$f(x)$ Protocol: Creation of a decentralized, scalable, and capital efficient stablecoin

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Abstract

$f(x)$ Protocol creates a new class of decentralized stablecoin paired with a new leveraged long ETH perpetual token by separating pure ETH collateral into a lower-volatility component ($\beta < 1$) called fractional ETH (fETH) and a higher-volatility ($\beta > 1$) one called leveraged ETH (xETH). By constraining $\beta_{\mathrm{f}} = 0.1$ the fractional ETH token captures some growth of the cryptocurrency market while limiting volatility enough to retain the characteristics of a stablecoin. The leveraged ETH component is essentially a long perpetual future contract with zero funding costs and variable leverage. With only pure ETH collateral the system is not exposed to centralized risk. Maximum liquidity of fETH can scale quickly compared with CDP-issued stablecoins because it is based on the high demand for leverage long ETH positions (xETH) rather than the relatively lower demand for CDPs, with their attendant maintenance and capital inefficiency.

1 Introduction

Stablecoins are an essential enabling technology for decentralized finance (DeFi). Stablecoins enable fully on-chain swaps in and out of fiat denomination as well as the creation of a wide array of powerful decentralized financial instruments. Additionally, due to the volatility of all cryptocurrencies, even the most crypto-native of organizations and individuals must plan for expenses in fiat terms, and must therefore hold at least some fiat-denominated reserve.

Despite the critical importance of these tokens, they have also introduced new risks and been responsible for some of the DeFi's most damaging failures. Pure algorithmic stablecoins have failed so spectacularly that little needs to be said about their risk, but current stablecoin implementations suffer from critical weaknesses which will ultimately hinder their long-term success.

In this document we will introduce a protocol uses a unique mechanism to create two new ETH derivative tokens: fETH, a low β near-stablecoin, and xETH, a high β leveraged long ETH perpetual contract. The leveraged long token offers a powerful new decentralized tool for on-chain trading while the stablecoin provides an alternative to current offerings that eliminates centralized risk while also being able to scale efficiently enough to meet the needs of the DeFi industry.

2 Stablecoin Risks and Scaling

The most obvious type of risk seen in recent years in stablecoins is in pure algorithmic (under- or un-collateralized) implementations, of which UST by Terra is the most obvious example. Making this type of stablecoin safe and reliable may very well be impossible, but for the purposes of this discussion we consider this class of stablecoin so risky as to immediately disqualify it from being a good choice for long-term reliability.

Among remaining designs, there are currently three broad categories of stablecoins in the market:

1. Fiat-backed stablecoins (USDC, USDT)
2. Algorithmic, but fully or partly collateralized by fiat-backed stablecoins (DAI, FRAX)
3. Fully decentralized CDP algorithmic stablecoins (LUSD)

Categories (1) and (2) both suffer from centralization risk, while category (3) has, as yet, struggled with scalability and capital efficiency compared to those in (1) and (2). These centralization risk and scaling challenges leave the door open for other solutions.

2.1 Centralization Risk

Centralization is the most insidious type of risk in DeFi because it can often look like a strength. Stablecoins based on the centralized reserve model are capital efficient since their collateral is denominated in the same currency as their issued token, thus they can scale very quickly with easy minting and redemption in and out of the reserve based entirely on direct demand for the stablecoin. The problem is that such protocols require a real-world entity to maintain those fiat cash reserves. These real-world entities and the custodied reserves themselves introduce attack surfaces and failure modes from outside of crypto. This was amply demonstrated with recent USDC depeg drama which occurred during the insolvency crisis of Silicon Valley Bank, which held a fraction of USDC's reserve.

Although that matter was resolved quickly, the whole episode highlighted the fact that any protocol with reserves stored at centralized institutions cannot avoid these types of risk and that even a small centralized risk can quickly become existential. This risk is also transmitted to any stablecoin which relies on fiat-backed stables to collateralize it. Avoiding centralized risk is the whole reason cryptocurrency was created in the first place, so it makes little sense to continue to build DeFi around such tokens.

