03 - Frações Parciais.

1. $\int \frac{4x^2+13x-9}{x^3+2x^2-3x}\,dx = \int \frac{4x^2+13x-9}{x(x+3)(x-1)}\,dx$

$$\frac{4x^2+13x-9}{x^3+2x^2-3x} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-1}$$

$$4x^2+13x-9 = A(x+3)(x-1) + B(x)(x-1) + C(x)(x+3)$$

$$4x^2+13x-9 = A(x^2+2x-3) + B(x^2-x) + C(x^2+3x)$$

$$4x^2+13x-9 = Ax^2+2Ax-3A + Bx^2-Bx + Cx^2+3Cx$$

$$4x^2+13x-9 = Ax^2+Bx^2+Cx^2+2Ax+3Cx-Bx-3A$$

$$4x^2+13x-9 = (A+B+C)x^2 + (2A+3C-B)x - 3A$$

$$\begin{cases} A+B+C=4 \rightsquigarrow 3+B+C=4 \rightsquigarrow B+C=1 \\ 2A-B+3C=13 \rightsquigarrow 6-B+3C=13 \rightsquigarrow -B+3C=7 \\ -3A=-9 \rightsquigarrow A=3 \end{cases}$$

$$4C=8$$
$$C=2$$

entonces.

$$3+B+2=4 \rightsquigarrow B=-1$$

$$\int \frac{3}{x} - \frac{1}{x+3} + \frac{2}{x-1}\,dx$$

$$3\ln|x| - \ln|x+3| + 2\ln|x-1| + C$$