Finite Difference Taylor Series

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Abstract. We rediscover Taylor's theorem and his definition of the finite difference operation, then prove equivalence of this new series definition to the difference operator by means of induction to the relationship of subset-to-term equality with respect to ascending degrees of input function derivatives. Implication of approximation to the derivative is more explicit, and as a result, direct claims made of the properties of the finite difference operation can be proven with greater ease.

1 Series Definition and Proof

Theorem 1.1. *Given some $x \in \mathbb{R}$ and $h \in \mathbb{R}$, let f be a function defined as $f : \mathbb{R} \to \mathbb{R}$ continueous and differentiable on the closed interval $[x, x+h]$; the following relationship then holds:*

$$\Delta_h [f](x) = f(x+h) - f(x) = \sum_{k=1}^{\deg f} \frac{h^k}{k!} f^{(k)}(x) \tag{1}$$

Proof. □