CDP Ordering is Maintained Batog-based Redistribution with Corrected Stakes

Here we show that ICR ordering is preserved with corrected stakes across a liquidation event.

We make use of the first-order equivalence result, namely, that with corrected stakes:

1) $S_N = S_1$

i.e:

Any N'th order system of CDPs is equivalent to a first-order system of CDPs. For a given fresh CDP with stake s_i *and collateral* c_i*, the stake* s_i *is equivalent to some hypothetical first-order stake* c_j *which has accumulated collateral reward* $x_j = (c_i - c_j)$ *and debt reward* $y_j = (d_i - d_j)$*.*

Due to this equivalence between first and N'th-order systems, if ordering is preserved for first-order systems, it is preserved for N'th order systems.

Now consider a first-order system of CDPs, with stakes equal to their initial collateral.

Let CDP_1 and CDP_2 be CDPs with initial collateral c_1, c_2 accumulated collateral and debt rewards x_1, y_1 and x_2, y_2 respectively:

2) $ICR_1 = (c_1 + x_1) / (d_1 + x_1)$
3) $ICR_2 = (c_2 + x_2) / (d_2 + y_2)$

Let their ICRs be such that:

4) $ICR_1 > ICR_2$

Since, a first-order CDP's collateral and debt rewards are always in direct proportion to its initial collateral, we can write the accumulated rewards as:

5) $x_1 = Ac_1$
6) $x_2 = Ac_2$

and

7) $y_1 = Bc_1$
8) $y_2 = Bc_2$

Where A is the sum of all 'collateral rewards per unit staked', and B is the sum of all 'debt rewards per unit staked'. This yields ICRs:

9) $\mathbf{ICR_1 = c_1(1 + A) / (d_1 + Bc_1)}$
10) $\mathbf{ICR_2 = c_2(1+A) / (d_2 + Bc_2)}$

And the initial ICR inequality becomes:

11) $\mathbf{c_1(1 + A) / (d_1 + Bc_1) > c_2(1 + A) / (d_2 + Bc_2)}$

Cross multiplying and cancelling the common denominator yields:

12) $\mathbf{c_1(1 + A)(d_2+Bc_2) > c_2(1+A)(d_1+Bc_1)}$

Then expanding:

13) $\mathbf{c_1(d_2 + Bc_2 + Ad_2 + ABc_2) > c_2(d_1 + Bc_1 + Ad_1 + ABc_1)}$
14) $\mathbf{c_1d_2 + Bc_1c_2 + Ac_1d_2 + ABc_1c_2 > c_2d_1 + Bc_1c_2 + Ad_1c_2 + ABc_1c_2}$

And cancelling terms:

15) $\mathbf{c_1d_2 + Ac_1d_2 > c_2d_1 + Ad_1c_2}$

16) $\mathbf{c_1d_2(1 + A) > c_2d_1(1 + A)}$

Finally yielding the result:

17) $\mathbf{d_2/c_2 > d_1/c_1}$

We will later use this to prove that the inequality of ICRs holds across a liquidation event.

Now consider a liquidation event occurs. Upon a CDP liquidation, r_c collateral and r_d debt are distributed to all active CDPs. Each active CDP earns rewards proportional to its initial collateral, thus:

18) $\mathbf{ICR_{1\ After} = (c_1(1+A) + ac_1) / ((d_1 + Bc_1) + bc_1)}$
19) $\mathbf{ICR_{2\ After} = (c_2(1+A) + ac_2) / ((d_2 + Bc_2) + bc_2)}$

Where:
20) $\mathbf{a = r_c /} \boldsymbol{totalStakes}$
21) $\mathbf{b = r_d /} \boldsymbol{totalStakes}$

Collecting terms:

22) $ICR_1 = (c_1(1 + a + A)) / (d_1 + (1 + B)c_1)$
23) $ICR_2 = (c_2(1 + a + A)) / (d_2 + (1 + B)c_2)$

And taking reciprocals:

24) $1 / ICR_{1\ After} = (d_1 + (1+ B)c_1) / (c_1(1 + a + A))$
25) $1/ ICR_{2\ After} = (d_2 + (1+ B)c_2) / (c_2(1 + a + A))$

Dividing all terms by c_1, and separating the constant term:

26) $1 / ICR_{1\ After} = [(d_1/c_1) / (1 + a + A)] + [(1+ B) / (1 + a + A)]$
27) $1 / ICR_{2\ After} = [(d_2/c_2) / (1 + a + A)] + [(1+ B) / (1 + a + A)]$

Recall our earlier result 17): $d_1/c_1 < d_2/c_2$. Thus:

28) $1 / ICR_{1\ After} < 1 / ICR_{2\ After}$

Then taking reciprocals, finally yields:

29) $ICR_{1\ After} > ICR_{2\ After}$

Therefore, CDP ordering holds across a liquidation event in first-order systems, and thus holds across a liquidation event in N'th order systems.