01 - límites por definición.

$$\lim_{x \to x_0} f(x) = L \longleftrightarrow \forall \varepsilon > 0. \ \exists \delta > 0 : 0 < |x - x_0| < \delta \to |f(x) - L| < \varepsilon$$

1. $\lim_{x \to 3} (2x - 1) = 5$

entonces:

$$|2x - 1 - 5| < \varepsilon$$

$$|2x - 6| < \varepsilon$$

$$|2(x-3)| < \varepsilon$$

$$2|x - 3| < \varepsilon \quad / \ 1/2$$

$$|x - 3| < \varepsilon/2$$

Por lo tanto:

$$0 < |x - 3| < \delta$$

$$\Rightarrow |x - 3| < \delta \ /(2)$$

$$2|x - 3| < 2\delta$$

$$|2(x-3)| < 2\delta$$

$$|2x - 6| < 2\,\varepsilon/2$$

$$\Rightarrow |2x - 1 - 5| < \varepsilon$$

$$|(2x-1) - 5| < \varepsilon$$

$$\Rightarrow |f(x) - L| < \varepsilon$$

$$\therefore \lim_{x \to 3} 2x - 1 = 5$$

2. $\lim_{x \to 2} (x^2 + 3x + 2) = 12$

$\varepsilon > 0$, $\delta > 0$ $\quad 0 < |x - 2| < \delta \Rightarrow |(x^2 + 3x + 2) - 12| < \varepsilon$

$|x^2 + 3x + 2 - 12| < \varepsilon$

$|x^2 + 3x - 10| < \varepsilon$

$|(x+5)(x-2)| < \varepsilon$

$|x+5||x-2| < \varepsilon$

Acotar el primer factor.

$(1, 3)$ $\quad \delta < 1$ $\quad 0 < |x - 2| < \delta$, $\delta = 1$

$x_0 = 2$

$< |x-2| < 1$

$\hookrightarrow -1 < x - 2 < 1 \quad / + 7$

$6 < x + 5 < 8$

$\longrightarrow \quad x + 5 < 8 \longrightarrow |x+5| < 8$

entonces.

$|x+5||x-2| < \varepsilon$

$8|x-2| < \varepsilon \quad / (1/8)$

$|x-2| < \varepsilon/8$

$\delta = \min(1, \varepsilon/8)$

Implica que:

$$0 < |x-2| < \delta$$

$$\Rightarrow \quad |x-2| < \delta \quad / \ |x+5|$$

$$|x-2|\,|x+5| < |x+5|\,\delta$$

$$|(x-2)(x+5)| < |x+5|\,\delta$$

$$|x^2+3x-10| < |x+5|\,\delta$$

$$|x^2+3x-10| < 8\delta$$

$$\Rightarrow \quad |(x^2+3x+2)-12| < 8\,\varepsilon/8$$

$$|(x^2+3x+2)-12| < \varepsilon$$

$$|f(x)-L| < \varepsilon$$

$$\therefore \quad \lim_{x\to 2}(x^2+3x+2) = 12$$