Lesson 6 | Mixed Practice With Fractions

Problem Solving: Box-and-Whisker Plots

Mixed Practice With Fractions

Vocabulary
reciprocal

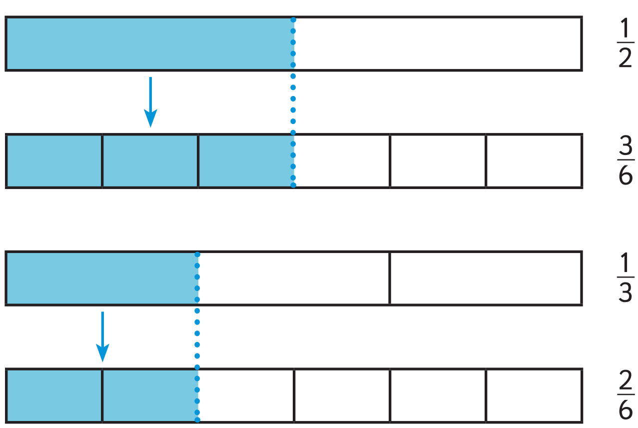
When do we change the denominator and when don't we?

Working with fractions can be confusing. Sometimes we change the denominator, and sometimes we don't. This is why we have spent so much time thinking about key ideas or observations behind each operation. Here is a brief review.

When we add or subtract fractions, we need to have the same fair shares.

If we do not have the same fair shares, or the same denominators, then we get an answer that we cannot describe. Adding $\frac{1}{2}$ and $\frac{1}{3}$ does not give us halves or thirds. The answer does not make sense. By using the same fair shares, we get an answer that is exact. Take a look at the fraction bars below.

$$\frac{1}{2} + \frac{1}{3}$$

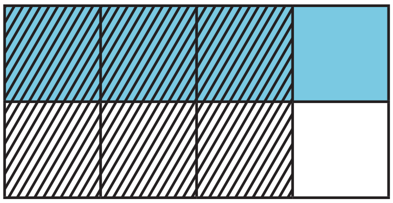


$$\begin{array}{ccccc} \frac{1}{2} & + & \frac{1}{3} & & \\ \downarrow & & \downarrow & & \\ \frac{3}{6} & + & \frac{2}{6} & = & \frac{5}{6} \end{array}$$

When we multiply fractions, we do not change the denominator.

When we multiply, we take a portion of the fraction. This is why the answer is usually smaller than the numbers we are multiplying.

$$\frac{3}{4} \cdot \frac{1}{2}$$

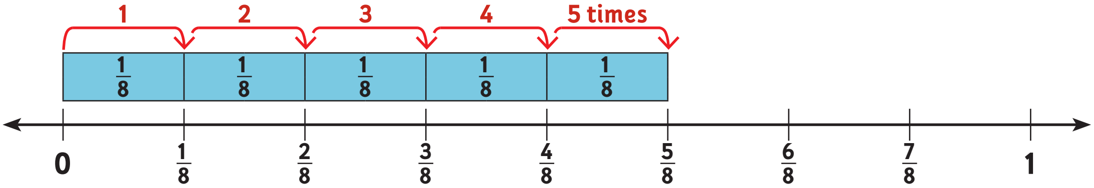


$$\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$$

When we divide fractions, we do not change the denominator.

We break up a fraction with another fraction. The answer to a division problem with two fractions is usually larger than the number we are dividing.

$$\frac{5}{8} \div \frac{1}{8}$$



The traditional way to divide fractions is to invert, or flip over, the second fraction, then multiply.

$$\frac{5}{8} \cdot \frac{8}{1} = \frac{40}{8} = 5$$

What are reciprocals?

When we use the traditional method for dividing fractions, we invert one of the fractions. When we invert a fraction, we make a **reciprocal**. A reciprocal is the inverse of the fraction. Reciprocals are important. We use them a lot in algebra.

