05 - Longitud de arco.

1. Hallar L de la curva $y = 2/3 x^{3/2}$ en $[0, 3]$

$y = \frac{2}{3} x^{3/2}$, $y' = x^{1/2}$

$$L = \int_0^3 \sqrt{1 + (\sqrt{x})^2}\, dx$$

$$= \int_0^3 \sqrt{1 + x}\, dx$$

$$= \frac{2}{3} (1 + x)^{3/2} \Big|_0^3 = \frac{2}{3} (1 + 3)^{3/2} - \frac{2}{3} (1)^{3/2}$$

$$= \frac{14}{3} u .$$

2. Hallar L de la curva $y = \frac{1}{3}(x^2+2)^{3/2}$ en $x=0$ y $x=3$

$$y = \frac{1}{3}(x^2+2)^{3/2} \quad \Rightarrow \quad L = \int_0^3 \sqrt{1 + \left(x(x^2+2)^{1/2}\right)^2}\, dx$$

$$y' = \frac{1}{2}(x^2+2)^{1/2} \cdot 2x$$

$$y' = x(x^2+2)^{1/2}$$

$$= \int_0^3 \sqrt{1 + x^2(x^2+2)}\, dx$$

$$= \int_0^3 \sqrt{x^4 + 2x^2 + 1}\, dx$$

$$= \int_0^3 \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^3 (x^2+1)\, dx$$

$$\left[\frac{x^3}{3} + x\right]\Big|_0^3 = 12 \text{ und.}$$

3. Hallar l de la curva $y = \ln|\cos x|$ entre $[0, \pi/3]$

$y = \ln|\cos x|$ $\Rightarrow$ $l = \int_0^{\pi/3} \sqrt{1 + \left(\frac{-\operatorname{sen} x}{\cos x}\right)^2}\, dx$

$y' = \frac{1}{\cos x} \cdot (-\operatorname{sen} x)$

$$= \int_0^{\pi/3} \sqrt{1 + \frac{\operatorname{sen}^2 x}{\cos^2 x}}\, dx$$

$y' = -\frac{\operatorname{sen} x}{\cos x}$

$$\int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx$$

$$\int_0^{\pi/3} \sqrt{\sec^2 x}\, dx$$

$$\int_0^{\pi/3} \sec x\, dx = \ln|\sec x + \tan x| \Big|_0^{\pi/3}$$

$$= \ln|\sec \pi/3 + \tan \pi/3| - \ln|\sec 0 + \tan 0|$$

$$= \ln|2 + \sqrt{3}| - \cancel{\ln(1)}^{0}$$

$\approx 1{,}32$ und.

4. Hallar L de la curva $y = \ln(1-x^2)$ entre $[0, 1/2]$

$$y = \ln(1-x^2)$$

$$y' = \frac{1}{1-x^2} \cdot (-2x)$$

$$\int_0^{1/2} \left(1 + \left(\frac{-2x}{1-x^2}\right)^2 \right)^{1/2} dx$$

$$\int_0^{1/2} \left(1 + \frac{4x^2}{(1-x^2)^2} \right)^{1/2} dx$$

$$\int_0^{1/2} \left(\frac{(1-x^2)^2 + 4x^2}{(1-x^2)^2} \right)^{1/2} dx$$

$$\int_0^{1/2} \frac{((1-x^2)^2 + 4x^2)^{1/2}}{1-x^2} dx$$

Obs: $(1-x^2)^2 + 4x^2$

$= 1 - 2x^2 + x^4 + 4x^2$

$= x^4 + 2x^2 + 1 = (x^2+1)^2$

$$\int_0^{1/2} \frac{\sqrt{(x^2+1)^2}}{1-x^2} dx$$

$$\int_0^{1/2} \frac{x^2+1}{1-x^2} dx$$

Obs: $\frac{x^2+1}{1-x^2} = \frac{2}{1-x^2} - 1$

$$\begin{array}{r|l} x^2+1 & 1-x^2 \\ \cline{2-2} -x^2+1 & -1 \\ \hline 2 & \end{array}$$

$$\int_0^{1/2} \left(\frac{2}{1-x^2} - 1 \right) dx$$

$$\frac{2}{2} \ln \left| \frac{x+1}{x-1} \right| - x \Big|_0^{1/2} = -\frac{1}{2} + \ln(3) \approx 0{,}59 \text{ und.}$$