Método de Sustitución

1. $\int \frac{dx}{2x-5} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(2x-5) + C$

$u = 2x-5$
$du = 2dx$
$\frac{du}{2} = dx$

2. $\int \frac{x+5}{x+3}\,dx = \int \frac{u+2}{u}\,du = \int 1 + \frac{2}{u}\,du = u + 2\ln(u) + C$

$= x+3 + 2\ln(x+3) + C$

$u = x+3 \rightsquigarrow \quad u+2 = x+5$
$du = dx$

$= x + 2\ln|x+3| + 3 + C$

3. $\int \frac{(2x^2+1)}{(2x^3+3x+1)^{2/3}}\,dx$

$u = 2x^3+3x+1 \quad \Rightarrow \frac{1}{3}\int \frac{du}{u^{2/3}} = \frac{1}{3}\int u^{-2/3}\,du = \frac{1}{3}u^{1/3}\cdot\frac{3}{1} + C$

$du = (6x^2+3)\,dx$

$= u^{1/3} + C$

$du = 3(2x^2+1)\,dx$

$= (2x^3+3x+1)^{1/3} + C$

$\frac{du}{3} = (2x^2+1)\,dx$

4. $\int \frac{\ln^2(x)}{x}\,dx$

$u = \ln(x) \quad \Rightarrow \quad \int u^2\,du = \frac{u^3}{3} + C$
$du = \frac{1}{x}\,dx$

$= \frac{\ln^3(x)}{3} + C$

5. $\int 4\cos 2x\,dx$

$$\Rightarrow \frac{4}{2}\int \cos(u)\,du = 2\int \cos u\,du = 2\,\mathrm{Sen}(u) + C = 2\,\mathrm{Sen}(2x) + C$$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

6. $\int (2x+1)\csc^2(x^2+x)\,dx$

$u = x^2 + x$

$du = (2x+1)dx$

$$\Rightarrow \int \csc^2(u)\,du$$

$$= \ln\left(\frac{\mathrm{Sen}\, u/2}{\cos u/2}\right) + C = \ln\left(\frac{\mathrm{Sen}\left(\frac{x^2+x}{2}\right)}{\cos\left(\frac{x^2+x}{2}\right)}\right) + C$$

7. $$\int \frac{3dx}{1+9x^2} = 3\int \frac{dx}{1+(3x)^2} = \frac{3}{3}\int \frac{du}{1+u^2}$$

$u = 3x$

$du = 3dx$

$\frac{du}{3} = dx$

$$= \frac{3}{3}\arctan(u) + C$$

$$= \arctan(3x) + C$$

8. $$\int \frac{x\,dx}{x^2+1} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+1| + C$$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

9. $\int \frac{9\,dy}{y} = 9\ln y + c$

10. $\int \frac{\cos x\,dx}{1+\operatorname{Sen}x} = \int \frac{du}{u} = \ln|u| + c = \ln|1+\operatorname{Sen}x| + c$

$u = 1+\operatorname{Sen}x$
$du = \cos x\,dx$

11. $\int \operatorname{Sen}^5 3x \cos(3x)\,dx = \frac{1}{3}\int u^5\,du$

$u = \operatorname{Sen}3x$ $\qquad = \frac{1}{3}\frac{u^6}{6} + C = \frac{1}{18}(\operatorname{Sen}^6(3x)) + c$
$du = 3\cos 3x\,dx$

$\frac{du}{3} = \cos(3x)\,dx$

12. $\int \tan x\,dx = -\ln|\cos(x)| + c$

$\int \frac{\operatorname{sen}x\,dx}{\cos x} = -\int \frac{du}{u} = -\ln|u| + c$

$= -\ln|\cos x| + c$

$u = \cos x$
$du = -\operatorname{sen}x\,dx$
$-du = \operatorname{Sen}x\,dx$

13. $\int \operatorname{Cot}(3y)\,dy = \int \frac{\cos(3y)\,dy}{\operatorname{Sen}(3y)} = \frac{1}{3}\int \frac{du}{u}$

$u = \operatorname{Sen}(3y)$ $\qquad = \frac{1}{3}\ln|u| + c$
$du = 3\cos(3y)\,dy$
$\frac{du}{3} = \cos(3y)\,dy$ $\qquad = \frac{1}{3}\ln|\operatorname{Sen}(3y)| + c$

