$(\mathbb{F}_{11}, +)$, $p = 11$, $t = 2$, $g = 2$ (any element generates)

Pairing $e(x, y) = x \cdot y \bmod 11$.

① $e(x_1 + x_2, y) = (x_1 + x_2)y = x_1y + x_2y = e(x_1, y) + e(x_2, y)$ ✓

② $e(g, g) = 2 \cdot 2 = 4 \neq 0$. ✓

Setup: ~~$\alpha = 3$~~

$PP = \langle g, \alpha g, \alpha^2 g \rangle = \langle 2, 3 \cdot 2, 3^2 \cdot 2 \rangle = \langle 2, 6, 18 \rangle \equiv \langle 2, 6, 7 \rangle \bmod 11$.
$g \quad \alpha g \quad \alpha^2 g$

Commit:

$\phi(x) = 3x^2 + 5x + 7$,

$\xi = [\phi(\alpha)] = 3\alpha^2 g + 5\alpha g + 7g = 3 \cdot 7 + 5 \cdot 6 + 7 \cdot 2 = 65 \equiv 10 \bmod 11$.

↳ $\xi = 10$.

Open at $i = 1$.

$\phi(1) = 3 + 5 + 7 = 15 \equiv 4 \bmod 11$.

$\rightarrow \psi = \dfrac{\phi(x) - \phi(1)}{x - 1} = \dfrac{3x^2 + 5x + 7 - 4}{x - 1} = \dfrac{3x^2 + 5x + 3}{x - 1}$

$3x^2 + 5x + 3 \;\big|\; x - 1$

$-3x^2 + 3x \qquad 3x + 8$

$8x + 3$

$-8x + 8$

$11 \equiv 0$ ✓

$\rightarrow \psi(x) = 3x + 8$

$\rightarrow \xi_\psi = 3\alpha g + 8g = 3 \cdot 6 + 8 \cdot 2$

$= 34 \equiv 1$.

Verification:

$i=1,\ \phi(1)=4,\ \mathcal{E}=10,\ \mathcal{E}_N=1,\ g=2,\quad \langle 2,6,\cancel{7}\rangle;\ e(x,y)=x\cdot y \pmod{11}$

$g\quad \alpha g\quad \alpha^2 g$

$$e(\mathcal{E}+[-\phi(i)],[1]) \overset{?}{=} e(\mathcal{E}_N,[\alpha-i])$$

① $e(\mathcal{E}+[-\phi(i)],[1]) = e(\mathcal{E}+[7],[1]) = e(10+(2\cdot 7),2)$

$-\phi(i) = -4 \equiv 7 \bmod 11.$

$= e(24,2) = 48 \equiv 4 \bmod 11.$

② $e(\mathcal{E}_N,[\alpha-i]) = e(1,[\alpha+10]) = e(1, \underbrace{g\alpha}_{6} + 10g)$

$-i = -1 \equiv 10 \bmod 11.$

$= e(1,\ 6+(10\cdot 2)) = 26 \equiv 4 \bmod 11$ ✓✓✓

What happens if α is locked? $(\alpha=3)$

$P_1(x) = 3x^2+5x+7 \longrightarrow P_1(\alpha) = 43$

$P_2(x) = 2x^2+7x+4 \longrightarrow P_2(\alpha) = 43.$