Practical Approach to Zero-Knowledge Basics: KZG Commitment

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Why commitments?

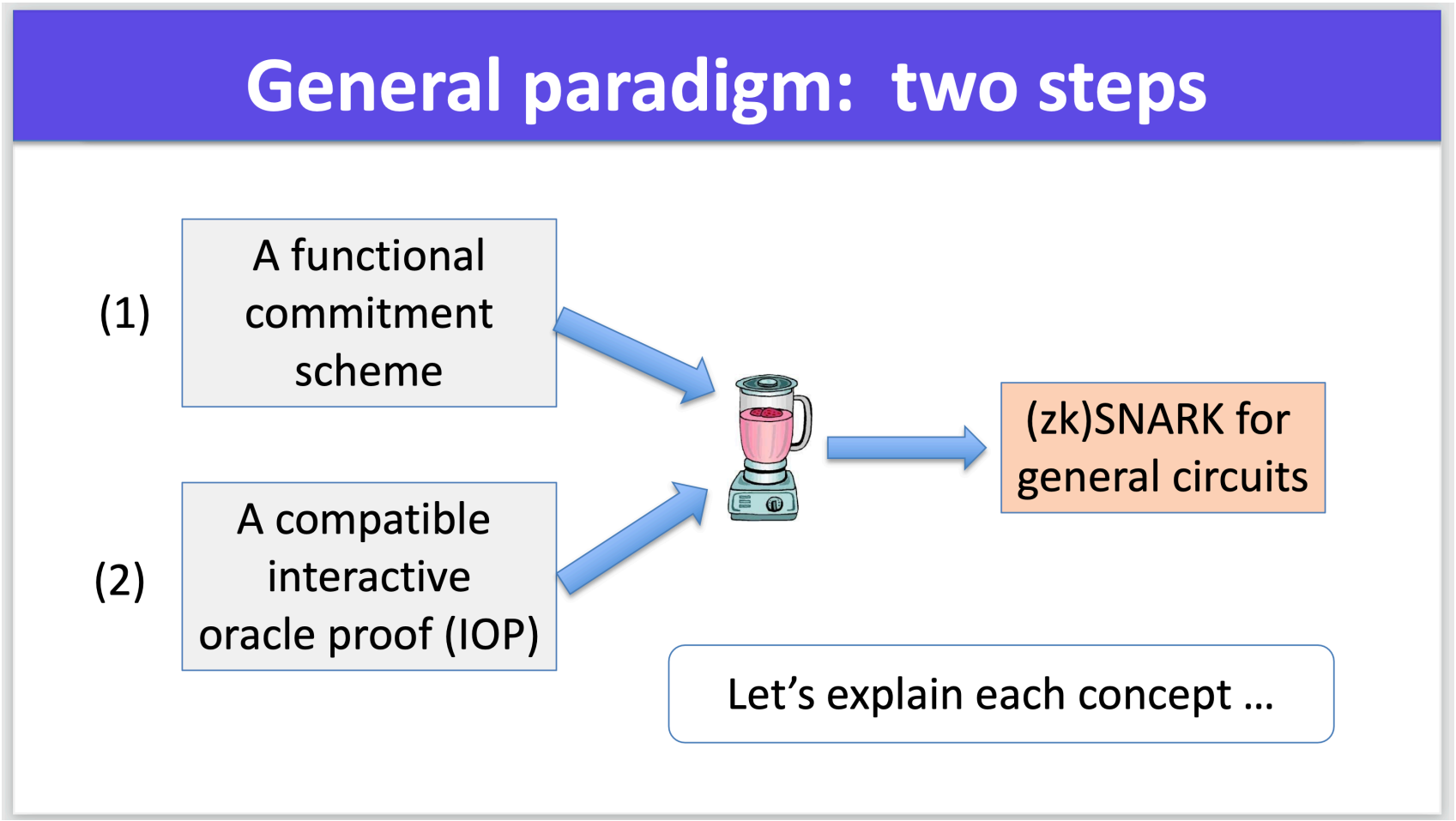

Figure: Building a SNARK, from Dan Boneh's zkWhiteboards session 2

