Área entre Curvas.

1. $y^2 = 2x$, $y = x - 4$

término y.

$\dfrac{y^2}{2} = x$, $y + 4 = x$

límites.

$\dfrac{y^2}{2} = y + 4$

$\dfrac{y^2}{2} - y - 4 = 0$ (2)
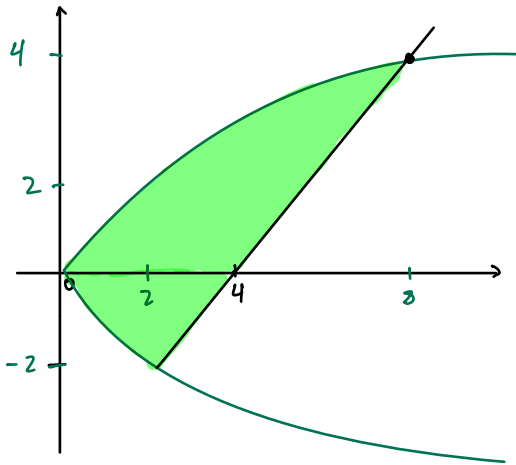
$y^2 - 2y - 8 = 0$

$(y - 4)(y + 2) = 0$

$y = 4$, $y = -2$

Área:

$A = \displaystyle\int_{-2}^{4} (y + 4) - \left(\dfrac{y^2}{2}\right)\, dy$

$A = \displaystyle\int_{-2}^{4} y + 4 - \dfrac{y^2}{2}\, dy$

$A = \left[\dfrac{y^2}{2} + 4y - \dfrac{y^3}{6} \right]\Big|_{-2}^{4} = \dfrac{4^2}{2} + 4(4) - \dfrac{4^3}{6} - \left(\dfrac{(-2)^2}{2} + 4(-2) - \dfrac{(-2)^3}{6} \right)$
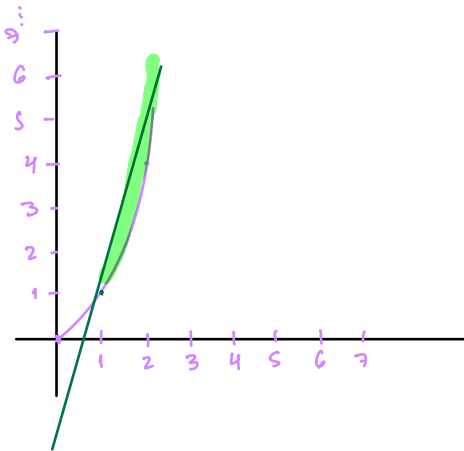
$= 18$

2. $y = x$, $y = x^2$
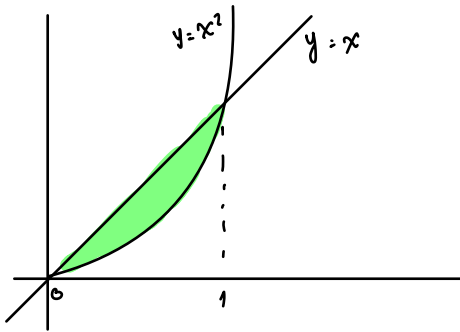
$x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$, $x = 1$

$A = \int_0^1 x - x^2 \, dx$

$= \dfrac{x^2}{2} - \dfrac{x^3}{3} \Big/_0^1 = \dfrac{1}{2} - \dfrac{1}{3} = \dfrac{1}{6}$

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3. $y = 4x - 3$, $y = x^2$

fórum y.

$y = 4x - 3 \qquad\qquad \sqrt{y} = x$

$\dfrac{y+3}{4} = x$

$\dfrac{y}{4} + \dfrac{3}{4} = \sqrt{y}$

$\dfrac{y}{4} - \sqrt{y} + \dfrac{3}{4} = 0 \qquad (4)$

$y - 4\sqrt{y} + 3 = 0 \qquad\qquad U = \sqrt{y}$

$U^2 - 4U + 3 = 0$

$(U - 3)(U - 1) = 0$

$U - 3 = 0 \qquad , \qquad U - 1 = 0$

$\sqrt{y} - 3 = 0 \qquad\qquad \sqrt{y} - 1 = 0$

$\sqrt{y} = 3 \qquad\qquad \sqrt{y} = 1$
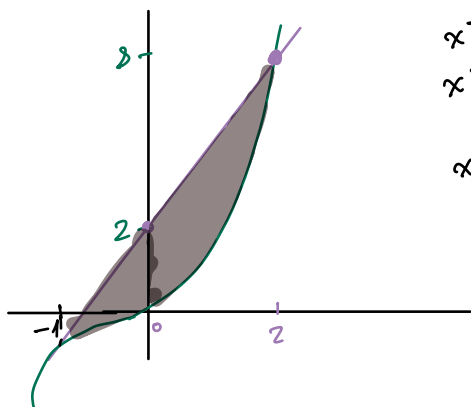
$y = 3^2 = 9 \qquad\qquad y = 1$

