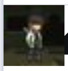
← Notes



Number Theory - 1

267 Number Theory Primes CodeMonk

Introduction:

Problems in competitive programming which involve Mathematics are are usually about number theory, or geometry. If you know number theory, that increases your ammo heavily in solving a lot of tougher problems, and helps you in getting a strong hold on a lot of other problems, too.

Problems in competitive programming require insight, so just knowing some topics is not enough at all. All of the problems requires more or less math tough. For instance, solving large systems of equations and approximating solutions to differential equations.

Modulo:
Modulo operation gives the remainder after division, when one number is divided by another. It is denoted by % sign.

Example:
We have two numbers 5 and 2, then 5%2 is 1 as when 5 is divided by 2, it leaves 1 as remainder.

Properties:
Some of the properties of modulo are:
(a+b)%c = (a%c + b%c)%c.
(a * b)%c = ((a%c) * (b%c))%c.

Example:
Let's say a = 5, b = 3, c = 2.
Then:
1) (5+3)%2 = 8%2 = 0.
Similarly (5%2 + 3%2)%2 = (1 + 1)%2 = 0.
2) (5 * 3)%2 = 15%2 = 1.
Similarly ((5%2) * (3%2))%2 = (1 * 1)%2 = 1.

Greatest Common Divisor
Greatest Common Divisor (GCD) of two or more numbers is the largest positive number that divides all the numbers which are being taken into consideration.
For example: GCD of 6, 10 is 2 since 2 is the largest positive number that divides both 6 and 10.

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Naive Approach:
We can traverse over all the numbers from min(A, B) to 1 and check if the current number divides both A and B or not. If it does, then it will be the GCD of A and B.

```
int GCD(int A, int B) {
    int m = min(A, B), gcd;
    for(int i = m; i > 0; --i)
        if(A % i == 0 && B % i == 0) {
            gcd = i;
            return gcd;
        }
}
```

Time Complexity: Time complexity of this function is O(min(A, B)).

Euclid's Algorithm:

Idea behind Euclid's Algorithm is GCD(A, B) = GCD(B, A % B). The algorithm will recurse until A % B = 0.

```
int GCD(int A, int B) {
    if(B==0)
        return A;
    else
        return GCD(B, A % B);
}
```

Let us take an example.
Let A = 16, B = 10.
GCD(16, 10) = GCD(10, 16 % 10) = GCD(10, 6)
GCD(10, 6) = GCD(6, 10 % 6) = GCD(6, 4)
GCD(6, 4) = GCD(4, 6 % 4) = GCD(4, 2)
GCD(4, 2) = GCD(2, 4 % 2) = GCD(2, 0)

Since B = 0 so GCD(2, 0) will return 2.

Time Complexity: The time complexity of Euclid's Algorithm is **O(log(max(A, B)))**.

Extended Euclid Algorithm:
This is the extended form of Euclid's Algorithm explained above. GCD(A,B) has a special property that it can always be represented in the form of an equation, i.e., Ax + By = GCD(A, B).

This algorithm gives us the coefficients (x and y) of this equation which will be later useful in finding the Modular Multiplicative Inverse. These coefficients can be zero or negative ir

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their value. This algorithm takes two inputs as A and B and returns GCD(A, B) and coefficients of the above equations as output.

Implementation:

```
#include < iostream >

int d, x, y;
void extendedEuclid(int A, int B) {
    if(B == 0) {
        d = A;
        x = 1;
        y = 0;
    }
    else {
        extendedEuclid(B, A%B);
        int temp = x;
        x = y;
        y = temp - (A/B)*y;
    }
}

int main( ) {
extendedEuclid(16, 10);
cout << ”The GCD of 16 and 10 is ” << d << endl;
cout << ”Coefficient x and y are: ”<< x <<  “and  “ << y << endl;
return 0;
}
```

Output:

```
The GCD of 16 and 10 is 2
Coefficient x and y are: 2 and -3
```

Initially, Extended Euclid Algorithm will run as Euclid Algorithm until we get GCD(A, B) or until B gets 0 and then it will assign x = 1 and y = 0. As B = 0 and GCD(A, B) is A in the current condition, so equation Ax + By = GCD(A, B) will be changed to A * 1 + 0 * 0 = A.

So the values of d, x, y in the whole process of extendedEuclid( ) function are:

1) d=2, x = 1, y = 0.
2) d=2, x = 0 , y = 1 - (4/2) * 0 = 1.
3) d=2, x = 1 , y = 0 - (6/4) * 1 = -1.

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4) d=2, x = -1 , y = 1 - (10/6) * -1 = 2.
5) d=2 , x= 2, y = -1 - (16/10) * 2 = -3.

Time Complexity: The time complexity of Extended Euclid’s Algorithm is **O(log(max(A, B)))**.

