Automatic differentiation to simultaneously identify nonlinear dynamics and extract noise probability distributions from data

Systèmes à modéliser

Problématique :

- Trouver les équations différentielles du modèle physique
- Utilisation d’une extension de LASSO (SINDy)
- Utilisatation de la différentiation automatique
- Trouver le bruit et comparer avec l’approche NN et une modélisation gaussienne

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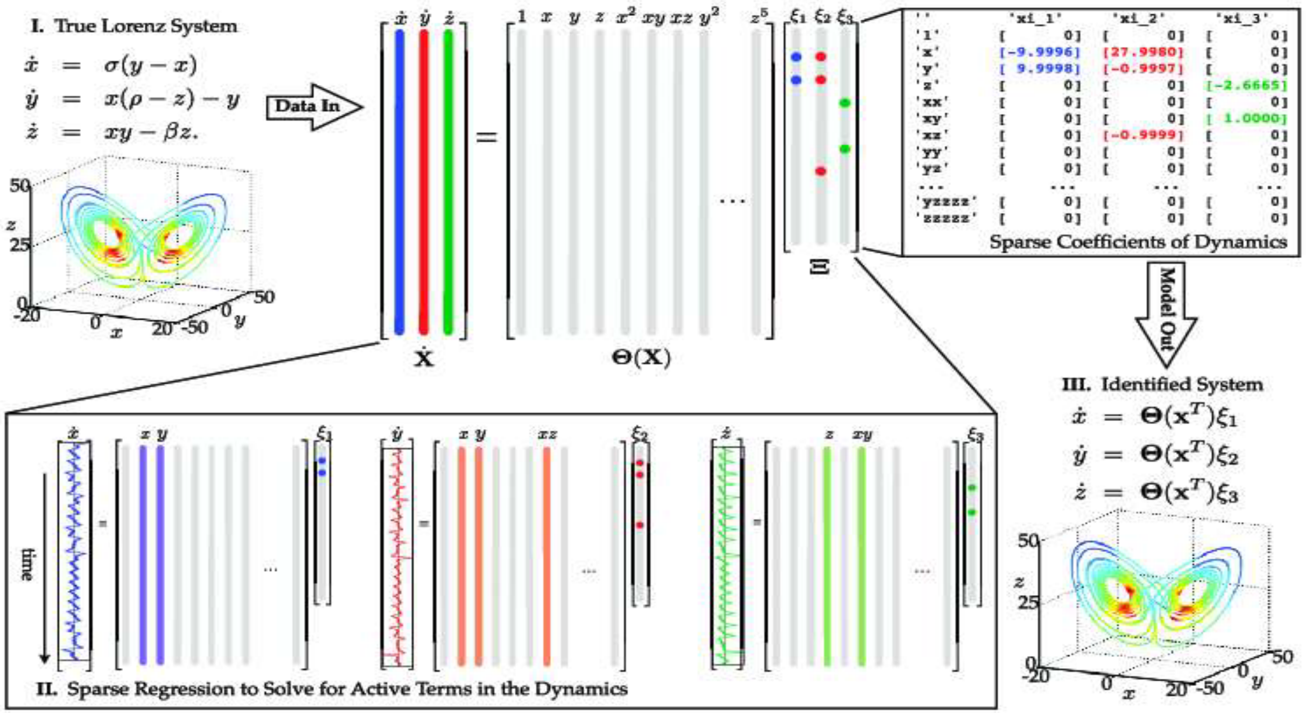
PySINDy is a sparse regression package with several implementations for the Sparse Identification of Nonlinear Dynamical systems (SINDy) method introduced in Brunton et al. (2016a), including the unified optimization approach of Champion et al. (2019), SINDy with control from Brunton et al. (2016b), Trapping SINDy from Kaptanoglu et al. (2021), SINDy-PI from Kaheman et al. (2020), PDE-FIND from Rudy et al. (2017), and so on. A comprehensive literature review is given in de Silva et al. (2020) and Kaptanoglu, de Silva et al. (2021).

