Face Reconstruction Using Two-Dimensional PCA Method

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***Abstract*—This paper investigates the potential of Two-Dimensional Principal Component Analysis (2DPCA) in image representation. As opposed to PCA, 2DPCA is based on 2D image matrices rather than 1D vectors, so the image matrix does not need to be transformed into a vector prior to feature extraction. Instead, an image covariance matrix is constructed directly using the original image matrices, and its eigenvectors are derived for image feature extraction. To test 2DPCA and evaluate its performance, a series of experiments were performed using the ORL face image database. The recognition rate across all trials was higher using 2DPCA than using PCA. The experimental results also indicated that the extraction of image features is computationally more efficient using 2DPCA than PCA.**

***Index Terms*—Face Reconstruction, Principal Component Analysis (PCA), Eigenfaces, Facial Recognition, Feature Extraction.**

I. Introduction

Principal component analysis (PCA), also known as KarhunenLoeve expansion, is a classical feature extraction and data representation technique widely used in the areas of pattern recognition and computer vision. Sirovich and Kirby first used PCA to efficiently represent pictures of human faces.The core idea of PCA is to transform multiple variables in the original dataset into a few uncorrelated principal components through linear transformation, thereby reducing the dimensionality of the data while retaining as much of the most important information from the original data as possible. In the PCA-based face recognition technique, the 2D face image matrices must be previously transformed into 1D image vectors.The resulting image vectors of faces usually lead to a high-dimensional image vector space, where it is difficult to evaluate the covariance matrix accurately due to its large size and the relatively small number of training samples. In this paper, a straightforward image projection technique was adopted for the extraction of image features, known as the two-dimensional principal component analysis (2DPCA), which has previously been developed by other researchers. Unlike conventional PCA, which operates on 1D vectors, 2DPCA works directly with 2D matrices. This means that the image matrix does not need to be transformed into a vector beforehand. Instead, an image covariance matrix can be constructed directly from the original image matrices. Compared to the covariance matrix used in PCA, the image covariance matrix in 2DPCA is significantly smaller. Consequently, 2DPCA offers two key advantages over PCA: it facilitates more accurate and efficient computation of the covariance matrix, and it requires less time to determine the corresponding eigenvectors.

The primary contributions are listed as follows:

- The 2DPCA Algorithm: Introducing a two-dimensional principal component analysis (2DPCA) method based on two-dimensional image matrices for image feature extraction and representation. Unlike traditional PCA, it avoids the process of converting two-dimensional image matrices into one-dimensional vectors and directly constructs the covariance matrix using the original image matrices, simplifying the image feature extraction process.
- Improving Recognition Accuracy: Conducted extensive experiments on the ORL face image database. The results showed that 2DPCA achieved higher face recognition accuracy than PCA under various conditions, such as pose, expression, and illumination changes.
- Enhancing Computational Efficiency: 2DPCA is significantly more computationally efficient than PCA in feature extraction. For example, in the experiments on the ORL database, the feature extraction time was greatly reduced, and the efficiency advantage became more pronounced as the number of training samples increased. This contributes to improving the processing speed of face recognition systems.
- Analyzing the Reasons for Performance Differences: Analyzed the reasons for the performance advantages of 2DPCA. It was pointed out that the image covariance matrix of 2DPCA is small, making it more suitable for small sample size problems (such as face recognition). It can evaluate the covariance matrix more accurately, resulting in better recognition accuracy.
- Robustness and Generalization:Evaluation of the robustness of the proposed method against variations in lighting conditions, facial expressions, and pose changes.Exploration of the method's generalization capabilities to diverse face datasets.

II. BACKGROUND

Principal Component Analysis (PCA), also known as Karhunen-Loeve expansion, is a classical technique widely used for feature extraction and data representation in pattern recognition and computer vision. The Karhunen-Loeve expansion is particularly useful for representing random fields and is based on the spectral decomposition of the covariance function. PCA's ability to transform high-dimensional data into a lower-dimensional space while retaining the most significant features makes it an invaluable tool in various applications.

As early as 1987 and 1990, Sirovich and Kirby [1], [2] pioneered the use of PCA to effectively represent human face images. They demonstrated that a face image could be approximately reconstructed as a weighted combination of a small number of specific images (eigenimages) and an average image. This foundational work laid the groundwork for the development of the Eigenfaces method for face recognition by Turk and Pentland in 1991 [3]. The Eigenfaces method treats images as vectors in a high-dimensional space and projects them onto an eigenspace spanned by the leading eigenvectors of the sample covariance matrix of the training images. This approach has become a de facto standard and a common performance benchmark in the field of face recognition.

