Lesson 1 | Fair Shares and Operations on Fractions

Problem Solving: Organizing Data

Fair Shares and Operations on Fractions

What are fair shares?

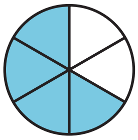
A fraction is the relationship between a part and a whole. Let's look at the fraction $\frac{4}{6}$. In a fraction, the part is called the numerator, and the whole is called the denominator.

$$\begin{matrix} \text{part} \rightarrow & 4 & \leftarrow \text{numerator} \\ \text{whole} \rightarrow & \overline{6} & \leftarrow \text{denominator} \end{matrix}$$

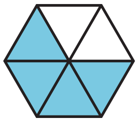
We can use shapes to show the part-to-whole relationship for $\frac{4}{6}$. In the models below, four parts are shaded out of a total, or whole, of six parts.

We can show the part-to-whole relationship for $\frac{4}{6}$ using shapes.

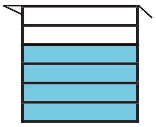
$\frac{4}{6}$



$\frac{4}{6}$



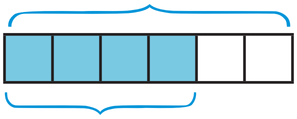
$\frac{4}{6}$



$\frac{4}{6}$

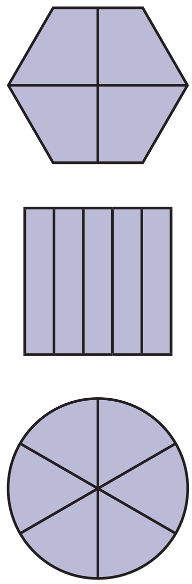
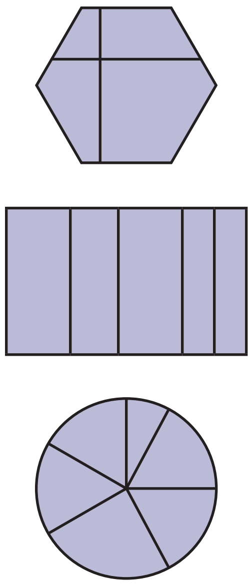
The parts of the fraction are equal in size or area within each shape. We call the equal parts within a shape "fair shares."

6 equal parts, or "fair shares"



4 parts-to-whole

The next model shows examples of shapes that have been divided into fair shares and shapes that have not been divided into fair shares. In the shapes that are not divided into fair shares, some of the parts in the shape are bigger than others. This is how we know they are not divided into fair shares.

Fair Shares	Not Fair Shares
	

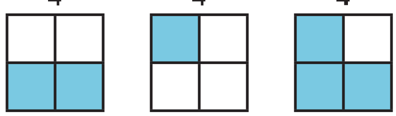
How do we add and subtract fractions with the same denominators?

Fair shares are essential to adding and subtracting fractions. They allow us to find exact sums or differences. The models below show how we add fractions that have the same fair shares. We start with $\frac{2}{4}$ and add $\frac{1}{4}$. We are adding fair shares of the same size or area. Example 1 shows that when we use fair shares to add or subtract fractions, we just add the numerators. The denominator stays the same.

Example 1

Add fractions using fair shares.

$$\frac{2}{4} + \frac{1}{4} = \mathbf{\frac{3}{4}}$$

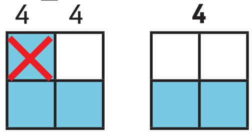


We subtract fractions in a similar way. We subtract, or take away, fair shares. We also demonstrate this with arrays. In Example 2, we subtract $\frac{3}{4} - \frac{1}{4}$. We begin with $\frac{3}{4}$ and then take away, or cross out, one of the parts.

Example 2

Subtract fractions using fair shares.

$$\frac{3}{4} - \frac{1}{4} = \mathbf{\frac{2}{4}}$$



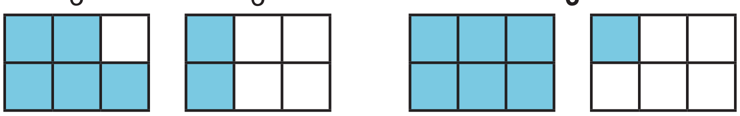
These operations apply to all kinds of fractions, even fractions that are greater than 1. The next two examples show addition and subtraction of fractions with a sum that is greater than 1. We use more than one square for each number because our fractions are greater than 1.

In the addition example, we add $\frac{5}{6}$ and $\frac{2}{6}$. The sum is represented by one array that is totally shaded and a second array with one part shaded.

Example 3

Add fractions where the sum is greater than 1.

$\frac{5}{6} + \frac{2}{6} = \mathbf{\frac{7}{6}}$

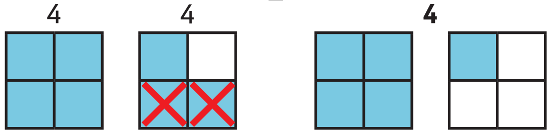


In the subtraction example, we start with two figures to show the problem. We subtract $\frac{2}{4}$ from $\frac{7}{4}$. We show this by shading one whole array and three of the four parts of the second array to represent $\frac{7}{4}$. Then we cross out two of the parts to get $\frac{5}{4}$. We still need two squares to demonstrate the answer. The answer $\frac{5}{4}$ is greater than 1.

Example 4

Subtract fractions greater than 1.

