attribute
- Must determine the *best split point* for A
  - Sort the value A in increasing order
  - Typically, the midpoint between each pair of adjacent values is considered as a possible *split point*
    - $(a_i + a_{i+1})/2$  is the midpoint between the values of  $a_i$  and  $a_{i+1}$
  - The point with the *minimum expected information requirement* for A is selected as the split-point for A
- Split:
  - D1 is the set of tuples in D satisfying  $A \leq \text{split-point}$ , and D2 is the set of tuples in D satisfying  $A > \text{split-point}$



# Gain Ratio for Attribute Selection (C4.5)

- Information gain measure is biased towards attributes with a large number of values
- C4.5 (a successor of ID3) uses gain ratio to overcome the problem (normalization to information gain)

$$SplitInfo_A(D) = -\sum_{j=1}^v \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

- $GainRatio(A) = Gain(A)/SplitInfo(A)$
- Ex.  $SplitInfo_A(D) = -\frac{4}{14} \times \log_2\left(\frac{4}{14}\right) - \frac{6}{14} \times \log_2\left(\frac{6}{14}\right) - \frac{4}{14} \times \log_2\left(\frac{4}{14}\right) = 0.926$ 
  - $gain\_ratio(income) = 0.029/0.926 = 0.031$
- The attribute with the maximum gain ratio is selected as the splitting attribute

# Gini index (CART, IBM IntelligentMiner)

- If a data set  $D$  contains examples from  $n$  classes, gini index,  $gini(D)$  is defined as

$$gini(D) = 1 - \sum_{j=1}^n p_j^2$$

where  $p_j$  is the relative frequency of class  $j$  in  $D$

- If a data set  $D$  is split on  $A$  into two subsets  $D_1$  and  $D_2$ , the  $gini$  index  $gini_A(D)$  is defined as

$$gini_A(D) = \frac{|D_1|}{|D|} gini(D_1) + \frac{|D_2|}{|D|} gini(D_2)$$

- Reduction in Impurity:

$$\Delta gini(A) = gini(D) - gini_A(D)$$

- The attribute provides the smallest  $gini_{split}(D)$  (or the largest reduction in impurity) is chosen to split the node (*need to enumerate all the possible splitting points for each attribute*)

# Gini index (CART, IBM IntelligentMiner)

- Ex. D has 9 tuples in buys\_computer = "yes" and 5 in "no"

$$gini(D) = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right)^2 = 0.459$$

- Suppose the attribute income partitions D into 10 in  $D_1$ : {low, medium} and 4 in  $D_2$


$$\begin{aligned} gini_{income \in \{low, medium\}}(D) &= \left(\frac{10}{14}\right)Gini(D_1) + \left(\frac{4}{14}\right)Gini(D_2) \\ &= \frac{10}{14} \left(1 - \left(\frac{6}{10}\right)^2 - \left(\frac{4}{10}\right)^2\right) + \frac{4}{14} \left(1 - \left(\frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) \\ &= 0.450 \\ &= Gini_{income \in \{high\}}(D) \end{aligned}$$

but  $gini_{\{medium, high\}}$  is 0.30 and thus the best since it is the lowest

- All attributes are assumed continuous-valued
- May need other tools, e.g., clustering, to get the possible split values
- Can be modified for categorical attributes

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# Bayesian Classification: Why?

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- A statistical classifier: performs *probabilistic prediction*, *i.e.*, predicts class membership probabilities
- Foundation: Based on Bayes' Theorem.
- Performance: A simple Bayesian classifier, *naïve Bayesian classifier*, has comparable performance with decision tree and selected neural network classifiers
- Incremental: Each training example can incrementally increase/decrease the probability that a hypothesis is correct — prior knowledge can be combined with observed data
- Standard: Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured



