Lesson 8 ▶Working With Decimal Numbers

Problem Solving:
▶Interpreting Box-and-Whisker Plots

▶Working With Decimal Numbers

Vocabulary

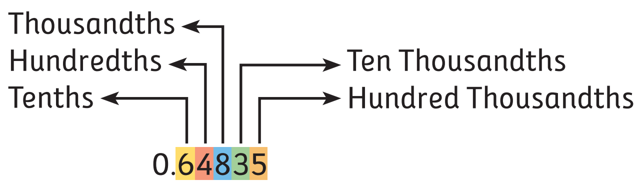
benchmark
repeating decimal
irrational number

How do we round decimal numbers?

In Lesson 7, we learned to change a fraction into a decimal number by dividing the denominator into the numerator. The result was often a decimal number with many digits to the right of the decimal point. Decimal numbers like this are difficult to read and understand. For example:

$$\frac{9}{14} = \begin{array}{r} 0.64286 \\ 14\overline{)9} \end{array}$$

Reading decimal numbers means knowing about place value.



This number is difficult to read. It is read "64,835 hundred thousandths." If we understand place value, we will know the meanings of the numbers.

But for most of us, in our day-to-day life, we do not work with such precise numbers. Instead, we round the decimal number to a more manageable decimal place, such as tenths, hundredths, or thousandths.

The rules for rounding decimal numbers are the same as we used for whole numbers. In this case, we will round 0.64835 to the hundredths place.

Example 1

Round 0.64835 to the hundredths place.

Remember: If the number is five or greater, we round up to the next digit.

0.6 4 8 3 5

We look at the number to the right of the hundredths place and circle it.

0.6 4 8 3 5

The 8 in the thousandths place is greater than 5, so we round up.

0.65

Now our decimal number is rounded to 0.65 or "65 hundredths."

Now let's change the decimal number slightly, to 0.64235, and round to the hundredths place. The number in the thousandths column is less than five. We do not round up because 2 is less than five.

Example 2

Round 0.64235 to the hundredths place.

0.6 4 2 3 5
0.64

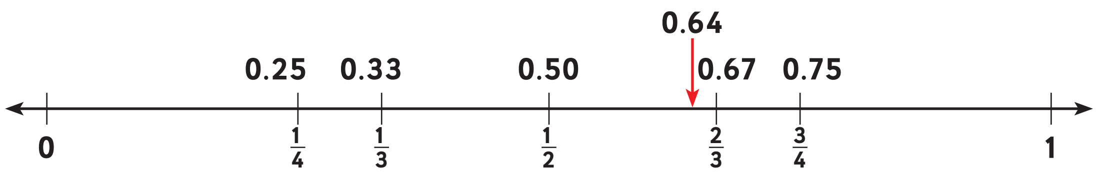
Because the number to the right of the hundredths place is less than five, we do not round up. We drop all the numbers to the right of the hundredths place and round to 0.64, or "64 hundredths."

How does rounding help us understand a number's meaning?

An important part of working with decimal numbers is knowing what a rational number is close to on the number line. There are some important rational numbers that we need to remember, called **benchmarks**. The number line below shows some of the most common benchmarks for fractions and decimal numbers. We remember these numbers because they help us understand how big a decimal number is.

Example 1

Find the approximate location of 0.64285 on the number line.

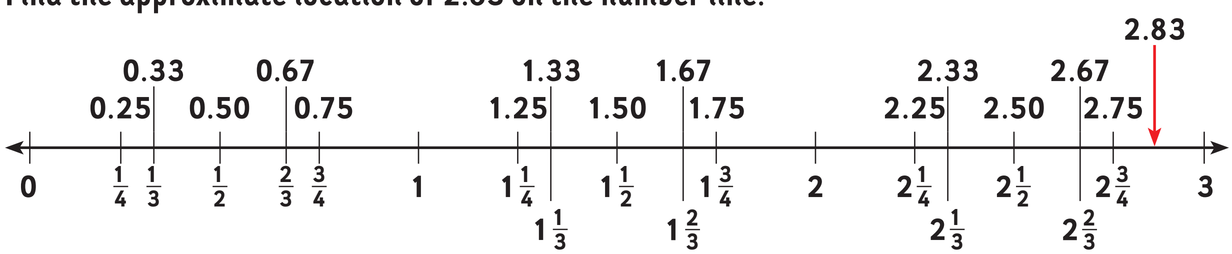


The decimal number 0.64 is close to 0.67, or $\frac{2}{3}$. That means 0.64285 is about $\frac{2}{3}$.