2.2 Scalability and Capital Efficiency

Although the third class of stablecoin listed in section 2 (CDP-backed) can avoid centralization risk by accepting only decentralized collateral, they struggle with scalability and capital efficiency. Since the collateral is denominated in a different currency than the minted token, these stablecoins provide far less than 100% capital efficiency. Since fully decentralized collateral is generally volatile, borrowing limits need to be conservative to allow the protocol sufficient time to liquidate vulnerable positions before they become insolvent. On top of this, individual borrowers must apply yet more conservatism on top of the protocol limits to avoid the risk of liquidation.

The main scalability-related challenge of CDP-backed stablecoins is that their liquidity in the market is driven entirely by demand for CDPs, rather than demand for stablecoins specifically. Holding a CDP requires users to remain completely exposed to the price of their collateral and also requires constant monitoring and periodic top-up to avoid liquidation risk. CDPs are very useful in certain circumstances, such as yield farming or tax optimization, but they are also relatively niche financial instruments whose demand does not scale quickly enough to produce the depth of liquidity in the market that would allow a stablecoin to achieve critical mass.

3 Design Goals of $f(x)$

$f(x)$ protocol is built with the aim of creating a symbiotic system that decomposes ETH into two useful tokens. Specifically for xETH we create a leveraged long ETH token which:

1. is fully decentralized and Ethereum-native
2. is composable, with liquidity on-chain

3. has extremely low risk of liquidation (i.e. NAV to 0)

For fETH, the goal is to produce a pseudo-stablecoin which:

1. is fully decentralized and Ethereum-native
2. minimizes volatility while retaining a small exposure to the market (10%)
3. can be minted and redeemed instantly in direct response to stablecoin demand
4. has maximum liquidity depth based on a multiple of demand for xETH, rather than a fraction of demand for CDPs

4 Decomposing ETH using $f(x)$ Protocol

$f(x)$ protocol accepts only ETH as collateral, and mints low and/or high volatility (β) tokens backed by that collateral. Supplying ETH allows a user to mint fETH and/or xETH, with quantities based on the price of ETH and the current net asset value (NAV) of each token. Conversely, users can redeem fETH or xETH for their NAV worth of ETH from the reserve at any time.

4.1 Cryptocurrency Volatility

In traditional finance, β is a measurement of the volatility of a given security or portfolio compared to the market. Since fiat is the denominator of these measurements, cash is said to have $\beta = 0$ while a portfolio with $\beta = 1$ would perfectly reflect the return of the market (an example of which might be a S&P 500 ETF). A portfolio that moves in the same direction as the market but with smaller relative magnitude has $\beta < 1$, while one which experiences larger-than-market movements in the same direction has $\beta > 1$.

For the purposes of $f(x)$, we define the price of ETH as "the market", and so β is the volatility of a given cryptocurrency compared to ETH. ETH itself, therefore, has $\beta = 1$ and a perfect stablecoin has $\beta = 0$.

fETH targets $\beta = 0.1$ which means that $f(x)$ constraints its price (via controlling its NAV) such that it gains or loses only a fraction of the value that ETH does in a given time period. If the price of ETH falls by 10%, fETH's NAV will decrease by only 1%. The same attenuation is true on the upside: a 10% price increase for ETH would result in a 1% NAV increase for fETH.

xETH captures all the ETH price movement which is shielded from fETH, so is has a NAV which moves by *more* than the price of ETH (i.e., $\beta > 1$). Its precise β at any given time is based on the total relative supplies of fETH and xETH, which are always changing. When xETH's $\beta = 2$, for example, a 10% price increase for ETH would create a 20% increase in the NAV of xETH. Another way to look at high-β is as a leveraged return on the market (i.e. ETH), since both gains and losses are amplified.

