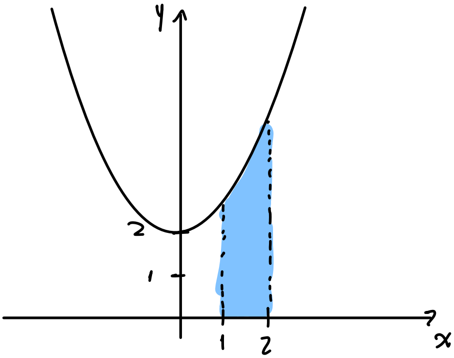
Section 14.9

9. $y = x^2 + 2 \; ; \quad x = 1 \, , \; x = 3$



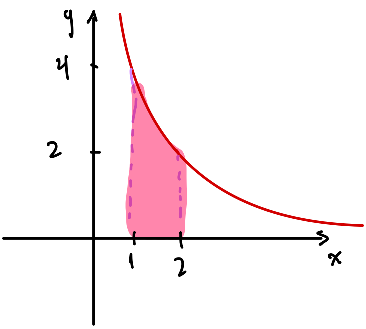
$$A = \int_1^3 (x^2+2)\,dx$$

$$= \left[\frac{x^3}{3} + 2x\right]\Big|_1^3 = \frac{3^3}{3} + 2(3) - \left(\frac{1}{3} + 2\right)$$

$$= \frac{27}{3} + 6 - \left(\frac{7}{3}\right)$$

$$= \frac{38}{3}$$

14. $y = \frac{4}{x} \; ; \quad x = 1 \, , \; x = 2$

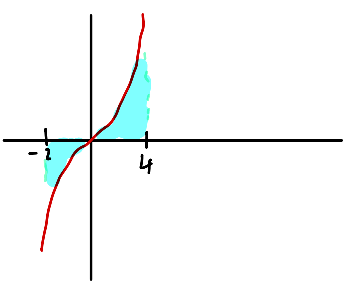


$$A = \int_1^2 \frac{4}{x}\,dx = 4\ln|x|\Big|_1^2 = 4\left(\ln(2) - \ln(1)\right)$$

$$= 4\ln(2)$$

$$\approx 2.77$$

31. $y = x^3 \, , \quad x = -2 \, , \quad x = 4$



$$A = \int_{-2}^{0} |x^3|\,dx + \int_0^4 x^3\,dx$$

$$A = \left|\frac{x^4}{4}\right|\Big|_{-2}^{0} + \frac{x^4}{4}\Big|_0^4$$

$$= \left| 0 - \left(\frac{(-2)^4}{4} \right) \right| + \frac{4^4}{4} - 0$$

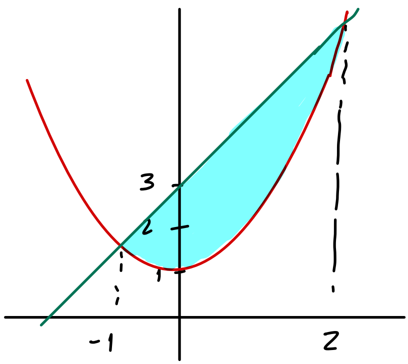
$$= |-4| + \frac{256}{4}$$

$$= 4 + 64 = 68$$

Sección 14.10

12, 13, 14, 22, 26, 28

12. $y = x^2 + 1 , \quad y = x + 3$



$x^2 + 1 = x + 3$

$x^2 - x + 1 - 3 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

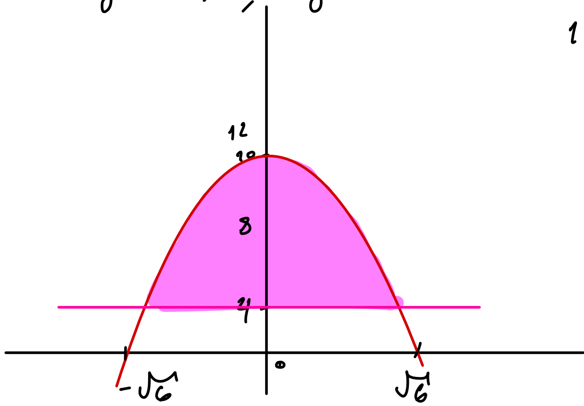
$x = 2 , \quad x = -1$

$$A = \int_{-1}^{2} (x + 3 - (x^2 + 1))\, dx$$

$$\frac{x^2}{2} + 3 - \frac{x^3}{3} - x \Big|_{-1}^{2} = \frac{2^2}{2} + 3 - \frac{(2)^3}{3} - 2 - \left(\frac{(-1)^2}{2} + 3 - \frac{(-1)^3}{3} - (-1) \right)$$

$$= \frac{4}{2} + 3 - \frac{8}{3} - 2 - \frac{1}{2} - 3 - \frac{1}{3} - 1 = \frac{9}{2}$$

