DeFi's Concentrated Liquidity From Scratch

Lecture 1 of 5

Mark. B. Richardson, Ph.D.
Project Lead, Bancor

Token Engineering Academy

CARBON DEFI

Bancor

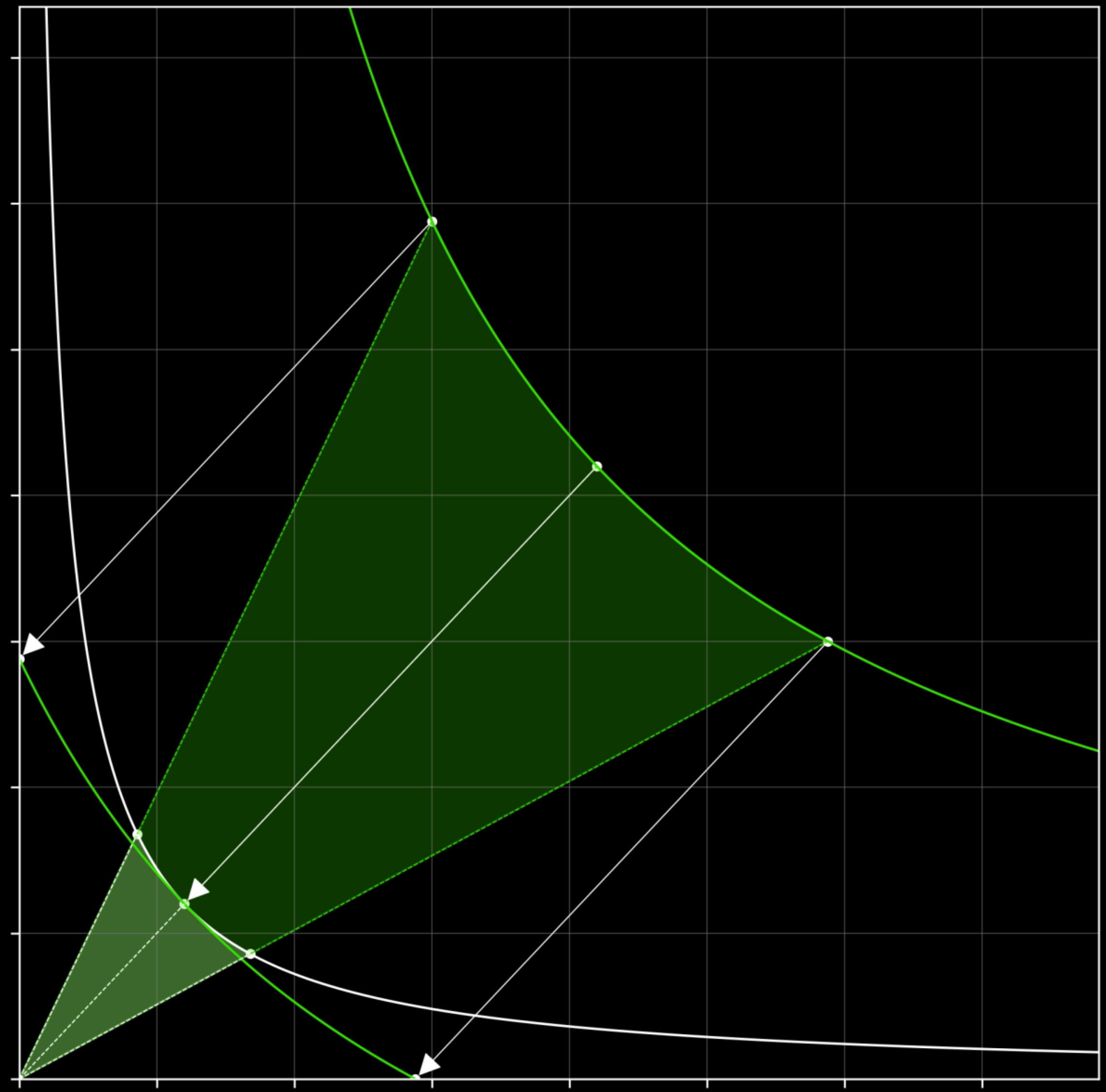
I am going to give you what I call an *elementary* demonstration.

