# 02 - l'Hopital.

1. $$\lim_{x\to 0} \frac{e^x - e^{-x}}{\operatorname{Sen} x} = \frac{e^0 - e^{-0}}{\operatorname{Sen}(0)} = \frac{1-1}{0} = \frac{0}{0}$$

entonces:

l'Hopital.

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{\operatorname{Sen} x} = \lim_{x\to 0} \frac{e^x + e^{-x}}{\operatorname{Cos} x} = \frac{1+1}{1} = 2$$

2. $$\lim_{x\to\infty} \frac{x}{\ln^3 x + 2x} = \frac{\infty}{\infty}$$

entonces por l'Hopital:

$$\lim_{x\to\infty} \frac{x}{\ln^3 x + 2x} = \lim_{x\to\infty} \frac{1}{\frac{3}{x} + 2} = \frac{1}{\frac{3}{\infty} + 2} = \frac{1}{2} \,//.$$

3. $$\lim_{x\to 0} \left( \frac{1}{x} - \frac{1}{\operatorname{Sen} x} \right) = \lim_{x\to 0} \frac{\operatorname{Sen} x - x}{x \operatorname{Sen} x} = \frac{0}{0}$$

entonces por l'Hopital:

$$\lim_{x\to 0} \frac{\operatorname{Cos} x - 1}{\operatorname{Sen} x + x \operatorname{Cos} x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{-\operatorname{Sen} x}{\operatorname{Cos} x + \operatorname{Cos} x - x \operatorname{Sen} x} = \frac{0}{2} = 0$$

4. $\lim_{x \to \frac{\pi}{4}} (\tan x - 1)\sec 2x = \lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{\cos 2x} = \frac{0}{0}$

entonces por l'Hopital:

$$\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x}{-2\operatorname{Sen} 2x} = \frac{2}{-2(1)} = -1$$

5. $\lim_{x \to 0} x^{\tan x} = 0^0$

entonces:

$$\lim_{x \to 0} e^{\ln x^{\tan x}} = \lim_{x \to 0} e^{\tan x \ln x}$$

Debido a propiedades:

$$e^{\lim_{x \to 0} \tan x \ln x} = e^{\left(\lim_{x \to 0} \frac{\ln x}{\cot x}\right)}$$

l'Hopital:

$$e^{\lim_{x \to 0} \frac{\frac{1}{x}}{-\csc^2 x}}$$

Obs: $\csc^2 x = \frac{1}{\operatorname{Sen}^2 x}$

Segunda l'Hopital.

$$e^{\lim_{x \to 0} -2\operatorname{Sen} x \cos x} = e^0 = 1$$

6. $\lim_{x \to 2} \left(\frac{x}{2}\right)^{\frac{1}{x-2}} = 1^{\frac{1}{0}}$

entonces:

$$\lim_{x \to 2} e^{\ln\left(\frac{x}{2}\right)^{\frac{1}{x-2}}} = e^{\lim_{x \to 2} \frac{\ln\left(\frac{x}{2}\right)}{x-2}}$$

l'Hopital

$$e^{\lim_{x \to 2} \frac{\ln(x) - \ln(2)}{x - 2}} = e^{\lim_{x \to 2} \frac{\frac{1}{x}}{1}} = e^{1/2} = \sqrt{e}$$