01 - Integración por Partes. Y Sust.

1. $\int \cos(\ln(x))\,dx \Rightarrow \int \cos(u) e^u du$

$u = \ln(x)$

$e^u = e^{\ln(x)}$

$e^u = x$

$du e^u = dx$

$= \int e^u \cos(u) du$

$= \int \cos(u) de^u$

$= \cos(u) e^u + \int e^u \operatorname{Sen}(u) du$

$= e^u \cos(u) + \int \operatorname{Sen}(u) de^u$

$\int e^u \cos u\, du = e^u \cos(u) + e^u \operatorname{Sen}(u) - \int e^u \cdot \cos(u) du$

$2\int e^u \cos u\, du = e^u \cos(u) + e^u \operatorname{Sen}(u)$

$\int e^u \cos u\, du = \dfrac{e^u \cos(u) + e^u \operatorname{Sen}(u)}{2}$

$= \dfrac{e^{\ln(x)} \cos(\ln(x)) + e^{\ln(x)} \operatorname{Sen}(\ln(x))}{2}$

2. $\int \sqrt{e^x - 1}\, dx$

$$\int \frac{e^x}{e^x} \sqrt{e^x - 1}\, dx$$

$$\int \frac{\sqrt{e^x - 1}}{e^x}\, de^x \qquad u = e^x$$

$$\int \frac{\sqrt{u - 1}}{u}\, du \qquad t^2 = u - 1 \rightsquigarrow t^2 + 1 = u$$

$$2t\,dt = du$$

$$2 \int \frac{t^2 + 1 - 1}{t^2 + 1}\, dt$$

$$2 \int 1 - \frac{1}{t^2 + 1}\, dt = 2\left[t - \arctan(t) \right] + C$$

$$= 2\left[\sqrt{e^x - 1} - \arctan\left(\sqrt{e^x - 1}\right) \right] + C$$

3.

$$\int x \arctan\left(\sqrt{x^2-1}\right)dx$$

$$\frac{1}{2}\int \arctan\left(\sqrt{x^2-1}\right)d[x^2-1]$$

$$\frac{1}{2}\int \arctan\left(\sqrt{u}\right)du$$

Obs: $\left(\arctan\left(\sqrt{u}\right)\right)' = \dfrac{1}{2\sqrt{u}\,(u+1)}$

$$\frac{1}{2}\left[u\arctan\left(\sqrt{u}\right) - \int u \cdot \frac{1}{2\sqrt{u}\,(u+1)}du\right]$$

$$\frac{1}{2}\left[u\arctan\left(\sqrt{u}\right) - \frac{1}{2}\int \frac{\sqrt{u}}{u+1}du\right]$$

Obs: $\displaystyle\int \frac{\sqrt{u}}{u+1}du$

$$\int \frac{\sqrt{u}}{\sqrt{u}} \cdot \frac{\sqrt{u}}{u+1}du$$

$$2\int \frac{u+1-1}{u+1}d\sqrt{u}$$

$$2\int 1 - \frac{1}{u+1}d\sqrt{u}$$

$$2\left(\sqrt{u} - \arctan\left(\sqrt{u}\right)\right)+C$$

$$\frac{1}{2}\left[u.\arctan\left(\sqrt{u}\right) - \frac{1}{2}.2\left(\sqrt{u} - \arctan\left(\sqrt{u}\right)\right)\right]+C$$

$$\frac{1}{2}\left[u.\arctan\left(\sqrt{u}\right) - \sqrt{u} + \arctan\left(\sqrt{u}\right)\right]+C$$

$$\frac{1}{2}\left[(x^2-1)\arctan\left(\sqrt{x^2-1}\right) - \sqrt{x^2-1} + \arctan\left(\sqrt{x^2-1}\right)\right]+C$$

4. $\displaystyle\int (x - x\ln|x|)^{-1}dx$

$$\int \frac{1}{x - x\ln|x|}dx = \int \frac{1}{x}\,\frac{1}{(1-\ln|x|)}dx$$

$$\int \frac{d\ln|x|}{1-\ln|x|} = -\ln|1-\ln|x|| + C$$