4.2 The $f(x)$ Invariant

The NAV of fETH and xETH vary with the price of ETH such that at all times the total value of all fETH plus the total value of all xETH are equal to the total value of the ETH reserve. In this way every fETH and xETH token is backed by and redeemable for its NAV at any time. Put mathematically, at all times the invariant will hold:

$$n_{\text{eth}} p_{\text{eth}} = n_f p_f + n_x p_x \tag{1}$$

where n_{eth} is number of ETH collateral, p_{eth} is ETH price in USD, n_f is total supply of fETH, p_f is the NAV of fETH, n_x is total supply of xETH, and p_x is the NAV of xETH.

The protocol constrains the volatility of fETH by adjusting its NAV in response to changes in ETH price such that 10% (for β_{f}=0.1) of the ETH return is reflected in the fETH price.

The protocol simultaneously adjusts the xETH NAV by more than the magnitude of the ETH return in order to satisfy the $f(x)$ invariant (equation 1). In this way xETH provides leveraged ETH returns (tokenized, and with zero funding costs, see section 4.4) while fETH exhibits low volatility, and both remain credibly decentralized.

4.3 Fractional ETH: A pseudo-stablecoin

The power of a stablecoin is based on three key factors: price volatility and intrinsic risk (which should be low) and its liquidity (which should be deep). Regarding the first two factors, it's clear that fractional ETH avoids centralized risk and keeps volatility low enough to retain the key characteristics of a stablecoin. The liquidity depth is a slightly more complex story.

Whereas a CDP-backed stablecoin is only minted in response to demand for a CDP, fETH is minted in response to direct demand. For fETH the only factor limiting the quantity that can be immediately minted is the supply of xETH, which must be sufficient to absorb the volatility of the total supply of fETH. As the leverage of xETH is designed to be variable, a relatively small amount of xETH can support a large supply of fETH. The ability of the system to maintain the price stability of fETH in the face of varying supply of xETH is discussed in section 5.1.

fETH minting does not affect NAV (i.e. price) and it can be minted for minimal fees so long as sufficient xETH supply exists. This means that while the demand for fETH determines the fETH supply at any time, the *effective* liquidity depth (i.e. the number of fETH that can be minted immediately for minimal fees in response to demand) is generally much higher, being limited only to a *multiple* of the supply of xETH, which is in turn limited by its demand.

4.4 Leveraged ETH: A perpetual contract with zero funding cost

The xETH token, also called leveraged ETH, is a decentralized, composable leveraged long ETH futures contract with low risk of liquidations and zero funding costs (and in extreme cases, xETH minters can actually earn fees). xETH holders in aggregate take on most of the volatility of the fETH supply, and using a ready on-chain AMM liquidity pool traders can swap in an out of positions at will.

We expect variable but sustained demand for xETH through all market conditions, because demand for ETH long positions never disappears. This is well illustrated by the combination of open interest (shown in figure 1) and funding rate (shown in figure 2). The ETH funding rate is virtually never negative, and those time when it does go negative the OI is still so large that there is clearly still demand for longs, even if they are (briefly) not dominant. Given this continuous demonstrated level of demand for long positions as well as the highly desirable DeFi traits of decentralization and composability, xETH will provide a powerful new DeFi primitive and tool in the trader's toolkit.

The precise effective leverage of the xETH token varies over time as the relatively supplies of xETH and fETH change from minting and redemption. A higher supply of xETH relative to fETH means a lower effective leverage for xETH, since excess volatility from the fETH is spread over more tokens. Conversely, a larger supply of fETH focusing volatility on fewer xETH tokens results in a higher effective leverage. For more detail, see section 7.3.1.

5 System Stability

From the system descriptions above it should be clear that there needs to be sufficient xETH supply for the system to operate correctly and constrain the volatility of the fETH NAV. A convenient way to think about the system's ability to do this is to consider the entire $f(x)$ system as one big CDP. The total ETH reserve represents the total CDP collateral. The total fETH supply represents the borrowed amount and the xETH supply, then, represents the difference between supplied collateral and the total borrowed amount.

