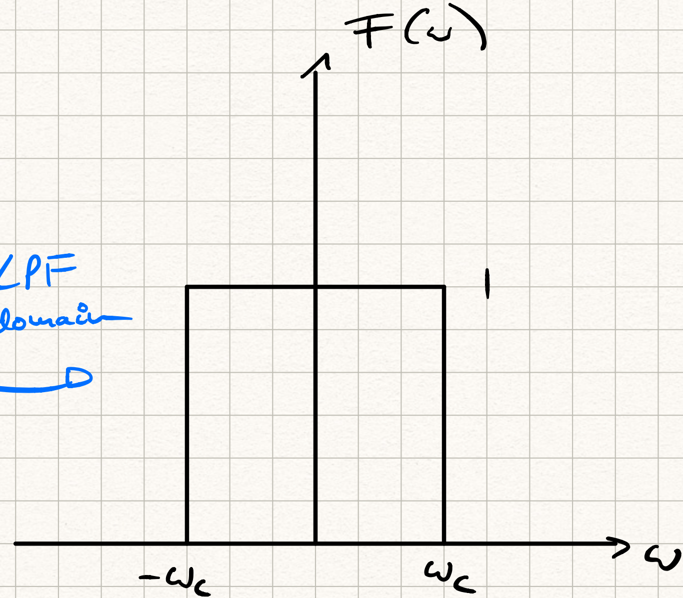
FIR Filter Design

e.g. Low-Pass Filter

Ideal LPF
in frequency domain



Inverse Fourier transform: $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega$

$\Rightarrow$ For LPF: $f(t) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega t} \, d\omega = \frac{1}{2\pi} \left[\frac{e^{j\omega t}}{jt} \right]_{-\omega_c}^{\omega_c}$

$$= \frac{1}{\pi t} \cdot \underbrace{\frac{1}{2j} \cdot \left(e^{j\omega_c t} - e^{-j\omega_c t} \right)}_{= \sin(\omega_c t)}$$

$$\Rightarrow \boxed{f(t) = \frac{\sin(\omega_c t)}{\pi t}}$$

⇒ Impulse response of ideal LPF:

Sinc function!

Problems:

↳ Non-causal!

↳ Infinite length!

↳ Continuous function!

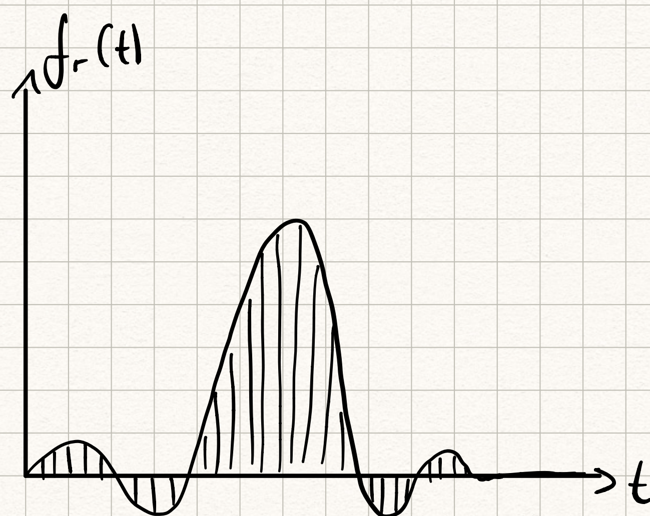
Solutions:

↳ Shift response!

↳ Truncate!

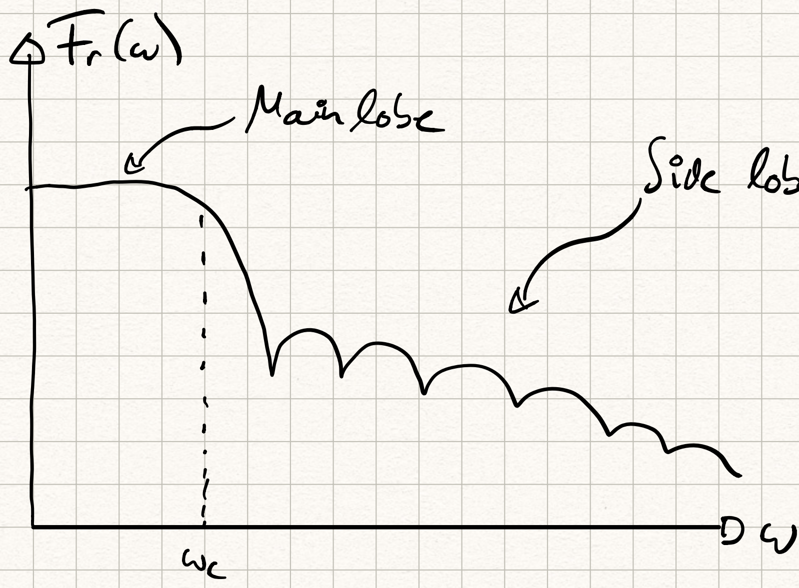
↳ Sample!

↳ "Realistic" response:



↳ However.... this response has an altered, non-ideal frequency response!

e.g.

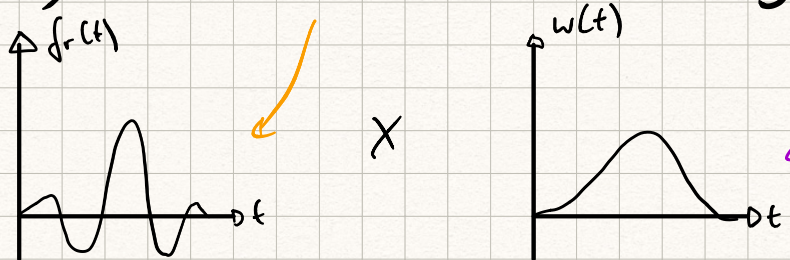


High-frequency side lobes due to truncation ⇒ creates discontinuities in time domain!

⇒ Solution: Windowing!

Different functions available with different frequency responses!

Multiply "realistic" impulse response by "smoothing" function!



Choices when designing a FIR filter:

- Sampling frequency
- Filter length (i.e. number of samples)
- Cut-off frequency
- Window function

Check frequency and time domain properties!

overshoot, rise time, ...

Stopband attenuation, passband ripple, ...

Similar procedures for other filter types, e.g. high-pass, band-pass, band-stop ...