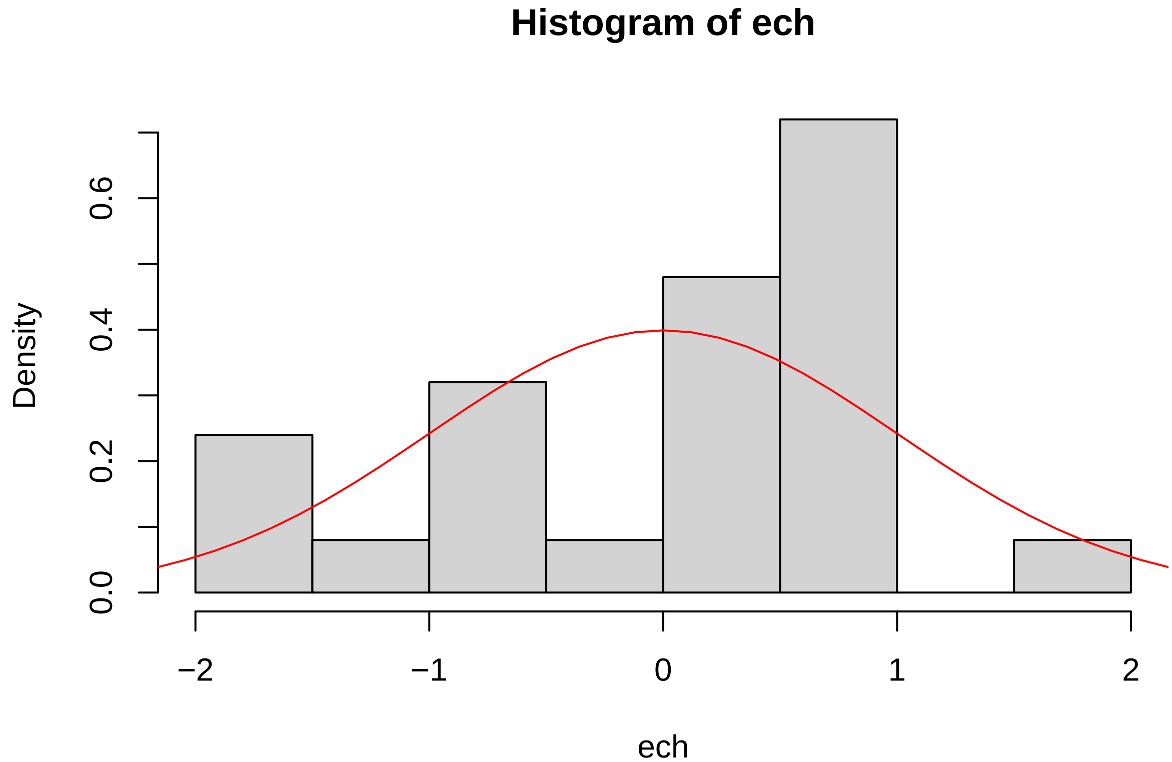
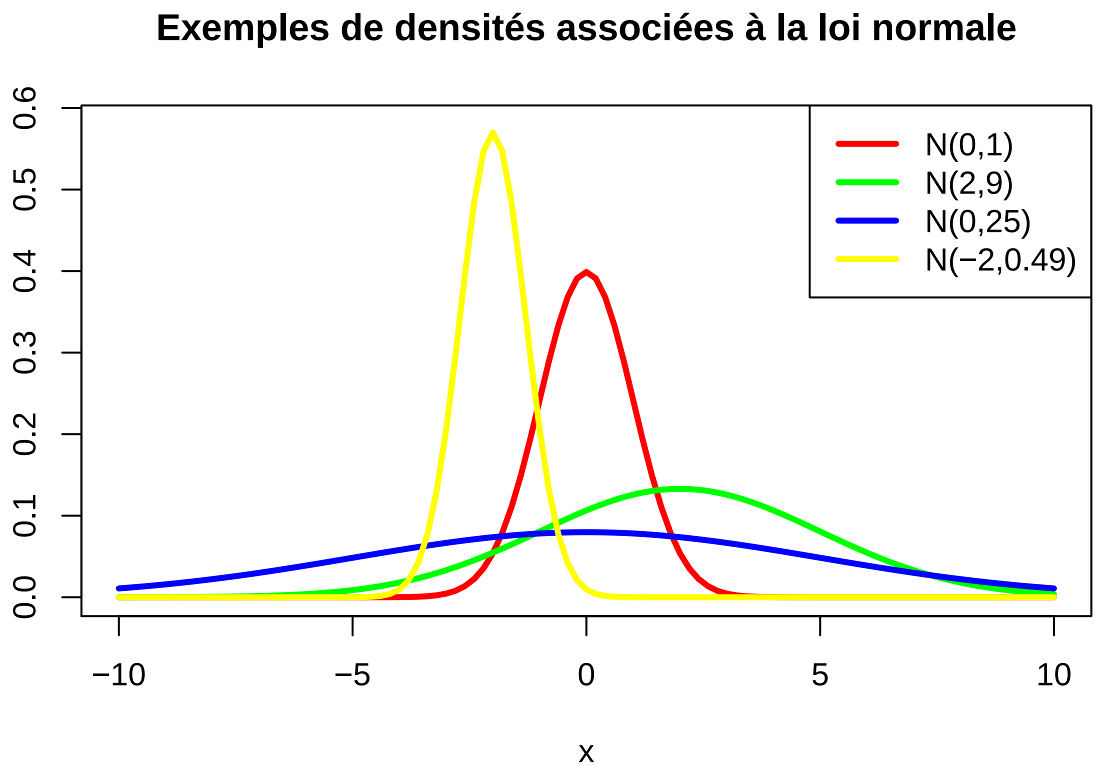
TP4