But elementary does not mean easy to understand.

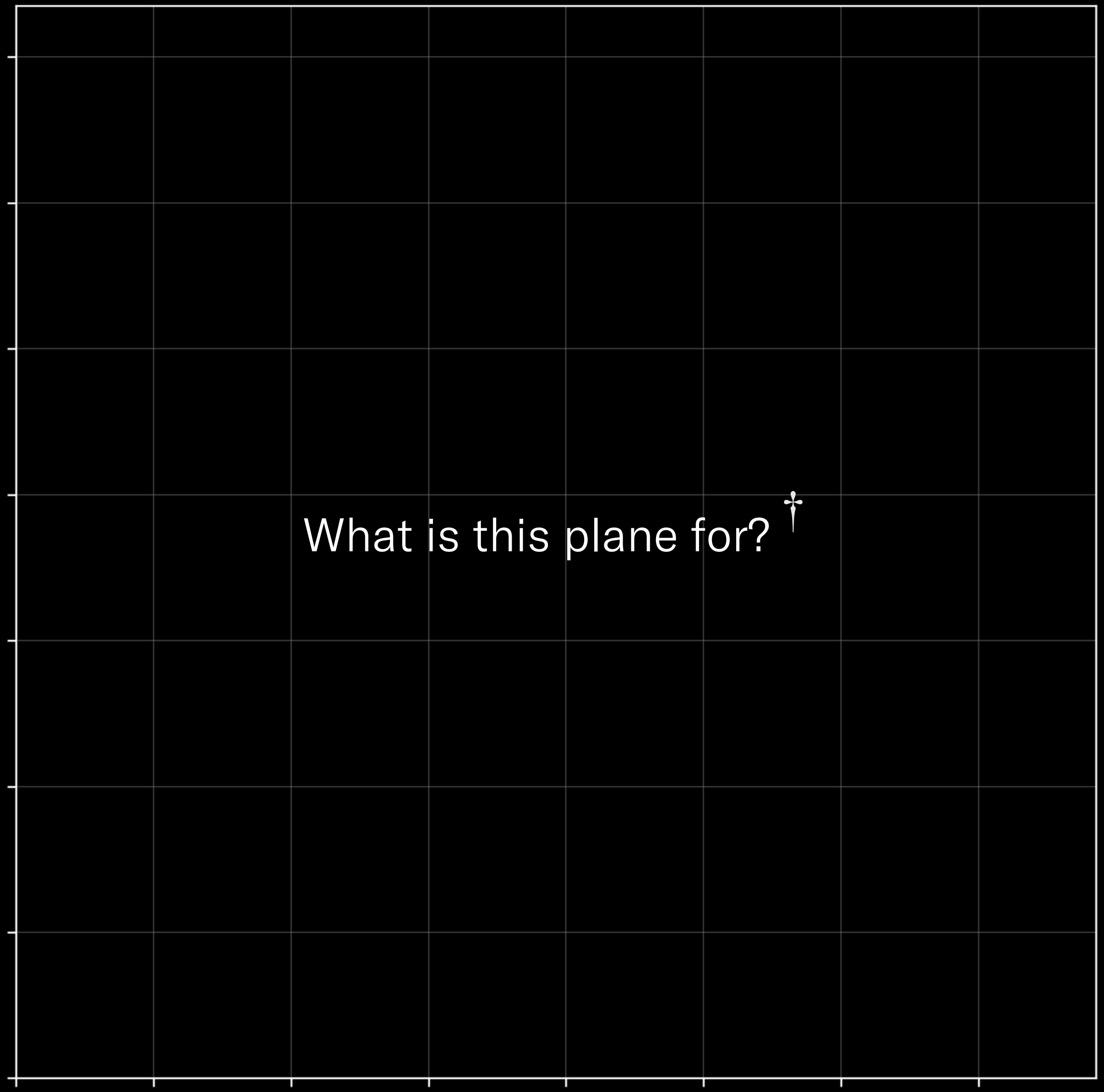
Elementary means that very little is required to know ahead of time in order to understand it, except to have an infinite amount of intelligence.