Figure 1: ETH open interest since 2020

Figure 2: ETH Funding Rate - Positive rate indicates greater demand for longs than shorts

Viewed through this lens, we can measure the health of the system in a similar way as a CDP, using the collateralization ratio (sometimes called a health factor). Another way to state that the system requires sufficient xETH supply is to say it requires a CR over 100% to function correctly. For $f(x)$ CR of the system is given as the total collateral value divided by the total NAV of fETH:

$$\mathrm{CR} = \frac{n_{\mathrm{eth}} p_{\mathrm{eth}}}{n_{\mathrm{f}} p_{\mathrm{f}}} \times 100\% \tag{2}$$

Minting fETH or xETH, as well as adjusting the NAV of either token as described in section 4.2, will have an impact on the CR. If the system CR were ever to fall to 100%, that would imply that the NAV of xETH would be zero. Although fETH would remain mintable and redeemable, its β would jump to 1, meaning it would be exposed to the full price movements of ETH (i.e. no longer be low volatility). $f(x)$ has a comprehensive 4-level risk management module which kicks in if the CR falls below certain thresholds to ensure that does not happen.

5.1 Risk Management Overview

If the CR of the system (equation 2) ever falls to a level where the ability to maintain $\beta_{\mathrm{f}} = 0.1$ becomes at risk, the system's risk management system with four progressively more powerful modes will kick in to guide the system back in the direction of over-collateralization. Each mode sets a CR threshold below which additional measures kick in to help maintain the overall system stability. The controls described for each mode remain active as long as the CR is below its specified level, so for example, if level 3 is active it implies that both level 1 and 2 are also.

5.1.1 Value at Risk and Risk Management Thresholds

Selection of the thresholds for each mode of the risk management module is done based on the probability of a one-day price drop so severe as to push the system CR to 100% (i.e. xETH NAV to 0). The lower the CR of the system gets, the smaller the magnitude of ETH price drop required to trigger such an event, and therefore the higher the probability of experiencing one. Using a value at risk (VaR) calculation based on historical price data for ETH from CoinGecko since 2017-01-01 we calculate the probability of experiencing such price drops and use those probabilities to set the risk management thresholds.

In plain language, if the probability of a destabilizing event rises above 0.10%, we engage stability mode. If it rises above 0.25% we enable user liquidation mode and if it rises above 1% we engage protocol liquidation mode, as shown in table 1. Each mode applies progressively more pressure and incentive to participants to push the CR back up. We use a one-day time period based on the assumption that is how long it will take the protocol's risk management response to deal with risks brought by market volatility.

Probability of Destabilizing Event	Req'd Magnitude for Destabilizing Event	Equivalent CR Threshold	Risk Management Module
0.10 %	-25.34 %	130.6	Stability Mode
0.25 %	-18.68 %	120.6	User Liquidation Mode
1.00 %	-13.78 %	114.4	Protocol Liquidation Mode

Table 1: We engage each risk management module when the probability of a destabilizing event (ETH price drop of given magnitude) rises above its specified threshold.

5.1.2 Level 1: Stability Mode

If CR falls below 130% the system enters Stability Mode. In this mode minting fees for fETH and redemption fees for xETH are increased (see sec 6), and xETH minters earn extra incentives from fETH holders in the form of a small stability fee. This stability fee is implemented as a small reallocation of fETH collateral (resulting in a slight NAV reduction), to new xETH minters who are topping up the system collateral.

5.1.3 Level 2: User Liquidation Mode

If CR falls below 120% the system enters User Liquidation Mode. In this mode, users can earn incentives by redeeming fETH for ETH, paid by the remaining fETH holders as stability fees in a similar way as Stability Mode. In this way users can receive a bit more than the NAV of fETH upon redemption. In this mode the redemption fee for fETH is set to zero.