# Bayesian Theorem: Basics

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- Let  $\mathbf{X}$  be a data sample ("*evidence*"): class label is unknown
- Let  $H$  be a *hypothesis* that  $X$  belongs to class  $C$
- Classification is to determine  $P(H|\mathbf{X})$ , the probability that the hypothesis holds given the observed data sample  $\mathbf{X}$
- $P(H)$  (*prior probability*), the initial probability
  - E.g.,  $\mathbf{X}$  will buy computer, regardless of age, income, ...
- $P(\mathbf{X})$ : probability that sample data is observed
- $P(\mathbf{X}|H)$  (*posteriori probability*), the probability of observing the sample  $\mathbf{X}$ , given that the hypothesis holds
  - E.g., Given that  $\mathbf{X}$  will buy computer, the prob. that  $X$  is 31..40, medium income

# Bayesian Theorem

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- Given training data  $\mathbf{X}$ , *posteriori probability of a hypothesis*  $H$ ,  $P(H|\mathbf{X})$ , follows the Bayes theorem

$$P(H | \mathbf{X}) = \frac{P(\mathbf{X} | H)P(H)}{P(\mathbf{X})}$$

- Informally, this can be written as  
posteriori = likelihood x prior/evidence
- Predicts  $\mathbf{X}$  belongs to  $C_2$  iff the probability  $P(C_i|\mathbf{X})$  is the highest among all the  $P(C_k|X)$  for all the  $k$  classes
- Practical difficulty: require initial knowledge of many probabilities, significant computational cost

# Towards Naïve Bayesian Classifier

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- Let  $D$  be a training set of tuples and their associated class labels, and each tuple is represented by an  $n$ -D attribute vector  $\mathbf{X} = (x_1, x_2, \dots, x_n)$
- Suppose there are  $m$  classes  $C_1, C_2, \dots, C_m$ .
- Classification is to derive the maximum posteriori, i.e., the maximal  $P(C_i|\mathbf{X})$
- This can be derived from Bayes' theorem

$$P(C_i|\mathbf{X}) = \frac{P(\mathbf{X}|C_i)P(C_i)}{P(\mathbf{X})}$$

- Since  $P(\mathbf{X})$  is constant for all classes, only

$$P(C_i|\mathbf{X}) = P(\mathbf{X}|C_i)P(C_i)$$

needs to be maximized

# Derivation of Naïve Bayes Classifier

- A simplified assumption: attributes are conditionally independent (i.e., no dependence relation between attributes):

$$P(\mathbf{X} | C_i) = \prod_{k=1}^n P(x_k | C_i) = P(x_1 | C_i) \times P(x_2 | C_i) \times \dots \times P(x_n | C_i)$$

- This greatly reduces the computation cost: Only counts the class distribution
- If  $A_k$  is categorical,  $P(x_k | C_i)$  is the # of tuples in  $C_i$  having value  $x_k$  for  $A_k$  divided by  $|C_i, D|$  (# of tuples of  $C_i$  in  $D$ )
- If  $A_k$  is continuous-valued,  $P(x_k | C_i)$  is usually computed based on Gaussian distribution with a mean  $\mu$  and standard deviation  $\sigma$

$$g(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

and  $P(x_k | C_i)$  is

$$P(\mathbf{X} | C_i) = g(x_k, \mu_{C_i}, \sigma_{C_i})$$

# Naïve Bayesian Classifier: Training Dataset

Class:

C1:buys\_computer = 'yes'

C2:buys\_computer = 'no'

Data sample

X = (age <=30,

Income = medium,

Student = yes

Credit\_rating = Fair)

age	income	student	credit_rating	comp
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
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31...40	high	yes	fair	yes
>40	medium	no	excellent	no