08/04/2022

#Question 1

```
##  [1]  0.5055155  0.8404445  0.6289685  0.5004590  0.5478886  0.5513299
##  [7] -0.4179851 -1.8808010  0.7642944  0.2137405  0.1164113 -1.8785862
## [13]  0.1024688  0.8290577 -1.7086217 -0.9184384  0.5765443  0.1239372
## [19] -0.6864768 -1.2643768
```





#Question 2

La densité d'une loi normale s'écrit

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

Avec

$$\theta = (m, \sigma)$$

, la vraisemblance s'écrit

$$L(x_1, ..., x_n, \theta) = \frac{1}{\sigma\sqrt{2\pi}} \prod e^{-\frac{(xi-m)^2}{2\sigma^2}}$$

d'où

$$lnL(x_1, ..., x_n, \theta) = -nln(\sigma) - n\frac{ln(2\pi)}{2} - \frac{1}{2\sigma^2} \sum (xi - m)^2$$

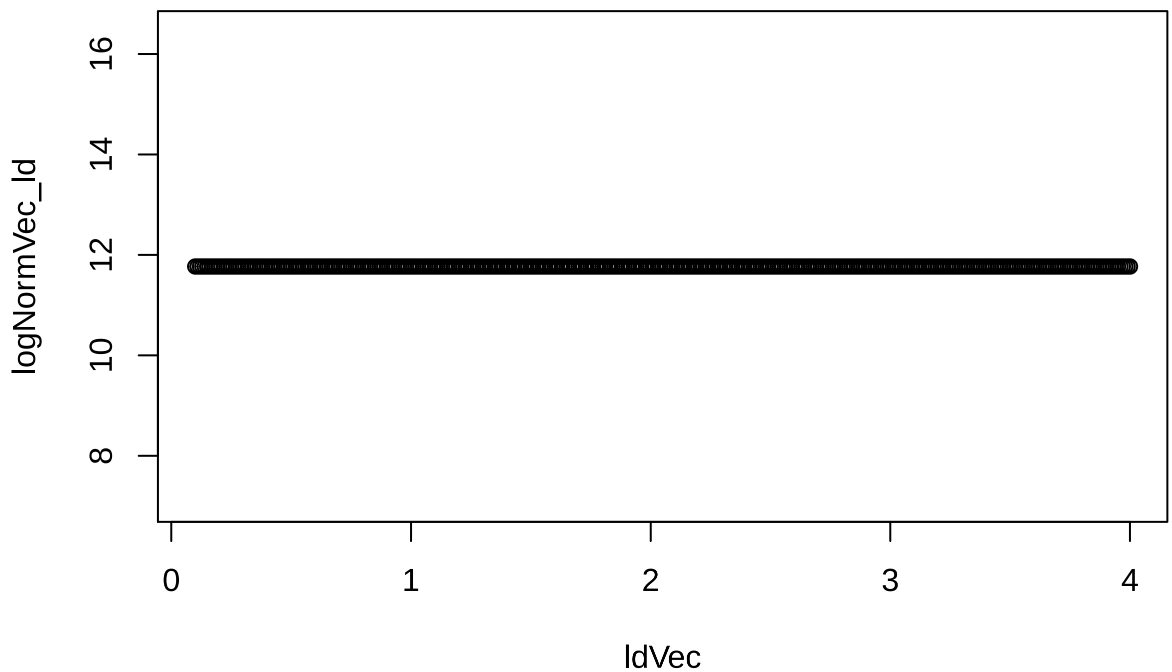
On trouve donc en résolvant les équations de ln L (dérivée nulle):

$$\hat{\mu} = \bar{xn}$$

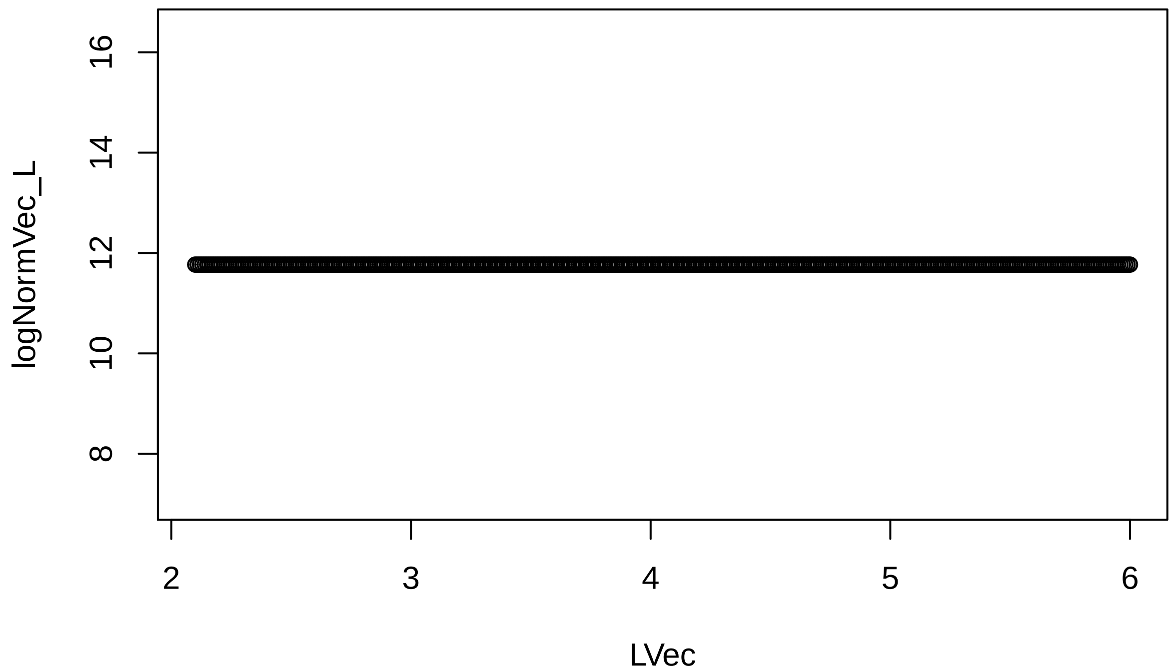
$$\hat{\sigma}\check{s} = \frac{1}{n} \sum (xi - \bar{xn})^2$$

```
log_norm = function(mu, std, X) {
  n = length(X)
  return(-n*log(std)+0.5*n*log(2*pi)-(1/(2*std**2))*sum((X-mu)**2))
}
```