Benchmarks can be found between any whole numbers or they can be the whole numbers themselves. Let's look at what happens with a decimal number greater than 1.

Example 2

Find the approximate location of 2.83 on the number line.



Look for the number 2 on the number line. Then find which benchmark 0.83 is closest to. The decimal part 0.83 is close to the benchmark 0.75.

So 2.83 is about 2.75, or $2\frac{3}{4}$.

How do we work with decimal numbers that never end?

Sometimes when we change a fraction into a decimal number, we get a pattern. The numbers keep repeating themselves. These are called **repeating decimals**.

Look at the repeating pattern we get when we convert $\frac{3}{11}$ and $\frac{4}{7}$ to decimal numbers. There is a special way to write this kind of number. We put a line over the top to show that the pattern repeats itself.

repeating pattern

$$11\overline{)3}\quad \overbrace{0.27272727272727}$$

written as

$$\overbrace{0.\overline{27}}$$

repeating pattern

$$7\overline{)4}\quad \overbrace{0.571428571428}$$

written as

$$\overbrace{0.\overline{571428}}$$

With repeating decimal numbers, we stop the decimal after the pattern is shown once. We put a line over the repeated part of the decimal.

There are also other kinds of decimal numbers that never end, but they do not have a pattern. These are called **irrational numbers**. One of the most famous irrational numbers is pi. We use pi to find the circumference and area of a circle. It is a comparison of the diameter and the circumference of a circle. The decimal number for pi is 3.14159265 . . . The three dots at the end mean that the decimal number goes on infinitely.

We can get close to pi by changing $\frac{22}{7}$ into a decimal number. There are even contests for people who can remember the most decimal places for pi. People have memorized pi to more than 1,000 decimal places. There is no pattern to irrational numbers like pi.

Apply Skills
Turn to ***Interactive Text***, page 21.

mBook **Reinforce Understanding**
Use the ***mBook Study Guide*** to review lesson concepts.

▶Problem Solving: Interpreting Box-and-Whisker Plots

How do we interpret box-and-whisker plots?

Now let's put together all that we have learned about box-and-whisker plots. We use the concepts of minimum, maximum, range, mean, and especially median to understand a set of data. Let's look at some data from two different sporting events. All of the numbers have been rounded to the closest decimal number benchmarks. We will interpret the data by entering information from tables into box-and-whisker plots.

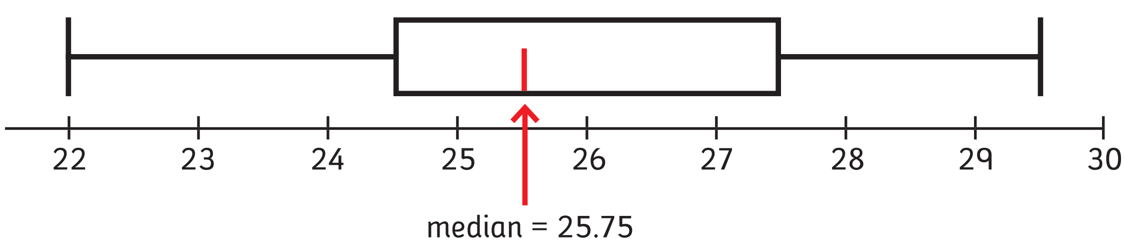
Example 1

Interpret the box-and-whisker plot for the hammer throw.

Hammer Throw	
Name	**Feet**
Marcus	22
Tracey	22.5
Oscar	23
Seth	24.25
Paige	24.25
Michael	24.5
Lamar	25.5
Amber	26
Ryan	26
Robert	26.5
Autumn	27.5
Brittany	29
DeAnne	29.25
Joshua	29.5
Total	**359.75**

Mean = 25.7

Median = 25.75



What does the data tell us?

- The mean and the median are about the same. In fact, the median is close to the middle of the box, which means it is in the center of the middle $\frac{1}{2}$ of the scores.
- We see from the box-and-whisker plot that the lower and upper quarters, or whiskers, are about the same. That means people in the lower and upper quarters are spread out about the same distance.
- The range is only 7.5 points and the median is just about halfway between the minimum and the maximum. We have a pretty evenly distributed set of data, beginning with a minimum of 22 and reaching a maximum of 29.5.