Prime Numbers

Prime numbers are the numbers greater than 1 that have only two factors, 1 and itself.

Composite numbers are the numbers greater that 1 that have at least one more divisor other than 1 and itself.

For example, 5 is prime number as 5 is divisible by 1 and 5 only. But, 6 is a composite number as 6 is divisible by 1, 2, 3 and 6.

There are different methods to check if the number is prime or not.

Naive Approach:

We will traverse through all the numbers from 1 to N and count the number of divisors. If the number of divisors are equal to 2 then the given number is prime otherwise it is not.

```
void checkprime(int N){
     int count = 0;
     for( int i = 1;i <= N;++i )
          if( N % i == 0 )
               count++;
     if(count == 2)
          cout << N << “ is a prime number.” << endl;
     else
          cout << N << “ is not a prime number.” << endl;
}
```

Time Complexity: Time complexity of this function is **O(N)** as we are traversing from 1 to N.

A slightly better approach:

Consider two positive numbers N and D such that N is divisible by D and D is less than the square root of N. Then, (N / D) must be greater than the square root of N. N is also divisible by (N / D). So, if there is a divisor of N that is less than square root of N then there will be a divisor of N that is greater than square root of N. Therefore, we just need to traverse till the square root of N.

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```
void checkprime(int N) {
     int count = 0;
     for( int i = 1;i * i <=N;++i ) {
          if( N % i == 0) {
          if( i * i == N )
                          count++;
                    else
// i < sqrt(N) and (N / i) > sqrt(N)
                          count += 2;
      }
     }
     if(count == 2)
          cout << N << “ is a prime number.” << endl;
     else
          cout << N << “ is not a prime number.” << endl;
}
```

Time complexity: Time complexity of this function is O(sqrt(N)) as we are traversing from 1 to sqrt(N).

Sieve of Eratosthenes:

The Sieve of Eratosthenes can be used to find all the prime numbers less than or equal to a given number N. It can also be used to find out if a number is prime or not, by just looking up at the sieve.

The basic idea behind the sieve of eratosthenes is that at each iteration, we pick one prime number and eliminate all the multiples of that prime number. After the elimination process ends, all the unmarked numbers which are left are prime!

Pseudo Code:

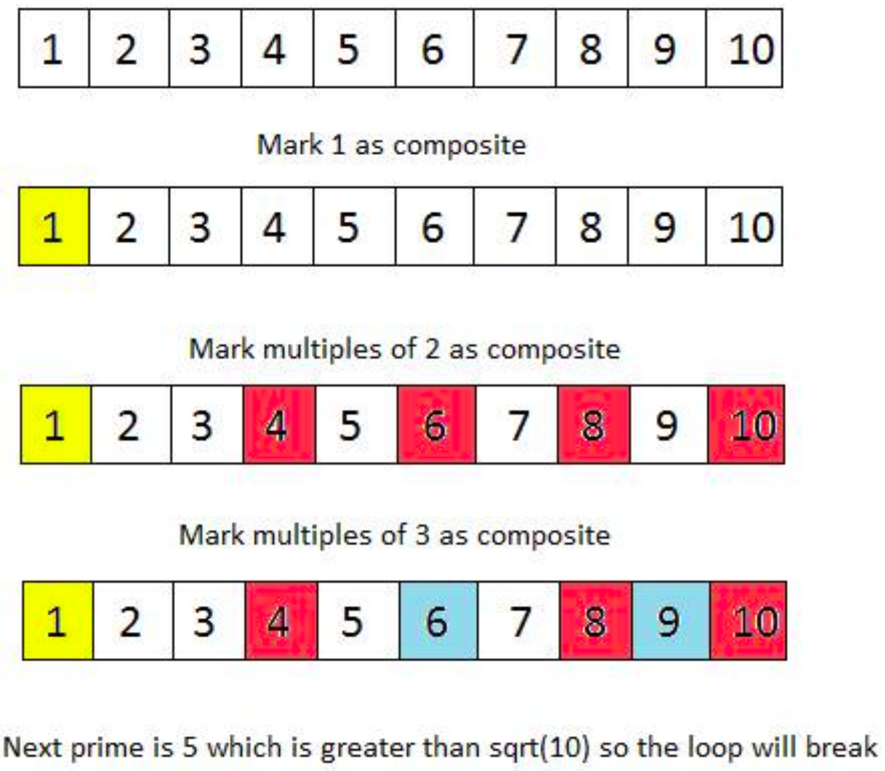
```
Mark all the numbers as prime numbers.
Traverse over all the prime numbers smaller than sqrt(N).
Mark the multiple of the prime numbers as composite numbers.
```

```
void sieve(int N) {
     bool isPrime[N+1];
     for(int i = 0; i <= N;++i) {
          isPrime[i] = true;
     }
     isPrime[0] = false;
     isPrime[1] = false;
     for(int i = 2; i * i <= N; ++i) {
```

?

