

Contents

<i>Preface</i>	xiii
Chapter 1 Introduction	1
Part I: Gasdynamics Review	3
Chapter 2 Governing Equations of Gasdynamics	5
2.0 Introduction	5
2.1 The Integral Form of the Euler Equations	5
2.1.1 Conservation of Mass	5
2.1.2 Conservation of Momentum	6
2.1.3 Conservation of Energy	7
2.1.4 Equations of State for a Perfect Gas	8
2.1.5 Entropy and the Second Law of Thermodynamics	10
2.1.6 Vector Notation	12
2.2 The Conservation Form of the Euler Equations	13
2.2.1 Vector and Vector–Matrix Notation	14
2.2.2 Rankine–Hugoniot Relations	15
2.3 The Primitive Variable Form of the Euler Equations	15
2.3.1 Vector–Matrix Notation	16
2.4 Other Forms of the Euler Equations	17
Chapter 3 Waves	21
3.0 Introduction	21
3.1 Waves for a Scalar Model Problem	22
3.2 Waves for a Vector Model Problem	26
3.3 The Characteristic Form of the Euler Equations	28
3.3.1 Examples	33
3.3.2 Physical Interpretation	36
3.4 Simple Waves	37
3.5 Expansion Waves	39
3.6 Compression Waves and Shock Waves	42
3.7 Contact Discontinuities	44
Chapter 4 Scalar Conservation Laws	48
4.0 Introduction	48
4.1 Integral Form	49

4.2	Conservation Form	49
4.3	Characteristic Form	50
4.4	Expansion Waves	51
4.5	Compression Waves and Shock Waves	51
4.6	Contact Discontinuities	52
4.7	Linear Advection Equation	52
4.8	Burgers' Equation	54
4.9	Nonconvex Scalar Conservation Laws	58
4.10	Entropy Conditions	61
4.11	Waveform Preservation, Destruction, and Creation	64
Chapter 5	The Riemann Problem	71
5.0	Introduction	71
5.1	The Riemann Problem for the Euler Equations	72
5.2	The Riemann Problem for Linear Systems of Equations	75
5.3	Three-Wave Linear Approximations – Roe's Approximate Riemann Solver for the Euler Equations	82
5.3.1	Secant Line and Secant Plane Approximations	82
5.3.2	Roe Averages	84
5.3.3	Algorithm	88
5.3.4	Performance	90
5.4	One-Wave Linear Approximations	94
5.5	Other Approximate Riemann Solvers	96
5.6	The Riemann Problem for Scalar Conservation Laws	96
Part II:	Computational Review	103
Chapter 6	Numerical Error	105
6.0	Introduction	105
6.1	Norms and Inner Products	105
6.2	Round-Off Error	108
6.3	Discretization Error	110
Chapter 7	Orthogonal Functions	117
7.0	Introduction	117
7.1	Functions as Vectors	118
7.2	Legendre Polynomial Series	120
7.3	Chebyshev Polynomial Series	123
7.4	Fourier Series	126
Chapter 8	Interpolation	132
8.0	Introduction	132
8.1	Polynomial Interpolation	133
8.1.1	Lagrange Form	134
8.1.2	Newton Form	135
8.1.3	Taylor Series Form	138