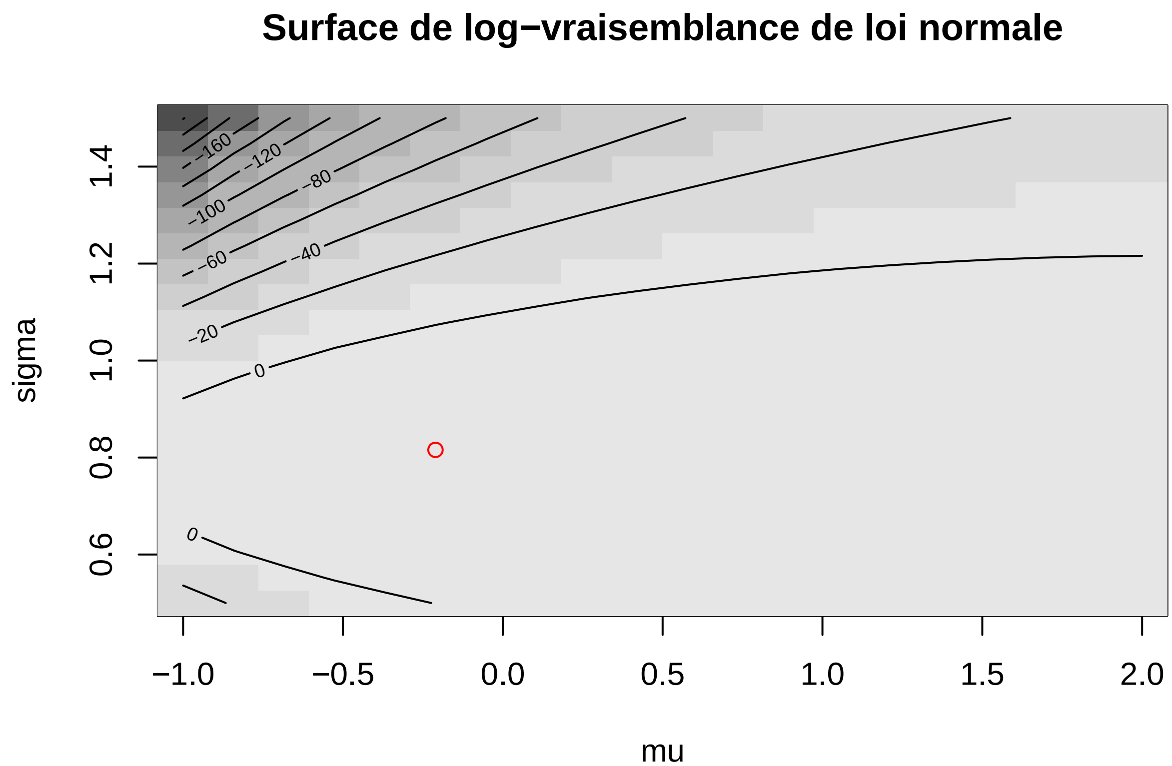
```
plot(ldVec, logNormVec_ld);
```




```
plot(LVec, logNormVec_L);
```