$A = \int_1^9 \left(\sqrt{y} - (y/4 + 3/4) \right) dy$

$A = \dfrac{2}{3} y^{3/2} - \dfrac{y^2}{8} - \dfrac{3}{4} y \Big/_1^9$

$A = \dfrac{2}{3}(9)^{3/2} - \dfrac{(9)^2}{8} - \dfrac{3}{4}(9) - \left(\dfrac{2}{3}(1)^{3/2} - \dfrac{(1)^2}{8} - \dfrac{3}{4}(1) \right)$

$A = 4/3$

4. $y = x^3$, $y = 3x + 2$
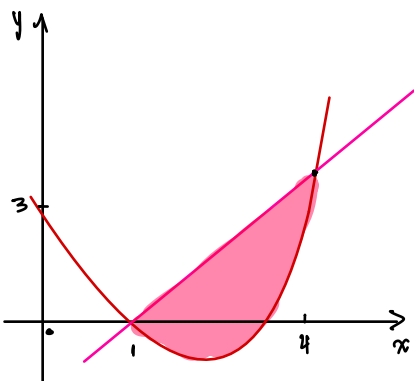

$x^3 = 3x + 2$

$x^3 - 3x + 2 = 0$

$x = 2$, $x = -1$

$$A = \int_{-1}^{2} \left| \left(x^3 - (3x+2) \right) \right| dx$$

$$A = \left| \left[\frac{x^4}{4} - \frac{3x^2}{2} - 2x \right] \Big/ \Big|_{-1}^{2} \right| = \frac{2^4}{4} - \frac{3}{2}(2)^2 - 2(2) - \left[\frac{(-1)^4}{4} - \frac{3}{2}(-1)^2 - 2(-1) \right]$$

$$= \left| -\frac{27}{4} \right| = \frac{27}{4}$$
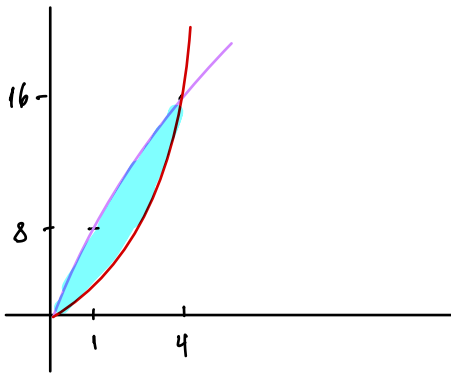
5. $y = x^2 - 4x + 3$, $x - y - 1 = 0$

$$y = x - 1$$

$$A = \int_{1}^{4} \left(x - 1 - (x^2 - 4x + 3) \right) dx$$

$$A = \int_{1}^{4} \left(x - 1 - x^2 + 4x - 3 \right) dx$$

$$A = \left[\frac{x^2}{2} - x - \frac{x^3}{3} + \frac{4x^2}{2} - 3x \right] \Big/ \Big|_{1}^{4}$$

$$A = \frac{(4)^2}{2} - (4) - \frac{(4)^3}{3} + 2(4)^2 - 3(4) - \left[\frac{1}{2} - 1 - \frac{1}{3} + 2 - 3 \right] = \frac{9}{2}$$

6. $y = x^2$, $y^2 = 64x \rightsquigarrow y = 8\sqrt{x}$

$$x^2 = 8\sqrt{x}$$

$$x^2 - 8\sqrt{x} = 0$$

$$x^2\left(1 - \frac{8}{x^{3/2}}\right) = 0$$

$x^2 = 0 \qquad , \qquad 1 - 8x^{-3/2} = 0$

$x = 0 \qquad\qquad\quad -8x^{-3/2} = -1$

$$x^{-3/2} = \frac{1}{8}$$

$$\frac{1}{x^{3/2}} = \frac{1}{8}$$
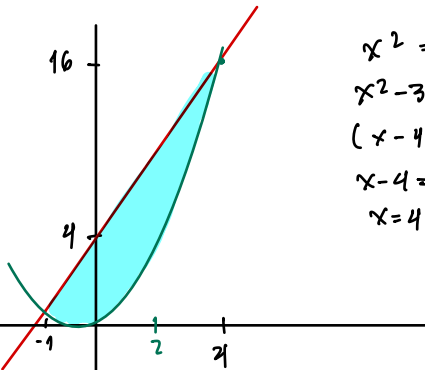
$$x^{3/2} = 8$$

