Homework

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1 Introduction

Derivation and Equivalence of P_0 in a Liquidity Pool Bonding Curve

Introduction

In this assignment, we aim to prove the equivalence of various expressions for the price P_0 of a liquidity pool token at a specific point on the bonding curve. This derivation is crucial for understanding the dynamics of liquidity pools, which are fundamental components of DeFi systems.

We will show the equivalence of the following expressions:

$$\frac{x_0 \cdot P_{high} - y_0}{y_0/P_{low} - x_0} = P_0 = \frac{y_0}{x_0} = \sqrt{P_{high} \cdot P_{low}} = \frac{y_{int}}{x_{int}}$$

Where: - P_0: Price at the point (x_0, y_0) - P_{high}: Price when x runs out (intercept with the y-axis) - P_{low}: Price when y runs out (intercept with the x-axis) - x_0: Balance of token x - y_0: Balance of token y - y_{int}: Y-intercept of the bonding curve - x_{int}: X-intercept of the bonding curve

Steps to Prove the Equivalence

Step 1: Starting with the Original Equation The given equation is:

$$P_0 = \frac{x_0 \cdot P_{high} - y_0}{y_0/P_{low} - x_0}$$

Step 2: Since P_0 Equal to $\frac{y_0}{x_0}$, we set:

$$\frac{y_0}{x_0} = \frac{x_0 \cdot P_{high} - y_0}{y_0/P_{low} - x_0}$$

Step 3: Cross Multiply By cross-multiplying, we obtain:

$$y_0 \left(\frac{y_0}{P_{low}} - x_0 \right) = x_0 \left(x_0 \cdot P_{high} - y_0 \right)$$

Step 4: Distribute Both Sides Distributing the terms inside the parentheses, we get:

$$\frac{y_0^2}{P_{low}} - y_0 x_0 = x_0^2 P_{high} - x_0 y_0$$

Step 5: Combine Like Terms Combine like terms on both sides of the equation:

$$\frac{y_0^2}{P_{low}} - x_0^2 P_{high} = 0$$

Adding $x_0^2 P_{high}$ to both sides gives:

$$\frac{y_0^2}{P_{low}} = x_0^2 P_{high}$$

Step 6: Solve for P_{high} and P_{low} Rewriting the equation:

$$\frac{y_0^2}{P_{low}} = x_0^2 P_{high}$$

Multiply both sides by P_{low}:

$$y_0^2 = x_0^2 P_{high} \cdot P_{low}$$

Step 7: Divide by x_0^2 To isolate the fraction on the left side:

$$\left(\frac{y_0}{x_0}\right)^2 = P_{high} \cdot P_{low}$$

Step 8: Take the Square Root of Both Sides Finally, taking the square root of both sides:

$$\frac{y_0}{x_0} = \sqrt{P_{high} \cdot P_{low}}$$

Equivalence with $\frac{y_{int}}{x_{int}}$
Given:

$$P_{high} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{low} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

From the bonding curve, the intercepts x_{int} and y_{int} can be defined as:

$$x_{int} = \frac{x_0}{A-1}$$

$$y_{int} = \frac{y_0}{A-1}$$

Therefore:

$$\frac{y_{int}}{x_{int}} = \frac{y_0/(A-1)}{x_0/(A-1)} = \frac{y_0}{x_0}$$

Thus:

$$\frac{y_0}{x_0} = \sqrt{P_{high} \cdot P_{low}} = \frac{y_{int}}{x_{int}}$$

Conclusion Thus, we have shown that:

$$\frac{x_0 \cdot P_{high} - y_0}{y_0/P_{low} - x_0} = P_0 = \frac{y_0}{x_0} = \sqrt{P_{high} \cdot P_{low}} = \frac{y_{int}}{x_{int}}$$

This completes the derivation and shows the equivalence of the given expressions for P_0.

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