Método de Integración por partes:

$$\int u\,dv = uv - \int v\,du$$

1. $\int x \cos x\,dx$

$u = x$ $\quad dv = \cos x$

$du = dx$ $\quad \int dv = \int \cos x\,dx$

$v = \operatorname{Sen} x$

$$\Rightarrow x \operatorname{Sen} x - \int \operatorname{Sen} x\,dx$$

$$x \operatorname{Sen} x + \operatorname{Cos} x + C$$

2. $\int x e^x\,dx$

$u = x$ $\quad dv = e^x\,dx$

$du = dx$ $\quad \int dv = \int e^x\,dx$

$v = e^x$

$$\Rightarrow x e^x - \int e^x\,dx$$

$$x e^x - e^x + C$$

3. $\int x \ln x\,dx$

$u = \ln|x|$ $\quad dv = x$

$du = \frac{1}{x}\,dx$ $\quad \int dv = \int x\,dx$

$v = \frac{x^2}{2}$

$$\Rightarrow x^2 \frac{\ln x}{2} - \frac{1}{2}\int x^2 \cdot \frac{1}{x}\,dx$$

$$\Rightarrow \frac{x^2}{2}\ln|x| - \frac{1}{2}\int x\,dx$$

$$\frac{x^2}{2}\ln|x| - \frac{1}{4}x^2 + C$$

4. $\int e^x \operatorname{Sen} 2x \, dx$

$u = \operatorname{Sen} 2x \qquad dv = e^x dx$

$du = 2\cos(2x)\,dx$

$\int dv = \int e^x dx$

$v = e^x$

$\Rightarrow e^x \operatorname{Sen} 2x - \int 2e^x \cos(2x)\,dx$

$e^x \operatorname{Sen}(2x) - 2\int e^x \cos(2x)\,dx$

$u = \cos(2x) \qquad dv = e^x$

$du = -2\operatorname{Sen}(2x)\,dx \qquad v = e^x$

$\Rightarrow e^x \operatorname{Sen}(2x) - 2\left[\cos(2x)e^x + 2\int e^x \operatorname{Sen}(2x)\,dx\right]$

$\int e^x \operatorname{Sen} 2x \, dx = e^x \operatorname{Sen}(2x) - 2e^x \cos(2x) - 4\int e^x \operatorname{Sen}(2x)\,dx$

$\int e^x \operatorname{Sen} 2x \, dx + 4\int e^x \operatorname{Sen} 2x \, dx = e^x \operatorname{Sen}(2x) - 2e^x \cos(2x)$

$5\int e^x \operatorname{Sen}(2x)\,dx = e^x \operatorname{Sen}(2x) - 2e^x \cos(2x)$

$\int e^x \operatorname{Sen}(2x)\,dx = \dfrac{e^x \operatorname{Sen}(2x) - 2e^x \cos(2x)}{5} + C$

5. $\int x^2 \cos x \, dx$

$u = x^2 \qquad dv = \cos x \, dx$

$du = 2x \, dx \qquad \int dv = \int \cos x \, dx$

$v = \operatorname{Sen} x$

$\Rightarrow x^2 \operatorname{Sen} x - 2 \int x \operatorname{Sen} x \, dx$

$u = x \qquad dv = \operatorname{Sen} x \, dx$

$du = dx \qquad \int dv = \int \operatorname{Sen} x \, dx$

$v = -\cos x$

$\Rightarrow x^2 \operatorname{Sen} x - 2 \left[-x \cos x + \int \cos x \, dx \right]$

$x^2 \operatorname{Sen} x + 2x \cos x - 2 \operatorname{Sen} x + C$

6. $\int x^3 e^x \, dx$

$u = x^3 \qquad \int dv = \int e^x \, dx$

$du = 3x^2 \, dx \qquad v = e^x$

$x^3 e^x - 3 \int e^x x^2 \, dx$

$u = x^2 \qquad \int dv = \int e^x \, dx$

$du = 2x \, dx \qquad v = e^x$

$x^3 e^x - 3 \left[x^2 e^x - 2 \int e^x x \, dx \right] = x^3 e^x - 3 \left[x^2 e^x - 2 \int x e^x \, dx \right]$