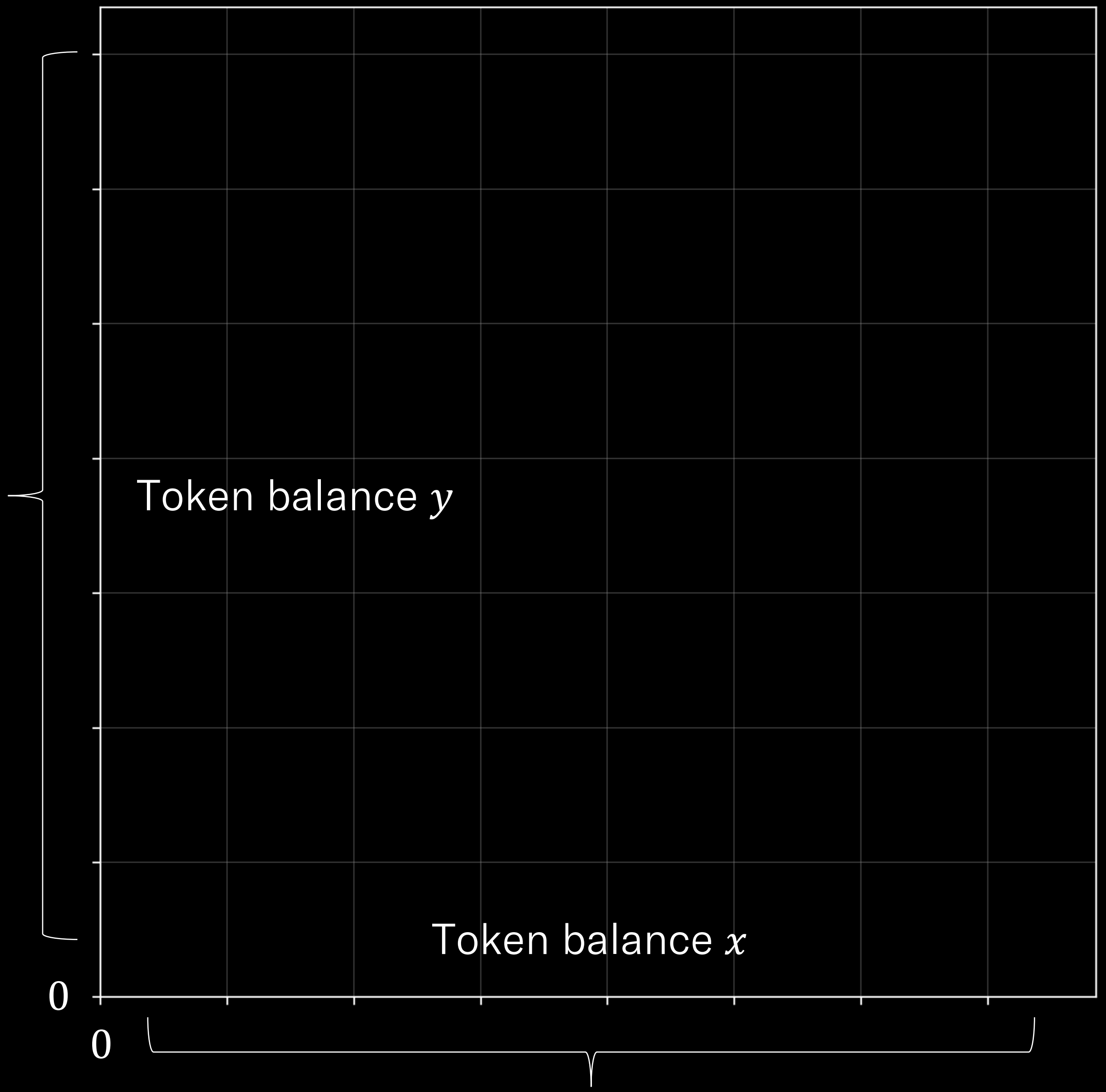
- Richard Feynman