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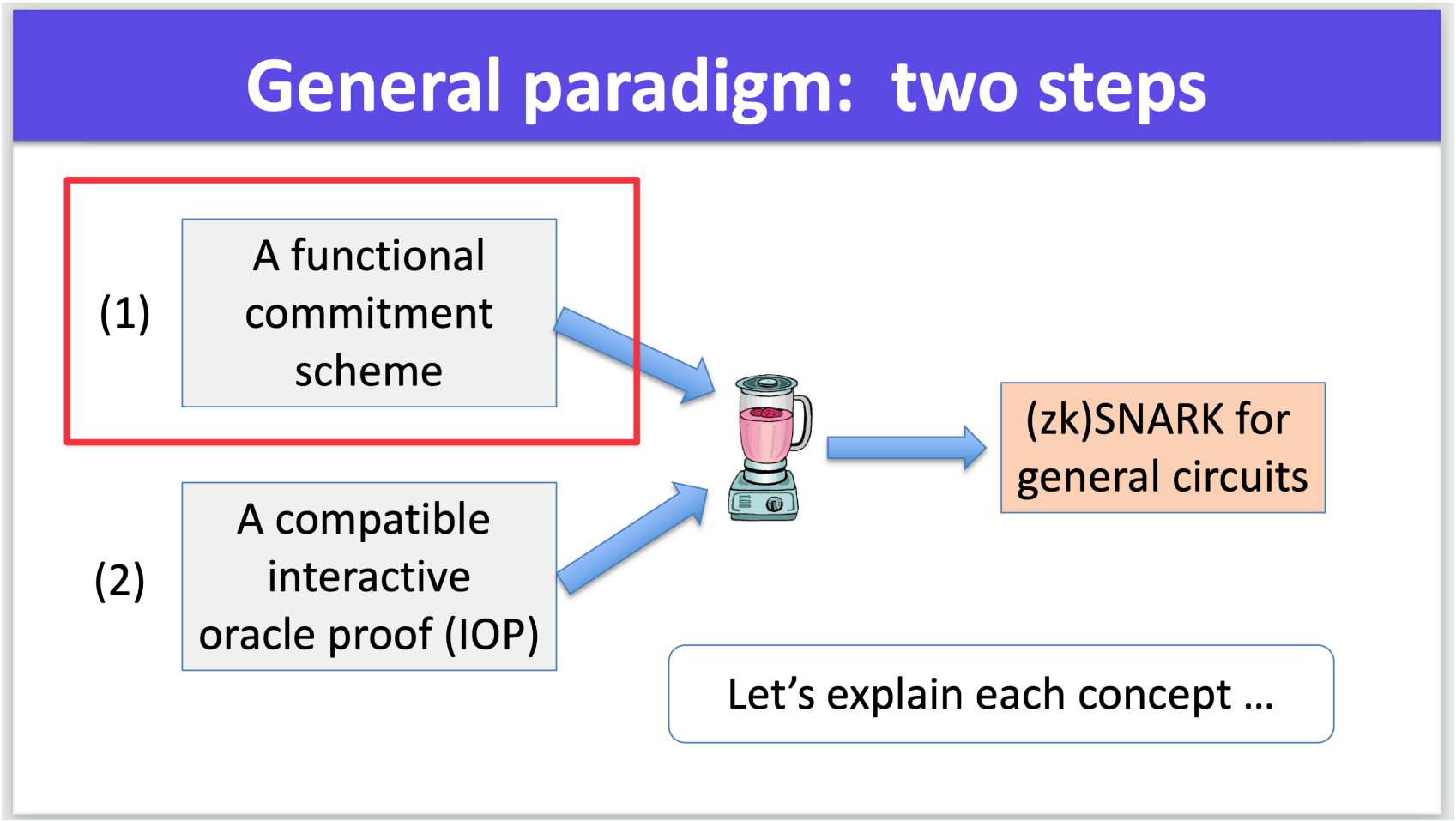

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Proto-Danksharding (a.k.a. EIP-4844)

Couldn't we use some other commitment scheme without a trusted setup?

