1. $f(x) = 10$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{10 - 10}{0} = 0$$

2. $f(x) = x - 1$

$$f'(x) = \lim_{h \to 0} \frac{(x+h) - 1 - (x-1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h \cancel{-1} \cancel{-x} \cancel{+1}}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = 1$$

3. $f(x) = -3x + 5$

$$f'(x) = \lim_{h \to 0} \frac{-3(x+h) + 5 - (-3x + 5)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-3x} - 3h \cancel{+5} \cancel{+3x} \cancel{-5}}{h}$$

$$= \lim_{h \to 0} \frac{-3h}{h}$$

$$= \lim_{h \to 0} -3 = -3$$

4. $f(x) = \pi x$

$$f'(x) = \lim_{h \to 0} \frac{\pi(x+h) - \pi x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{\pi x} + \pi h - \cancel{\pi x}}{h}$$

$$= \lim_{h \to 0} \frac{\pi h}{h} = \pi$$

5. $f(x) = 3x^2$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - (3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}} = 6x$$

6. $f(x) = -x^2 + 1$

$$f'(x) = \lim_{h \to 0} \frac{-(x+h)^2 + 1 - (-x^2 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{-(x^2 + 2xh + h^2) + 1 + x^2 - 1}{h}$$

$$\lim_{h \to 0} \frac{\cancel{-x^2} - 2xh - h^2 \cancel{+1} \cancel{+x^2} \cancel{-1}}{h}$$

$$\lim_{h \to 0} \frac{-2xh - h^2}{h}$$

$$\lim_{h \to 0} \frac{h(-2x - h)}{h} = \lim_{h \to 0} -2x - h = -2x$$

7. $f(x) = -x^2 + 4x + 1$

$$f'(x) = \lim_{h \to 0} \frac{-(x+h)^2 + 4(x+h) + 1 - (-x^2 + 4x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{-(x^2 + 2xh + h^2) + 4x + 4h + 1 + x^2 - 4x - 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-x^2} - 2xh - h^2 \cancel{+4x} + 4h \cancel{+1} \cancel{+x^2} \cancel{-4x} \cancel{-1}}{h}$$

$$= \lim_{h \to 0} \frac{-2xh - h^2 + 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-2x - h + 4)}{\cancel{h}}$$

$$= \lim_{h \to 0} -2x - h + 4 = -2x + 4$$

8. $f(x) = \frac{1}{2}x^2 + 6x - 7$

$$f'(x) = \lim_{h \to 0} \frac{1/2\,(x+h)^2 + 6\,(x+h) - 7 - \left[\,1/2\,x^2 + 6x - 7\right)}{h}$$

$$= \lim_{h \to 0} \frac{1/2\,(x^2 + 2xh + h^2) + 6x + 6h - 7 - 1/2\,x^2 - 6x + 7}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1/2\,x^2} + xh + 1/2\,h^2 + \cancel{6x} + 6h \cancel{-7} \cancel{-1/2\,x^2} \cancel{-6x} \cancel{+7}}{h}$$

$$= \lim_{h \to 0} \frac{xh + 1/2\,h^2 + 6h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(x + 1/2\,h + 6)}{\cancel{h}}$$

$$= \lim_{h \to 0} x + 1/2\,h + 6 = x + 6$$

9. $y = (x+1)^2$

$$y' = \lim_{h \to 0} \frac{((x+h)+1)^2 - (x+1)^2}{h}$$

$$= \lim_{h \to 0} \frac{\left((x+h)^2 + 2(x+h) + 1\right) - \left(x^2 + 2x + 1\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h \cancel{+1} \cancel{-x^2} \cancel{-2x} \cancel{-1}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h + 2)}{h} = \lim_{h \to 0} 2x + h + 2 = 2x + 2$$

10. $f(x) = (2x-5)^2$

$$f'(x) = m_{tan} = \lim_{h \to 0} \frac{(2(x+h)-5)^2 - (2x-5)^2}{h}$$

$$\lim_{h \to 0} \frac{(2x+2h)^2 - 20(x+h) + 25 - (4x^2 - 20x + 25)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 \cancel{-20x} - 20h \cancel{+25} - \cancel{4x^2} \cancel{+20x} \cancel{-25}}{h}$$

$$\lim_{h \to 0} \frac{8xh + 4h^2 - 20h}{h}$$

$$\lim_{h \to 0} \frac{h(8x + 4h - 20)}{h}$$

$$\lim_{h \to 0} 8x + 4h - 20 = 8x - 20$$

11. $f(x) = x^3 + x$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 + (x+h) - (x^3 + x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{x} + h \cancel{-x^3} \cancel{-x}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + h}{h}$$