13. $y = 10 - x^2, \quad y = 4$



$$10 - x^2 = 4$$
$$10 - x^2 - 4 = 0$$
$$-x^2 + 6 = 0 \quad (-1)$$
$$x^2 - 6 = 0$$
$$x^2 = 6$$
$$x = \pm\sqrt{6}$$

$$A = \int_{-\sqrt{6}}^{\sqrt{6}} (10 - x^2 - 4)\, dx = \int_{-\sqrt{6}}^{\sqrt{6}} (6 - x^2)\, dx = \left[6x - \frac{x^3}{3} \right]\Big|_{-\sqrt{6}}^{\sqrt{6}}$$

$$= 6(\sqrt{6}) - \frac{(\sqrt{6})^3}{3} - \left[-6(\sqrt{6}) - \frac{(-\sqrt{6})^3}{3} \right]$$

$$= 6\sqrt{6} - \frac{6^{3/2}}{3} + 6\sqrt{6} - \frac{6\sqrt{6}}{3}$$

$$= 6\sqrt{6} - \frac{6\sqrt{6}}{3} + 6\sqrt{6} - \frac{6\sqrt{6}}{3}$$

$$= 12\sqrt{6} - \frac{6\sqrt{6}}{3} - \frac{6}{3}\sqrt{6}$$

$$= 12\sqrt{6} - 4\sqrt{6}$$

$$= 8\sqrt{6}$$

14. $y^2 = x+1$, $x = 1$ Calcular por el eje y.

$y = \sqrt{x+1}$, $x = 1$

$y^2 = x + 1$; $x = 1$

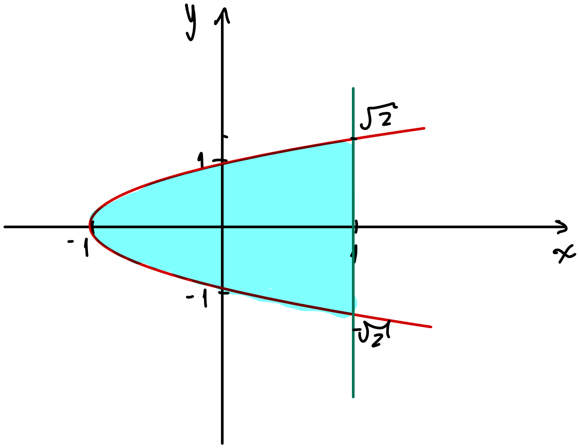
$y^2 - 1 = x$; $x = 1$

$y^2 - 1 = 1$

$y^2 - 1 - 1 = 0$

$y^2 - 2 = 0$

$y = \pm\sqrt{2}$



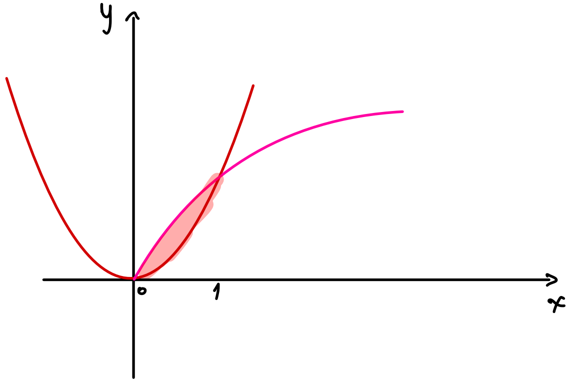
$$A = \int_{-\sqrt{2}}^{\sqrt{2}} \left| 1 - (y^2 - 1) \right| dy = \int_{-\sqrt{2}}^{\sqrt{2}} \left| 2 - y^2 \right| dy = 2y - \frac{y^3}{3} \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= 2\sqrt{2} - \frac{2\sqrt{2}}{3} - \left[-2\sqrt{2} + \frac{2\sqrt{2}}{3} \right]$$

