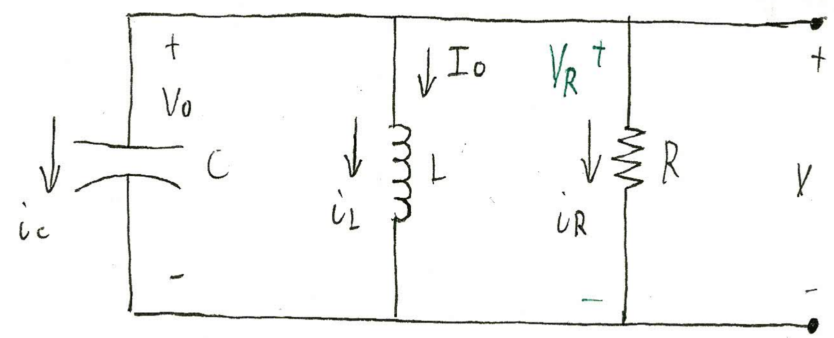
1)



Given
$R = 2k\Omega$
$L = 250mH$
$C = 10nF$
$V_0 = 0V$
$I_0 = -4A$

↳ The initial voltage of capacitor, V_0, is 0V
So

$$V_R = V_0 = 0V = V(0^-) = V(0^+)$$

↳ Find $i_R(0^+)$

$$\left[i_R(0^+) = \frac{V_R}{R} = 0A \right] \text{ a)}$$

↳ The I_0 is given
So we have $I_0 = -4A = i_L(0^+) = i_L(0^-)$

↳ Find $i_c(0^+)$

i_c going down

$$[i_c(0^+) = i_L(0^+) + i_R(0^+) = -(-4A + 0A) = 4A] \text{ b)}$$

↳ Find $\frac{dV(0^+)}{dt}$ using i-v equation for capacitor

$$\left[i_c(0^+) = C\frac{dV(0^+)}{dt} \rightarrow \frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{4A}{10nF} = 400M\ V/s = 4 \times 10^8\ V/s \right] \text{ c)}$$

↳ Find s_1, s_2 using values of RLC

$$\alpha = \frac{1}{2(2000\Omega)(10 \times 10^{-9}F)} = 25000 \text{ rad/s},\ \omega_0^2 = \frac{(10^3)(10^9)}{(250H)(10F)} = 400M \text{ rad/s}$$

$$s_1 = -25000 + \sqrt{25000^2 - (400 \times 10^6)} = -10k \text{ rad/s}$$
$$s_2 = -25000 - \sqrt{25000^2 - (400 \times 10^6)} = -40k \text{ rad/s}$$

↳ Since $\alpha > \omega_0$, overdamped

$$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}\ V,\ t \geq 0^+$$

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L Solving For A_1 and A_2, use $V(0^+)$, $\frac{dv}{dt}(0^+)$

$$V(0^+) = A_1 e^{-10000(0)} + A_2 e^{-40000(0)} \ V$$

$$0 = A_1 + A_2 \quad (1)$$

$$\frac{dV}{dt}(0^+) = -10000A_1 + -40000A_2$$

$$4\times10^8 = -10000A_1 + -40000A_2$$

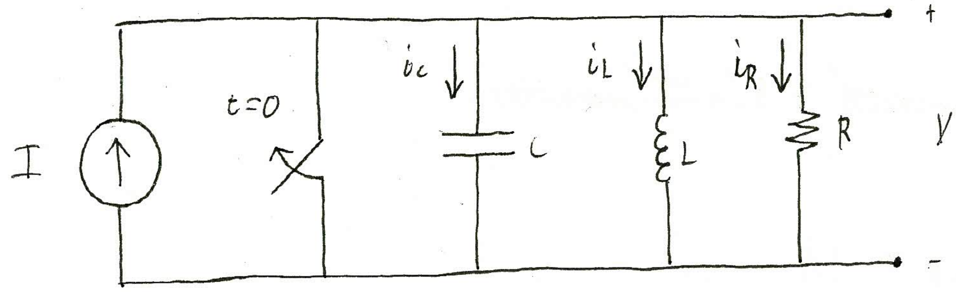
L By calculator

$$\boxed{A_1 = 13.3333\,kV} \ d)$$
$$\boxed{A_2 = -13.3333\,kV} \ e)$$

L Combining everything

$$\boxed{V(t) = 13.3333e^{-10000t} - 13.3333e^{-40000t} \ V, \ t \geq 0s} \ F)$$

2)