# Naïve Bayesian Classifier: An Example

- $P(C_i)$ :  
 $P(\text{buys\_computer} = \text{"yes"}) = 9/14 = 0.643$   
 $P(\text{buys\_computer} = \text{"no"}) = 5/14 = 0.357$
- Compute  $P(X|C_i)$  for each class  
 $P(\text{age} = \text{"<=30"} \mid \text{buys\_computer} = \text{"yes"}) = 2/9 = 0.222$   
 $P(\text{age} = \text{"<= 30"} \mid \text{buys\_computer} = \text{"no"}) = 3/5 = 0.6$   
 $P(\text{income} = \text{"medium"} \mid \text{buys\_computer} = \text{"yes"}) = 4/9 = 0.444$   
 $P(\text{income} = \text{"medium"} \mid \text{buys\_computer} = \text{"no"}) = 2/5 = 0.4$   
 $P(\text{student} = \text{"yes"} \mid \text{buys\_computer} = \text{"yes"}) = 6/9 = 0.667$   
 $P(\text{student} = \text{"yes"} \mid \text{buys\_computer} = \text{"no"}) = 1/5 = 0.2$   
 $P(\text{credit\_rating} = \text{"fair"} \mid \text{buys\_computer} = \text{"yes"}) = 6/9 = 0.667$   
 $P(\text{credit\_rating} = \text{"fair"} \mid \text{buys\_computer} = \text{"no"}) = 2/5 = 0.4$
- **$X = (\text{age} \leq 30, \text{income} = \text{medium}, \text{student} = \text{yes}, \text{credit\_rating} = \text{fair})$**

$$P(X|C_i) : P(X|\text{buys\_computer} = \text{"yes"}) = 0.222 \times 0.444 \times 0.667 \times 0.667 = 0.044$$

$$P(X|\text{buys\_computer} = \text{"no"}) = 0.6 \times 0.4 \times 0.2 \times 0.4 = 0.019$$

$$P(X|C_i) * P(C_i) : P(X|\text{buys\_computer} = \text{"yes"}) * P(\text{buys\_computer} = \text{"yes"}) = 0.028$$

$$P(X|\text{buys\_computer} = \text{"no"}) * P(\text{buys\_computer} = \text{"no"}) = 0.007$$

**Therefore, X belongs to class ("buys\_computer = yes")**

# Avoiding the 0-Probability Problem

- Naïve Bayesian prediction requires each conditional prob. be non-zero. Otherwise, the predicted prob. will be zero

$$P(X | C_i) = \prod_{k=1}^n P(x_k | C_i)$$

- Ex. Suppose a dataset with 1000 tuples, income=low (0), income=medium (990), and income = high (10),
- Use Laplacian correction (or Laplacian estimator)
  - Adding 1 to each case
$$\text{Prob}(\text{income} = \text{low}) = 1/1003$$
$$\text{Prob}(\text{income} = \text{medium}) = 991/1003$$
$$\text{Prob}(\text{income} = \text{high}) = 11/1003$$
  - The “corrected” prob. estimates are close to their “uncorrected” counterparts

# Naïve Bayesian Classifier: Comments

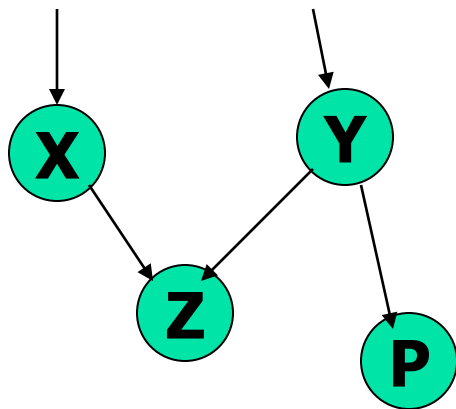
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- Advantages
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Assumption: class conditional independence, therefore loss of accuracy
  - Practically, dependencies exist among variables
    - E.g., hospitals: patients: Profile: age, family history, etc.  
Symptoms: fever, cough etc., Disease: lung cancer, diabetes, etc.
    - Dependencies among these cannot be modeled by Naïve Bayesian Classifier
- How to deal with these dependencies?
  - Bayesian Belief Networks

