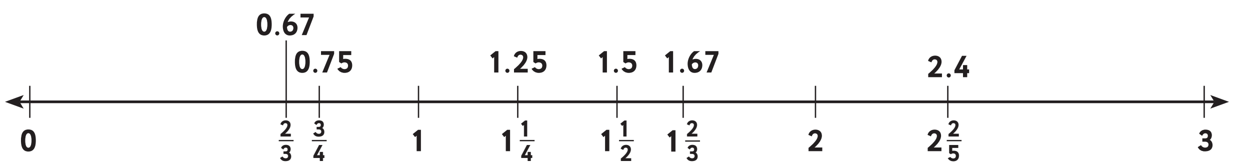
Lesson 15

Unit Review
▶Fractions and Decimal Numbers
Problem Solving:
▶Statistics

▶Fractions and Decimal Numbers

What are basic fraction and decimal number concepts?

Fractions and decimal numbers point to the same location on a number line. They are just different ways of describing the same number.



The concept of fractions is based on fair shares. That means we can take a shape and break it into equal parts. Fair shares are important when we add or subtract fractions because they allow us to add or subtract the same units.

When we change fractions into decimal numbers, we divide the denominator into the numerator. This changes the fraction into a base-10 number. Look at Review 1. We listen for the place value when we read the number.

Review 1

How are fractions and decimal numbers related?

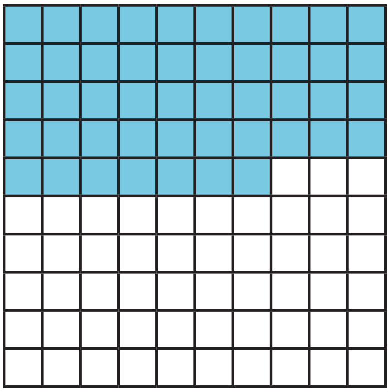
Fraction	Decimal Number	We Say	Place Value
$\frac{4}{10}$	0.4	four tenths	Tenths
$\frac{1}{4}$	0.25	twenty-five hundredths	Hundredths
$\frac{3}{8}$	0.375	three hundred and seventy-five thousandths	Thousandths

There are different ways to show fractions and decimal numbers. Review 2 shows fraction bars and a 100-square grid. Each one helps us better understand these two kinds of rational numbers.

Review 2

How do we use models to show how fractions and decimal numbers are related?

$\frac{3}{5}$



0.47

We use different models for fractions and decimal numbers. Fraction bars help us see fractions. We use a 100-square grid to show decimal numbers up to the hundredths place.

What do we need to remember when we work with fractions?

The rules for how to add, subtract, multiply, and divide fractions are different. It is important to be able to keep track of what we need to do. We begin by looking at the operation.

Review 1

What do we need to remember when we add or subtract fractions?

When adding and subtracting fractions, we need to have the same denominators. This is because we can only add or subtract the same fair shares. Without fair shares, the answer would not make sense.

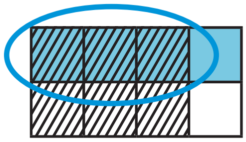
$\frac{2}{3} + \frac{1}{2} = ?$ $\quad$ $\mathbf{\frac{4}{6} + \frac{3}{6} = \frac{7}{6}}$

Review 2

What do we need to remember when we multiply fractions?

When multiplying fractions, we use one fraction to take a portion of another fraction. If we work with fractions less than 1, the answer is usually smaller than the numbers we started with. We use a model to show what this looks like.

$\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$



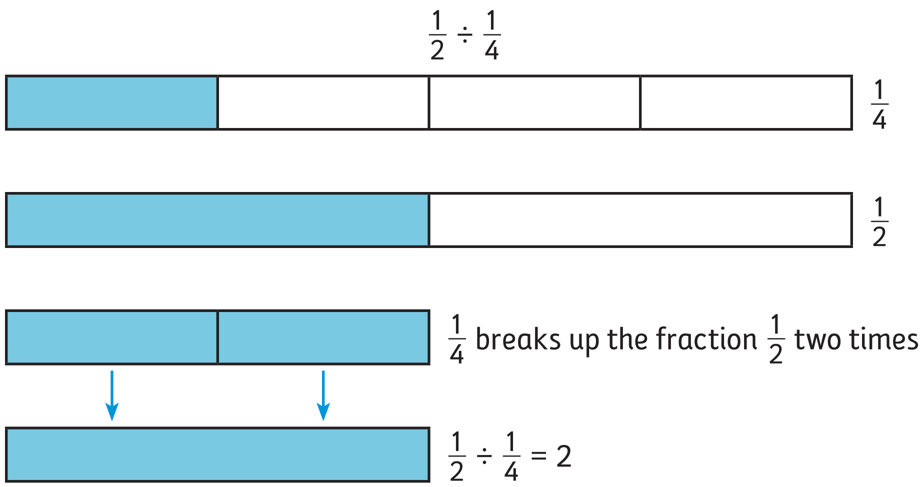
The overlap is 3 out of 8 sections, or $\mathbf{\frac{3}{8}}$.

