

FR. PARCIALES

→ Es una técnica de integración que se usa para convertir una expresión racional en fracciones parciales.

Ej: $\int \frac{dx}{x^2 + 2x + 1}$

① Factorizar el denominador.

$$\int \frac{dx}{(x+1)^2}$$

② $\int \frac{dx}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$

③ Plantear la ecuación.

$$1 = (x+1)A + B$$

$$1 = (-1+1)A + B \Rightarrow B = 1$$

$$1 = (1+1)A + 1 \rightarrow 0 = 2A$$
$$\frac{0}{2} = A \rightarrow A = 0$$

④ $\int \frac{dx}{(x+1)^2} = \int \frac{0}{(x+1)} + \frac{1}{(x+1)^2} dx$

$$\int (x+1)^{-2} dx = \int (x+1)^{-2} dx$$

$$\frac{(x+1)^{-1}}{-1} + C = \frac{(x+1)^{-1}}{-1} + C$$

$$\int \frac{dx}{(x+1)^2} = -\frac{1}{(x+1)} + C$$

Método No. 2 Igualar los coeficientes.

$$1 = (x+1)A + B$$

$$1 = Ax + A + B$$

$$0 + 1 = Ax + (A + B)$$

$$\begin{cases} 0 = A \\ 1 = A + B \end{cases}$$

Integrales f. Parciales

① $\int \frac{3x^3 - 18x^2 + 29x - 4}{x^4 - 5x^3 + 6x^2 + 4x - 8} dx$

División Sintética

$$\begin{array}{r|rrrrrr}
 1 & -5 & 6 & 4 & -8 & -1 \\
 & -1 & 6 & -12 & 8 & \\
 \hline
 1 & -6 & 12 & -8 & 0 & 2 \\
 & 2 & -8 & 8 & & \\
 \hline
 1 & -4 & 4 & 0 & & 2 \\
 & 2 & -4 & & & \\
 \hline
 1 & -2 & 0 & & 2 & \\
 & 2 & & & & \\
 \hline
 1 & 0 & & & &
 \end{array}$$

← término indep.

¿Cuáles son múltiplos de 8?

$D_v 8 \{ \pm 1, \pm 2, \pm 4, \pm 8 \}$

$(x+1) = 0$

$(x-2) = 0$

$(x-2) = 0$

$(x-2) = 0$

$(x+1)(x-2)^3 = 0$

$$\frac{3x^3 - 18x^2 + 29x - 4}{(x+1)(x-2)^3} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

$3x^3 - 18x^2 + 29x - 4 = (x-2)^3 A + (x+1)(x-2)^2 B + (x+1)(x-2) C + (x+1) D$

$x = 2$

$3(2)^3 - 18(2)^2 + 29(2) - 4 = (2+1)D \rightarrow D = 2$

$x = -1$

$3(-1)^3 - 18(-1)^2 + 29(-1) - 4 = (-1-2)^3 A \rightarrow A = 2$

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$$\frac{3x^3 - 18x^2 + 29x - 4}{(x+1)(x-2)^3} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

$$3x^3 - 18x^2 + 29x - 4 = (x-2)^3 A + (x+1)(x-2)^2 B + (x+1)(x-2)C + (x+1)D$$

$x = 2$

$$3(2)^3 - 18(2)^2 + 29(2) - 4 = (2+1)D \rightarrow D = 2$$

$x = -1$

$$3(-1)^3 - 18(-1)^2 + 29(-1) - 4 = (-1-2)^3 A \rightarrow A = 2$$

2. $3x^3 - 18x^2 + 29x - 4 = (x-2)^3 A + (x+1)(x-2)^2 B + (x+1)(x-2)C + (x+1)D$

$$3x^3 - 18x^2 + 29x - 4 = (x^3 A - 6x^2 A + 12xA - 8A) + (Bx^3 - 3x^2 B + 4B) + (x^2 C - Cx - 2C) + (xD + D)$$

$$3x^3 - 18x^2 + 29x - 4 = (x^3 A + x^3 B) + (x^2 C - 6x^2 A - 3x^2 B) + (12xA + xC + xD) + (-8A + 4B + D - 2C)$$

$$3x^3 - 18x^2 + 29x - 4 = (A+B)x^3 + (C - 6A - 3B)x^2 + (12A + C + D)x + (-8A + 4B + D - 2C)x^0$$

③ Sistema de Ecuaciones.

$$\begin{cases} A + B + 0 + 0 = 3 \\ -6A - 3B + C + 0 = -18 \\ 12A + 0 + C + D = 29 \\ -8A + 4B - 2C + D = -4 \end{cases}$$

$A = 2$
 $D = 2$

$$\begin{cases} 2 + B + 0 + 0 = 3 \\ -12 - 3B + C + 0 = -18 \\ 24 + 0 + C + 2 = 29 \\ -16 + 4B - 2C + 2 = -4 \end{cases}$$

$$\begin{cases} B = 1 \\ -3B + C = 4 \\ C = -3 \\ 4B - 2C = 10 \end{cases}$$

Incógnitas

$$\begin{aligned} A &= 2 \\ B &= 1 \\ C &= -3 \\ D &= 2 \end{aligned}$$

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$$\frac{3x^3 - 18x^2 + 29x - 4}{(x+1)(x-2)^3} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

Integrar a ambos lados.

$$\int \frac{3x^3 - 18x^2 + 29x - 4}{(x+1)(x-2)^3} dx = \int \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3} dx$$

$$= \int \frac{2}{(x+1)} + \frac{1}{(x-2)} + \frac{-3}{(x-2)^2} + \frac{2}{(x-2)^3} dx$$

$$= 2 \ln|x+1| + \ln|x-2| + \frac{3}{(x-2)} - \frac{1}{(x-2)^2} + C$$

Incógnitas

$$A = 2$$

$$B = 1$$

$$C = -3$$

$$D = 2$$

