1. Decoupling tension and path

Can we cleanly separate tension control from path control during hover? Obviously, tension and path are coupled because for any given position of the wing, we must supply a horizontal tension to keep the tether from touching the ground. Moreover, we would like to avoid supplying excessive horizontal tension to save power, reduce motor heating, and leave more thrust available for control. A final constraint is that we always want to supply a minimum horizontal tension to ensure that the tether remains wrapped around the drum and the wing maintains roll stability. Here we show that for minimum tensions around 3000 N, increasing horizontal tension and reducing altitude will not decrease thrust. And, thus we can simply set a minimum horizontal tension and decouple the tension and path loops.

To make the math easy, we model the tether catenary using a simple parabola:

$$h(r) = ar^2 + br \tag{1}$$

Here the coefficients may be related to the horizontal tension, t_x, the tether linear weight density, μ, and the minimum height of the tether parabola, h_{min}.

$$a = \frac{\mu}{2t_x}, b = \sqrt{-4ah_{\text{min}}} \tag{2}$$

Making the approximation that the weight of the tether supported by the wing is equal to the fraction of the radial distance of the tether after the tether minimum, $r_{\text{min}} = -b/2a$, multiplied by the total tether weight, W_T, we find that the thrust, T, required to support the tether and the weight of the wing, W_W, is:

$$T = \sqrt{(W_T(1 - r_{\text{min}}/r) + W_W)^2 + t_x^2} \tag{3}$$

For some combination of μ, W_T, W_W, h_{min}, and t_x it may be advantageous to increase horizontal tension above the minimum required horizontal tension in order to decrease altitude and support less tether weight. To determine where this point occurs, we calculate the derivative of the thrust with respect to horizontal tension:

$$\frac{dT}{dt_x} = \frac{1}{2T}\left[2t_x - W_{\text{supp.}}\sqrt{\frac{-2h_{\text{min}}\mu}{t_x}}\right] \tag{4}$$

Here $W_{\text{supp.}} = W_T(1 - r_{\text{min}}/r) + W_W$ is the total weight supported by the thrust. Thus, the point when $\frac{dT}{dt_x}$ becomes negative, and thus it is advantageous to increase t_x beyond the minimum, occurs when:

$$t_x < \left(\frac{-W_{\text{supp.}}^2 h_{\text{min}}\mu}{2}\right)^{1/3} \tag{5}$$

Inserting some representative numbers, $W_{\text{supp.}} = W_T/2 + W_W = 17000$ N, $h_{\text{min}} = -10$ m, $\mu = 10$ N/m, we find that the critical horizontal tension is about 2435 N. This is somewhat below our current minimum horizontal tension, so for most situations simply setting a mimimum horizontal tension irrespective of path should suffice.