Lesson 15

Review 3

What do we need to remember when we divide fractions?

Division is about breaking up one number with another number. We use one fraction as the unit to break up another fraction. In the problem $\frac{1}{2} \div \frac{1}{4}$, we break up $\frac{1}{2}$ by the unit $\frac{1}{4}$. Fraction bars help us see this.



We saw the traditional methods for operating with fractions. We learned that the traditional methods are shortcuts for solving difficult problems.

What are the traditional methods for working with fractions?

Models help us see why addition, subtraction, multiplication, and division of fractions work. They help us build good number sense about these operations. They are not always the most efficient way to find an answer. It is faster to use the traditional methods for these operations. It's important for us to have good number sense and good computational skills.

Addition and Subtraction:

Use equivalent fractions to make the denominators the same.

$$\frac{2}{3} + \frac{1}{2}$$

$$\frac{2}{3} \cdot \frac{2}{2} = \frac{4}{6}$$

$$\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$$

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$$

Multiplication:

Multiply across. Do not change denominators.

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

Division:

Invert the second fraction and multiply across. Do not change denominators.

$$\frac{1}{2} \div \frac{1}{4}$$

$$\frac{1}{2} \cdot \frac{4}{1} = \frac{4}{2}, \text{ or } 2$$

What do we need to remember when working with decimal numbers?

Operations on decimal numbers are less of a problem. We do not have to convert or simplify as we do with fractions. Here are some key ideas for working with decimal numbers.

Adding or subtracting decimal numbers:

We make sure to line up the decimal points. An easy way to do this is to add zeros to the end of the decimal number if the numbers are not the same length.

Original problem	Add a zero to make the same place values
$\begin{array}{r} 3.242 \\ -\ 1.15 \\ \hline \end{array}$	$\begin{array}{r} 3.242 \\ -\ \ 1.150 \\ \hline 2.092 \end{array}$

Multiplying decimal numbers using the traditional method:

We multiply the two numbers as if they were whole numbers. When we get the answer, we count the number of places to the right of the decimal point in our problem, then count over that many places in the answer and place the decimal point.

$$\begin{array}{r} 3.25 \\ \times\ \ 0.7 \\ \hline 2.275 \end{array}$$

three numbers to the right of the decimal point

Start all the way to the right and count over three places to the left and place the decimal point.

Dividing decimal numbers:

We start by making the divisor a whole number. If we move the decimal point in the divisor, we have to do the same thing with the dividend. We add a zero to the dividend if we need to.

divisor dividend

$0.13\overline{)5.2}$ $\qquad$ $\begin{array}{r} 40 \\ 13\overline{)520} \end{array}$

Lesson 15

How do we check our answers?

Number sense with fractions and decimal numbers is important. We use estimations to see if we have the right answer. This is especially true with decimal numbers. One of the easiest ideas to remember is that the decimal part of a number is always less than 1. If we want to find an approximate answer to a decimal number problem, we round to the nearest whole number. Let's look at an example with exact and estimated answers to different decimal number problems.

Review 1

How do we check our answers using estimation?

Exact Answer	Approximate Answer
$\begin{array}{r} 33.75 \\ 2.103 \\ +\ \ 4.7 \\ \hline 40.553 \end{array}$	$\begin{array}{r} 34 \\ 2 \\ +\ \ 5 \\ \hline 41 \end{array}$
$\begin{array}{r} 3.39 \\ \times\ \ \ 5.2 \\ \hline 17.628 \end{array}$	$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$
$\begin{array}{r} 5.1 \\ 2.4\overline{)12.24} \end{array}$	$\begin{array}{r} 6 \\ 2\overline{)12} \end{array}$
$\begin{array}{r} 5.1 \\ 24\overline{)122.4} \end{array}$	$\begin{array}{r} 6 \\ 20\overline{)120} \end{array}$

The estimated answers are very close to the exact answers.

Apply Skills
Turn to ***Interactive Text***, page 41.

Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Lesson 15

▶Problem Solving: Statistics

What is important about the median?

There are many ways to think about a set of data. The first job to do with any set of data is to organize it from low to high or high to low. It makes it much easier to find important information when it is organized this way.

We also need to find the average, or mean. This number is the best way to predict any other number that might be added to the set of data. For example, let's say we are keeping track of average car sales for 11 months. We find out that on average 280 cars were sold each month. The best prediction for the 12th month would be that 280 cars would be sold.

Review 1 shows a set of data that has been organized from low to high. Now we can find the minimum, maximum, range, and median. We add the total data and divide by the number of scores to find the mean.

Review 1

What are the min, max, range, mean, and median in the set of data?

27, 35, 37, 41, 46, 52, 133

Mean = $7\overline{)371}$ = 53

Min = 27

Max = 133

Range = 133 – 27 = 106

Median = 41

In Review 1 the mean is 53 and the median is 41. There is a noticeable difference between the two. This is because the max of 133 is extreme compared to the other numbers. Extreme values like this are called outliers. If we took out the 133, we would have just six scores. As Review 2 shows, we would have a mean of 39.67 and a median of 39. Now both numbers are good indicators of the data set.

Review 2

What are the min, max, range, mean, and median in the set of data?

27, 35, 37, 41, 46, 52

Mean = $6\overline{)238}$ = 39.67

Min = 27

Max = 52

Range = 52 – 27 = 25

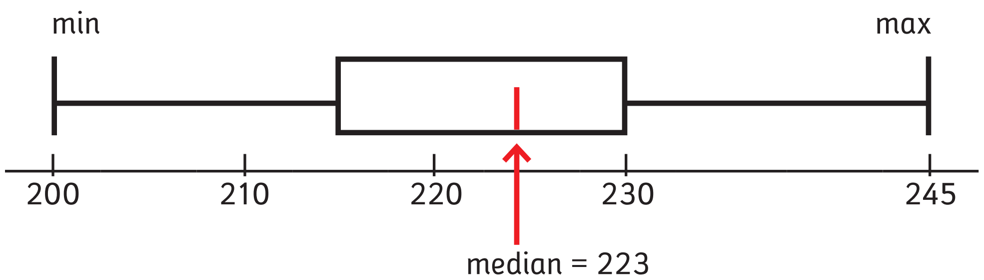
Median = 39

How do we represent data visually?

We can show data in a number of ways. Sometimes when we look at data on a graph or plot, it is easier to make sense of it. One way to show data is the box-and-whisker plot. We learn a lot about data by looking at the length of the three different parts of the plot. Let's look at two plots. Each plot tells a different story about the lower $\frac{1}{4}$, middle $\frac{1}{2}$, and upper $\frac{1}{4}$ of the data set.

Review 1

How do we analyze the data on the box-and-whisker plot?



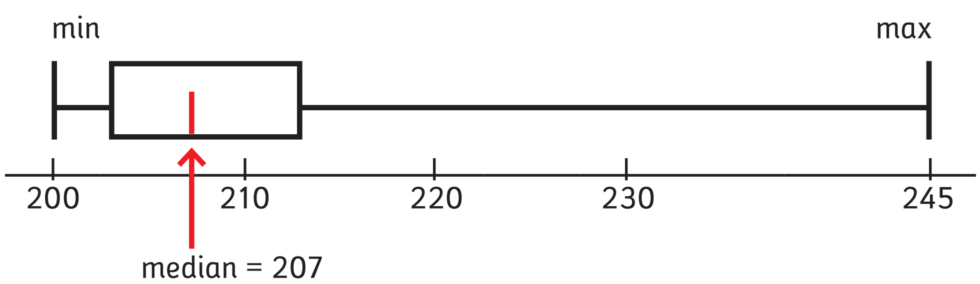
Let's look at what this plot tells us:

- This plot shows an evenly distributed set of data between the minimum (200) and the maximum (245).
- The median is near the middle of the box, which means it is about halfway in the middle $\frac{1}{2}$ of the data.
- Also, the whiskers are about equal in length, which means there is about as much of a spread in the lower $\frac{1}{4}$ of the data as there is in the upper $\frac{1}{4}$ of the data.

Not every box-and-whisker plot will be as evenly distributed as the plot in Review 1. Sometimes the plot shows us that the data is not distributed evenly.

Review 2

How do we analyze the data on the box-and-whisker plot?



