Quantum Electrodynamics in Motion: Commentary on the `SpacetimeQEDScene` Animation

Prepared for Christian H. Cooper, CFA, FRM

April 16, 2025

Abstract

This document provides a comprehensive, graduate-level exegesis of each visual vignette in the Manim animation *SpacetimeQEDScene*. For every scene we review the historical milestones, derive (or recall) the requisite mathematics, and sketch the conceptual bridges that connect seemingly disparate scales—from femtometres to gigaparsecs—within the framework of relativistic quantum field theory.

Contents

1 Scene I: Cosmic Overture — Star-Field Genesis

1.1 Historical Context

The opening star field evokes humanity's earliest cosmological questions, but also recalls modern precision cosmology. The uniform sprinkling of points is suggestive of the near-isotropy of the *Cosmic Microwave Background* (CMB), first measured by Penzias and Wilson (1965), whose black-body spectrum is evidence for a hot Big-Bang origin.

1.2 Mathematical Framework

On largest scales the Universe is well-described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric,

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}\chi^2 + S_k^2(\chi)\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{1}$$

where $a(t)$ is the scale factor and $S_k(\chi) = \{\sin\chi, \chi, \sinh\chi\}$ for $k = +1, 0, -1$. In the animation we deliberately defer this curved geometry until later; the apparent flatness of the star field is an allusion to the observational fact $\Omega_k \approx 0$.

1.3 Spacetime Connection

The fade-in from darkness mirrors the epoch of *recombination* ($z \sim 1100$) when photons decouple, allowing first light to propagate freely—a prelude to all subsequent electromagnetic phenomena, including those depicted in later scenes.

2 Scene II: Title Reveal and the Minkowski Scaffold

2.1 Historical Context

Hermann Minkowski's 1908 lecture "Raum und Zeit" cemented spacetime as a four-dimensional arena for physics. Einstein (1905) had already unified space and time kinematically; Minkowski gave the union a geometric dignity.

2.2 Metric Signature and Notation

The animation displays the pseudo-Riemannian line element in mostly-plus convention

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 \;+\; \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2, \tag{2}$$

where the sign of the temporal component encodes causal hierarchy via light cones. The *wire-frame* grid and rotating double cone visually implement the null surface $\mathrm{d}s^2 = 0$ that separates time-like from space-like intervals.

2.3 Connections Across Scale

Although (2) is locally valid even in curved spacetimes (by the equivalence principle), its global depiction reminds the viewer that special relativity remains an indispensable scaffold inside every tangent space of the larger FLRW manifold introduced implicitly in Scene I.

3 Scene III: Electromagnetic Plane Waves

3.1 Historical Context

Maxwell unified electricity and magnetism in 1865; Hertz's experiments (1887) vindicated their wave nature. The orthogonal red/blue oscillations visualise a monochromatic solution to the source-free Maxwell equations.

3.2 Mathematics

Using natural units ($c = 1$) for brevity, pick wave-vector $k^\mu = (\omega, \boldsymbol{k})$ with $\omega = \|\boldsymbol{k}\|$. A plane wave propagating in the $+\hat{z}$ direction satisfies

$$\boldsymbol{E}(\boldsymbol{x}, t) = E_0\,\hat{x}\,\sin(kz - \omega t), \qquad \boldsymbol{B}(\boldsymbol{x}, t) = E_0\,\hat{y}\,\sin(kz - \omega t). \tag{3}$$

Both fields are transverse ($\boldsymbol{k}\cdot\boldsymbol{E} = 0$), mutually orthogonal ($\boldsymbol{E}\cdot\boldsymbol{B} = 0$), and satisfy $\boldsymbol{B} = \hat{k} \times \boldsymbol{E}$. In four-tensor language one packages them as $F_{\mu\nu} = k_\mu A_\nu - k_\nu A_\mu$ with gauge potential A^μ subject to $k_\mu A^\mu = 0$.

3.3 Cosmological Echo

Electromagnetic waves fill the Universe from the 21-cm line to gamma-ray bursts. The depicted classical plane wave is the macroscopic limit of the quantised photon field introduced in later scenes.

4 Scene IV: Maxwell Morph — From Vector Calculus to Lorentz Tensors

4.1 Historical Milestones

The compact equation $\partial_\mu F^{\mu\nu} = \mu_0 J^\nu$ emerged gradually: Heaviside (1884) vectorised the original quaternionic notation; Minkowski (1908) cast the set into tensor form, revealing manifest Lorentz covariance.