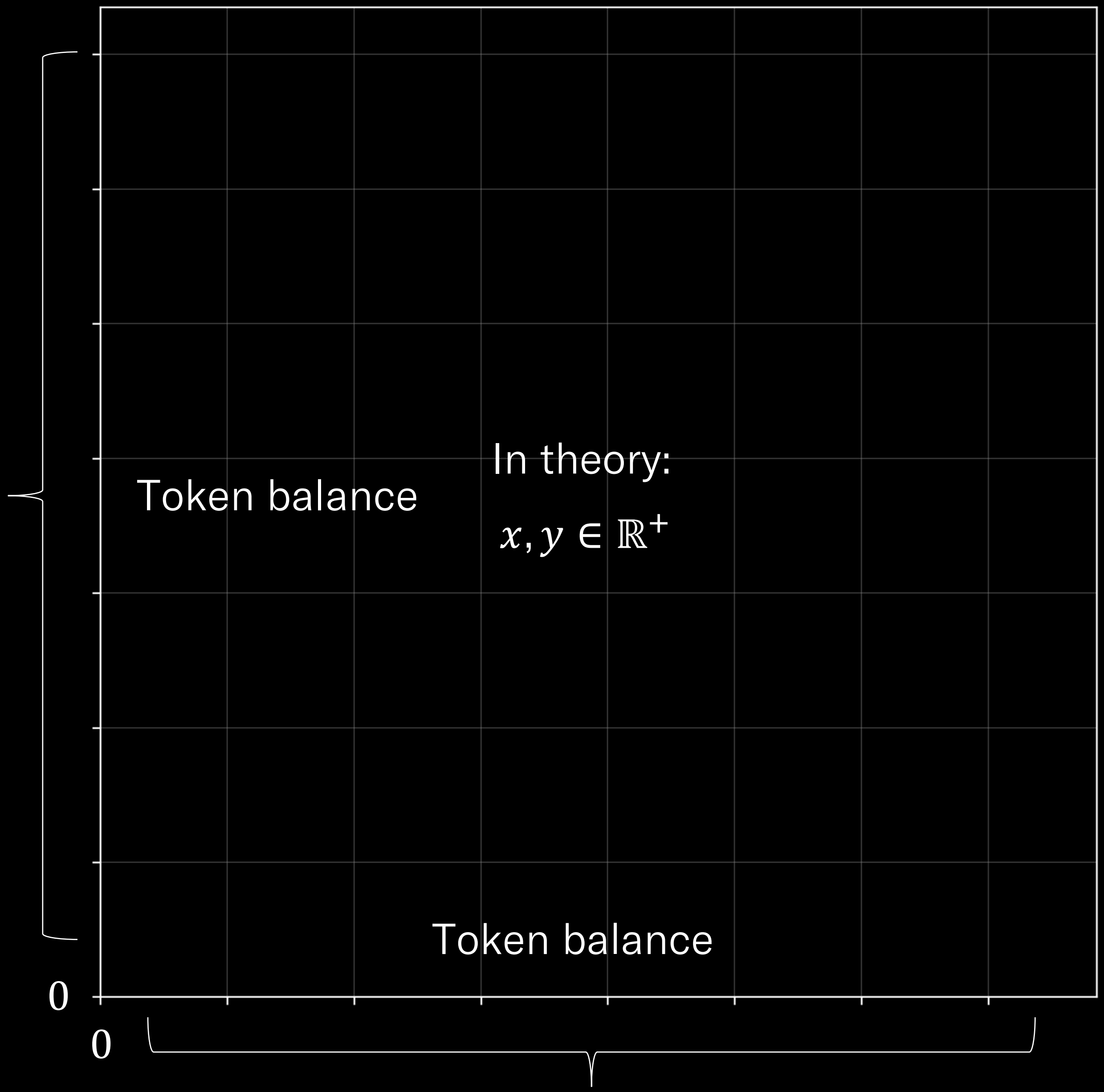
The focus of this lecture series is this object.