$$x = 8^{2/3} = 4$$

$$A = \int_0^4 \left(8\sqrt{x} - x^2\right) dx$$

$$= 8 \cdot \frac{2}{3} x^{3/2} - \frac{x^3}{3} \Big|_0^4$$

$$= \frac{16}{3}(4)^{3/2} - \frac{(4)^3}{3} - \left(0\right)$$

$$= \frac{64}{3}$$

7. $y = x^2$, $3x - y + 4 = 0 \rightsquigarrow 3x + 4 = y$

$x^2 = 3x + 4$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x - 4 = 0 \qquad x + 1 = 0$
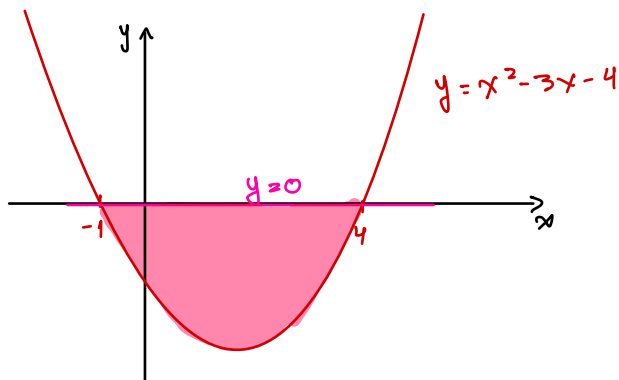
$x = 4 \qquad\quad x = -1$

$$A = \int_{-1}^4 \left(3x + 4 - (x^2)\right) dx$$

$$A = \left[\frac{3}{2}x^2 + 4x - \frac{x^3}{3}\right]\Big|_{-1}^4$$

$$A = \frac{3}{2}(4)^2 + 4(4) - \frac{(4)^3}{3} - \left(\frac{3}{2}(-1)^2 + 4(-1) - \frac{(-1)^3}{3}\right)$$
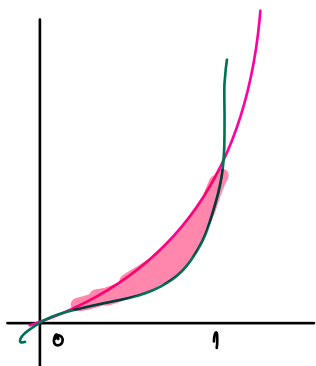
$$A = 125/6$$

8. $y = x^2 - 3x - 4$, $y = 0$

$y = x^2 - 3x - 4$

$y = 0$

-1   4

$A = \int_{-1}^{4} \left(-(x^2 - 3x - 4) \right) dx$

$= \int_{-1}^{4} \left(-x^2 + 3x + 4 \right) dx$

$= \left[-\frac{x^3}{3} + \frac{3}{2}x^2 + 4x \right] \Big|_{-1}^{4} = -\frac{(4)^3}{3} + \frac{3}{2}(4)^2 + 4(4) - \left(-\frac{(-1)^3}{2} + \frac{3}{2}(-1)^2 + 4(-1) \right)$

$= \frac{125}{6}$

9   $y = x^2$, $y = x^3$

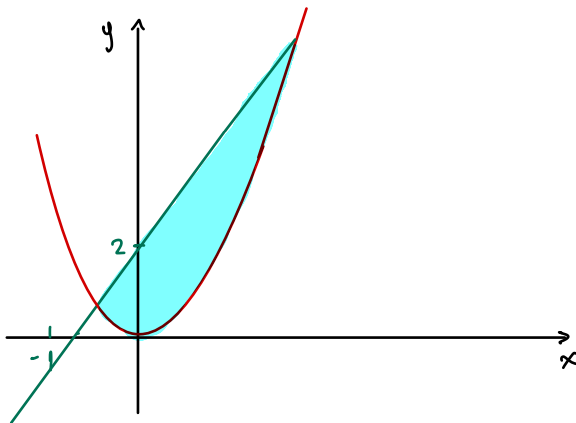0   1

$x^2 = x^3$

$x^3 - x^2 = 0$

$x^2 (x - 1) = 0$

$x^2 = 0 , \quad x - 1 = 0$

$x = 0 \qquad x = 1$

$A = \int_{0}^{1} \left(x^2 - x^3 \right) dx$

$A = \frac{x^3}{3} - \frac{x^4}{4} \Big|_{0}^{1} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

10. $y = 3x + 2$, $y = x^2$

$x^2 = 3x + 2$

$x^2 - 3x - 2 = 0$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2}$

$x_1 = \dfrac{3 + \sqrt{17}}{2}$

$x_2 = \dfrac{3 - \sqrt{17}}{2}$

$A = \displaystyle\int_{\frac{3-\sqrt{17}}{2}}^{\frac{3+\sqrt{17}}{2}} \left(3x + 2 - x^2\right) dx = \dfrac{3x^2}{2} + 2x - \dfrac{x^3}{3} \Big|_{\frac{3-\sqrt{17}}{2}}^{\frac{3+\sqrt{17}}{2}}$

$= \dfrac{3}{2}\left(\dfrac{3+\sqrt{17}}{2}\right)^2 + 2\left(\dfrac{3+\sqrt{17}}{2}\right) - \dfrac{\left(\frac{3+\sqrt{17}}{2}\right)^3}{3} - \left[\dfrac{3}{2}\left(\dfrac{3-\sqrt{17}}{2}\right)^2 + 2\left(\dfrac{3-\sqrt{17}}{2}\right) - \dfrac{\left(\frac{3-\sqrt{17}}{2}\right)^3}{3}\right]$

$= 11,68$