Given
$R = 500\Omega$
$L = 0.64H$
$C = 1\mu F$
$I = -1A$
$V_{0C} = 40V$
$I_{0L} = 0.5A$

∟ since $V_{0C}(0^+) = V_R(0^+)$

$$\left[i_R(0^+) = \frac{V_{0C}(0^+)}{R} = \frac{40V}{500\Omega} = 80\,mA = 0.08A \right] \text{ a)}$$

∟ Use i_R, $i_L = I_{0L}$, and I to find i_C

$$\left[i_C = -(-I + i_R + i_L) = -(1A + 0.08A + 0.5A) = -1.58A \right] \text{ b)}$$

i_C going down

I in opposite direction

∟ Since $V_{0C} = V_L$

$$\left[V_L = L\frac{di_L(0^+)}{dt} \rightarrow \frac{di_L(0^+)}{dt} = \frac{V_L}{L} = \frac{40V}{0.64H} = 62.5\ A/s \right] \text{ c)}$$

∟ Find α and ω_0 with given values of RLC

$$\alpha = \frac{1}{2RC} = \frac{1}{2(500\Omega)(1\times10^{-6}F)} = 1000\ rad/s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.64H)(1\times10^{-6}F)}} = 1250\ rad/s$$

∟ The response is underdamped since $\omega_0 > \alpha$, Find Damped Radian Frequency ωd

$$\omega d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1250^2 - 1000^2} = 750\ rad/s$$

∟ Find s_1 and s_2 with ωd and α

$$\left[\begin{array}{l} s_1 = -\alpha + j\omega d = -1000 + j750\ rad/s \\ s_2 = -\alpha - j\omega d = -1000 - j750\ rad/s \end{array} \right] \text{ d)}$$

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L The general solution for underdamped response current

$$i_L(t) = I_F + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$$

at $t = 0^+$

$$i_L(0^+) = I_F + B_1' \cancel{e^{-\alpha(0)}}^{1} \cancel{\cos \omega_d(0)}^{1} + \cancel{B_2' e^{-\alpha(0)} \sin \omega_d(0)}^{0}$$

$$i_L(0^+) = I_F + B_1'$$

$$* \; i_L(0^+) = I_{0L} = 0.5A \;,\; I_F = -1A$$

$$B_1' = i_L(0^+) - I_F = 0.5 - (-1) = 1.5A$$

$$\left[\frac{di_L(0)}{dt} = \omega_d B_2' - \alpha B_1'\right] \frac{1}{\omega_d}$$

$$\frac{\frac{di_L(0)}{dt}}{\omega_d} = B_2' - \frac{\alpha B_1'}{\omega_d}$$

$$B_2' = \frac{di_L(0)}{dt} \cdot \frac{1}{\omega_d} + \alpha \frac{B_1'}{\omega_d} = 62.5 \cdot \frac{1}{750} + 1000\left(\frac{1.5}{750}\right) = \frac{25}{12} A$$

so

$$\left[i_L(t) = -1 + 1.5 e^{-1000t} \cos 750t + \frac{25}{12} e^{-1000t} \sin 750t \;\; A,\; t \geq 0_s\right] \;e)$$

L The general solution for underdamped response voltage

$$V(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$$

at $t = 0^+$

$$V(0) = B_1 = 40 V$$

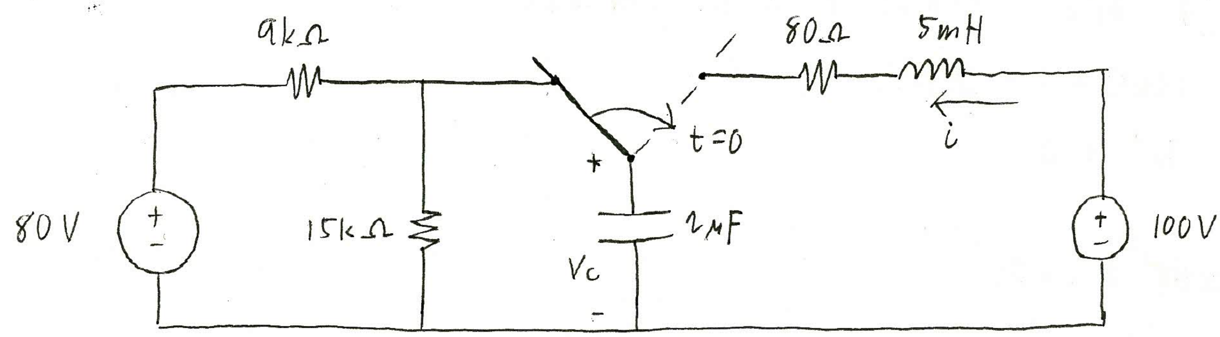
$$\frac{dV(0)}{dt} = -\alpha B_1 \cancel{e^{-\alpha(0)}}^{1} \cancel{\cos \omega_d(0)}^{1} + \omega_d B_2 \cancel{e^{-\alpha(0)}}^{1} \cancel{\cos \omega_d(0)}^{1} \cancel{- B_1 e^{-\alpha(0)} \sin \omega_d(0)} - \alpha B_2 e^{-\alpha(0)} \cdot \cancel{\sin \omega_d(0)}^{0}$$

