Part I

Gasdynamics Review

This part of the book concerns gasdynamics. One might think that the term "gasdynamics" could refer to any sort of flow of any sort of gas. However, by tradition, unless specifically stated otherwise, the terms *gasdynamics* or *compressible flow* refers to a relatively simple type of gas flow, affected only by pressure and flux, neither too dense nor too rare. A more precise definition appears in Chapter 2.

The treatment of gasdynamics found in Part I varies from the traditional gasdynamics treatment in several ways, due mainly to the demands of the numerical approximations studied later in this book, as opposed to the demands of the simple hand calculations studied in traditional gasdynamics texts. For example, traditional gasdynamics texts consider linearized potential flow approximations such as the the Prandtl–Glauert equation; while many people still use linear approximations, modern computing power has made them increasingly unnecessary. This book will not discuss linearized approximations. For another example, traditional gasdynamics texts focus mainly on steady two-dimensional flows, whereas this book focuses mainly on unsteady one-dimensional flows. These two model problems are equally difficult: Both model problems involve the same number of dependent and independent variables; and many solutions to one problem have an analogous solution in the other problem such as, for example, the steady two-dimensional expansion fan versus the unsteady one-dimensional expansion fan. The steady two-dimensional model problem has one major positive aspect: Many gas flows of practical interest are approximately steady and two dimensional. Unfortunately, the steady two-dimensional model problem has at least two critical cons. First, the nature of the steady problem changes dramatically when the flow shifts from subsonic to supersonic or vice versa, requiring similarly dramatic changes in the solution procedure. In particular, the equations governing steady two-dimensional supersonic flows exhibit a wavelike or *hyperbolic* behavior, whereas the equations governing steady subsonic two-dimensional flows exhibit a polar opposite nonwavelike or *elliptic* behavior. Because of this, solving steady two-dimensional gas flows requires a great deal of trouble and expense around the sonic lines separating subsonic and supersonic flows. As the second major con, techniques for solving steady two-dimensional flows do not readily extend to steady three-dimensional flows, nor to unsteady flows in any number of dimensions.

By contrast, consider the model problem of unsteady one-dimensional flow. This model problem has one major con: Few gas flows of practical interest are truly one dimensional. However, the model problem of unsteady one-dimensional flow has several major pros. First, unsteady one-dimensional flows have wavelike hyperbolic behaviors regardless of flow speed. Second, techniques for solving unsteady one-dimensional flows lead naturally to techniques for solving unsteady two- and three-dimensional flows and, in fact, most numerical techniques for simulating unsteady multidimensional flows are heavily based on numerical techniques for simulating unsteady one-dimensional flows, as discussed in Chapter 24. Third, if one can simulate unsteady flows in any given dimension, then one can also simulate steady flows in the same number of dimensions simply by letting the time run out to large values, until the unsteadiness settles out. Taken on balance, unless you need immediately useful solutions for a limited class of simple problems, the steady one-dimensional model problem proves far more fruitful than the unsteady two-dimensional model problem. Indeed, the unsteady one-dimensional model problem appears nearly universally in the modern computational gasdynamics literature as part of the basic construction and explanation of numerical methods, while the steady two-dimensional model problem usually appears only in passing, as part of the verification and testing phase.

Having decided on the model problem of unsteady one-dimensional flow, let us discuss some of the particulars. The governing equations of unsteady one-dimensional flows follow from the application of simple commonsense notions of *conservation*, as discussed in Chapter 2. Conservation of mass says that mass is neither created nor destroyed; conservation of momentum is Newton's second law; and conservation of energy is the first law of thermodynamics. *Characteristic* theory, coupled with *shock wave* theory, describes the wavelike nature of unsteady one-dimensional flows, as seen in Chapter 3. Chapter 4 describes simple models of unsteady one-dimensional flows called *scalar conservation equations*. Chapter 5 concerns the *Riemann problem*, which has uniform initial conditions punctuated by a single jump discontinuity. These simple initial conditions allow for an exact solution exemplifying most of the principles studied in Part I. The solution to the Riemann problem has assumed a primary role in modern computational gasdynamics, not only as a simple exact solution to compare numerical approximations against, but also as a fundamental element of the numerical methods themselves.