```
        if(isPrime[i] == true) {
            // Mark all the multiples of i as composite numbers
            for(int j = i * i; j <= N ;j += i)
                isPrime[j] = false;
        }
    }
}
```

The above code will compute all the prime numbers that are smaller than or equal to N. Let us compute prime numbers where N = 10. First mark all the numbers as prime. Now mark 1 as composite. Then in each iteration we will select a prime and then mark its multiple as composite.


1 2 3 4 5 6 7 8 9 10
Mark 1 as composite
Mark multiples of 2 as composite
Mark multiples of 3 as composite
Next prime is 5 which is greater than sqrt(10) so the loop will break

So the prime numbers are 2, 3, 5 and 7.

Time Complexity:
The inner loop runs for each element.
if i = 2, inner loop runs N / 2 times
if i = 3, inner loop runs N / 3 times
if i = 5, inner loop runs N / 5 times
.

.

.

Total complexity: N * (½ + ⅓ + ⅕ + ...) = O(NloglogN)

Modular Exponentiation

Consider a problem in which you need to calculate a^b%c, where **%** is a modulo operator and b can be very large (i.e. in order of 10^{18}).

Naive Approach:

a^b%c can be rewritten as a * a * a * a *... upto b times. So in this approach we will multiply **a**, **b** times.

```
long long exponentiation(long long a, long long b, long long c) {
        long long ans = 1;
        for(int i = 1;i <= b;i++) {
            ans *= a;                           //multiplying
a, b times.
            ans %= c;
        }
    return ans;
}
```

In each iteration the variable **ans** will get multiplied by a. Also, special care needs to be taken that this value never exceeds 'c' in any iteration. Hence we take modulo of 'ans' with 'c' in each iteration. i.e., ans = ans%c.
The basic property that we exploit in this is: (x*y) mod n = ((x mod n) * (y mod n)) mod n.

So in the above code we have to calculate (ans*a)%c which is equal to ((ans%c)*(a%c))%c .

Complexity: O(b)

Modular exponentiation:
We can now clearly see that this approach is very inefficient, and we need to come up with something better. We can take care of this problem in O($\log_2$b) by using a technique called exponentiation by squaring. This uses only O($\log_2$b) squarings and O($\log_2$b) multiplications. This is a major improvement over the most naive method.

The process we would follow is:

```
  ans=1                               //Final answer which will be
displayed
  while(b !=0 ) {
     /*Finding the right most digit of 'b' in binary form, if it
is 1 , then multiply the current value of a
      in  ans. */
             if(b%2 == 1) {           //as if b%2 == 1,means last
/rightmost digit of b in binary form is 1.
                ans = ans*a ;
                 ans = ans%c;       //at each iteration if value of
ans exceeds then reduce it to modulo c.
             }
   a = a*a;                    // This is explained below
   a %= c;                     //at each iteration if value of a
exceeds then reduce it to modulo c.
   b /= 2;                     //Trim the right-most digit of b in
binary form.
}
```

To understand the above code : Let's say b = 45, then express 'b' in binary form. Note the positions of '1' in 'b' .
As in binary form b changes to $\{101101\}_2$

$a^b = a^{\{101101\}_2}$
As if we split the binary representation of b ,
$b = 1 * 2^5 + 0 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0$.
Then $a^b = (1 * (a^{2^5}) * (0 * (a^{2^4})) * (1 * (a^{2^3})) * (1 * (a^{2^2})) * (0 * (a^{2^1})) * (1 * (a^{2^0}))$

Now this can be computed easily in 5 iterations.

In every iteration the value of 'a' is made equal to 'a * a'.
As,
$a * (2^0) = a$
$a * (2^1) = a^2 = a * a$
$a * (2^2) = a^4 = ((a * a) * (a * a))$
and so on..

So, at 1st iteration a changes to a * a, then a * a changes to (a * a) * (a * a) and so on.
And also in each iteration, we multiply current value of a in ans, whenever we encounter a digit of b in binary form, which is equal to 1.

Processing:
1. Calculate $(5^{59})\%19$, where '%' stands for modulo operator.
express 59 in binary = $\{111011\}_2$
$5^{59} = 5^{\{111011\}_2} = 5^{2^5} * 5^{2^4} * 5^{2^3} * 5^{2^3} * 5^{2^1} * 5^{2^0}$

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Let’s compute its value in 6 iterations:

Let ans=1, a=5, b=59, c=19;

Iteration 1:
Since the rightmost digit of ‘b’(111011) is 1:
ans=(ans * a)%c = (1 * 5)%19 = 5
a=(a * a)%c = (5 * 5)%19 = 6
b /= 2 (b = 11101)

Iteration 2:
Since the rightmost digit of ‘b’(11101) is 1:
ans=(ans * a)%c = (5 * 6)%19 = 11
a=(a * a)%c = (6 * 6)%19 = 17
b /= 2 (b = 1110)

Iteration 3:
Since the rightmost digit of ‘b’(1110) is 0:
We won’t multiply anything to ans
a=(a * a)%c = (17 * 17)%19 = 4
b /= 2 (b = 111)

Iteration 4:
Since the rightmost digit of ‘b’(111) is 1:
ans=(ans * a)%c = (11 * 4)%19 = 6
a=(a * a)%c = (4 * 4)%19 = 16
b /= 2 (b = 11)

Iteration 5:
Since rightmost digit of ‘b’(11) is 1:
ans=(ans * a)%c = (6 * 16)%19 = 1
a=(a * a)%c = (16 * 16)%19 = 9
b /= 2 (b = 1)

Iteration 6:
Since rightmost digit of ‘b’(1) is 1:
ans=(ans * a)%c = (1 * 9)%19 = 9
a=(a * a)%c = (9 * 9)%19 = 5
b /= 2 (b = 0) //break

Hence, the final answer is ans = 9,

therefore (5^{59})%19 = 9

Time limit:
O($\log_2$(b))

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Memory limit:

O(1)

Complexity: O($\log_2(b)$) (number of digits present in binary notation of number ‘b’)

Implementation:

