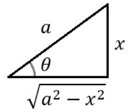
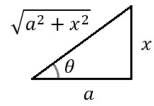
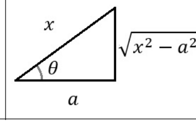
02 - Sustitución Trigonométrica.

<table>
<tr><td>$\sqrt{a^2 - x^2}$</td><td>$\sqrt{a^2 + x^2}$</td><td>$\sqrt{x^2 - a^2}$</td></tr>
<tr><td>$x = a \operatorname{sen} \theta$</td><td>$x = a \tan \theta$</td><td>$x = a \sec \theta$</td></tr>
<tr><td>$dx = a \cos \theta \, d\theta$</td><td>$dx = a \sec^2 \theta \, d\theta$</td><td>$dx = a \sec \theta \tan \theta \, d\theta$</td></tr>
<tr><td>$\sqrt{a^2 - x^2} = a \cos \theta$</td><td>$\sqrt{a^2 + x^2} = a \sec \theta$</td><td>$\sqrt{x^2 - a^2} = a \tan \theta$</td></tr>
<tr><td></td><td></td><td></td></tr>
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1. $\displaystyle\int \sqrt{4 - x^2}\, dx$ Sust:

$x = 2\operatorname{Sen}\theta \qquad \sqrt{4 - x^2} = 2\operatorname{Cos}\theta$

$dx = 2\operatorname{Cos}\theta\, d\theta$

$$\Rightarrow \int 2\operatorname{Sec}\theta \cdot 2\operatorname{Cos}\theta\, d\theta$$

$$4\int \operatorname{Cos}\theta \operatorname{Cos}\theta\, d\theta$$

$$4\int \operatorname{Cos}^2\theta\, d\theta = 4\int \frac{1 + \operatorname{Cos}2\theta}{2}\, d\theta = 4\int \frac{1}{2} + \frac{\operatorname{Cos}2\theta}{2}\, d\theta$$

$$4\left[\frac{1}{2}\theta + \frac{\operatorname{Sen}2\theta}{4}\right] + C$$

$$2\theta + 2\operatorname{Sen}\theta\operatorname{Cos}\theta + C \Rightarrow$$

$$\operatorname{Sen}\theta = \frac{x}{2}$$

$$\operatorname{Cos}\theta = \frac{\sqrt{4 - x^2}}{2}$$

$$= 2\operatorname{arcSen}\left(\frac{x}{2}\right) + 2 \cdot \frac{x}{2} \cdot \frac{\sqrt{4 - x^2}}{2} + C$$

$$= 2\operatorname{arcSen}\left(\frac{x}{2}\right) + \frac{x\sqrt{4 - x^2}}{2} + C$$

2.

$$\int \frac{dx}{(x^2+16)^2} \qquad x = 4\tan\theta \rightsquigarrow \quad x^2+16 = 16\tan^2\theta + 16$$
$$= 16(\sec^2\theta)$$
$$dx = 4\sec^2\theta \, d\theta$$

$$4\int \frac{\sec^2\theta \, d\theta}{(16\sec^2\theta)^2} \qquad \frac{x}{4} = \tan\theta \rightsquigarrow \quad \theta = \tan^{-1}\left(\frac{x}{4}\right)$$

$$\frac{4}{256}\int \frac{\sec^2\theta}{\sec^4\theta} \, d\theta$$

$$\frac{1}{64}\int \frac{1}{\sec^2\theta} \, d\theta$$

$$\frac{1}{64}\int \cos^2\theta \, d\theta = \frac{1}{64}\int \frac{1}{2} + \frac{\cos(2\theta)}{2} \, d\theta$$

$$= \frac{1}{128}\int 1 + \cos(2\theta) \, d\theta$$

$$= \frac{1}{128}\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$$

$$= \frac{1}{128}\left[\tan^{-1}\left(\frac{x}{4}\right) + \sin(\theta)\cos(\theta)\right] + C$$

$$= \frac{1}{128}\left[\tan^{-1}\left(\frac{x}{4}\right) + \frac{x}{\sqrt{x^2+16}} \cdot \frac{4}{\sqrt{x^2+16}}\right] + C$$