# Bayesian Belief Networks

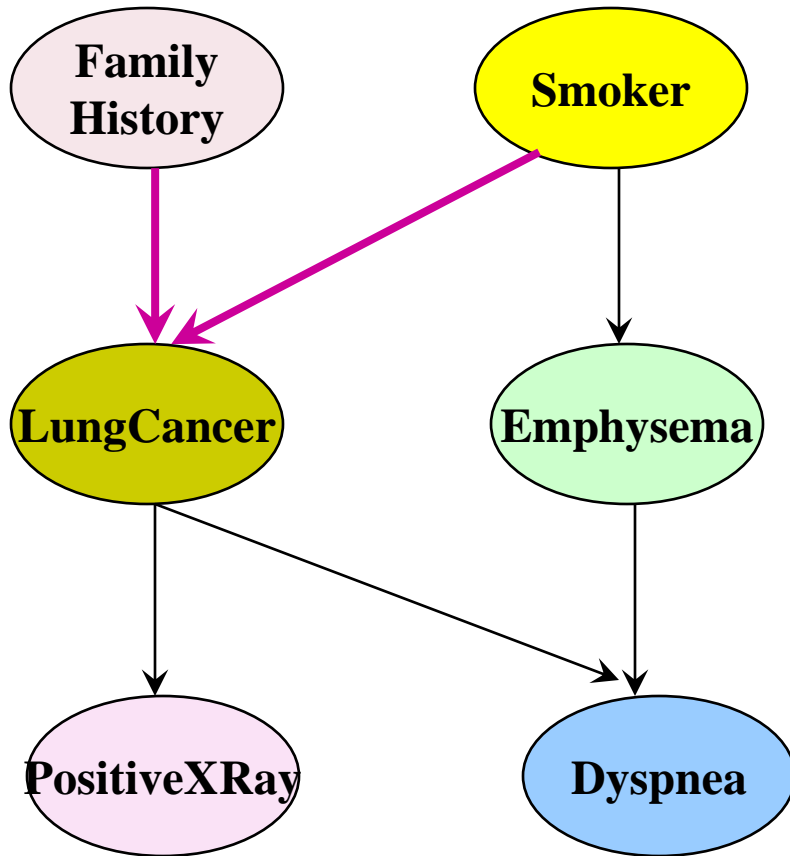
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- Bayesian belief network allows a *subset* of the variables conditionally independent
- A graphical model of causal relationships
  - Represents dependency among the variables
  - Gives a specification of joint probability distribution



- ☐ Nodes: random variables
- ☐ Links: dependency
- ☐ X and Y are the parents of Z, and Y is the parent of P
- ☐ No dependency between Z and P
- ☐ Has no loops or cycles

# Bayesian Belief Network: An Example



The **conditional probability table (CPT)** for variable LungCancer:

	(FH, S)	(FH, ~S)	(~FH, S)	(~FH, ~S)
LC	0.8	0.5	0.7	0.1
~LC	0.2	0.5	0.3	0.9

CPT shows the conditional probability for each possible combination of its parents

Derivation of the probability of a particular combination of values of  $\mathbf{X}$ , from CPT:

$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i \mid \text{Parents}(Y_i))$$

## Bayesian Belief Networks


# Training Bayesian Networks

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- Several scenarios:
  - Given both the network structure and all variables observable: *learn only the CPTs*
  - Network structure known, some hidden variables: *gradient descent* (greedy hill-climbing) method, analogous to neural network learning
  - Network structure unknown, all variables observable: search through the model space to *reconstruct network topology*
  - Unknown structure, all hidden variables: No good algorithms known for this purpose
- Ref. D. Heckerman: Bayesian networks for data mining

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# Using IF-THEN Rules for Classification