14. $\int \frac{dx}{a+x^2} = \int \frac{dx}{(\sqrt{a})^2+x^2} = \frac{1}{\sqrt{a}} \arctan\left(\frac{x}{\sqrt{a}}\right) + C$

15. $\int \frac{dx}{2x-3} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x-3| + C$

$u = 2x-3$

$du = 2dx$

$\frac{du}{2} = dx$

16. $\int \frac{x^2dx}{1-2x^3} = -\frac{1}{6}\int \frac{du}{u} = -\frac{1}{6}\ln|u| + C = -\frac{1}{6}\ln|1-2x^3| + C$

$u = 1-2x^3$

$du = -6x^2dx$

$\frac{du}{-6} = x^2dx$

17. $\int e^{-x}dx = -\int e^u du = -e^u + C$

$= -e^{-x} + C$

$= -\frac{1}{e^x} + C$

$u = -x$

$du = -dx$

$-du = dx$

18. $\int a^{2x}dx = \frac{1}{2}\int a^u du$

$= \frac{1}{2}\frac{a^u}{\ln(a)} + C$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$= \frac{1}{2}\frac{a^{2x}}{\ln(a)} + C$

19. $\int \cos(3x)\,dx = \frac{1}{3}\int \cos(u)\,du$

$u = 3x$
$du = 3dx$
$\frac{du}{3} = dx$

$= \frac{1}{3}\operatorname{Sen}(u) + C = \frac{1}{3}\operatorname{Sen}(3x) + C$

20. $\int \frac{dx}{\sqrt{9 - x^2}} = \operatorname{arcSen}\left(\frac{x}{3}\right) + C$

21. $\int \frac{\operatorname{Sen} x}{1 + \cos x}\,dx = -\int \frac{du}{u} = -\ln|u| + C$

$= -\ln|1 + \cos x| + C$

$u = 1 + \cos x$
$du = -\operatorname{Sen} x\,dx$

$-du = \operatorname{Sen} x\,dx$

22. $\int \frac{x}{9 + x^2}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|9 + x^2| + C$

$u = 9 + x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

23. $\int \frac{7}{x^2 - 6x + 25}\,dx = \int \frac{7}{(x-3)^2 + 4^2}\,dx = \int \frac{7}{u^2 + 4^2}\,du = \frac{7}{4}\arctan\left(\frac{u}{4}\right) + C$

$x^2 - 6x + 9 - 9 + 25$

$\downarrow \quad \downarrow$

$x \cdot 2 \quad 3$

$= \frac{7}{4}\arctan\left(\frac{x-3}{4}\right) + C$

$u = x - 3$
$du = dx$

$(x-3)^2 + 16$

24. $\int \frac{3}{1+u^2} du = 3\arctan(u) + C$

25. $\int 3\sec(3x)\tan(3x)\,dx = \frac{3}{3}\int \sec(u)\tan(u)\,du$

$u = 3x$

$\frac{du}{3} = dx$

$= \int \frac{1}{\cos(u)} \cdot \frac{\sin(u)}{\cos(u)}\,du$

$= \int \frac{\sin(u)}{\cos^2(u)}\,du$

$t = \cos(u)$

$dt = -\sin(u)\,du$

$-dt = \sin(u)\,du$

$= -\int \frac{dt}{t^2}$

$= -\int t^{-2}\,dt = \frac{1}{t} + C = \frac{1}{\cos(u)} + C$

$= \frac{1}{\cos(3x)} + C$

26. $\int x\sqrt{x^2+1}\,dx = \frac{1}{2}\int \sqrt{u}\,du$

$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \frac{1}{3}(x^2+1)^{3/2} + C$

27. $\int x^{1/3}(x^{4/3}+1)^{20}\,dx = \frac{3}{4}\int u^{20}\,du$

$u = x^{4/3} + 1$

$du = \frac{4}{3}x^{1/3}\,dx$

$\frac{3}{4}du = x^{1/3}\,dx$

$= \frac{3}{4}\frac{u^{21}}{21} + C$

$= \frac{1}{28}(x^{4/3}+1)^{21} + C$