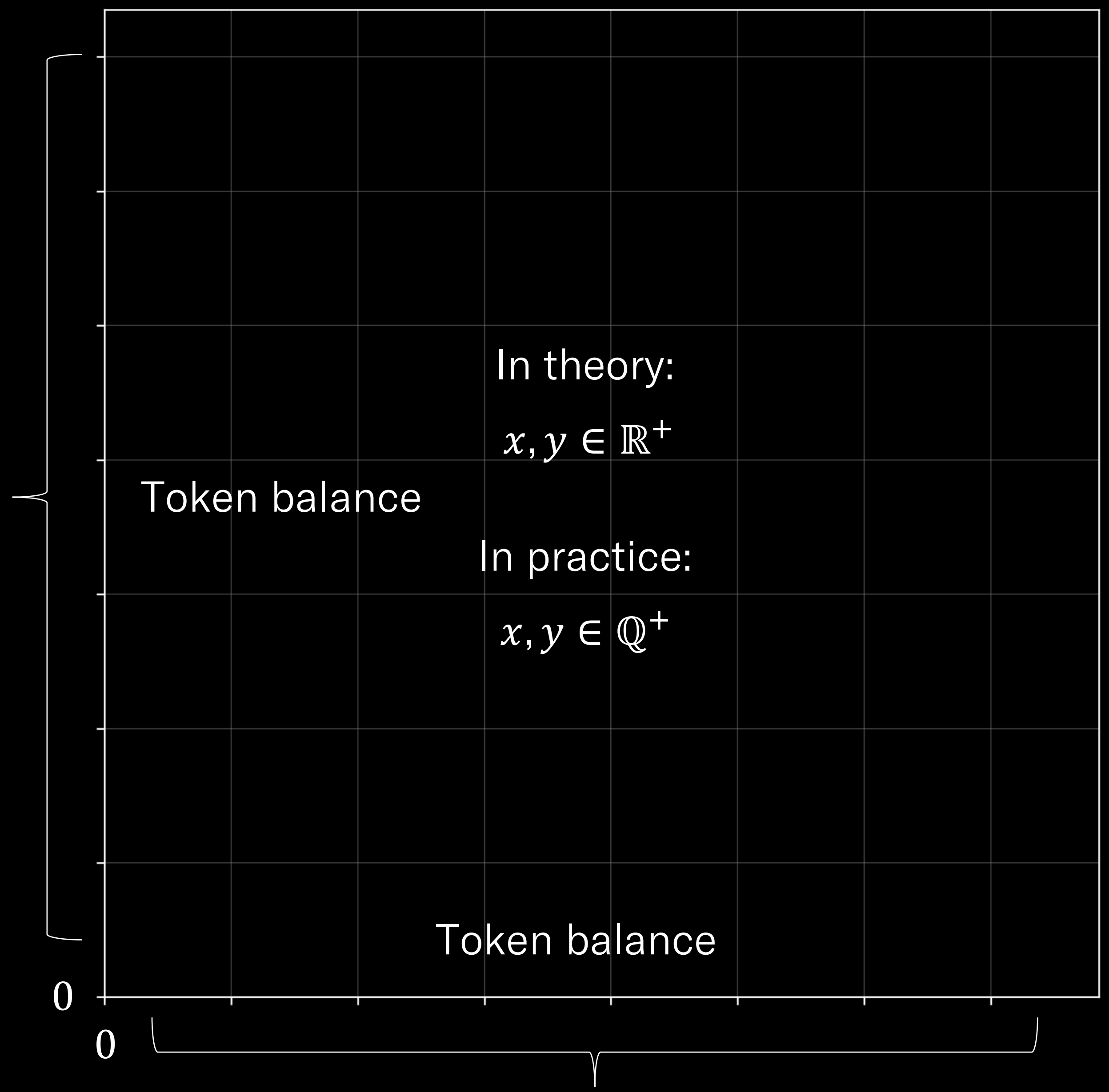