I) Algorithme SINDy

$$\min_{\Theta} \|Y - X\Theta\|^2 + \lambda \|\Theta\|_1$$



Ajout différentiation automatique

$$L = ||x(t+1) - x(t) - \Delta t \cdot f(x(t))||^2 + \lambda||\Theta||_1$$

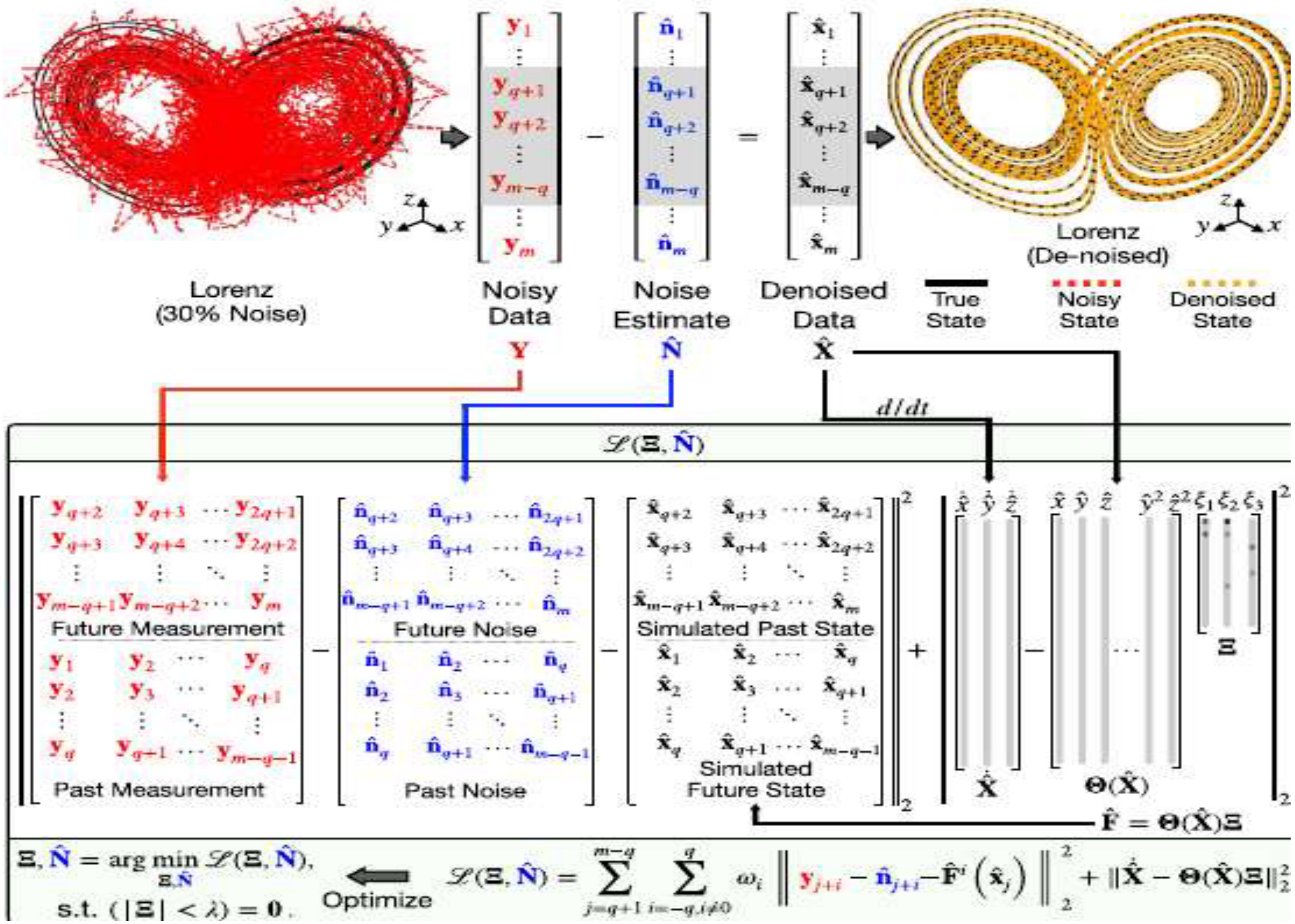
$$\begin{aligned} \boldsymbol{\Xi}, \hat{\mathbf{N}} &= \arg\min_{\boldsymbol{\Xi}, \hat{\mathbf{N}}} \mathcal{L}(\boldsymbol{\Xi}, \hat{\mathbf{N}}), \\ \text{s.t.} &\quad (|\boldsymbol{\Xi}| < \lambda) = 0. \end{aligned}$$

On a ainsi **initialisé** dans notre algorithme la valeur :
$\Xi = \mathrm{SINDy}(X, \Theta(X), \lambda)$

Ensuite, on cherche **en itérant** à **optimiser** la gestion du bruit en minimisant la distance :

$$\mathcal{E}_d = |\hat{X}' - \Theta(\hat{X})\Xi|_2^2 \tag{1}$$

II) Prise en compte du bruit y(t) = x(t)+n(t)