```
#include < cstdio >
#include < iostream >
using namespace std;

int modpowIter(int a, int b, int c) {
        int ans=1;
        while(b != 0) {
                if(b%2 == 1)
                        ans=(ans*a)%c;

                a=(a*a)%c;
                b /= 2;
        }
        return ans;
}
int main() {
        int a=5,b=59,c=19,ans,ans1;
        ans = modpowIter(a,b,c);
        cout << ans << endl;
}
```

Output:

```
9
```

You can use another recursive approach, to find $(a^b)\%c$.

We know that a^b can be written as:

$a^b = a^{2(b/2)}$ - If b is even, and b>0.

a^b= (a) * $a^{2(b-1/2)}$ - If b is odd.

a^b= 1 - If b is 0.

```
int modRecursion(int a, int b, int c)
{
```

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```
    if(b == 0)
    return 1;
    if(b == 1)
    return a%c;
    else if( b%2 == 0)
//if b is even
    {
        return modRecursion((a *a)%c,b/2,c);
    }
    else
// if b is odd.
    {
        return (a*modRecursion((a*a%c),b/2,c))%c;
    }
}
```

Let's take a = 2 ,b = 5 , c = 5, so modRecusrion(2, 5, 5).
1) As b is odd, it will be 2 * modRecusriion(2 * 2, 2, 5) = modRecursion(4, 2, 5).
2) Now in modRecursion(4 , 2, 5) as b is even, it will move to modRecusrion(16%5, 1, 5) = modRecursion(1, 1, 5).
3) Now as in modRecursion(1, 1, 5), B =1 ,therefore it will return 1.
4) Therefore modRecusrion(4, 2, 5) will return 1 and as in first statement modRecusrion(2, 5, 5) will return 2 * modRecusrion(4, 2, 5) = 2 * 1 = 2.
So final answer will be 2 here.

Complexity: O($\log_2(b)$)

//Change the data types according to the problem statement

In many problems ‘c’ might be given as 10^9+7. Hence, data types of all variables must be kept “long long int”.

Solve Problems

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Prerak Atolia 5 years ago

The below expression in Modular Exponentiation is wrong :
b = 1 * 25 + 0 * 24 + 1 * 23 + 1 * 22+ 0 * 21 + 1 * 20 .
Then a^b = (1 * (a ^2^5) * (0 * (a ^2^4)) * (1 * (a^ 2^3)) * (1 * (a ^2^2)) * (0 * (a ^2^1)) * (1 * (a ^2^0))
This makes a^b = 0.

The correct expression should be
Then a^b = (a ^ (1*2^5) * (a ^ (0 * 2^4)) * (a ^ (1 * 2^3)) * (a ^ (1 * 2^2)) * (a ^ (0 * 2^1)) * (a ^ (1 * 2^0))

▲ 12 votes ● Reply ● Message ● Permalink

Om AnirudhChellapilla 4 years ago

Yup!

▲ 0 votes ● Reply ● Message ● Permalink

Nichit Bodhak Goel 4 years ago

isn't there a typo in Sieve of Eratosthenes code initial value of j should be j=i+i or j=2*i?