This is a very different kind of plot. This is what the plot tells us:

- We see right away the two whiskers are not balanced.
- This means there is a big spread in the scores in the upper $\frac{1}{4}$ of the distribution compared to the lower $\frac{1}{4}$ where all the scores are packed together.
- Half of the scores are between 200 and 207, compared with the other half, which are between 207 and 245.

These data are unbalanced, and most scores are at the lower end of the range.

Now let's look at scatter plots.

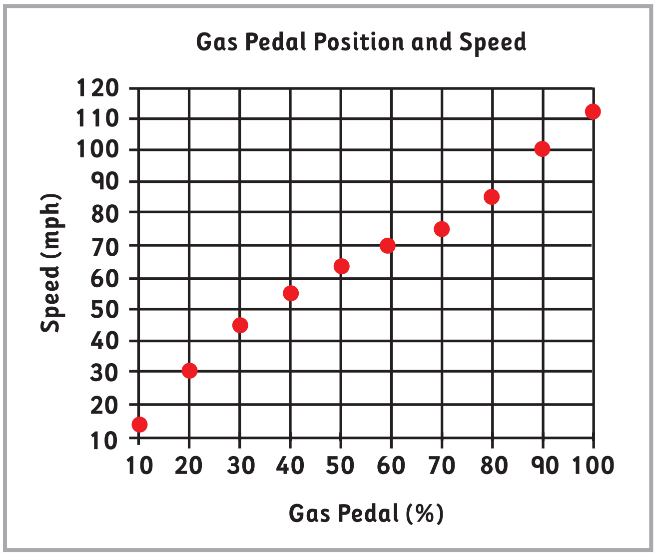
What do we need to remember about scatter plots?

Scatter plots are another way to understand data. In scatter plots, we look for relationships between variables.

Review 1

What is a direct relationship?

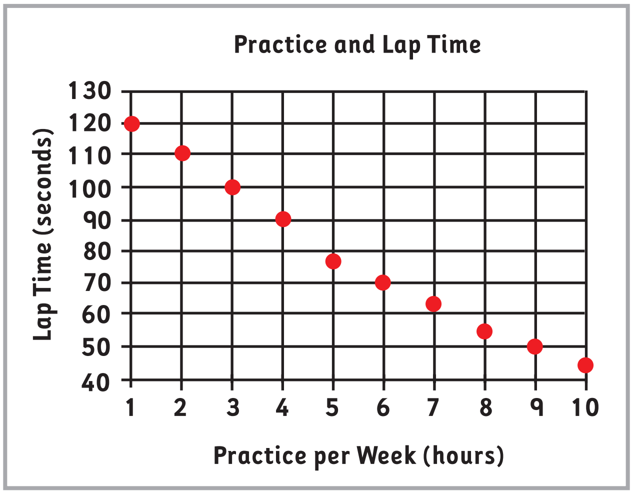
In one type of relationship called a direct relationship, "the more we have of one, the more we have of the other." A good example of this is, "The more you push down on the gas pedal in a car, the faster the car goes." Points on a scatter plot tend to move from the lower left corner to the upper right corner.



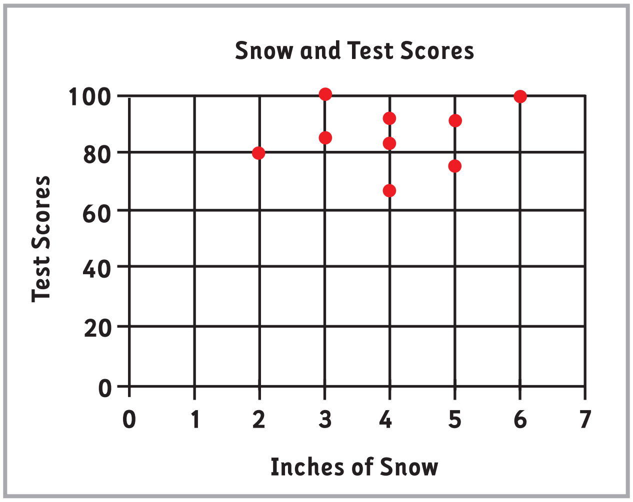
Review 2

What is an indirect relationship?

The opposite is an indirect relationship where "the more we have of one, the less we have of another." A good example of this is, "The more you practice, the less time it takes to run a lap." The points are high on the left and move lower as we go to the right on the scatter plot.



Of course, there are always times when there is no relationship at all. For example, eating more spinach does not make you smarter. Also, lifting more weights does not make you taller.



Problem-Solving Activity
Turn to ***Interactive Text***, page 43.

Reinforce Understanding
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