3. Por partes:

$$\int x^4 \ln|x|\, dx = \frac{x^3}{3}\ln|x| - \int \frac{x^5}{5}\cdot\frac{1}{x}\, dx$$

$u = \ln|x|$ $\quad dv = x^4\, dx$

$du = \frac{1}{x}\, dx$ $\quad \int dv = \int x^4\, dx$

$v = \frac{x^5}{5}$

$$\frac{x^5}{5}\ln|x| - \frac{1}{5}\int x^4\, dx$$

$$\frac{x^5}{5}\ln|x| - \frac{1}{5}\frac{x^5}{5} + C$$

$$= \frac{x^5}{5}\ln|x| - \frac{1}{25}x^5 + C$$

$$\cdot \int \frac{\ln(x)}{x^2}\, dx$$

$u = \ln|x|$ $\quad dv = x^{-2}\, dx$ $\quad \Rightarrow -\frac{\ln(x)}{x} + \int \frac{1}{x}\cdot\frac{1}{x}\, dx$

$du = \frac{1}{x}\cdot dx$ $\quad \int dv = \int x^{-2}\, dx$

$$= -\frac{\ln(x)}{x} + \int \frac{1}{x^2}\, dx$$

$v = -\frac{1}{x}$

$$= -\frac{\ln(x)}{x} - \frac{1}{x} + C$$

$$\int x^5 e^{-2x}\, dx$$

$u = x^5 \qquad dv = e^{-2x}\, dx$

$du = 5x^4\, dx \qquad \int dv = \int e^{-2x}\, dx$

$v = -\frac{1}{2} e^{-2x} + C$

Obs: $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$

$$-\frac{x^5}{2} e^{-2x} + \frac{1}{2}\int e^{-2x} \cdot 5x^4\, dx$$

$$-\frac{x^5}{2} e^{-2x} + \frac{2}{5}\int x^4 e^{-2x}\, dx$$

$u = x^4 \qquad dv = e^{-2x} dx$

$du = 4x^3\, dx$

$\int dv = \int e^{-2x}\, dx$

$v = -\frac{1}{2} e^{-2x} + C$

$$-\frac{x^5}{2} e^{-2x} + \frac{2}{5}\left[-\frac{x^4}{2} e^{-2x} + \frac{4}{2}\int e^{-2x} \cdot x^3\, dx\right]$$

$u = x^3 \qquad dv = e^{-2x}\, dx$

$du = 3x^2\, dx$

$\int dv = \int e^{-2x}\, dx$

$v = -\frac{1}{2} e^{-2x}$

$$-\frac{x^5}{2} e^{-2x} + \frac{2}{5}\left[-\frac{x^4}{2} e^{-2x} + \frac{4}{2}\left[-\frac{x^3}{2} e^{-2x} + \frac{3}{2}\int e^{-2x} \cdot x^2\, dx\right]\right]$$

$u = x^2$

$du = 2x\,dx$

$dv = e^{-2x}dx$

$\int dv = \int e^{-2x}dx$

$v = -\frac{1}{2}e^{-2x}$

$$-\frac{x^5}{2}e^{-2x} + \frac{2}{5}\left[-\frac{x^4}{2}e^{-2x} + \frac{4}{2}\left[-\frac{x^3}{2}e^{-2x} + \frac{3}{2}\left[-\frac{x^2}{2}e^{-2x} + \frac{2}{2}\int e^{-2x}\cdot x\,dx\right]\right.\right.$$

$u = x$ $\quad \int dv = \int e^{-2x}dx$

$du = dx$ $\quad v = -\frac{1}{2}e^{-2x}$

$$-\frac{x^5}{2}e^{-2x} + \frac{2}{5}\left[-\frac{x^4}{2}e^{-2x} + \frac{4}{2}\left[-\frac{x^3}{2}e^{-2x} + \frac{3}{2}\left[-\frac{x^2}{2}e^{-2x} + \frac{2}{2}\left[-\frac{x}{2}e^{-2x} + \int e^{-2x}dx\right]\right]\right]\right]$$

$$-\frac{x^5}{2}e^{-2x} + \frac{2}{5}\left[-\frac{x^4}{2}e^{-2x} + \frac{4}{2}\left[-\frac{x^3}{2}e^{-2x} + \frac{3}{2}\left[-\frac{x^2}{2}e^{-2x} + \frac{2}{2}\left[-\frac{x}{2}e^{-2x} - \frac{1}{2}e^{-2x}\right]\right]\right]\right] + C$$

$\int \frac{t}{e^t}dt = \int te^{-t}dt$

$z = -t$

$dz = -dt$

$-dz = dt$

$\Rightarrow -\int(-z)e^z d \;=\; \int ze^z dz \;\rightsquigarrow\; ze^z - \int e^z dz$

$ze^z - e^z + C$

$-te^{-t} - e^{-t} + C$

$u = z$

$du = dz$

$dv = e^z dz$

$\int dv = \int e^z dz$

$v = e^z$