Once $f(x)$ is well enough established to be on the radar of arbitrageurs and MEV-searchers, this mode will be sufficient to stabilize the system immediately from nearly any state. These liquidations can be run in a single transaction block and with a flash loan if necessary, using ETH to buy fETH from the market, and redeeming it for more ETH.

5.1.4 Level 3: Protocol Liquidation Mode

If CR falls below 114%, the system enters Protocol Liquidation Mode. This mode is equivalent to Level 2, except that the protocol itself can run the liquidation using the reserve. This mode is unlikely to ever be triggered due to the profitability of liquidations in Level 2 and users' ability to response more quickly than the protocol, however it creates an extra layer of protection.

In this mode, the protocol uses ETH from the fETH reserve to market buy, and then burn, fETH from an AMM. Using this mechanism the NAV of fETH is only reduced by the amount of the liquidation fee, which the protocol earns in this case.

5.1.5 Level 4: Recapitalization

In the most extreme case, the protocol has the ability to issue governance tokens to raise ETH to recapitalize, either by minting xETH or by buying and redeeming fETH.

6 Fees

$f(x)$ earns revenue by charging fees on the minting and redemption of fETH and xETH. Additional stability fees are charged to the NAV of fETH when the risk management module is engaged, but those are paid to other users of the system who help rebalance it, based on the prescription described in the risk management module.

Risk Mode	fETH Mint Fee	fETH Redeem Fee	xETH Mint Fee	xETH Redeem Fee	fETH Stability Fee
Normal Mode	normal	normal	normal	normal	n/a
L1 Stability	above norm.	normal	earns bonus	above norm.	xETH Mint
L2 User Liq.	above norm.	earns bonus	earns bonus	above norm.	xETH Mint fETH Redeem
L3 Protocol Liq.	above norm.	earns bonus	earns bonus	above norm.	xETH Mint fETH Redeem

Table 2: The fee structure changes depending on whether system is in normal mode or one of the risk management modes is active. Specific fees are an operational parameter which will be finalized at launch.

7 Calculations

7.1 Protocol Genesis and Seed Liquidity

At the genesis of the protocol, the NAV of both fETH and xETH are set at $1 USD. As part of launching $f(x)$ a reserve of ETH will be raised through a token sale. Some of that ETH will be the first collateral supplied to the system, and equal parts of fETH and xETH will be minted with it, then and paired with other ETH from the token sale to seed liquidity pools.

7.2 Reserve Allocation based on the $f(x)$ Invariant

The total ETH reserve can be thought of as divided into two allocations, one backing fETH and the other backing xETH, and the fraction of the reserve backing each token varies with the price of ETH. It is useful to define λ_f and λ_x as the current fractional allocation to fETH and xETH, respectively

$$\lambda_f = \frac{n_f p_f}{n_{\text{eth}} p_{\text{eth}}}, \lambda_x = \frac{n_x p_x}{n_{\text{eth}} p_{\text{eth}}} \tag{3}$$

Using equation 3, the invariant defined in equation 1 can be rewritten as $\lambda_f + \lambda_x = 1$. When the price of ETH changes, the NAV of fETH and xETH are both updated such that:

1. the NAV of fETH only changes by 10% of the change in ETH price (i.e. $\beta_f = 0.1$)
2. the NAV of xETH changes such that the $f(x)$ invariant (equation 1) is satisfied

7.3 Changes to xETH, fETH NAV as ETH price varies

When the price of ETH changes, the NAV of fETH is updated such that $\beta_{\text{f}} = 0.1$. If the fractional return of ETH during the period from time $t-1$ until time t is given as

$$r_{\text{ETH}} = \frac{p_{\text{eth}}(t)}{p_{\text{eth}}(t-1)} - 1 \tag{4}$$

then the return of fETH is

$$r_{\text{f}} = (1 + \beta_{\text{f}}) r_{\text{ETH}} \tag{5}$$

and therefore the NAV of fETH will be updated as follows

$$p_{\text{f}}(t) = r_{\text{f}} p_{\text{f}}(t-1) \tag{6}$$

The new NAV of xETH is easily calculated using the new $p_{\text{f}}(t)$ by rearranging equation (1).