4.2 Derivation Sketch

Write the field tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Then charge conservation $\partial_\nu J^\nu = 0$ follows identically from $\partial_\nu \partial_\mu F^{\mu\nu} = 0$, underscoring the topological underpinning of Gauss's law when phrased covariantly.

4.3 Geometric Insight

The "shape-shifting" symbols in the animation visually encode the gauge-invariant *bundle* structure: $F^{\mu\nu}$ lives on the Lie algebra of U(1) while A^μ is a connection 1-form. Curvature $\mathrm{d}A$ becomes force.

5 Scene V: The QED Lagrangian Density

5.1 Historical Context

- 1928 — Dirac introduces relativistic wave equation for spin-$\frac{1}{2}$.
- 1940s — Tomonaga, Schwinger, Feynman, Dyson formulate renormalised QED.
- 1950s — Ward identities link gauge symmetry to charge conservation.

5.2 Lagrangian Anatomy

$$\boxed{\mathcal{L}_{\text{QED}} = \underbrace{\bar{\psi}\big(i\gamma^\mu D_\mu - m\big)\psi}_{\substack{\text{Dirac kinetic term}\\ \text{and Yukawa mass}}} \; - \; \underbrace{\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu}}_{\text{Maxwell term}}} \tag{4}$$

with $D_\mu = \partial_\mu + ieA_\mu$. Colour coding in the animation emphasises distinct algebraic roles—*spinor*, *gamma matrices*, *covariant derivative*, and *field tensor*.

5.3 Gauge Symmetry and Noether Charge

Under a local phase $\psi \mapsto e^{i\alpha(x)}\psi$,

$$A_\mu \;\longmapsto\; A_\mu - \frac{1}{e}\,\partial_\mu\alpha(x), \qquad \mathcal{L}_{\text{QED}} \text{ invariant.} \tag{5}$$

Noether's theorem then yields $\partial_\mu J^\mu = 0$ with $J^\mu = e\bar{\psi}\gamma^\mu\psi$, which the animation flags via a pulsating phase overlay.

6 Scene VI: The Feynman Vertex

6.1 Perturbative Picture

The lowest-order scattering amplitude for $e^-e^- \to e^-e^-$ reads

$$\mathcal{M} = \big[\bar{u}(p_3)(-ie\gamma^\mu)u(p_1)\big]\, \frac{-ig_{\mu\nu}}{q^2 + i\varepsilon}\, \big[\bar{u}(p_4)(-ie\gamma^\nu)u(p_2)\big], \tag{6}$$

graphically depicted by two fermion lines exchanging a photon propagator.

6.2 Fine-Structure Constant

At low energy the effective coupling is

$$\alpha \equiv \frac{e^2}{4\pi\varepsilon_0\hbar c} \;\simeq\; \frac{1}{137.035999\ldots}. \tag{7}$$

The animation's numeric $\to$ symbolic morph encodes the experimentalist-theorist dialogue.

7 Scene VII: Running of α

7.1 Renormalisation Group (RG) Flow

Vacuum polarisation from virtual e^+e^- pairs screen the electric charge. To one-loop order the β-function is

$$\beta(\alpha) \equiv \mu\frac{\mathrm{d}\alpha}{\mathrm{d}\mu} = \frac{2}{3\pi}\,\alpha^2 \;+\; \mathcal{O}(\alpha^3), \tag{8}$$

yielding the Landau-pole behaviour $\alpha(\mu) = \alpha(\mu_0)/\big[1 - \frac{2\alpha(\mu_0)}{3\pi}\ln(\mu/\mu_0)\big]$.

7.2 Graphical Depiction

The upward-sloping yellow curve in the film plots $\alpha(\mu)$ on log-energy abscissa, with dots representing *g-2*, deep-inelastic, and LEP data points.

8 Scene VIII: Grand Collage and Conceptual Synthesis

8.1 Unification Narrative

The closing tableau layers classical geometry (light cone), quantum matter (spinor field), and interaction carriers (photons), visually reinstating the principle that "*gauge symmetry dictates interaction*"—a cornerstone of the Standard Model.

8.2 Across Space and Time

Electromagnetism permeates all epochs:

- **Recombination (380 000 yr):** photon decoupling $\to$ CMB.
- **Stellar epochs:** nuclear fusion needs Coulomb tunnelling.
- **Terrestrial technology:** every screen pixel dancing to QED.

9 Scene IX: Cosmic Fade-Out

The return to an unadorned star field is an homage to *inflationary* initial conditions—structure seeds grown from quantum fluctuations. The epilogue word "Finis" reminds us that our theoretical edifice, though elegant, is inevitably provisional pending deeper quantum-gravitational insight.

Acknowledgements & Further Reading

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