

12.2 – Conditional Probability

Conditional Probability

- Conditional Probability contains a condition that may limit the sample space for an event.
- You can write a conditional probability using the notation

$$P(B|A)$$

- This reads “the probability of event B, given event A”

Conditional Probability

The table shows the results of a class survey.

Find $P(\text{own a pet} \mid \text{female})$

Do you own a pet?

	yes	no
female	8	6
male	5	7

14 females;

13 males

The condition female limits the sample space to 14 possible outcomes.

Of the 14 females, 8 own a pet.

Therefore, $P(\text{own a pet} \mid \text{female})$ equals $\frac{8}{14}$.

Conditional Probability

The table shows the results of a class survey.

Find $P(\text{wash the dishes} \mid \text{male})$

Did you wash the dishes last night?

	yes	no
female	7	6
male	7	8

13 females;

15 males

The condition male limits the sample space to 15 possible outcomes.

Of the 15 males, 7 did the dishes.

Therefore, $P(\text{wash the dishes} \mid \text{male}) = \frac{7}{15}$

Let's Try One

Using the data in the table, find the probability that a sample of not recycled waste was plastic. $P(\text{plastic} \mid \text{non-recycled})$

The given condition limits the sample space to non-recycled waste.

A favorable outcome is non-recycled plastic.

Material	Recycled	Not Recycled
Paper	34.9	48.9
Metal	6.5	10.1
Glass	2.9	9.1
Plastic	1.1	20.4
Other	15.3	67.8

$$\begin{aligned} P(\text{plastic} \mid \text{non-recycled}) &= \frac{20.4}{48.9 + 10.1 + 9.1 + 20.4 + 67.8} \\ &= \frac{20.4}{156.3} \\ &\approx 0.13 \end{aligned}$$

The probability that the non-recycled waste was plastic is about 13%.

Conditional Probability Formula

- For any two events A and B from a sample space with $P(A)$ does not equal zero

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

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Conditional Probability

Researchers asked people who exercise regularly whether they jog or walk. Fifty-eight percent of the respondents were male. Twenty percent of all respondents were males who said they jog. Find the probability that a male respondent jogs.

Relate: $P(\text{male}) = 58\%$
 $P(\text{male and jogs}) = 20\%$

Define: Let A = male.
Let B = jogs.

Write:
$$P(A|B) = \frac{P(A \text{ and } B)}{P(A)}$$

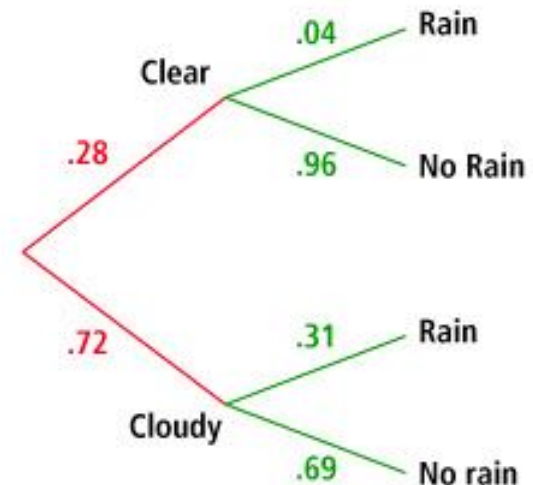
$$= \frac{0.2}{0.58}$$
$$\approx 0.344$$

Substitute 0.2 for $P(A \text{ and } B)$ and 0.58 for $P(A)$.
Simplify.

The probability that a male respondent jogs is about 34%.

Using Tree Diagrams

Jim created the tree diagram after examining years of weather observations in his hometown. The diagram shows the probability of whether a day will begin clear or cloudy, and then the probability of rain on days that begin clear and cloudy.



- a. Find the probability that a day will start out clear, and then will rain.

The path containing clear and rain represents days that start out clear and then will rain.

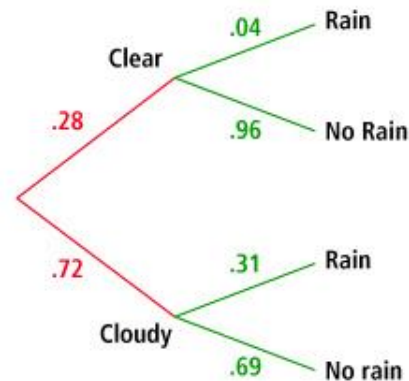
$$\begin{aligned} P(\text{clear and rain}) &= P(\text{rain} \mid \text{clear}) \cdot P(\text{clear}) \\ &= 0.04 \cdot 0.28 \\ &= 0.011 \end{aligned}$$

The probability that a day will start out clear and then rain is about 1%.

Conditional Probability

(continued)

- b. Find the probability that it will not rain on any given day.



The paths containing clear and no rain and cloudy and no rain both represent a day when it will not rain. Find the probability for both paths and add them.

$$\begin{aligned} P(\text{clear and no rain}) + P(\text{cloudy and no rain}) &= \\ P(\text{clear}) \cdot P(\text{no rain} \mid \text{clear}) + P(\text{cloudy}) \cdot P(\text{no rain} \mid \text{cloudy}) &= \\ = 0.28(.96) + .72(.69) &= \\ = 0.7656 \end{aligned}$$

The probability that it will not rain on any given day is about 77%.

Let's Try One

Pg 688

- A survey of Pleasanton Teenagers was given.
 - 60% of the responders have 1 sibling; 20% have 2 or more siblings
 - Of the responders with 0 siblings, 90% have their own room
 - Of the respondents with 1 sibling, 20% do not have their own room
 - Of the respondents with 2 siblings, 50% have their own room

Create a tree diagram and determine

A) $P(\text{own room} \mid 0 \text{ siblings})$

B) $P(\text{share room} \mid 1 \text{ sibling})$

- 60% of the responders have 1 sibling; 20% have 2 or more siblings
- Of the responders with no siblings, 90% have their own room
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Create a tree diagram and determine

- A) $P(\text{own room} \mid 0 \text{ siblings})$
- B) $P(\text{share room} \mid 1 \text{ sibling})$