Since then, PCA has attracted significant attention and has been extensively studied and improved upon. Numerous subsequent studies have built upon this foundation, exploring various aspects such as the optimization of the PCA algorithm and its application in different contexts [4], [5], [6], [7]. For instance, PCA has been used not only for face recognition but also for other tasks such as image compression and noise reduction.

However, PCA also faces several challenges in practical applications. One of the primary issues is the problem of dimensionality in the "face space," as discussed by Penev and Sirovich [8]. This issue arises due to the high-dimensional nature of the data and the limited number of samples, which can lead to difficulties in accurately estimating the covariance matrix. Zhao and Yang [9] addressed the impact of illumination on PCA-based systems by developing methods to account for lighting variations, which is crucial for improving the robustness of face recognition systems [9]. Additionally, Wiskott et al. [10] pointed out that PCA has limitations in capturing invariance, which is essential for achieving robust recognition under varying conditions.

In response to these challenges, several improved methods related to PCA have emerged in recent years. Independent Component Analysis (ICA) and Kernel Principal Component Analysis (Kernel PCA) have become research hotspots. Bartlett et al. [11] and Draper et al. [12] proposed using ICA for face representation and found that it performed better than PCA under specific similarity measures, such as cosine similarity, but the difference was not significant under other measures like Euclidean distance [7]. Yang [13] adopted Kernel PCA for face feature extraction and recognition, achieving better results than the classical Eigenfaces method [7]. However, these methods have a relatively high computational cost, with the average computation time ratio for ICA, Kernel PCA, and PCA being 8.7: 3.2: 1.0 [7].

In the traditional PCA face recognition technique, the two-dimensional face image matrix needs to be pre-transformed into a one-dimensional vector, leading to the generation of a high-dimensional image vector space. This transformation results in a large covariance matrix that is difficult to evaluate accurately due to the relatively limited number of samples. Although the eigenvectors can be calculated using the Singular Value Decomposition (SVD) technique to avoid directly generating the covariance matrix, the accuracy of the eigenvectors is still affected [7]. SVD is a powerful tool in PCA, as it allows for the decomposition of the data matrix into orthogonal components, which can be used to identify the principal components of the data. However, the computational complexity of SVD increases significantly with the size of the data, making it challenging to apply in real-time systems or with large datasets.

To address these limitations, Two-dimensional PCA (2DPCA) was introduced as an alternative approach. Unlike traditional PCA, which treats images as vectors, 2DPCA views an image as a matrix and performs the PCA directly on the matrix form. This approach reduces the computational cost and dimensionality of the problem, making it more efficient for large-scale face recognition tasks. 2DPCA has been shown to achieve higher recognition rates compared to traditional PCA in certain scenarios, particularly when dealing with large datasets.

In conclusion, while PCA remains a fundamental technique in face recognition and other pattern recognition tasks, ongoing research continues to address its limitations and improve its performance. The development of methods like ICA, Kernel PCA, and 2DPCA highlights the ongoing efforts to enhance the effectiveness and efficiency of PCA in various applications.

III. Two-Dimensional Principal Component Analysis (2DPCA)

A. Overview of the Algorithm

In Two-Dimensional Principal Component Analysis (2DPCA), consider an n-dimensional unitary column vector X. The core idea of 2DPCA involves projecting an image A, which is an $m \times n$ random matrix, onto X using the following linear transformation:

$$Y = AX. \tag{1}$$

This results in an m-dimensional projected vector Y, commonly referred to as the projected feature vector of image A. How do we determine a suitable projection vector X? One approach is to evaluate the total scatter of the projected samples, which measures the discriminative power of the projection vector X. This can be characterized by the trace of the covariance matrix of the projected feature vectors. Thus, the criterion used is:

$$J(X) = \mathrm{tr}(S_x), \tag{2}$$

where S_x denotes the covariance matrix of the projected feature vectors of the training samples, and $\mathrm{tr}(S_x)$ represents the trace of S_x. Maximizing the criterion in (2) aims to find a projection direction X such that the total scatter of the resulting projected samples is maximized.