- Represent the knowledge in the form of **IF-THEN** rules

R: IF *age* = youth AND *student* = yes THEN *buys\_computer* = yes

- Rule antecedent/precondition vs. rule consequent

- Assessment of a rule: *coverage* and *accuracy*

- $n_{\text{covers}}$  = # of tuples covered by R

- $n_{\text{correct}}$  = # of tuples correctly classified by R

$\text{coverage}(R) = n_{\text{covers}} / |D|$  /\* D: training data set \*/

$\text{accuracy}(R) = n_{\text{correct}} / n_{\text{covers}}$

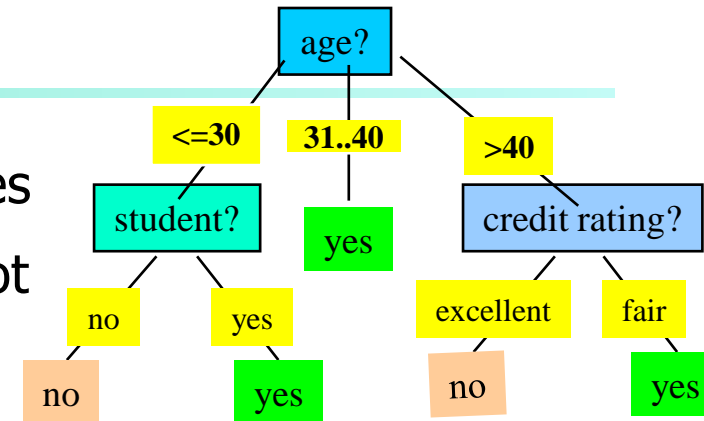
- If more than one rule is triggered, need **conflict resolution**

- Size ordering: assign the highest priority to the triggering rules that has the “toughest” requirement (i.e., with the *most attribute test*)
- Class-based ordering: decreasing order of *prevalence or misclassification cost per class*
- Rule-based ordering (**decision list**): rules are organized into one long priority list, according to some measure of rule quality or by experts



# Rule Extraction from a Decision Tree

- Rules are easier to understand than large trees
- One rule is created for each path from the root to a leaf
- Each attribute-value pair along a path forms a conjunction: the leaf holds the class prediction
- Rules are mutually exclusive and exhaustive
- Example: Rule extraction from our *buys\_computer* decision-tree



IF *age* = young AND *student* = *no*

THEN *buys\_computer* = *no*

IF *age* = young AND *student* = *yes*

THEN *buys\_computer* = *yes*

IF *age* = mid-age

THEN *buys\_computer* = *yes*

IF *age* = old AND *credit\_rating* = *excellent* THEN *buys\_computer* = *yes*

IF *age* = young AND *credit\_rating* = *fair* THEN *buys\_computer* = *no*

# Rule Extraction from the Training Data

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- Sequential covering algorithm: Extracts rules directly from training data
- Typical sequential covering algorithms: FOIL, AQ, CN2, RIPPER
- Rules are learned *sequentially*, each for a given class  $C_i$  will cover many tuples of  $C_i$  but none (or few) of the tuples of other classes
- Steps:
  - Rules are learned one at a time
  - Each time a rule is learned, the tuples covered by the rules are removed
  - The process repeats on the remaining tuples unless *termination condition*, e.g., when no more training examples or when the quality of a rule returned is below a user-specified threshold
- Comp. w. decision-tree induction: learning a set of rules *simultaneously*

# How to Learn-One-Rule?

- Star with the most general rule possible: condition = empty
- Adding new attributes by adopting a greedy depth-first strategy
  - Picks the one that most improves the rule quality
- Rule-Quality measures: consider both coverage and accuracy
  - Foil-gain (in FOIL & RIPPER): assesses info\_gain by extending condition

$$FOIL\_Gain = pos' \times (\log_2 \frac{pos'}{pos' + neg'} - \log_2 \frac{pos}{pos + neg})$$

It favors rules that have high accuracy and cover many positive tuples

- Rule pruning based on an independent set of test tuples

$$FOIL\_Prune(R) = \frac{pos - neg}{pos + neg}$$

Pos/neg are # of positive/negative tuples covered by R.

If *FOIL\_Prune* is higher for the pruned version of R, prune R