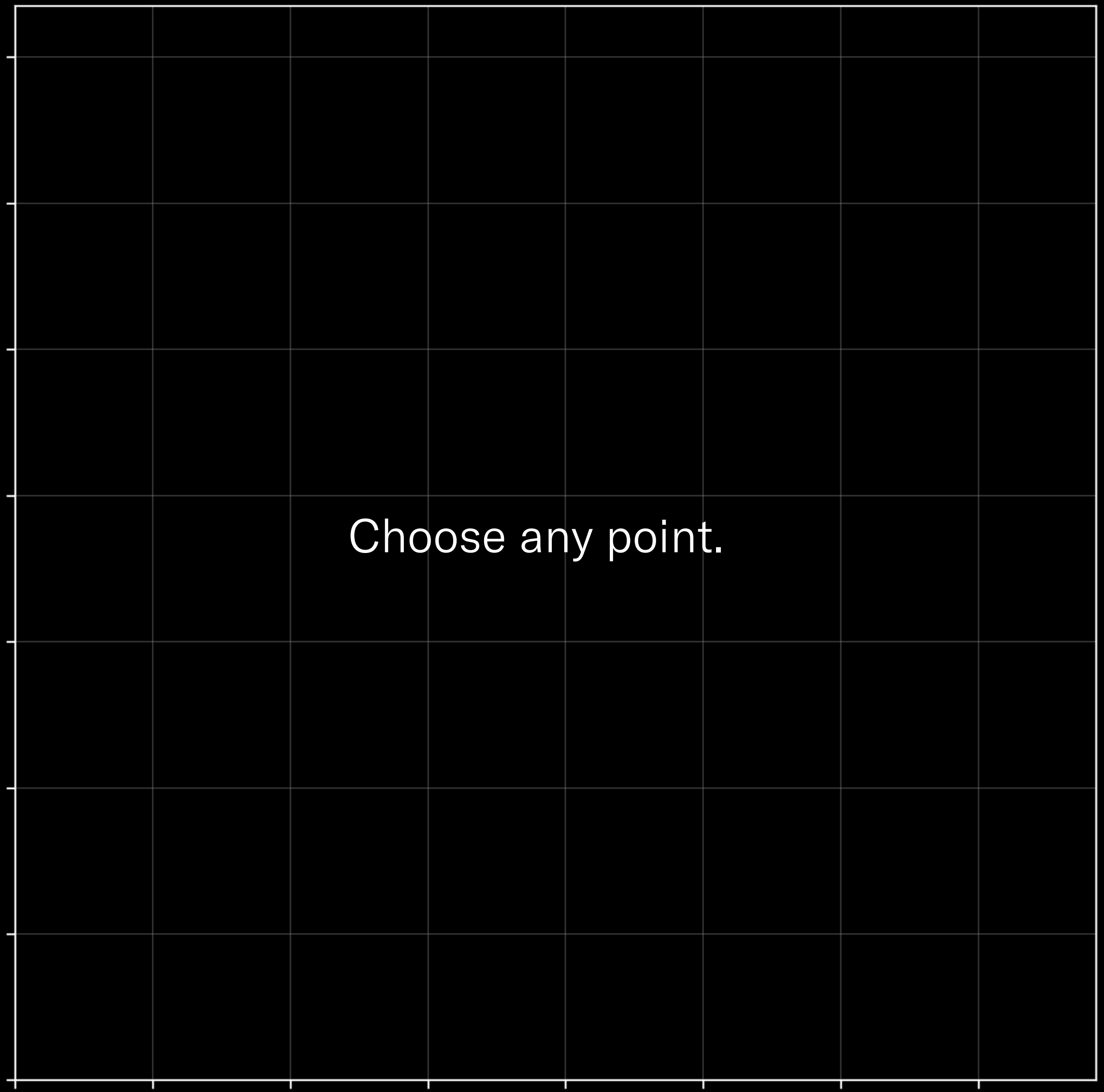
† Usually.

