Lista de Participantes.

Integrales Impropias.

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1). $\int_1^{\infty} \frac{dx}{x}$

Recordar:

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int_1^{\infty} \frac{dx}{x} = \lim_{b\to\infty} \int_1^{b} \frac{dx}{x}$$

$$= \lim_{b\to\infty} \left[\ln|x| \Big|_1^{b} \right]$$

$$= \lim_{b\to\infty} \left[\ln(b) - \ln(1) \right]$$

0

$$= \infty$$

El límite no existe entonces la integral impropia diverge.

2. $\int_0^\infty e^{-x}\,dx$ T=5min.

$$\int_0^\infty e^{-x}\,dx = \lim_{b\to\infty}\int_0^b e^{-x}\,dx$$

$$= \lim_{b\to\infty}\left[-\frac{1}{e^x}\right]\Big|_0^b$$

$$= \lim_{b\to\infty}\left[-e^{-b}+1\right]$$

$$= \frac{-1}{e^\infty}+1$$

$$= 1$$

3. $\int_0^\infty \frac{1}{x^2+1}\,dx$ T: 5min.

Recordar:

$$\int \frac{1}{x^2+1}\,dx = \arctan(x)+C$$

$$\lim_{b\to\infty}\arctan(b) = \frac{\pi}{2}$$

$$\lim_{b\to\infty}\int_0^b \frac{1}{x^2+1}\,dx$$

$$= \lim_{b\to\infty}\left[\arctan(x)\right]\Big|_0^b$$

$$= \lim_{b\to\infty}\left[\arctan(b)-\cancel{\arctan(0)}^{\,0}\right]$$

$$= \pi/2$$

4. $$\int_1^{\infty} (1-x)e^{-x}\,dx$$

T= 6min.

$u = 1-x \qquad dv = e^{-x}\,dx$

$du = -dx \qquad \int dv = \int e^{-x}\,dx$

$v = -e^{-x}$

$$\int u\,dv = uv - \int v\,du = (1-x)(-e^{-x}) - \int -e^{-x}(-dx)$$

$$= -(1-x)e^{-x} - \int e^{-x}\,dx$$

$$= (x-1)e^{-x} + e^{-x} + C$$

$$= xe^{-x} - e^{-x} + e^{-x} + C$$

$$= xe^{-x} + C$$

$$\int_1^{b} (1-x)e^{-x}\,dx = \lim_{b\to\infty} \left[xe^{-x} \right]_1^b$$

$$= \lim_{b\to\infty} \left[be^{-b} - (1)e^{-(1)} \right]$$

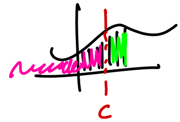
$$= \lim_{b\to\infty} \left[\frac{b}{e^b} - \frac{1}{e} \right]$$

$$= \lim_{b\to\infty} \left[\frac{(1)}{e^b} - \frac{1}{e} \right]$$

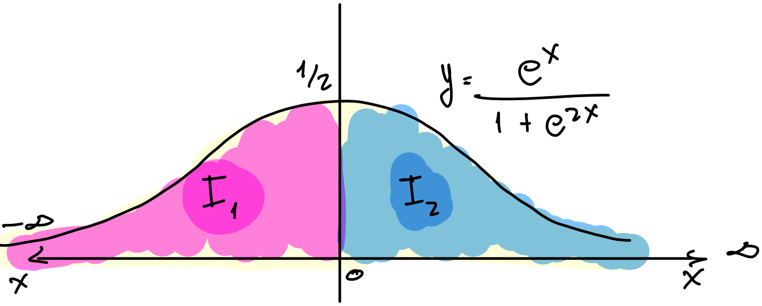
$$= \left[\frac{1}{e^{\infty}} - \frac{1}{e} \right] = -\frac{1}{e}.$$

5.

$$\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$$



$$\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx = \int_{-\infty}^{0} \frac{e^x}{1+e^{2x}}\,dx + \int_{0}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$$



$$\int \frac{e^x}{1+e^{2x}}\,dx$$

tip:

$$(x^n)^m = x^{n\cdot m}$$

$$= e^{2x} = (e^x)^2$$

$$\int \frac{e^x}{1+(e^x)^2}\,dx$$

T: 4 min.

$u = e^x$

Recordar:

$$\int \frac{1}{1+u^2}\,du = \arctan(u) + C$$

$du = e^x\,dx$

$$\int \frac{du}{1+u^2}$$

$$\arctan(u) + C$$

$$\arctan(e^x) + c$$

$$\int_{-\infty}^{0} \frac{e^x}{1+e^{2x}}\,dx + \int_{0}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$$

$$\lim_{a\to-\infty} \left[\arctan(e^x) \right]\Big|_a^0 + \lim_{b\to\infty} \left[\arctan(e^x) \right]\Big|_0^b$$

$$\lim_{a\to-\infty} \left[\arctan(e^0) - \arctan(e^a) \right] + \lim_{b\to\infty} \left[\arctan(e^b) - \arctan(e^0) \right]$$