Unfortunately, using anything other than KZG (eg. IPA or SHA256) would make the sharding roadmap much more difficult. This is for a few reasons:

- Non-arithmetic commitments (eg. hash functions) are not compatible with data availability sampling, so if we use such a scheme we would have to change to KZG anyway when we move to full sharding.
- IPAs may be compatible with data availability sampling, but it leads to a much more complex scheme with much weaker properties (eg. self-healing and distributed block building become much harder)
- Neither hashes nor IPAs are compatible with a cheap implementation of the point evaluation precompile. Hence, a hash or IPA-based implementation would not be able to effectively benefit ZK rollups or support cheap fraud proofs in multi-round optimistic rollups.
- One way to keep data availability sampling and point evaluation but introduce another commitment is to store multiple commitments (eg. KZG and SHA256) per blob. But this has the problem that either (i) we need to add a complicated ZKP proof of equivalence, or (ii) all consensus nodes would need to verify the second commitment, which would require them to download the full data of all blobs (tens of megabytes per slot).

Hence, the functionality losses and complexity increases of using anything but KZG are unfortunately much greater than the risks of KZG itself. Additionally, any KZG-related risks are contained: a KZG failure would only affect rollups and other applications depending on blob data, and leave the rest of the system untouched.

Figure: From Proto-Danksharding FAQ by Vitalik.

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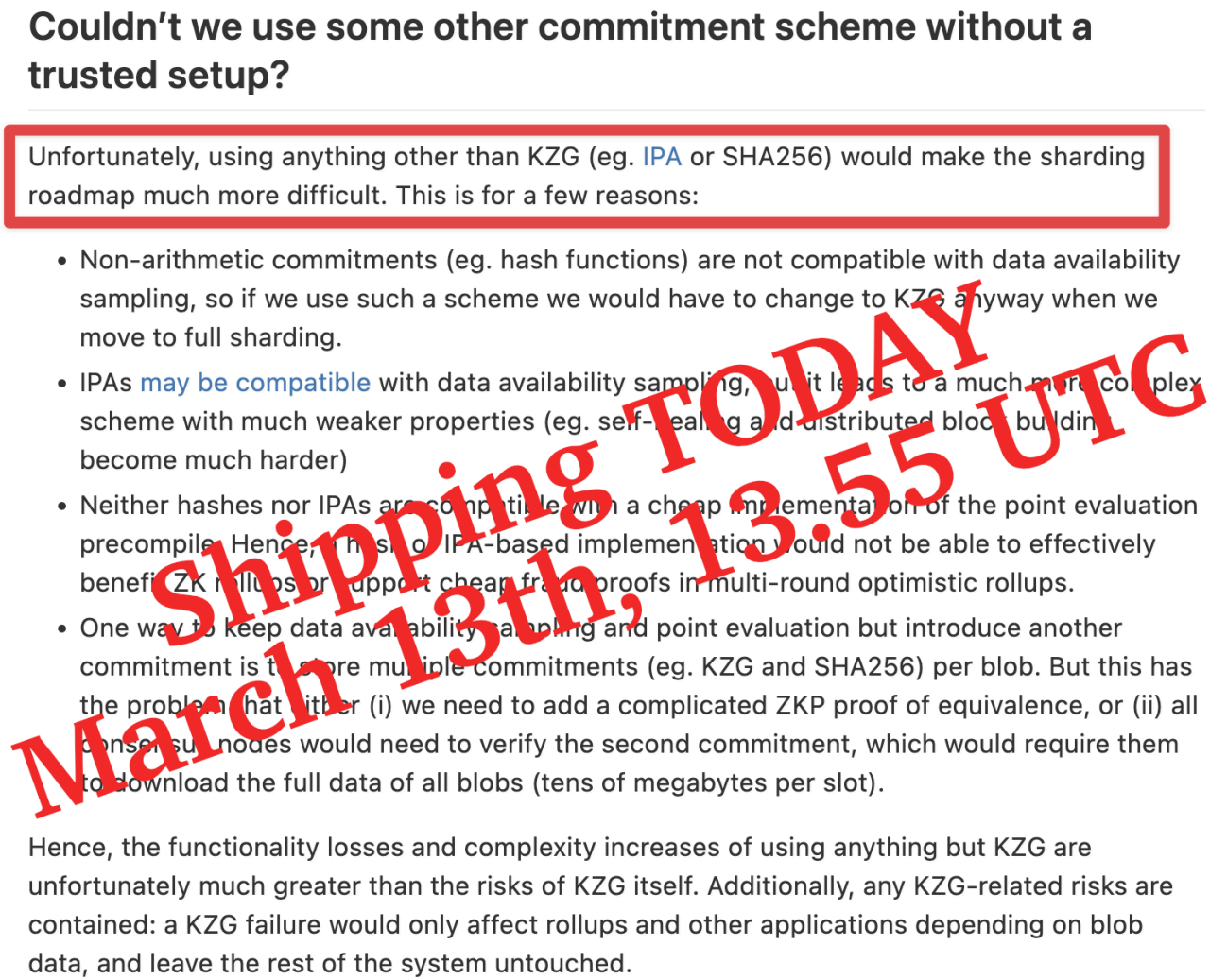

