Integrales directas.

1. $\int x^5 dx = \dfrac{x^6}{6} + C$

2. $\int x^{3/2} dx = \dfrac{2}{5} x^{5/2} + C$

3. $\int x\sqrt{x^1}\, dx = \int x^{3/2}\, dx = \dfrac{2}{5} x^{5/2} + C$

4. $\int \dfrac{dx}{x^3} = \int x^{-3} dx = \dfrac{x^{-2}}{-2} + C = \dfrac{-1}{2x^2} + C$

5. $\int \dfrac{x^3 + 4x^2 - 3}{x^2}\, dx = \int \left(\dfrac{x^3}{x^2} + \dfrac{4x^2}{x^2} - \dfrac{3}{x^2} \right) dx$

$= \int \left(x + 4 - \dfrac{3}{x^2} \right) dx = \dfrac{x^2}{2} + 4x + \dfrac{3}{x} + C$

6. $\int \sqrt[3]{z}\, dz = \int z^{1/3}\, dz = \dfrac{3}{4} z^{4/3} + C$

7. $\int (x+3)^2\, dx = \int (x^2 + 6x + 9)\, dx$

$\dfrac{x^3}{3} + 3x^2 + 9x + C$

8. $\int (4x^3 + 3x^2 + 1)\, dx = \dfrac{4}{4} x^4 + \dfrac{3x^3}{3} + x + C$

$= x^4 + x^3 + x + C$

9.
$$\int (x-1)^2 \, dx = \int (x^2 - 2x + 1) \, dx = \frac{x^3}{3} - \frac{2x^2}{2} + x + C$$

$$= \frac{x^3}{3} - x^2 + x + C$$

10.
$$\int \left(4x^{1/2} - 2x^{2/3} \right) dx = 4 \cdot \frac{2}{3} x^{3/2} - 2 \cdot \frac{3}{5} x^{5/3} + C$$

$$= \frac{8}{3} x^{3/2} - \frac{6}{5} x^{5/3} + C$$

11.
$$\int x(x-1) \, dx = \int (x^2 - x) \, dx = \frac{x^3}{3} - \frac{x^2}{2} + C$$

12.
$$\int (2x+3)^2 \, dx = \int (4x^2 + 12x + 9) \, dx = \frac{4}{3} x^3 + 6x^2 + 9x + C$$

13.
$$\int \left(x^3 - \sqrt{x} \right) dx = \int \left(x^3 - x^{1/2} \right) dx = \frac{x^4}{4} - \frac{2}{3} x^{3/2} + C$$

14.
$$\int \frac{x^{2/3} - x^{1/2}}{x^{1/3}} \, dx = \int \left(\frac{x^{2/3}}{x^{1/3}} - \frac{x^{1/2}}{x^{1/3}} \right) dx = \int \left(x^{1/3} - x^{4/6} \right)$$

Obs.
$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3} \quad ; \quad \frac{1}{2} - \frac{1}{3} = \frac{6-2}{6} = \frac{4}{6}$$

15.
$$\int 3^x \, dx = \frac{3^x}{\ln(3)} + C$$

16.
$$\int 2e^x \, dx = 2e^x + C$$

17.
$$\int \frac{dz}{z} = \ln|z| + C$$

18.
$$\int \left(2\cos x + 3\sin x \right) dx = 2\sin x - 3\cos x + C$$

19.
$$\int \sin(x + \pi) \, dx = \int -\sin(x) \, dx = -\int \sin(x) \, dx$$

Obs: $\sin(x+\pi) = -\sin(x)$
$$= -(-\cos x) + C$$
$$= \cos x + C$$

20.
$$\int \cos\left(x + \frac{\pi}{2} \right) dx$$

Obs: $\cos(x + \pi/2) = -\sin(x)$
$$-\int \sin(x) \, dx = -(-\cos x) + C = \cos(x) + C$$

21.
$$\int 2 \sec x \tan x \, dx = 2\int \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \, dx = \int \frac{\sin x}{\cos^2 x} \, dx$$

$u = \cos x$
$du = -\sin x \, dx$
$$-2\int \frac{du}{u^2} = -2\int u^{-2} \, du = \frac{2}{u} + C$$

$-du = \sin x \, dx$
$$= \frac{2}{\cos x} + C$$

22. $\displaystyle\int \frac{3\,dx}{\sqrt{1-x^2}}$. $3\arcsin(x) + C$

23. $\displaystyle\int \left[\frac{1}{2}\csc(x)\right]^2 = \int \frac{1}{4}\csc^2(x)\,dx = -\frac{1}{4}\cot(x) + C$

24. $\displaystyle\int x^2\cos(\pi)\sin\left(\pi/2\right)\,dx = \frac{x^3}{3}(-1)(1) + C = -\frac{x^3}{3} + C$