

# DeFi's Concentrated Liquidity From Scratch

Lecture 2 of 5

Mark. B. Richardson, Ph.D.

Project Lead, Bancor



CARBON DEFI



Bancor

# <Homework Discussion>

# <Homework Discussion>

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$$

# <Homework Discussion>

We discussed these in the last lecture

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

We did not discuss this one, but you should be able to derive it.

$$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$$

$$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$$

# <Homework Discussion>

Inputs:	$y, x_0, y_0$	$x, y$	$x, x_0, y_0$
$\Delta y$	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
$\Delta x$	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

# <Homework Discussion>

Inputs:	$y, x_0, y_0$	In theory: $x, y$	$x, x_0, y_0$
	$x_0, y_0, x, y, \Delta x, \Delta y \in \mathbb{R}$		
$\Delta y$			
	$x_0, y_0, x, y, \Delta x, \Delta y \in \mathbb{Q}$		
$\Delta x$			

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$$

$$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

# <Homework Discussion>

Inputs:

$y, x_0, y_0$

$x, y$

$x, x_0, y_0$

$\Delta y$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$$

Do these equations always agree?

In practice:

$\Delta x$

$$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$x_0, y_0, x, y, \Delta x, \Delta y \in \mathbb{Q}$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

# <Homework Discussion>

Inputs:	$y, x_0, y_0$	$x, y$	$x, x_0, y_0$
$\Delta y$	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
$\Delta x$	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$



# <Homework Discussion>

inputs	'y_and_invariant'	'x_and_y'	'x_and_invariant'
'Dy'	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
'Dx'	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

# <Homework Discussion>

inputs	'y_and_invariant'	'x_and_y'	'x_and_invariant'
'Dy'	<pre>self.swap_functions = {     'Dy_from_Dx' : {         'x_and_y' : self.calculate_Dy_from_x_y_Dx,         'x_and_invariant' : self.calculate_Dy_from_x_invariant_Dx,         'y_and_invariant' : self.calculate_Dy_from_y_invariant_Dx,     },     'Dx_from_Dy' : {         'x_and_y' : self.calculate_Dx_from_x_y_Dy,         'x_and_invariant' : self.calculate_Dx_from_x_invariant_Dy,         'y_and_invariant' : self.calculate_Dx_from_y_invariant_Dy,     }, }</pre>		
'Dx'			

# <Homework Discussion>

```
create_static_trade_plot(  
    token_pair = {'x' : 'F00', 'y' : 'BAR'},  
    x_0 = 1234,  
    y_0 = 5678,  
    trade_actions = [('Dx', +100),  
                     ('Dy', +321),  
                     ('Dx', -123),  
                     ('Dy', -321),  
                     ('Dx', +150),  
                     ('Dy', +500),  
                     ('Dx', -150),  
                     ('Dy', -600)],  
)
```

## Input Information:

[illegible]

## Output Calculations:

Function	Output
x_and_y	-425.63718140929535138639039359986782073974609375000000000000000000
x_and_invariant	-425.63718140929535138639039359986782073974609375000000000000000000
y_and_invariant	-425.63718140929535138639039359986782073974609375000000000000000000

No discrepancies detected.

Discrepancy check completed.

## Input Information:

[illegible]

## Output Calculations:

Function	Output
x_and_y	-76.83224902775651798947365023195743560791015625000000000000000000
x_and_invariant	-76.83224902775651798947365023195743560791015625000000000000000000
y_and_invariant	-76.83224902775651798947365023195743560791015625000000000000000000

No discrepancies detected.

Discrepancy check completed.

## Input Information:

[illegible]

## Output Calculations:

Function	Output
x_and_y	604.42877704731472476851195096969604492187500000000000000000000000
x_and_invariant	604.42877704731483845534967258572578430175781250000000000000000000
y_and_invariant	604.42877704731483845534967258572578430175781250000000000000000000

Selected most positive value: 604.428777047314838

Discrepancy check completed.

## Input Information:

[illegible]

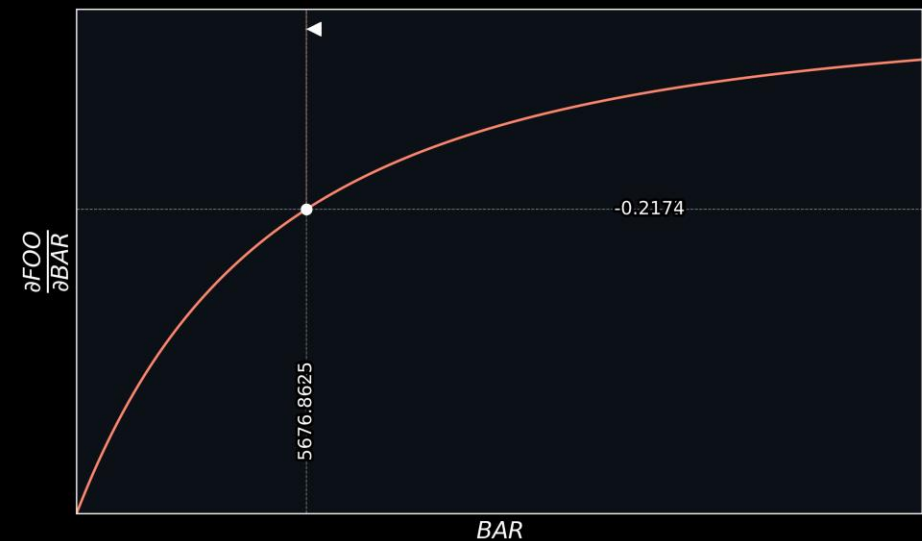
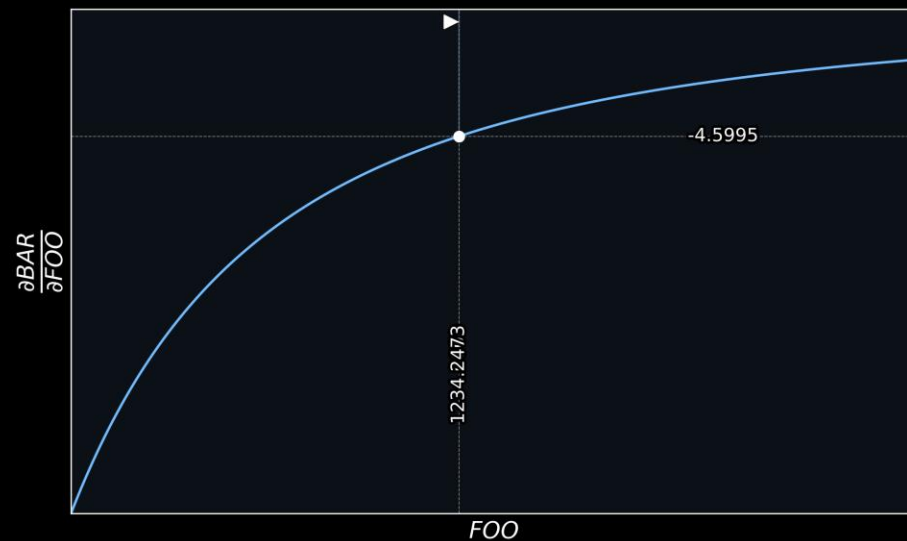
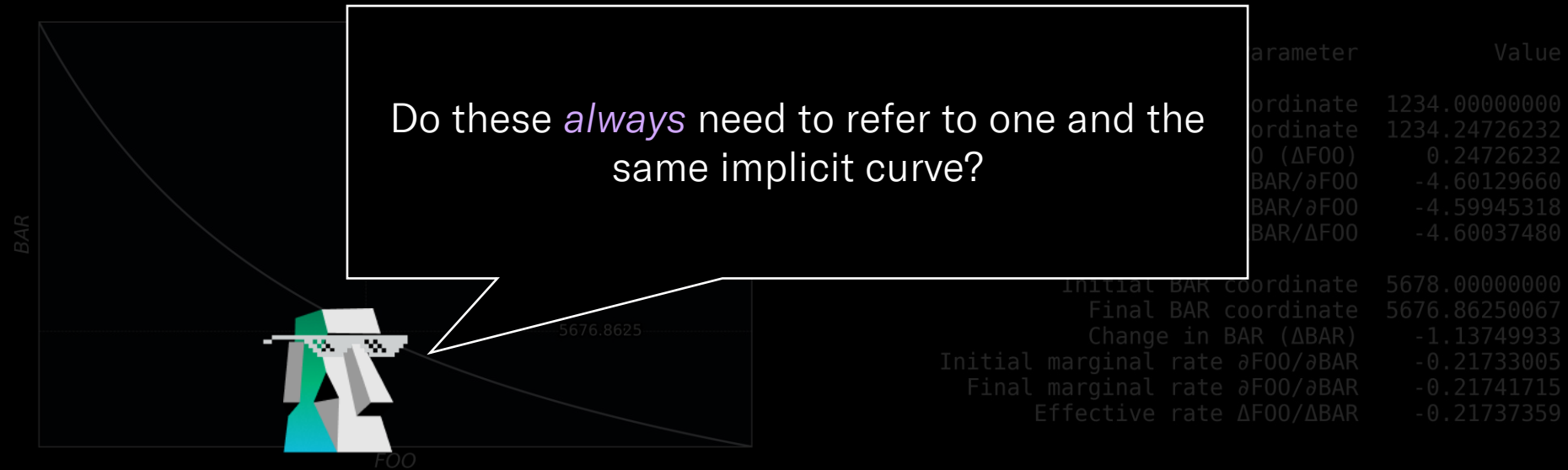
## Output Calculations:

	Function	Output
x_and_y		109.701225262349609579359821509569883346557617187500000000000000000000
x_and_invariant		109.701225262349595368505106307566165924072265625000000000000000000000
y_and_invariant		109.701225262349595368505106307566165924072265625000000000000000000000

```
Selected most positive value: 109.701225262349610
```

Discrepancy check completed.

[We will return to this point in Lecture 4]





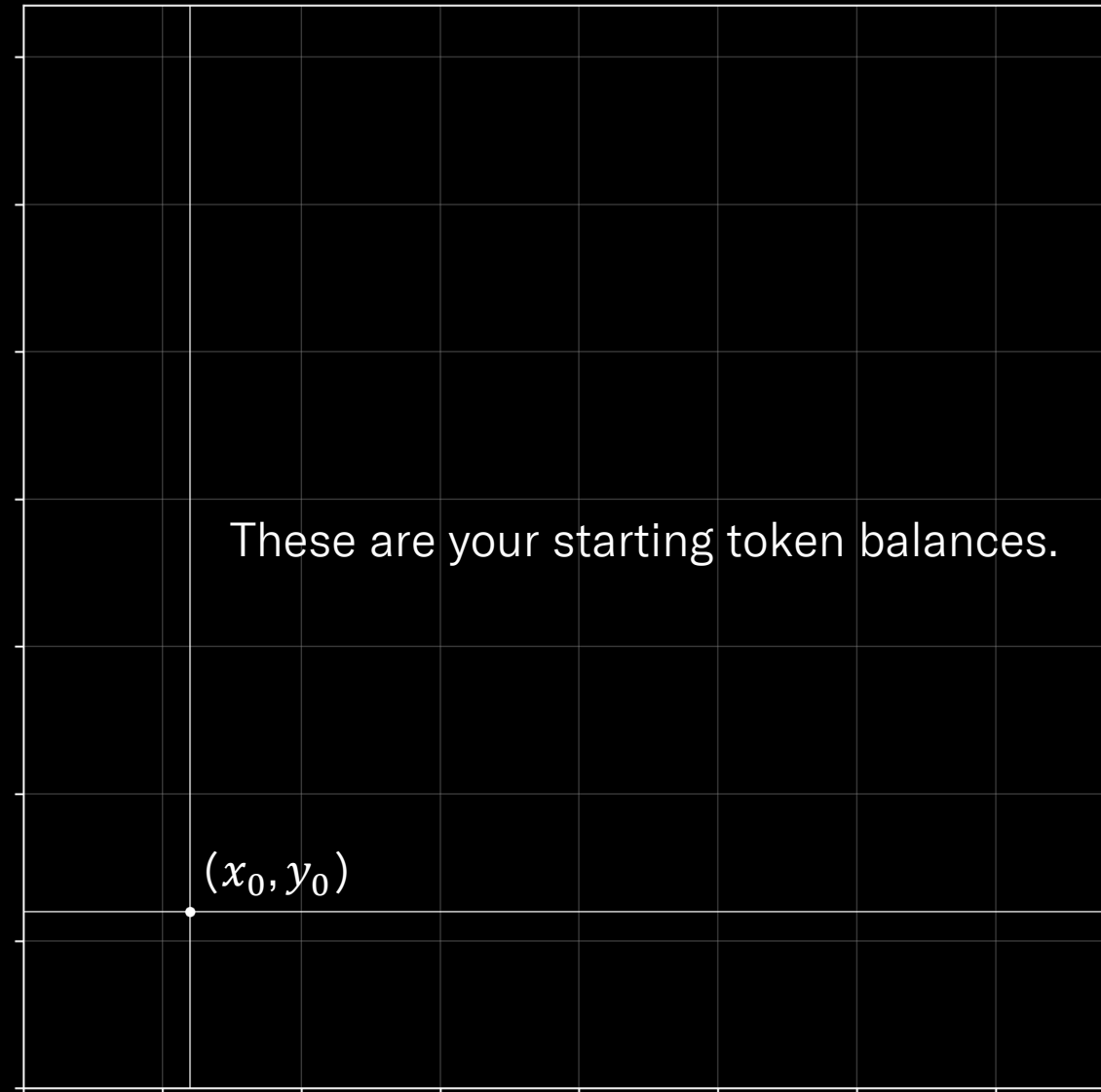
**</Homework Discussion>**

# <Lecture 2>

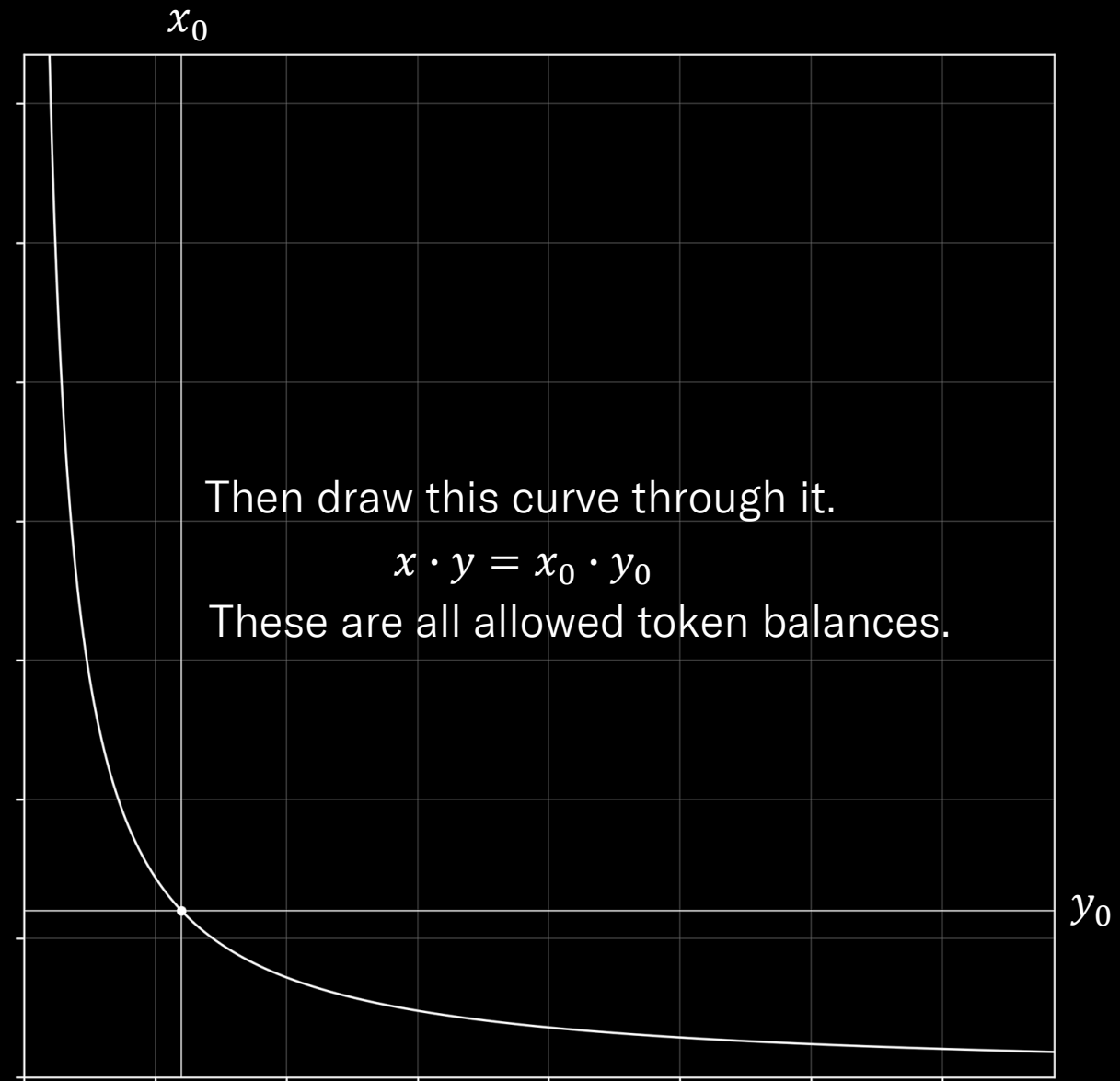
Recall from the first lecture:



Recall from the first lecture:

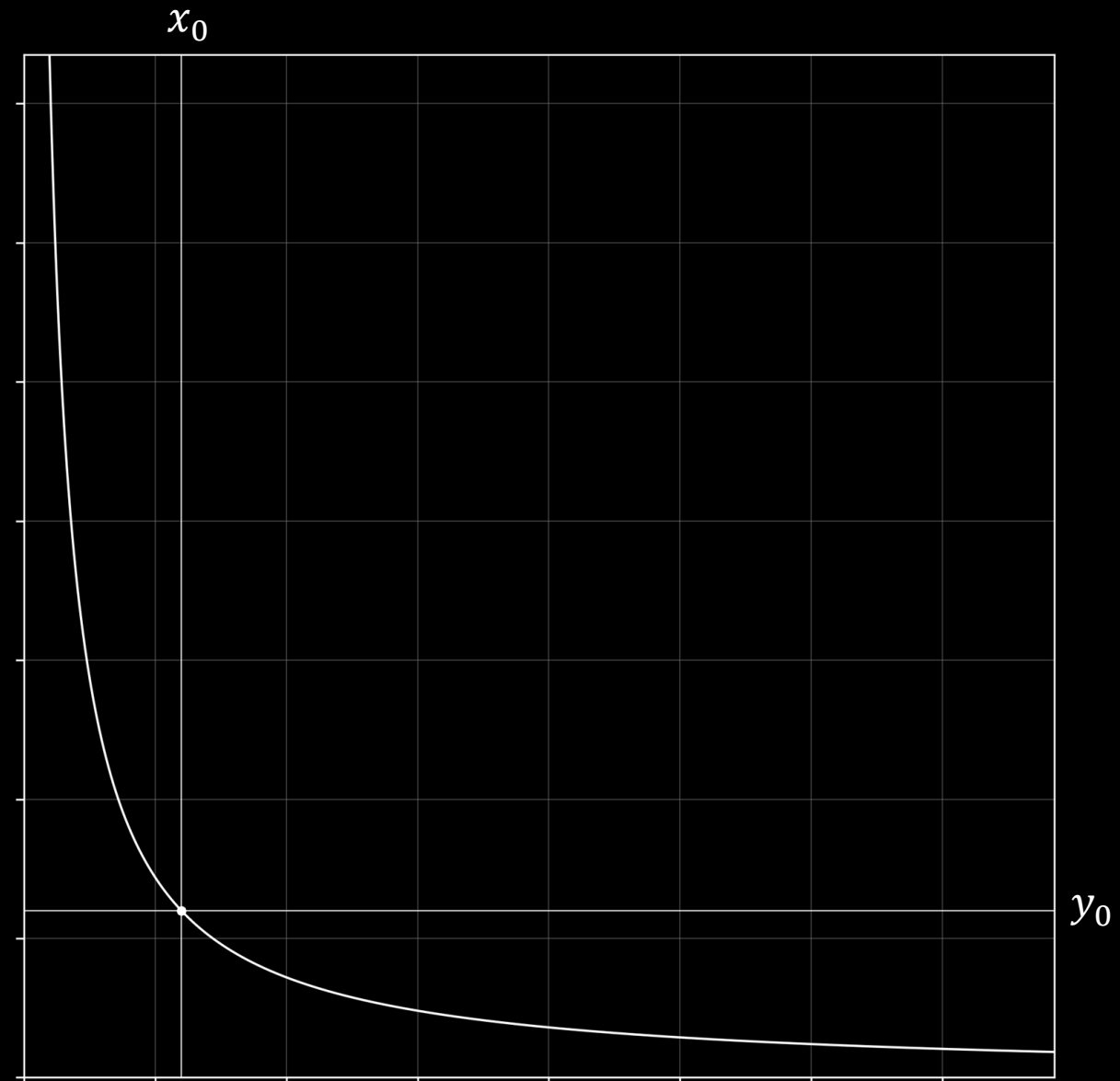


Recall from the first lecture:



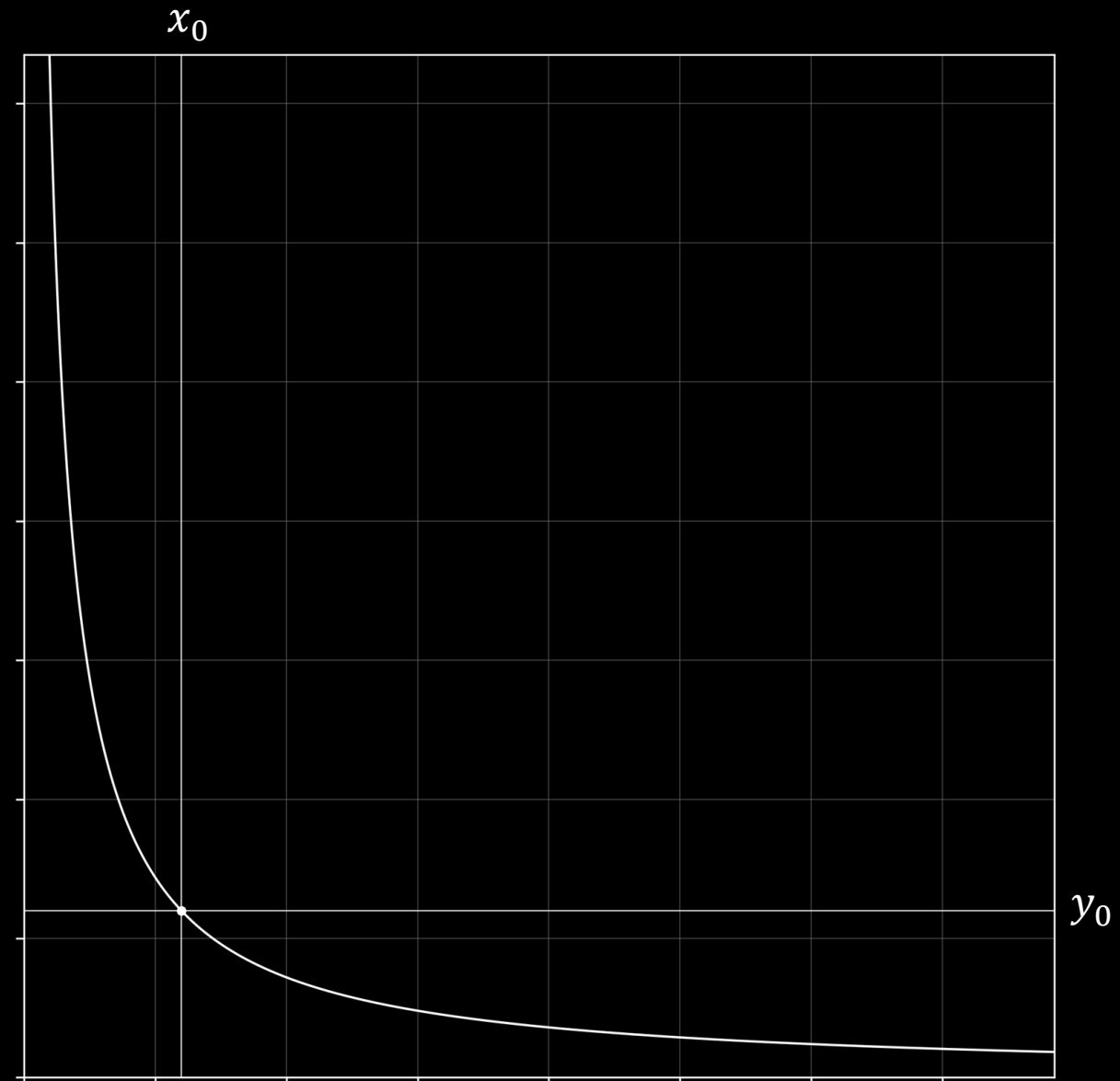
Recall from the first lecture:

$$x \cdot y = x_0 \cdot y_0$$



Recall from the first lecture:

$$x = \frac{x_0 \cdot y_0}{y}$$

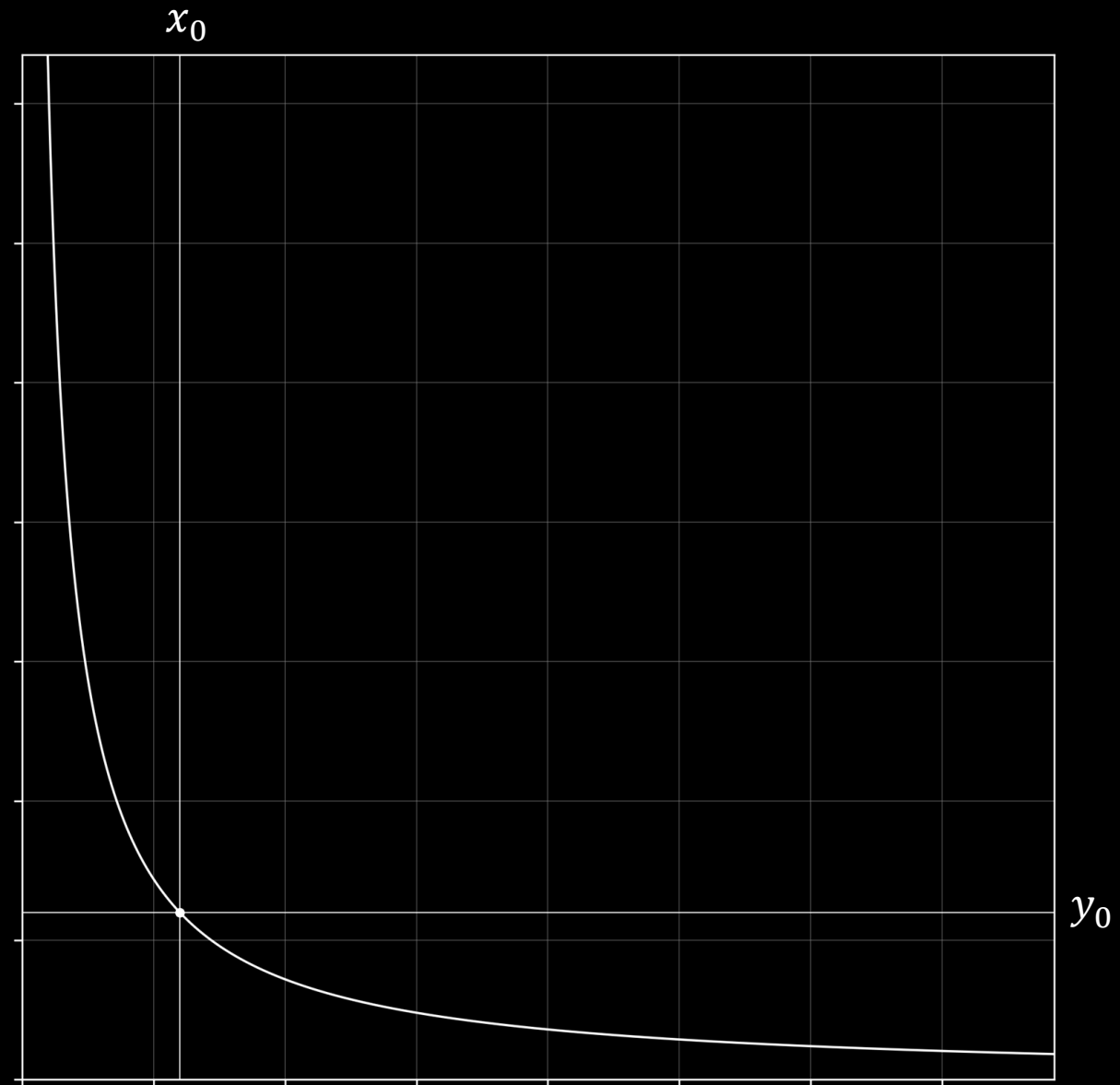


Recall from the first lecture:

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

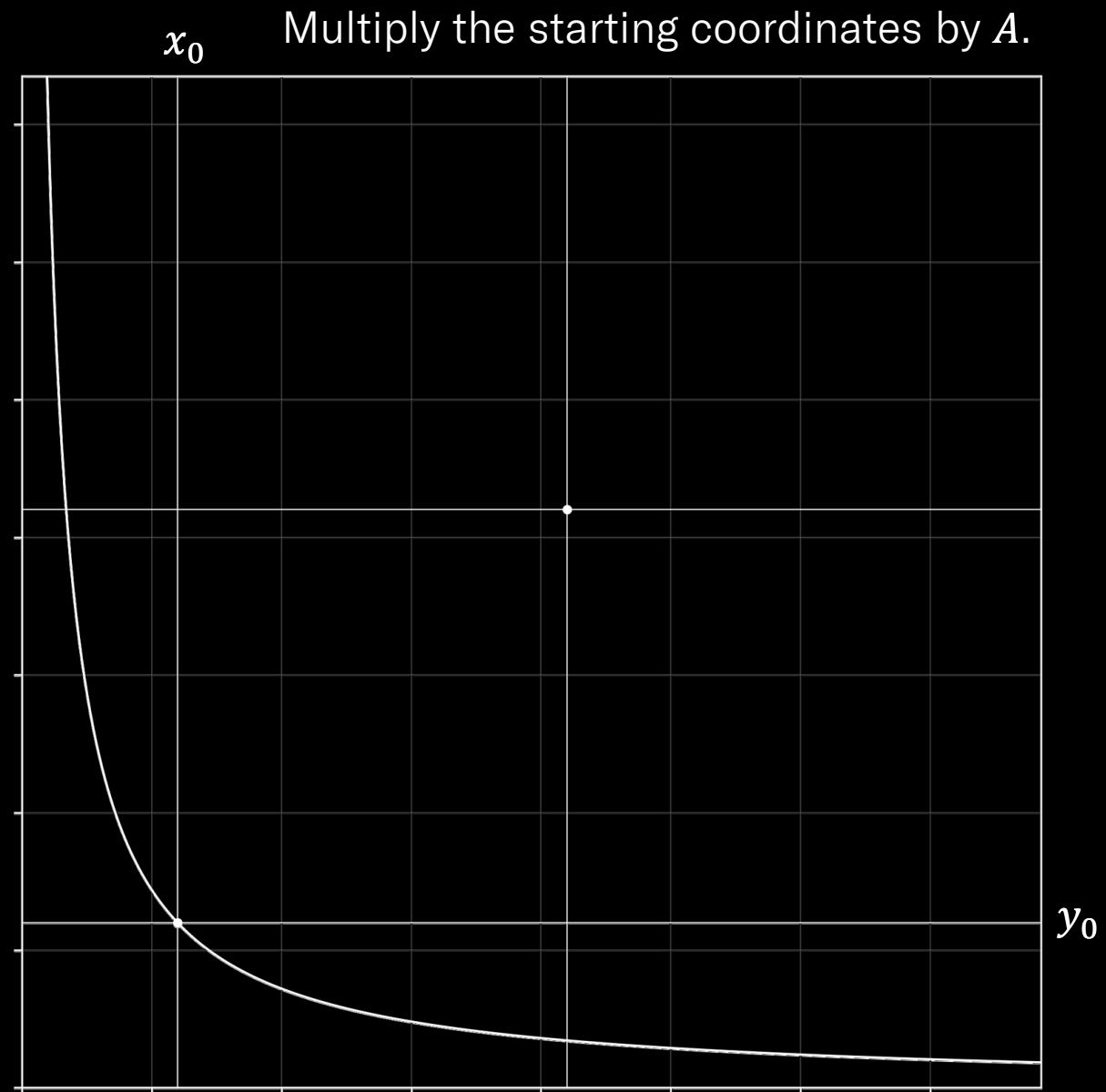




$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

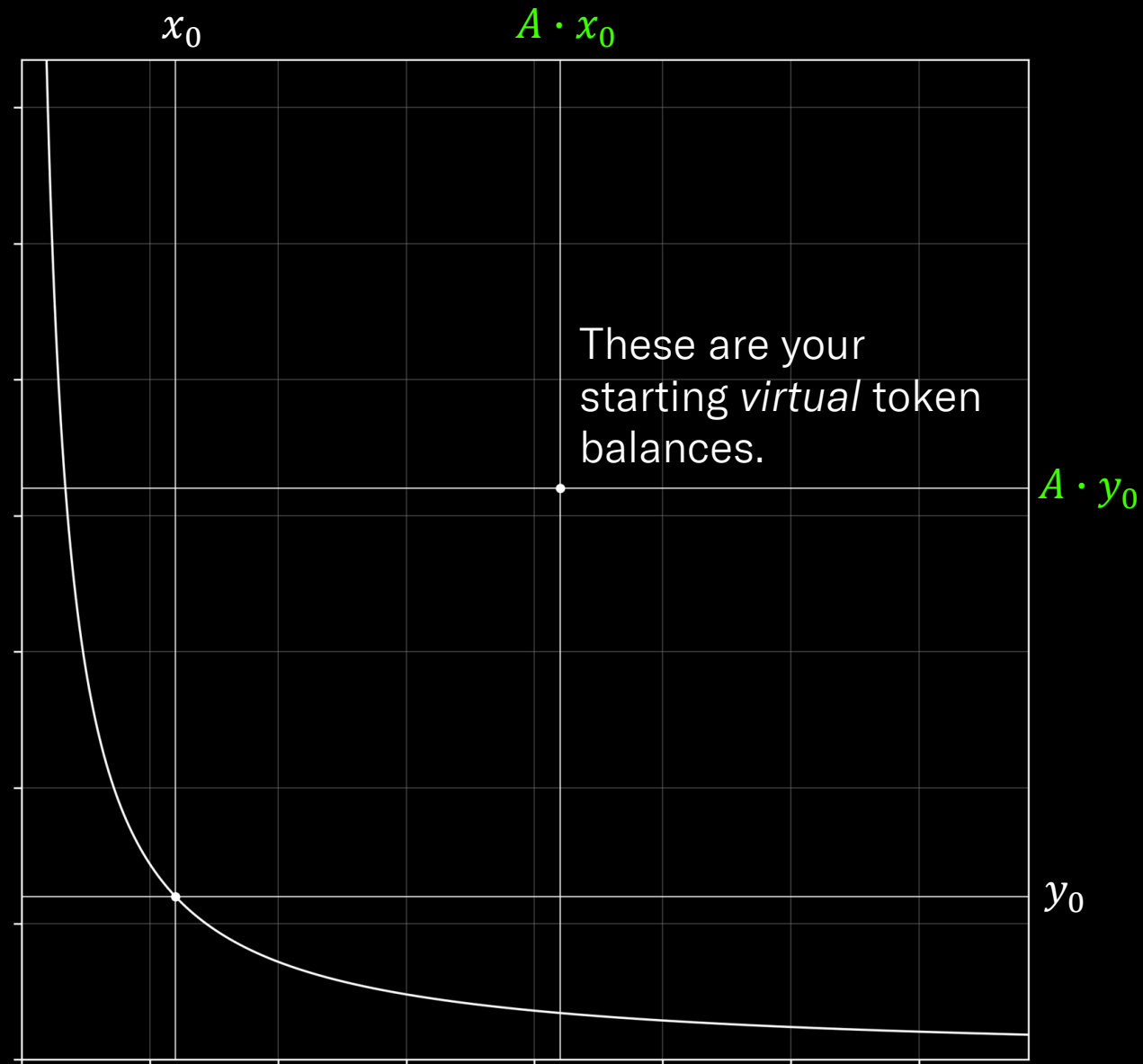
$$y = \frac{x_0 \cdot y_0}{x}$$



$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

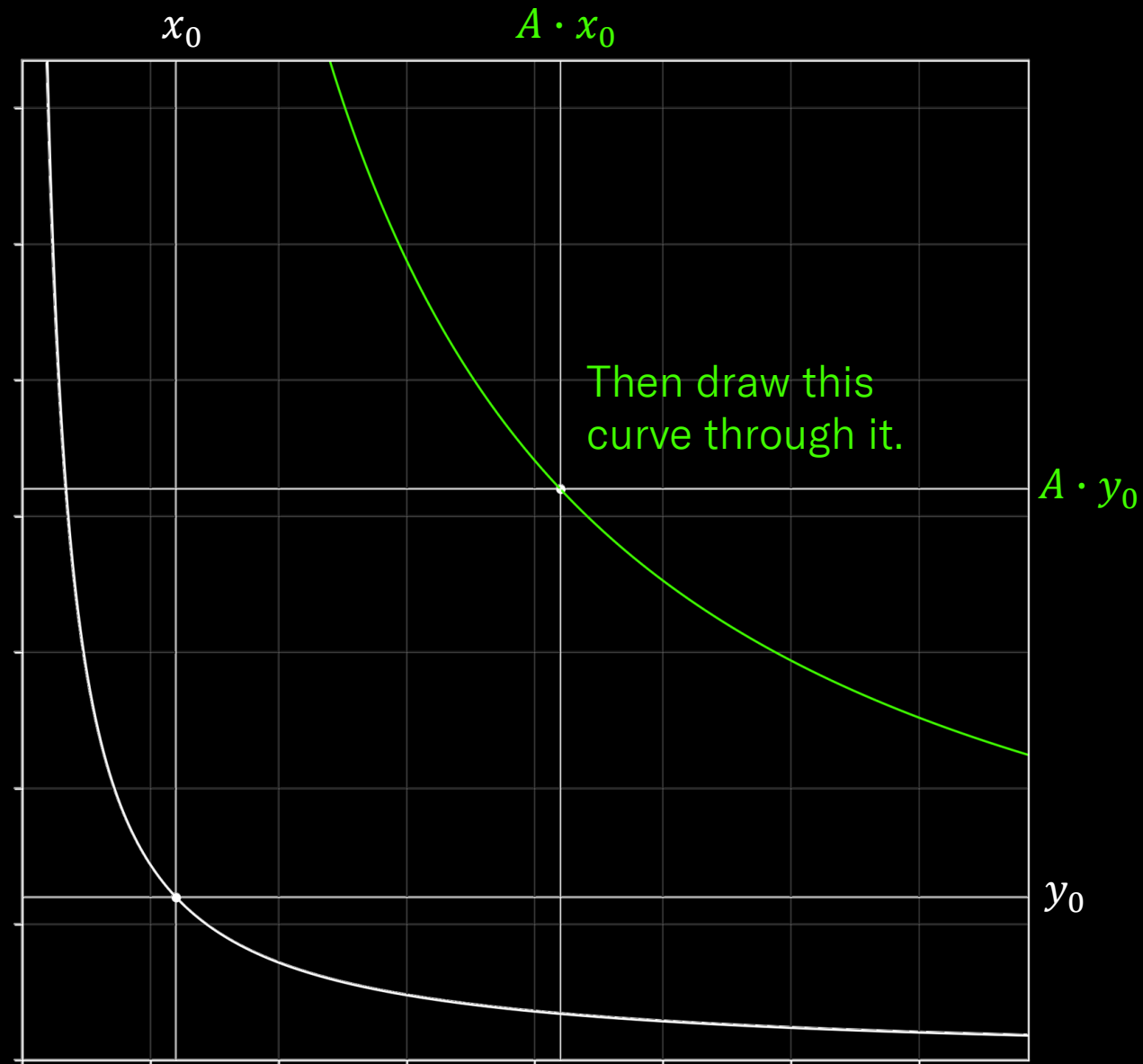
$$y = \frac{x_0 \cdot y_0}{x}$$



$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

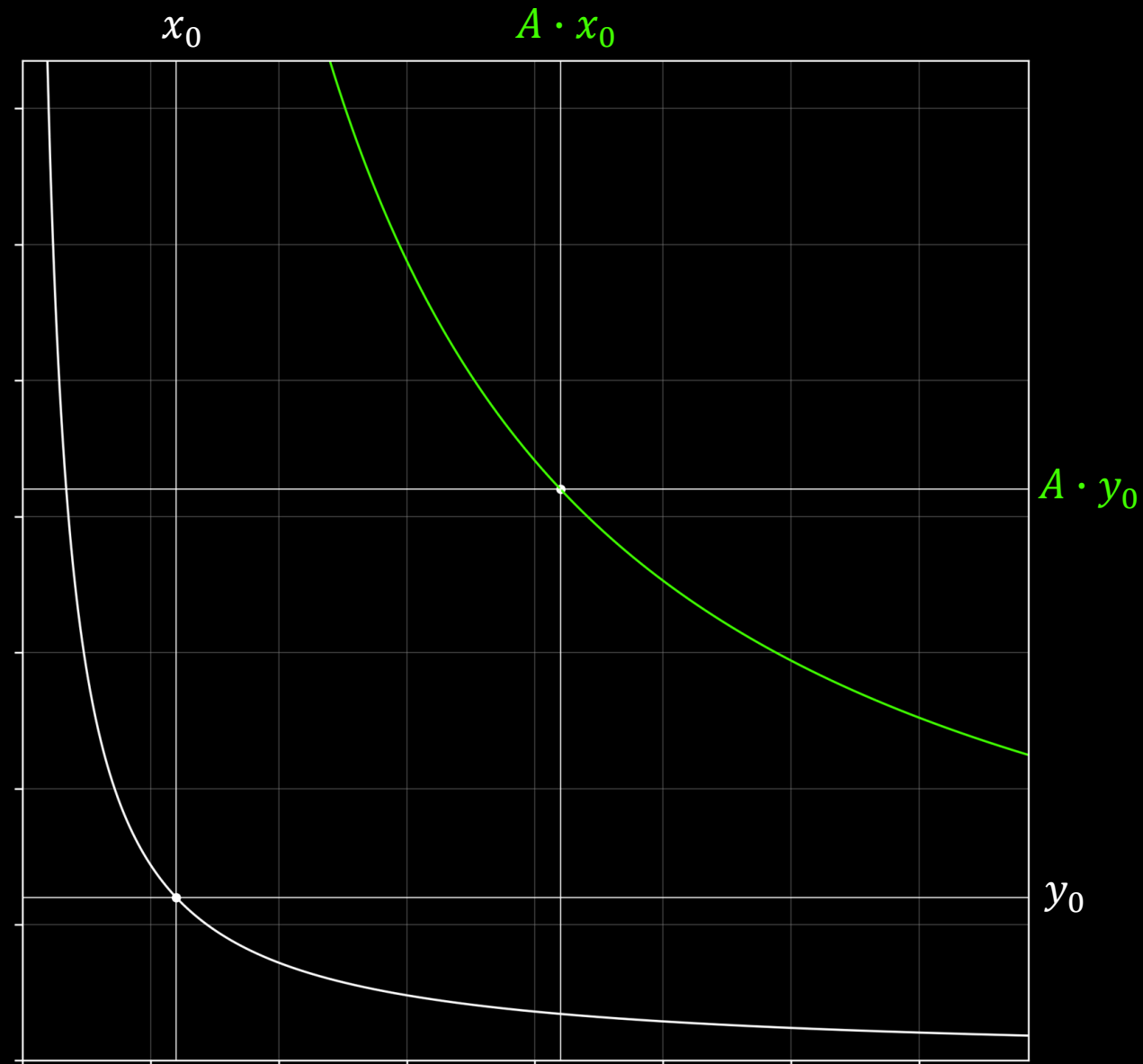


$$x \cdot y = A \cdot x_0 \cdot A \cdot y_0$$

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$



$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$A \cdot y_0$$

$$y_0$$

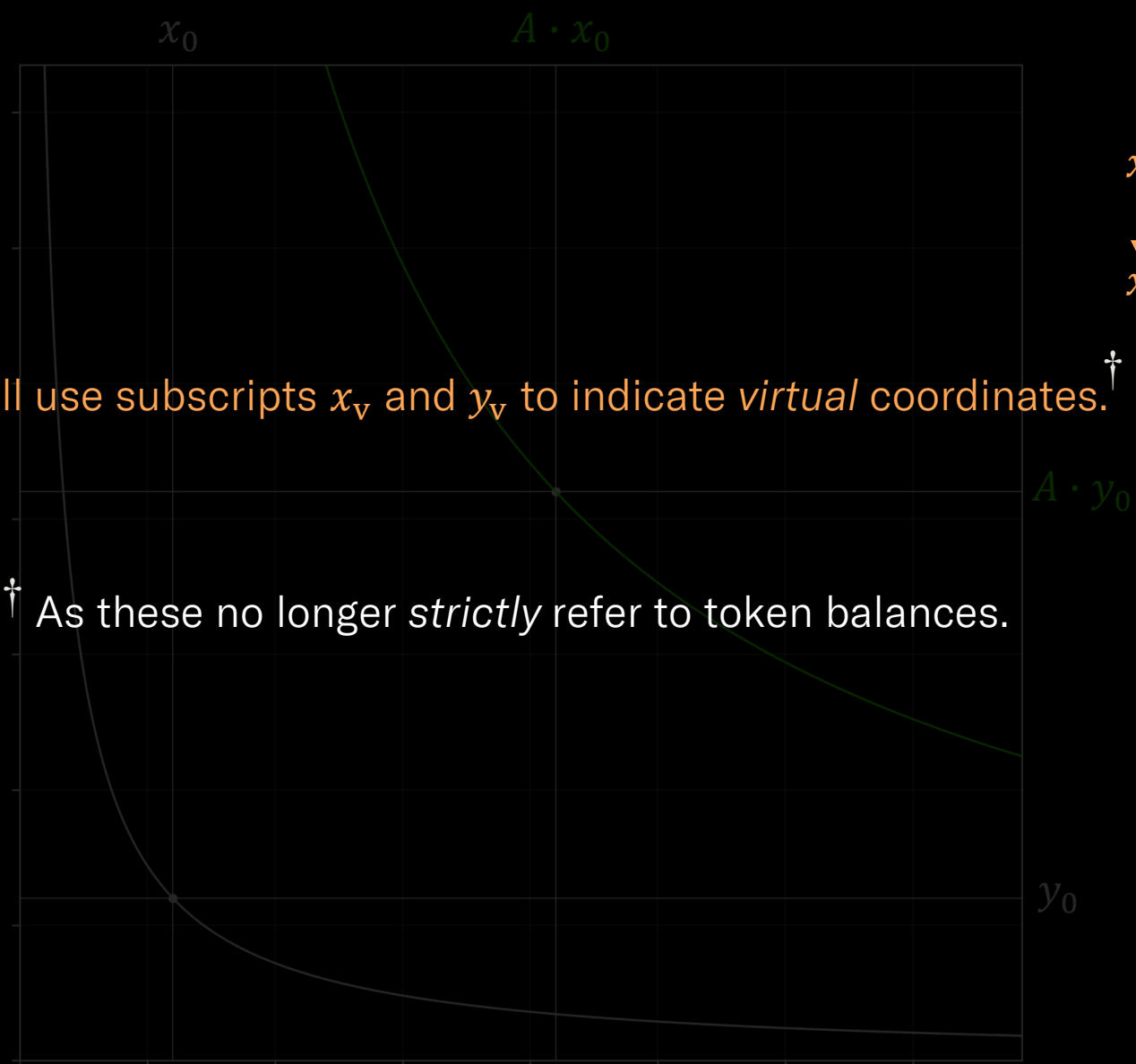
$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

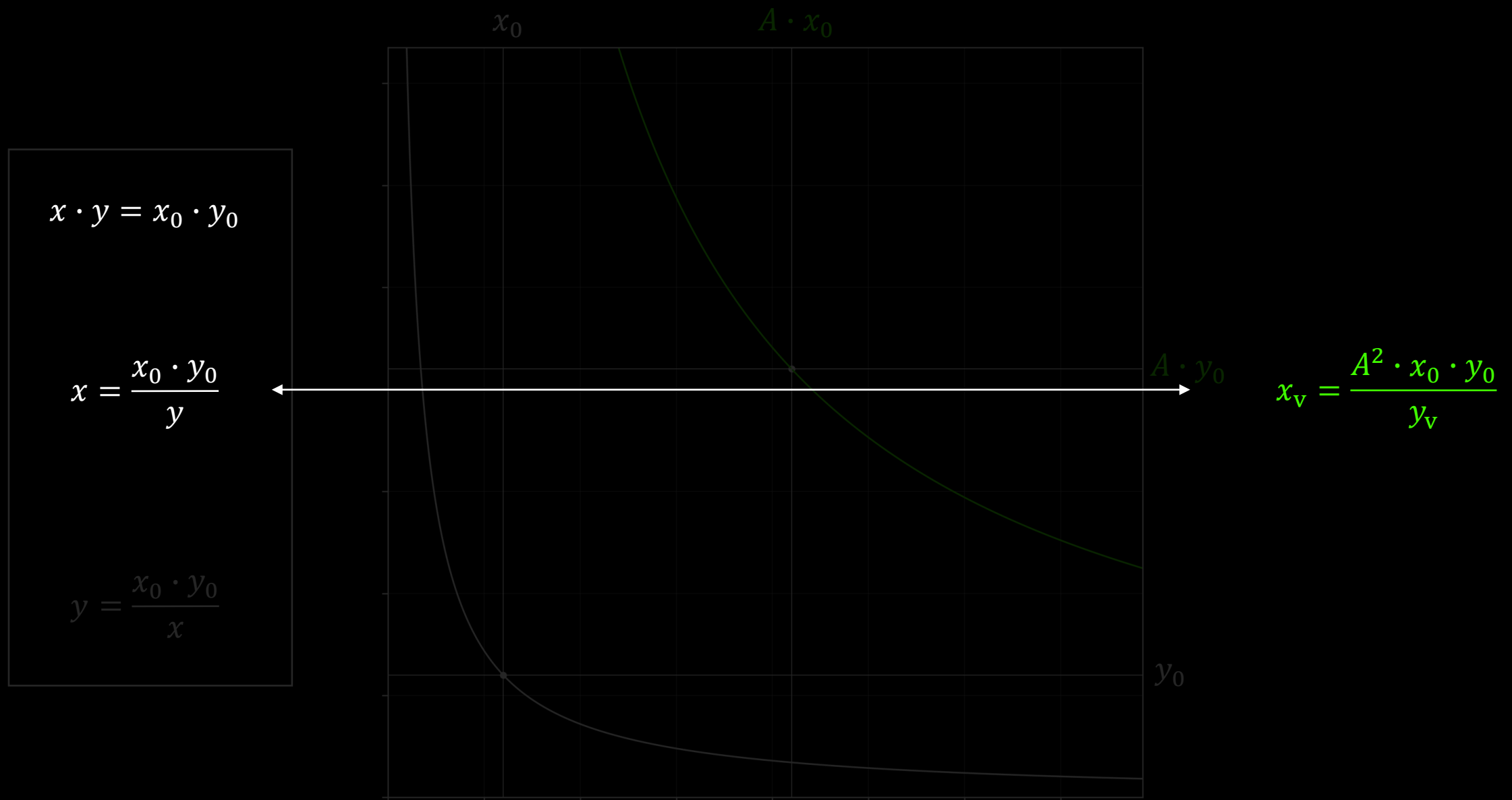
$$y = \frac{x_0 \cdot y_0}{x}$$

I will use subscripts  $x_v$  and  $y_v$  to indicate *virtual* coordinates.<sup>†</sup>

<sup>†</sup> As these no longer *strictly* refer to token balances.



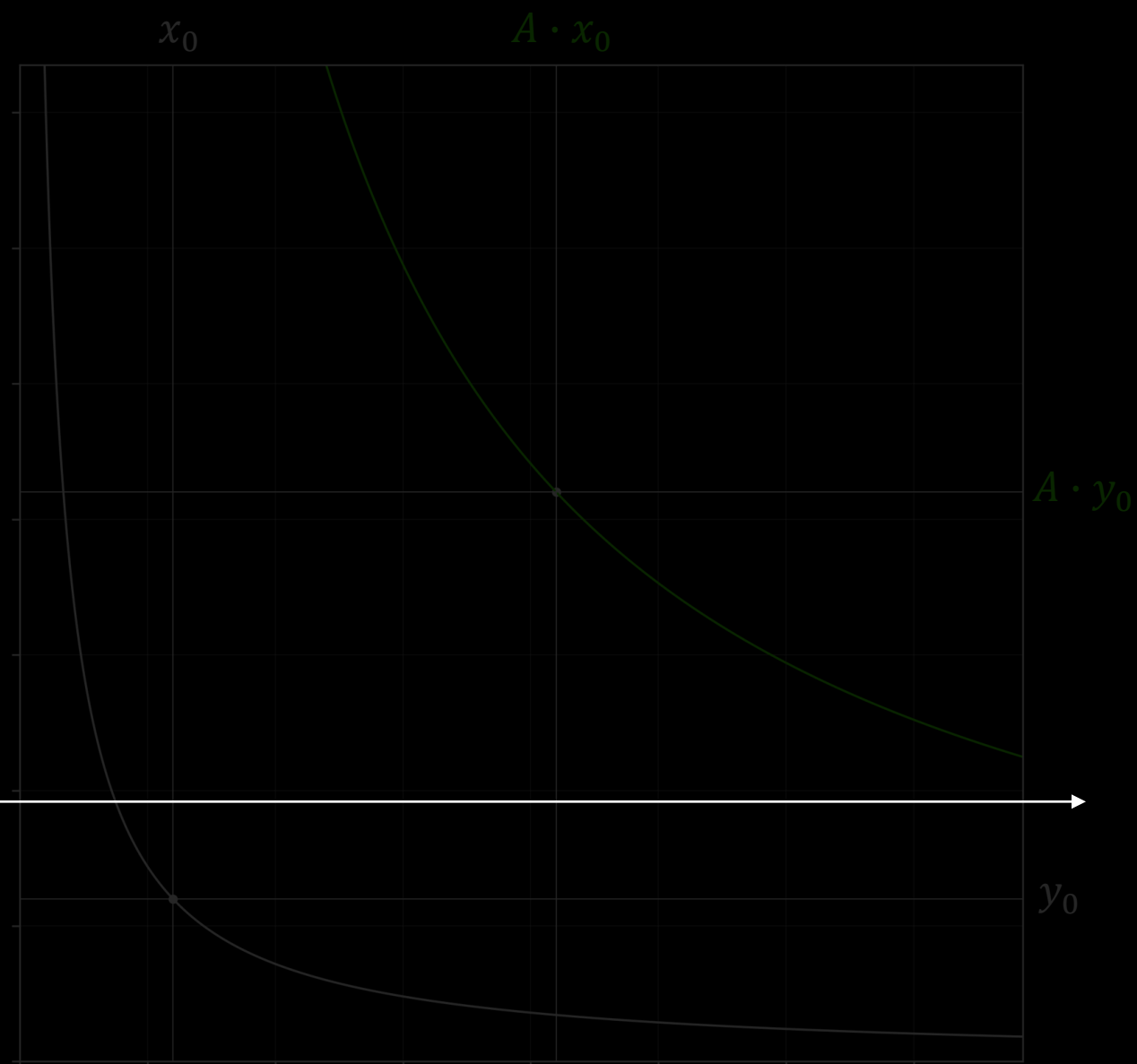




$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$



$$y_v = \frac{A^2 \cdot x_0 \cdot y_0}{x_v}$$



$$x \cdot y = x_0 \cdot y_0$$

Inputs:	$y, x_0, y_0$	$x, y$	$x, x_0, y_0$
$\Delta y$	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
$\Delta x$	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

$$x_v \cdot y_v = A^2 \cdot x_0 \cdot y_0$$

Inputs:	$y_v, A, x_0, y_0$	$x_v, y_v$	$x_v, A, x_0, y_0$
$\Delta y$	$\Delta x = -\frac{\Delta y \cdot A^2 \cdot x_0 \cdot y_0}{y_v \cdot (y_v + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x_v}{y_v + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x_v^2}{\Delta y \cdot x_v + A^2 \cdot x_0 \cdot y_0}$
$\Delta x$	$\Delta y = -\frac{\Delta x \cdot y_v^2}{\Delta x \cdot y_v + A^2 \cdot x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y_v}{x_v + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot A^2 \cdot x_0 \cdot y_0}{x_v \cdot (x_v + \Delta x)}$

$$x_v \cdot y_v = A^2 \cdot x_0 \cdot y_0$$

The only difference here is that the new invariant is larger by a factor of  $A^2$ .

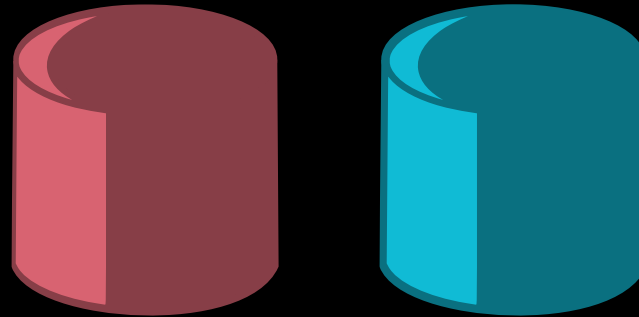
Inputs:	$y_v, A, x_0, y_0$	$x_v, y_v$	$x_v, A, x_0, y_0$
$\Delta y$	$\Delta x = -\frac{\Delta y \cdot A^2 \cdot x_0 \cdot y_0}{y_v \cdot (y_v + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x_v}{y_v + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x_v^2}{\Delta y \cdot x_v + A^2 \cdot x_0 \cdot y_0}$
$\Delta x$	$\Delta y = -\frac{\Delta x \cdot y_v^2}{\Delta x \cdot y_v + A^2 \cdot x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y_v}{x_v + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot A^2 \cdot x_0 \cdot y_0}{x_v \cdot (x_v + \Delta x)}$

$$x_v \cdot y_v = A^2 \cdot x_0 \cdot y_0$$

Inputs:	$y_v, A, x_0, y_0$	$x_v, y_v$	$x_v, A, x_0, y_0$
$\Delta y$	$\Delta x = -\frac{\Delta y \cdot A^2 \cdot x_0 \cdot y_0}{y_v \cdot (y_v + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x_v}{y_v + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x_v^2}{\Delta y \cdot x_v + A^2 \cdot x_0 \cdot y_0}$
$\Delta x$	$\Delta y = -\frac{\Delta x \cdot y_v^2}{\Delta x \cdot y_v + A^2 \cdot x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y_v}{x_v + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot A^2 \cdot x_0 \cdot y_0}{x_v \cdot (x_v + \Delta x)}$

What does it mean?

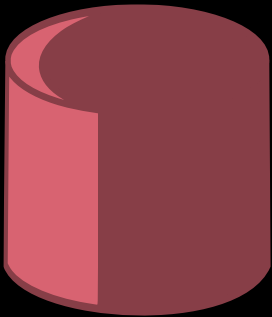
Imagine there are two smart contract systems (i.e. “liquidity pools”), using the same pricing algorithms we are studying here.



Both contain the tokens FOO, and BAR.

One of them has 100 tokens in each balance.

FOO	BAR
100	100



And the other has 10 × more of both.

FOO	BAR
1,000	1,000



One of them has 100 tokens in each balance.

And the other has 10 × more of both.

FOO	BAR
100	100

$$\frac{\partial \text{FOO}}{\partial \text{BAR}} = -\frac{100}{100} = -1$$



FOO	BAR
1,000	1,000

$$\frac{\partial \text{FOO}}{\partial \text{BAR}} = -\frac{1,000}{1,000} = -1$$

Both systems are quoting the same marginal rate.

*But* - consider the rate for a non-zero swap amount.

$$\Delta \text{FOO} = +25$$



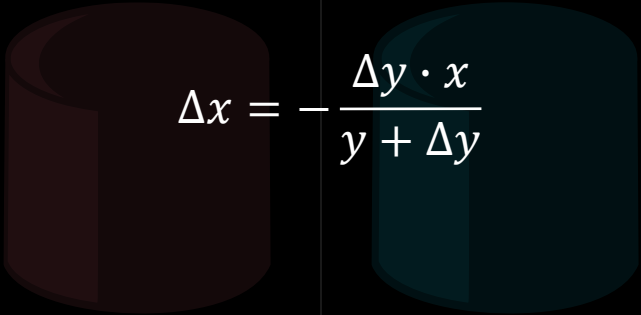
One of them has 100 tokens in each balance.

And the other has 10 × more of both.

FOO	BAR
100	100

$$\Delta \text{FOO} = +25$$

FOO	BAR
1,000	1,000


$$\Delta x = - \frac{\Delta y \cdot x}{y + \Delta y}$$

One of them has 100 tokens in each balance.

And the other has 10 × more of both.

FOO	BAR
100	100

$$\Delta\text{FOO} = +25$$

FOO	BAR
1,000	1,000


$$\Delta\text{BAR} = -\frac{\Delta\text{FOO} \cdot \text{BAR}}{\text{FOO} + \Delta\text{FOO}}$$

One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta \text{BAR} = - \frac{\Delta \text{FOO} \cdot \text{BAR}}{\text{FOO} + \Delta \text{FOO}}$$

$$\Delta \text{FOO} = +25$$



And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta \text{BAR} = - \frac{\Delta \text{FOO} \cdot \text{BAR}}{\text{FOO} + \Delta \text{FOO}}$$

One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta \text{BAR} = - \frac{25 \cdot \text{BAR}}{\text{FOO} + 25}$$



And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta \text{BAR} = - \frac{25 \cdot \text{BAR}}{\text{FOO} + 25}$$



One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta \text{BAR} = -\frac{25 \cdot 100}{100 + 25}$$



And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta \text{BAR} = -\frac{25 \cdot 1,000}{1,000 + 25}$$



One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta \text{BAR} = -\frac{2,500}{125}$$



And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta \text{BAR} = -\frac{25,000}{1,025}$$



One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta \text{BAR} = -\frac{2,500}{125} = -20$$



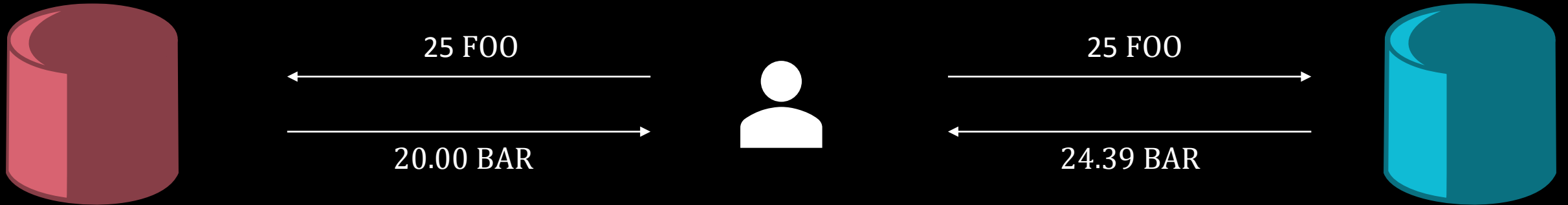
And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta \text{BAR} = -\frac{25,000}{1,025} = -24.39$$

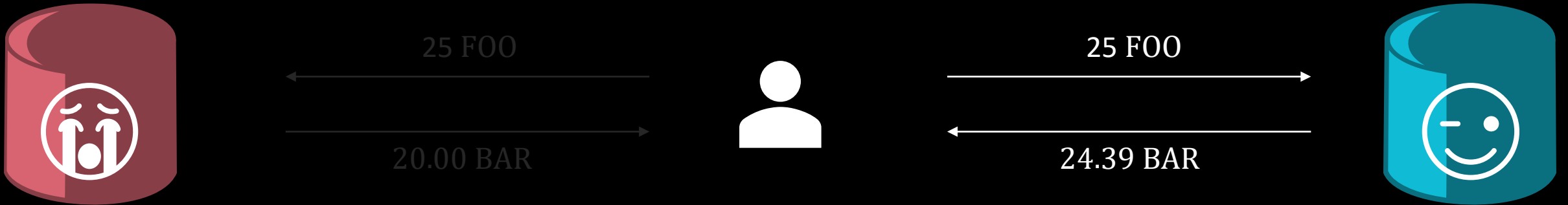


Which offer is better for the consumer?





Which offer is better for the consumer?



Will you come back if I offer the same price?



25 F00



20.00 BAR



25 F00



24.39 BAR





What if I always offer the same price as a competitor with 10× more liquidity than me?



25 F00

24.39 BAR



25 F00

24.39 BAR



What if I just use 10% of my liquidity and offer the same prices?



25 F00

24.39 BAR



25 F00

24.39 BAR



What if I always offer the same price as a competitor with 10× more liquidity than me?

What if I just use 10% of my liquidity and offer the same prices?



25 FOO  
24.39 BAR



25 FOO  
24.39 BAR



The short version is this:

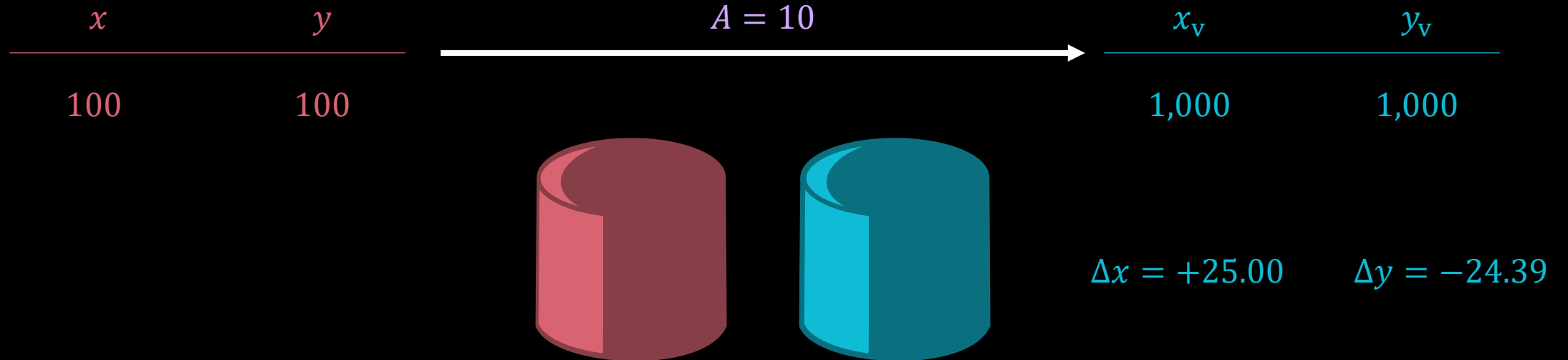
We can re-work the pricing algorithm to allow *any* liquidity pool to emulate an *arbitrary size*.

What's the catch?

REAL

VIRTUAL

“Amplification Coefficient” or “Capital Efficiency Term”



REAL

VIRTUAL

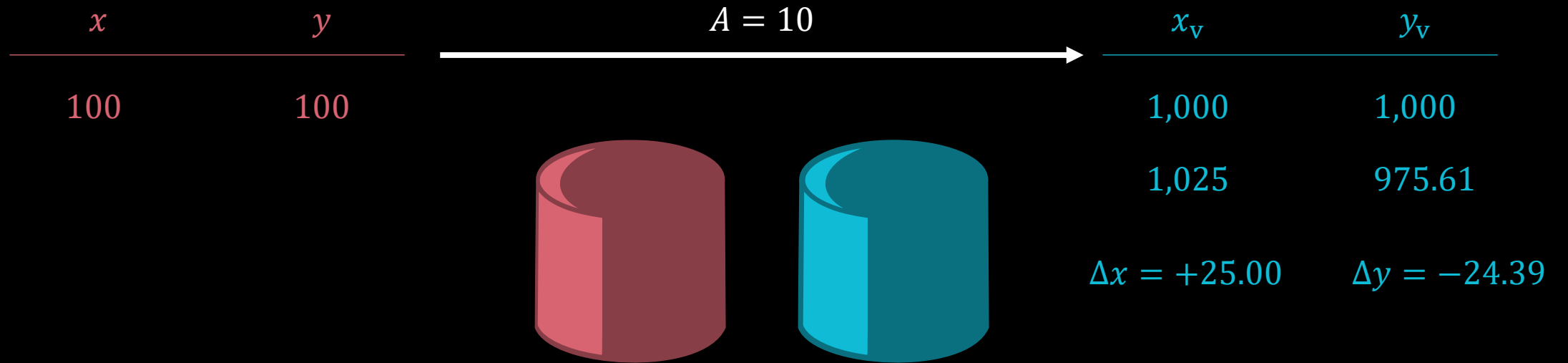


The trade is calculated on the virtual side.



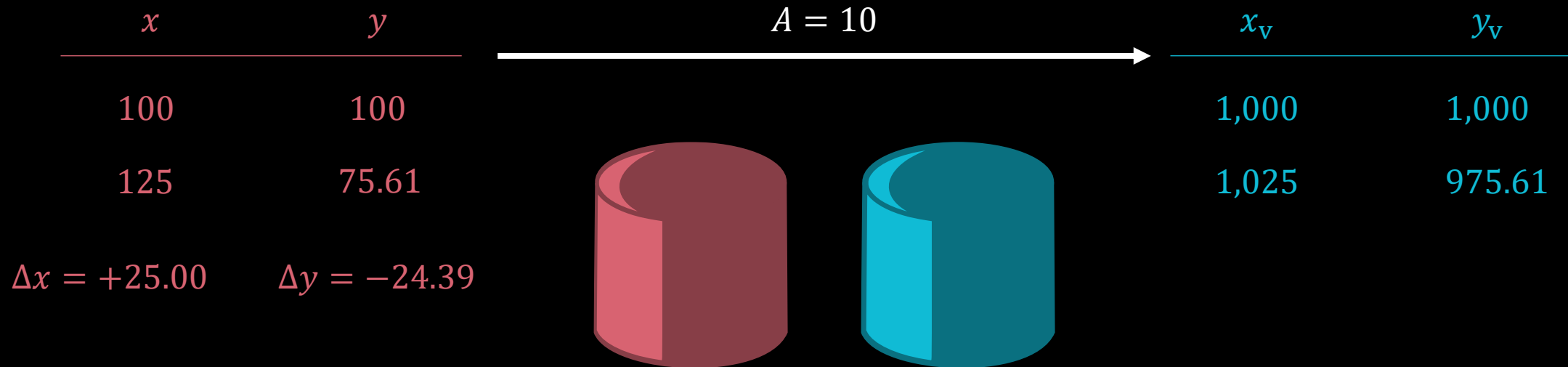
REAL

VIRTUAL



REAL

VIRTUAL



REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
$\Delta x = +25.00$	$\Delta y = -24.39$				



Then, the trade amounts are applied verbatim to the real token balances.



REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61

$10,000 \neq 9451.22$

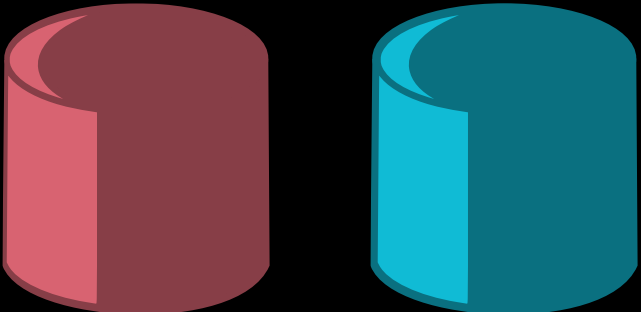


Note that this means the general invariant  
no longer applies.

(We need to determine a new one.)

REAL

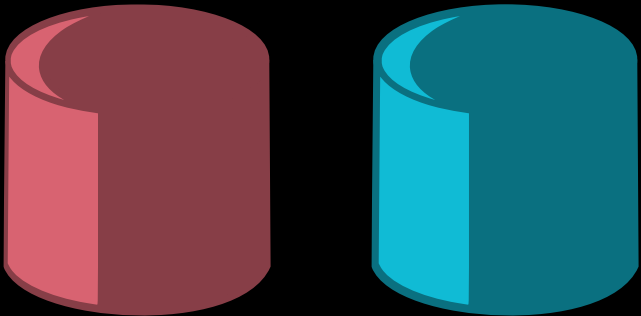
VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
				1,111.11	900.00
				$\Delta x = +86.11$	$\Delta y = -75.61$

REAL

VIRTUAL


$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
$\Delta x = +86.11$	$\Delta y = -75.61$				



REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
$\Delta x = +86.11$	$\Delta y = -75.61$				




Since the general invariant is no longer applicable, token balances can be driven to zero.



REAL

VIRTUAL

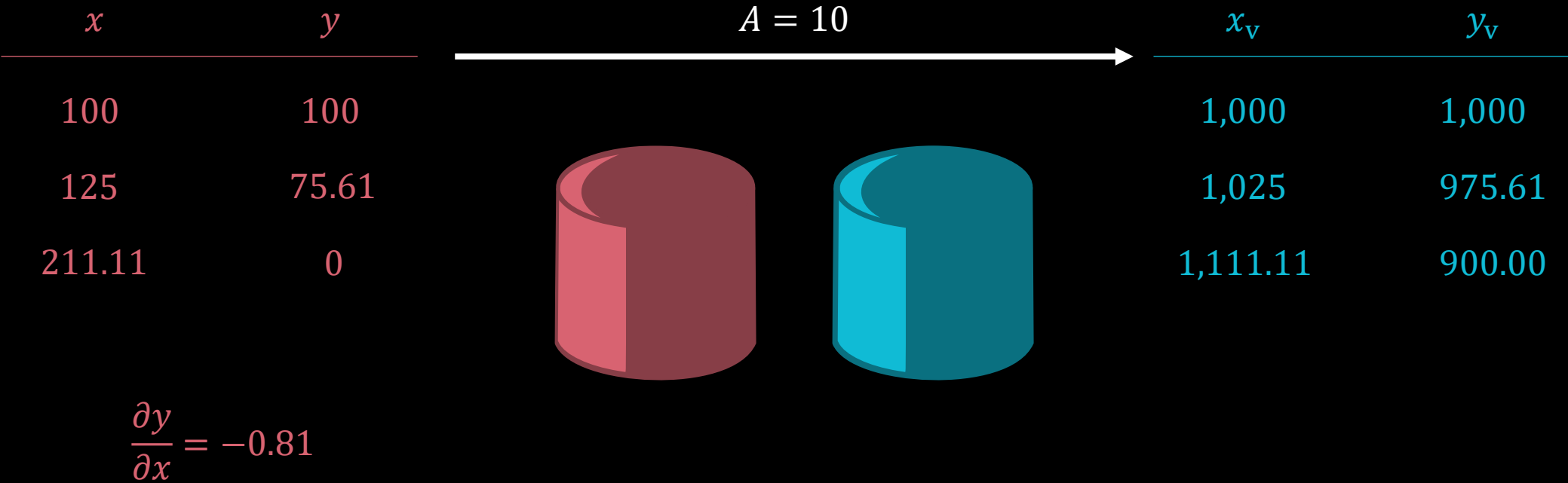
$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00



$$\frac{\partial y}{\partial x} = -\frac{900.00}{1,111.11} = -0.81$$

REAL

VIRTUAL



REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00

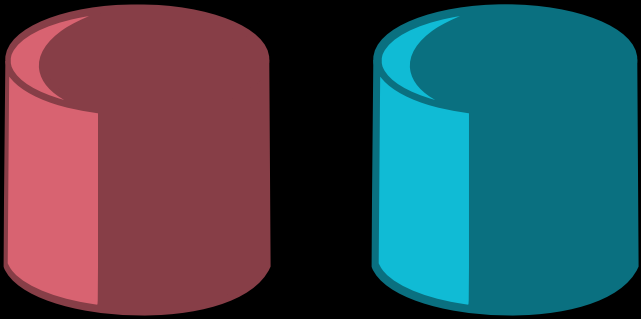
$$\frac{\partial y}{\partial x} = -0.81$$

As the  $x$  balance is now zero, marginal rates beyond this value are impossible to reach.

REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00



$$\frac{\partial y}{\partial x} = -0.81$$

REAL

VIRTUAL

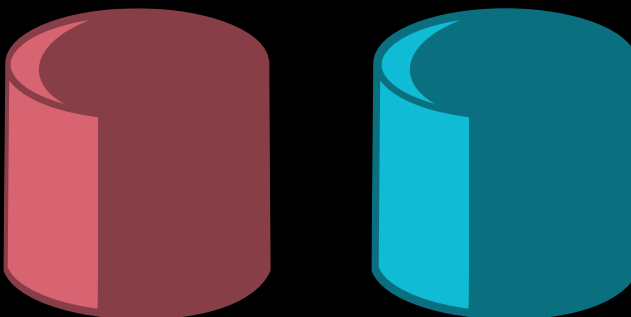
$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
				900.00	1,111.11
				$\Delta x = -211.11$	$\Delta y = +211.11$

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
0	211.11			900.00	1,111.11



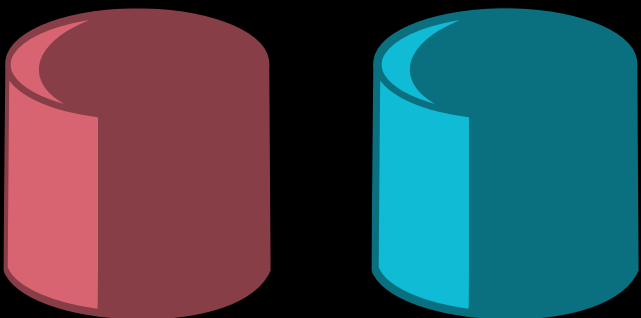
$$\Delta x = -211.11 \quad \Delta y = +211.11$$

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
0	211.11			900.00	1,111.11



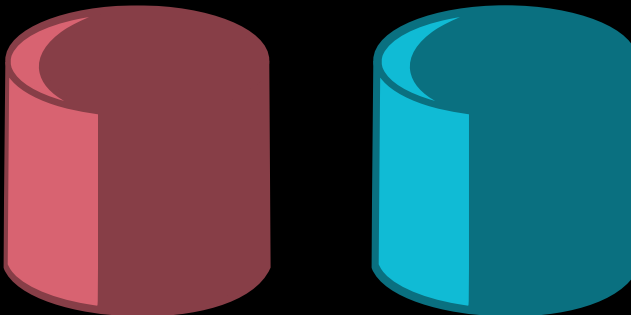
$$\frac{\partial y}{\partial x} = -\frac{1,111.11}{900.00} = -1.23$$

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
0	211.11			900.00	1,111.11



$$\frac{\partial y}{\partial x} = -1.23$$

$$\frac{\partial y}{\partial x} = -0.81$$





REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
0	211.11			900.00	1,111.11



The amplified pool can only service the market between these price bounds.

$$\frac{\partial y}{\partial x} = -1.23$$

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

VIRTUAL

$x$	$y$	$A = 10$		$x_v$	$y_v$
100	100			1,000	1,000
125	75.61			1,025	975.61
211.11	0			1,111.11	900.00
0	211.11			900.00	1,111.11



$$\frac{\partial y}{\partial x} = -1.23$$

$$\frac{\partial y}{\partial x} = -0.81$$

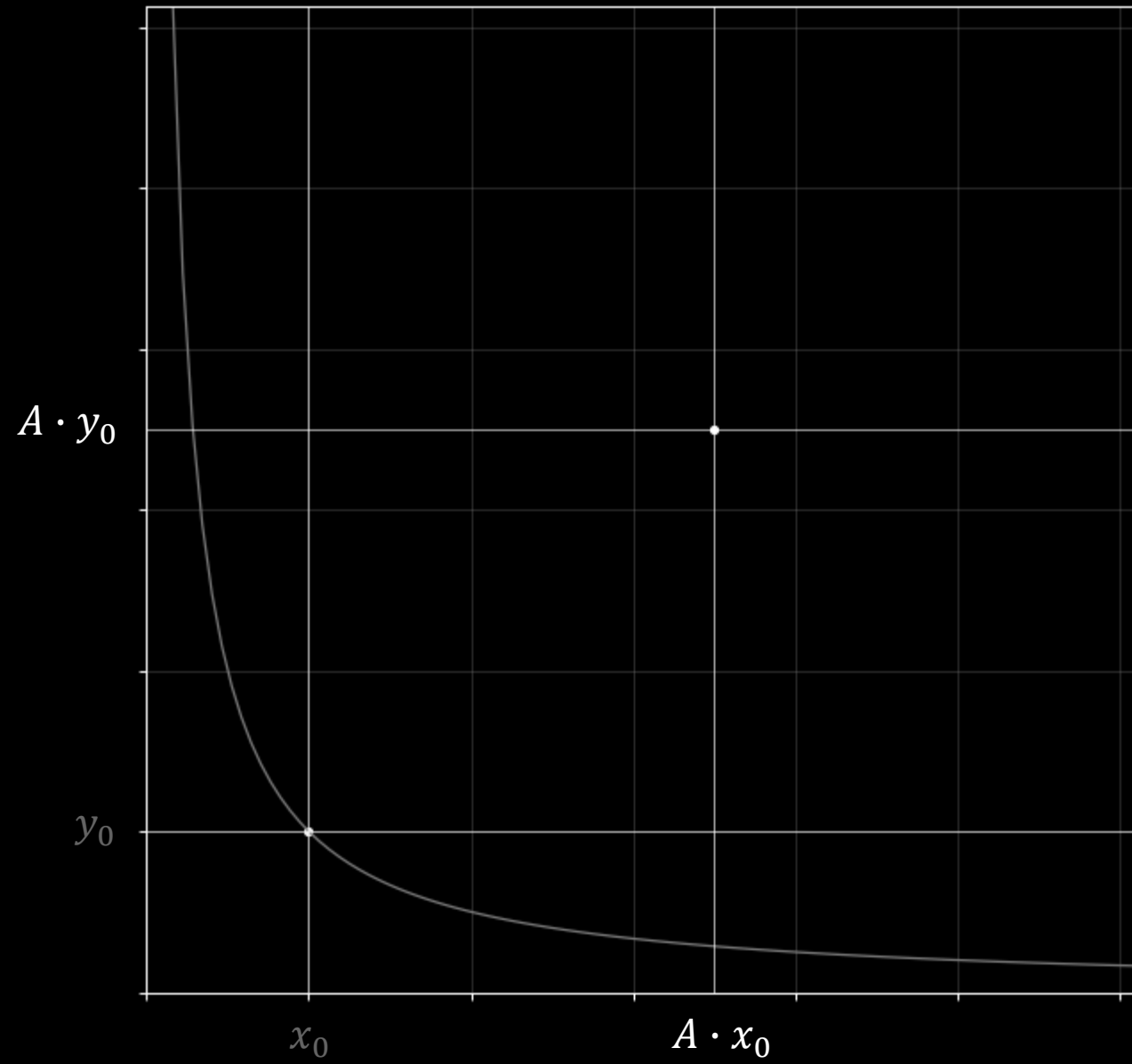
$$-P_{\text{high}} = -1.23$$

$$-P_{\text{low}} = -0.81$$

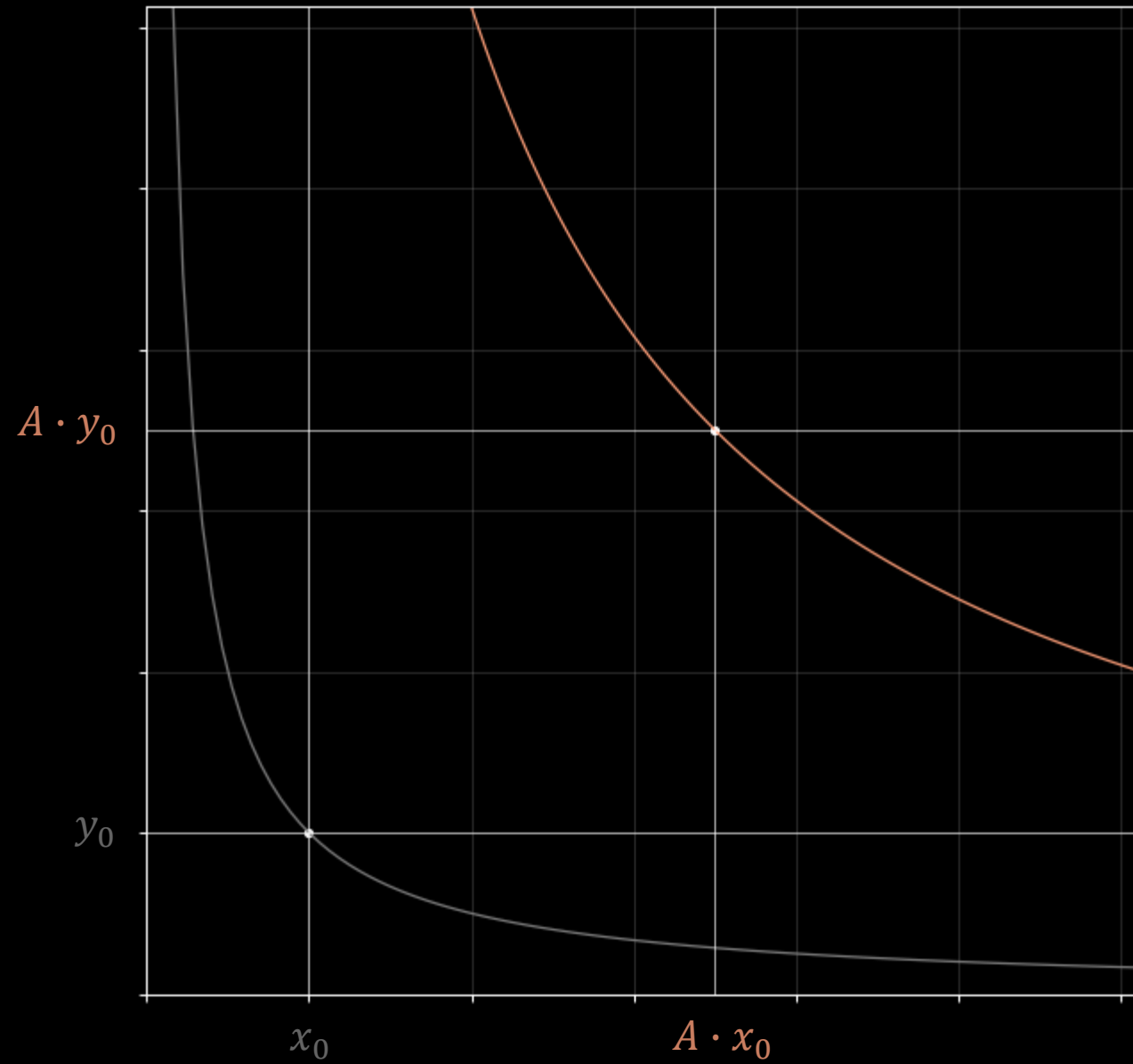


What is the geometric and algebraic intuition?

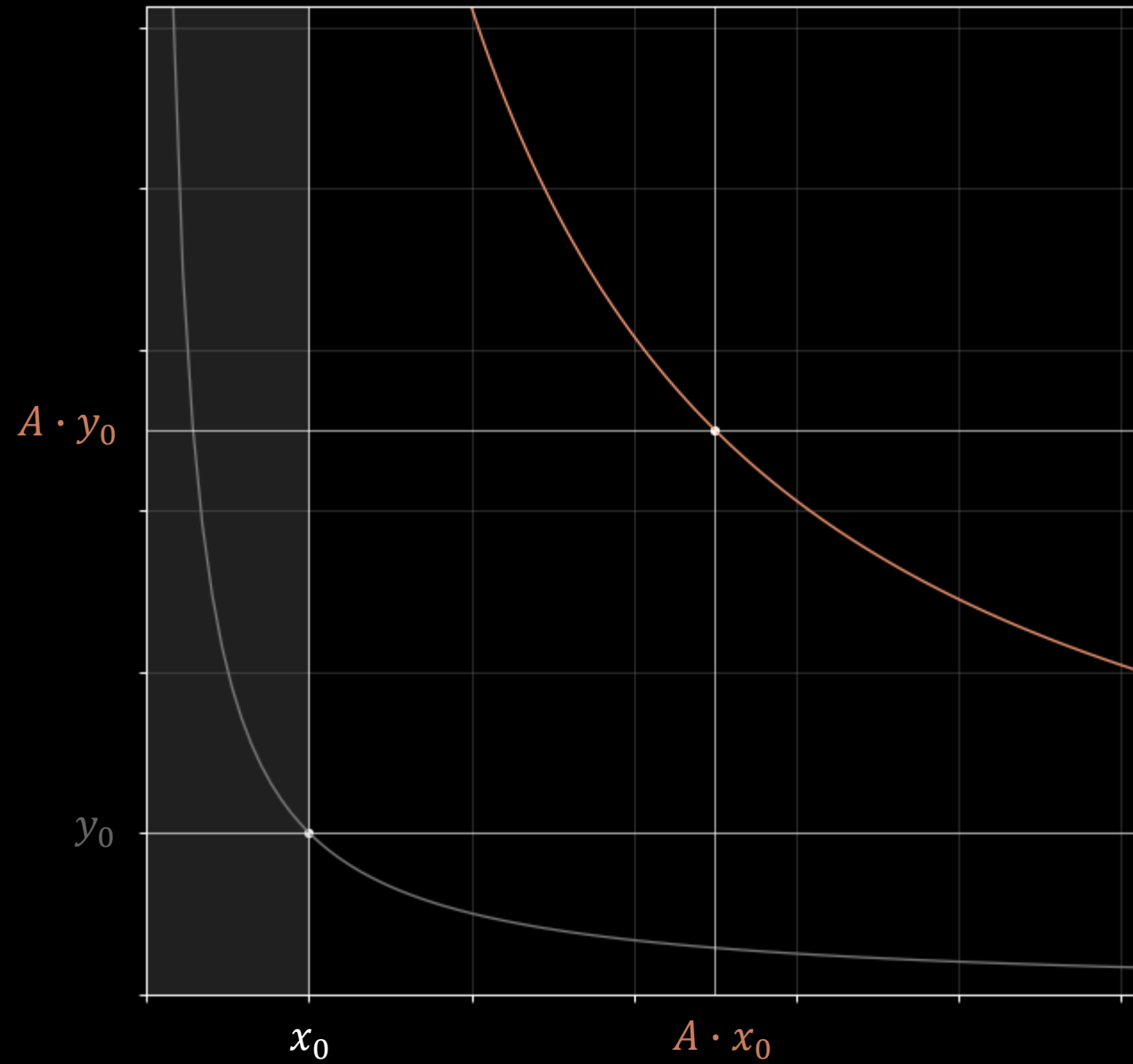
$$x \cdot y = x_0 \cdot y_0$$



$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$

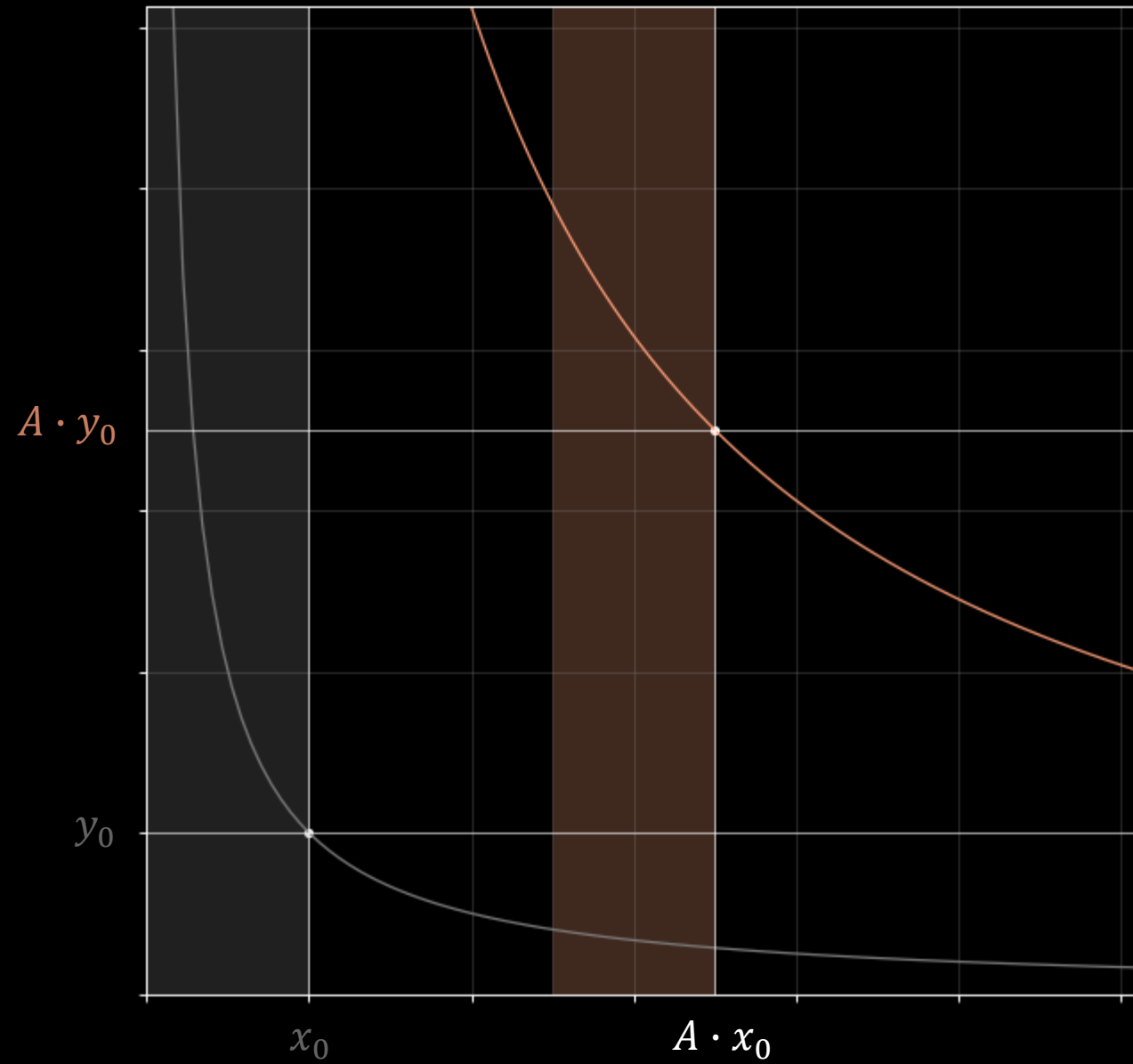


$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$

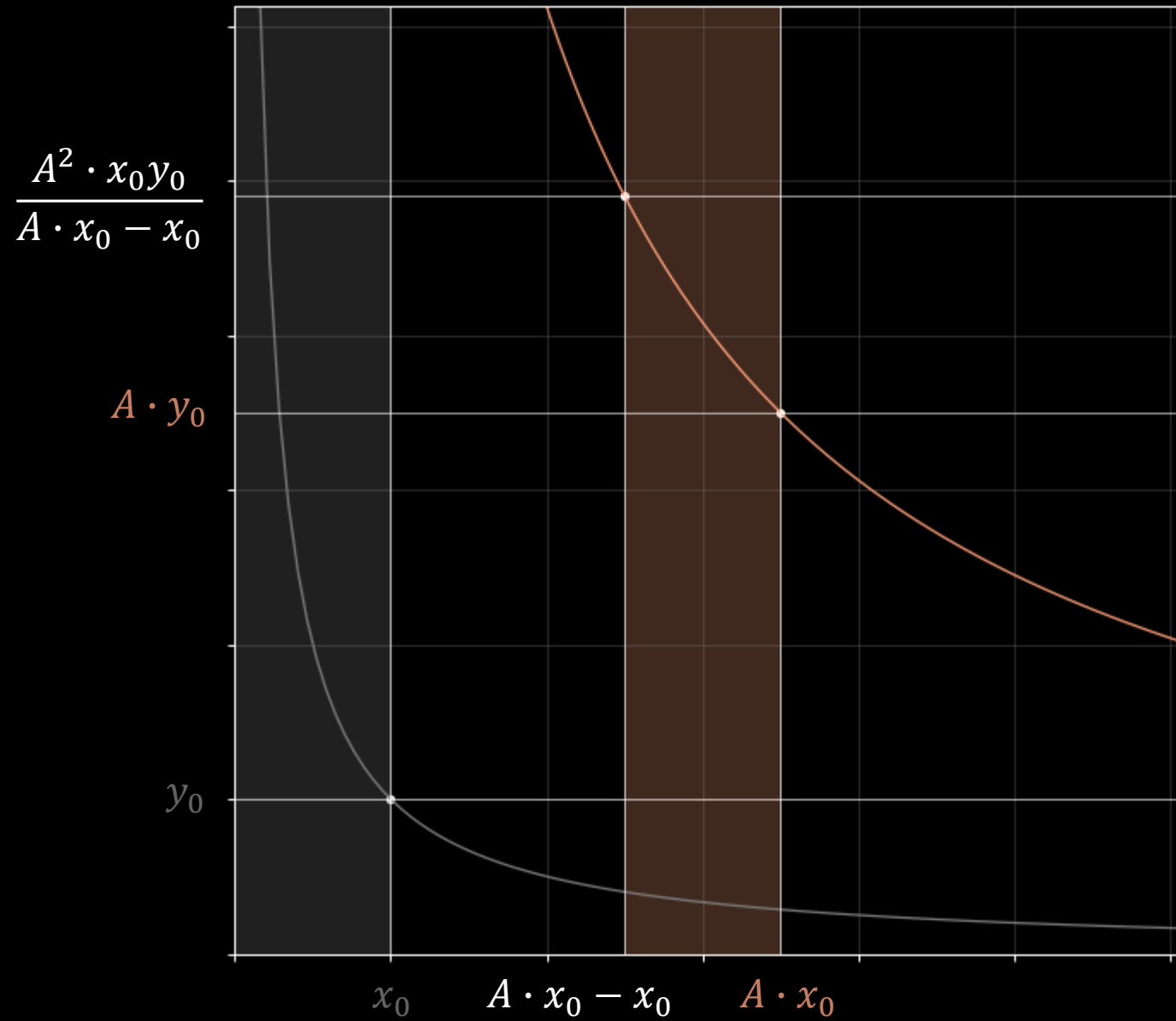




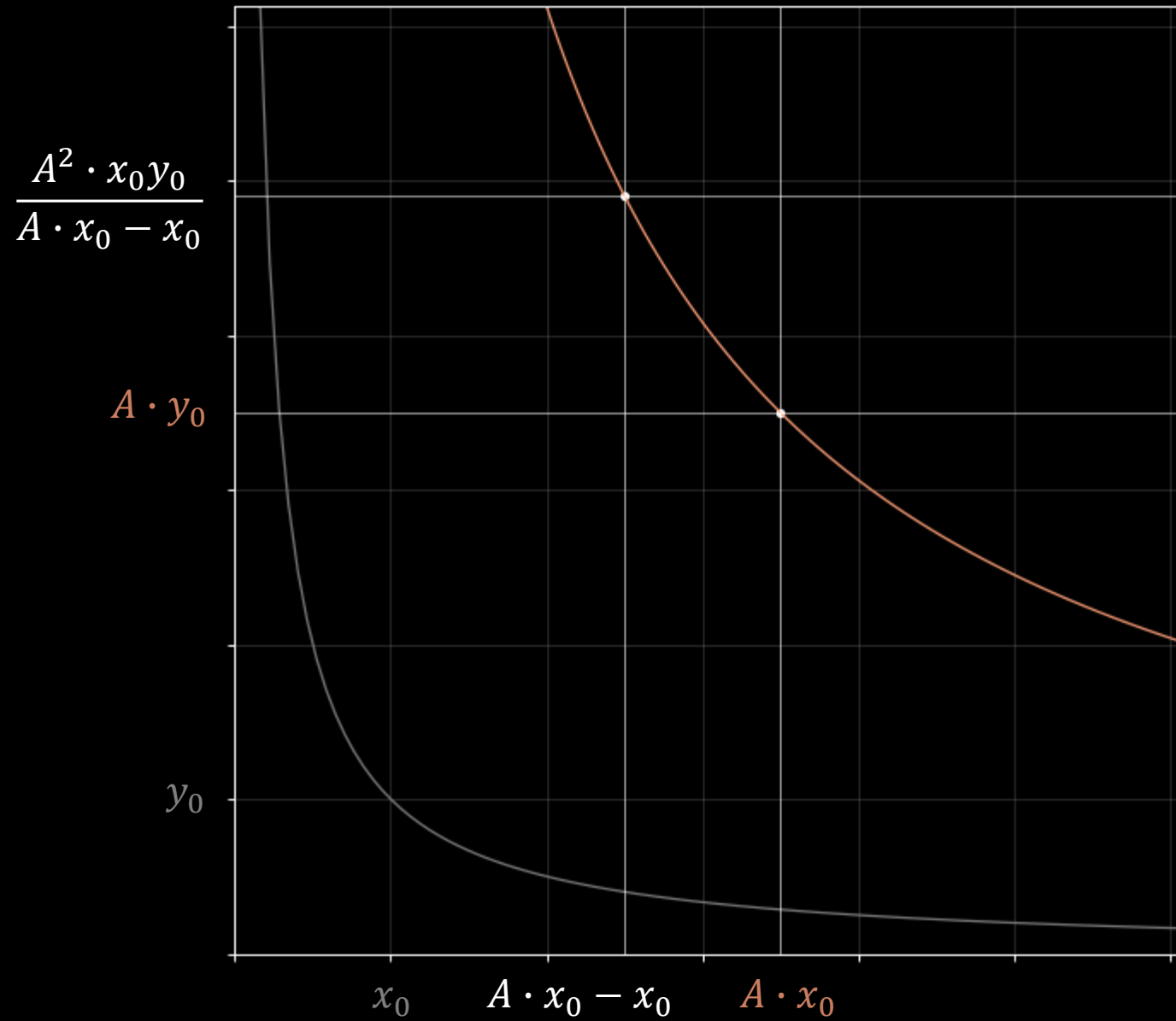
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



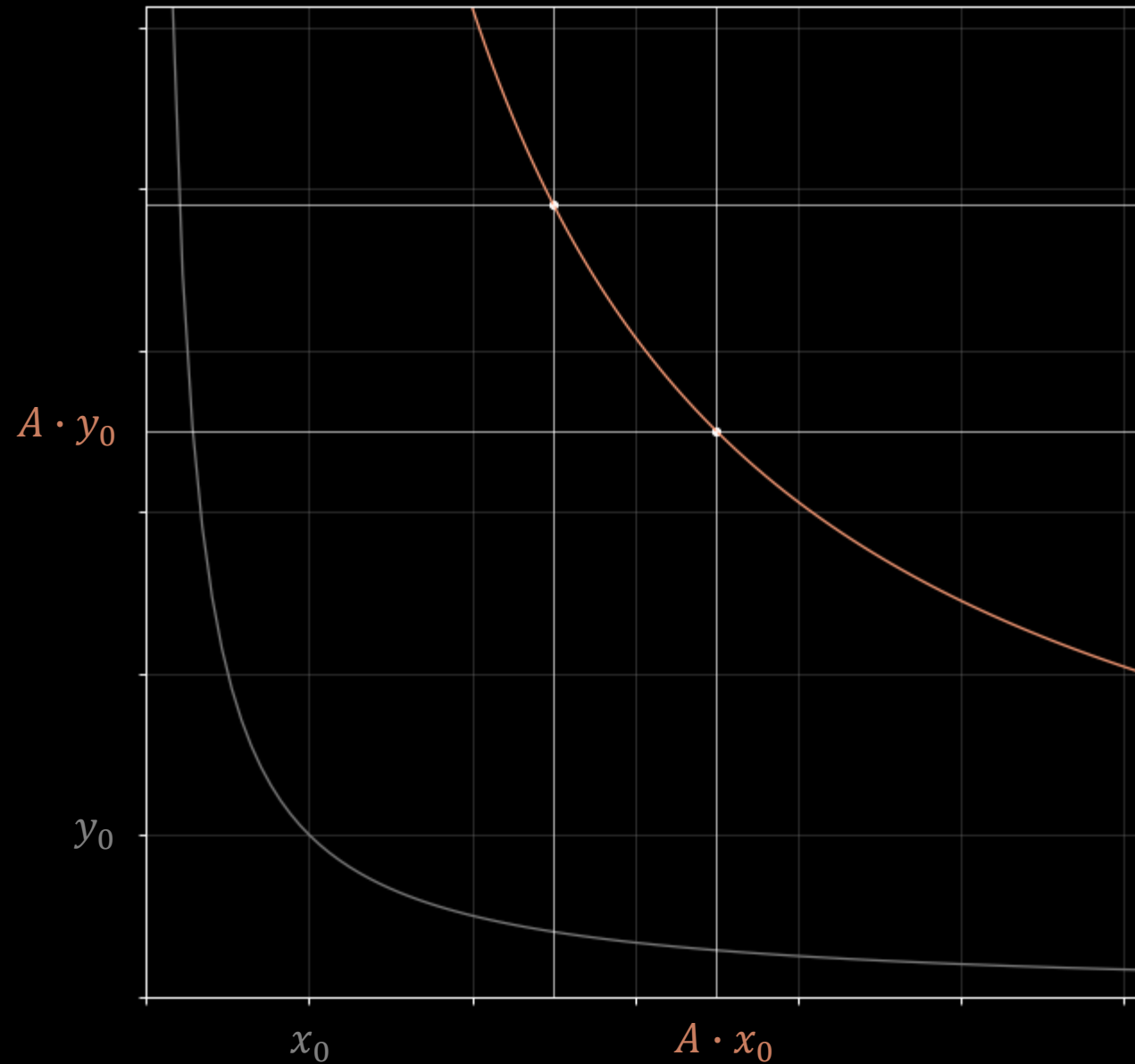
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



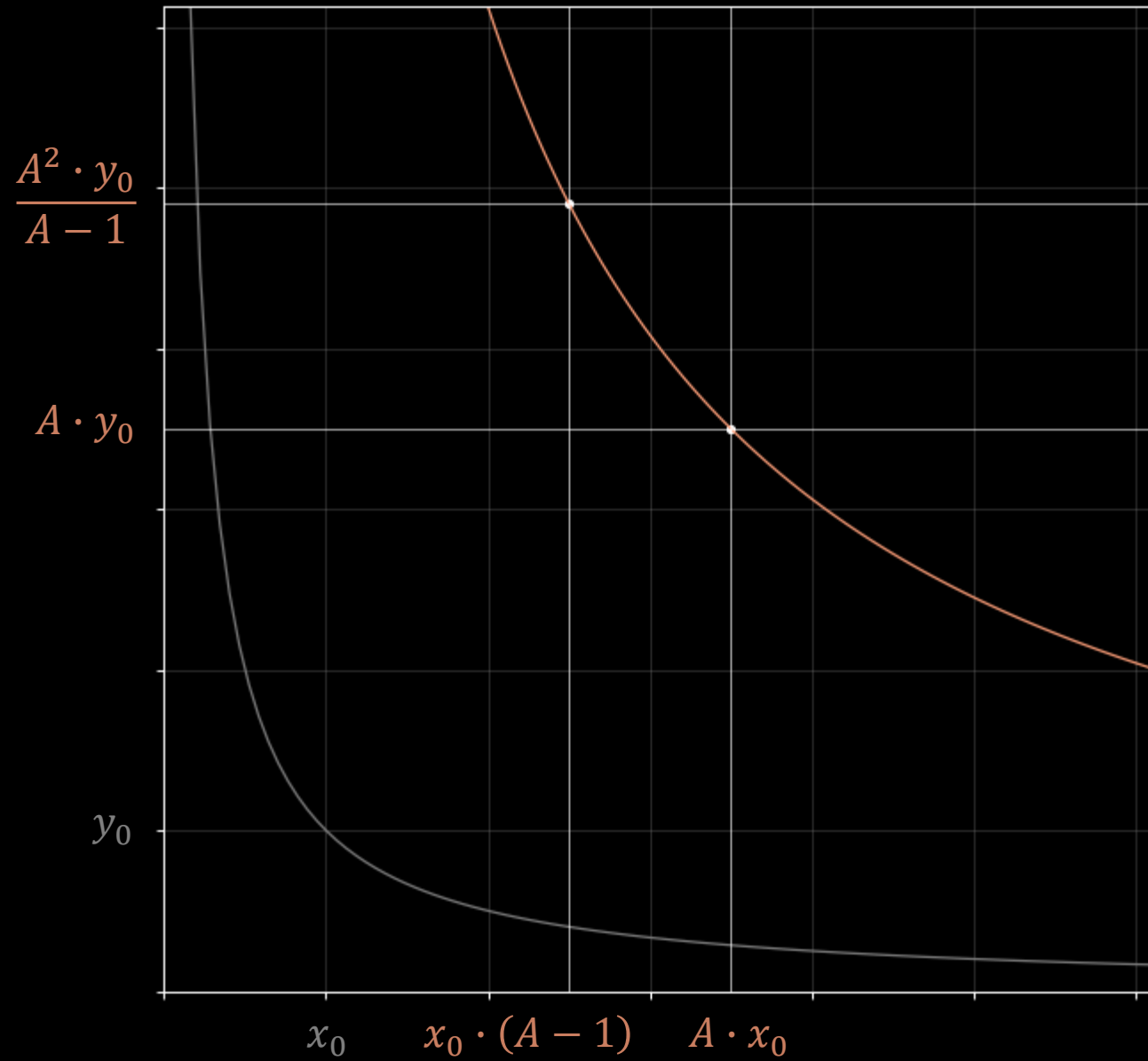
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



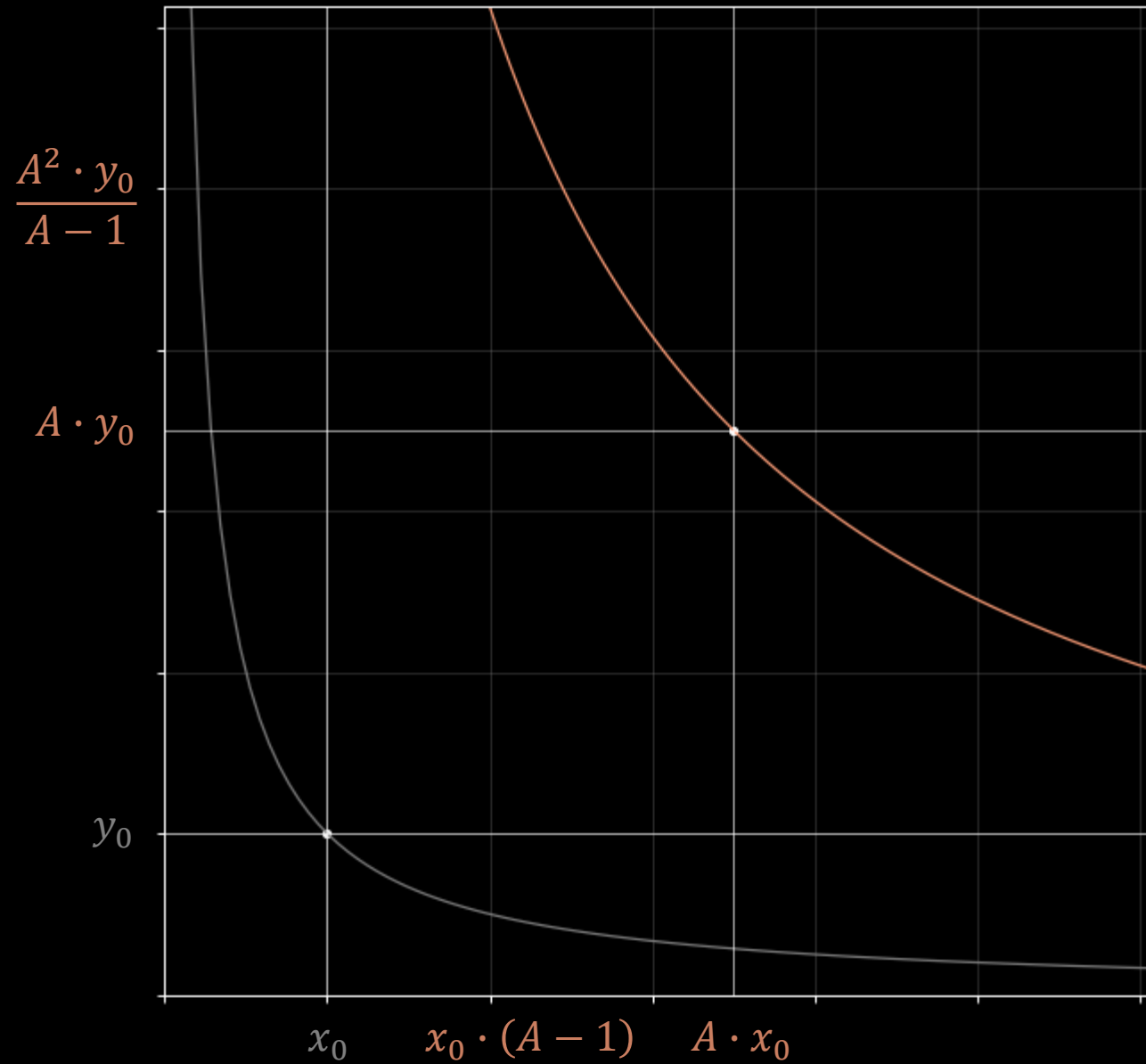
$$\frac{A^2 \cdot x_0 y_0}{A \cdot x_0 - x_0} = \frac{A^2 \cdot y_0}{A - 1}$$

$$A \cdot x_0 - x_0 = x_0 \cdot (A - 1)$$

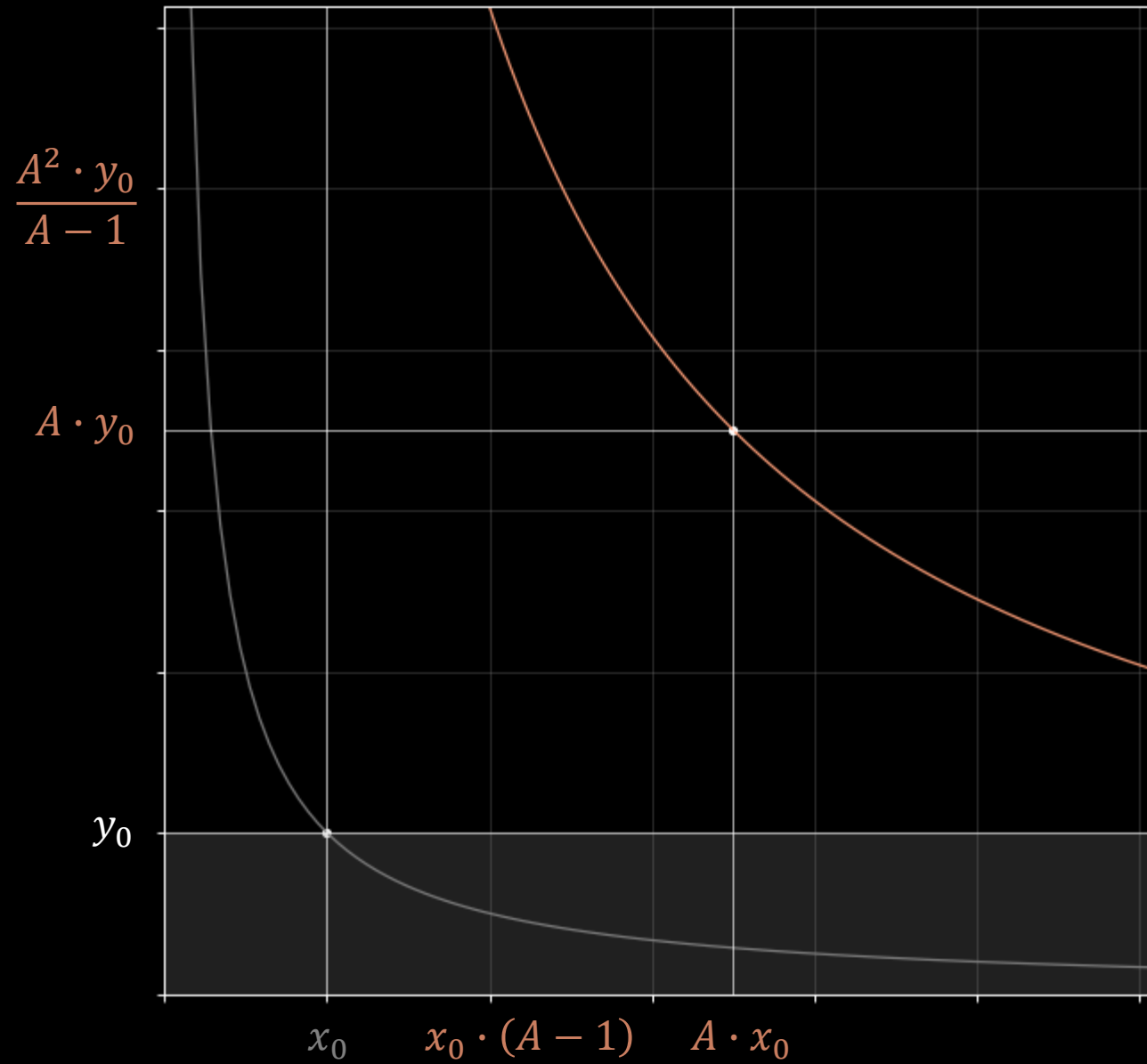
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



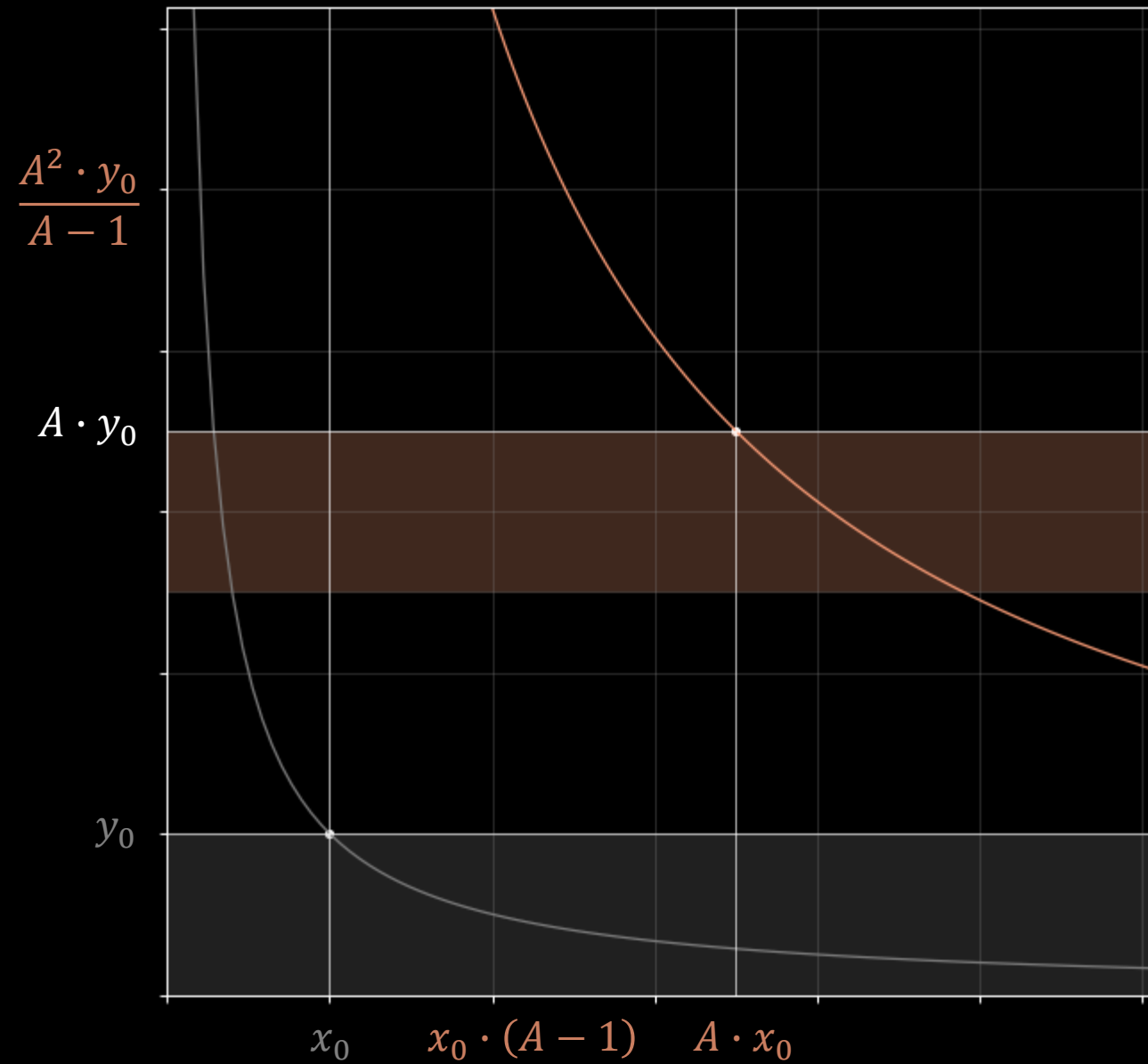
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$