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Shipping TODAY March 13th, 13.55 UTC

Figure: From Proto-Danksharding FAQ by Vitalik. Countdown.

Polynomial commitments

Polynomial Commitments*

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December 01, 2010

Abstract

We introduce and formally define polynomial commitment schemes, and provide two efficient constructions. A polynomial commitment scheme allows a committer to commit to a polynomial with a short string that can be used by a verifier to confirm claimed evaluations of the committed polynomial. Although the homomorphic commitment schemes in the literature can be used to achieve this goal, the sizes of their commitments are linear in the degree of the committed polynomial. On the other hand, polynomial commitments in our schemes are of constant size (single elements). The overhead of opening a commitment is also constant; even opening multiple evaluations requires only a constant amount of communication overhead. Therefore, our schemes are useful tools to reduce the communication cost in cryptographic protocols. On that front, we apply our polynomial commitment schemes to four problems in cryptography: verifiable secret sharing, zero-knowledge sets, credentials and content extraction signatures.

Figure: KZG ("Kate") Polynomial commitments article.

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Goal

- Prove that we know a Polynomial $\phi(x) := a_0 + a_1 x + a_2 x^2 + \cdots + a_t x^t \in \mathbb{F}_p[X]$ without revealing it.

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  - In particular, prove that we know an EVALUATION of a polynomial $\phi(x)$.

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- $\deg(\phi(x)) =$ (at most) t $(< p)$.

Cryptographic Assumptions

Cryptographic Assumption 1: Discrete Logarithm (informal)

Given a generator g of $\mathbb{G}^*$ then, given $g^x \in \mathbb{F}_p^*$, finding x is **HARD** (i.e. computationally unfeasible).

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Cryptographic Assumption 2: Strong Diffie-Hellman (informal)

Given a generator g of $\mathbb{G}^*$ and given g^a and g^b, where $a, b \in \mathbb{F}_p^*$, then g^a and g^b is **"indistinguishable"** from g^{ab}.

How can we do that?