8.1.4	Accuracy of Polynomial Interpolation	142
8.1.5	Summary of Polynomial Interpolation	145
8.2	Trigonometric Interpolation and the Nyquist Sampling Theorem	145
Chapter 9	Piecewise-Polynomial Reconstruction	150
9.0	Introduction	150
9.1	Piecewise Interpolation-Polynomial Reconstructions	152
9.2	Averaged Interpolation-Polynomial Reconstructions	158
9.3	Reconstruction via the Primitive Function	161
9.4	Reconstructions with Subcell Resolution	167
Chapter 10	Numerical Calculus	172
10.0	Introduction	172
10.1	Numerical Differentiation	172
10.1.1	Linear Approximations	172
10.1.2	Quadratic Approximations	173
10.2	Numerical Integration	175
10.2.1	Constant Approximations	176
10.2.2	Linear Approximations	177
10.3	Runge–Kutta Methods for Solving Ordinary Differential Equations	178
Part III:	Basic Principles of Computational Gasdynamics	185
Chapter 11	Conservation and Other Basic Principles	187
11.0	Introduction	187
11.1	Conservative Finite-Volume Methods	187
11.1.1	Forward-Time Methods	193
11.1.2	Backward-Time Methods	196
11.1.3	Central-Time Methods	201
11.2	Conservative Finite-Difference Methods	203
11.2.1	The Method of Lines	205
11.2.2	Formal, Local, and Global Order of Accuracy	206
11.3	Transformation to Conservation Form	208
Chapter 12	The CFL Condition	214
12.0	Introduction	214
12.1	Scalar Conservation Laws	215
12.2	The Euler Equations	219
Chapter 13	Upwind and Adaptive Stencils	222
13.0	Introduction	222
13.1	Scalar Conservation Laws	223
13.2	The Euler Equations	228
13.3	Introduction to Flux Averaging	228
13.4	Introduction to Flux Splitting	231

13.4.1 Flux Split Form	234
13.4.2 Introduction to Flux Reconstruction	237
13.5 Introduction to Wave Speed Splitting	237
13.5.1 Wave Speed Split Form	239
13.6 Introduction to Reconstruction–Evolution Methods	243
Chapter 14 Artificial Viscosity	249
14.0 Introduction	249
14.1 Physical Viscosity	249
14.2 Artificial Viscosity Form	250
Chapter 15 Linear Stability	255
15.0 Introduction	255
15.1 von Neumann Analysis	257
15.2 Alternatives to von Neumann Analysis	264
15.3 Modified Equations	265
15.4 Convergence and Linear Stability	268
Chapter 16 Nonlinear Stability	272
16.0 Introduction	272
16.1 Monotonicity Preservation	276
16.2 Total Variation Diminishing (TVD)	277
16.3 Range Diminishing	282
16.4 Positivity	285
16.5 Upwind Range Condition	289
16.6 Total Variation Bounded (TVB)	292
16.7 Essentially Nonoscillatory (ENO)	293
16.8 Contraction	294
16.9 Monotone Methods	294
16.10 A Summary of Nonlinear Stability Conditions	295
16.11 Stability and Convergence	297
16.12 The Euler Equations	298
16.13 Proofs	299
Part IV: Basic Methods of Computational Gasdynamics	307
Chapter 17 Basic Numerical Methods for Scalar Conservation Laws	309
17.0 Introduction	309
17.1 Lax–Friedrichs Method	312
17.2 Lax–Wendroff Method	316
17.3 First-Order Upwind Methods	323
17.3.1 Godunov’s First-Order Upwind Method	325
17.3.2 Roe’s First-Order Upwind Method	329
17.3.3 Harten’s First-Order Upwind Method	332

17.4	Beam–Warming Second-Order Upwind Method	336
17.5	Fromm’s Method	343
Chapter 18	Basic Numerical Methods for the Euler Equations	351
18.0	Introduction	351
18.1	Flux Approach	355
18.1.1	Lax–Friedrichs Method	355
18.1.2	Lax–Wendroff Methods	355
18.2	Wave Approach I: Flux Vector Splitting	362
18.2.1	Steger–Warming Flux Vector Splitting	373
18.2.2	Van Leer Flux Vector Splitting	374
18.2.3	Liou–Steffen Flux Vector Splitting	378
18.2.4	Zha–Bilgen Flux Vector Splitting	379
18.2.5	First-Order Upwind Methods	379
18.2.6	Beam–Warming Second-Order Upwind Method	381
18.3	Wave Approach II: Reconstruction–Evolution	406
18.3.1	Godunov’s First-Order Upwind Method	410
18.3.2	Roe’s First-Order Upwind Method	410
18.3.3	Harten’s First-Order Upwind Method	417
18.3.4	First-Order Upwind Method Based on One-Wave Solver	417
18.3.5	Second- and Higher-Order Accurate Methods	420
Chapter 19	Boundary Treatments	430
19.0	Introduction	430
19.1	Stability	433
19.2	Solid Boundaries	434
19.3	Far-Field Boundaries	441
Part V:	Advanced Methods of Computational Gasdynamics	455
Chapter 20	Flux Averaging I: Flux-Limited Methods	459
20.0	Introduction	459
20.1	Van Leer’s Flux-Limited Method	463
20.2	Sweby’s Flux-Limited Method (TVD)	466
20.2.1	The Linear Advection Equation with $a > 0$	466
20.2.2	The Linear Advection Equation with $a < 0$	472
20.2.3	Nonlinear Scalar Conservation Laws with $a(u) > 0$	473
20.2.4	Nonlinear Scalar Conservation Laws with $a(u) < 0$	474
20.2.5	Nonlinear Scalar Conservation Laws at Sonic Points	475
20.2.6	The Euler Equations	482
20.3	Chakravarthy–Osher Flux-Limited Methods (TVD)	482
20.3.1	A Second-Order Accurate Method: A Semidiscrete Version of Sweby’s Flux-Limited Method	485
20.3.2	Another Second-Order Accurate Method	487

