Lesson 4

Multiply and Simplify

Problem Solving:

Putting It All Together

Multiply and Simplify

Vocabulary

improper fraction

What is the traditional method for multiplying fractions?

Multiplying fractions is a relatively easy process. It is easy because we do not have to change denominators. We just multiply straight across.

$\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$ $\frac{7}{2} \cdot \frac{2}{7} = \frac{14}{14}$ $\frac{6}{8} \cdot \frac{3}{7} = \frac{18}{56}$ $\frac{6}{3} \cdot \frac{4}{2} = \frac{24}{6}$

Remember:

When we add or subtract fractions, we are looking for an exact answer. That is why we must have the same denominators when we add or subtract:

$$\frac{1}{4} + \frac{2}{3}$$

- Is the answer written in fourths, or is it written in thirds? We have to have fair shares.
- We change $\frac{1}{4} + \frac{2}{3}$ into $\frac{3}{12} + \frac{8}{12}$ to get fair shares.
- Now we add so that the problem makes sense.

$$\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

When we multiply fractions, we are taking a portion of one of the fractions. When we multiply $\frac{2}{3} \cdot \frac{1}{2}$, we are taking $\frac{2}{3}$ of $\frac{1}{2}$. Think of it as taking a "fraction of a fraction."

$$\frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$$

How do we simplify the answer?

In most cases, we need to simplify the answers to problems. The larger fractions can be broken down into simpler fractions. We simplify by pulling out a fraction equal to one. The fraction we find contains the greatest common factor of the numerator and the denominator placed over itself. In a fraction, any number over itself is always equal to 1. Also, any number times 1 is the same number.

Example 1

Simplify the answer.

$$\frac{3}{4} \cdot \frac{2}{4} = \frac{6}{16}$$

$$\frac{6}{16} = \frac{3}{8} \cdot \frac{2}{2}$$

Because 2 is the greatest common factor of 6 and 16, we use $\frac{2}{2}$ as our fraction.

$$= \frac{3}{8} \cdot 1$$

$$= \mathbf{\frac{3}{8}}$$

Sometimes the numerator is bigger than the denominator. These are called **improper fractions**. We usually change improper fractions to mixed numbers when we simplify the answer.

Example 2

Simplify the answer when it is an improper fraction.

$$\frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8}$$

The answer, $\frac{9}{8}$, is an improper fraction.

We make a mixed number this way:

$$\frac{9}{8} = \frac{8}{8} + \frac{1}{8}$$

We need to find the whole number in the fraction.

$$= \mathbf{1\frac{1}{8}}$$

When do we simplify more than once?

There are times when we have to change our answer to a proper fraction or mixed number and still need to simplify the answer. In Example 1, we simplify the answer more than once.

Example 1

Write the answer in simplest form.

$$\frac{4}{5} \cdot \frac{3}{2} = \frac{12}{10}$$

First, we change the answer to a mixed number.

$$\frac{12}{10} = \frac{10}{10} + \frac{2}{10}$$

$$= 1\frac{2}{10}$$

Next, we simplify $\frac{2}{10}$

Next, we simplify $\frac{2}{10}$. We find the greatest common factor of the numerator and the denominator. It is 2.

$$\frac{2}{10} = \frac{1}{5} \cdot \frac{2}{2}$$

$$= \frac{1}{5} \cdot 1$$

$$= \frac{1}{5}$$

Be sure to simplify all answers. The products of multiplication problems, as well as the answers to all operations with fractions, should be simplified.

So, $\frac{12}{10} = 1\frac{1}{5}$

The final answer after simplifying is $1\frac{1}{5}$.

When working with mixed numbers, we simplify all of our answers.

Apply Skills
Turn to ***Interactive Text***, page 10.

Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

▶Problem Solving: Putting It All Together

Vocabulary
outlier

How do we use statistics to make sense of data?

We have worked with the minimum, maximum, mean, mode, and median. The mean, or average, helps us predict the next number in the set of data.

But the mean alone can sometimes give us an incomplete picture of the data. We need to look at all the numbers (min, max, range, mean, and median) if we want to have a good understanding of a set of data.

The data set below shows the number of cars sold at Wilson's Autos. The data are written in order from low to high. This makes it easier to find the *min*, *max*, *mode*, and *median*. It also helps us compare the mean with the other data.

Example 1

Find the mean, median, mode, and range of the data in the table.

Week	Number of Cars Sold	
July 9 to July 15	21	**min**
June 4 to June 10	29	**mode**
June 18 to June 24	29	**mode**
July 16 to July 22	37	
July 23 to July 29	38	**median**
July 2 to July 8	39	
July 30 to Aug. 5	40	
June 11 to June 17	44	
June 25 to July 1	47	**max**
Total	**324**	

The table shows:

- the min = 21, the max = 47
- the range = 26 (47 − 21)
- the mode = 29 (the number that occurs most frequently in the list)
- the mean = 36 (the total, 324, divided by the number of data points, 9)

The median of 38 is close to the mean of 36. In a data set, when the mean and the median are close together, then either one is a good indication of what is "about average."

Let's look at what happens when we have some extreme numbers.

Let's say that Wilson's Autos wants to sell all the cars it has before new cars come in October. It decides to run a big sale. The sale was so successful that Wilson's Autos started to run out of cars by the week of September 17. That is why the number of auto sales per week drops off so much for the last two weeks of September.

