Lesson 2 ▶Equivalent Fractions

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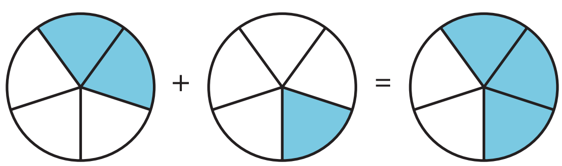
Vocabulary

common denominator
least common denominator (LCD)

Why do we need the same fair shares?

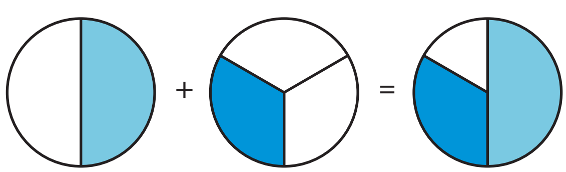
Let's look again at adding and subtracting fractions with the same denominators. We learned that fractions have a part-to-whole relationship. When we add, we combine the parts. The model shows how the whole, or denominator, stays the same. It is because the circles are divided into fair shares. What changes is the number of parts because we are adding them together.

Look what happens when we add fractions with the same denominator.



$$\frac{2}{5} + \frac{1}{5} = \mathbf{\frac{3}{5}}$$

This process does not work as easily when the wholes, or denominators, are not the same. Let's look at the next model. We can combine the parts, but we cannot tell what the answer is. Should the answer be in halves or thirds or something else?



$$\frac{1}{2} + \frac{1}{3} = \mathbf{?}$$

We see what the answer looks like. We could probably give an approximate answer to the problem if we had to. But we cannot give an exact answer because of the way the fractions are set up.

Lesson 2

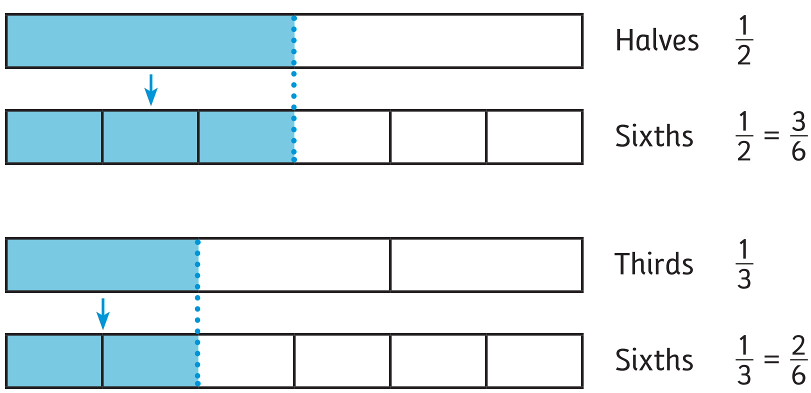
To get an exact answer when we add and subtract fractions with uncommon denominators, we need to use the same fair share for each fraction. This is the **common denominator**.

Let's look at how we change fractions to make common denominators.

Example 1

Make common denominators using fraction bars.

$$\begin{array}{r} \frac{1}{2} \\ +\ \frac{1}{3} \\ \hline \end{array}$$

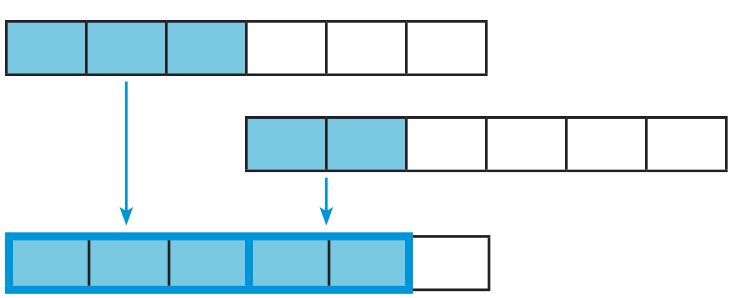


When we add and subtract fractions with unlike denominators, we need to get the same fair share for each fraction.

Both halves and thirds line up with sixths. We turn $\frac{1}{2}$ into $\frac{3}{6}$ and we turn $\frac{1}{3}$ into $\frac{2}{6}$. This way, the denominators will be the same, or common.