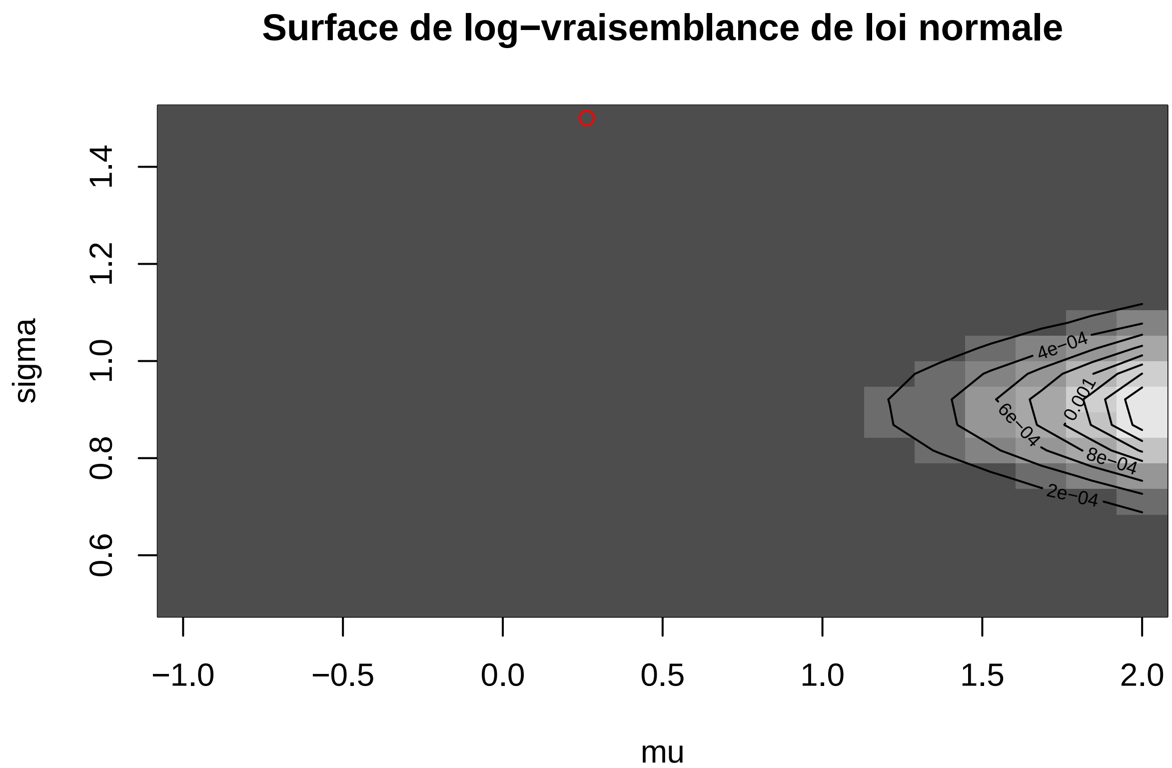


logNormVec_L
8
10
12
14
16
2
3
4
5
6
LVec



On remarque que le maximum se situe légèrement au dessus d'un axe de symétries entre une parabole concave entourées par des paraboles convexes #Question 3