Fraction	Reciprocal
$\frac{1}{5}$	$\frac{5}{1}$
$\frac{7}{4}$	$\frac{4}{7}$
$\frac{25}{17}$	$\frac{17}{25}$
$\frac{2}{1}$	$\frac{1}{2}$

When we multiply a fraction and its reciprocal, we always get 1.

Multiplying by the Reciprocal
$\frac{1}{5} \cdot \frac{5}{1} = \frac{5}{5} = 1$
$\frac{7}{4} \cdot \frac{4}{7} = \frac{28}{28} = 1$
$\frac{25}{17} \cdot \frac{17}{25} = \frac{425}{425} = 1$
$\frac{2}{1} \cdot \frac{1}{2} = \frac{2}{2} = 1$

Apply Skills
Turn to ***Interactive Text***, page 15.

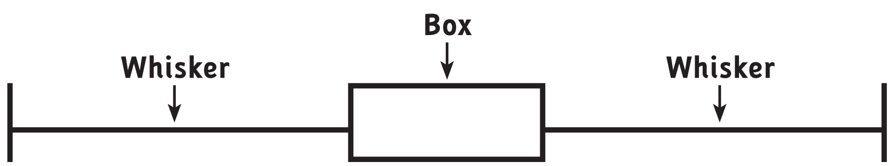
Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

▶Problem Solving: Box-and-Whisker Plots

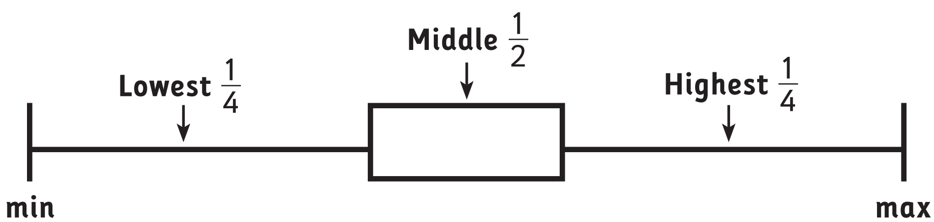
Vocabulary
box-and-whisker plot

What are box-and-whisker plots?

We know that the median is a useful statistic when analyzing data. It is the halfway point in a set of data when the numbers are listed from low to high. Sometimes we want to find out how all the numbers in the data set relate to the median. **Box-and-whisker plots**, a kind of graph, help us understand how all the data are distributed from low to high.



When we make box-and-whisker plots, we break the data into three parts. The first whisker contains the lowest $\frac{1}{4}$ of the data. The box in the middle contains the middle $\frac{1}{2}$ of the data, and the last whisker contains the highest $\frac{1}{4}$ of the data.



How do we make a box-and-whisker plot from a table of data?

Let's see how a box-and-whisker plot helps us organize data.

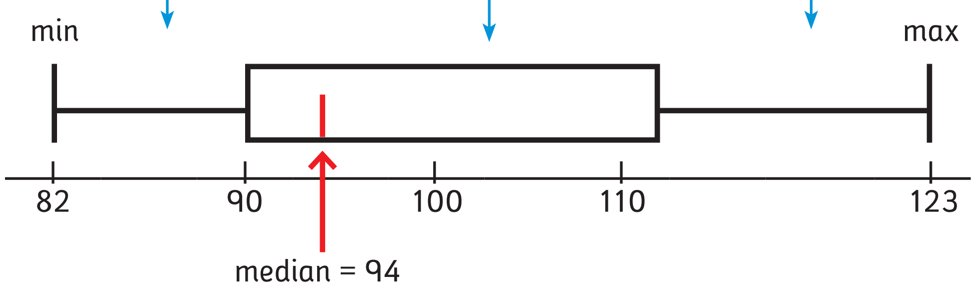
It is Sports Day at Union High School. There are many track-and-field events, and all students are encouraged to try them. Fifteen students try the long jump. Their longest jumps are recorded.