The covariance matrix S_x can be represented as:

$$\begin{aligned} S_x &= E[(Y - EY)(Y - EY)^T] \\ &= E[(AX - E(AX))(AX - E(AX))^T] \\ &= E[(A - EA)X][(A - EA)X]^T. \end{aligned} \tag{3}$$

Thus,

$$\operatorname{tr}(S_x) = X^T \left[E[(A - EA)^T (A - EA)] \right] X. \tag{4}$$

We define the following matrix:

$$G_t = E[(A - EA)^T (A - EA)], \tag{5}$$

which is known as the image covariance (scatter) matrix. It is straightforward to verify that G_t is an $n \times n$ nonnegative definite matrix from its definition. We can estimate G_t directly using the training image samples. Assuming there are M training image samples, where the jth training image is denoted by an $m \times n$ matrix A_j ($j = 1, 2, \ldots, M$), and the average image of all training samples is denoted by $\bar{A}$, then G_t can be computed as:

$$G_t = \frac{1}{M} \sum_{j=1}^{M} (A_j - \bar{A})^T (A_j - \bar{A}). \tag{6}$$

Consequently, the criterion in (2) can be expressed as:

$$J(X) = X^T G_t X, \tag{7}$$

where X is a unitary column vector. This criterion is known as the generalized total scatter criterion. The unitary vector X that maximizes this criterion is called the optimal projection axis. Intuitively, this means that the total scatter of the projected samples is maximized after projecting an image matrix onto X.

The optimal projection axis X_{opt} is the unitary vector that maximizes $J(X)$, i.e., the eigenvector of G_t corresponding to the largest eigenvalue. Generally, one optimal projection axis is not sufficient; we usually need to select a set of projection axes, $X_1, \ldots, X_d$, subject to orthonormal constraints and maximizing the criterion $J(X)$, that is,

$$\begin{cases} \{\mathbf{X}_1, \cdots, \mathbf{X}_d\} = \arg\max J(\mathbf{X}) \\ \mathbf{X}_i^T \mathbf{X}_j = 0, \quad i \neq j, \quad i, j = 1, \cdots, d. \end{cases} \tag{8}$$

In fact, the optimal projection axes $X_1, \ldots, X_d$ are the orthonormal eigenvectors of G_t corresponding to the first d largest eigenvalues.

B. Feature Extraction

The optimal projection vectors of 2DPCA, $X_1, \cdots, X_d$, are utilized for feature extraction. For a given image sample A, let

$$Y_k = AX_k, k = 1, 2, \cdots, d. \tag{9}$$

This yields a series of projected feature vectors, $Y_1, \cdots, Y_d$, which are referred to as the principal component (vectors) of the sample image A. It's important to note that each principal component in 2DPCA is a vector, whereas in PCA, it is typically a scalar.

These principal component vectors of 2DPCA are used to form an $m \times d$ matrix $B = [Y_1, \cdots, Y_d]$, which is called the feature matrix or feature image of the image sample A.

C. Classification Method

After transforming images using 2DPCA, each image obtains a feature matrix. To perform classification, a nearest neighbor classifier is employed. The distance between two arbitrary feature matrices $\mathbf{B}_i = [Y_1^{(i)}, Y_2^{(i)}, \ldots, Y_d^{(i)}]$ and $\mathbf{B}_j = [Y_1^{(j)}, Y_2^{(j)}, \ldots, Y_d^{(j)}]$ is defined as follows:

$$d(\mathbf{B}_i, \mathbf{B}_j) = \sum_{k=1}^{d} \|Y_k^{(i)} - Y_k^{(j)}\|^2 \tag{10}$$

where $\|Y_k^{(i)} - Y_k^{(j)}\|^2$ denotes the Euclidean distance between the two principal component vectors $Y_k^{(i)}$ and $Y_k^{(j)}$.

Suppose the training samples are $\{\mathbf{B}_1, \mathbf{B}_2, \ldots, \mathbf{B}_M\}$, where M is the total number of training samples, and each sample is assigned a specific identity (class) ω_k. For a given test sample $\mathbf{B}$, if

$$d(\mathbf{B}, \mathbf{B}_l) = \min_j d(\mathbf{B}, \mathbf{B}_j) \tag{11}$$

and $\mathbf{B}_l \in \omega_k$, then the decision is that $\mathbf{B} \in \omega_k$.

In simpler terms, this process finds the training sample most similar to the test sample and classifies the test sample into the same class as the most similar training sample. This method leverages the feature matrices extracted by 2DPCA to measure the similarity between different images, thereby achieving effective classification.

