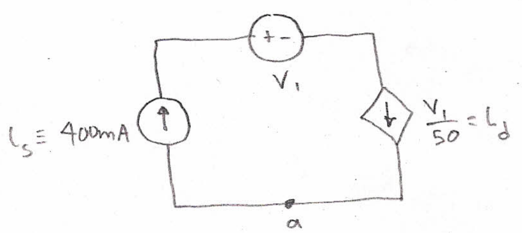
Sergio Aguilar
2-16-2015
ENGR 17

HOMEWORK # 2

Problems 1-9

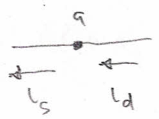
Problem 2.6 in text

1:) Given the following circuit



a) Define $i_s = 400\,mA$, $i_d = \frac{V_1}{50}$

Using KCL at node a, labeled above:



$$\sum_{\substack{node\\a}} i_{in} = \sum_{\substack{node\\a}} i_{out}$$

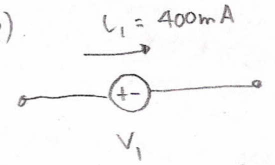
(1) $i_d = i_s$

$$\frac{V_1}{50} = 400mA \qquad V_1 = 50(400\times10^{-3})$$

$$V_1 = 20000\times10^{-3}V$$

$$\boxed{V_1 = 20V}$$

b)

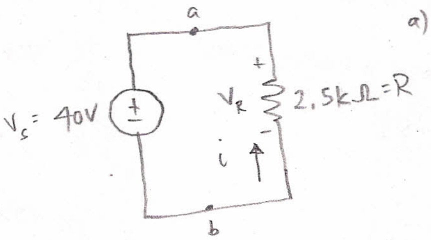


$P_1 = +V_1 i_1 = +(400mA)(20V) = 8000\,mW$

$\boxed{P_1 = 8.0W}$ absorbed / dissipated

Problem 2.12 in text

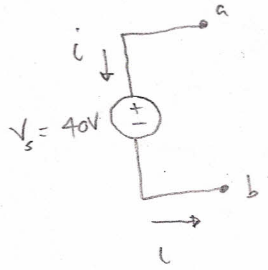
2:) Given the following circuit



a) Label the voltage across the resistor as V_R with the polarity shown. With this $V_R = +40V$. Also by the passive sign convention, $V_R = -iR$. $i = -\frac{V_R}{R}$

$$i = -\frac{40V}{2.5\times10^3\Omega} = -16\times10^{-4}A = \boxed{-0.016A = i}$$

b) For the power supplied by the source;

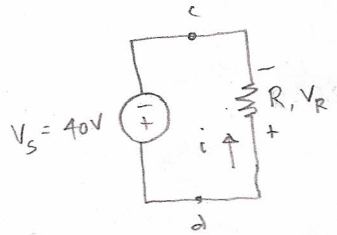


By the passive sign convention

$P_s = +V_s i = (40V)(-0.016A)$

$\boxed{P_s = -0.64W}$ supplied

c) Reverse the polarity of the voltage source



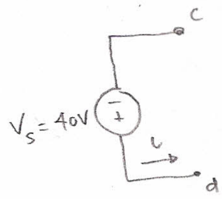
Note the new chosen polarity of V_R. Also note that $V_s = V_R = 40V$.

By Ohm's Law and the passive sign convention we have

$V_R = +iR \qquad i = \frac{V_R}{R}$

$$i = \frac{40V}{2.5\times10^3\Omega} = \boxed{+0.016A = i}$$

For the power supplied by the source:



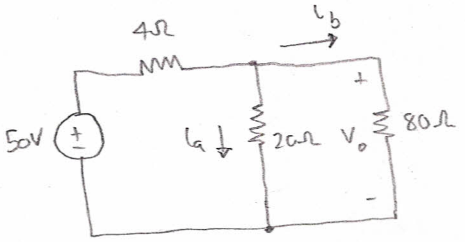
By the passive sign convention

$P_s = -V_s i = -(40V)(0.016A)$

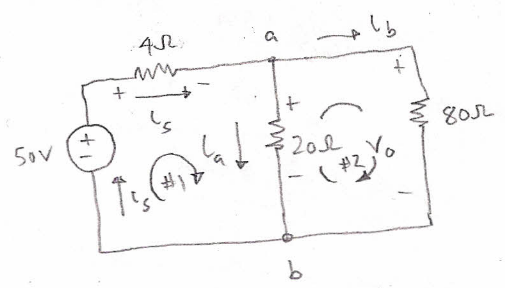
$\boxed{P_s = -0.64W}$ supplied

Problem 2.18 in text

3:) Given the following circuit



Define a current i_s as follows and add the following voltage polarities across the resistors as follows.