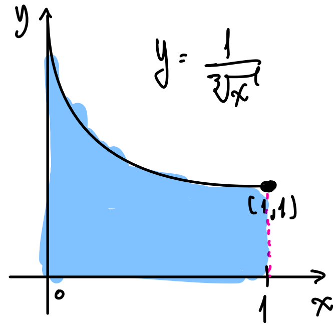
$$\left[\arctan(1) - \arctan\left(\frac{1}{e^\infty}\right) \right] + \left(\arctan(e^\infty) - \arctan(1) \right]$$

$$\frac{\pi}{4} - 0 + \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{2}$$

Integral impropia con una discontinuidad infinita

6. $\displaystyle\int_0^1 \frac{dx}{\sqrt[3]{x}}$



$y = \frac{1}{\sqrt[3]{x}}$

$y(0) = \frac{1}{\sqrt[3]{0}} = \frac{1}{0}$

$$\int_0^1 \frac{dx}{\sqrt[3]{x}} = \int_0^1 x^{-1/3}\,dx$$

Propiedades de la potencia

- $x^{n/m} = \sqrt[m]{x^n}$
- $\frac{1}{x^n} = x^{-n}$

T: 3min.

$$\int x^{-1/3}\,dx = \frac{3}{2}x^{2/3} + C$$

Recordar:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \;;\; n \neq -1$$

$$\lim_{a \to 0^+} \left[\frac{3}{2}x^{2/3}\right]\Big|_a^1$$

$$= \frac{3}{2}\lim_{a\to 0^+}\left[(1)^{2/3} - (a)^{2/3}\right]$$

$$= \frac{3}{2}\left[1 - 0\right]$$

$$= \frac{3}{2} \;//$$

7. $\int_0^2 \frac{dx}{x^3}$ T: 5 min.

$$\int_0^2 x^{-3}\,dx$$

$$\lim_{a\to 0^+}\left[-\frac{1}{2x^2}\right]\Big|_a^2 = -\frac{1}{2}\lim_{a\to 0^+}\left[-\frac{1}{8}+\frac{1}{2a^2}\right]$$

$$= \infty.$$

8. Discontinudad interior. t: 6 min.

$$\int_{-1}^2 \frac{dx}{x^3}$$

$$\int_{-1}^2 \frac{dx}{x^3} = \int_{-1}^0 \frac{dx}{x^3} + \int_0^2 \frac{dx}{x^3}$$

$$= \lim_{b\to 0^+} -\frac{1}{2}\left[\frac{1}{x^2}\right]\Big|_{-1}^b + \lim_{a\to 0^-}\left[-\frac{1}{2x^2}\right]\Big|_a^2$$

$$= \left[-\frac{1}{2}\left(\frac{1}{(-1)^2}-\frac{1}{b^2}\right)+\left(-\frac{1}{2}\right)\left(\frac{1}{a^2}-\frac{1}{2^2}\right)\right]$$

$$= \left[-\frac{1}{2}\left(1-\frac{1}{0}\right)+(-1/2)\left(\frac{1}{0}-\frac{1}{4}\right)\right]$$

$$= -\frac{1}{2}(1-\infty)+(-1/2)(\infty-1/4)$$

$$= \infty - \infty$$ Divergente

9. Integral doblemente impropia.

$t: 10_{min}$

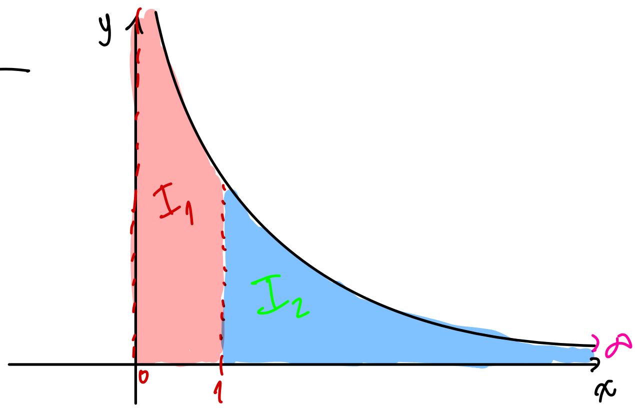
$$\int_0^{\infty} \frac{dx}{\sqrt{x}\,(x+1)}$$

$$\int \frac{dx}{\sqrt{x}(x+1)} \qquad u = \sqrt{x} \rightsquigarrow u^2 = x$$

$$u^2 + 1 = x + 1$$

$$du = \frac{1}{2\sqrt{x}}\,dx$$

$$2\int \frac{du}{u^2+1} \qquad 2du = \frac{dx}{\sqrt{x}}$$



$$2\arctan(u) + C$$

$$2\arctan(\sqrt{x})$$

$$\int_0^1 \frac{dx}{\sqrt{x}(x+1)} + \int_1^{\infty} \frac{dx}{\sqrt{x}(x+1)}$$

$$2\lim_{a\to 0^+}\left[\arctan(\sqrt{x})\right]\Big|_a^1 + 2\lim_{b\to\infty}\left[\arctan(\sqrt{x})\right]\Big|_1^b$$

$$2\lim_{a\to 0^+}\left[\arctan(1) - \arctan(\sqrt{a})\right] + 2\lim_{b\to\infty}\left[\arctan(\sqrt{b}) - \arctan(1)\right]$$

$$2\left[\frac{\pi}{4} - 0\right] + 2\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$$

$$= \frac{\pi}{2} + \pi - \frac{\pi}{2} = \pi$$