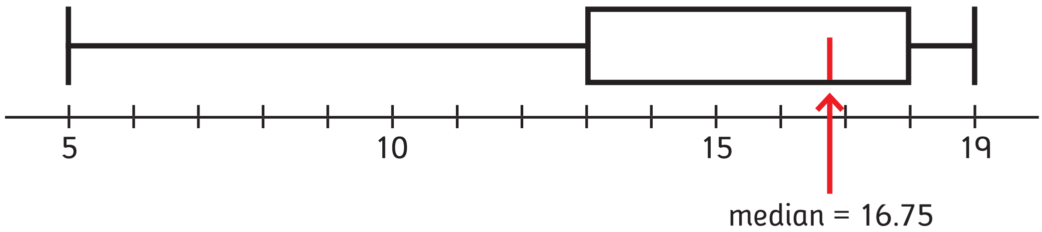
Example 2

Interpret the box-and-whisker plot for the shot put.

Shot Put	
Name	**Feet**
Brittany	5
Seth	6
DeAnne	6.5
Amber	13
Paige	14.5
Lamar	14.75
Robert	15.5
Tracey	16.75
Autumn	17
Erica	17.25
Joshua	17.5
Ryan	18
Oscar	18.25
Michael	18.5
Marcus	19
Total	**217.5**

Mean = 14.5

Median = 16.75



What does the data tell us?

- The lower $\frac{1}{4}$ whisker is a lot longer than the upper $\frac{1}{4}$ whisker. That means there is a bigger spread of scores between the very lowest distance, or minimum of 5, and the bottom of the box, 13. It is a much bigger distance than the top of the box, 18, and the maximum of 19. So, some students could not throw the shot very far.
- The range in the lower $\frac{1}{4}$ was from 5 feet to 13 feet. That is quite a difference than the highest $\frac{1}{4}$ of the scores. These students were very close to each other. The top three students threw within 1 foot of each other.
- There is a bit of a difference between the mean and the median, but it isn't as clear as the differences between the lower and upper quarters of the scores.

In summary, the set of scores for the hammer throw show us that students are fairly evenly distributed from a minimum of 22 to a maximum of 29.5.

The shot put data tell us that there are some students in the lower $\frac{1}{4}$ who couldn't throw very far and the best students in the group were very close together in their throws.

The shape of the box-and-whisker plot helps us think about the data in the tables.

Problem-Solving Activity
Turn to ***Interactive Text***, page 22.

mBook **Reinforce Understanding**
Use the ***mBook Study Guide*** to review lesson concepts.

Lesson 8

Homework

Activity 1

Look at the calculator display for each of the fraction-to-decimal number conversions. Round the numbers to the nearest hundredths place.

Model $\frac{1}{9}$ 0.111111111 Answer: 0.11

1. $\frac{2}{3}$ 0.66666666666666
2. $\frac{3}{8}$ 0.375
3. $\frac{13}{7}$ 1.857142857142857
4. $\frac{6}{11}$ 0.545454545454545
5. $\frac{1}{3}$ 0.333333333333333
6. $\frac{6}{7}$ 0.857142857142857

Activity 2

Rewrite each of the repeating decimal numbers using the line over the top to represent the repeating part.

Model $\frac{3}{11} = 0.272727272727272 = 0.\overline{27}$

1. $\frac{2}{7} = 0.285714285714285714$
2. $\frac{4}{11} = 0.3636363636363636$
3. $\frac{5}{9} = 0.5555555555555555555$
4. $\frac{1}{6} = 0.1666666666666666666$

Homework

Activity 3

Select the best answer for the questions about decimal numbers.

1. When a decimal number does not seem to have an end and there is no pattern to it, we call this a(n) _______ number.
   (a) rational
   (b) whole
   (c) irrational

2. If you were asked to round 0.275 to the nearest hundredths place, the answer would be _______.
   (a) 0.28
   (b) 0.27
   (c) 0.3

3. Pi is an example of an irrational number because _______.
   (a) it repeats but doesn't end
   (b) it doesn't end and it doesn't repeat
   (c) it ends but doesn't repeat

4. If you were asked to round 0.119 to the nearest tenths place, the answer would be _______.
   (a) 0.1
   (b) 0.2
   (c) 0.12

Activity 4 • Distributed Practice

Solve.

1. $480 \div 12$
2. $999 + 1{,}011$
3. $47 \cdot 9$
4. $3{,}201 - 1{,}987$
5. $\frac{3}{5} + \frac{2}{4}$
6. $\frac{3}{5} - \frac{1}{3}$
7. $\frac{2}{3} \cdot \frac{4}{5}$
8. $\frac{3}{8} \div \frac{1}{4}$
9. $\frac{9}{5} - \frac{7}{5}$
10. $\frac{5}{12} + \frac{4}{6}$