Note that i_s passes through the voltage source AND the 4Ω resistor.

Using KCL at node a: $\sum_{\text{node a}} i_{in} = \sum_{\text{node a}} i_{out}$

(1) $i_s = i_a + i_b$

By Ohms Law applied to each resister we have

$V_{4\Omega} = +i_s(4\Omega) = 4i_s$

$V_{20\Omega} = +i_a(20\Omega) = 20i_a$

$V_{80\Omega} = +i_b(80\Omega) = 80i_b$

Using KVL around loop #1 labeled above

(2) $-50V + 4i_s + 20i_a = 0$

$4i_s + 20i_a = 50$

Using KVL around loop #2 labeled above

(3) $-20i_a + 80i_b = 0$

solving for i_a

$20i_a = 80i_b$

$i_a = 4i_b$

The system of equations we have to solve is

(1) $i_s = i_a + i_b$

(2) $4i_s + 20i_a = 50$

(3) $i_a = 4i_b$

This is a system of 3 equations and three unknowns.

Use (1) to eliminate from (2)

$4(i_a + i_b) + 20i_a = 50$

(4) $24i_a + 4i_b = 50$

Use (3) to eliminate i_a from (4)

$24(4i_b) + 4i_b = 50$

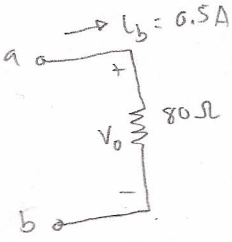
$96i_b + 4i_b = 50$

b) $100i_b = 50$ $\boxed{i_b = 0.5A}$

Also $i_a = 4i_b$ from (3). But $i_b = 0.5A$ so

a) $i_a = 4(0.5A) = \boxed{2.0A = i_a}$

c) Using Ohm's Law by looking at the 80Ω resister



$V_o = i_b R_{80\Omega}$

$V_o = (0.5A)(80\Omega) = 40V$

$\boxed{V_o = +40V}$

d) For each resistor use the passive sign convention and $P = iV$, $P = i^2R$, $P = \frac{V^2}{R}$

$P_{4\Omega} = +i_s^2 R_{4\Omega}$ but by (1) $i_s = i_a + i_b = 2.5A$

$P_{4\Omega} = +(2.5A)^2(4\Omega) = \boxed{+25W = P_{4\Omega}}$

$P_{20\Omega} = i_a^2 R_{20\Omega} = (2.0A)^2(20\Omega) = \boxed{+80W = P_{20\Omega}}$

$P_{80\Omega} = i_b^2 R_{80\Omega} = (0.5A)^2(80\Omega) = \boxed{+20W = P_{80\Omega}}$

e) $P_{50V} = -i_s V_{50V} = -(2.5A)(50V) = \boxed{-125W = P_{50V}}$

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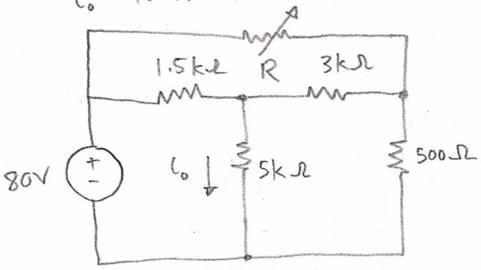
HOMEWORK #2

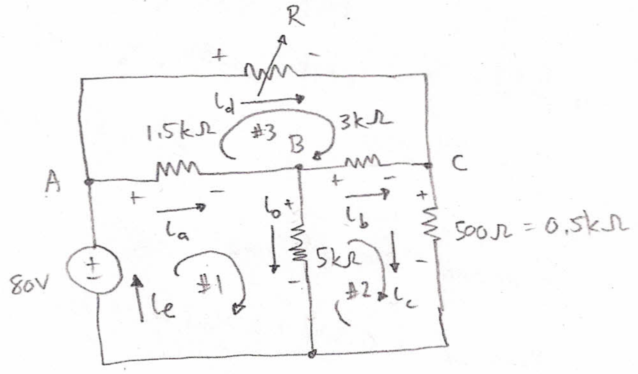
Problems 1-9

Problem 2.23 in text

4:) Given The following circuit along with $i_o = 10mA$. Determine The value of R.