▲ 0 votes ● Reply ● Message ● Permalink

Atul Kumar Edited 4 years ago

Nope it's correct, as the loop progress you can see that first multiple of current j is i*i. Consider an example the first multiple of 3 after all the multiple of 2 which are already marked is 9 (3*3) because 6 has been already marked false by 2.

▲ 5 votes ● Reply ● Message ● Permalink

Arjit Srivastava 6 years ago

This is a well written post! :)

▲ 4 votes ● Reply ● Message ● Permalink

Gaston Fontenla Nuñez 5 years ago

I have a few improvements over your prime sieve. Here's the code:

1)Declaring the bool array outside the function allow make queries to the array from anywhere.
2)Making it dynamic is a good choice, because it can be bigger (see code)
3)Use memset to set the array to 1. It's faster.

```
bool *isPrime; //Can be bigger, and support queries
void sieve(int N)
{
isPrime = new(bool[N+1]); ///Dynamic is better
memset(isPrime, 1, sizeof(bool)*(N+1)); ///Save time (use <string.h>)
isPrime[0] = 0, isPrime[1] = 0;
for(int i=2; i*i<=N; i++)
if(isPrime[i])
for(int j=i*i; j<=N; j+=i) // Mark all the multiples of i as composite numbers
isPrime[j] = 0;
}
```

▲ 3 votes ● Reply ● Message ● Permalink

Rishab Ladha 3 months ago

great!!!!

▲ 0 votes ● Reply ● Message ● Permalink

Vishal Anand 4 years ago

Is there any relationship for expression (a - b)% c = [a%c - b%c]%c ? I know this is wrong but is there a correct equation that would give (a - b) % c just like (a+b)%c = (a%c + b%c)%c.

?

▲ 0 votes ● Reply ● Message ● Permalink

Sachin Chauhan 3 years ago

It is right if a>b

▲ 0 votes ● Reply ● Message ● Permalink

Umang Panchal 3 years ago

if a=10, b=8 and c=5 then it will go wrong ,....

▲ 1 vote ● Reply ● Message ● Permalink

Sachin Chauhan 3 years ago

No it's not.
See..
(10-8)%5 = 2%5 = 2
and
((10%5)-(8%5))%5 = (0-3)%5 = -3%5 = 2 (We should treat -3 and 2 same in modulo 5 arithmetic)

SO... (a - b)% c = [a%c - b%c]%c is right provided we treat -b = -b+c. :)

▲ 1 vote ● Reply ● Message ● Permalink

Mridul Bhatt 2 years ago

so we could ultimately generalise the formula as (a-b)%c=(a%c-b%c+c)%c;

▲ 0 votes ● Reply ● Message ● Permalink

Abhinav Koul Edited 2 years ago

if the no we take modulo of is -ve then only we add c. (But yes it will provide the same answer even if we add c in the expression)

▲ 1 vote ● Reply ● Message ● Permalink

saksham agarwal 2 years ago

complexity of euclid theorem will log(min(A,B)).

▲ 1 vote ● Reply ● Message ● Permalink

Deepanshu Thakur 6 years ago

You have a typo in example of Modular exponentiation Iteration 3:
Since the rightmost digit of ‘b’(1110) is 1:
it should be
Iteration 3:
Since the rightmost digit of ‘b’(1110) is 0:

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg Author 6 years ago

Thank You for the information Deepanshu, will fix it soon :)

▲ 0 votes ● Reply ● Message ● Permalink

Manoj Kumar Regar 6 years ago

can you provide a link for more explained proof of time complexity for sieve of Eratosthenes ?

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg Author 6 years ago

https://en.wikipedia.org/wiki/Sieve_of_Eratosthenes
You can visit this link for more information on complexity of this algorithm .

▲ 0 votes ● Reply ● Message ● Permalink

?

D Rajiv Lochan Patra 6 years ago

how the else block worked in recursion and value of A and B are assigned in else block while computing the rcursion?

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg Author 6 years ago

recursion in which algorithm?

▲ 0 votes ● Reply ● Message ● Permalink

D Rajiv Lochan Patra 6 years ago

Extended Euclid algoritm

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg Author 6 years ago

Try to understand this using Euclid Algorithm. In Extended Euclid, the values of A and B are computed as were in Euclid Algorithm. You can see in the Example explained above for Euclid Algorithm :
Let A = 16, B = 10.
GCD(16, 10) = GCD(10, 16 % 10) = GCD(10, 6)
GCD(10, 6) = GCD(6, 10 % 6) = GCD(6, 4)
GCD(6, 4) = GCD(4, 6 % 4) = GCD(4, 2)
GCD(4, 2) = GCD(2, 4 % 2) = GCD(2, 0)
Here the values changes from {16,10} - > {10, 6} -> {6, 4} -> {4, 2} -> {2, 0}.
Now in Extended Algortihm, once you got GCD , then in each level of recursion,these values are used in reverse order.For this you can refer Example explained for Extended Euclid.
I hope it will help you more :) .