$u = x \qquad \int dv = \int e^x dx$

$du = dx \qquad v = e^x$

$\Rightarrow e^x x^3 - 3e^x x^2 + 6\left[xe^x - \int e^x dx\right]$

$e^x x^3 - 3e^x x^2 + 6xe^x - 6e^x + C$

7. $\int \frac{xe^x}{(1+x)^2} dx$

$u = xe^x \qquad dv = \frac{1}{(1+x)^2} dx$

$du = (e^x + xe^x) dx$

$du = e^x (x+1) dx \qquad \int dv = \int \frac{dx}{(1+x)^2}$

$v = -\frac{1}{1+x}$

$\Rightarrow -\frac{xe^x}{1+x} + \int \frac{e^x \cancel{(x+1)}}{\cancel{(1+x)}} dx$

$= -\frac{xe^x}{1+x} + e^x + C$

$= e^x - \frac{xe^x}{1+x} + C$

8. $\int e^x (x+1)^2 dx$

$u = (x+1)^2 \qquad dv = e^x dx$

$du = 2(x+1) dx \qquad \int dv = \int e^x dx$

$v = e^x$

$\Rightarrow e^x (x+1)^2 - 2\int (x+1) e^x dx$

$$e^x(x+1)^2 - 2\left[\int x e^x + e^x\, dx\right]$$

$$e^x[x+1)^2 - 2\left[e^x(x-1) + e^x\right] + C$$

$$e^x(x+1)^2 - 2e^x(x-1) - 2e^x + C$$

9 $\int \arctan x\, dx$

$u = \arctan x \qquad dv = dx$

$du = \frac{1}{x^2+1}\, dx \qquad \int v\, dv = \int dx$

$v = x$

$$\Rightarrow x\arctan x - \int \frac{x}{x^2+1}\, dx$$

$$= x\arctan x - \frac{1}{2}\ln|x^2+1| + C$$

10. $\int x\sqrt{x+1}\, dx$

$u = x \qquad dv = \sqrt{x+1}$

$du = dx \qquad \int dv = \int \sqrt{x+1}\, dx$

$v = \frac{2}{3}(x+1)^{3/2}$

$$\Rightarrow \frac{2}{3}x(x+1)^{3/2} - \frac{2}{3}\int (x+1)^{3/2}\, dx =$$

$$\frac{2}{3}x(x+1)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}\left[x+1\right]^{5/2} + C$$

$$= \frac{2}{3}x(x+1)^{3/2} - \frac{4}{15}(x+1)^{5/2} + C$$

11. $\int e^{-x} \operatorname{Sen} x \, dx$

$u = \operatorname{Sen} x \qquad dv = e^{-x} dx$

$du = \operatorname{Cos} x \, dx \qquad \int dv = \int e^{-x} dx$

$v = -e^{-x}$

$\Rightarrow -e^{-x} \operatorname{Sen} x + \int e^{-x} \operatorname{Cos} x \, dx$

$u = \operatorname{Cos} x \qquad dv = e^{-x}$

$du = -\operatorname{Sen} x \, dx \qquad \int dv = \int e^{-x} dx$

$v = -e^{-x}$

$\Rightarrow -e^{-x} \operatorname{Sen} x - e^{-x} \operatorname{Cos} x - \int (-e^{-x})(-\operatorname{Sen} x \, dx)$

$-e^{-x} \operatorname{Sen} x - e^{-x} \operatorname{Cos} x - \int e^{-x} \operatorname{Sen} x \, dx$

$= \int e^{-x} \operatorname{Sen} x \, dx = -e^{-x} \operatorname{Sen} x - e^{-x} \operatorname{Cos} x - \int e^{-x} \operatorname{Sen} x \, dx$

$\int e^{-x} \operatorname{Sen} x \, dx + \int e^{-x} \operatorname{Sen} x \, dx = -e^{-x} \operatorname{Sen} x - e^{-x} \operatorname{Cos} x$

$2 \int e^{-x} \operatorname{Sen} x \, dx = -e^{-x} \operatorname{Sen} x - e^{-x} \operatorname{Cos} x$

$$\int e^{-x} \operatorname{Sen} x \, dx = \frac{-e^{-x} \operatorname{Sen} x - e^{-x} \operatorname{Cos} x}{2}$$

12. $\int x a^x \, dx$

$u = x \qquad \int dv = \int a^x \, dx$

$du = dx \qquad v = \dfrac{a^x}{\ln(a)}$

$$\Rightarrow \frac{x a^x}{\ln(a)} - \int \frac{a^x}{\ln(a)} \, dx$$

$$= \frac{x a^x}{\ln(a)} - \frac{1}{\ln(a)} \left[\frac{a^x}{\ln(a)} \right] + C$$

$$= \frac{x a^x}{\ln(a)} - \frac{a^x}{\ln^2(a)} + C$$