III) Résultat SINDy modifié

$$E_N = \frac{1}{m}\sum_{i=1}^{m} \|n_i - \hat{n}_i\|_2^2$$

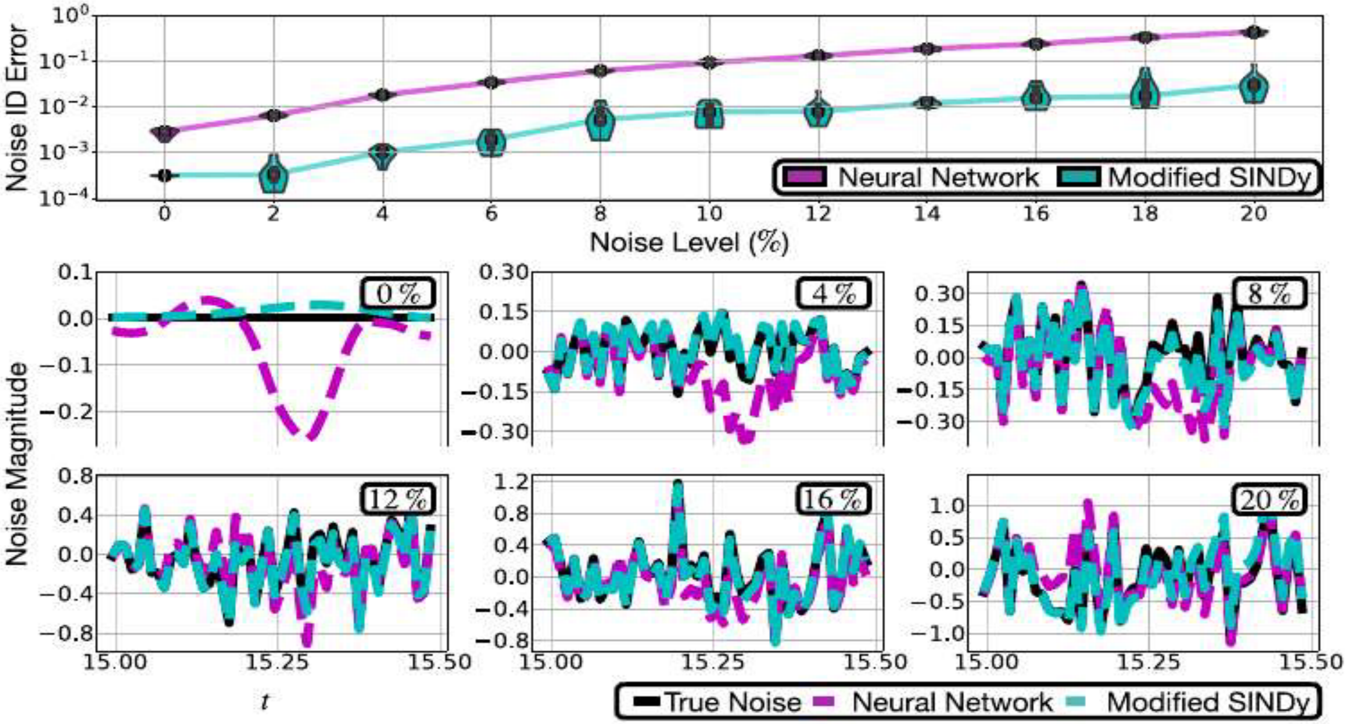


Figure 2. Top: Noise identification error of modified SINDy (labeled as SINDy) and NN denoising approach by Rudy *et al* [32]. The black circle represents the median of ten runs while the violin shape represents the distribution of error. The modified SINDy approach shows better noise identification error. Bottom: Comparison between the average noise applied to the Lorenz system and the noise identified by the two approaches. As shown on the left, both approaches can not produce the correct zero noise result when no noise is applied, which happens since there is a tiny difference between the learned dynamics and true dynamics.

Influence de l'hyperparamètre lambda (parcimonie vs exactitude)