20.3.3	Second- and Third-Order Accurate Methods	488
20.3.4	Higher-Order Accurate Methods	490
20.4	Davis–Roe Flux-Limited Method (TVD)	491
20.4.1	Scalar Conservation Laws	491
20.4.2	The Euler Equations	494
20.5	Yee–Roe Flux-Limited Method (TVD)	495
20.5.1	Scalar Conservation Laws	495
20.5.2	The Euler Equations	498
Chapter 21	Flux Averaging II: Flux-Corrected Methods	504
21.0	Introduction	504
21.1	Boris–Book Flux-Corrected Method (FCT)	506
21.2	Zalesak’s Flux-Corrected Methods (FCT)	515
21.3	Harten’s Flux-Corrected Method (TVD)	517
21.3.1	Scalar Conservation Laws	517
21.3.2	The Euler Equations	521
21.4	Shu–Osher Methods (ENO)	523
Chapter 22	Flux Averaging III: Self-Adjusting Hybrid Methods	541
22.0	Introduction	541
22.1	Harten–Zwas Self-Adjusting Hybrid Method	542
22.2	Harten’s Self-Adjusting Hybrid Method	544
22.3	Jameson’s Self-Adjusting Hybrid Method	550
22.3.1	Scalar Conservation Laws	550
22.3.2	The Euler Equations	558
Chapter 23	Solution Averaging: Reconstruction–Evolution Methods	565
23.0	Introduction	565
23.1	Van Leer’s Reconstruction–Evolution Method (MUSCL)	565
23.1.1	Linear Advection Equation	565
23.1.2	The Lagrange Equations	572
23.2	Colella–Woodward Reconstruction–Evolution Method (PPM)	574
23.3	Anderson–Thomas–Van Leer Reconstruction–Evolution Methods (TVD/MUSCL): Finite-Volume Versions of the Chakravarthy–Osher Flux-Corrected Methods	577
23.3.1	A Second-Order Accurate Method	578
23.3.2	Second- and Third-Order Accurate Methods	579
23.4	Harten–Osher Reconstruction–Evolution Method (UNO)	582
23.4.1	The Linear Advection Equation	582
23.4.2	Nonlinear Scalar Conservation Laws	583
23.4.3	The Euler Equations	585
23.5	Harten–Engquist–Osher–Chakravarthy Reconstruction–Evolution Methods (ENO)	585
23.5.1	Second-Order Accurate Temporal Evolution for Scalar Conservation Laws	587

23.5.2 Third-Order Accurate Temporal Evolution for Scalar Conservation Laws	589
23.5.3 Second-Order Accurate Temporal Evolution for the Euler Equations	591
Chapter 24 A Brief Introduction to Multidimensions	597
24.0 Introduction	597
24.1 Governing Equations	597
24.2 Waves	599
24.3 Conservation and Other Numerical Principles	602
<i>Index</i>	605