▲ 0 votes ● Reply ● Message ● Permalink

D Rajiv Lochan Patra 6 years ago

This comment has been deleted.

D Rajiv Lochan Patra 6 years ago

and how the reverse order is working?

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg Author 6 years ago

Recursion always gives the result by computing values in reverse order. Try to recurse on example manually. It will make this more clear to you.

▲ 0 votes ● Reply ● Message ● Permalink

Karan Kapoor 6 years ago

hey @prateek it would be great if you could include a bit about CRT and ncr modulo m.. I always get confused with them :(

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg Author 6 years ago

Hi Karan, as this is Part I, so I have only included basic topics. The topics like CRT and ncr modulo m, will surely be included in further parts of Number Theory. Stay tuned :)

▲ 0 votes ● Reply ● Message ● Permalink

Karan Kapoor 6 years ago

@prateek..Thanks :) i was trying to solve a problem using extended euclid algorithm on spoj but was stuck.. Can you help me out a bit ?

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg ϟ Author 6 years ago

Sure, share the link !

▲ 0 votes ● Reply ● Message ● Permalink

Karan Kapoor 6 years ago

This was the problem. http://www.spoj.com/problems/POUR1/ this is my solution.. http://ideone.com/fntnEP i have written the problem i am facing in comments in the code.

I am writing it again here as well..
i considered the problem similar to the solving the diophantine equation by extended euclid algorithm
and in case the value of c is equal to gcd(a,b) we can simply return the value of abs(x) + abs(y) where x and
y are the coefficients calculated by extended gcd however in the other case.. when c is a multiple of gcd(a,b)
one of the solution will be x*(c/gcd(a,b) and y*(c/gcd(a,b)) however it may not be the minimum no of moves.
How do we minimize the number of moves. ????
I am stuck at this point..
How can we solve this problem by using extended gcd algorithm in this case when c is not gcd but multiple of it..???

▲ 0 votes ● Reply ● Message ● Permalink

u Utkarsh Bansal a year ago

u can minimize the number of moves by not moving .

▲ 0 votes ● Reply ● Message ● Permalink

Pradyumna Bang 6 years ago

You guys must use LaTeX for writing mathematical formulae.

▲ 0 votes ● Reply ● Message ● Permalink

Nikhil Kumar Singh 5 years ago

I support you for this

▲ 0 votes ● Reply ● Message ● Permalink

Dharmesh Jogadia 6 years ago

Awesome post :D

▲ 0 votes ● Reply ● Message ● Permalink

Anirud Samala 6 years ago

Awesome post!! Can you also suggest some Practice Problems too? Thanks

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Sandeep KD 6 years ago

Just Awesome coding tutorial with example ... courage to learn Many many thanks for share and making such a great site . Thank again......

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Pushkar 6 years ago

In the explanation of modular exponentiation Binary of b=55 is wrongly calculated as 101101(binary of 45). It should be 110111.

▲ 0 votes ● Reply ● Message ● Permalink

sathiyaseelan 6 years ago

Yes. Confused for long time.And now verified manually. Change b=45 or binary form of 55 = 110111

?

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg ϟ Author 6 years ago

Yes, it was a typo. It is 45 not 55. It is fixed now.

▲ 0 votes ● Reply ● Message ● Permalink

Anurag Sharma 6 years ago

@Prateek please tell me if i want to print almost 3000 prime numbers by Sieve of Eratosthenes algorithm..then what should be the length of array ?

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Prateek Garg ϟ Author 6 years ago

As you are using Sieve of Eratosthenes , you do not need an extra array. You can use isPrime[ ] array to print prime numbers. Like :

if (isPrime[i] == true)
cout << i << endl;

And if you want to print exactly 3000 primes, then you can use counter for that :) .

▲ 0 votes ● Reply ● Message ● Permalink

Anurag Sharma 6 years ago

i tried but couldn't get 3000 primes...as j=i*i then even n=1000000 is not working for my code.... please help

▲ 0 votes ● Reply ● Message ● Permalink

sathiyaseelan 6 years ago

Is there any specific reason for counting the divisors in checkPrime?

We can return immediately after finding the first divisor,right?

boolean checkPrime(int N){
for (i=2; i*i<=N;i++){
if(N%i==0)
return false;
}
return true;

▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg ϟ Author 6 years ago

Yes for checking prime number, you can use this code also. The complexity of this algorithm will also be O(sqrt(N)) . I have used algorithm which is explained in article, because by using that, user will also learn, how to calculate the total number of divisors of any number.