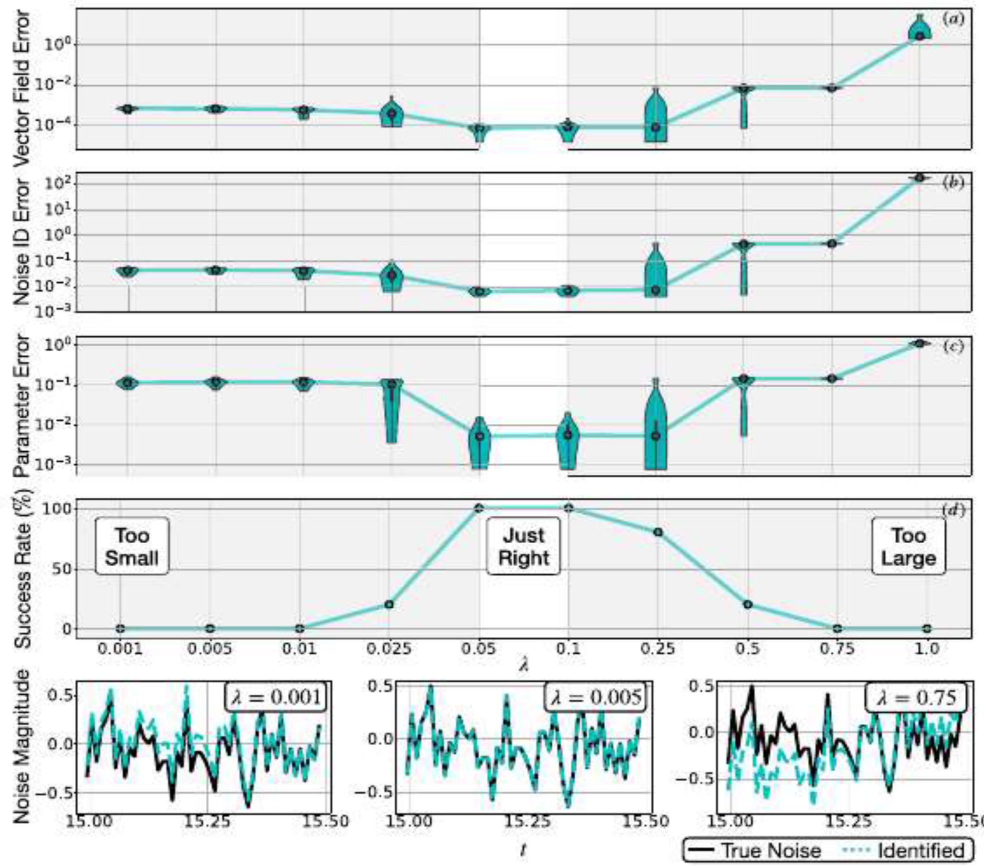


Figure 11. This figure shows how the choice of sparsity parameter λ will effect modified SINDy performance. If λ is too small, modified SINDy will not converge to the correct model in a short range of time and will be stuck in the local minimum. On the contrary, if the sparsity parameter is too large, the identified model will miss the necessary term to build the correct model. Thus, the value of λ needs to be tuned properly to determine the accurate model.

Tests avec différentes distributions

represents the scale.

True distribution	State	True parameter	Identified distribution	Identified Parameters
Gaussian	x	$\mu = 0, \sigma = 0.1413$	Gaussian	$\hat{\mu} = 0.003, \hat{\sigma} = 0.1451$
	y	$\mu = 0, \sigma = 0.1439$	Gaussian	$\hat{\mu} = 0.009, \hat{\sigma} = 0.1439$
Uniform	x	$\mu = 0, \sigma = 0.1413$	Uniform	$\hat{\mu} = -0.0717, \hat{\sigma} = 0.1438$
	y	$\mu = 0, \sigma = 0.1439$	Uniform	$\hat{\mu} = -0.0729, \hat{\sigma} = 0.1466$
Gamma	x	$k = 1, \text{loc} = 0, \theta = 0.1413$ $\mu = 0.1413, \sigma = 0.02$	Gamma	$k = 3.2714, \text{loc} = -0.095, \theta = 0.0722$ $\hat{\mu} = 0.1409, \hat{\sigma} = 0.0211$
	y	$k = 1, \text{loc} = 0, \theta = 0.1439$ $\mu = 0.1439, \sigma = 0.021$	Gamma	$k = 10.49, \text{loc} = -0.3105, \theta = 0.0432$ $\hat{\mu} = 0.1419, \hat{\sigma} = 0.0217$
Dweibull	x	$c = 2.07, \text{loc} = 0,$ $\text{scale} = 0.1413$	Dweibull	$\hat{c} = 2.064, \text{loc} = 0.8 \times 10^{-5},$ $\text{scale} = 0.1408$
	y	$c = 2.07, \text{loc} = 0,$ $\text{scale} = 0.1439$	Dweibull	$\hat{c} = 2.048, \text{loc} = -2.8 \times 10^{-5},$ $\text{scale} = 0.1438$
Rayleigh	x	$\mu = 0.1775, \sigma = 0.0085$	Rayleigh	$\hat{\mu} = 0.1775, \hat{\sigma} = 0.0085$
	y	$\mu = 0.1779, \sigma = 0.0086$	Rayleigh	$\hat{\mu} = 0.1779, \hat{\sigma} = 0.0086$

Merci de votre attention