$\frac{7}{4} - \frac{2}{4} = \mathbf{\frac{5}{4}}$



Apply Skills
Turn to ***Interactive Text***, page 2.

mBook Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

▶Problem Solving: Organizing Data

Vocabulary

minimum
maximum
range
mode
mean

How do we organize data in a way that is meaningful?

Carmen is a manager at a shoe store. She is in charge of buying shoes from shoe companies in New York and Los Angeles to sell in her store.

Carmen needs to figure out what kinds of shoes to buy based on what is popular. The most popular running shoe is QuikTrax. She has to be sure that she has enough pairs of QuikTrax to sell.

Here is Carmen's spreadsheet showing sales of QuikTrax shoes for the last nine weeks.

Week	Number of Pairs of Shoes Sold
May 1 to May 7	47
May 8 to May 14	39
May 15 to May 21	31
May 22 to May 28	39
May 29 to June 4	49
June 5 to June 11	57
June 12 to June 18	50
June 19 to June 25	48
June 26 to July 2	54
Total	**414**

Carmen wants to look at different parts of the data. She wants to find the lowest number of sales, the greatest number of sales, and the average sales for all nine weeks. This will help her determine how many pairs of running shoes to buy.

The data above are organized by date. It is not easy to find the data Carmen needs. We need to organize the data so that Carmen can make sense of the numbers.

Here are five words that we use to describe the important data Carmen is interested in.

Word	Definition
Minimum (Min)	Smallest value
Maximum (Max)	Largest value
Range	Difference between the min and the max
Mode	Number that appears the most
Mean	Average

The table below lists the number of pairs of shoes sold from smallest to largest. Now it is easy to see the **minimum** and the **maximum**.

Week	Number of Pairs of Shoes Sold	
May 15 to May 21	31	**minimum (min)**
May 8 to May 14	39	the number that appears the most—the **mode**
May 22 to May 28	39	
May 1 to May 7	47	
June 19 to June 25	48	
May 29 to June 4	49	
June 12 to June 18	50	
June 26 to July 2	54	
June 5 to June 11	57	**maximum (max)**
Total	**414**	

Now it is easy to see that the *minimum* sales occurred the week of May 15 to May 21, when the store only sold 31 pairs of shoes. The *maximum* sales occurred the week of June 5 to June 11, when the store sold 57 pairs of shoes.

From this, we determine the **range**. The *range* is how far the data stretch from lowest to highest. To find the range, we subtract the minimum from the maximum. In this set of data, the range is 57 – 31, or 26.

Ordering numbers this way is also helpful for seeing the **mode**. The *mode* is the number that appears the most in the set of data. In these data, it's 39. Every other number only appears once. The number 39 appears twice.

Another number that is helpful for Carmen is the average number of QuikTrax shoes sold. Another word for average is **mean**. We find the mean by dividing the total number of pairs of shoes sold during the nine weeks by the number of weeks.

Total Pairs of Shoes ÷ Number of Weeks = Mean

Example 1

Find the mean, or the average, number of pairs of QuikTrax shoes sold over nine weeks. Use the table on the previous page.

$$\underbrace{\text{Total Pairs of Shoes}}_{414} \div \underbrace{\text{Number of Weeks}}_{9} = \underbrace{\text{Mean}}_{46}$$

The average, or mean, is 46 pairs of shoes per week.

Different ways of looking at data tell us different things. The minimum and the maximum tell us about what is happening at the extremes of the data set—the largest data and the smallest data. Sometimes data can be extreme, and the minimum and the maximum stand out from the rest of the data.

We look at the mean because it tells us about what is generally happening over time. The mean is the average number. It is our best prediction of what will happen next.

POWER CONCEPT

Mean, mode, and range tell us different things about a set of data.

Problem-Solving Activity
Turn to *Interactive Text*, page 3.

mBook **Reinforce Understanding**
Use the ***mBook Study Guide*** to review lesson concepts.

Lesson 1

Homework

Activity 1

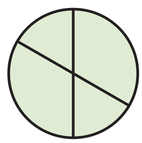
Add and subtract the fractions with like denominators.

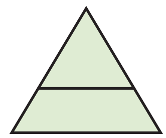
1. $\frac{3}{5} + \frac{2}{5}$
2. $\frac{1}{3} + \frac{1}{3}$
3. $\frac{7}{9} - \frac{1}{9}$
4. $\frac{8}{10} - \frac{7}{10}$
5. $\frac{4}{6} + \frac{8}{6}$
6. $\frac{15}{3} - \frac{14}{3}$

Activity 2

Tell which of the following shapes are divided into fair shares. Write the letter on your paper.

(a)


(b)


(c)


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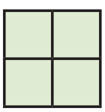

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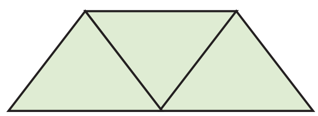

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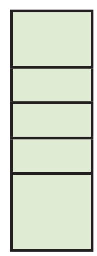

Activity 3

Tell the minimum, maximum, range, mode, and mean for the sets of data. Remember to put the data in order.

1. 6, 5, 2, 6, 3, 8, 1, 4, 6, 9
2. 1, 1, 9, 8, 2, 4, 3
3. 10, 30, 50, 70, 90, 50

Activity 4 • Distributed Practice

Solve.

1. 27 + 39
2. 23 – 19
3. 34 • 2
4. 270 ÷ 5
5. 409 – 118
6. 125 + 778
7. 123 • 4
8. 226 ÷ 2