$$p_x(t) = \frac{n_{\text{eth}} p_{\text{eth}}(t) - p_{\text{f}}(t) n_{\text{f}}}{n_x} \tag{7}$$

7.3.1 xETH Leverage

The NAV change to xETH during any given period will depend on the fraction of fETH supply vs xETH supply over the previous period. To calculate the effective leverage of xETH, we refer to equation (3) .

$$L_x(t) = \frac{1 - \beta \lambda_f(t-1)}{1 - \lambda_f(t-1)} \tag{8}$$

If there are zero fETH minted (assuming a nonzero number of xETH) then λ_f equals zero and $L_x = 1$, i.e. xETH becomes a 1X ETH long. As the supply of fETH rises vs xETH λ_{f} increases (which implies a decreasing CR), and the subsequent leverage of xETH tokens increases. If λ_{f} rises too far the risk management module described in section 5.1 kicks in to force the system back towards more balance between xETH and fETH. The first response

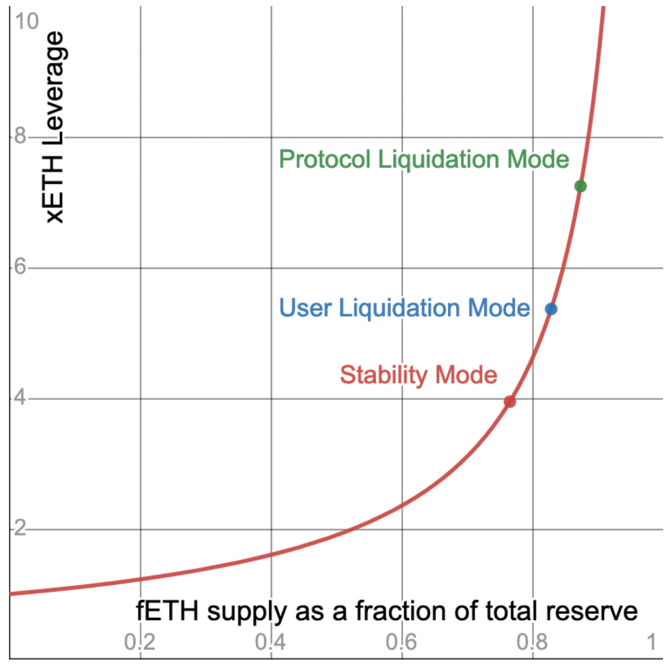

Figure 3: Leverage of the xETH token vs. λ_{f} showing the risk module trigger points

to this, Stability Mode, occurs around a leverage of 4X and so during normal operations (i.e. when the risk management module is *not* engaged) the leverage of xETH varies from 1X-4X. The xETH leverage can climb higher while the risk management module is engaged, though it should only be temporary until the stabilization measures push λ_{f} back down.

7.4 Minting and Redemption of xETH and fETH

Any user may mint or redeem fETH or xETH based on their NAV (i.e. p_x and p_{f}) at any time. In normal operations mode (i.e. when the risk management module described in sec 5.1 is not engaged), minting and redemption does not affect the NAV of fETH or xETH (p_{f} or p_x), but instead only changes the supply of fETH or xETH (n_{f} and n_x). Different risk management modules can create an effect on NAV due to minting and redemption through the application of stability fees.

Since single-sided (i.e. only fETH or only xETH) minting and redemption are allowed, and those operations will affect the CR of the system (and therefore also λ_x and λ_{f}).

8 Conclusion

We have proposed a new stablecoin paired with a composable perpetual long ETH token. If the market shows sufficient demand for xETH then fETH should be able to scale faster than any fully decentralized, fully collateralized stablecoin has ever been capable of achieving. As this work develops there are several natural extensions and new directions that could be examined, including investigating other forms collateral such as WBTC, and building $f(x)$-style systems that separate tokens into a wider variety of tokens.