$$\lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2 + 1)}{h}$$

$$\lim_{h \to 0} 3x^2 + 3xh + h^2 + 1 = 3x^2 + 1$$

12. $f(x) = 2x^3 + x^2$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^3 + (x+h)^2 - \left(2x^3 + x^2\right)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) + (x^2 + 2xh + h^2) - 2x^3 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 + \cancel{x^2} + 2xh + h^2 - \cancel{2x^3} - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3 + 2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(6x^2 + 6xh + 2h^2 + 2x + h)}{h}$$

$$= \lim_{h \to 0} 6x^2 + 6xh + 2h^2 + 2x + h = 6x^2 + 2x$$

13. $f(x) = -x^3 + 15x^2 - x$

$$f'(x) = m_{tan} = \lim_{h \to 0} \frac{-(x+h)^3 + 15(x+h)^2 - (x+h) - \left(-x^3 + 15x^2 - x\right)}{h}$$

$$= \lim_{h \to 0} \frac{-(x^3 + 3x^2h + 3xh^2 + h^3) + 15(x^2 + 2xh + h^2) - x - h + x^3 - 15x^2 + x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-x^3} - 3x^2h - 3xh^2 - h^3 + \cancel{15x^2} + 30xh + 15h^2 \cancel{-x} - h + \cancel{x^3} \cancel{-15x^2} \cancel{+x}}{h}$$

$$= \lim_{h \to 0} \frac{-3x^2h - 3xh^2 - h^3 + 30xh + 15h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-3x^2 - 3xh - h^2 + 30x + 15h - 1)}{\cancel{h}}$$

$$= \lim_{h \to 0} -3x^2 - 3xh - h^2 + 30x + 15h - 1 = -3x^2 + 30x - 1$$

$$f'(x) = -3x^2 + 30x - 1$$

14. $f(x) = 3x^4$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^4 - 3x^4}{h}$$

$$= \lim_{h \to 0} \frac{3(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - 3x^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^4} + 12x^3h + 18x^2h^2 + 12xh^3 + 3h^4 - \cancel{3x^4}}{h}$$

$$= \lim_{h \to 0} \frac{12x^3h + 18x^2h^2 + 12xh^3 + 3h^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(12x^3 + 18x^2h + 12xh + 3h^3)}{\cancel{h}}$$

$$= \lim_{h \to 0} 12x^3 + 18x^2h + 12xh + 3h^3 = 12x^3$$

15. $$y = \frac{2}{x+1}$$

$$y' = \lim_{h \to 0} \frac{\frac{2}{(x+h)+1} - \frac{2}{x+1}}{h}$$

$$y' = \lim_{h \to 0} \frac{2(x+1) - (2(x+h)+2)}{h((x+h)+1)(x+1)}$$

$$y' = \lim_{h \to 0} \frac{\cancel{2x} \cancel{+2} - \cancel{2x} - 2h - \cancel{2}}{h((x+h)+1)(x+1)}$$

$$y' = \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(x^2+x+xh+h+x+1)}$$

$$y' = \lim_{h \to 0} \frac{-2}{x^2+2x+xh+1} = \frac{-2}{x^2+2x+1}$$

16. $$y = \frac{x}{x-1}$$

$$y' = \lim_{h \to 0} \frac{\frac{(x+h)}{(x+h)-1} - \frac{x}{(x-1)}}{h}$$

$$y' = \lim_{h \to 0} \frac{(x+h)(x-1) - x((x+h)-1)}{h(x+h-1)(x-1)}$$

$$y' = \lim_{h \to 0} \frac{\cancel{x^2} - \cancel{x} + \cancel{xh} - h - \cancel{x^2} - \cancel{xh} + \cancel{x}}{h(x^2+xh-x-x-h+1)}$$

$$y' = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x^2+xh-2x-h+1)} = \frac{-1}{x^2-2x+1}$$

17. $y = \dfrac{2x+3}{x+4}$

$$y' = \lim_{h \to 0} \frac{\dfrac{2(x+h)+3}{(x+h)+4} - \dfrac{2x+3}{x+4}}{h}$$

$$y' = \lim_{h \to 0} \frac{(2x+2h+3)(x+4) - (2x+3)(x+h+4)}{h(x+4)(x+h+4)}$$

$$= \lim_{h \to 0} \frac{2x^2+8x+2xh+8h+3x+12 \quad - (2x^2+2xh+8x+3x+3h+12)}{h(x^2+xh+4x+4x+4h+16)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2}\cancel{+8x}+\cancel{2xh}+8h\cancel{+3x}\cancel{+12} \quad \cancel{-2x^2}\cancel{-2xh}\cancel{-8x}\cancel{-3x}-3h\cancel{-12}}{h(x^2+xh+4x+4x+4h+16)}$$

$$= \lim_{h \to 0} \frac{5\cancel{h}}{\cancel{h}(x^2+xh+4x+4x+4h+16)}$$

$$= \lim_{h \to 0} \frac{5}{x^2+xh+8x+4h+16}$$

$$= \frac{5}{x^2+x(0)+8x+4(0)+16}$$

$$= \frac{5}{x^2+8x+16}$$