▲ 0 votes ● Reply ● Message ● Permalink

sathiyaseelan 6 years ago

ok cool :)

▲ 0 votes ● Reply ● Message ● Permalink

Jeffry Cpps 6 years ago

In Sieve algorithm,
-The inner loop starts from i*i?
Then how come i=3, will select 6(shown in the example)?
please correct me if I am wrong.

▲ 0 votes ● Reply ● Message ● Permalink

sathiyaseelan 6 years ago

6 would be already covered by 2.

▲ 0 votes ● Reply ● Message ● Permalink

?

Jeffry Cpps 6 years ago
Exactly. You are right. But it was colored in orange which means it was affected by 3. So i got confused.
▲ 0 votes ● Reply ● Message ● Permalink

sathiyaseelan 6 years ago
ok :)
▲ 0 votes ● Reply ● Message ● Permalink

shubham rajput 4 years ago
But it must be starting from i*2 instead of i*i ,because in case of 3 it will be starting from 9 (3*3).According to your algorithm, with i=3,the inner loop will run only twice but you are stating that time complexity is N/3 which should be 3 as int(10/3)=3.
▲ 0 votes ● Reply ● Message ● Permalink

RISHABH TRIPATHI 6 years ago
i think Binary GCD algo is better .
▲ 0 votes ● Reply ● Message ● Permalink

Sunil Yadav 5 years ago
good post ! I request to add a link to download these notes as a pdf .It will be of great help to take print and read any time . Thanks .
▲ 0 votes ● Reply ● Message ● Permalink

Ashutosh Mangalekar 5 years ago
▲ 0 votes ● Reply ● Message ● Permalink

Gaurav Shankar 5 years ago
The time complexity of both Euclid's and extended Euclid's algorithm is same. Any advantages of one over the other?
▲ 0 votes ● Reply ● Message ● Permalink

Prateek Garg ϟ Author 5 years ago
Yes, extended euclid's also gives two variables of the above equation. They are very helpful in various cases, which will be discussed in future articles of number theory. Stay tuned :)
▲ 0 votes ● Reply ● Message ● Permalink

Abdullah Hashem 5 years ago
You can implement the modular exponentiation using binary operation to make the code run faster:
Code:
```
ans = 1;
while(b)
{
if(b & 1) ans = (ans * a) % c; // b & 1 return 1 if the number is odd (most right bit is 1) otherwise 0;
a = (a * a) % c;
b >> = 1; // integer division by 2;
}
return ans;
```
▲ 0 votes ● Reply ● Message ● Permalink

Sharvin Jondhale 4 years ago
Compilers are optimized to do that automatically, using them explicitly increases the chances of bugs.

▲ 0 votes ● Reply ● Message ● Permalink

Mohit Singh 5 years ago

Hii [developer:ptk23] after that post i am become your fan dear....
and this is request.. can you please write a post on "RECURSION WITH MEMOIZATION"..??

▲ 0 votes ● Reply ● Message ● Permalink

Shivasurya S 5 years ago

Really gud article !

▲ 0 votes ● Reply ● Message ● Permalink

Ishpreet Singh 5 years ago

Great post .... Keep writing such post .....

▲ 0 votes ● Reply ● Message ● Permalink

Jyotman Singh 5 years ago

a = 999999, b=999999 gives negative answer.

▲ 0 votes ● Reply ● Message ● Permalink

Sreenidhi 5 years ago

Thank you so much! brilliant explanation

▲ 0 votes ● Reply ● Message ● Permalink

Shubham Aggarwal 5 years ago

can you put binary exponentiation algorithm?

▲ 0 votes ● Reply ● Message ● Permalink

Renganatha 5 years ago

Good Easy to understand :)

▲ 0 votes ● Reply ● Message ● Permalink

Abhishek Kumar 5 years ago

In every iteration the value of ‘a’ is made equal to ‘a * a’.
As,
a * (20) = a
a * (21) = a2 = a * a
a * (22) = a4 = ((a * a) * (a * a))
and so on..
in the above phrase, how powers of two are being multiplied with a to get powers of a.
a must be with powers of two's not with multiple of two's.

▲ 0 votes ● Reply ● Message ● Permalink

Akash Garg 5 years ago

In extended euclid's algorithm, its okay how we get d, but for x and y, x and y get their values assigned as 0 or1 only after we get b=0.
and after that, there is no further looping, so how does the values of x and y start changing? please elaborate.

▲ 0 votes ● Reply ● Message ● Permalink

Abhishek Inamdar 5 years ago

sir,please can you provide some notes of basic mathematics for computer science and also a guidance to built a logic some mathematical programs and object oriented programs..:)

▲ 0 votes ● Reply ● Message ● Permalink

Bhishma Raj 5 years ago

Can you please write the tutorials in LATEX

?

