Integrales definidas.

1. $\int_0^1 (4x-1)\,dx = \frac{4}{2}x - x \Big|_0^1 = 2(1)-(1)-0$

$= 1$

2. $\int_0 (ax+b)\,dx$ Obs: No esta definida en el límite superior.

3. $\int_1^4 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big/_1^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$

$= \frac{14}{3}$

4. $\int_1^1 \sqrt{x+1}\,dx = 0$

Obs: Si los límites son iguales el resultado es cero.

5. $\int_0^{\pi/2} \operatorname{Sen} x\,dx = -\operatorname{Cos} x\Big/_0^{\pi/2} = -\operatorname{Cos}(\pi/2) - (-\operatorname{Cos}(0))$

$= 1$

6. $\int_2^4 \frac{dx}{x} = \ln(x)\Big|_2^4 = \ln(4) - \ln(2)$

$= \ln\left(\frac{4}{2}\right) = \ln(2)$

7. $\int_1^4 \frac{x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1)\Big/_1^4 = \frac{1}{2}\ln(4^2+1) - \frac{1}{2}\ln(1^2+1) = \frac{1}{2}\ln\left(\frac{17}{2}\right)$

$\int \frac{x}{x^2+1}\,dx = \int \frac{du}{u+1} = \ln|u+1| + c$

$= \frac{1}{2}\ln(x^2+1) + c$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

8. $\int^{3} x(x^2-4)\,dx$ Obs: No esta definida en el límite inferior.

9. $\int_0^1 (x^2-2x+3)\,dx = \frac{x^3}{3} - x^2 + 3x \Big|_0^1 = \frac{(1)^3}{3} - (1)^2 + 3(1) - 0 = \frac{7}{3}$

10. $\int_a^{2a} (a+z)\,dz = az + \frac{z^2}{2}\Big|_a^{2a} = a(2a) + \frac{(2a)^2}{2} - \left(a(a) + \frac{(a)^2}{2}\right)$

$= 2a^2 + \frac{4a^2}{2} - a^2 - \frac{a^2}{2}$

$= 2a^2 + 2a^2 \cancel{\frac{2a^2}{2}} - \frac{a^2}{2}$

$= 4a^2 - \frac{3a^2}{2}$

$= \frac{5a^2}{2}$

11. $\int_0^2 (4x+1)^{1/2}\,dx = \frac{1}{6}(4x+1)^{3/2}\Big|_0^2 = \frac{1}{6}\left(4(2)+1\right)^{3/2} - \left(1/6\,(4(0)+1)^{3/2}\right)$

$= \frac{13}{3}$

$\int (4x+1)^{1/2}\,dx$

$u = 4x$

$du = 4dx$

$\frac{du}{4} = dx$

$\int u^{1/2}\,du = \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{1}{6}(4x+1)^{3/2} + C$

12. $\int_2^5 \left(x^2 + \frac{1}{x^2}\right)dx = \frac{x^3}{3} - \frac{1}{x}\Big|_2^5 = \frac{(5)^3}{3} - \frac{1}{2} = \frac{393}{10}$

13. $\int_0^a \frac{x^4}{a^4}\,dx = \frac{1}{a^4}\frac{x^5}{5}\Big|_0^a = \frac{1}{a^4}\cdot\frac{a^5}{5} - 0$

$= \frac{a}{5}$

14. $\int_0^2 x^4\,dx = \frac{x^5}{5}\Big|_0^2 = \frac{(2)^5}{5} - 0$

$= \frac{32}{5}$

15. $\int_0^8 \sqrt[3]{x}\,dx = \frac{3}{4}x^{4/3}\Big|_0^8 = \frac{3}{4}(8)^{4/3} - 0$

$= 12$

16. $\int_0^{\pi/4} \operatorname{Sen}(2x)\,dx = -\frac{1}{2}\operatorname{Cos}(2x)\Big|_0^{\pi/4} = -\frac{1}{2}\operatorname{Cos}(2(\pi/4)) - \left(-\frac{1}{2}\operatorname{Cos}(2(0))\right)$

$= \frac{1}{2}$

$\int \operatorname{Sen}(ax)\,dx = -\frac{1}{a}\operatorname{Cos}(ax) + C$

17. $\int_0^{\pi/2} \operatorname{Cos}^2 x \operatorname{Sen} x\,dx = -\frac{1}{3}\operatorname{Cos}^3 x\Big|_0^{\pi/2} = -\frac{1}{3}\operatorname{Cos}^3(\pi/2) - \left(-\frac{1}{3}\operatorname{Cos}^3(0)\right)$

$= \frac{1}{3}$

$\int \operatorname{Cos}^2 x \operatorname{Sen} x\,dx$

$u = \operatorname{Cos} x$

$du = -\operatorname{Sen} x\,dx$

$-du = \operatorname{Sen} x\,dx$

$-\int u^2\,du = -\frac{u^3}{3} + C$

$= -\frac{\operatorname{Cos}^3 x}{3} + C$

18. $\int_0^{\pi/2} (12x^2 + 4\operatorname{Cos} x)\,dx = \frac{12}{3}x^3 + 4\operatorname{Sen} x\Big|_0^{\pi/2}$

$= \frac{12}{3}(\pi/2)^3 + 4\operatorname{Sen}(\pi/2) - \left(\frac{12}{3}(0) + 4\operatorname{Sen}(0)\right)$

$= \frac{12\pi^3}{24} + 4$

$= \frac{\pi^3}{2} + 4$

19. $\int_1^2 4x(\text{Cos}x^2+2)\,dx$

$\int_1^2 (4x\,\text{Cos}x^2 + 8x)\,dx$

$\int_1^2 4x\,\text{Cos}x^2 + \int_1^2 8x\,dx = \left[\frac{4}{2}\text{Sen}(x^2) + 4x^2\right]\Big|_1^2$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int 4x\,\text{Cos}x^2\,dx$

$= \frac{4}{2}\int \text{Cos}\,u\,du = 2\,\text{Sen}(u) + C$

$= 2\,\text{Sen}(x^2) + C$

$\int 8x\,dx = \frac{8x^2}{2} + C = 4x^2 + C$

$\int_1^2 4x(\text{Cos}x^2+2)\,dx = 2\text{Sen}(x^2) + 4x^2\Big|_1^2$

$= 2\,\text{Sen}(2^2) + 4(2)^2 - (2\,\text{Sen}(1) + 4(1)^2)$

$= 2\,\text{Sen}(4) + 16 - 2\,\text{Sen}(1) - 4$

$= 2\,\text{Sen}(4) - 2\,\text{Sen}(1) + 12$

20. $\int_0^{\pi/2} \text{Sen}x\,dx = -\text{Cos}x\Big|_0^{\pi/2} = -\text{Cos}(\pi/2) - (-\text{Cos}(0))$

$= 1$