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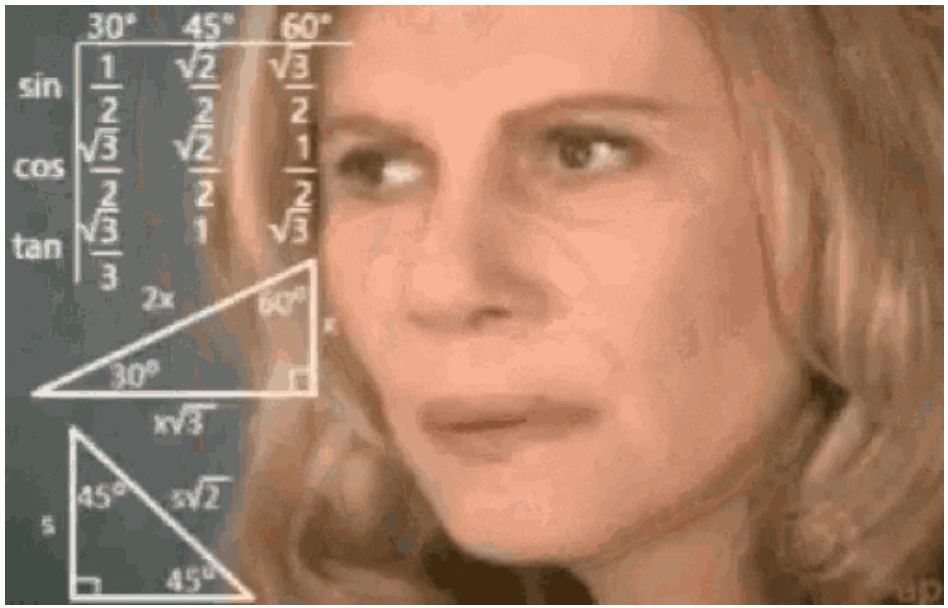

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  - **Binding:** once you commit a message, you cannot change your mind.
  - **Hiding:** the commitment does not reveal the message until you OPEN it.

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$$e(g, g) \neq 1.$$

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- Sends Public Parameters (PP) to the Prover (P) and Verifier (V).

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- So, P sends $\mathcal{C} = g^{\phi(\alpha)}$ to the V computed using the PP.

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- P sends $(i, \phi(i), \mathcal{C}_{\psi_i})$ to V.

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Nope!

Pairings come to rescue

Pairing Verification

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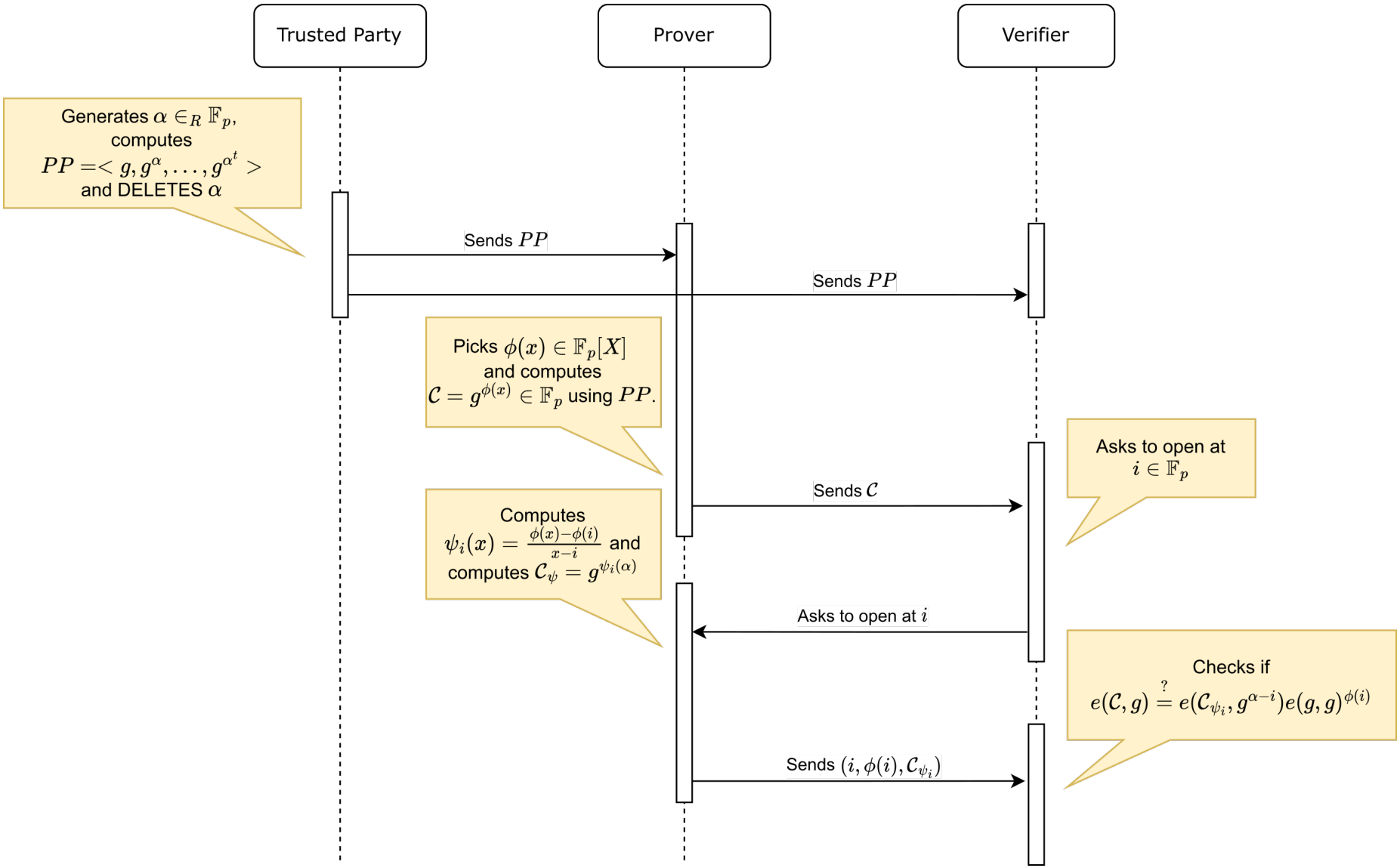
The Protocol: Summary