Once we have denominators where the whole, or total, is divided into the same fair shares, we can find an exact answer to the problem.

We use fraction bars to add our new fractions and get an exact answer.



$$\begin{array}{r} \frac{3}{6} \\ +\ \frac{2}{6} \\ \hline \mathbf{\frac{5}{6}} \end{array}$$

When we make the denominators the same, we are finding equivalent fractions for each fraction in the original problem. We are not just changing the denominator. We are also changing the numerator.

Fraction bars help us make common denominators. We get the exact answer when we add or subtract fractions with common denominators. This process works with fractions that can be easily converted, like the conversion from $\frac{1}{2}$ to $\frac{3}{6}$.

But with most fractions, we need a method other than fraction bars to find common denominators.

POWER CONCEPT

When we make denominators the same, we also make equivalent fractions.

How do we make equivalent fractions?

Let's look at two ways to find a common denominator.

Example 1

Use a table of multiples to find the common denominator.

1 2 3 4 5 6 7 times

3	3 6 9 12 15 18 21 24 27 30
7	7 14 21 28

1 2 3 times

A common denominator for the denominators 3 and 7 is 21.

A table of multiples of two numbers helps us find the **least common denominator**, or **LCD**. The LCD is the first multiple that is the same for both 3 and 7. Each row shows how we count by the multiple of the number. We count by each number until we get to the first multiple that is the same for 3 and 7. In this case, the LCD is 21.

When we add or subtract fractions, we convert each fraction so that the denominators are the same. We can use fraction bars, or we can use tables of multiples. We can also simply multiply by a fraction equal to 1 to make an equivalent fraction. Let's see how to multiply a fraction by a fraction equal to 1 to make an equivalent fraction.

Remember, a fraction equal to 1 is any fraction with the same number in the numerator and in the denominator.

Example 2

Multiply by a fraction equal to 1 to find the common denominator, then solve the problem.

$\frac{2}{3} - \frac{1}{7}$

$$\frac{2}{3} \cdot \frac{7}{7} = \frac{14}{21}$$

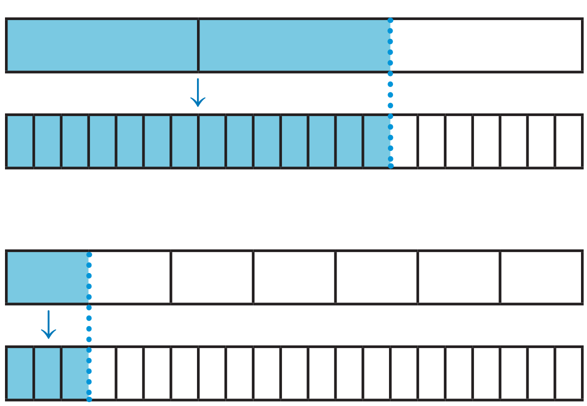
$$\frac{1}{7} \cdot \frac{3}{3} = \frac{3}{21}$$

$\frac{14}{21} - \frac{3}{21} = \frac{11}{21}$

We can check our work using fraction bars. In this case, we see that the fraction bars for thirds and sevenths line up with the fraction bar for twenty-firsts. But it is sometimes hard to work with fraction bars because the fair shares can be very small.

Example 3

Use fraction bars to check the answer.



$\frac{2}{3} = \frac{14}{21}$

$\frac{1}{7} = \frac{3}{21}$

The answer is the same: $\frac{14}{21} - \frac{3}{21} = \frac{11}{21}$.

Another method for finding the common denominator involves multiplying the two denominators together. There are cases when this method is the best one to use. Other times multiplying denominators gives us a denominator that is far larger than what we need.

Example 4

Multiply two denominators to get a common denominator.

$$\frac{7}{10} - \frac{3}{20} \rightarrow \frac{\quad}{10} \cdot \frac{\quad}{20} = \frac{\quad}{200}$$

A common denominator for these fractions is 200.