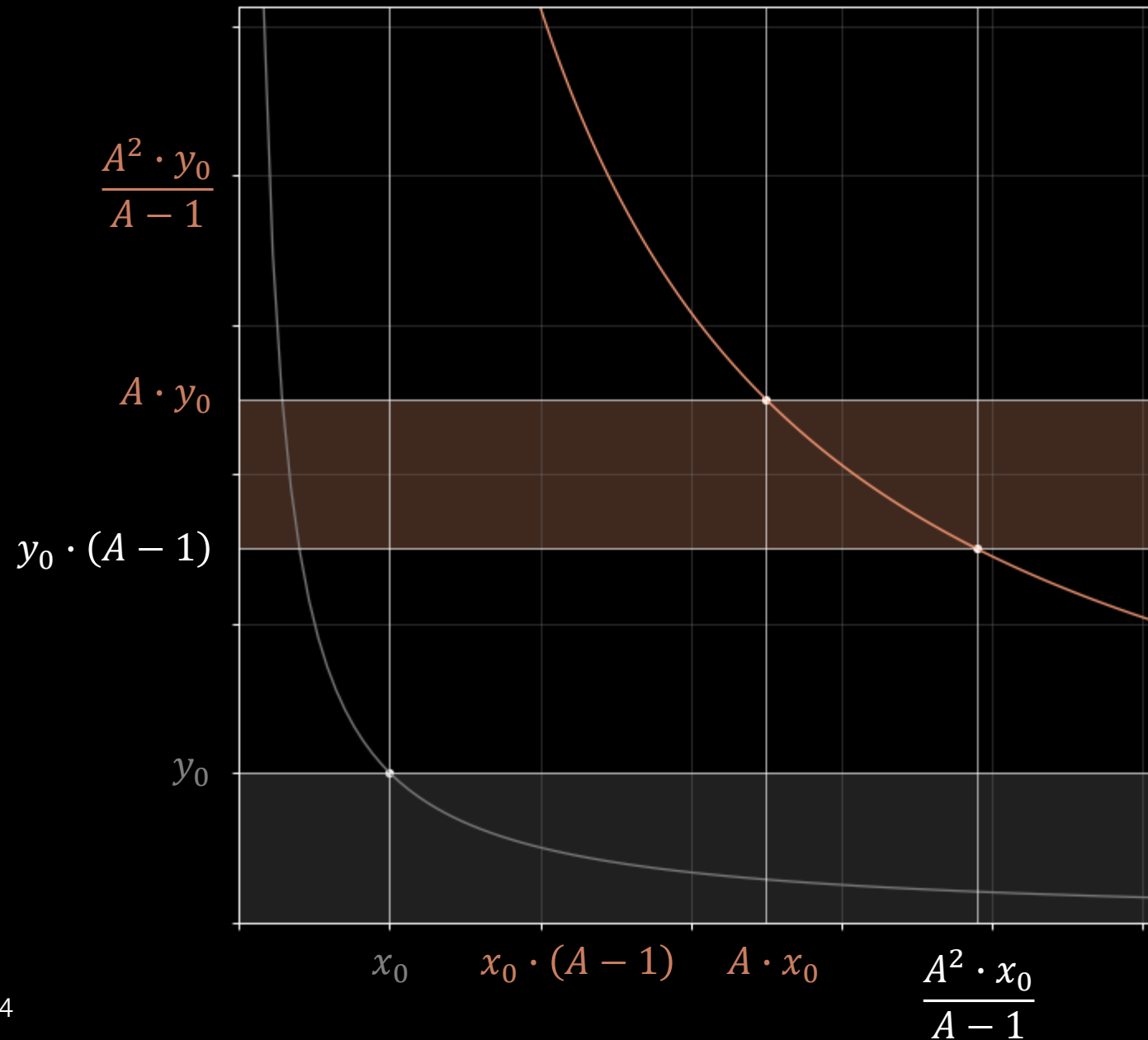


$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$

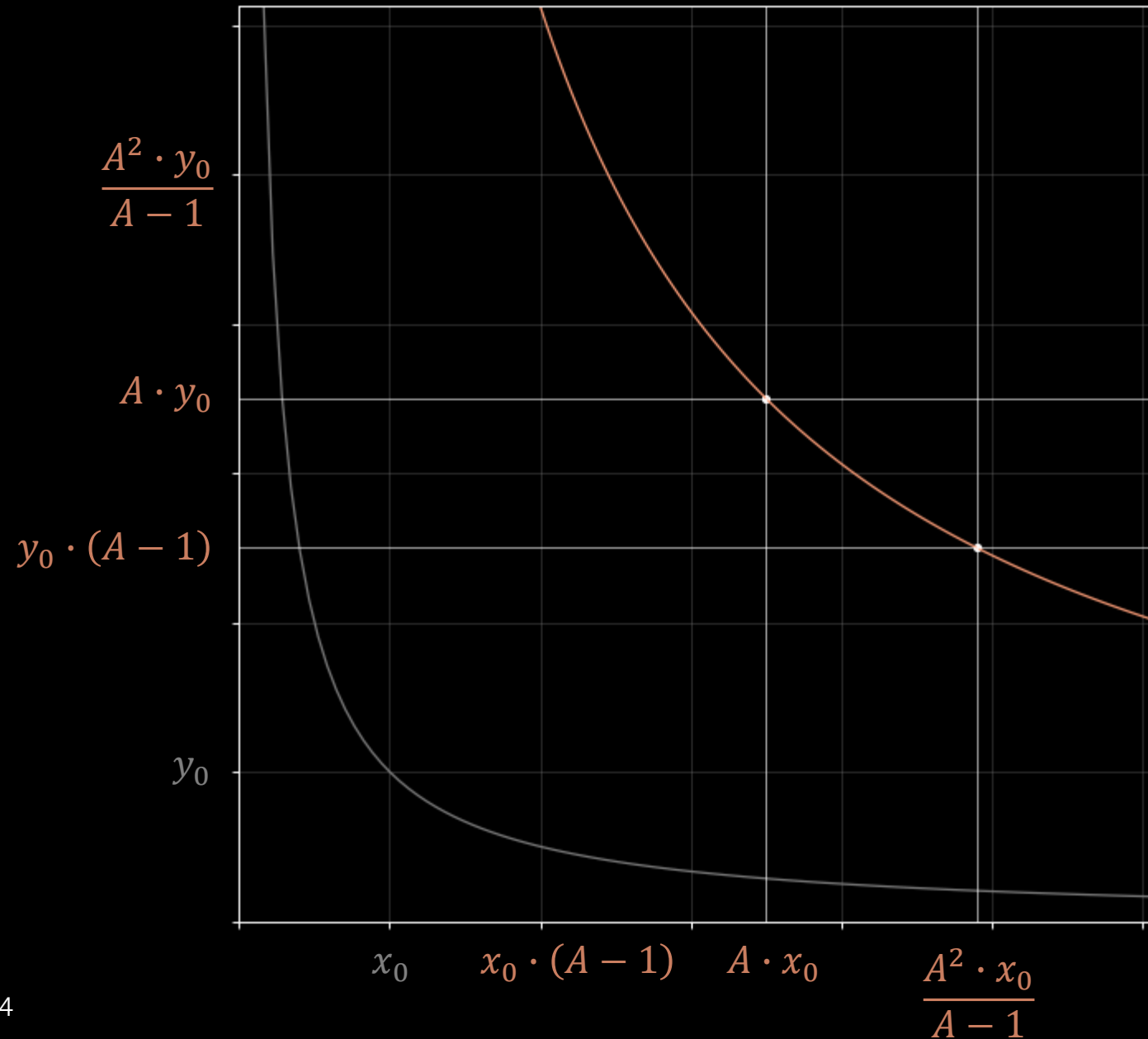




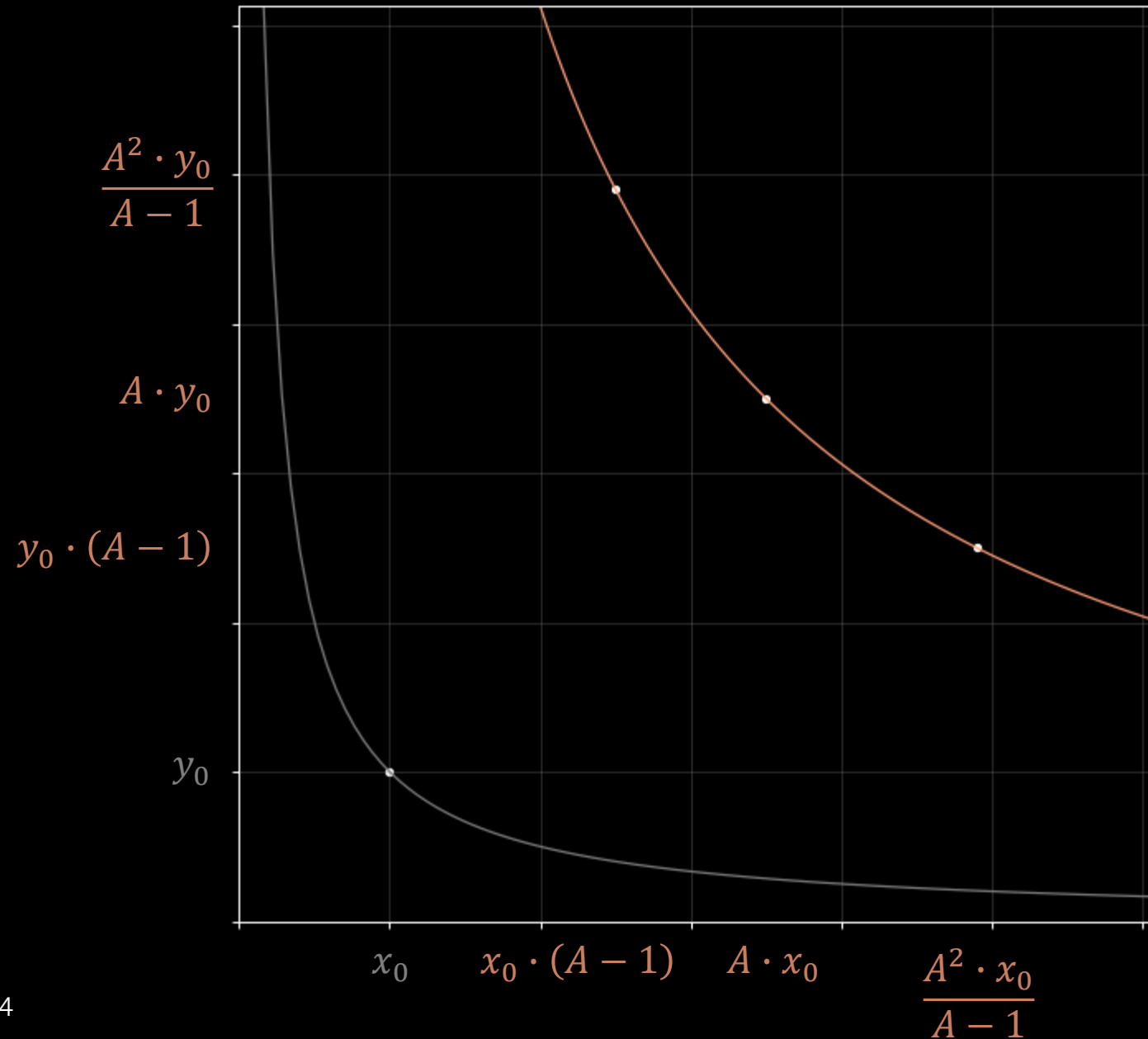
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



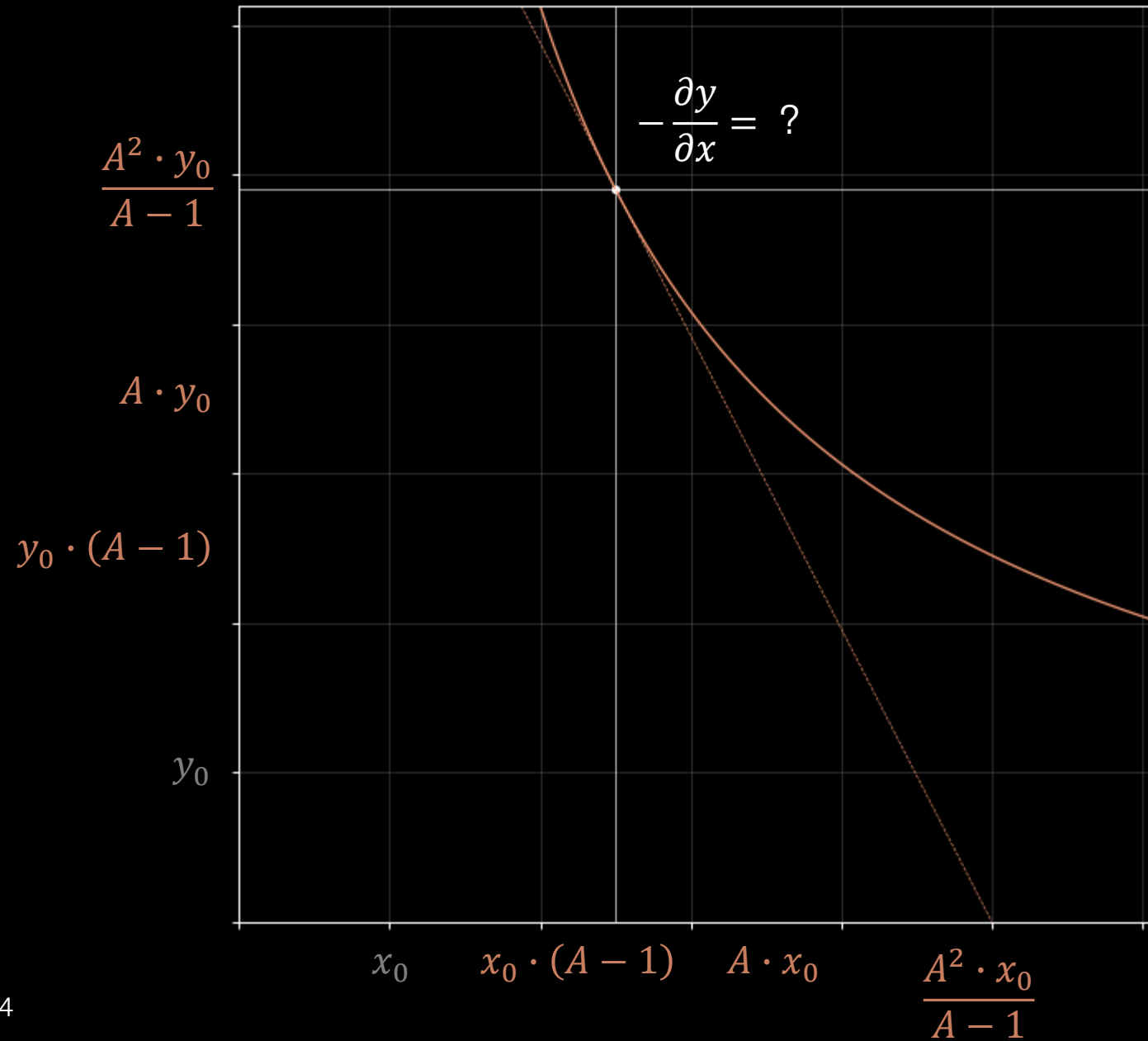
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



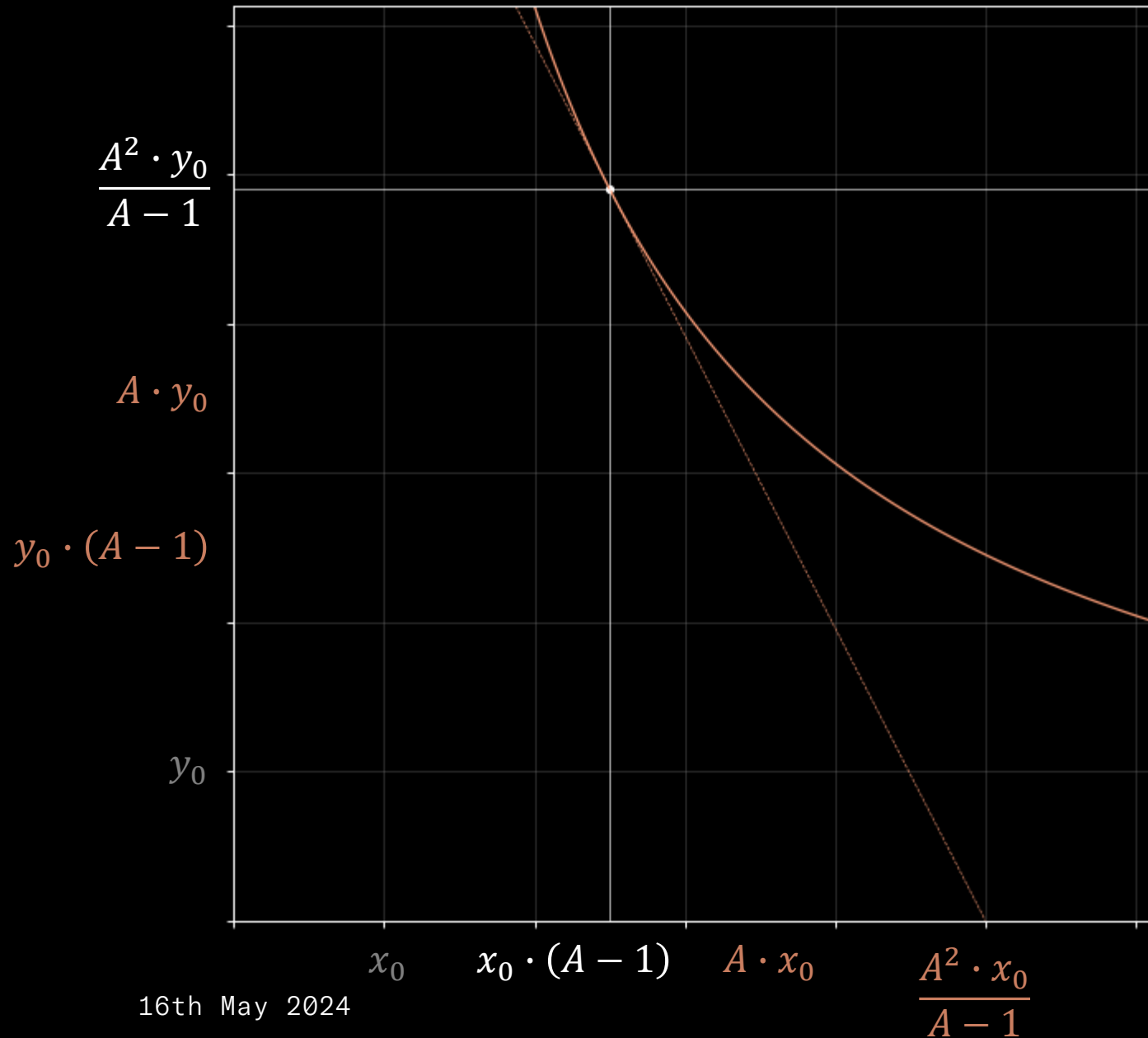
$$x \cdot y = x_0 \cdot y_0 \quad x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

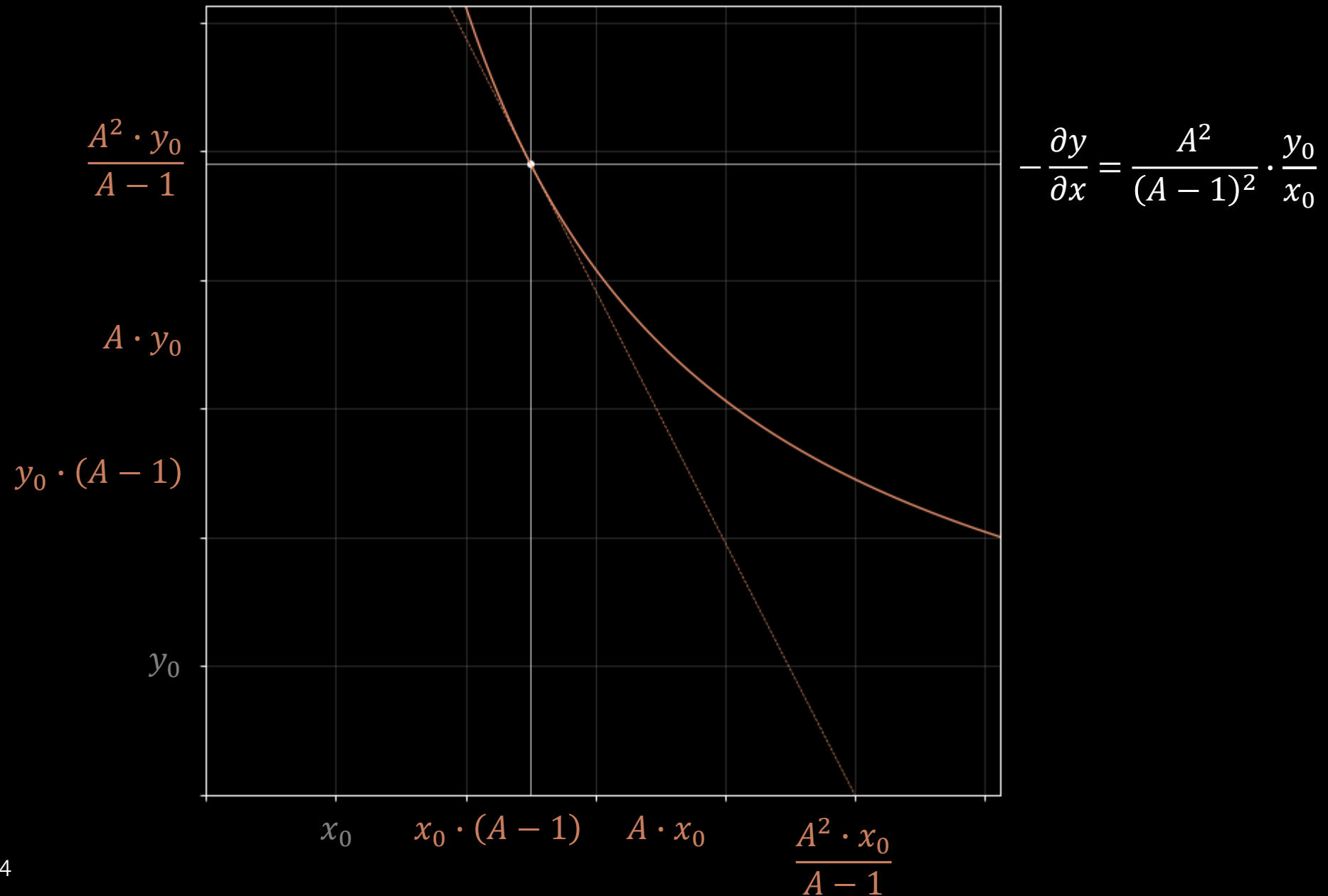


$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

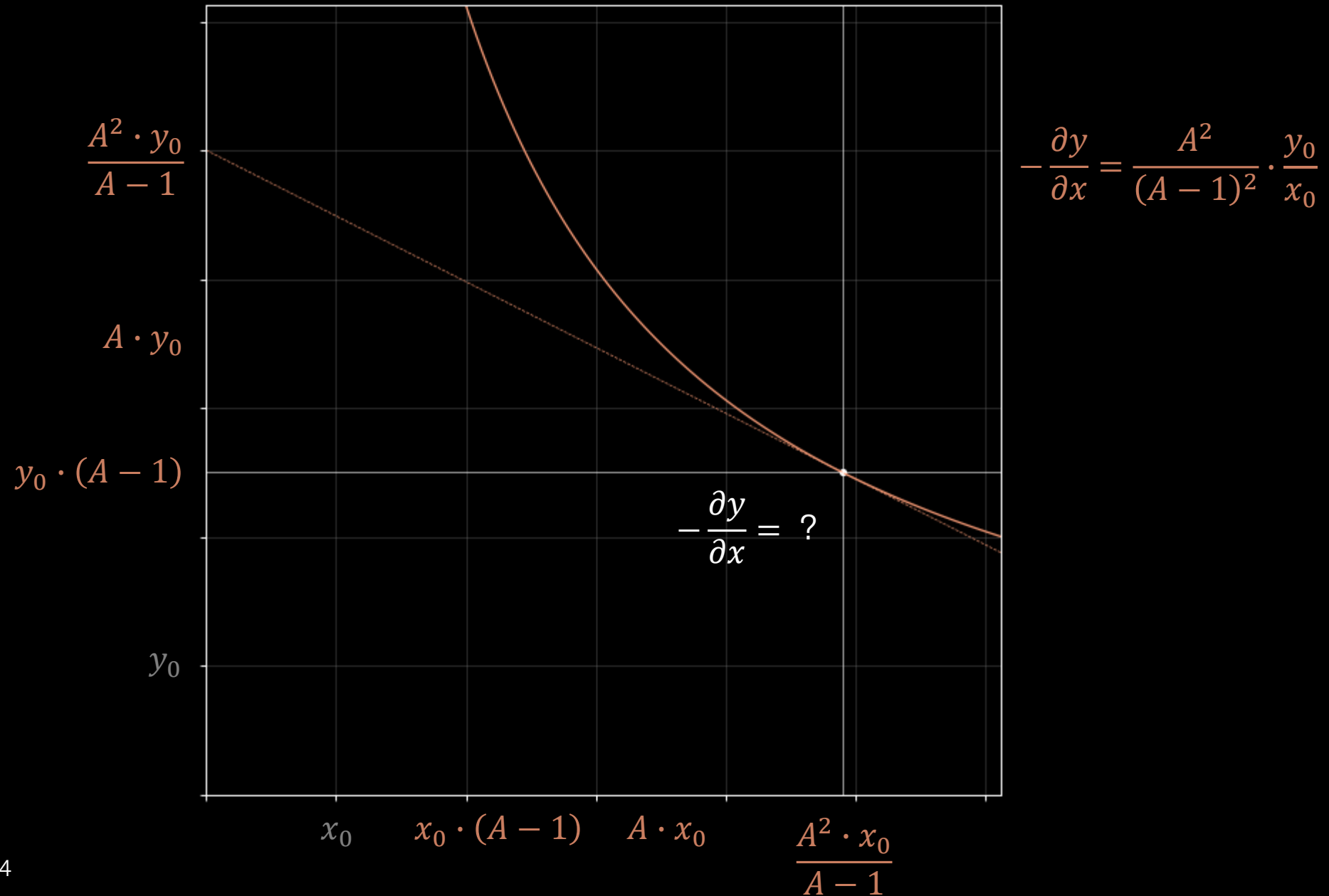


$$-\frac{\partial y}{\partial x} = \frac{\frac{A^2 \cdot y_0}{A - 1}}{x_0 \cdot (A - 1)} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

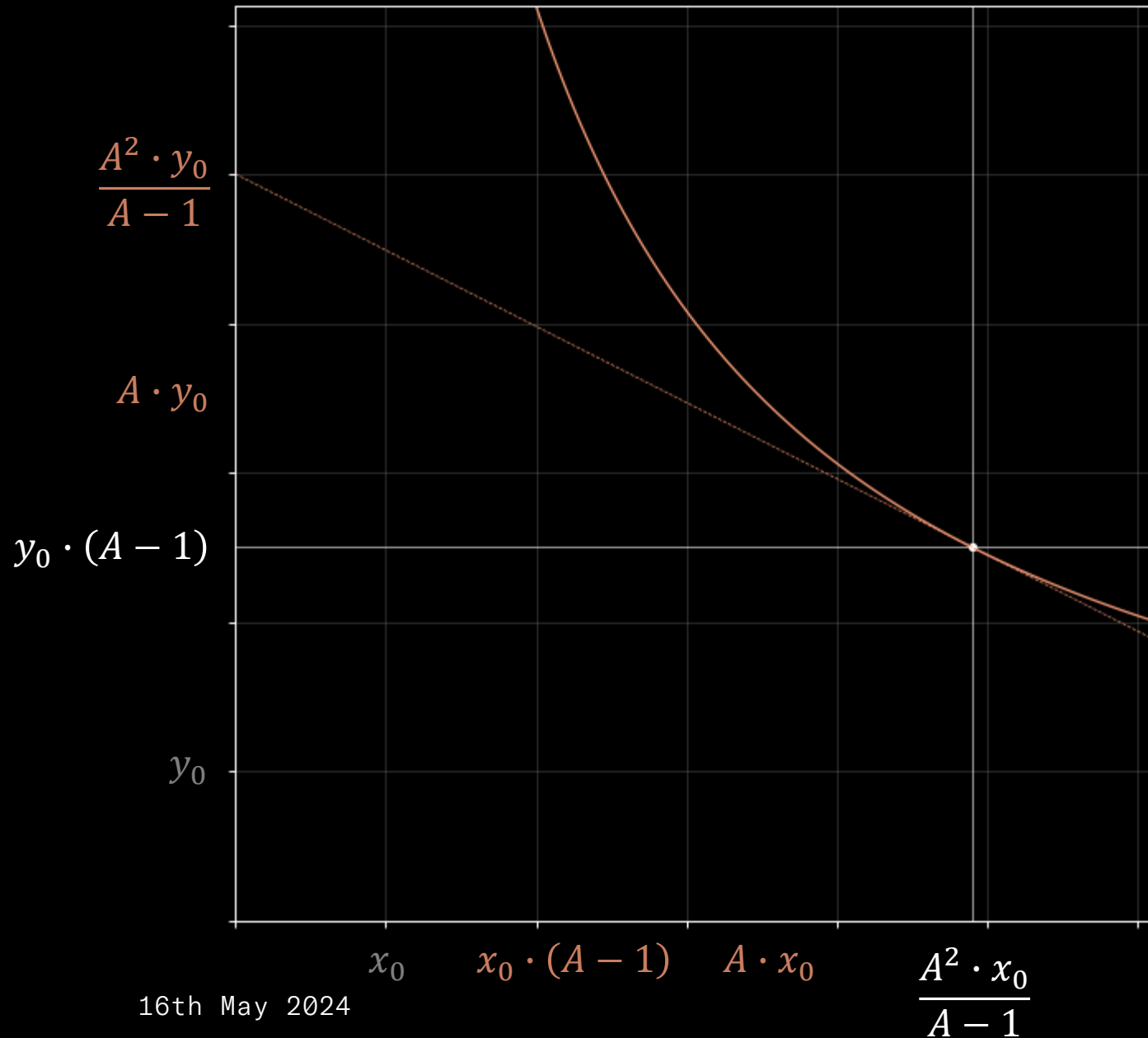
$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

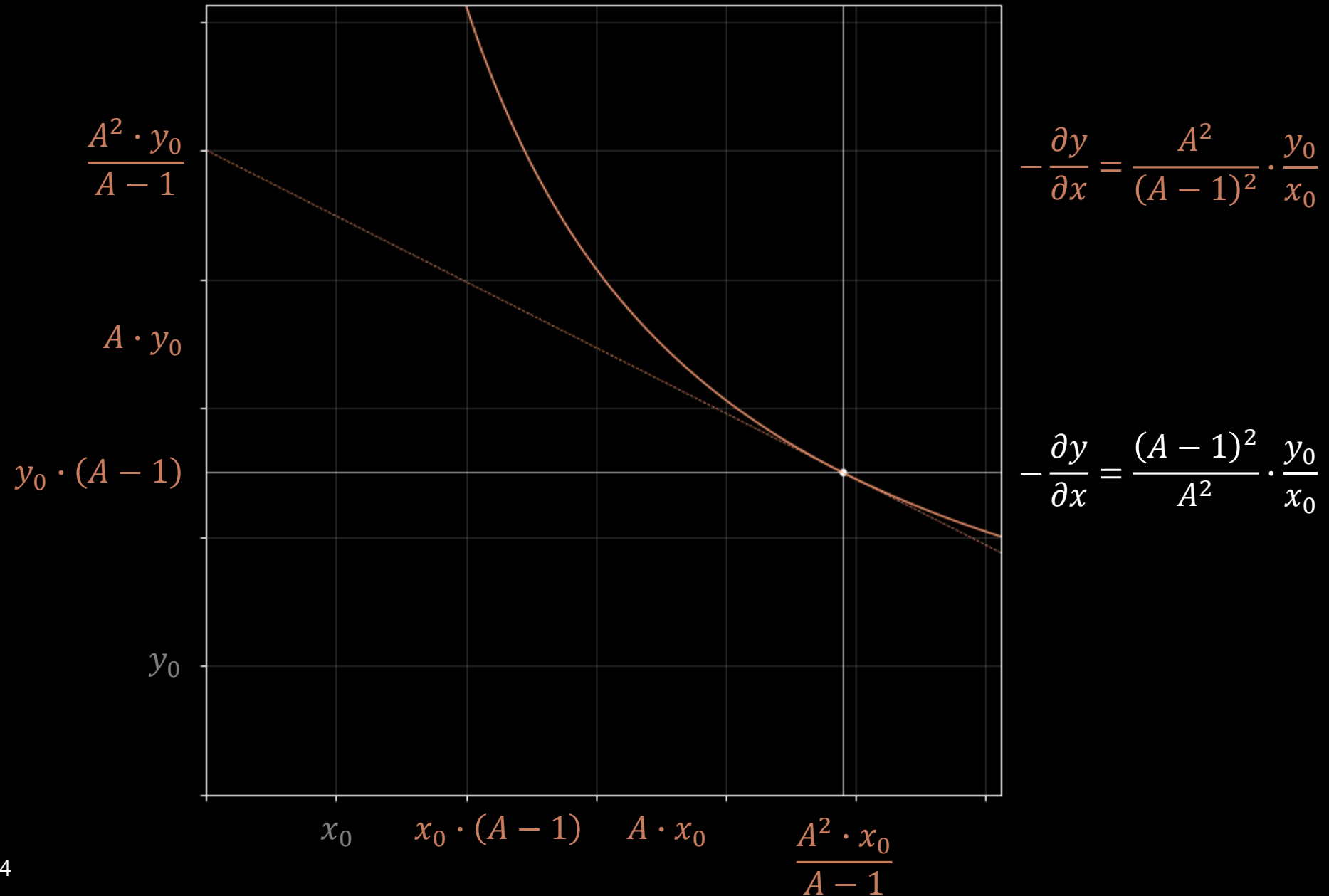


$$-\frac{\partial y}{\partial x} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

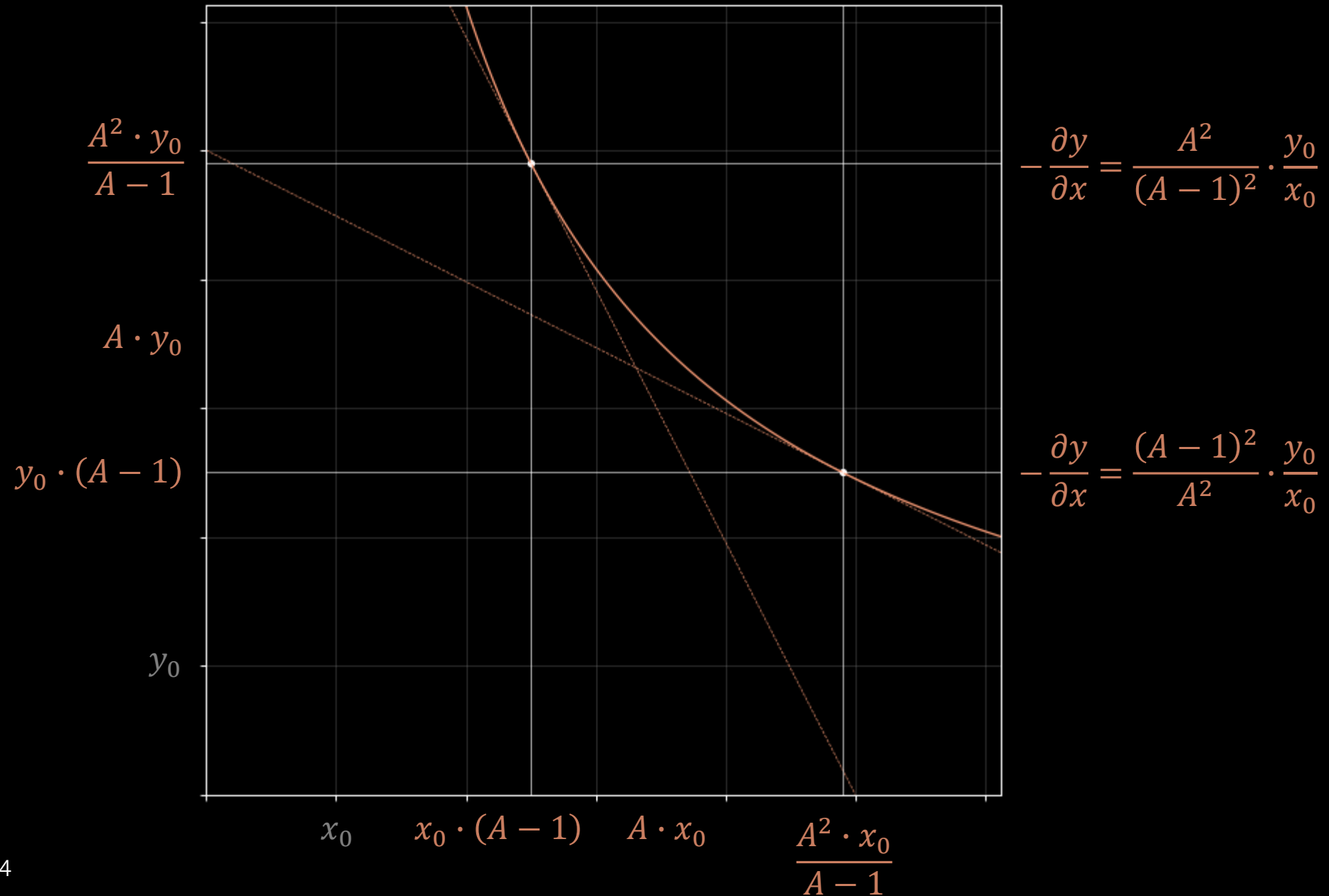
$$-\frac{\partial y}{\partial x} = \frac{y_0 \cdot (A - 1)}{\frac{A^2 \cdot x_0}{A - 1}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



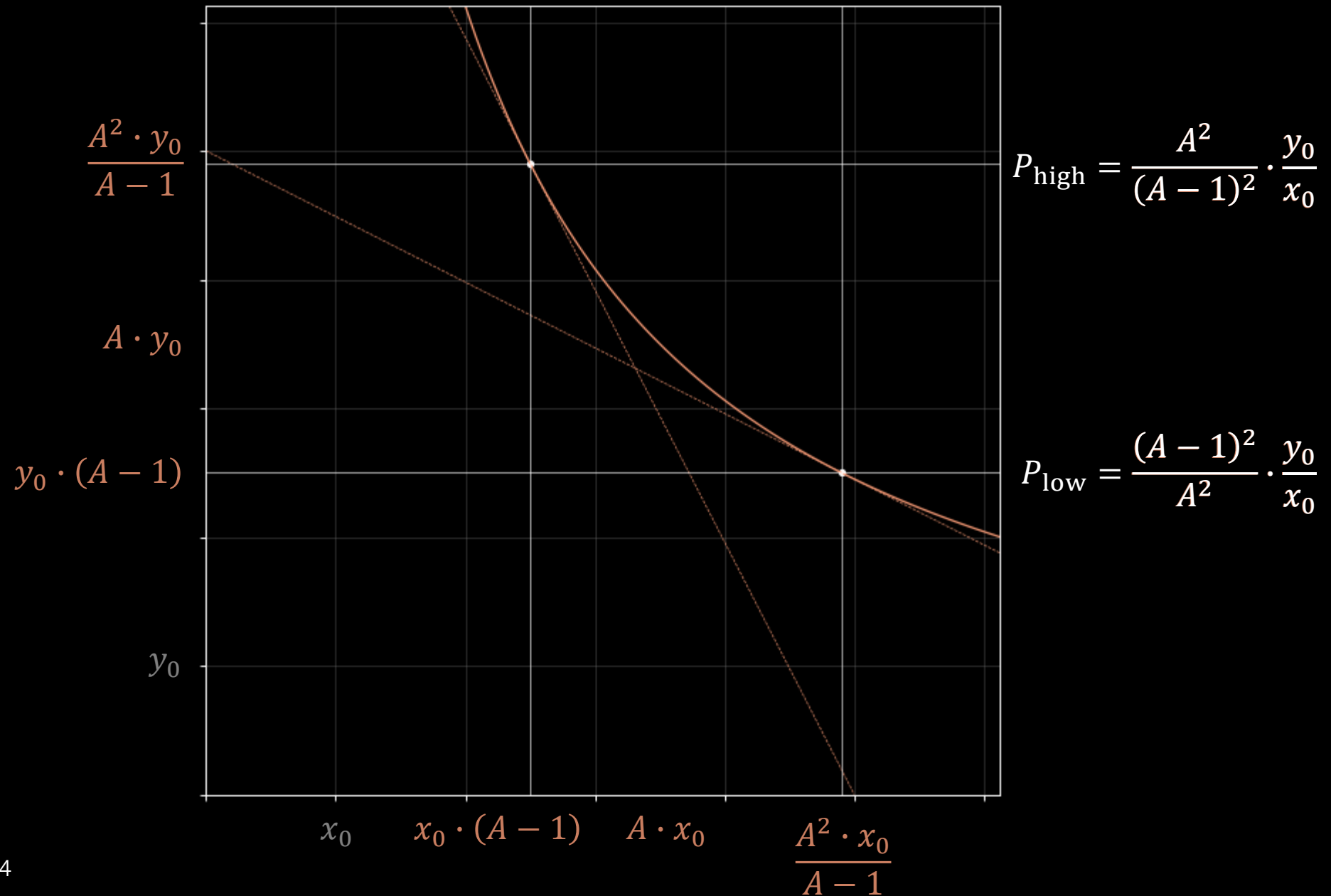
$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