Example 2

Find the mean, median, mode, and range of the data with extreme numbers.

Week	Number of Cars Sold
Sept. 24 to Sept. 30	1
Sept. 17 to Sept. 23	7
Sept. 10 to Sept. 16	78
Aug. 20 to Aug. 26	83
Aug. 13 to Aug. 19	85
Aug. 6 to Aug. 12	92
Sept. 3 to Sept. 9	95
Aug. 27 to Sept. 2	103
Total	**544**

minimum

median = 84
mean = 68

maximum

The table shows:

- The min and the max are easy to see. The min is 1, and the max is 103.
- The range is 103 – 1, or 102.
- The median is the halfway point in the set of numbers. We have an even number of weeks. To determine the halfway point, we find the midway point between 83 and 85. The median is 84.
- Because there is no number that occurs most often, or more than once, there is no mode.

To get the mean, we add all the cars sold and divide by the total number of weeks:

$544 \div 8 = 68$

There are two weeks of very low sales, but most of the numbers in the data set are between 78 and 103.

Notice the gap between the mean, 68, and the median, 84.

Now let's look at what happens when we take out the two extreme numbers, 1 and 7. Numbers like this in a data set are called **outliers**. An outlier is any number that is significantly larger or smaller than the other numbers in the data set.

Example 3

Find the mean, median, mode, and range of the data after removing the outliers.

Week	Number of Cars Sold
Sept. 10 to Sept. 16	78
Aug. 20 to Aug. 26	83
Aug. 13 to Aug. 19	85
Aug. 6 to Aug. 12	92
Sept. 3 to Sept. 9	95
Aug. 27 to Sept. 2	103
Total	**536**

median = 88.5
mean = 89.3

Let's look at what happened when we removed the outliers:

- Now the minimum is 78 and the maximum is still 103.
- The range is 103 – 78, or 25. This is a much smaller range than before.
- The median did not change that much from before, from 84 to 88.5.
- The mean changed a great deal, from 68 to 89.3.

When we have extreme numbers, the mean is affected. In this case, it is less when the outliers 1 and 7 are included. This is why the median is a better overall indicator of week-by-week auto sales throughout the eight weeks.

Another way to understand the effect of extremes on the mean and the median is to change one of the numbers to an extremely large number. Suppose that instead of selling one car for the week of September 24–September 30, Wilson's Autos sold 1,000 cars. Now the max is the most extreme number.

The table in Example 4 shows the change in the data.

Example 4

Compare the median and mean when an outlier has been added to the data set.

Week	Number of Cars Sold
Sept. 17 to Sept. 23	7
Sept. 10 to Sept. 16	78
Aug. 20 to Aug. 26	83
Aug. 13 to Aug. 19	85
Aug. 6 to Aug. 12	92
Sept. 3 to Sept. 9	95
Aug. 27 to Sept. 2	103
Sept. 24 to Sept. 30	1,000
Total	**1,543**

median = 88.5
mean = 192.9

The table shows:

- The outlier 1,000 has a great effect on the mean, causing it to jump from 68 to 192.9.
- The change in the median is very small, moving from 84 to 88.5. This makes the median a much better description of this data than the mean.

When we think about data, we look at both the mean and the median. We also look at the min and max and see if they are extreme compared to the other numbers. When the mean and the median are close, then either number is a good description of the data. When they are far apart, the median tends to be a better description of the data. There is less of a change to the median based on outliers.

POWER CONCEPT

When the mean and the median are close, either number is a good description of the center of the data. When they are far apart, the median tends to be a better description of the center of the data.

Problem-Solving Activity
Turn to *Interactive Text*, page 11.

mBook Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Lesson 4

Homework

Activity 1

Add and subtract.

1. $\frac{1}{4} + \frac{1}{8}$
2. $\frac{5}{6} - \frac{2}{3}$
3. $\frac{1}{3} + \frac{1}{9}$
4. $\frac{5}{8} - \frac{1}{4}$
5. $\frac{2}{3} + \frac{3}{4}$
6. $\frac{4}{9} - \frac{1}{6}$

Activity 2

Multiply across and simplify the answer.

Model $\frac{1}{3} \cdot \frac{2}{4}$ Answer: $\frac{1 \cdot 2}{3 \cdot 4} = \frac{2}{12} = \frac{2}{2} \cdot \frac{1}{6} = \frac{1}{6}$

1. $\frac{1}{5} \cdot \frac{2}{10}$
2. $\frac{3}{4} \cdot \frac{2}{3}$
3. $\frac{4}{6} \cdot \frac{1}{4}$
4. $\frac{1}{2} \cdot \frac{3}{9}$

Activity 3

Select the true statement.

1. Data Set: 1, 2, 3, 4, 5
   (a) The mean is bigger than the median.
   (b) The mean is the same as the median.
   (c) The mean is smaller than the median.
2. Data Set: 4, 6, 9, 11, 14
   (a) The mode is 9.
   (b) The median is 9.
   (c) The range is 9.

Activity 4 • Distributed Practice

Solve.

1. What is the least common denominator for the problem $\frac{1}{6} + \frac{1}{9}$?
2. 45 + 22 + 37
3. 267 − 199
4. 48 • 2
5. 236 ÷ 4
6. 629 + 481
7. 222 • 3