Now we want to find the equivalent numerators for each of the fractions. We multiply the original fractions by a fraction equal to 1 that will get us 200 for a denominator.

$$\frac{7}{10} \cdot \frac{20}{20} = \mathbf{\frac{140}{200}} \qquad \frac{3}{20} \cdot \frac{10}{10} = \mathbf{\frac{30}{200}}$$

$$\mathbf{\frac{140}{200} - \frac{30}{200} = \frac{110}{200}}$$

A simpler approach is to find the LCD.

In this case, although 200 is a common denominator, it is not the *least common denominator*. Using 200 for the common denominator makes the numbers large and the computations more difficult.

What are some common situations that require addition and subtraction of fractions?

There are times when we have to add or subtract fractions with denominators that are not the same. Let's look at an example.

Problem:

Hector rides his bicycle to school every day. He rides $2\frac{1}{3}$ miles down Perry Street to Grove Street. From Grove Street he rides $\frac{1}{4}$ mile to the school. How far does Hector ride to school?

We need to add $2\frac{1}{3} + \frac{1}{4}$.

Steps for Adding Fractions With Unlike Denominators

STEP 1

Find the LCD.

The simplest method is to count up by each number. The least common denominator is 12.

3	3	6	9	12	15	18
4	4	8	12	16	20	24

STEP 2

Find equivalent fractions by multiplying each fraction by a number equal to 1.

We start with $\frac{1}{3}$. Because 3 goes into 12 four times, we multiply our fraction by $\frac{4}{4}$.

$$\frac{1}{3} \cdot \frac{4}{4} = \frac{4}{12}$$

Next, we multiply $\frac{1}{4}$ by a fraction equal to 1. Because 4 goes into 12 three times, we use $\frac{3}{3}$.

$$\frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}$$

STEP 3

Add the new numbers together.

We can't forget the whole number that came before the fraction.

$$\begin{array}{r} 2\frac{4}{12} \\ + \ \frac{3}{12} \\ \hline 2\frac{7}{12} \end{array}$$

Hector rides his bike $2\frac{7}{12}$ miles to school.

Apply Skills
Turn to ***Interactive Text***, page 5.

mBook Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Homework

Activity 1

Add and subtract.

1. $\frac{1}{5} + \frac{3}{5}$
2. $\frac{7}{8} - \frac{2}{8}$
3. $\frac{1}{2} + \frac{3}{4}$
4. $\frac{4}{8} - \frac{1}{4}$
5. $\frac{2}{3} + \frac{1}{5}$
6. $\frac{7}{9} - \frac{1}{6}$

Activity 2

Select the fraction that is equivalent.

1. $\frac{1}{2}$
   (a) $\frac{2}{5}$
   (b) $\frac{2}{4}$
   (c) $\frac{1}{4}$
2. $\frac{3}{4}$
   (a) $\frac{9}{12}$
   (b) $\frac{6}{10}$
   (c) $\frac{1}{3}$
3. $\frac{2}{5}$
   (a) $\frac{2}{9}$
   (b) $\frac{4}{10}$
   (c) $\frac{3}{7}$
4. $\frac{5}{7}$
   (a) $\frac{5}{9}$
   (b) $\frac{3}{7}$
   (c) $\frac{10}{14}$

Activity 3

Select the least common denominator (LCD) for each of the problems.

1. $\frac{1}{2} + \frac{2}{5}$
   (a) The LCD is 5.
   (b) The LCD is 10.
   (c) The LCD is 2.
2. $\frac{3}{8} - \frac{1}{4}$
   (a) The LCD is 8.
   (b) The LCD is 32.
   (c) The LCD is 4.
3. $\frac{4}{6} + \frac{2}{9}$
   (a) The LCD is 54.
   (b) The LCD is 18.
   (c) The LCD is 9.

Activity 4 • Distributed Practice

Solve.

1. Find the missing numbers in the lists of multiples. Write the answers on your paper.

3	3	**(a)**	9	12	**(b)**	**(c)**	21	
4	4	8	**(d)**	16	**(e)**	24	**(f)**	32

2. What is the LCD for the problem $\frac{2}{3} + \frac{5}{4}$?
3. Write the multiples for 5 starting at 5 and ending at 50.
4. Write the multiples for 10 starting at 10 and ending at 100.
5. What is the LCD for the problem $\frac{3}{5} - \frac{3}{10}$?