Interpolation.

Known: ZSW, ZSE, ZNE, ZNW: values of Field Z at locations {i,j}, {i,j+1}, {i+1,j+1}, {i+1,j} (see following figure).

Required: $Z(\theta, \Phi)$, value of field Z at longitude θ, latitude Φ.

Modus operandi:

Find indexes i and j such that:

$Lat(i) \le \theta < Lat(i+1);$ $Long(j) \le \Phi < Long(j+1);$

$\Delta Lat = Lat(i+1) - Lat(i);$ $\Delta Long = Long(j+1) - Long(j);$

$a(\Phi) = (\Phi - Lat(i)) / \Delta Lat;$ $b(\theta) = (\theta - Long(j)) / \Delta Long;$

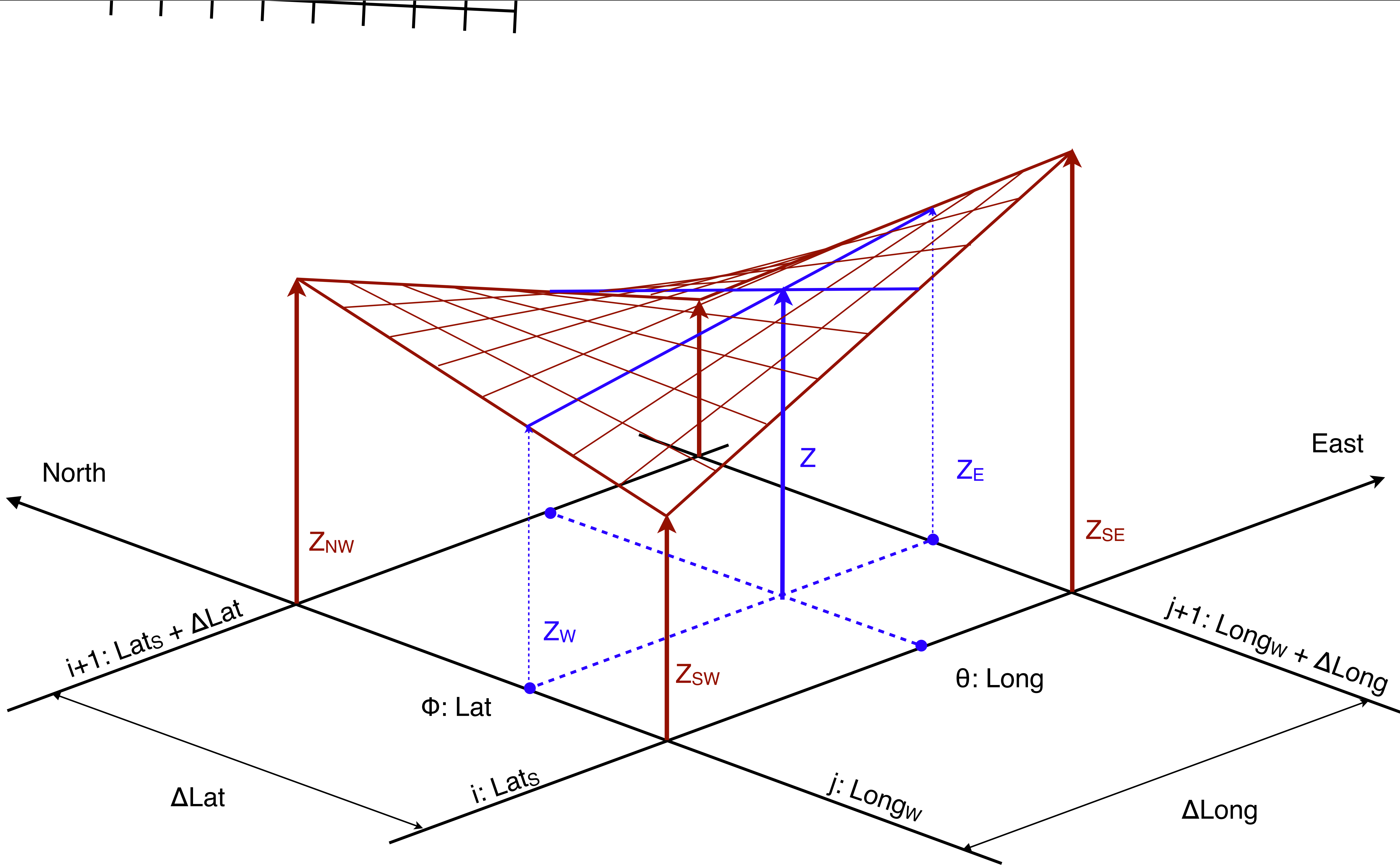
then $ZW = (1 - a)\ ZSW + a\ ZNW;$ $ZE = (1 - a)\ ZSE + a\ ZNE;$

finally $Z(\theta, \Phi) = (1 - b)\ ZW + b\ ZE.$

However, after replacing for ZE and ZW in $Z(\theta, \Phi)$ and expanding the expression, one gets:

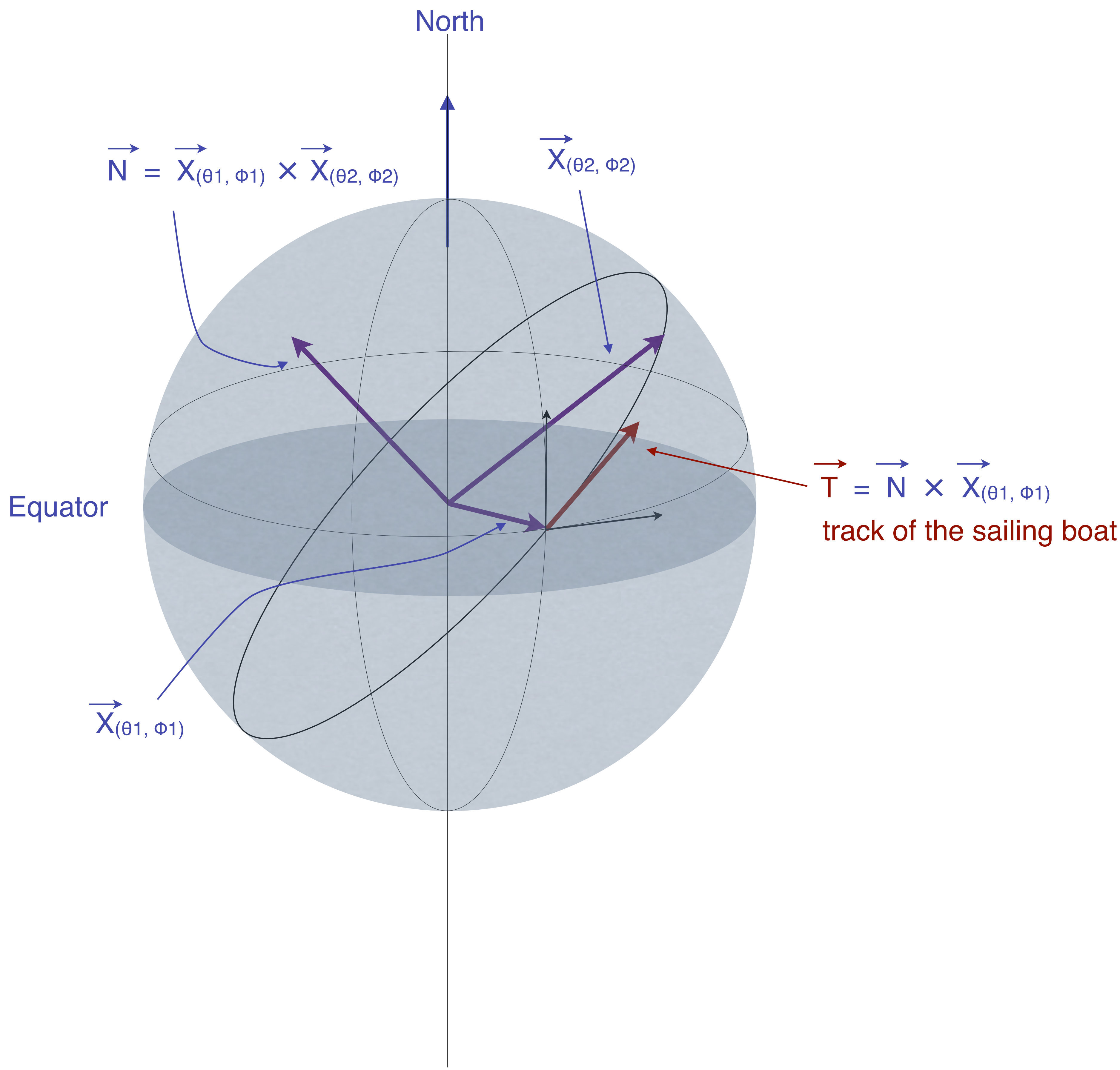
$$Z(\theta, \Phi) = a(\Phi)\ (ZNW - ZSW) + a(\Phi)\ b(\theta)\ (ZSW - ZSE + ZNE - ZNW) + b(\theta)\ (ZSE - ZSW) + ZSW.$$

This is a quadratic form in a and b whose geometric representation is the hyperbolic paraboloid, the doubly ruled surface seen on the following figure. I invite you to use this last expression in your programme.


North
East
Z_{NW}
Z_{SE}
Z_{SW}
Z_W
Z_E
Z
i+1: Lat_S + ΔLat
Φ: Lat
θ: Long
j+1: $Long_W$ + ΔLong
ΔLat
i: Lat_S
j: $Long_W$
ΔLong

Spherical navigation

The next figure will be discussed at our next meeting ;-))


North
$\vec{N} = \vec{X}_{(\theta 1,\, \Phi 1)} \times \vec{X}_{(\theta 2,\, \Phi 2)}$
$\vec{X}_{(\theta 2,\, \Phi 2)}$
Equator
$\vec{T} = \vec{N} \times \vec{X}_{(\theta 1,\, \Phi 1)}$
track of the sailing boat
$\vec{X}_{(\theta 1,\, \Phi 1)}$