Part II

Computational Review

The terms computational or numerical apply to almost all computer calculations found in science, mathematics, and engineering. However, over the years, the terms computational methods, numerical methods, and numerical analysis have come to refer to a standard repertoire of basic numerical techniques including root solvers, linear systems of equations, least squares fitting, Fourier series, differentiation, numerical quadrature, optimization, Runge–Kutta methods, and so forth. There is nothing magical about the standard numerical repertoire; it is simply a grab bag selection of material that has proven generally useful in modern science, mathematics, and engineering.

This second part of the text reviews computational methods. Of course, it would be neither appropriate nor possible, given space constraints, to cover the entire standard computational repertoire in a text devoted specifically to computational gasdynamics. Instead, this part of the book concerns only the standard topics of greatest relevance to the rest of the book. Speaking in broadbrush terms, Part II deals with polynomial and piecewisepolynomial approximations. More specifically, Chapter 6 concerns computer numbers and numerical errors, which allow one to measure the quality of polynomial and piecewisepolynomial approximations. Chapter 7 describes orthogonal functions and Fourier series, which allow one to find the best possible polynomial approximation to a given function. Chapter 8 concerns polynomial interpolation. Putting aside Chapter 9 for a moment, Chapter 10 covers numerical differentiation and finite-difference formulae, derived from polynomial and piecewise-polynomial interpolations; numerical integration and numerical quadrature, also derived from polynomial and piecewise-polynomial interpolations; and numerical solutions of ordinary differential equations including Runge-Kutta methods, derived from numerical differentiation and integration techniques. Beyond these standard topics, Chapter 9 describes something unusual: piecewise-polynomial essentially nonoscillatory (ENO) reconstruction and reconstruction via the primitive function, both of which are heavily based on polynomial interpolation. Although developed specifically for CFD, these techniques have wide applicability. Most classic reconstruction and interpolation procedures exhibit large spurious oscillation nears jumps, such as shocks, but ENO reconstructions handle discontinuous and nonsmooth functions with aplomb. Ordinary reconstructions and interpolations transform samples into functions. Reconstruction via the primitive function extends these techniques in order to transform integral averages into functions.