Define The following currents, loops and nodes



With these definitions we can apply Ohm's Law to each resistor noting The passive sign convention as $V_a = 1.5 i_a$, $V_b = +3 i_b$, $V_c = +0.5 i_c$

$V_d = i_d R$, $V_o = 5 i_o$ where all resistances are in kΩ.

- Using KVL around loop #1:

(1) $-80V + 1.5 i_a + 5 i_o = 0$

$1.5 i_a = 80 - 5 i_o$

$i_a = \dfrac{80 - 5i_o}{1.5}$ but $i_o = 10mA$

$i_a = \dfrac{80V - (5k\Omega)(10mA)}{1.5k\Omega} = \dfrac{80-50}{1.5} = 20mA$

- Using KCL at node B:

$i_a = i_o + i_b$ but $i_o = 10mA$, $i_a = 20mA$

so $i_b = i_a - i_o = 20mA - 10mA = 10mA = i_b$

- Using KVL around loop #3:

$i_d R - 3 i_b - 1.5 i_a = 0$

But $i_a = 20mA$, $i_b = 10mA$

$i_d R - (3k\Omega)(10mA) - (1.5k\Omega)(20mA) = 0$

(3) $i_d R = 30V + 30V = 60V$ $R = \dfrac{60V}{i_d}$

Using KCL at node C:

(4) $i_b + i_d = i_c$ but $i_b = 10mA$

$10mA + i_d = i_c$, or $10 + i_d = i_c$

Using KVL around loop #2:

$-5 i_o + 3 i_b + 0.5 i_c = 0$

But $i_o = 10mA$, $i_b = 10mA$

$-(5k\Omega)(10mA) + (3k\Omega)(10mA) + \frac{1}{2} i_c = 0$

$-50V + 30V = -\frac{1}{2} i_c$ $i_c = +40mA$

Calculations: Use $i_c = 40mA$ in (4) above

$10mA + i_d = i_c$ $10mA + i_d = 40mA$

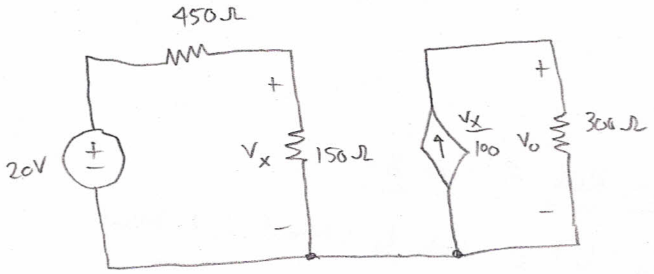
$i_d = 30mA$

Use This value in (3) above $R = \dfrac{60V}{i_d}$

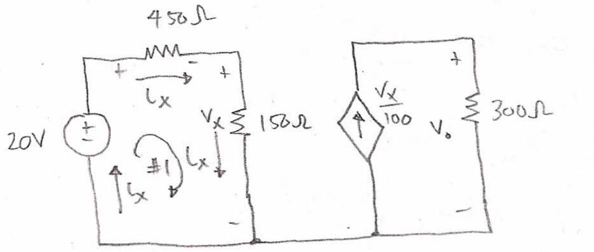
$R = \dfrac{60V}{30mA} = 2k\Omega$ so $\boxed{R = 2k\Omega}$

Problem 2.33 in text

5:) For the given circuit, find V_o and $P_{absorbed}$



Define a current i_x in the diagram below



Note i_x passes through the 20V source, the 450Ω resistor and the 150Ω resistor.

Using KVL around loop #1:

$$-20 + 450 i_x + 150 i_x = 0$$

$$600 i_x = 20 \qquad i_x = \frac{1}{30} A$$

Ohm's Law on the 150Ω resistor:

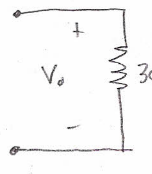
$$V_x = + i_x R_{150\Omega} = \left(\frac{1}{30} A\right)(150\Omega) = 5V$$

Dependent source current: Since $V_x = 5V$, the current provided by the dependent source

$$i_{source} = \frac{V_x}{100} = \frac{5V}{100} = \frac{1}{20} A$$

Looking at the 300Ω resistor:

$i_{source} = \frac{1}{20} A$



$$V_o = + i_{source} R_{300\Omega}$$

$$V_o = \left(\frac{1}{20} A\right)(300\Omega)$$

$$V_o = + \frac{30}{2} V = 15V$$

$$\boxed{V_o = 15V}$$

b) Note that the resistors will aborb power

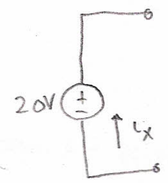
$$P_{450\Omega} = i_x^2 R_{450\Omega} = \left(\frac{1}{30} A\right)^2 (450\Omega) = +0.5W$$

$$P_{150\Omega} = i_x^2 R_{150\Omega} = \left(\frac{1}{30} A\right)^2 (150\Omega) = \frac{1}{6} W$$

$$P_{150\Omega} = 0.1667 W$$

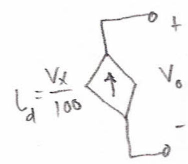
$$P_{300\Omega} = \frac{V_o^2}{R_{300\Omega}} = \frac{(15V)^2}{300\Omega} = 0.75W$$

For the sources:



$$P_{20V} = - i_x V_{20V} = -\left(\frac{1}{30} A\right)(20V)$$

$$P_{20V} = -\frac{2}{3} W = -0.6667 W$$



$$P_{dep} = -i_d V_o = -\frac{V_x}{100} V_o$$

but $V_x = 5V$, $V_o = 15V$

$$P_{dep} = -\left(\frac{5}{100}\right)(15) = -0.75W$$

$$P_{absorbed} = P_{450\Omega} + P_{150\Omega} + P_{300\Omega}$$

$$P_{absorbed} = 0.5W + 0.1667W + 0.75W$$

$$P_{absorbed} = 1.4167 W$$

$$P_{generated} = P_{20V} + P_{dep} = -0.6667W - 0.75W$$

$$P_{generated} = -1.4167 W$$

P 2.12 [a] Using the passive sign convention and Ohm's law,

$$i = -\frac{v}{R} = -\frac{40}{2500} = -0.016 = -16 \text{ mA}$$

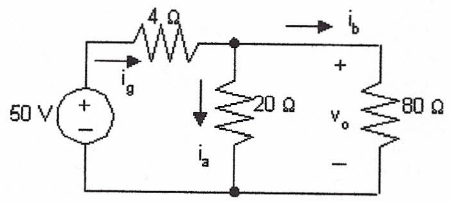
[b] $P_R = Ri^2 = (2500)(-0.016)^2 = 0.64 = 640 \text{ mW}$

[c] Using the passive sign convention with the voltage polarity reversed,

$$i = \frac{v}{R} = \frac{40}{2500} = 0.016 = 16 \text{ mA}$$

$$P_R = Ri^2 = (2500)(0.016)^2 = 0.64 = 640 \text{ mW}$$

P 2.18 [a]



$$\begin{aligned} 20i_a &= 80i_b \qquad i_g = i_a + i_b = 5i_b \\ i_a &= 4i_b \\ 50 &= 4i_g + 80i_b = 20i_b + 80i_b = 100i_b \\ i_b &= 0.5 \text{ A, therefore, } i_a = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A} \end{aligned}$$

[b] $i_b = 0.5$ A

[c] $v_o = 80i_b = 40$ V

[d]
$$\begin{aligned} p_{4\Omega} &= i_g^2(4) = 6.25(4) = 25 \text{ W} \\ p_{20\Omega} &= i_a^2(20) = (4)(20) = 80 \text{ W} \\ p_{80\Omega} &= i_b^2(80) = 0.25(80) = 20 \text{ W} \end{aligned}$$

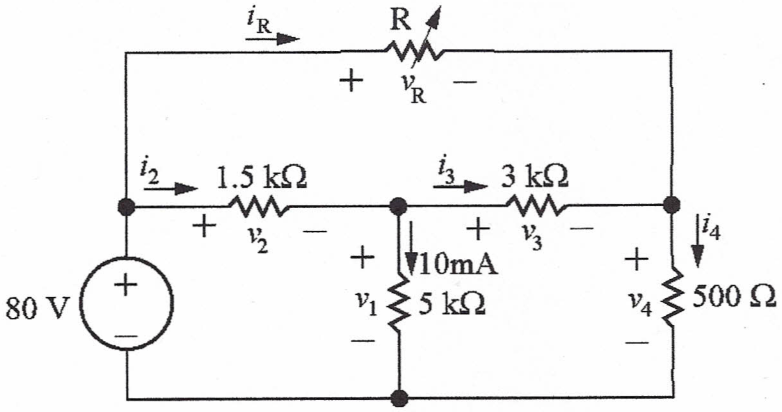
[e] p_{50V} (delivered) $= 50i_g = 125$ W