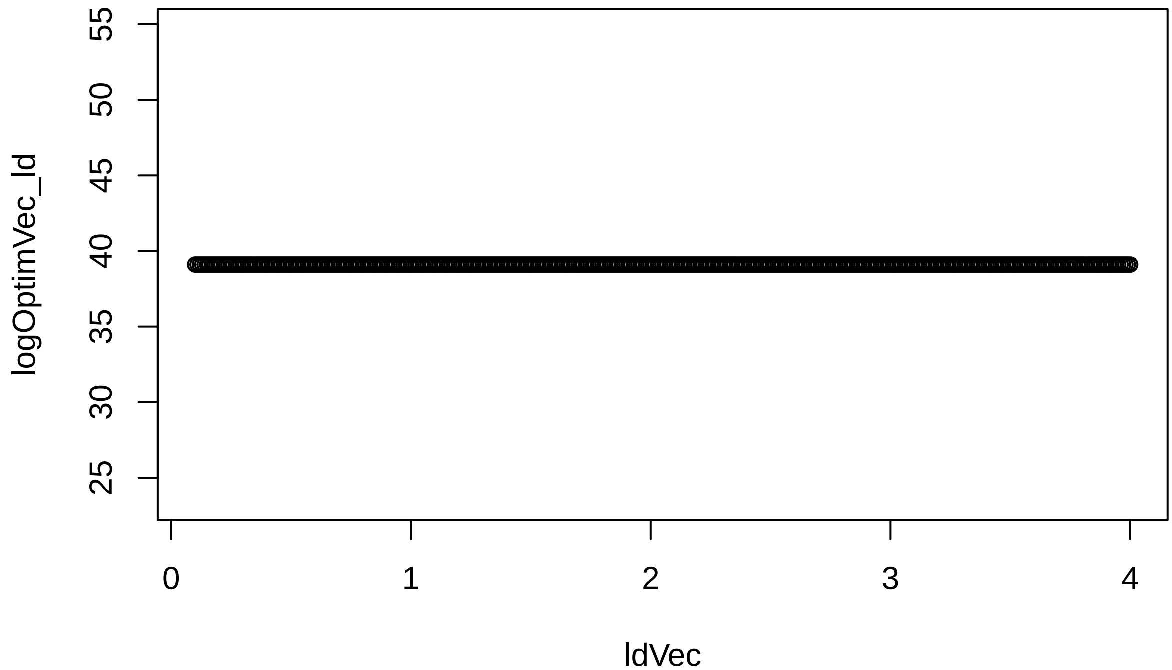
```
vraisemblance_norm = function(mu, std, X) {
  n = length(X)
  return(1/(std*((2*pi)**(1/2)))*prod(exp(-(X-mu)**2/(2*std**2))))
}
```