▲ 0 votes ● Reply ● Message ● Permalink

Samyak Jain 5 years ago

There is a mistake in the Sieve of Eratosthenes.
In the inner for loop to mark all multiples of the prime numbers as composite
Your code:
for(int j = i * i; j <= N ;j += i)
isPrime[j] = false;
Correction:
for(int j = 2 * i; j <= N ;j += i)
isPrime[j] = false;

▲ 0 votes ● Reply ● Message ● Permalink

Atul Kumar Gupta Edited 5 years ago

No it is not. j should start from i*i only.

▲ 0 votes ● Reply ● Message ● Permalink

shubham rajput 4 years ago

Exactly,I was also thinking the same ,it is wrong as it doesn't match with the given time complexity

▲ 0 votes ● Reply ● Message ● Permalink

Atul Kumar 4 years ago

Yeah, but it's little improvement . i*i is right

▲ 0 votes ● Reply ● Message ● Permalink

Gopish Monga 5 years ago

In the above given Euclid's Algorithm, variable A must always be greater than or equal to B.
If otherwise then A%B will become equal to zero(0), which is not desired and algorithm fails.
Also the time complexity should be O(log(A)) as A must be the larger element.

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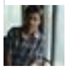
Himanshu Gupta 4 years ago

gcd(10,16) = gcd(16, 10%16) = gcd(16,10) = ...
So, it really does not matter if a>b or not. It is automatically taken care of.

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Ankur Shah Edited 5 years ago

If you have explained for GCD, you should have also written for LCM too! But why didn't you do that?

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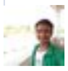
Sachin Chauhan 3 years ago

LCM(A,B) is simply ((A*B)/GCD(A,B).

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Krushi Gada 5 years ago

In the Extended Euclid implementation, shouldn't the variables x and y be initialized to 1 and 0 respectively? Considering that the given value of B will not be 0, we enter the else part of the function whereby the values of x and y are needed but we don't have them in the first go.

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Rezoan Shakil 4 years ago

thanks a lot to post this
i want more post in mathematics for competitive programming ...

?

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Rishabh Agarwal 4 years ago

Great Post !
Just wanted more explanation for extended euclidean theorem as it is important.

▲ 0 votes ● Reply ● Message ● Permalink

Dhruvil Bhuptani 4 years ago

how to multiply 2 10 digit numbers using modular arithmetic

▲ 0 votes ● Reply ● Message ● Permalink

Atul Kumar 4 years ago

Can anyone explain to me why we are using this expression in modular exponentiation "a=(a * a)%c" , it can be kept simple by counting the bit's traversed in some counter k , then calculating each (a^k)%c !

▲ 0 votes ● Reply ● Message ● Permalink

kaushik raj 4 years ago

extended euclid gives wrong answer for
a=2;
b=1000000007;
the correct answer should be 50000004

▲ 0 votes ● Reply ● Message ● Permalink

Sharvin Jondhale 4 years ago

You are correct, the problem with the code is.... it requires a>b at input stage too, as the recursions proceed with that assumption.. swap the variables i.e.
a=1e9+7
b=2
It will give correct answer, I had made this error when I had written it :p

▲ 0 votes ● Reply ● Message ● Permalink

Suraj Kumar Patel 3 years ago

thanks hackerearth...

▲ 0 votes ● Reply ● Message ● Permalink

Tanvir Hossain Yuv Edited 2 years ago

Please update the article in Modular Exponentiation. There are some mistakes making confused.
1. a^b = (1 * (a ^2^5) * (0 * (a ^2^4)) * (1 * (a ^2^3)) * (1 * (a ^2^2)) * (0 * (a ^2^1)) * (1 * (a ^2^0))
This statement is wrong.
2. 559 = 5 ^{111011}2 = 5^2^5 * 5^2^4 * 5^2^3 * 5^2^3 * 5^2^1 * 5^2^0
This statement is also wrong. Here, 5^2^3 occurs twice which should not be.
Please update.

▲ 0 votes ● Reply ● Message ● Permalink

Rishabh Rai 2 years ago

learned a lot from the post thanks

▲ 0 votes ● Reply ● Message ● Permalink

AUTHOR

?

Prateek Garg
SDE-1 at Flipkart
BENGALURU
7 notes

TRENDING NOTES

Python Diaries Chapter 3 Map | Filter | For-else | List Comprehension
written by Divyanshu Bansal

Bokeh | Interactive Visualization Library | Use Graph with Django Template
written by Prateek Kumar

Bokeh | Interactive Visualization Library | Graph Plotting
written by Prateek Kumar

Python Diaries chapter 2
written by Divyanshu Bansal

Python Diaries chapter 1
written by Divyanshu Bansal

more ...

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