Check:

$$\sum P_{\text{dis}} = 25 + 80 + 20 = 125 \text{ W}$$

$$\sum P_{\text{del}} = 125 \text{ W}$$

P 2.23 Label all unknown resistor voltages and currents:

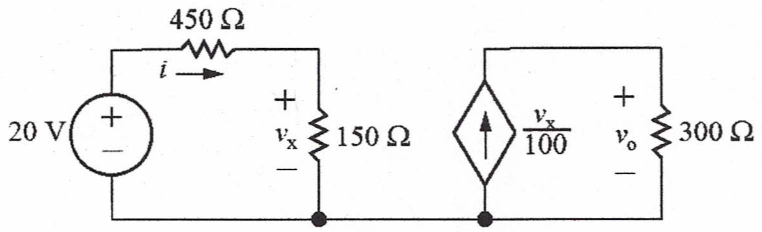


Ohms' law for 5 kΩ resistor: $v_1 = (0.01)(5000) = 50$ V
KVL for lower left loop: $-80 + v_2 + 50 = 0 \quad \rightarrow \quad v_2 = 80 - 50 = 30$ V
Ohm's law for 1.5 kΩ resistor: $i_2 = v_2/1500 = 30/1500 = 20$ mA
KCL at center node:
$i_2 = i_3 + 0.01 \quad \rightarrow \quad i_3 = i_2 - 0.01 = 0.02 - 0.01 = 0.01 = 10$ mA
Ohm's law for 3 kΩ resistor $v_3 = 3000 i_3 = 3000(0.01) = 30$ V
KVL for lower right loop:
$-v_1 + v_3 + v_4 = 0 \quad \rightarrow \quad v_4 = v_1 - v_3 = 50 - 30 = 20$ V

Ohm's law for 500 Ω resistor: $i_4 = v_4/500 = 20/500 = 0.04 = 40$ mA
KCL for right node:
$i_3 + i_R = i_4 \quad \rightarrow \quad i_R = i_4 - i_3 = 0.04 - 0.01 = 0.03 = 30$ mA
KVL for outer loop:
$-80 + v_R + v_4 = 0 \quad \rightarrow \quad v_R = 80 - v_4 = 80 - 20 = 60$ V
Therefore,

$$R = \frac{v_R}{i_R} = \frac{60}{0.03} = 2000 = 2\text{ k}\Omega$$

P 2.33 Label unknown current:



$$-20 + 450i + 150i = 0 \qquad \text{(KVL and Ohm's law)}$$

$$\text{so} \qquad 600i = 20 \quad \rightarrow \quad i = 33.33\,\text{mA}$$

$$v_x = 150i = 150(0.0333) = 5\,\text{V} \qquad \text{(Ohm's law)}$$

$$v_o = 300\left(\frac{v_x}{100}\right) = 300(5/100) = 15\,\text{V} \qquad \text{(Ohm's law)}$$

Calculate the power for all components:

$$p_{20\text{V}} = -20i = -20(0.0333) = -0.667\,\text{W}$$

$$p_{\text{d.s.}} = -v_o\left(\frac{v_x}{100}\right) = -(15)(5/100) = -0.75\,\text{W}$$

$$p_{450} = 450i^2 = 450(0.033)^2 = 0.5\,\text{W}$$

$$p_{150} = 150i^2 = 150(0.033)^2 = 0.1667\,\text{W}$$

$$p_{300} = \frac{v_o^2}{300} = \frac{15^2}{300} = 0.75\,\text{W}$$

Thus the total power absorbed is

$$p_{\text{abs}} = 0.5 + 0.1667 + 0.75 = 1.4167\,\text{W}$$

P 2.36 [a] $-50 - 20i_\sigma + 18i_\Delta = 0$

$-18i_\Delta + 5i_\sigma + 40i_\sigma = 0$ so $18i_\Delta = 45i_\sigma$

Therefore, $-50 - 20i_\sigma + 45i_\sigma = 0,$ so $i_\sigma = 2\,\text{A}$

$18i_\Delta = 45i_\sigma = 90;$ so $i_\Delta = 5\,\text{A}$

$v_o = 40i_\sigma = 80\,\text{V}$

[b] i_g = current out of the positive terminal of the 50 V source
v_d = voltage drop across the $8i_\Delta$ source