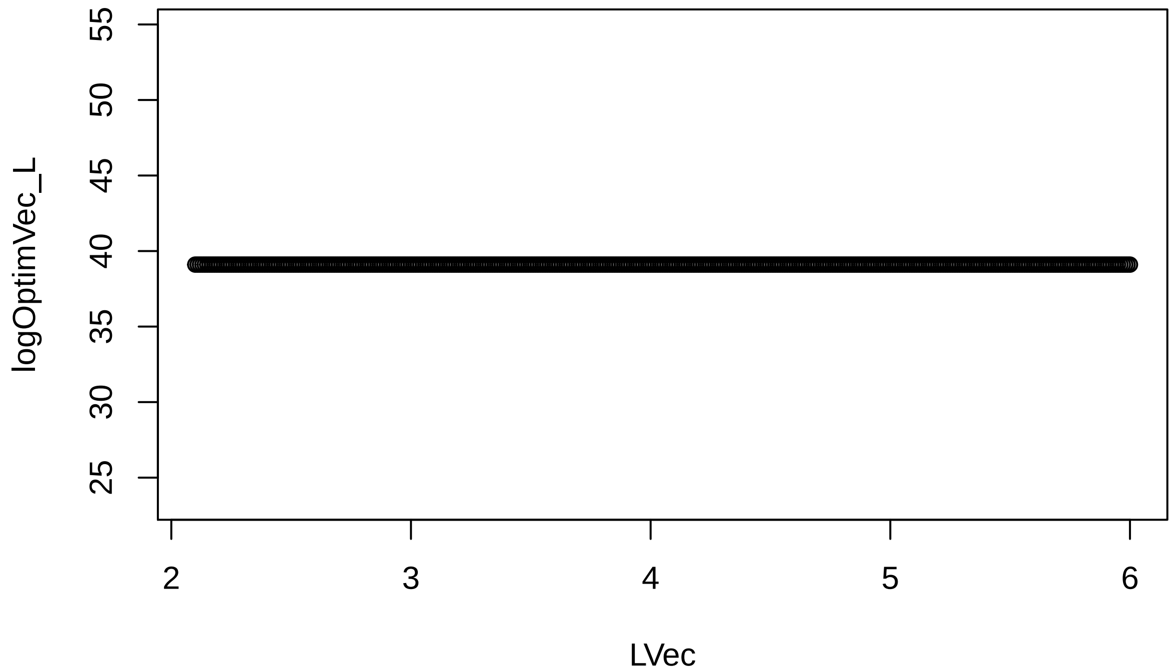


#Question 4

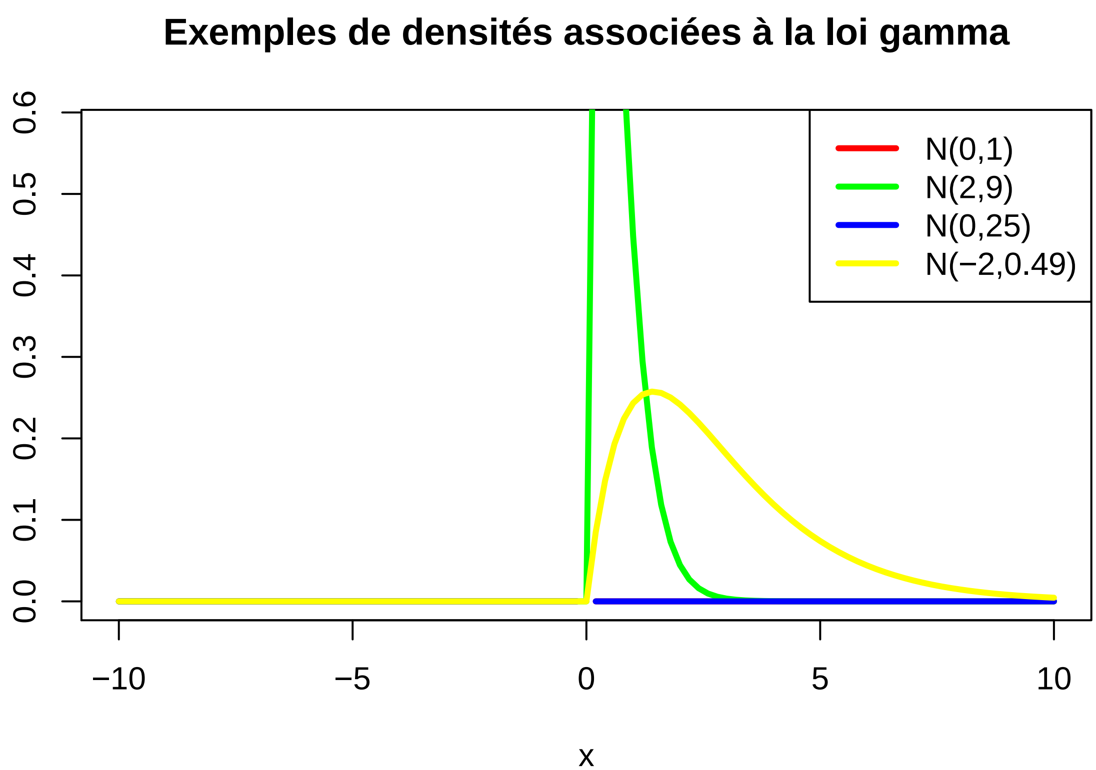
#Question 5

```
log_optim <- function(mu, std, X){
    accu <- 0
    n=length(X)
    for (i in seq(from=1, to=n, by=1)) {
      accu = accu + (X[i] - mu)**2
    }
   return(0.5*(n * log(2*pi*(std**2)) + 1/(std**2) * accu))
}
```