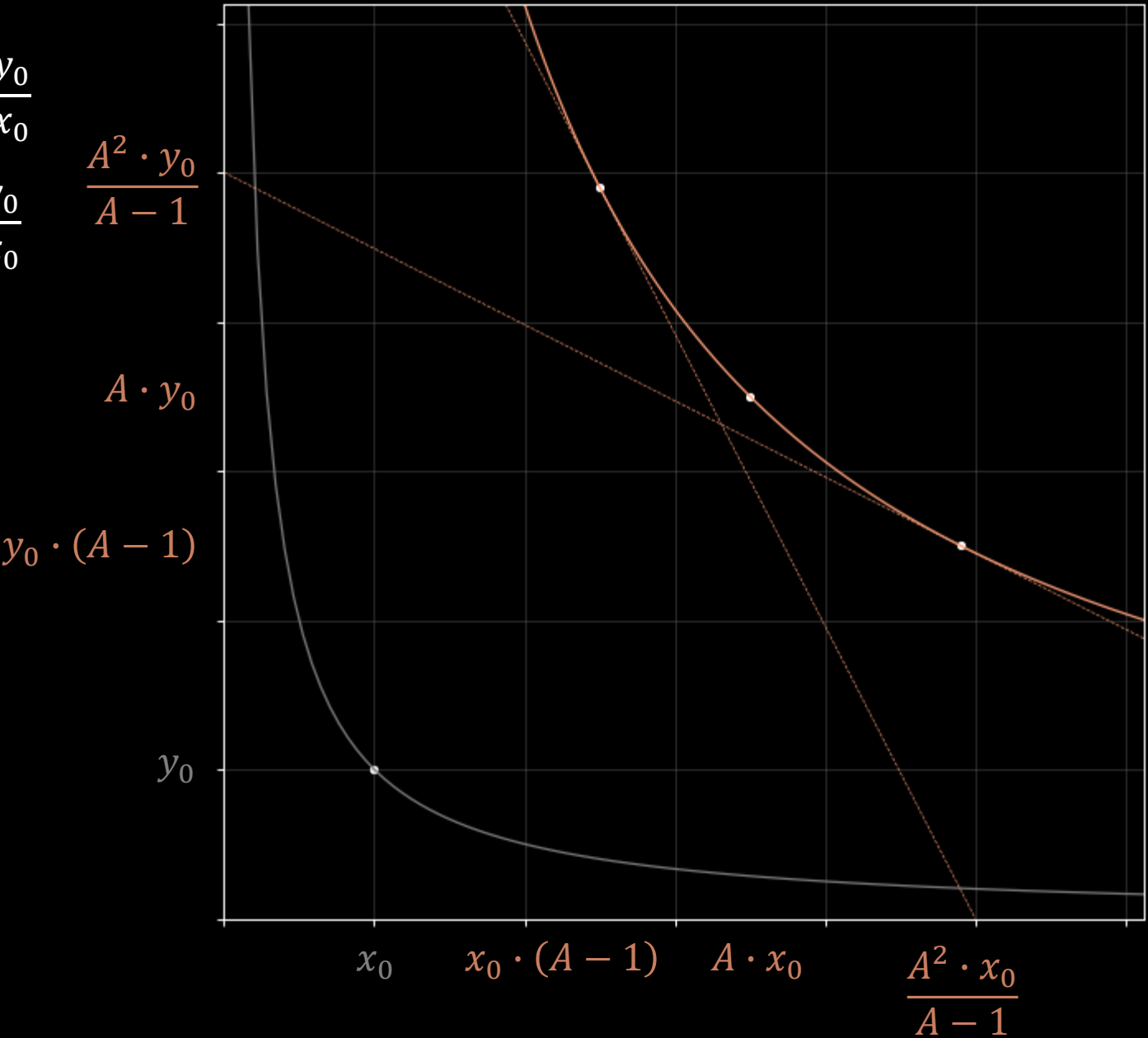


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

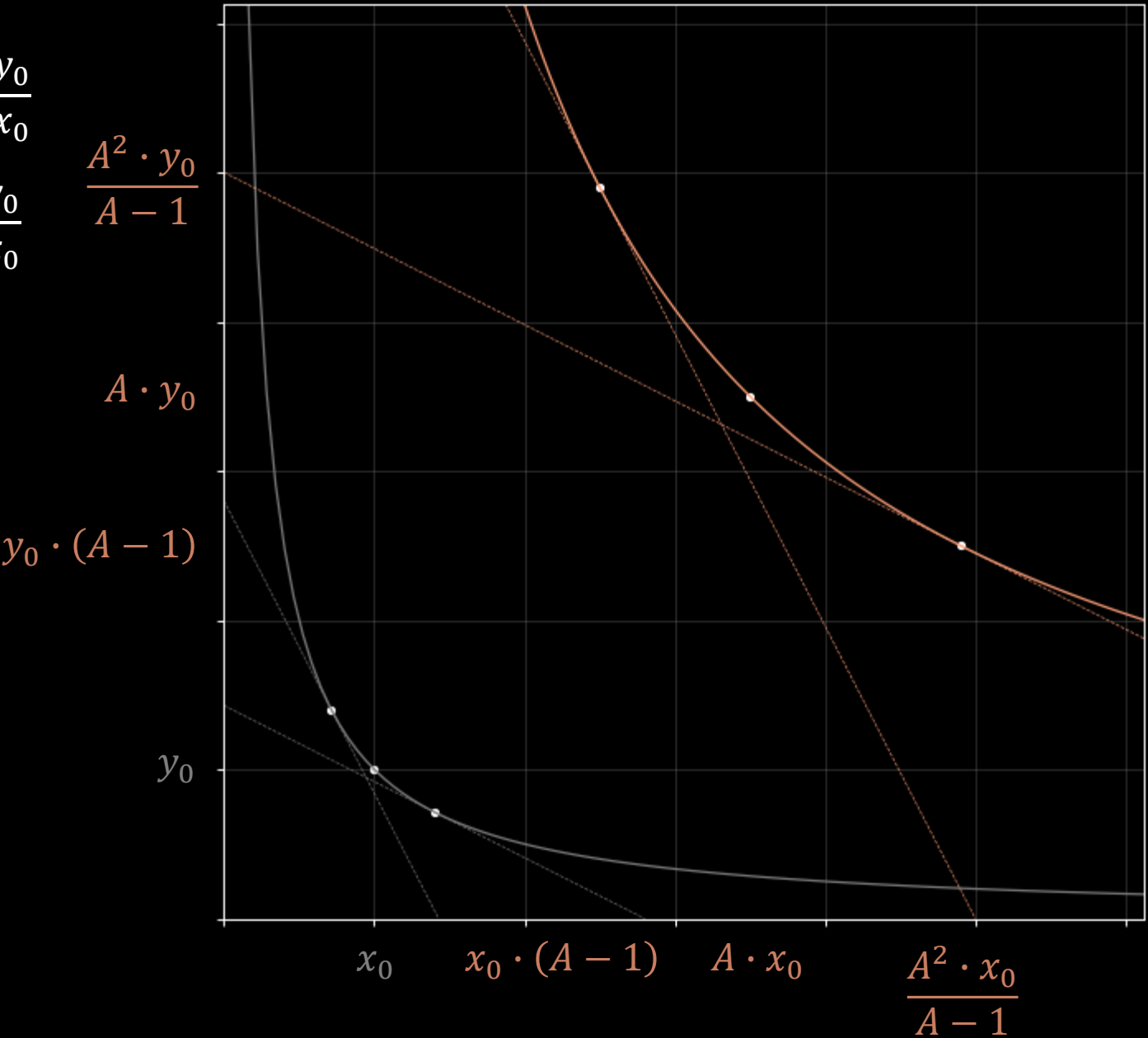


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

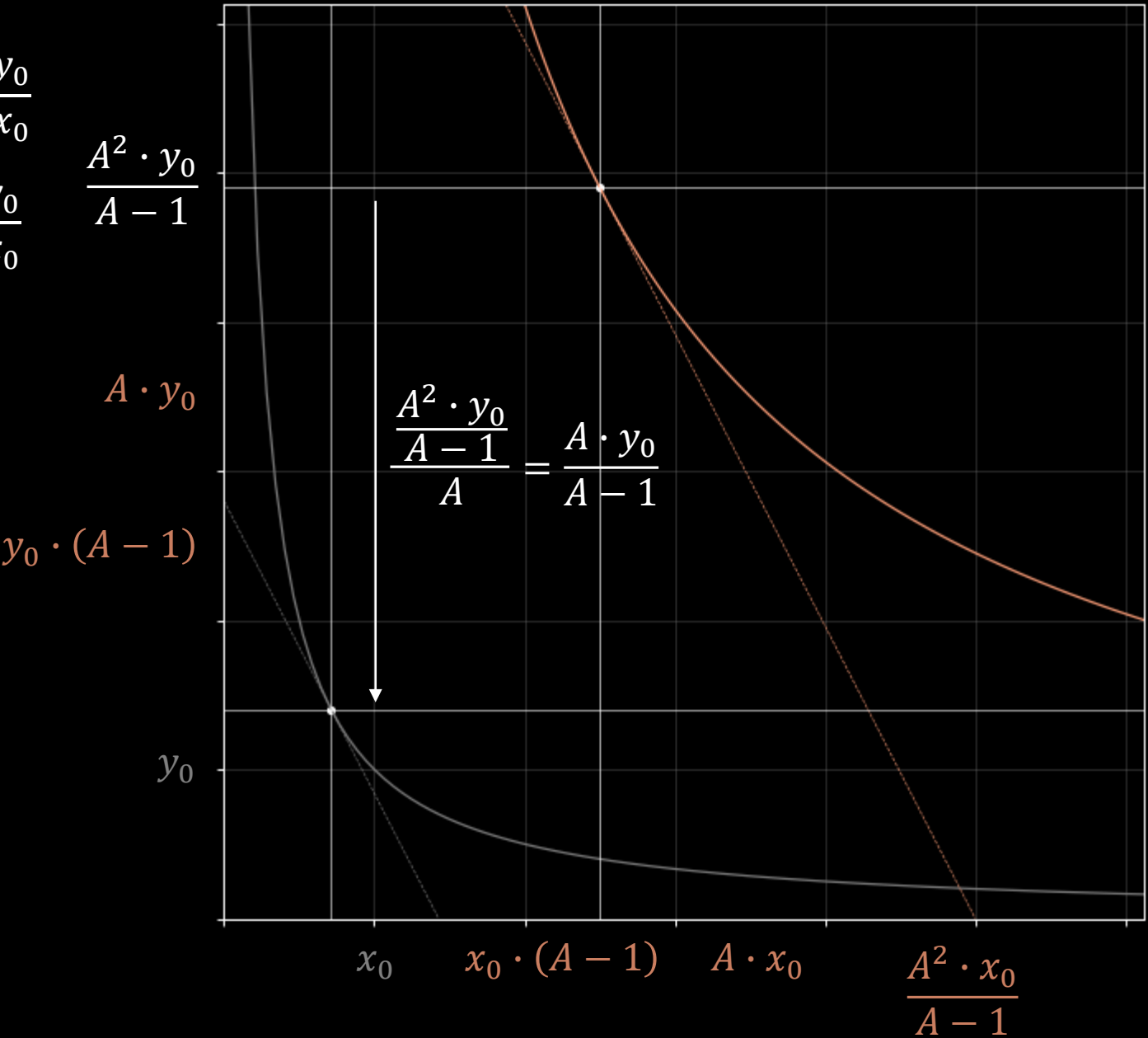


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

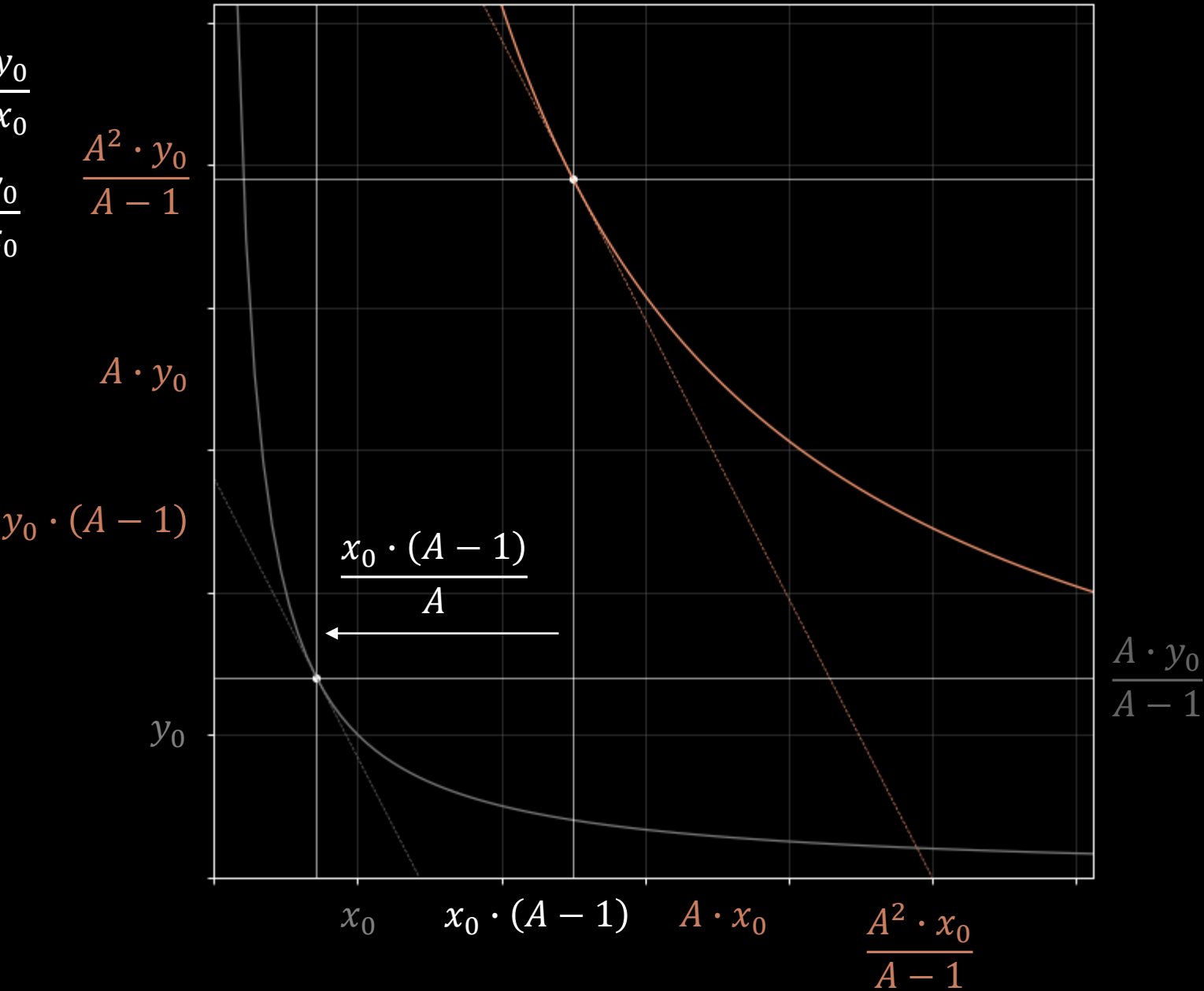


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

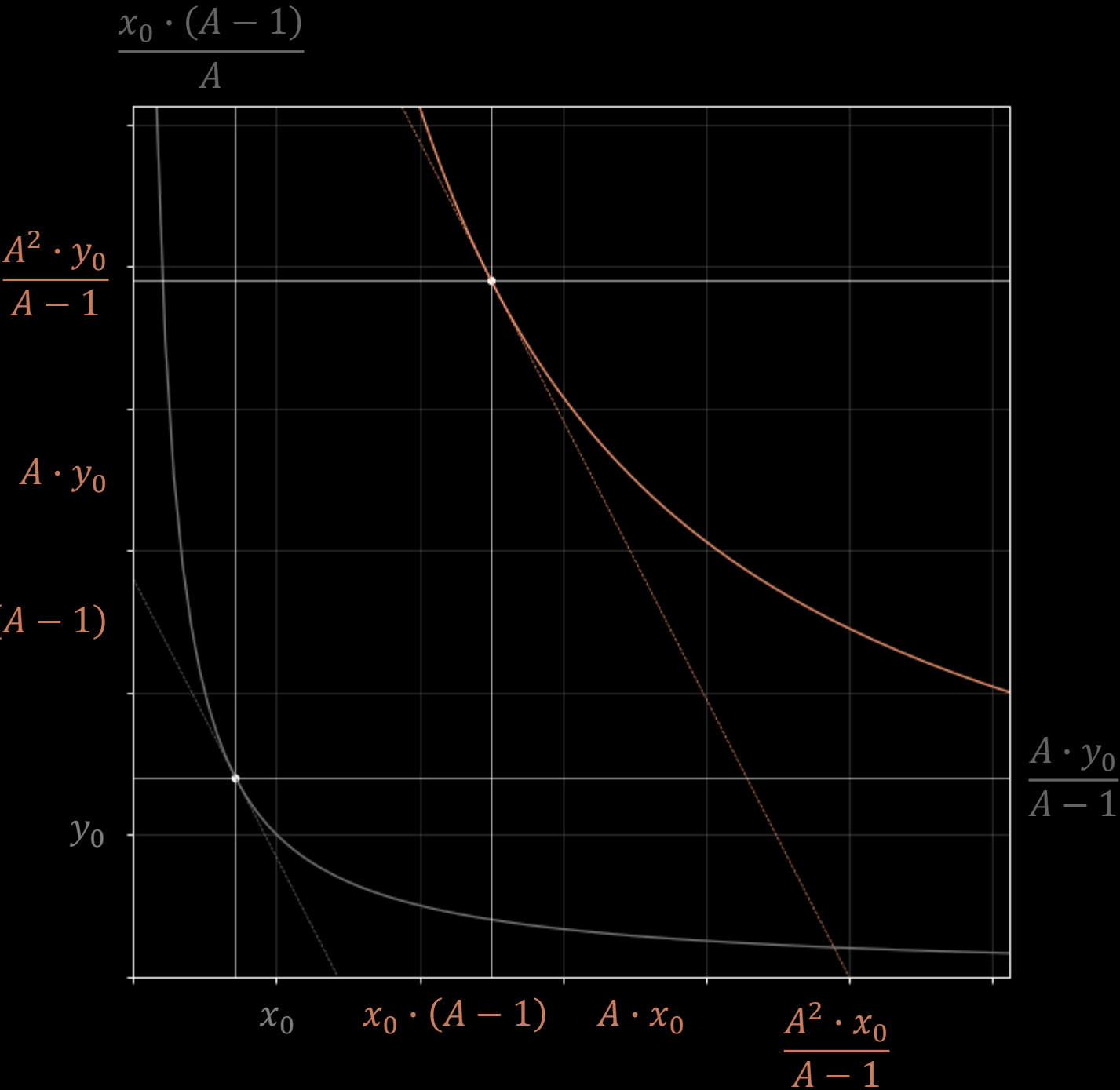


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



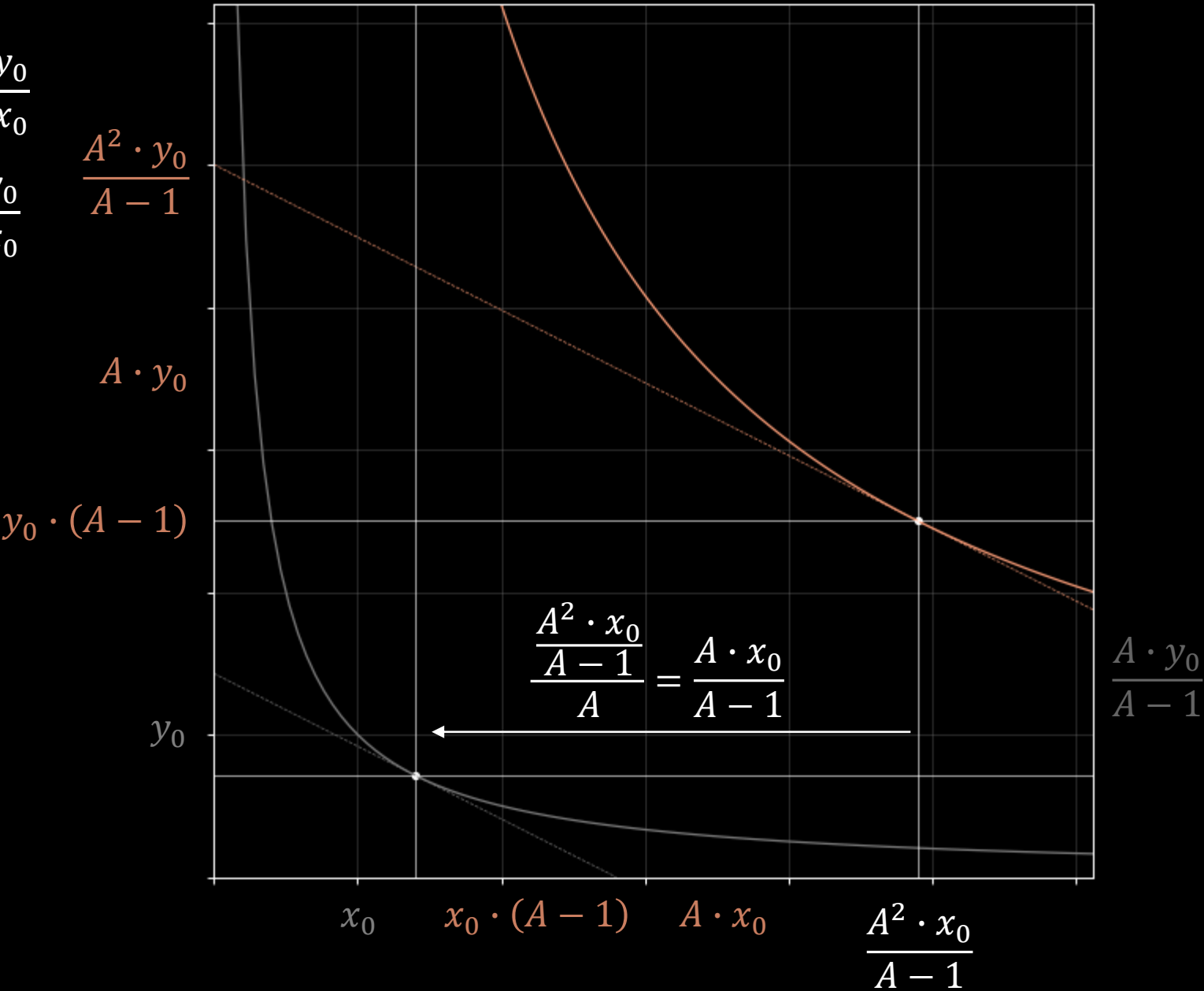


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

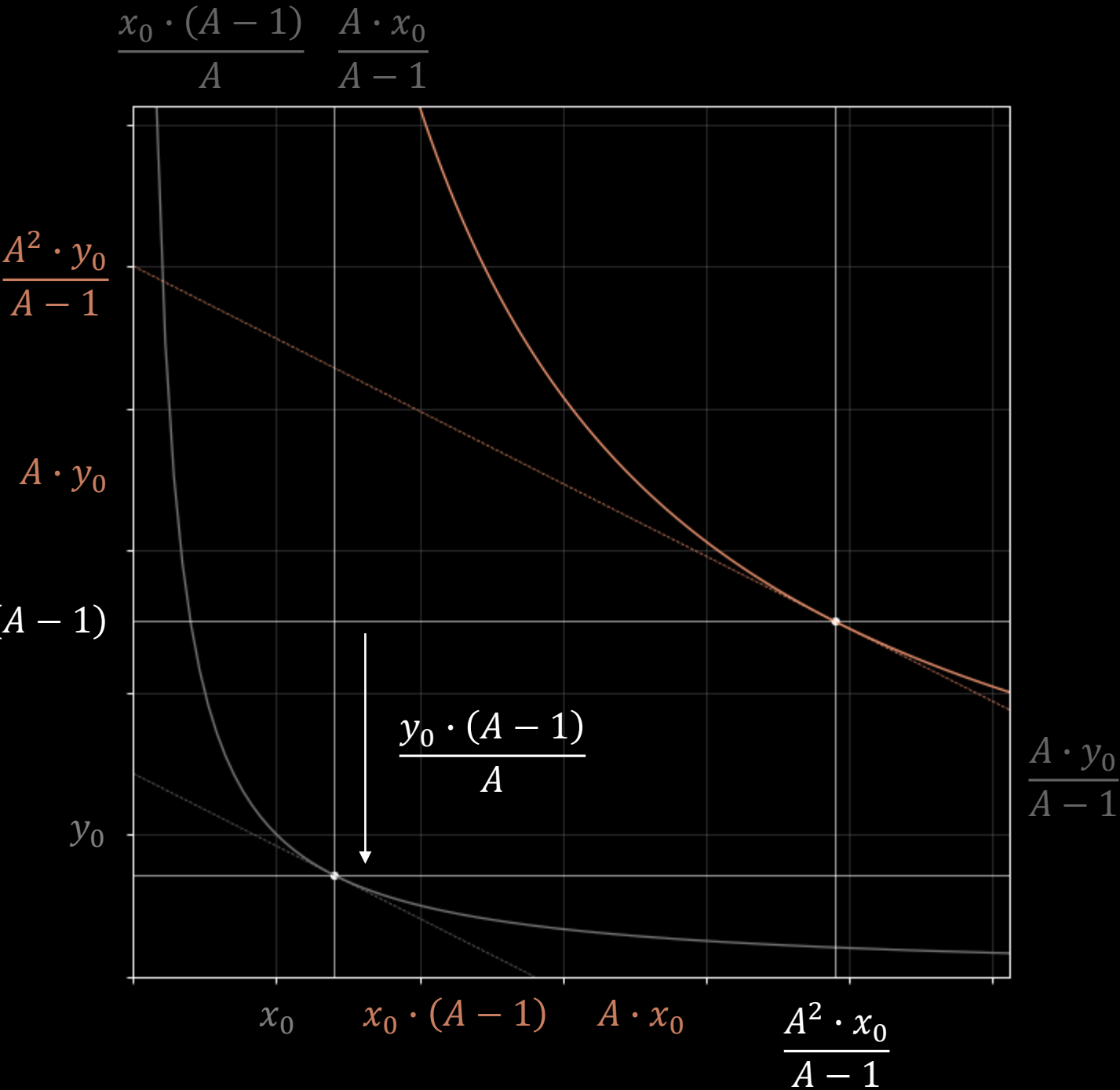


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

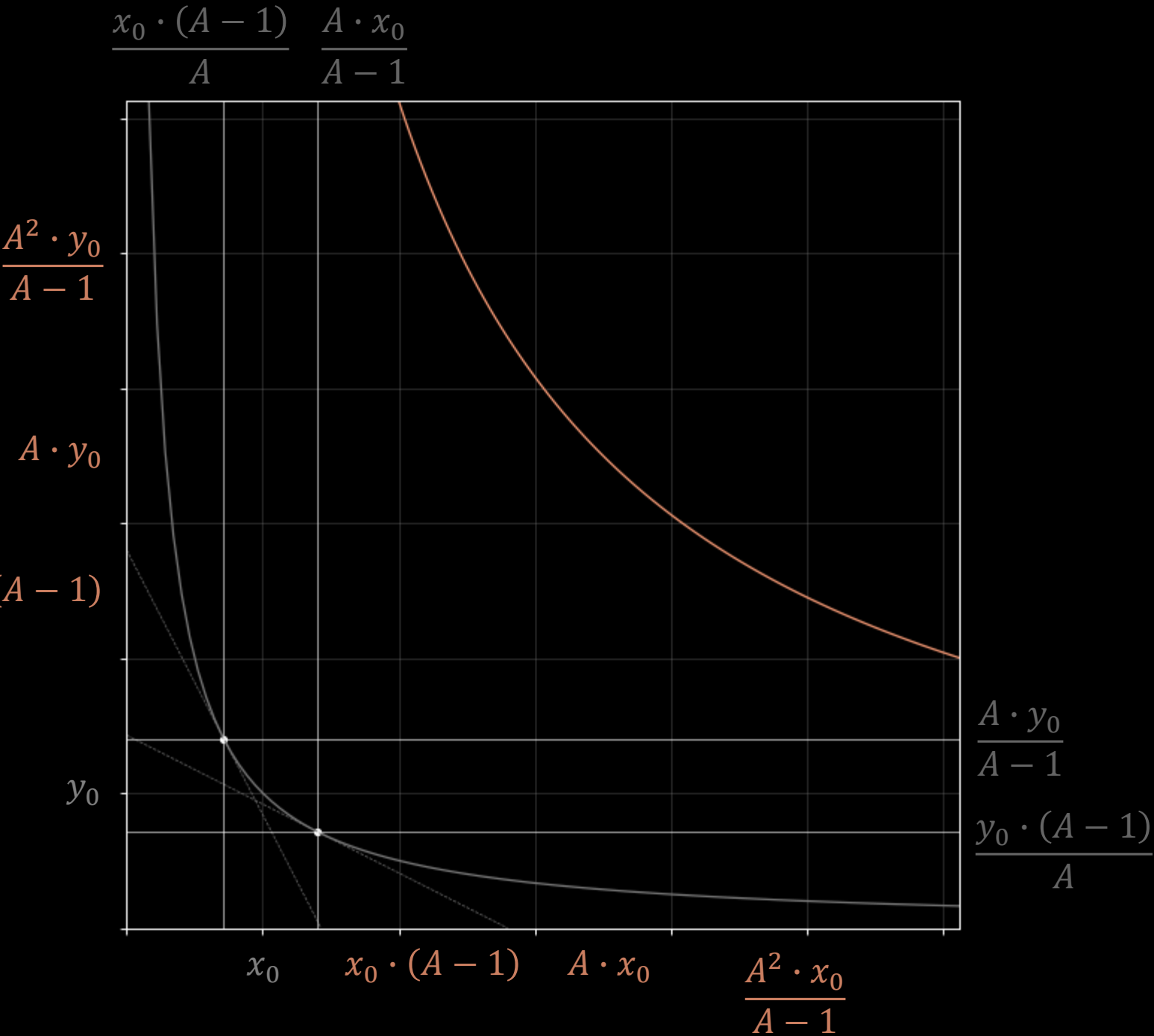


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



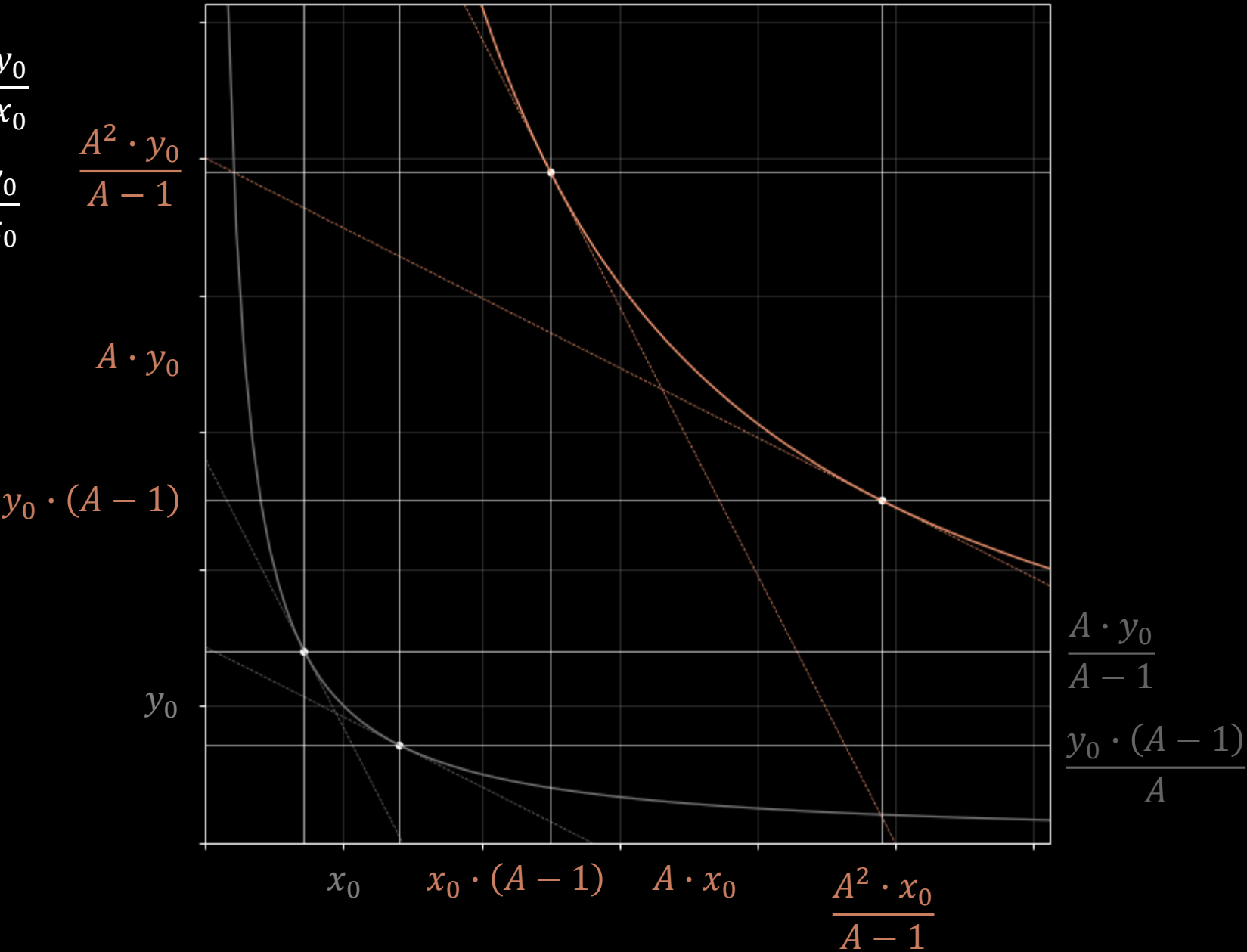
$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