$$B_2 = \frac{dV(0)}{dt} \cdot \frac{1}{\omega_d} + \frac{\alpha B_1}{\omega_d} = \frac{i_c(0)}{C} \cdot \frac{1}{\omega_d} + \frac{\alpha B_1}{\omega_d} = -\frac{6160}{3} V$$

So

$$\left[V(t) = 40 e^{-1000t} \cos 750t - \frac{6160}{3} e^{-1000t} \sin 750t \;V,\; t \geq 0_s\right] \;F)$$

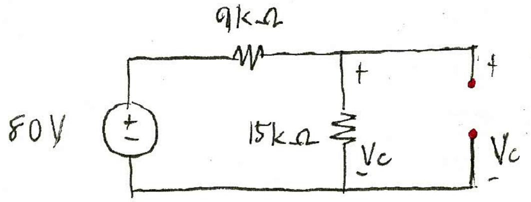
3)



L Since $i_L(0^-) = i_L(0^+)$,

[$i(0^+) = 0$, no current source flowing initially at the inductor] a)

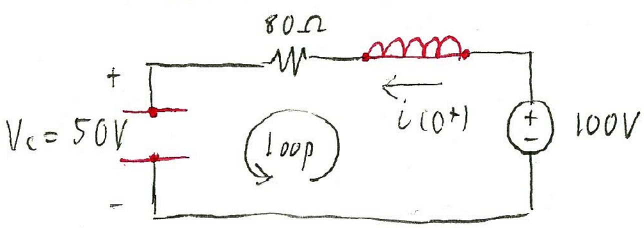
L Circuit at $t = 0^-$



By Voltage Division

$$\left[V_c(0^-) = \frac{15k\Omega}{9k\Omega + 15k\Omega}(80V) = 50\ V = V_c(0^+) \right] \text{ b)}$$

L Circuit at $t = 0^+$



Using KVL @ loop

$$-100 + L\frac{di(0^+)}{dt} \cancel{+80i(0^+)}^{0} + 50 = 0$$

$$\left[\frac{di(0^+)}{dt} = \frac{50}{L} = 10000\ A/s \right] \text{ c)}$$

L Find α and ω_0 at $t = 0^+$

$$\alpha = \frac{R}{2L} = 8000\ rad/s\ , \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5\times10^{-3}H)(2\times10^{-6}F)}} = 10000\ rad/s$$

L Voltage Response is underdamped $\omega_0 > \alpha$, find Damped Radian Frequency ωd

$$\omega d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(100\times10^6) - (64\times10^6)} = 6000\ rad/s$$

So

$$\left[\begin{aligned} s_1 &= -\alpha + j\omega d = -8000 + j6000\ rad/s \\ s_2 &= -\alpha - j\omega d = -8000 - j6000\ rad/s \end{aligned} \right] \text{ d)}$$

L General solution for $i(t)$, underdamped response

$$i(t) = I_F + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$$

* Know that $i(0^+) = 0 = I_F$

$$0 = 0 + B_1' + 0$$

$$B_1' = 0$$

$$\frac{di(0^+)}{dt} = \cancel{-\alpha B_1'}^{\,0} + \omega_d B_2'$$

$$10000 = 6000 B_2'$$

$$B_2' = 1.67 \text{ A}$$

So

$$\left[i(t) = 1.67 e^{-8000t} \sin 6000t \text{ A}, \quad t \geq 0_s \right] \quad e)$$