IV. EXPERIMENTS AND ANALYSIS

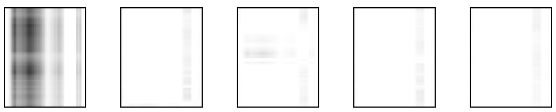
A. Dataset

The ORL Database of Faces was used in this experiment, which contains 10 different photos of 40 individuals. For some subjects, the images were taken at different times, varying the lighting, facial expressions (open / closed eyes, smiling / not smiling) and facial details (glasses / no glasses). All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position (with tolerance for some side movement).The size of each picture is 64 * 64, and an example of the dataset is shown in Fig.1.

B. Feature Extraction

First, the 2DPCA algorithm was used for feature extraction. The eigenvectors corresponding to the 10 largest eigenvalues were selected as the projection axes. After projecting the image samples onto these axes using formula (8), ten principal component vectors were obtained. Taking the last image in Fig. 1 as an example, its 10 reconstructed sub-images could be determined. Some of these sub-images are shown in Fig. 2. As observed in Fig. 2, the first subimage contains most of the energy of the original image. The other ones show the detailed local information from different levels. As the value of

Fig. 1. Sample images of one subject in the ORL face database



k=1 k=2 k=4 k=7 k=15

Fig. 2. Some reconstructed subimages

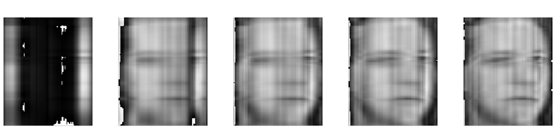
k increases, the information(the energy of image) contained in A_k becomes gradually weaker.Thus, we can conclude that the energy of an image is concentrated on its first small number of component vectors. Therefore, it is reasonable to use these component vectors to represent the image for recognition purposes.

C. Face Reconstruction

On the other hand, by adding up the first d subimages together, we obtain an approximate reconstruction of the original image. Fig.3 shows five reconstructed images of the first image in Fig.1 by adding the first d ($d = 2, 4, 6, 8, 10$) subimages together. The reconstructed images become clearer as the number of subimages is increased. For comparison, the PCA (Eigenfaces) was also used to represent and reconstruct the same face image.

D. Comparison

I ultimately conducted experiments to compare the performance of PCA and 2DPCA. I used the ORL Database of Faces, splitting it into a training set and a test set in an 8:2 ratio. Using the KNN classifier, we performed multiple experiments and selected the best-performing feature dimensions for each method: for 2DPCA, the optimal dimension was 112*2, while for PCA, it was 60. Additionally, we recorded the runtime of the programs to evaluate the efficiency differences between the two methods. The final results are shown in Table I.



d=2 d=4 d=6 d=8 d=10

Fig. 3. Some reconstructed images

TABLE I
PERFORMANCE COMPARISON BETWEEN PCA AND 2DPCA

Method	Accuracy	Times	dimension of feature vectors
PCA	96.25%	0.358s	60
2DPCA	97.5%	0.09s	112*5

CONCLUSION

In this experiment, I investigated an enhanced version of Principal Component Analysis (PCA), known as Two-Dimensional Principal Component Analysis (2DPCA). This method offers several advantages over traditional PCA, particularly in the context of image processing and analysis.

Firstly, 2DPCA operates directly on the image matrix rather than flattening the image into a one-dimensional vector before applying PCA. This approach simplifies the feature extraction process, making it more intuitive and straightforward. By maintaining the two-dimensional structure of images, 2DPCA can capture spatial relationships within the data that might be lost during the vectorization step required by traditional PCA.

Secondly, across multiple experiments, 2DPCA consistently demonstrated superior recognition accuracy compared to PCA. While this trend held true across various databases and experimental conditions, it is important to note that in some specific cases, the performance differences were not statistically significant. These findings highlight the robustness of 2DPCA but also suggest areas for further investigation to understand the nuances of its performance relative to PCA.

Thirdly, 2DPCA exhibits greater computational efficiency, significantly enhancing the speed of image feature extraction. This advantage is particularly valuable in real-time applications where rapid processing is essential. However, it is crucial to acknowledge that 2DPCA has certain limitations. Specifically, in terms of storage requirements, 2DPCA's image representation is less efficient compared to PCA because it necessitates a larger number of coefficients to accurately represent an image.

All in all, the Face Reconstruction Using PCA method remains a powerful tool for reducing the dimensionality of facial image data while retaining critical information. The visualizations and comparative analyses conducted in this study provide valuable insights into the quality of reconstruction and the contribution of each principal component to the overall variance in the dataset. This method holds potential applications in diverse fields such as facial recognition, image compression, and computer vision.

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