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Your turn

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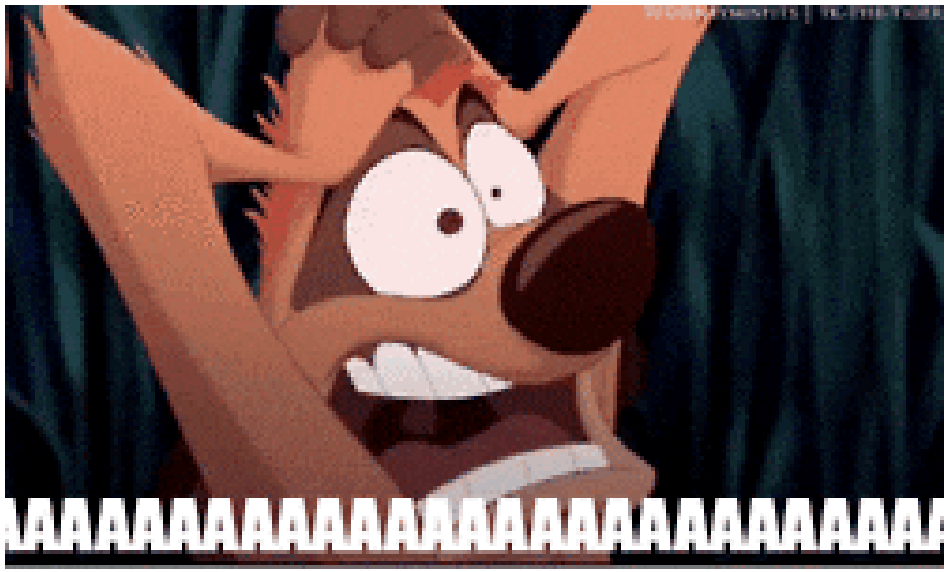

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  - It breaks the binding property!!
- Use Multi-Party Computation to generate α :)

Thank you for your attention!

@0xAdriaTorralba

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atorralbaag@uoc.edu

QR Code to my research