$$\frac{x_0 \cdot (A-1)}{A} \qquad \frac{A \cdot x_0}{A-1}$$

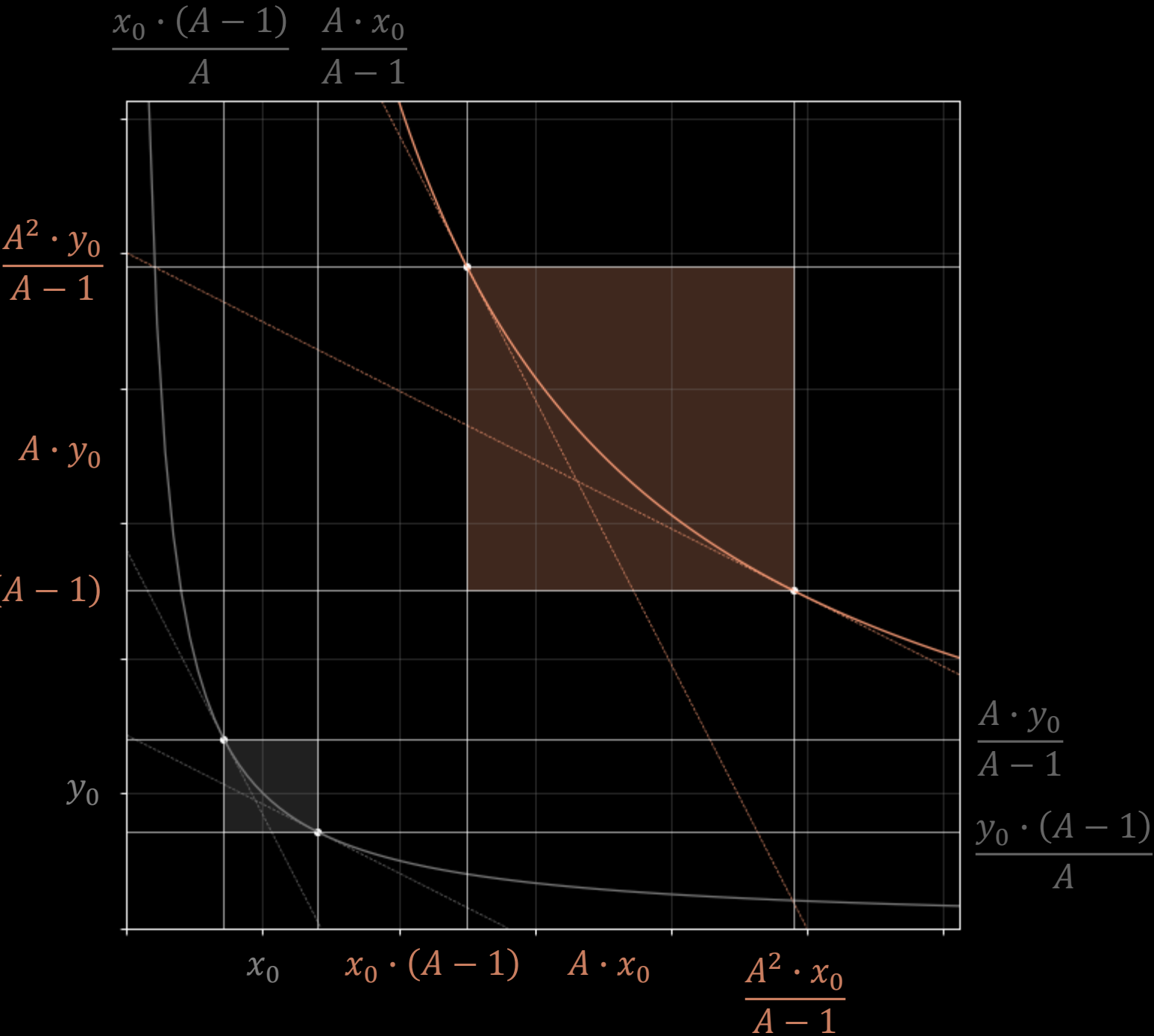


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

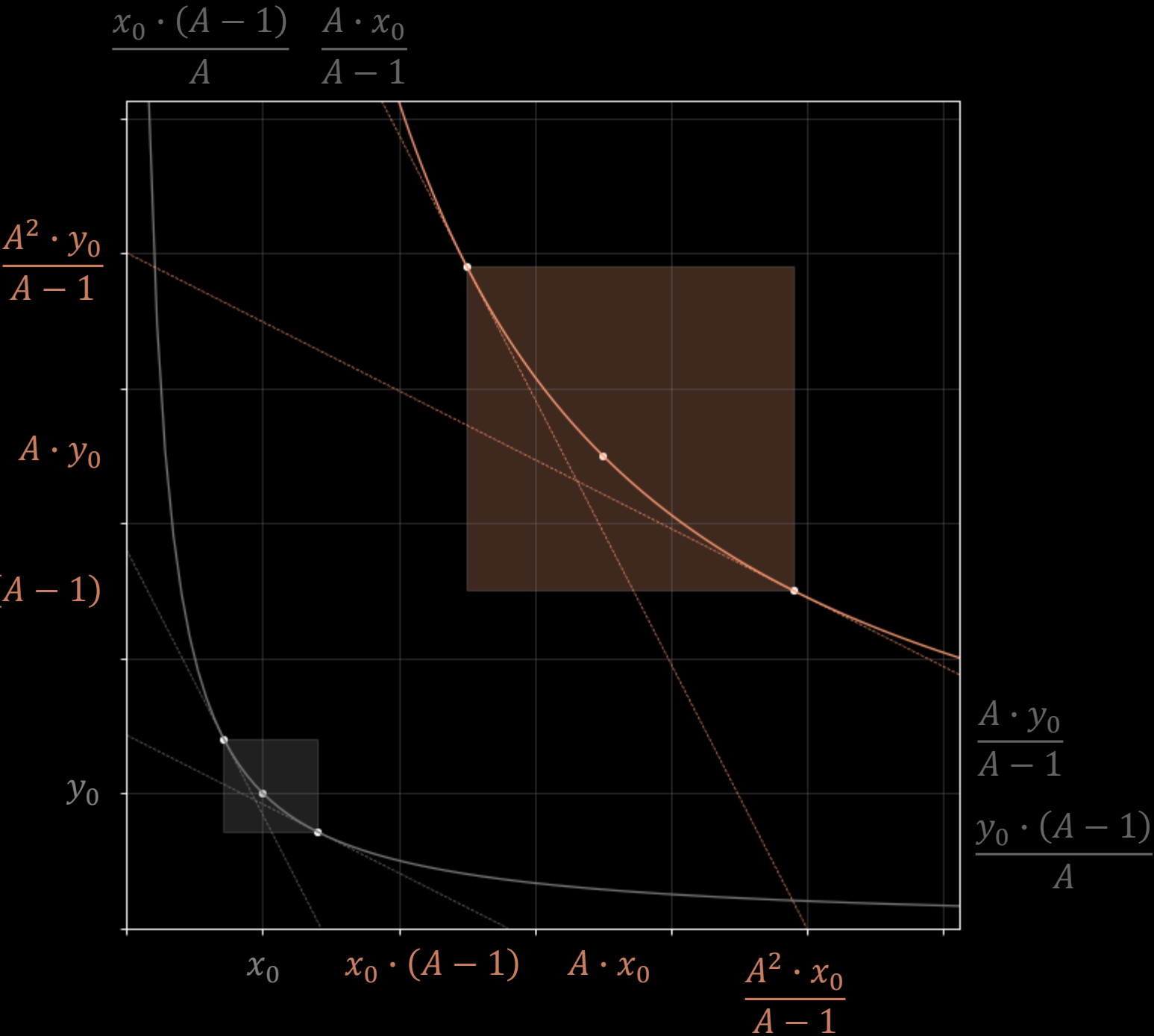


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

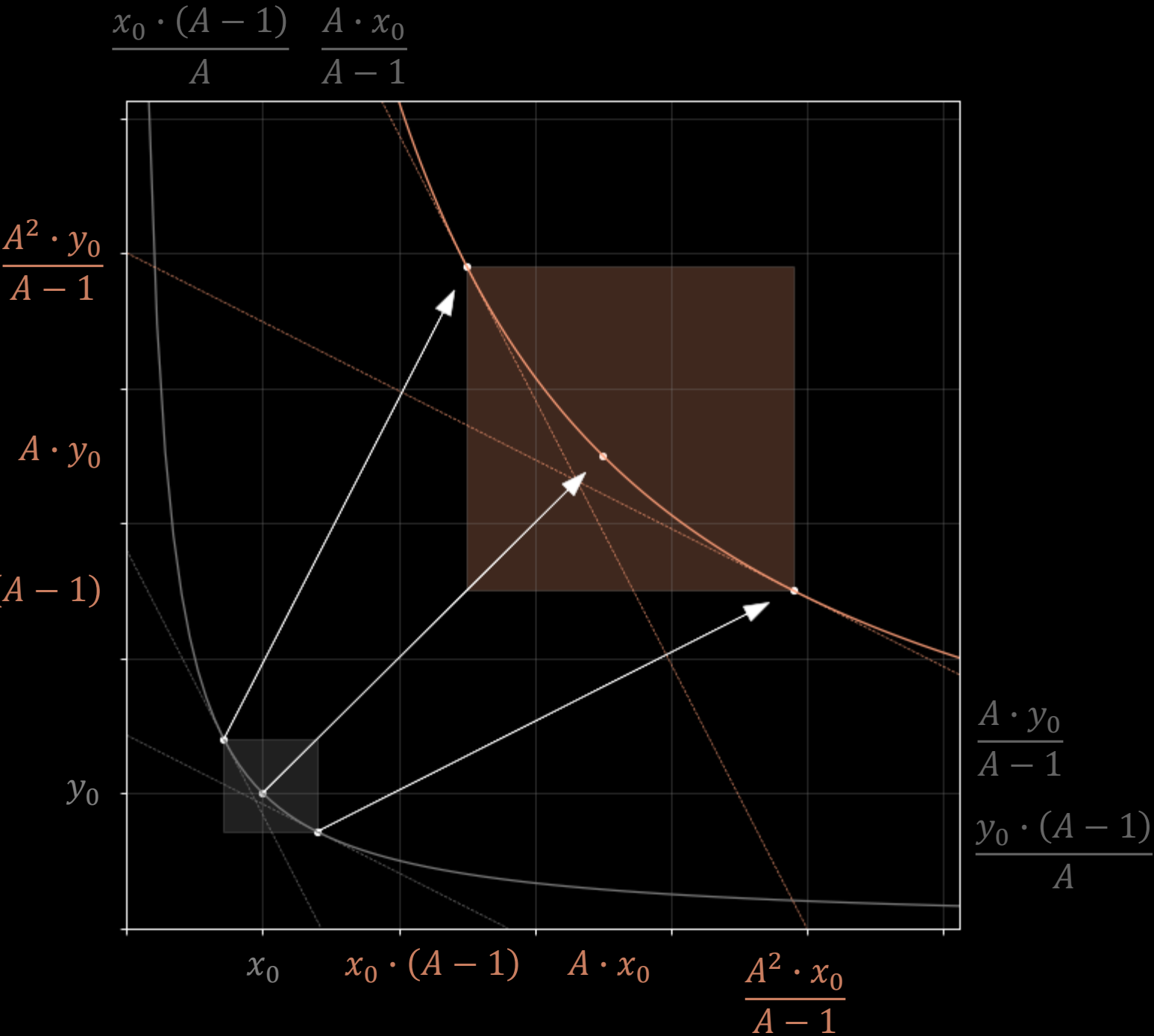


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



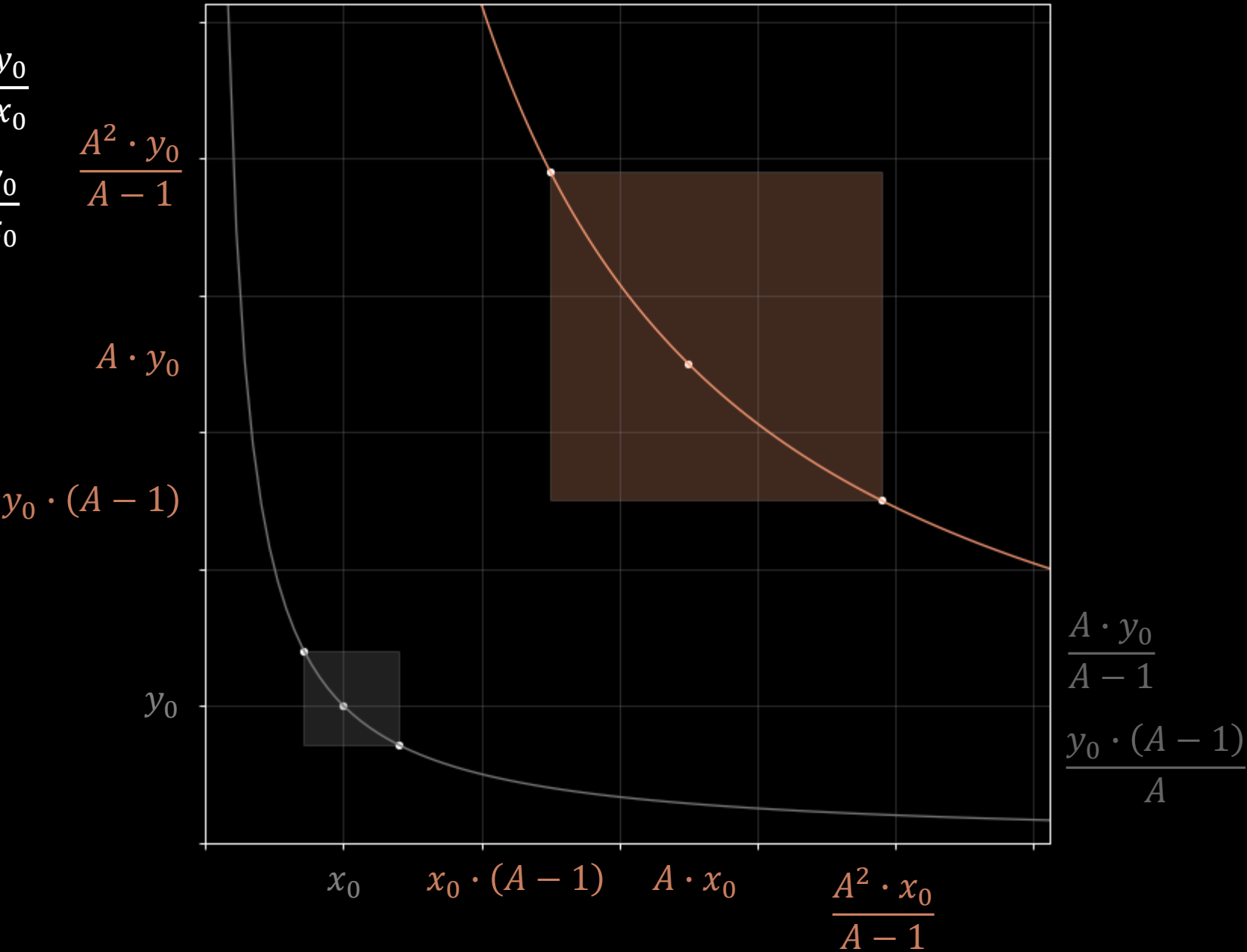
$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

$$\frac{x_0 \cdot (A-1)}{A} \qquad \frac{A \cdot x_0}{A-1}$$



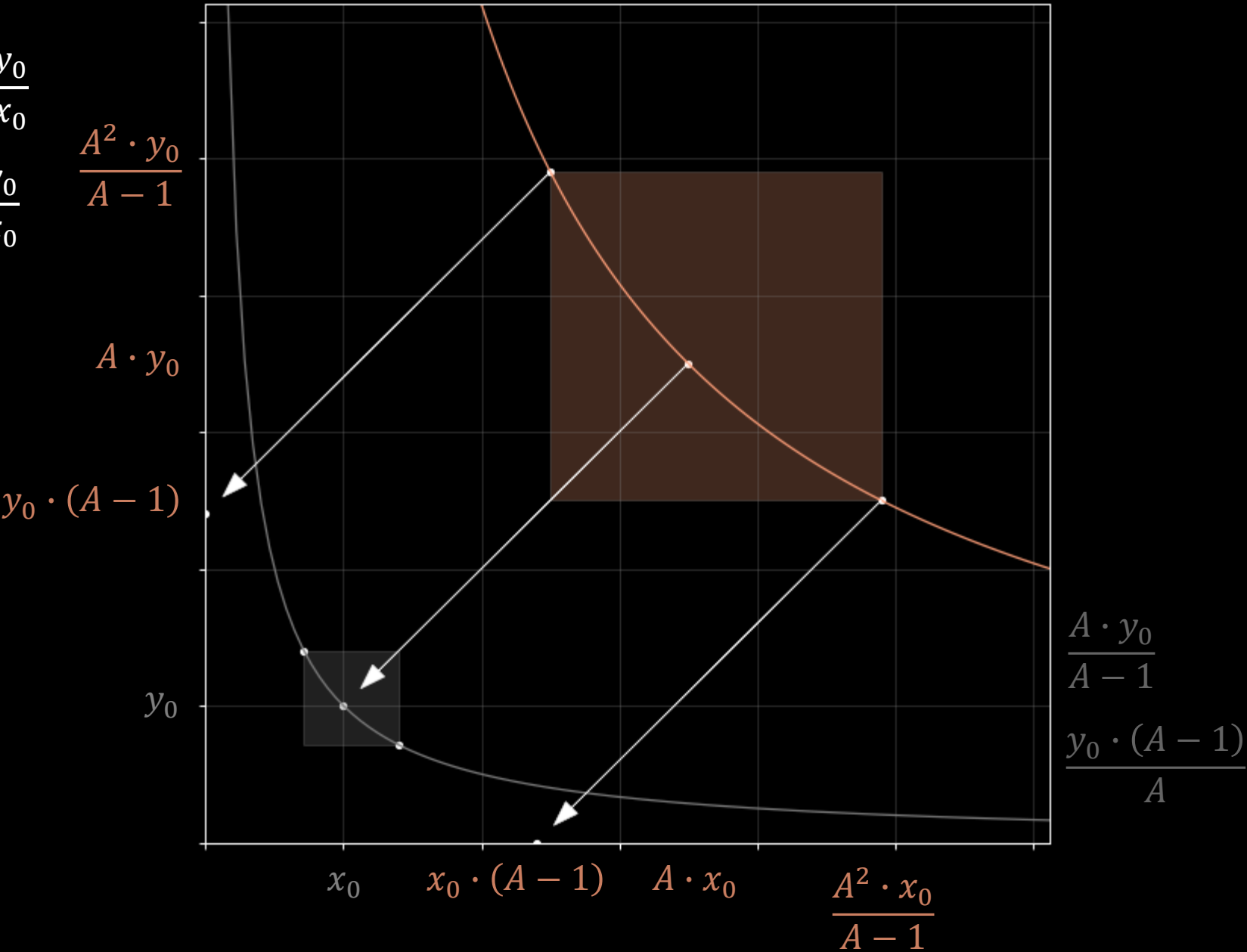


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

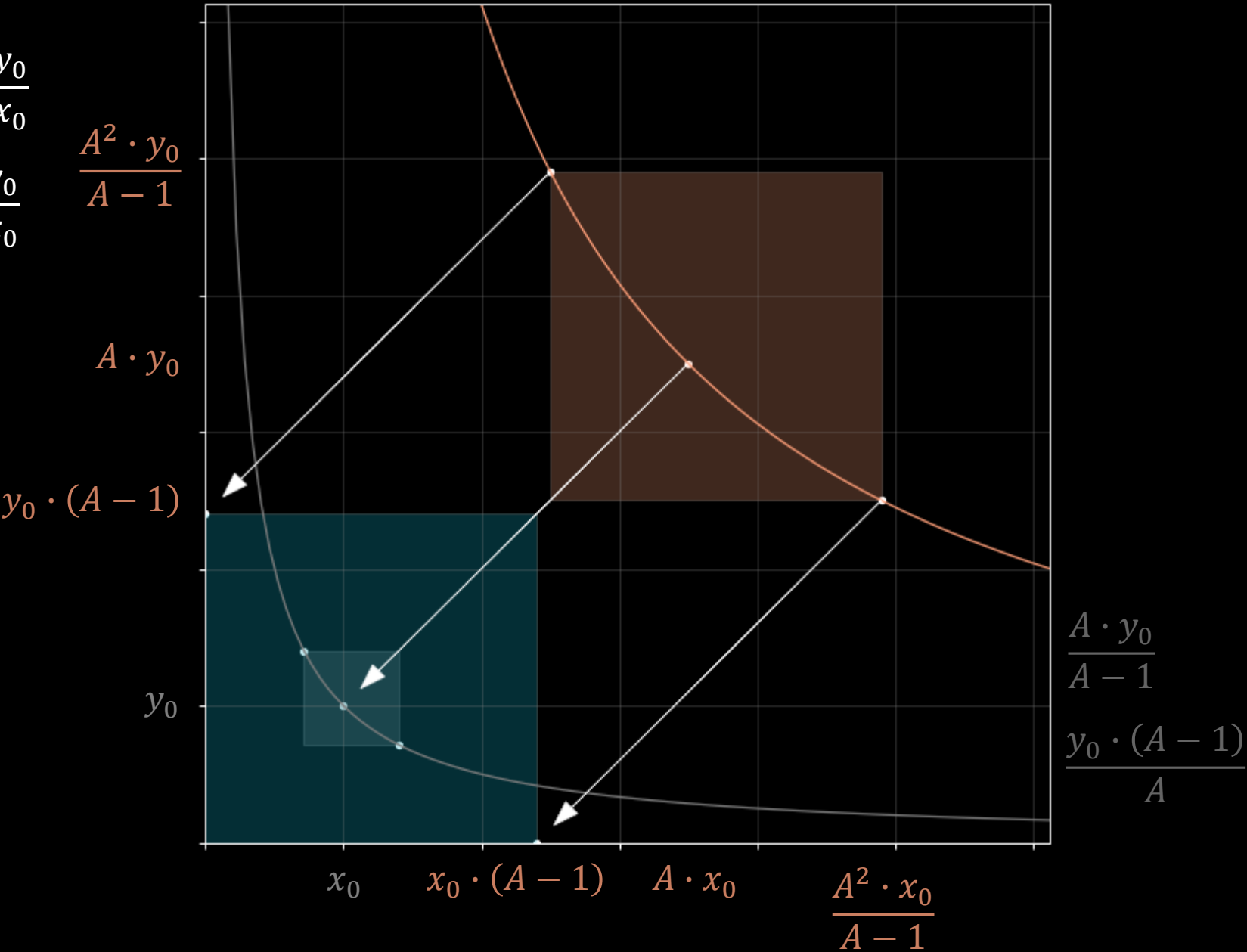


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

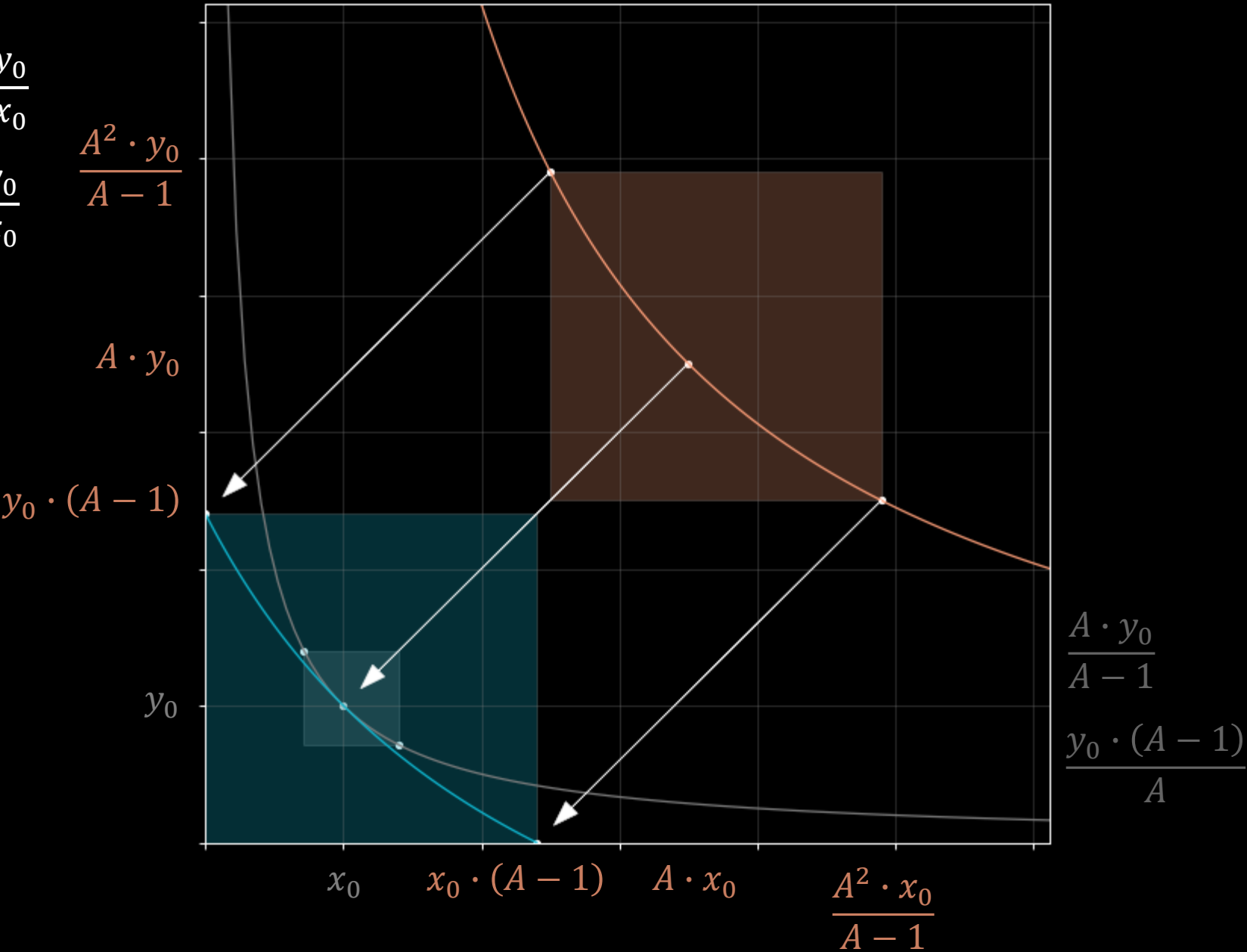


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

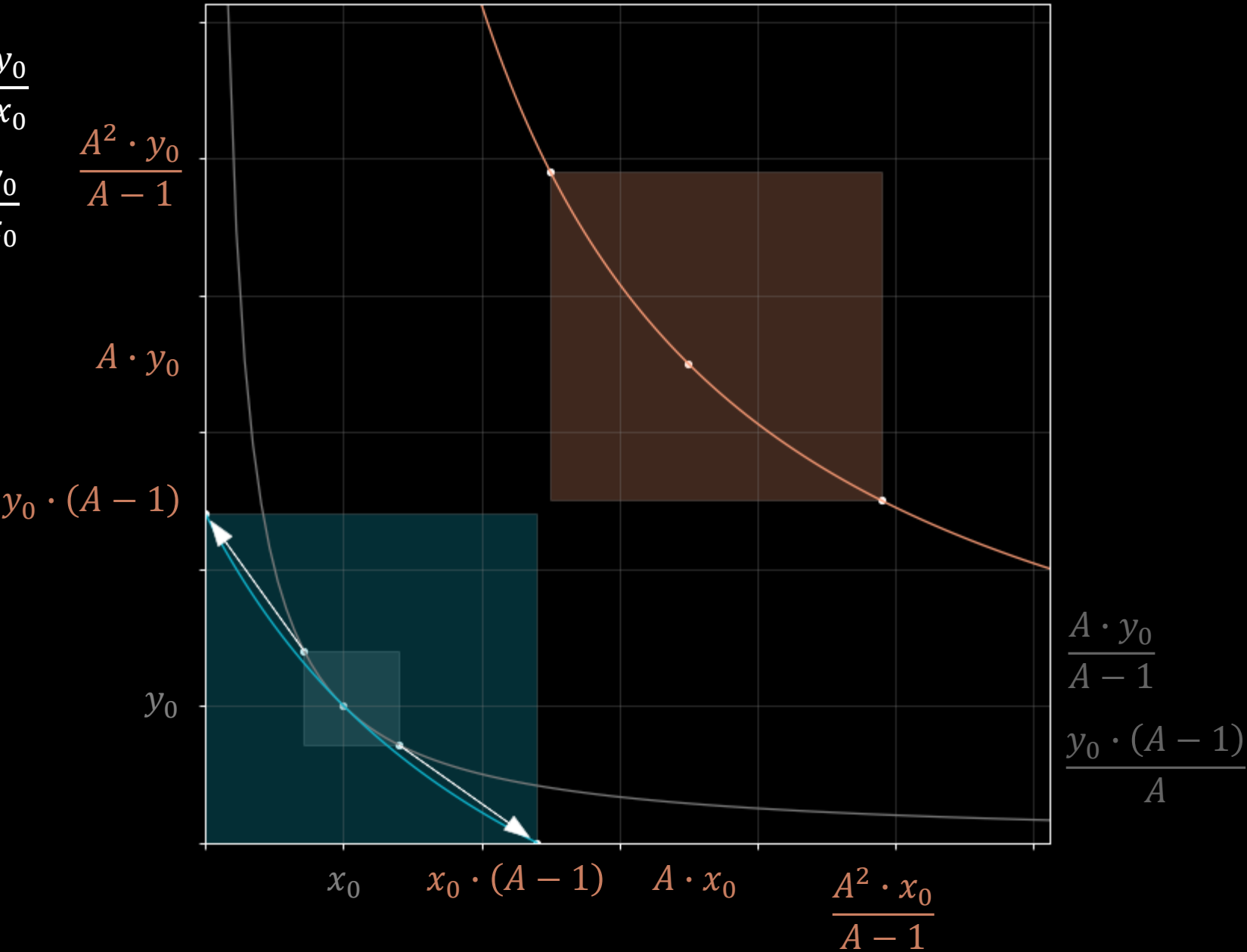


$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

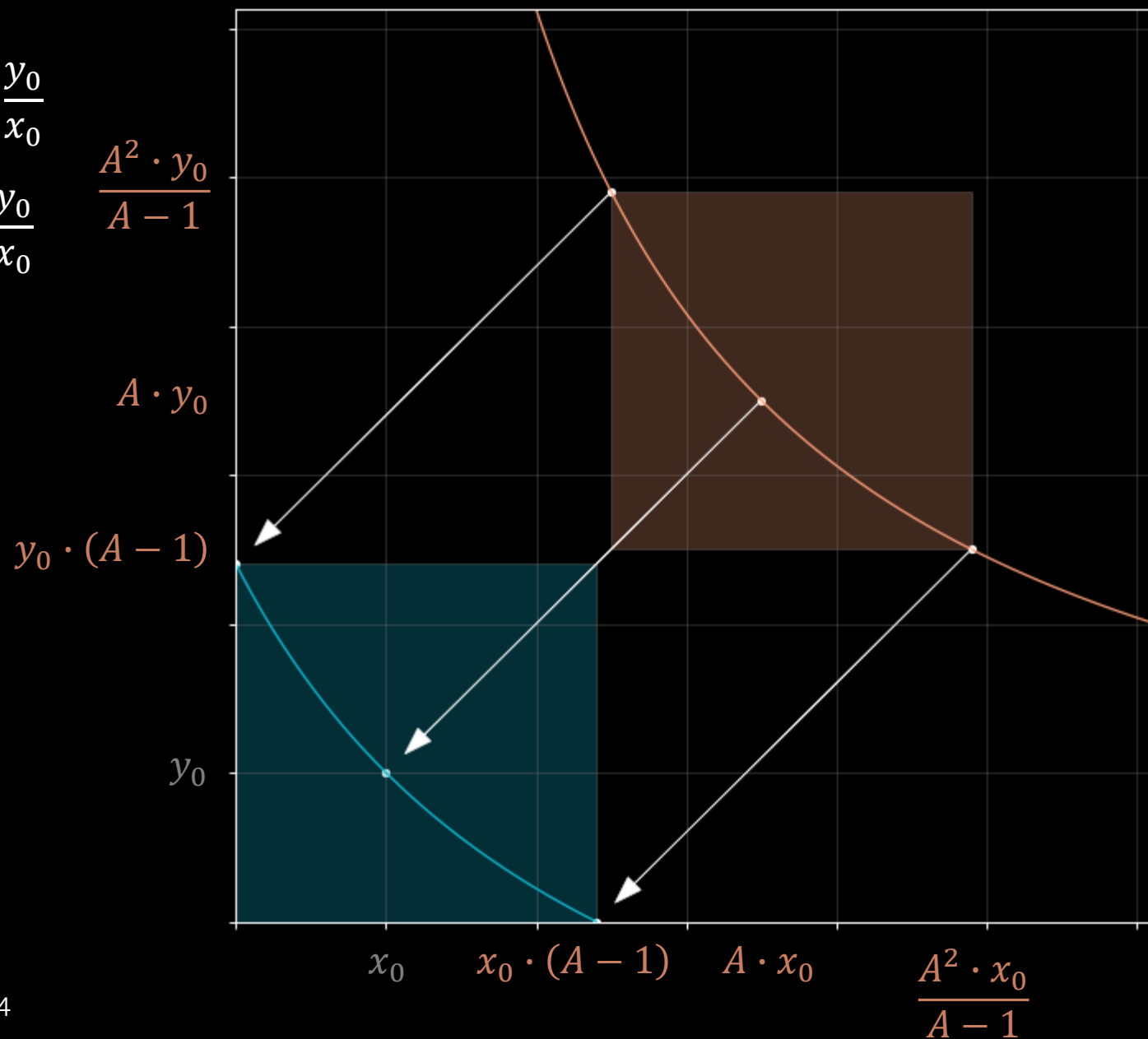
$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$





$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

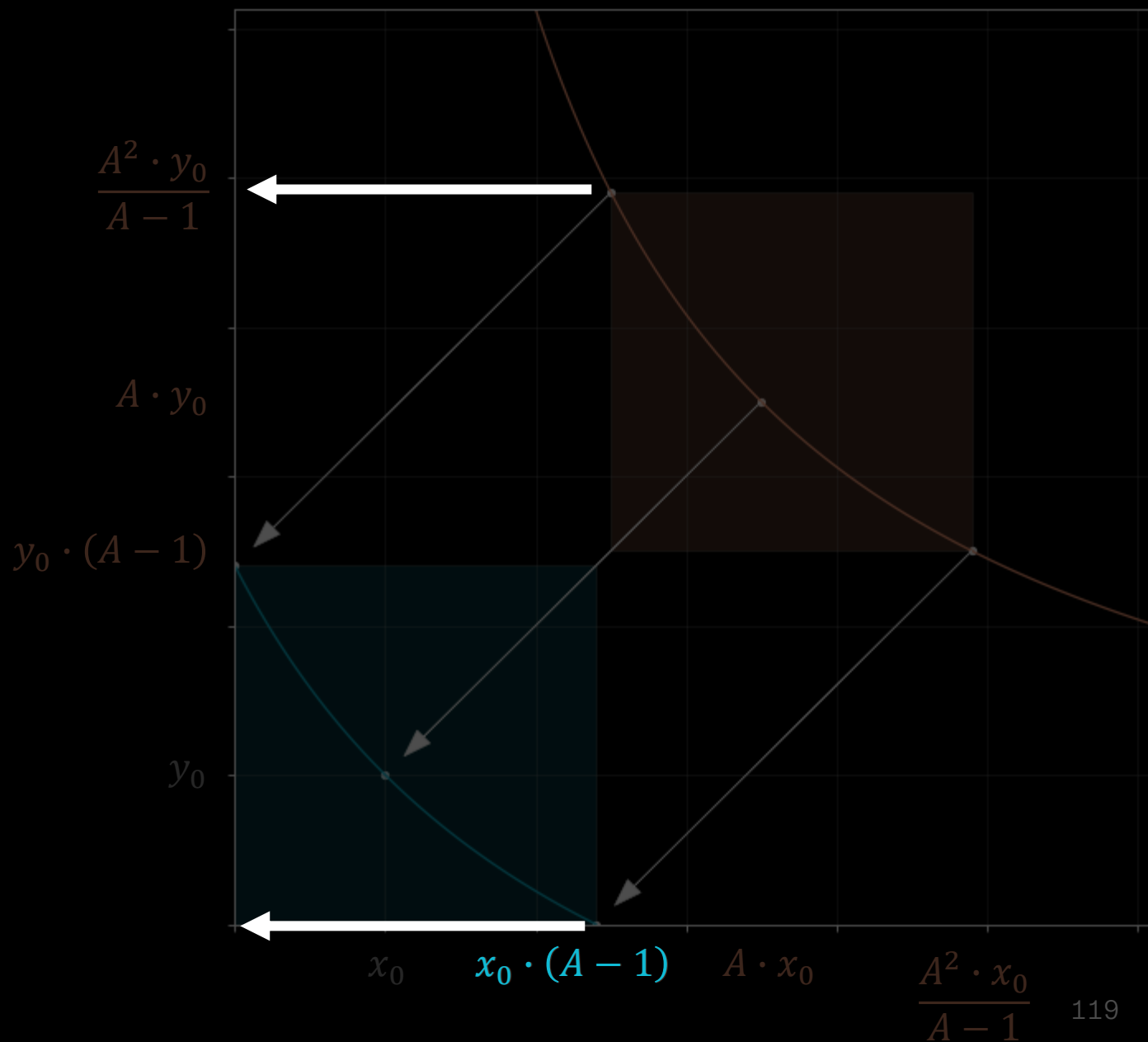
$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$(x+h) \cdot (y+v) = A^2 \cdot x_0 \cdot y_0$$

$h$  = horizontal shift

$v$  = vertical shift



$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

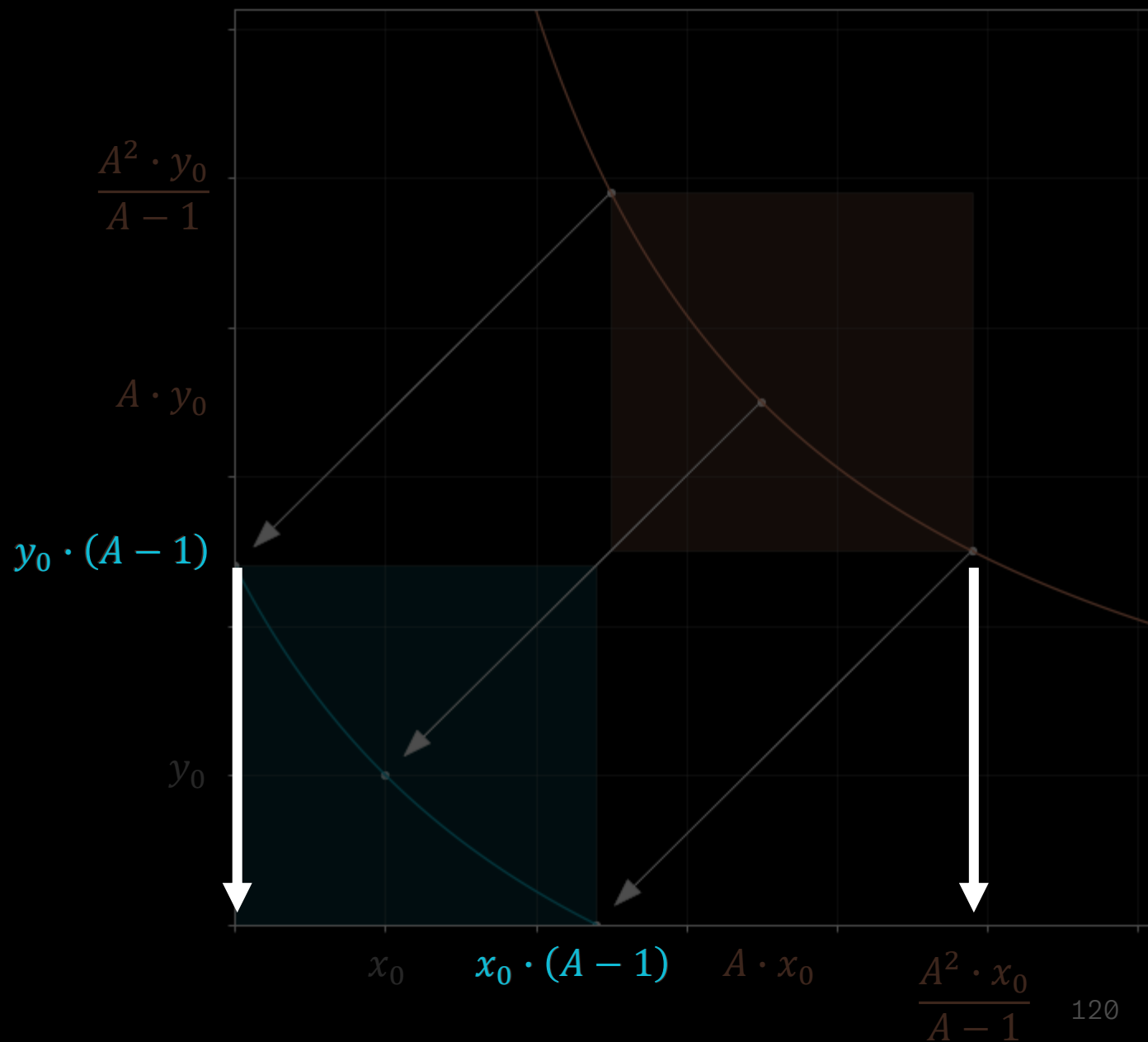
$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$(x+h) \cdot (y+v) = A^2 \cdot x_0 \cdot y_0$$

$h$  = horizontal shift

$v$  = vertical shift





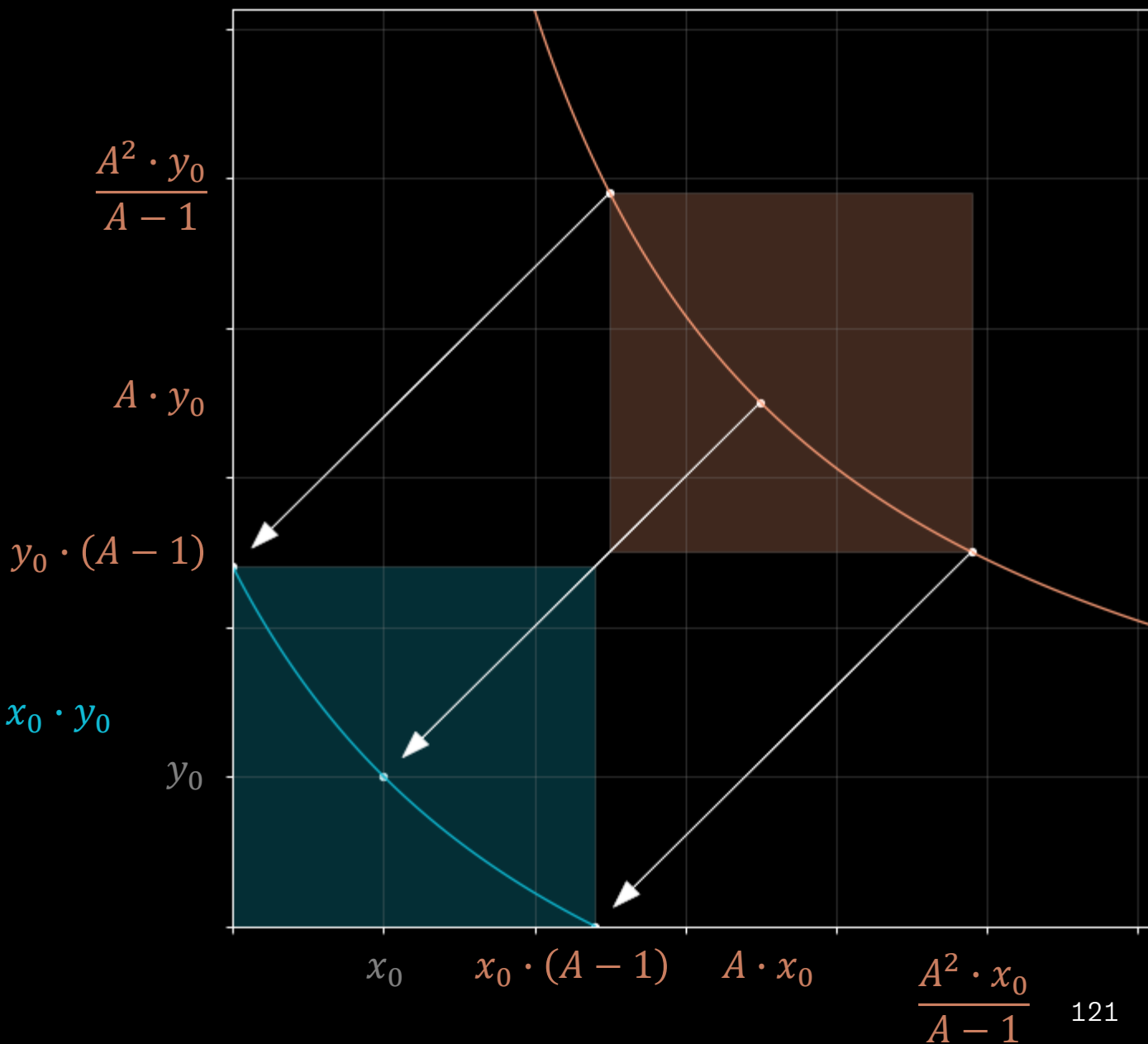
$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$(x+h) \cdot (y+v) = A^2 \cdot x_0 \cdot y_0$$

$$(x + x_0 \cdot (A-1)) \cdot (y + y_0 \cdot (A-1)) = A^2 \cdot x_0 \cdot y_0$$

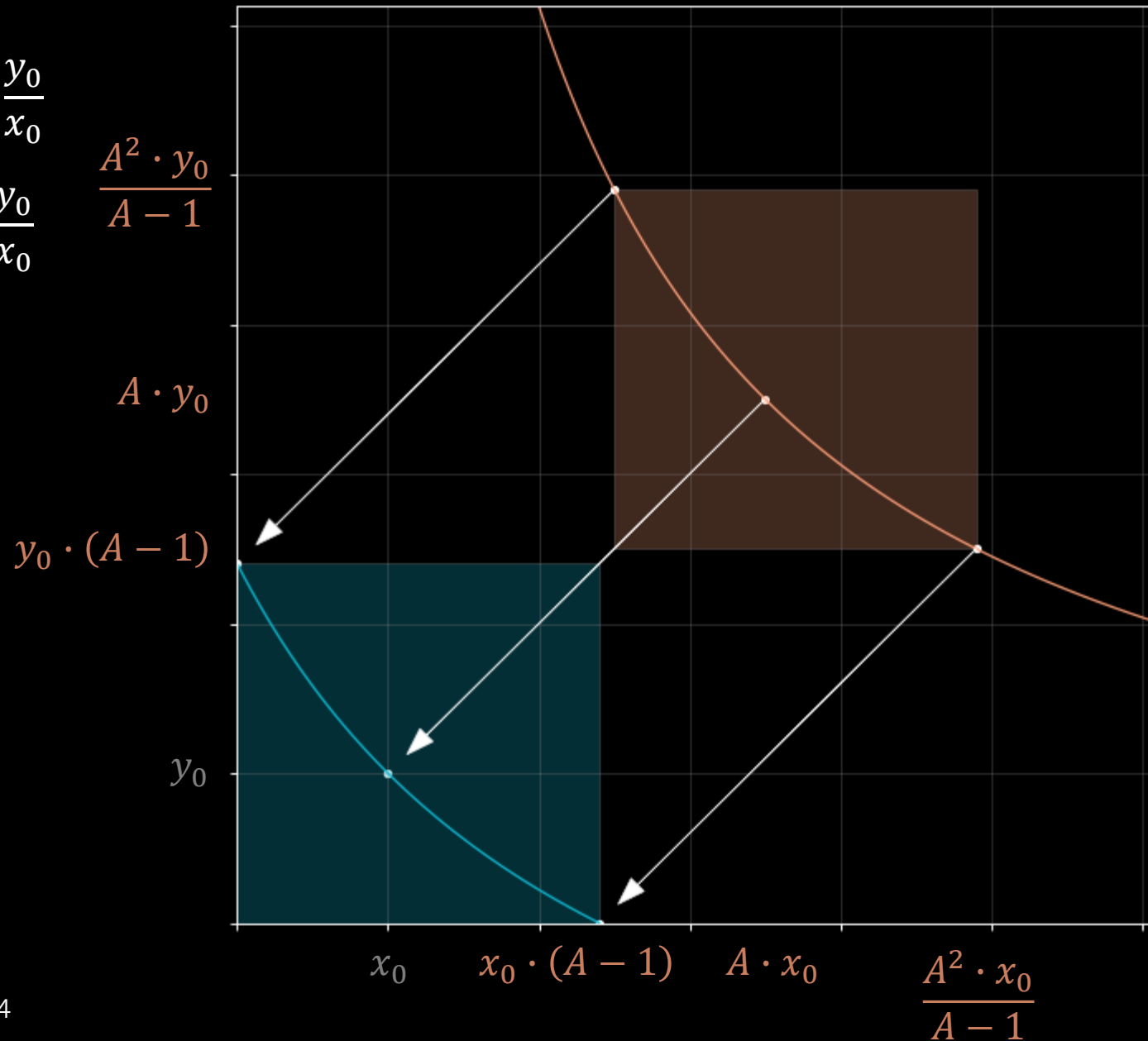


$$(x + x_0 \cdot (A - 1)) \cdot (y + y_0 \cdot (A - 1)) = A^2 \cdot x_0 \cdot y_0$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