The data are in order from shortest to longest jumps. It is easier to find the median when we organize the data this way.

Student	Distance (in inches)
Amber	82
Oscar	87
Autumn	88
Brittany	90
Joshua	91
Marcus	92
Mikaela	93
Ryan	94 ← median
Pablo	98
Seth	108
Tracey	110
Miguel	115
Paige	118
DeAnne	121
Lamar	123

Now we break the numbers into three different parts: (1) the lowest $\frac{1}{4}$, (2) the middle $\frac{1}{2}$, and (3) the highest $\frac{1}{4}$.

Long Jump Data (in inches)		
Lowest $\frac{1}{4}$	Middle $\frac{1}{2}$	Highest $\frac{1}{4}$
Amber: 82	Joshua: 91	Miguel: 115
Oscar: 87	Marcus: 92	Paige: 118
Autumn: 88	Mikaela: 93	DeAnne: 121
Brittany: 90	Ryan: 94	Lamar: 123
	Pablo: 98	
	Seth: 108	
	Tracey: 110	



We see where the median is inside the box. It is not in the middle of the box but toward the lower end.

Now we use the box-and-whisker plot to see how each student did compared to the median and to each other.

- The lowest $\frac{1}{4}$ of the jumpers jumped between 82 and 90 inches. Some of the students are not that far from the median of 94 inches.
- The middle $\frac{1}{2}$ of the students jumped between 90 and 110 inches. Some of the jumpers in this group, like Seth and Tracey, are in the middle $\frac{1}{2}$, but they are a good distance away from the median.
- Finally, the highest $\frac{1}{4}$ of the students jumped between 110 and 123 inches. They are the top $\frac{1}{4}$ of the jumpers in the group. DeAnne and Lamar are at the top, and they are a long way from the median.

What is the difference between the median and the halfway point between the min and the max?

When we compare the numbers below the median with those above the median in the box-and-whisker plot for the long jump data, we see right away that the range is smaller in the lower half. The box-and-whisker plots help us see how the numbers are distributed in a list of numbers, even if the list is ordered from low to high.

The range in the lower half is from the minimum of 82 to the median of 94. That is only 12 points. In the upper half, though, there is a range of 29 points from the median to the maximum.

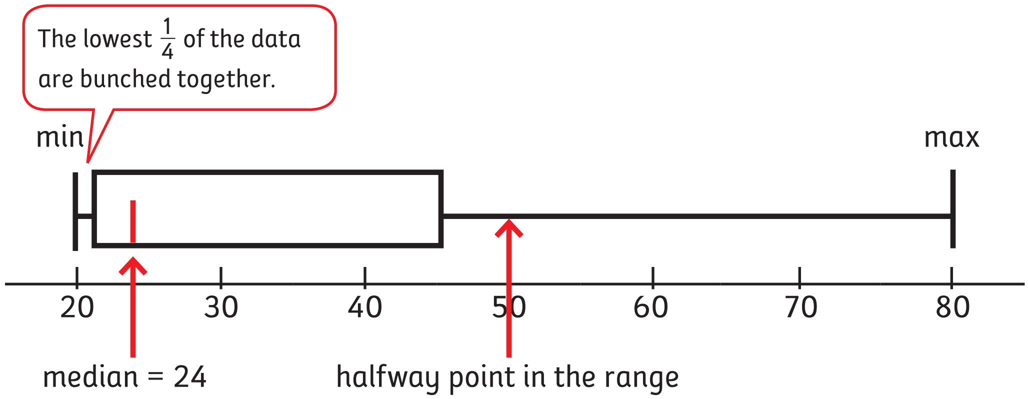
This plot shows that even though the median is the halfway point in a set of data, it does not mean that it is exactly halfway between the minimum and the maximum.

Let's draw another box-and-whisker plot to illustrate this.