logOptimVec_ld
55
50
45
40
35
30
25
0
1
2
3
4
ldVec



#Question 7



#Question 8

Soit

$$a \geq 0, b \geq 0$$

La densité d'une loi gamma s'écrit

$$f_{(a,b)}(x) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}, x \geq 0$$

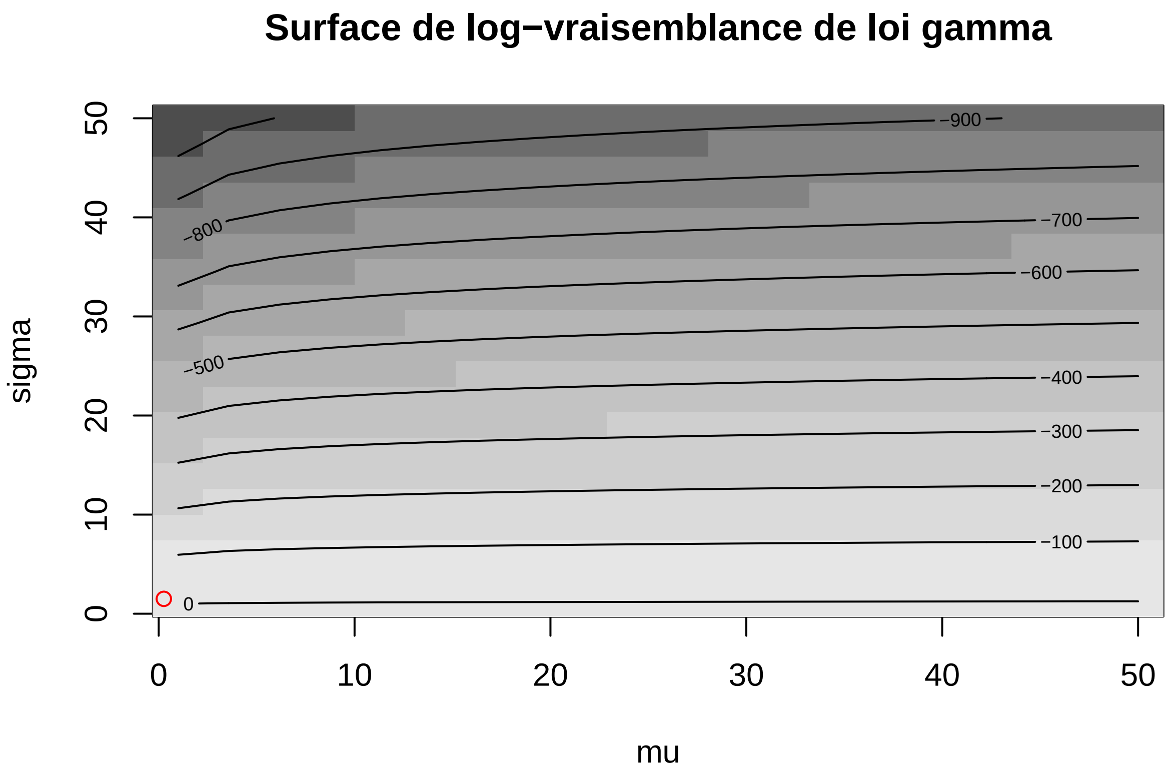
$$lnL(x_1, ..., x_n, a, b) = naln(b) - nln(\Gamma(a)) + (a-1)\sum ln(xi) - b\sum xi$$

On trouve donc on résolvant les équations de ln L:

$$\hat{a} = \frac{\bar{xn}^2}{\frac{1}{n}\sum(xi - \bar{xn})^2}$$

$$\hat{b} = \frac{\bar{xn}}{\frac{1}{n}\sum(xi - \bar{xn})^2}$$

```
log_gamma = function(a, b, X) {
  n = length(X)
  return(n*a*log(b)-n*log(gamma(a)) + (a-1)*sum(log(X)) -b *sum(X))
}
```



On remarque des lignes courbées sans maximum

#Question 9