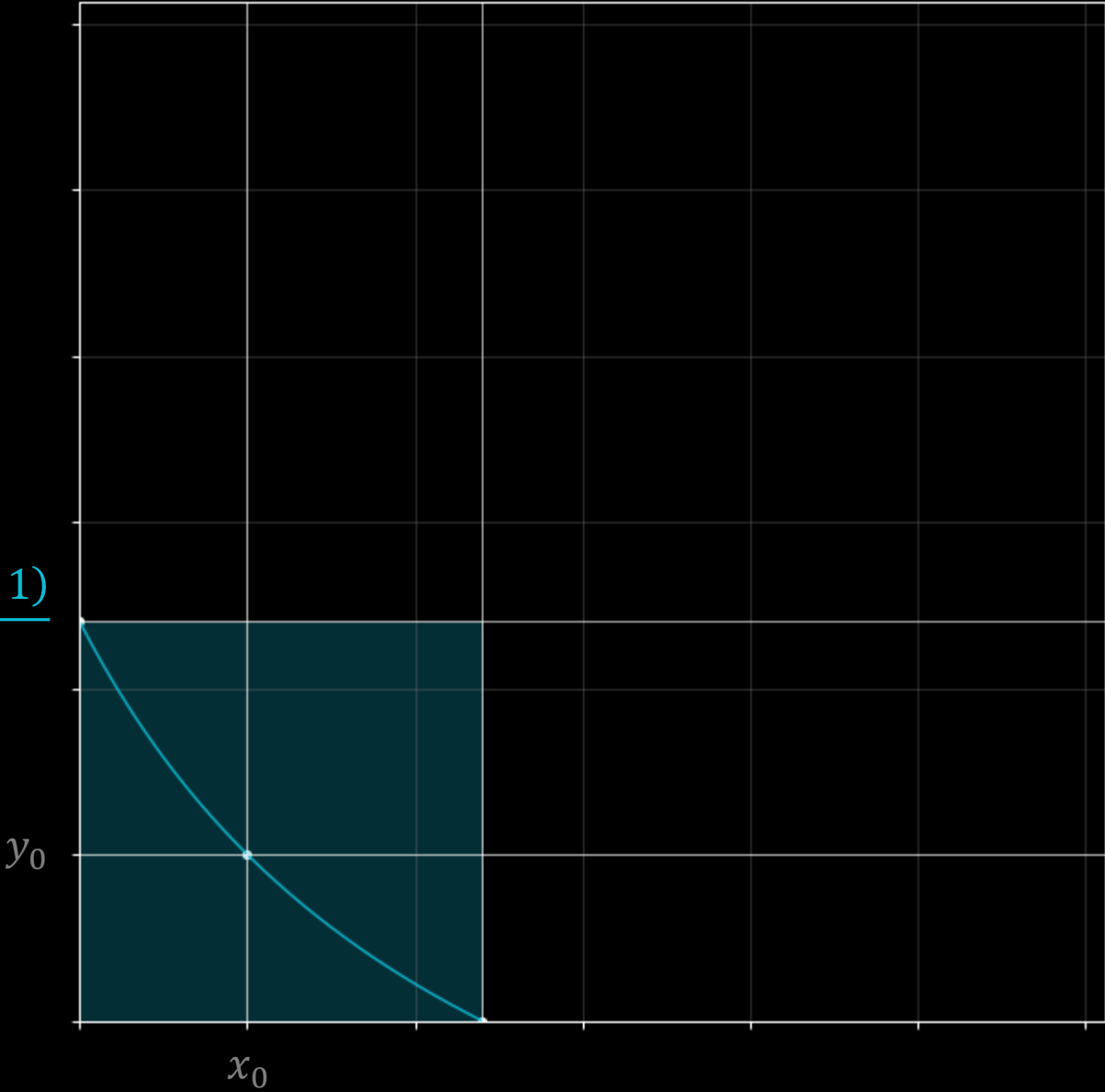
$$(x + x_0 \cdot (A - 1)) \cdot (y + y_0 \cdot (A - 1)) = A^2 \cdot x_0 \cdot y_0 \Big|_{x=0}$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

$$\frac{y_0 \cdot (2A - 1)}{A - 1}$$



$$(x + x_0 \cdot (A - 1)) \cdot (y + y_0 \cdot (A - 1)) = A^2 \cdot x_0 \cdot y_0 \Big|_{y=0}$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A - 1)^2} \cdot \frac{y_0}{x_0}$$

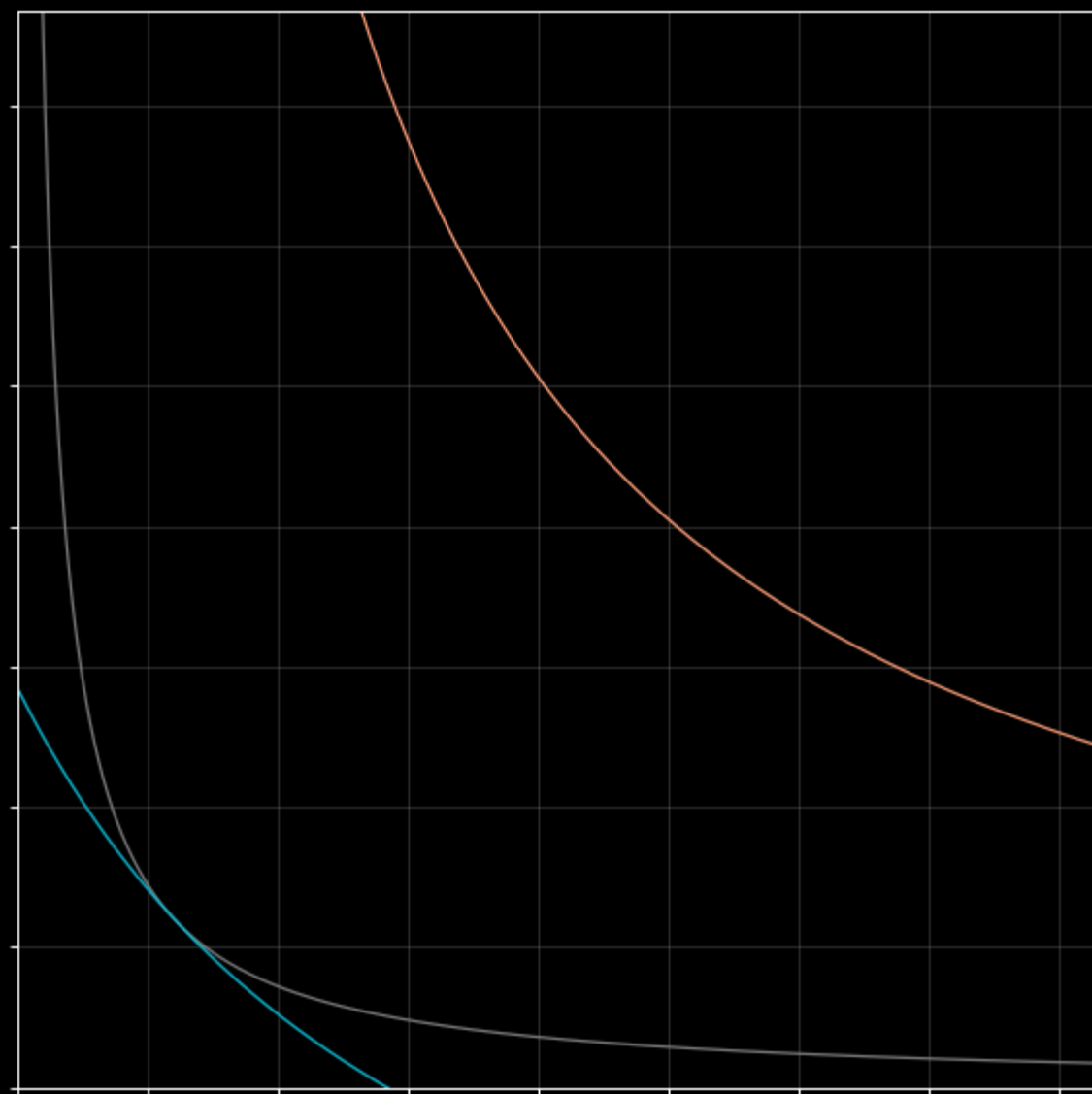
$$P_{\text{low}} = \frac{(A - 1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

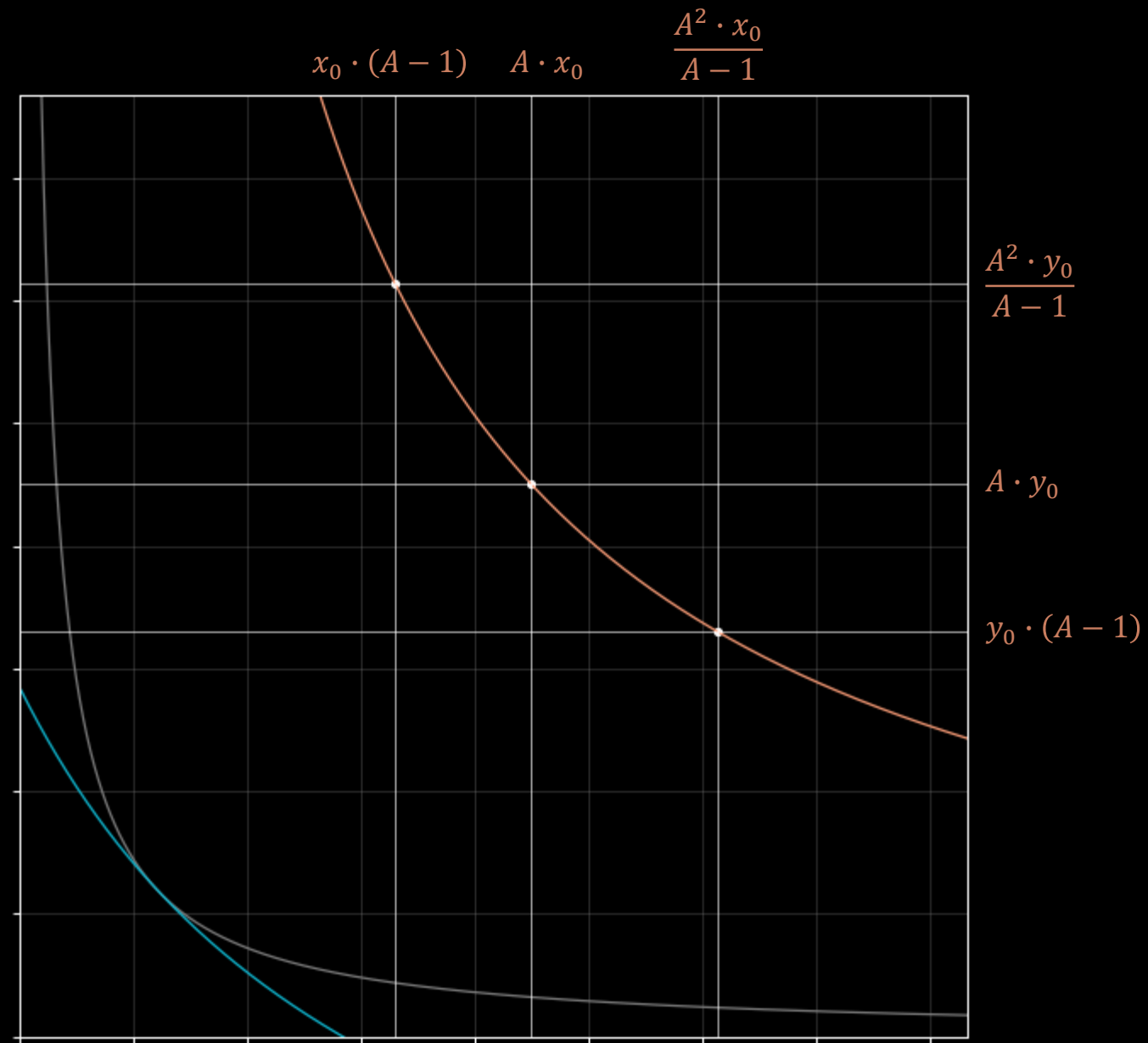
$$\frac{y_0 \cdot (2A - 1)}{A - 1}$$

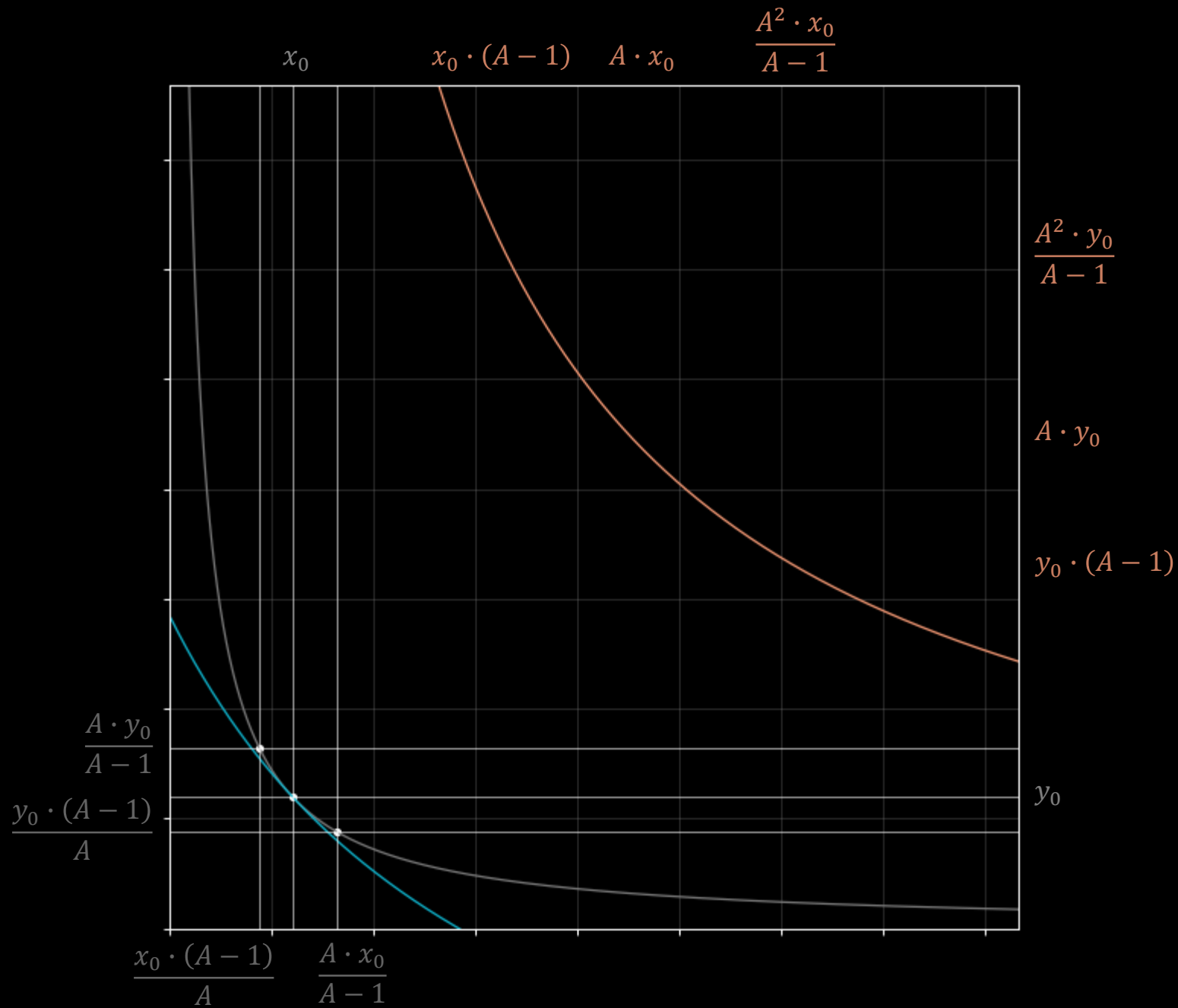
$$y_0$$

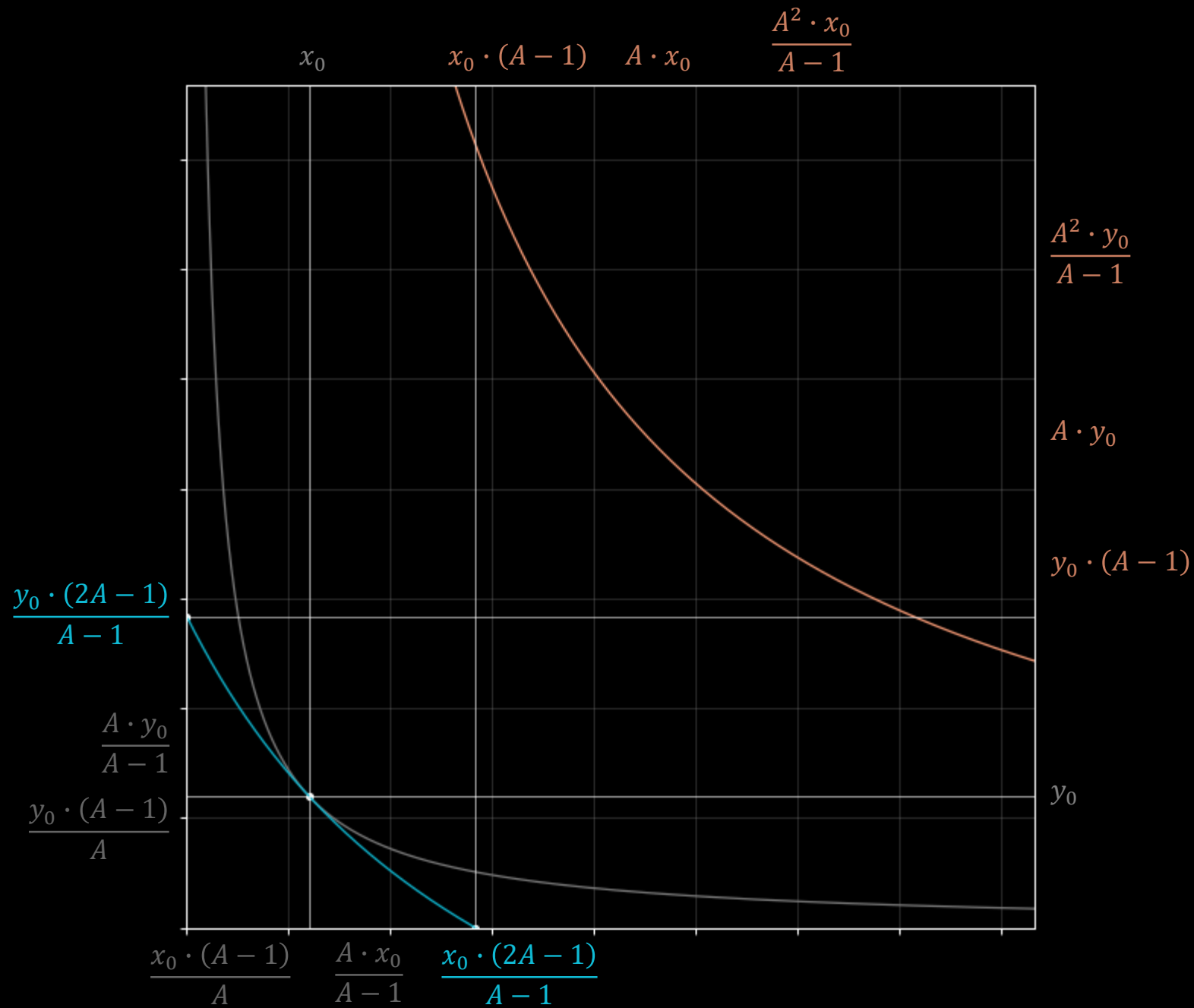
$$x_0$$

$$\frac{x_0 \cdot (2A - 1)}{A - 1}$$

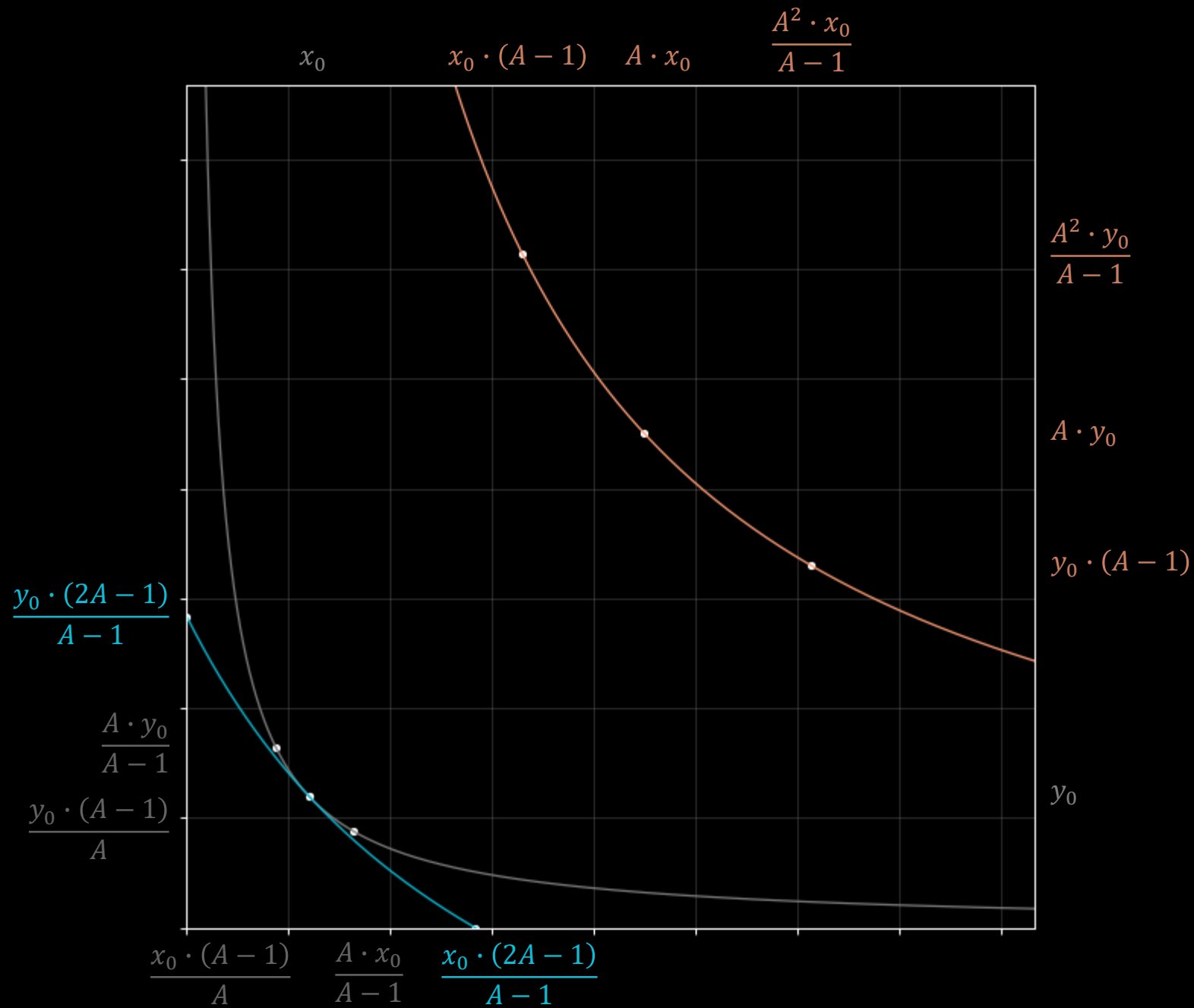




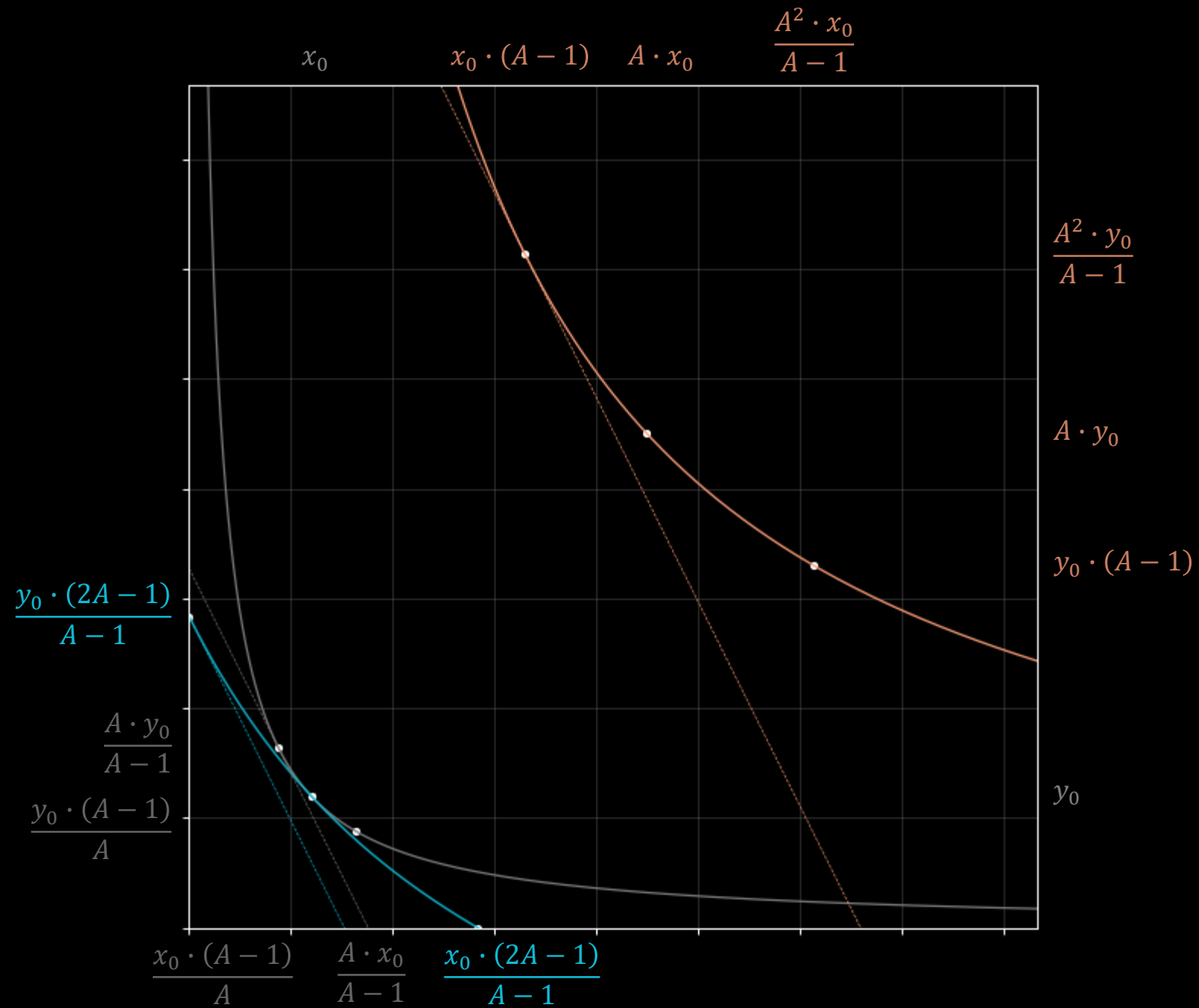




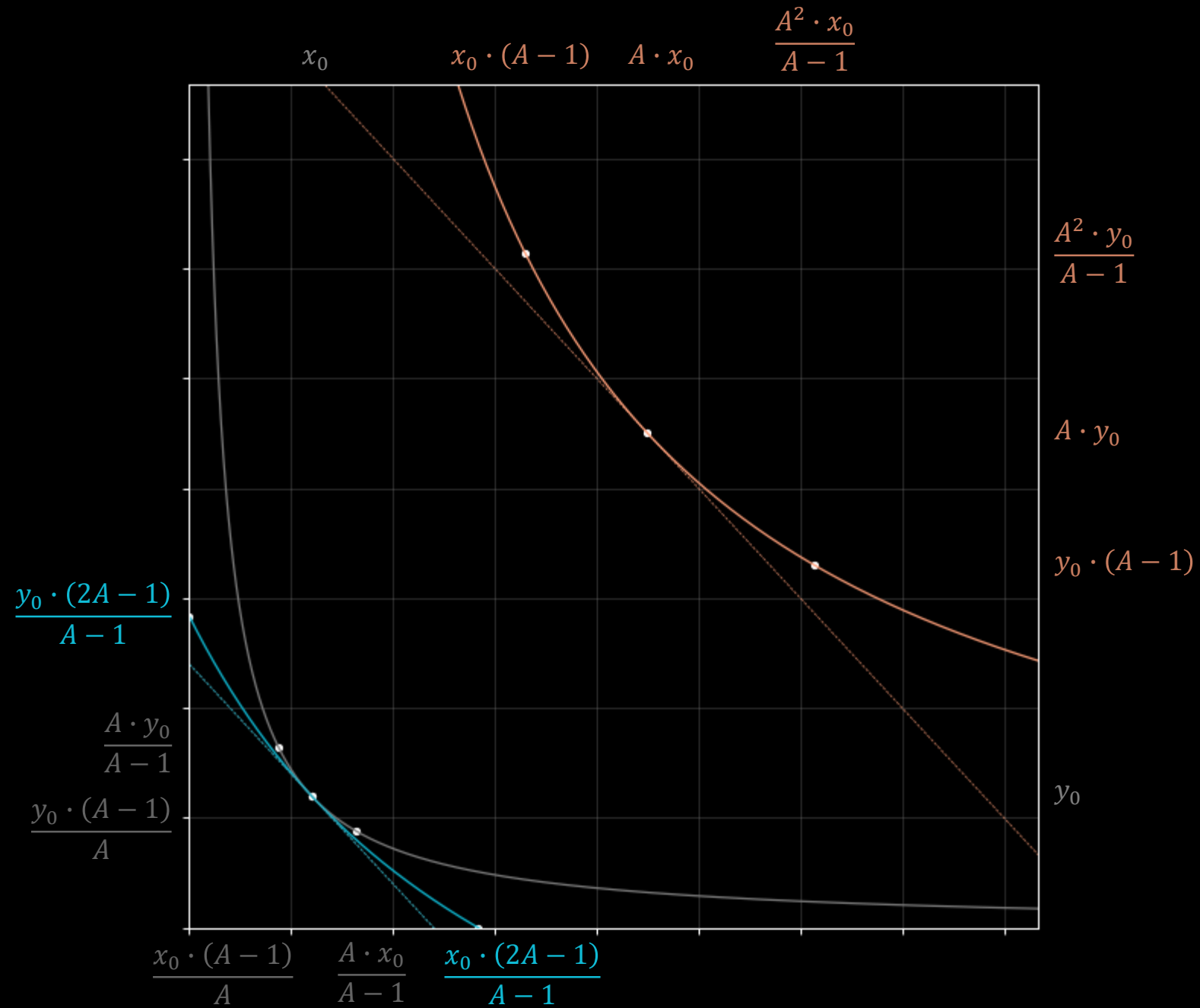




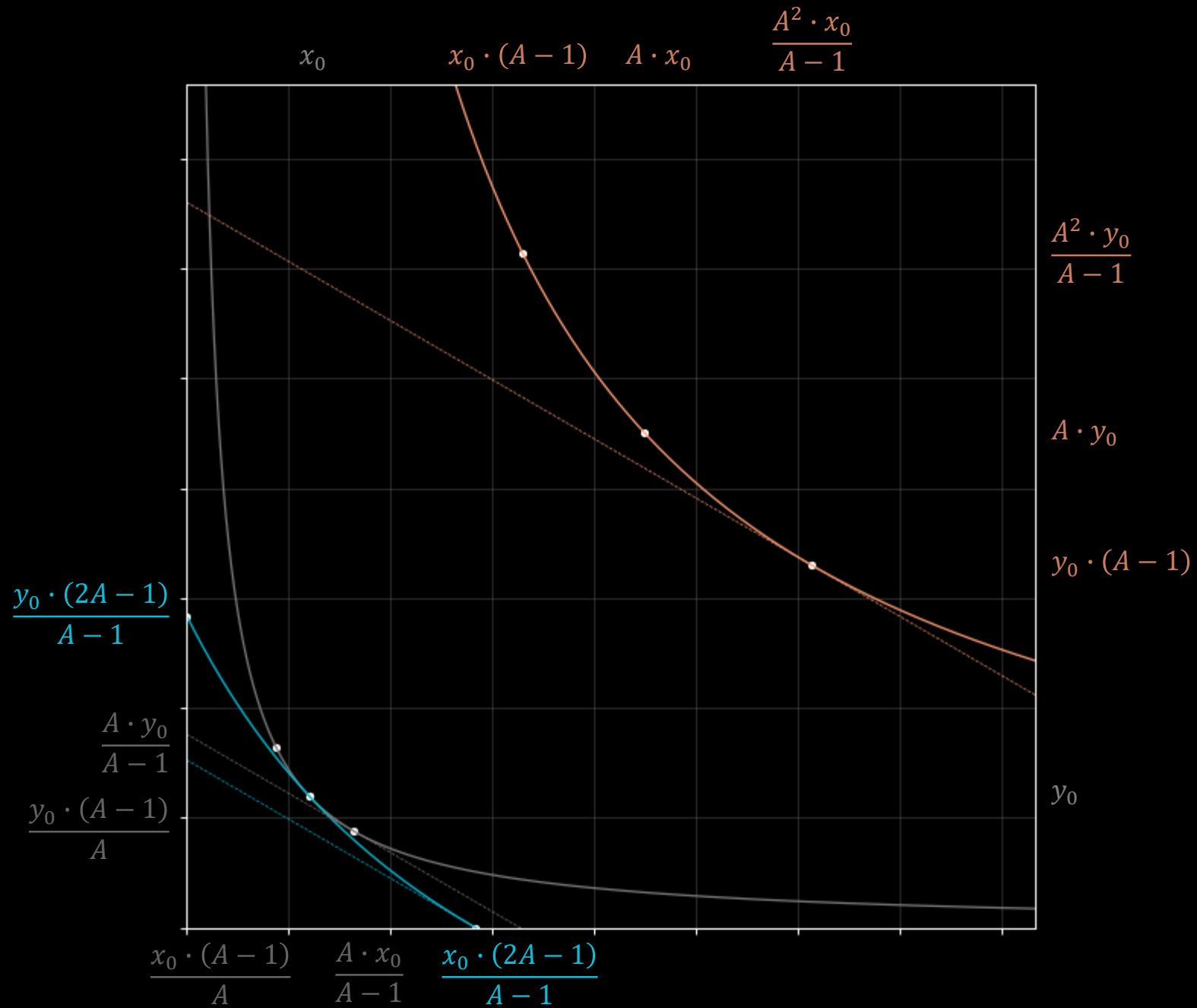
$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$



$$P_0 = \frac{y_0}{x_0}$$



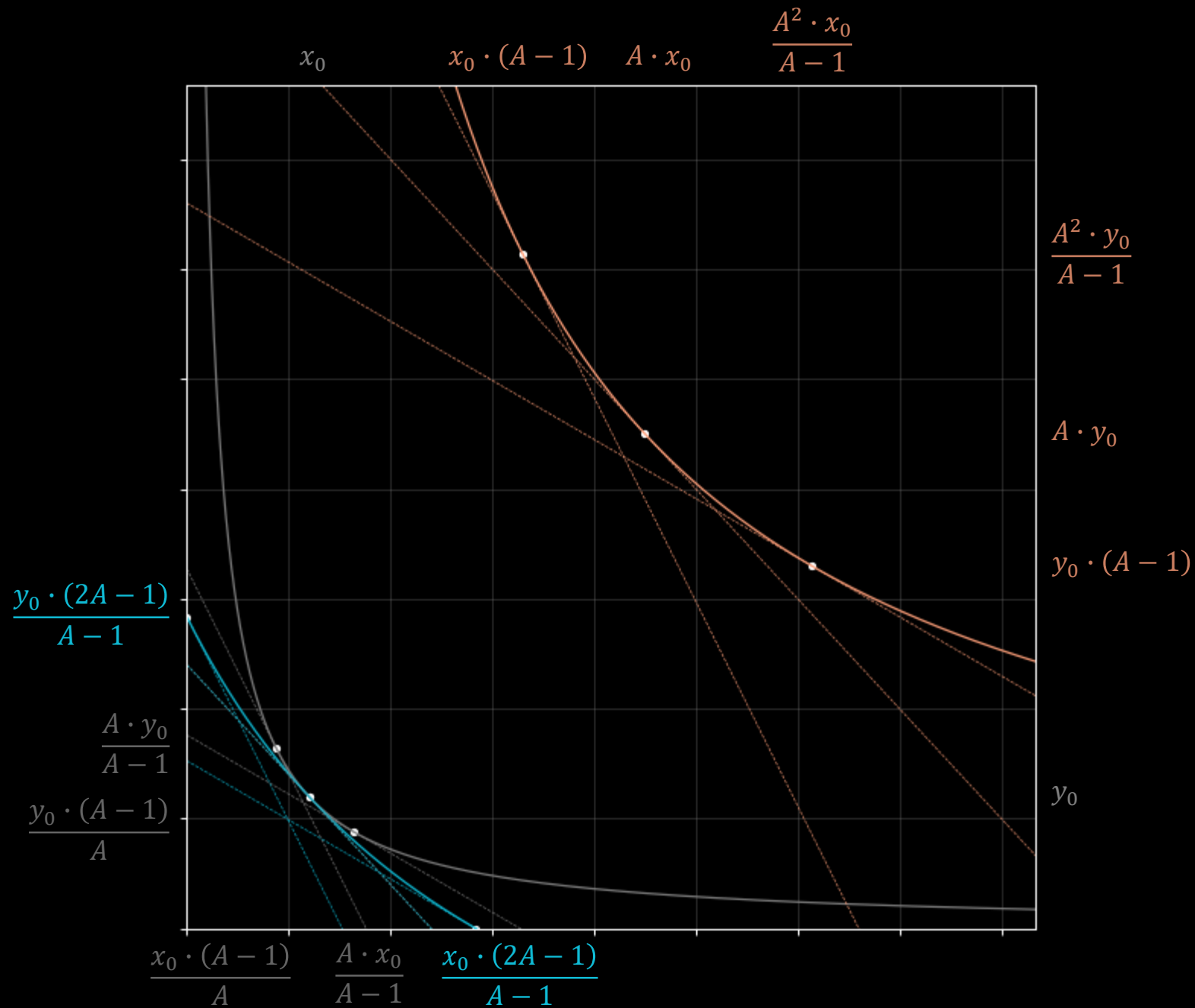
$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