$i_g = i_\Delta + i_\sigma + 8i_\Delta = 9i_\Delta + i_\sigma = 47\,\text{A}$

$v_d = 80 - 20 = 60\,\text{V}$

$$\begin{aligned}
\sum P_\text{gen} &= 50i_g + 20i_\sigma i_g = 50(47) + 20(2)(47) = 4230 \text{ W} \\
\sum P_\text{diss} &= 18i_\Delta^2 + 5i_\sigma(i_g - i_\Delta) + 40i_\sigma^2 + 8i_\Delta v_d + 8i_\Delta(20) \\
&= (18)(25) + 10(47 - 5) + 4(40) + 40(60) + 40(20) \\
&= 4230 \text{ W; Therefore,} \\
\sum P_\text{gen} &= \sum P_\text{diss} = 4230 \text{ W}
\end{aligned}$$

P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] Circuit in Fig. P3.3(a):

$R_\text{eq} = [(7000 + 5000)\|6000] + 8000 = 12{,}000\|6000 + 8000$

$= 4000 + 8000 = 12 \text{ k}\Omega$

Circuit in Fig. P3.3(b):

$R_\text{eq} = [500\|(800 + 1200)] + 300 + 200 = (500\|2000) + 300 + 200$

$= 400 + 300 + 200 = 900\,\Omega$

Circuit in Fig. P3.3(c):

$R_\text{eq} = (35 + 15 + 25)\|(10 + 40) = 75\|50 = 30\,\Omega$

Circuit in Fig. P3.3(d):

$R_\text{eq} = ([(70 + 80)\|100] + 50 + 90)\|300 = [(150\|100) + 50 + 90]\|300$

$= (60 + 50 + 90)\|300 = 200\|300 = 120\,\Omega$

[b] Note that in every case, the power delivered by the source must equal the power absorbed by the equivalent resistance in the circuit. For the circuit in Fig. P3.3(a):

$$P = \frac{V_s^2}{R_\text{eq}} = \frac{18^2}{12{,}000} = 0.027 = 27 \text{ mW}$$

For the circuit in Fig. P3.3(b):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{27^2}{900} = 0.81 = 810 \text{ mW}$$

For the circuit in Fig. P3.3(c):

$$P = \frac{V_s^2}{R_{\text{eq}}} = \frac{90^2}{30} = 270 \text{ W}$$

For the circuit in Fig. P3.3(d):

$$P = I_s^2(R_{\text{eq}}) = (0.03)^2(120) = 0.108 = 108 \text{ mW}$$

P 3.9 Write an expression for the resistors in series and parallel from the right side of the circuit to the left. Then simplify the resulting expression from left to right to find the equivalent resistance.

[a] $R_{\text{ab}} = [(26 + 10)\|18 + 6]\|36 = (36\|18 + 6)\|36 = (12 + 6)\|36 = 18\|36 = 12\,\Omega$

[b] $R_{\text{ab}} = [(12 + 18)\|10\|15\|20 + 16]\|30 + 4 + 14 = (30\|10\|15\|20 + 16)\|30 + 4 + 14$

$$= (4 + 16)\|30 + 4 + 14 = 20\|30 + 4 + 14 = 12 + 4 + 14 = 30\,\Omega$$

[c] $R_{\text{ab}} = (500\|1500\|750 + 250)\|2000 + 1000 = (250 + 250)\|2000 + 1000$

$$= 500\|2000 + 1000 = 400 + 1000 = 1400\,\Omega$$

[d] Note that the wire on the far right of the circuit effectively removes the $60\,\Omega$ resistor!

$$R_{\text{ab}} = [([(30 + 18)\|16 + 28]\|40 + 20)\|24 + 25 + 10]\|50$$

$$= ([(48\|16 + 28)\|40 + 20]\|24 + 25 + 10)\|50$$

$$= ([(12 + 28)\|40 + 20]\|24 + 25 + 10)\|50 = [(40\|40 + 20)\|24 + 25 + 10]\|50$$

$$= [(20 + 20)\|24 + 25 + 10]\|50 = (40\|24 + 25 + 10)\|50 = (15 + 25 + 10)\|50$$

$$= 50\|50 = 25\,\Omega$$

P 3.12 [a] $v_o = \dfrac{160(3300)}{(4700 + 3300)} = 66 \text{ V}$

[b] $i = 160/8000 = 20 \text{ mA}$

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88 \text{ W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32 \text{ W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160 - 66}{R_1}, \qquad \text{Therefore, } \left(\frac{94}{R_1}\right)^2 R_1 \leq 0.5$$

$$\text{Thus, } R_1 \geq \frac{94^2}{0.5} \quad \text{or} \quad R_1 \geq 17{,}672\,\Omega$$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17{,}672}(160) = 66$$

Thus, $R_2 = 12{,}408\,\Omega$