$$= 2\sqrt{2} - \frac{2}{3}\sqrt{2} + 2\sqrt{2} - \frac{2}{3}\sqrt{2}$$

$$= 4\sqrt{2} - \frac{4}{3}\sqrt{2} = \frac{8}{3}\sqrt{2} \approx 3{,}77$$

22. $y = \sqrt{x}$, $y = x^2$



$\sqrt{x} = x^2$

$x^2 - \sqrt{x} = 0$

$x^2 - x^{1/2} = 0$

$x^{1/2}\left(x^{3/2} - 1\right) = 0$

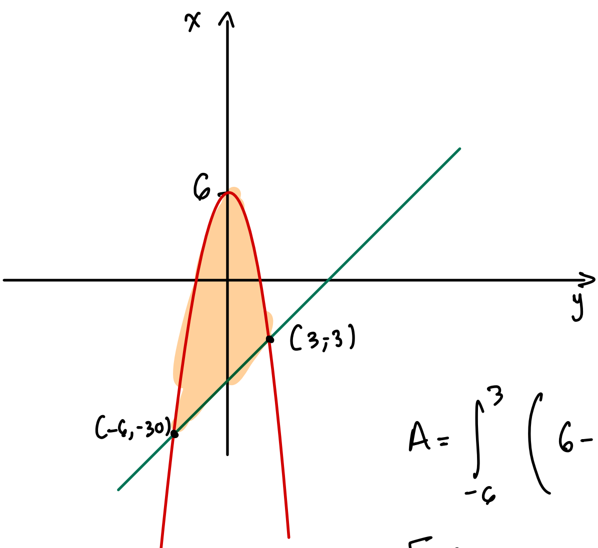
$x = 0$, $x = 1$

$$A = \int_0^1 \left(\sqrt{x} - x^2\right) dx = \frac{2}{3}x^{3/2} - \frac{x^3}{3} \Big|_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

26. $y^2 = 6 - x$, $3y = x + 12$

$y^2 - 6 = -x$ $\qquad$ $3y - 12 = x$

$x = 6 - y^2$



$3y - 12 = 6 - y^2$

$y^2 + 3y - 12 - 6 = 0$

$y^2 + 3y - 18 = 0$

$(y+6)(y-3) = 0$

$y = -6$, $y = 3$

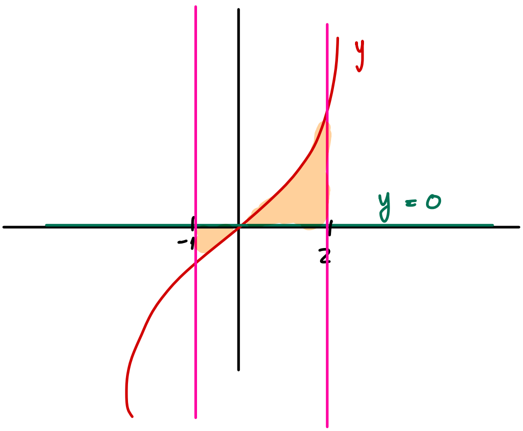
$$A = \int_{-6}^{3} \left(6 - y^2 - (3y - 12)\right) dy$$

$$A = \left[6y - \frac{y^3}{3} - \frac{3y^2}{2} + 12y\right]\Big|_{-6}^{3}$$

$$A = 6(3) - \frac{(3)^3}{3} - \frac{3}{2}(3)^2 + 12(3) - \left[6(-6) - \frac{(-6)^3}{3} - \frac{3}{2}(-6)^2 + 12(-6)\right]$$

$$A = 121.5$$

28. $y = x^3 + x \quad ; \quad y = 0 \quad ; \quad x = -1, \; x = 2$



$$A = \int_{-1}^{0} |x^3 + x|\, dx + \int_{0}^{2} x^3 + x\, dx$$

$$\left|\frac{x^4}{4} + \frac{x^2}{2}\right| \Big/_{-1}^{0} + \left[\frac{x^4}{4} + \frac{x^2}{2}\right] \Big/_{0}^{2}$$

$$\frac{(-1)^4}{4} + \frac{(-1)^2}{2} + \frac{2^4}{4} + \frac{2^2}{2}$$

$$= \frac{1}{4} + \frac{1}{2} + 4 + 2 = \frac{27}{4}$$