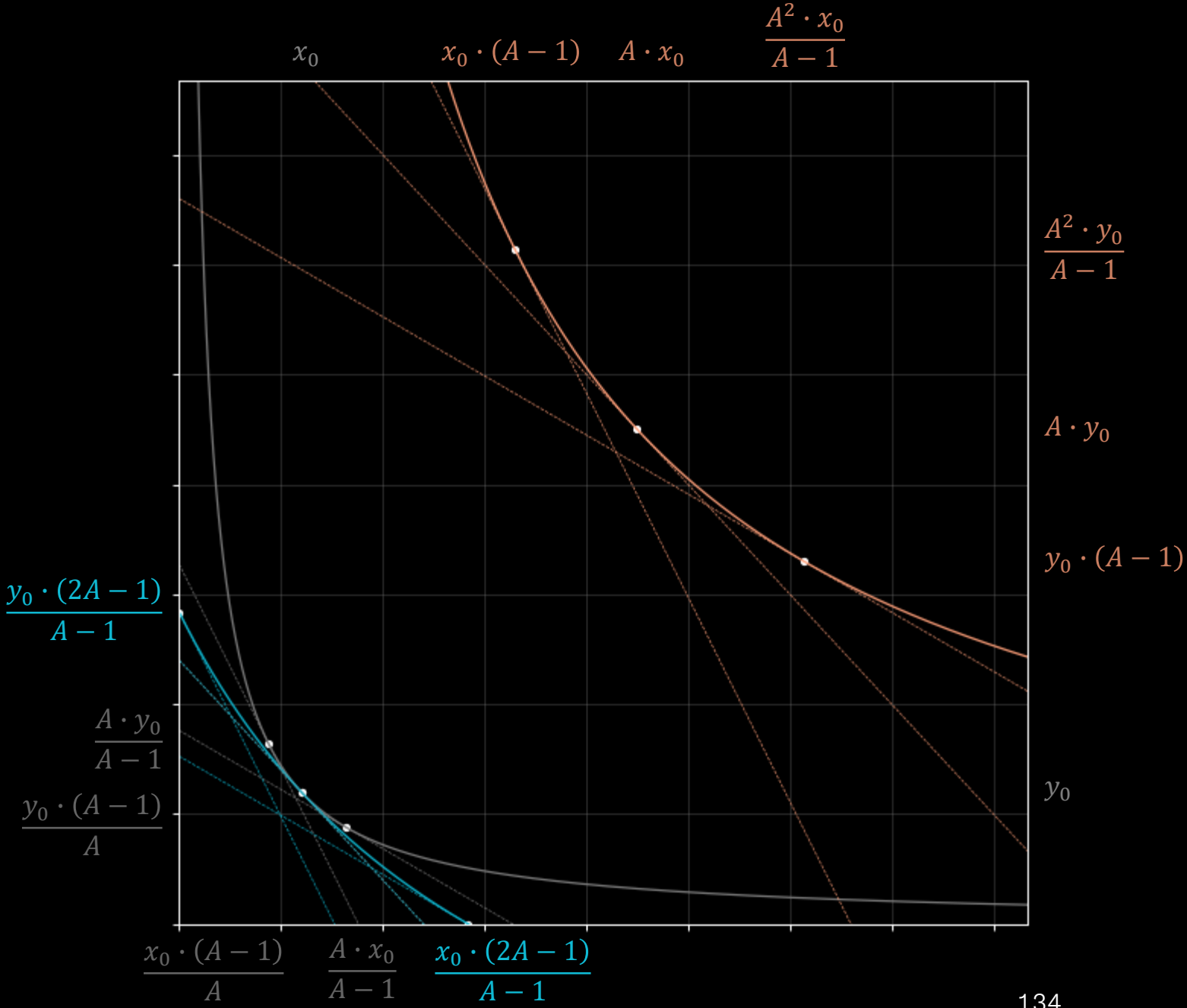
$$P_0 = \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



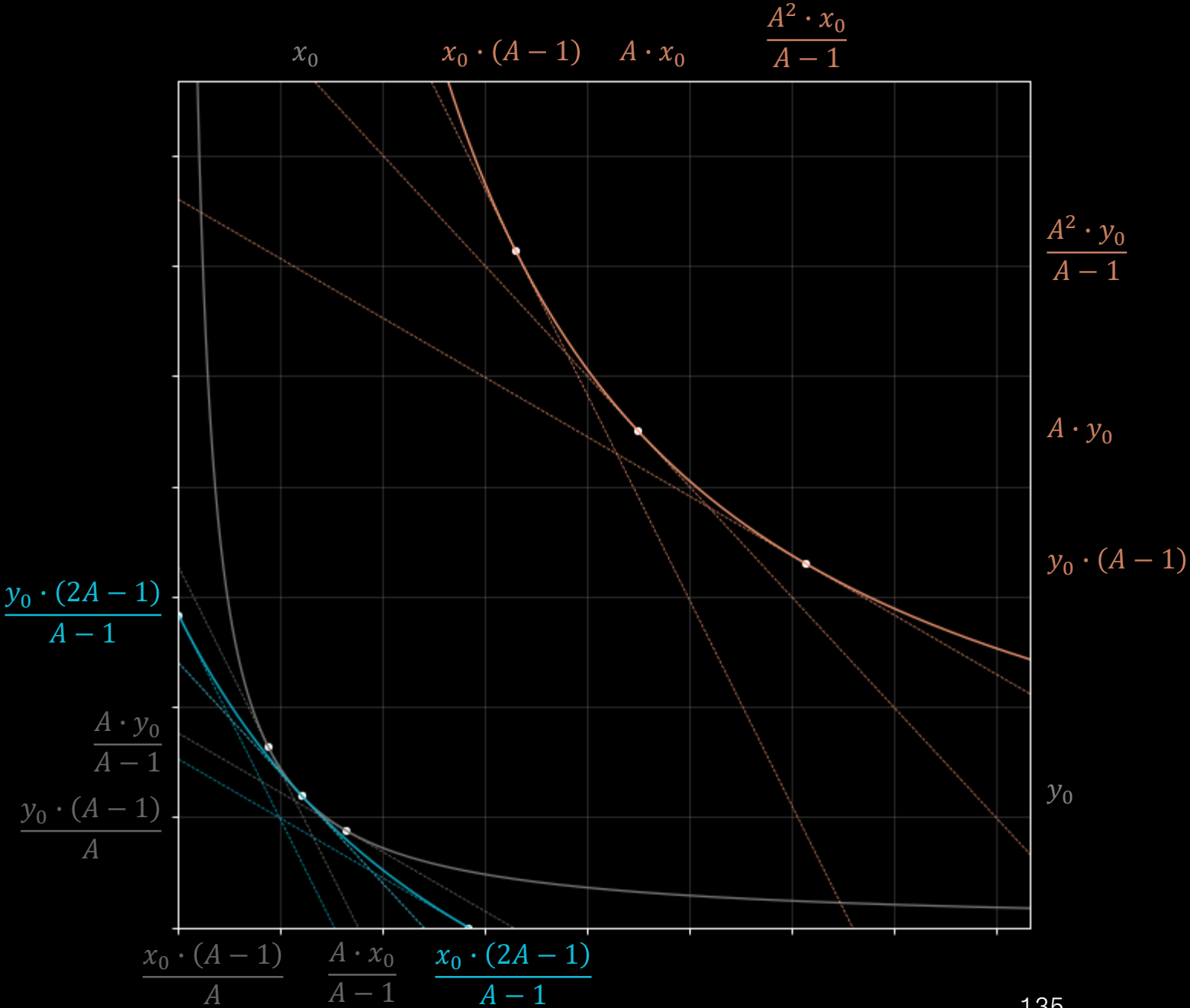
$$P_{\text{high}} \cdot P_{\text{low}} = \frac{A^2}{(A-1)^2} \cdot \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0} \cdot \frac{y_0}{x_0}$$

$$P_0 = \frac{y_0}{x_0}$$



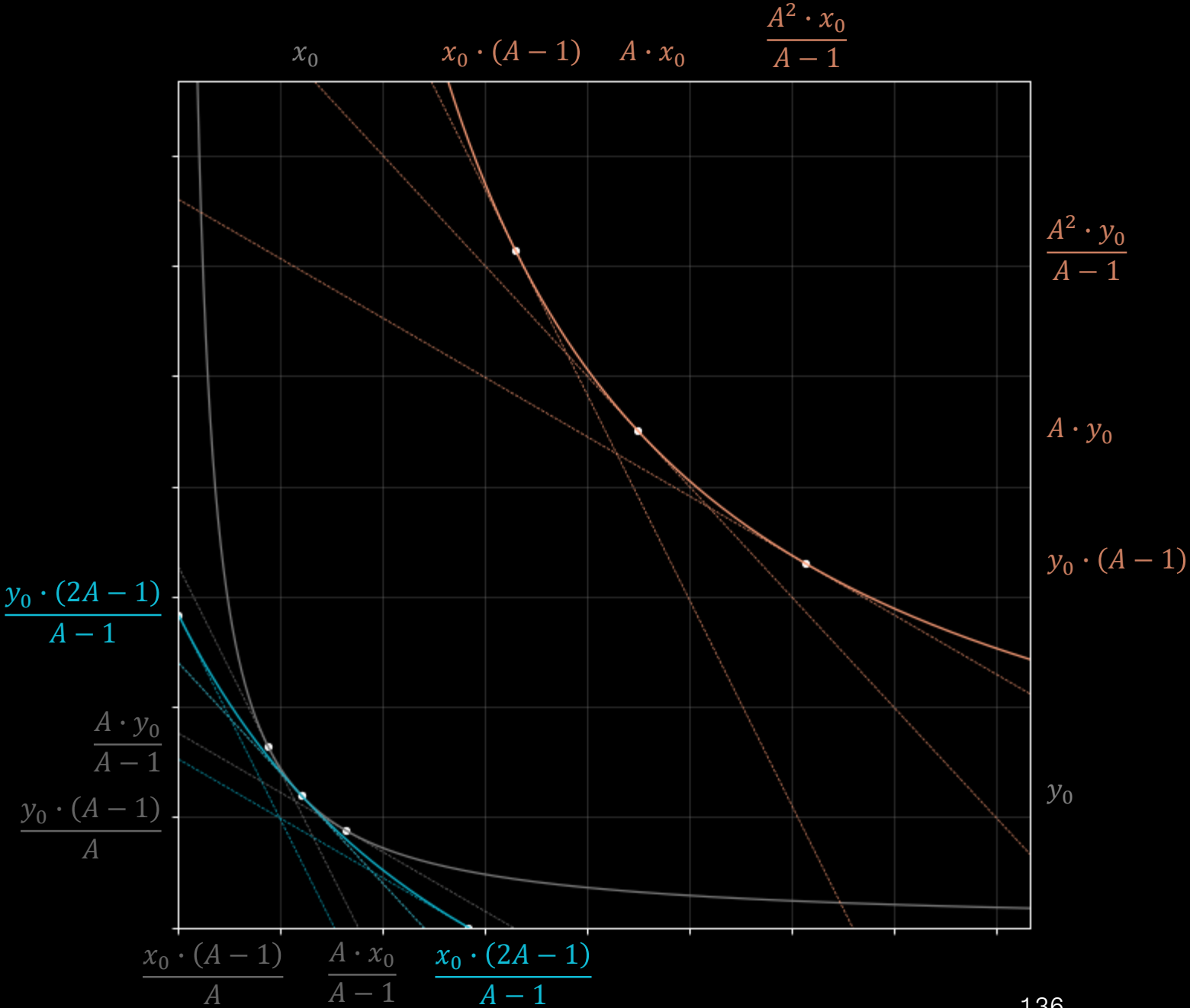
$$P_{\text{high}} \cdot P_{\text{low}} = \frac{A^2}{(A-1)^2} \cdot \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0} \cdot \frac{y_0}{x_0}$$

$$P_0 = \frac{y_0}{x_0}$$



$$P_{\text{high}} \cdot P_{\text{low}} = \frac{y_0}{x_0} \cdot \frac{y_0}{x_0}$$

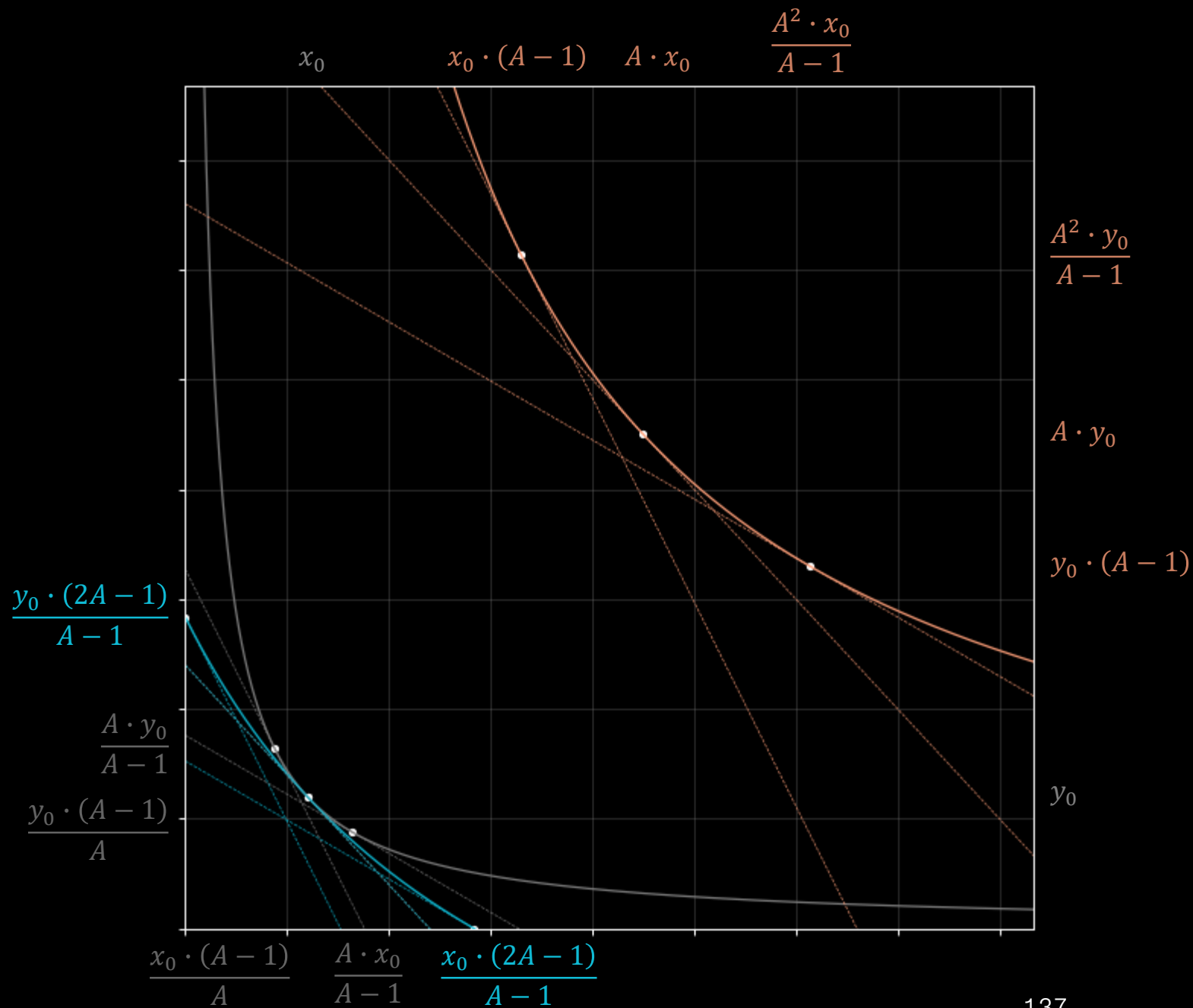
$$P_0 = \frac{y_0}{x_0}$$



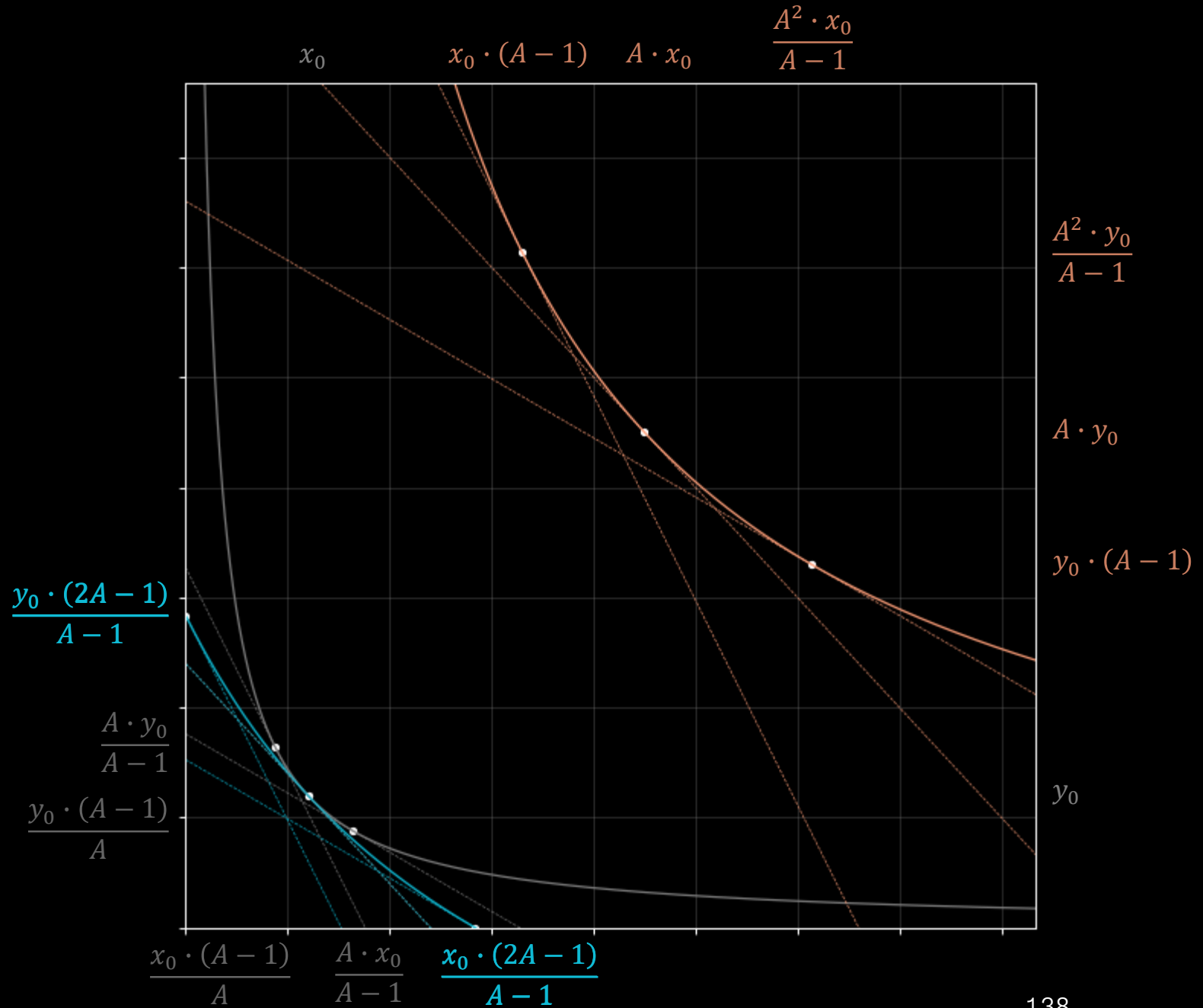


$$\sqrt{P_{\text{high}} \cdot P_{\text{low}}} = \frac{y_0}{x_0}$$

$$P_0 = \frac{y_0}{x_0}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$



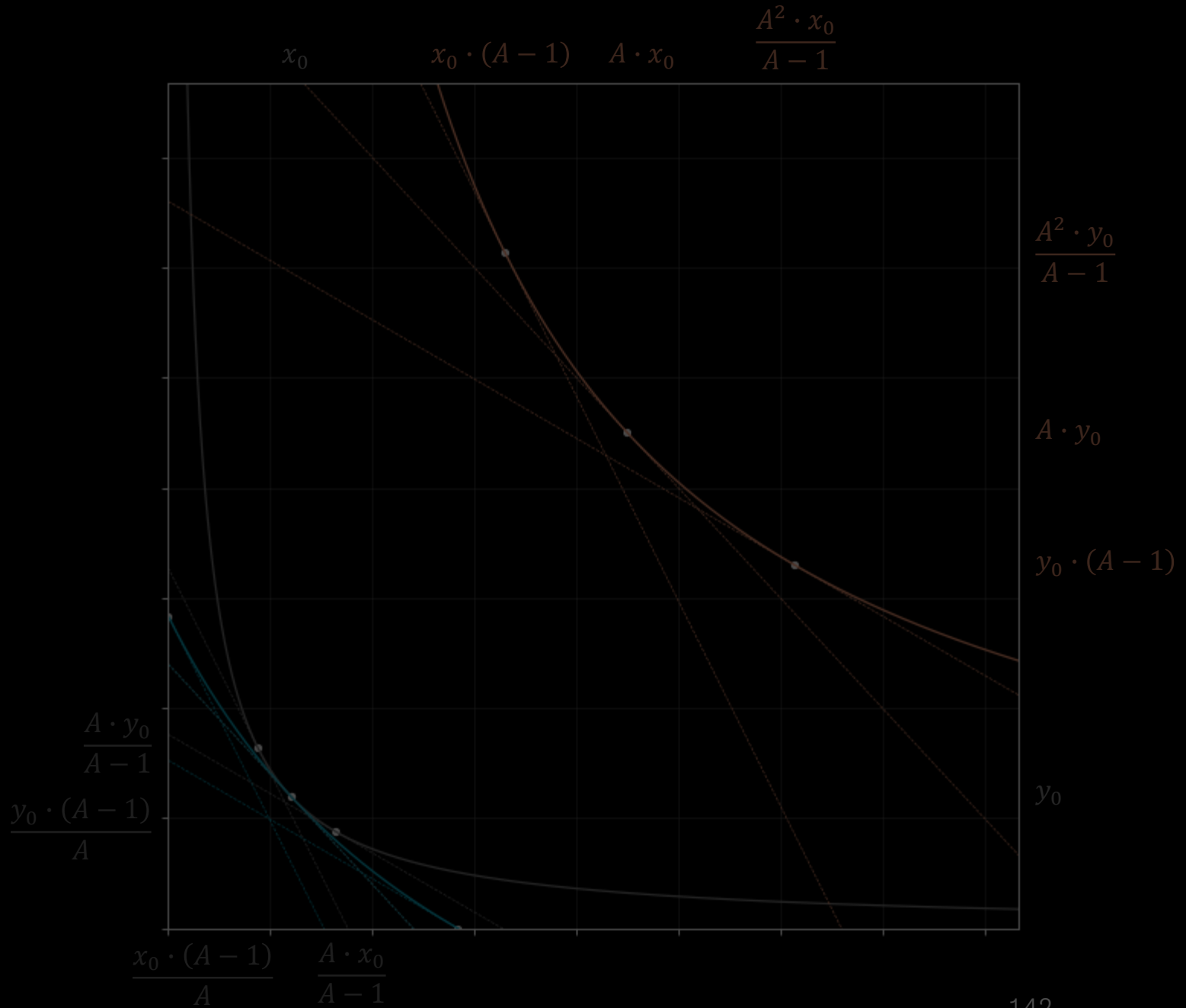




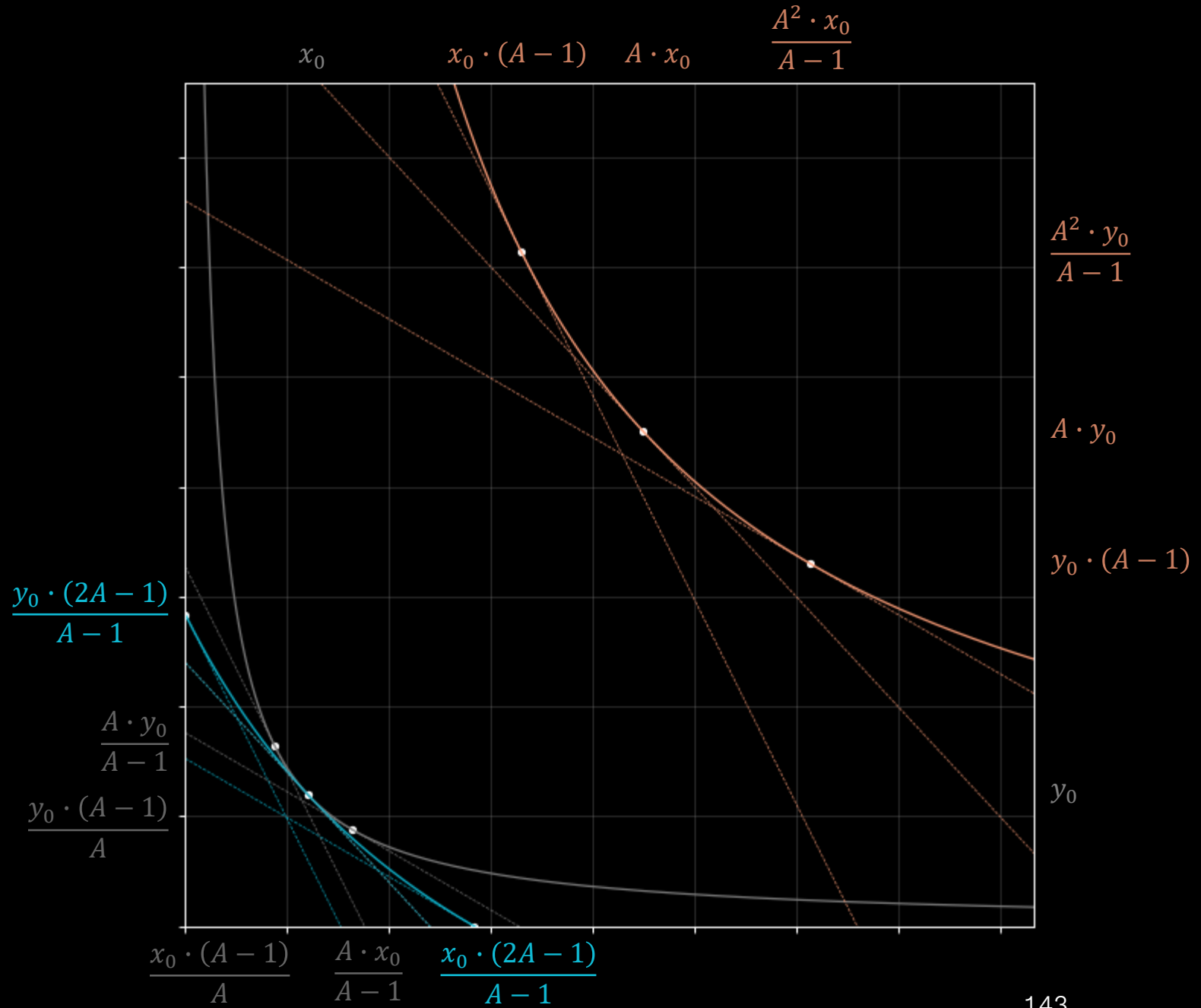


$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

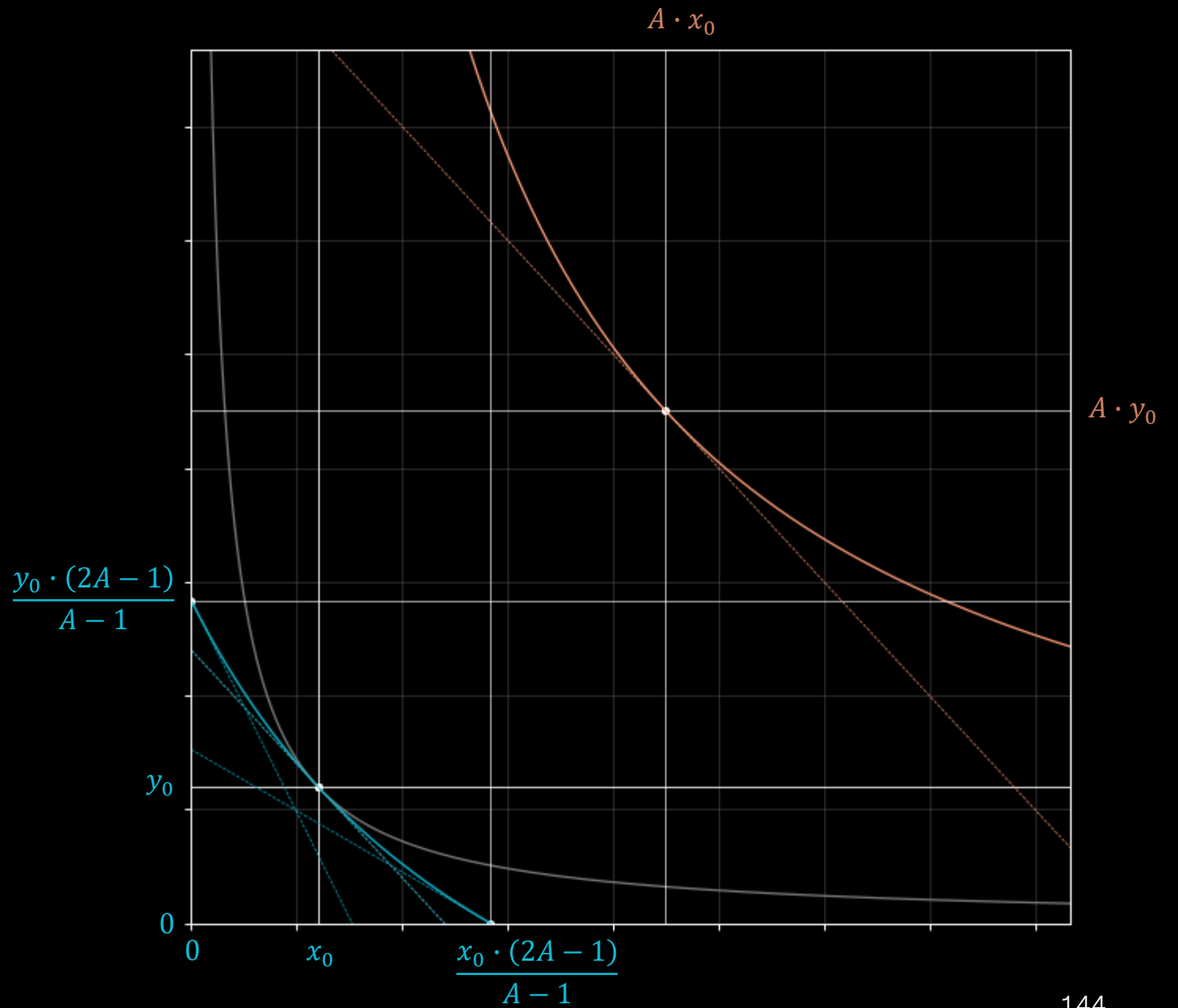
$$\frac{y_0}{x_0}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

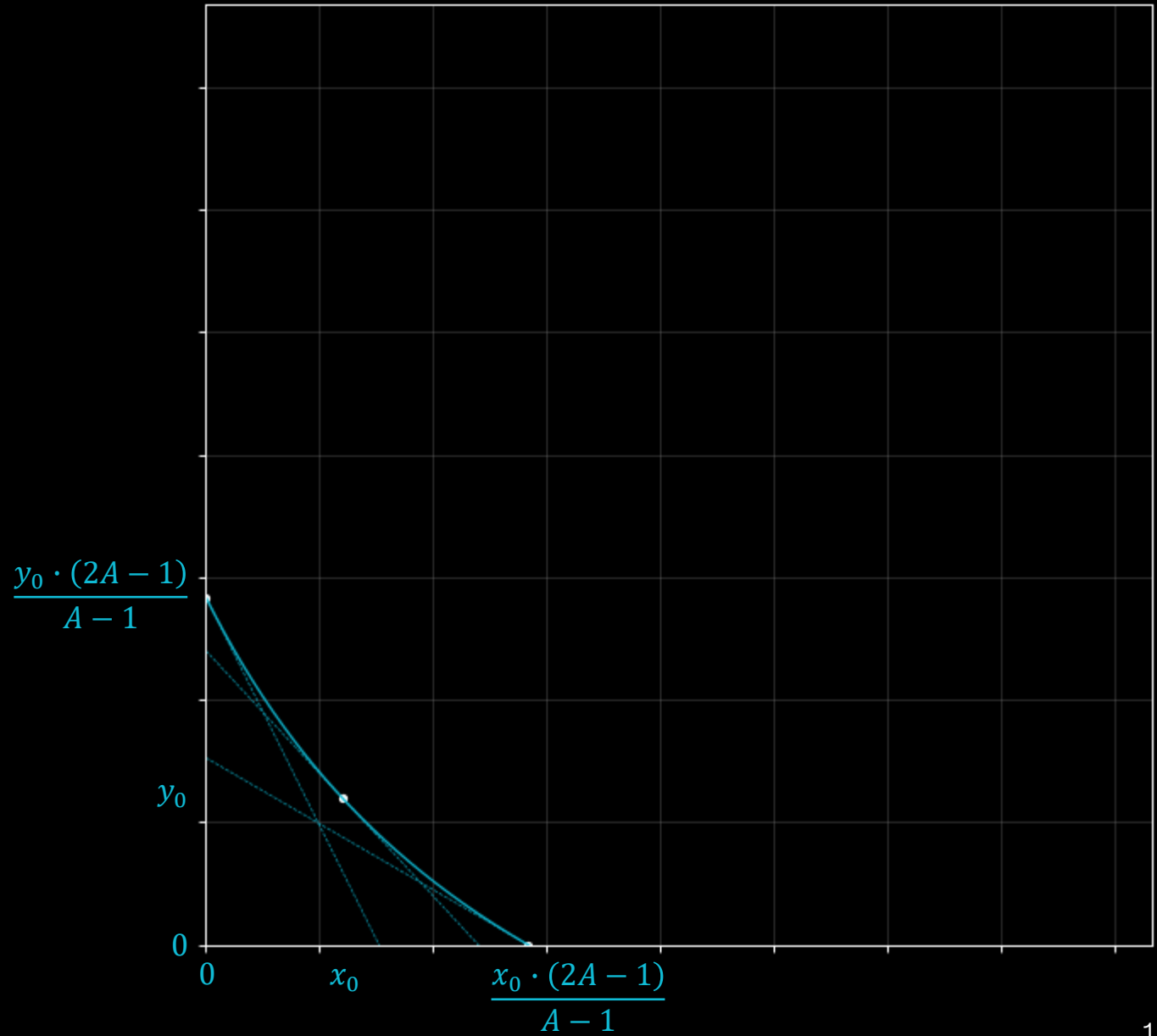


$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$





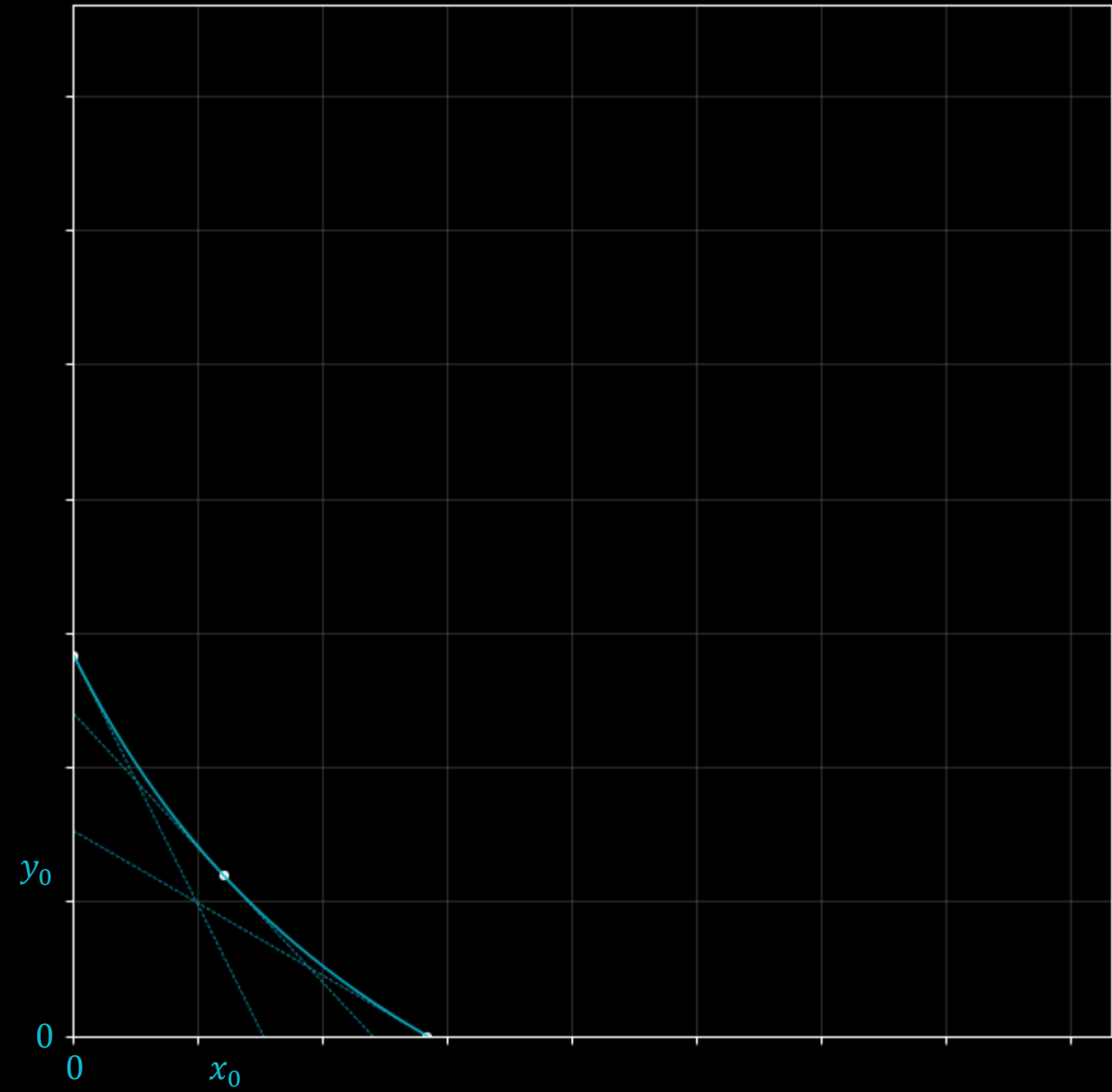
$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

$$y_{\text{int}} = \frac{y_0 \cdot (2A - 1)}{A - 1}$$

$$x_{\text{int}} = \frac{x_0 \cdot (2A - 1)}{A - 1}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}} = \frac{y_{\text{int}}}{x_{\text{int}}}$$

# Homework, 16<sup>th</sup> May

Show that:  $\frac{x_0 \cdot P_{\text{high}} - y_0}{\frac{y_0}{P_{\text{low}}} - x_0} = P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}} = \frac{y_{\text{int}}}{x_{\text{int}}}$



MB\_Richardson



MBRichardson87



mrichardson87



mark@bancor.network

# DeFi's Concentrated Liquidity From Scratch

Lecture 2 of 5

Mark. B. Richardson, Ph.D.

Project Lead, Bancor



CARBON DEFI



Bancor