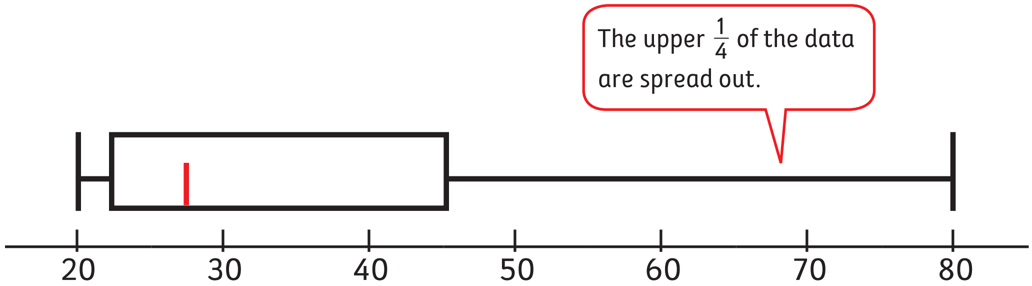
During Sports Day, some of the students take part in the discus throw. These are their results:

Discus Throw (in feet)		
Samuel	20	**min**
Bud	21	
Aubrey	22	
Nate	23	**median**
Jordan	25	
Jesus	40	
Tom	50	
Patrice	80	**max**

- The median is the halfway point of the data. Because we have an even number of throwers, we find the point between Nate's and Jordan's scores, 24.
- The range is the difference between the largest and smallest scores, 80 – 20 = 60. The halfway point from the min to the max is 20 + 30 = 50. There is a difference of 26 between the median and the halfway point in the range. Most of the scores fall well below the halfway point.



A box-and-whisker plot clearly shows where the extreme scores are and how numbers in a set of data can be bunched together or spread out.



Box-and-whisker plots help us understand the data better and interpret it properly.

POWER CONCEPT

Box-and-whisker plots help us understand how data are distributed.

Problem-Solving Activity
Turn to ***Interactive Text***, page 16.

mBook Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Homework

Activity 1

Add, subtract, multiply, and divide the following fractions. Be careful to use the correct strategy. Simplify the answers if necessary.

1. $\frac{1}{2} + \frac{3}{4}$ **2.** $\frac{4}{5} - \frac{2}{3}$ **3.** $\frac{1}{6} \div \frac{1}{2}$

4. $\frac{2}{5} \cdot \frac{1}{3}$ **5.** $\frac{2}{6} - \frac{1}{9}$ **6.** $\frac{3}{5} \div \frac{1}{5}$

Activity 2

Give the reciprocal for each of the numbers.

Model 8
Answer: $\frac{1}{8}$

1. $\frac{1}{3}$ **2.** $\frac{4}{5}$ **3.** 9

4. $\frac{6}{8}$ **5.** 10 **6.** $\frac{2}{7}$

Activity 3

Give the missing part in the problems involving reciprocals.

Model $4 \cdot$ ______ $= 1$
Answer: $\frac{1}{4}$

1. ______ $\cdot \frac{4}{5} = 1$ **2.** $\frac{3}{2} \cdot$ ______ $= 1$ **3.** $5 \cdot \frac{1}{5} =$ ______

4. $\frac{7}{8} \cdot$ ______ $= 1$ **5.** ______ $\cdot \frac{1}{8} = 1$ **6.** $\frac{4}{3} \cdot \frac{3}{4} =$ ______

Activity 4 • Distributed Practice

Solve.

1. Find the first six multiples of 5 and 10. Give the common multiples.
2. What are the common factors of 8 and 12?
3. What is the least common denominator for the problem $\frac{1}{3} + \frac{1}{4}$?
4. What is the greatest common factor of 56 and 64?
5. 160 – 78
6. $32 \cdot 9$
7. 150 ÷ 4
8. 1,027 + 873
9. $411 \cdot 3$
10. 600 ÷ 30

Copyright 2010 by Cambium Learning Sopris West®. All rights reserved. Permission is granted to reproduce this page for student use.