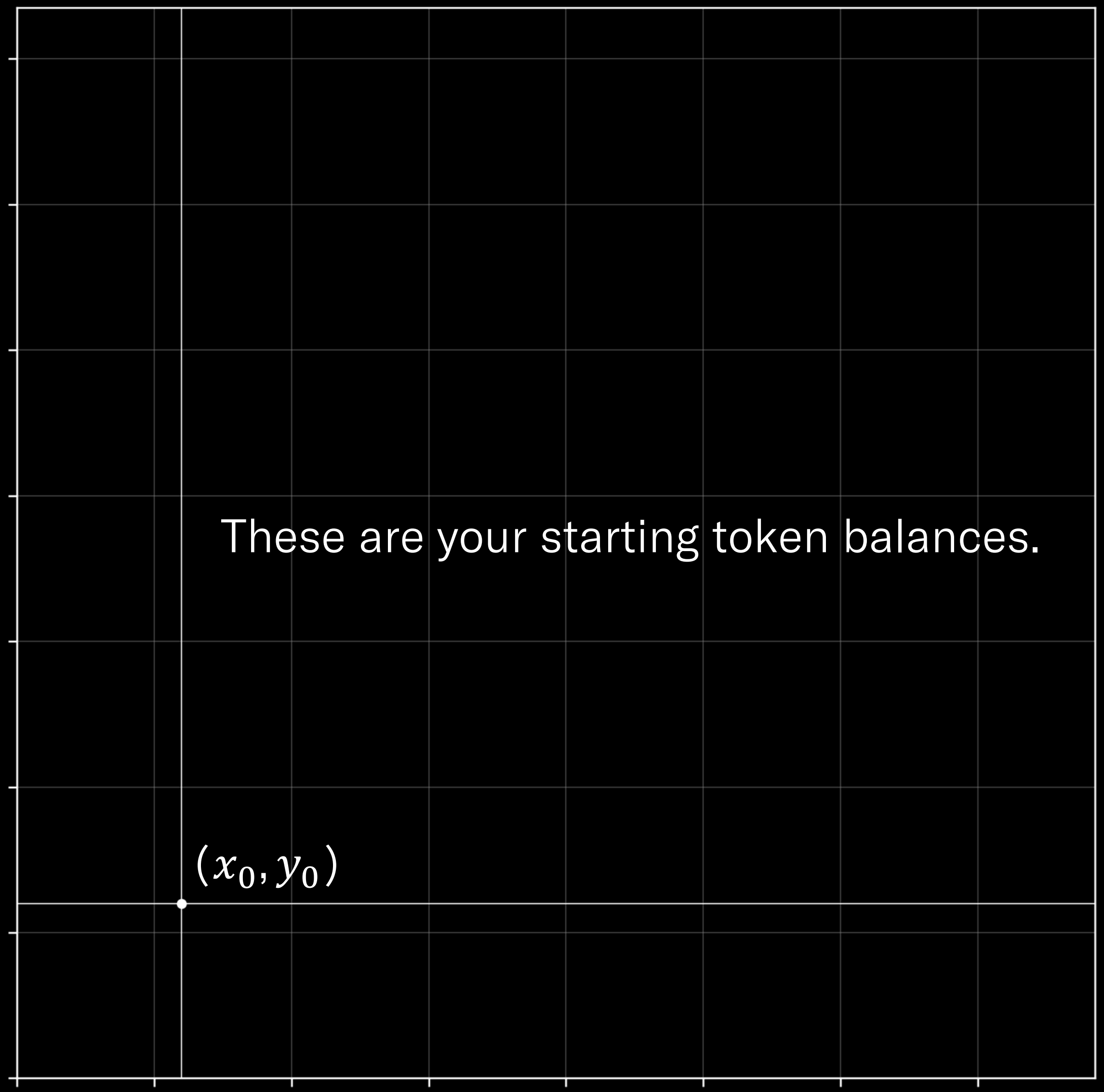


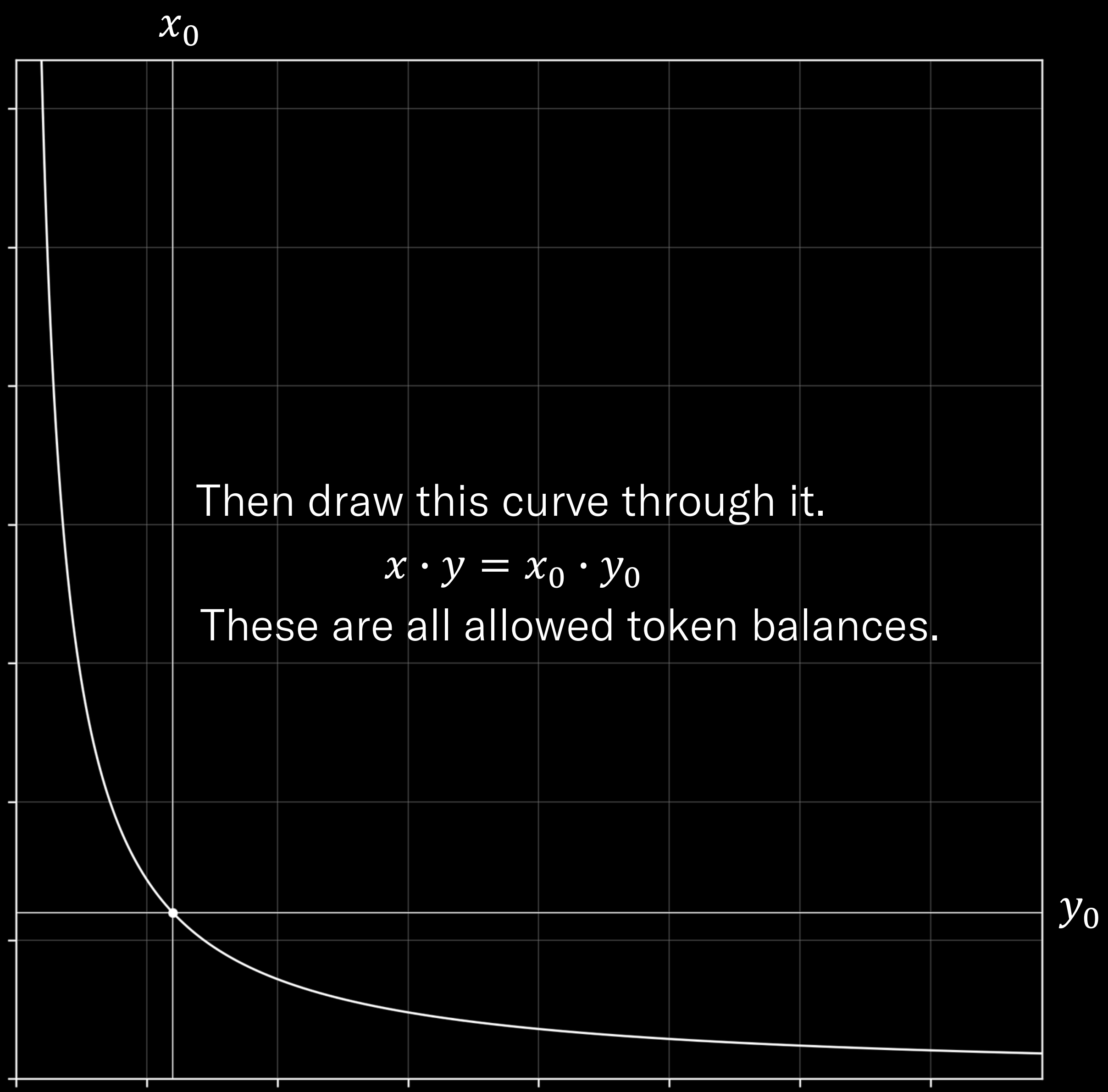

Token balance y
Token balance x
0
0


Token balance
In theory:
$x, y \in \mathbb{R}^+$
Token balance
0
0

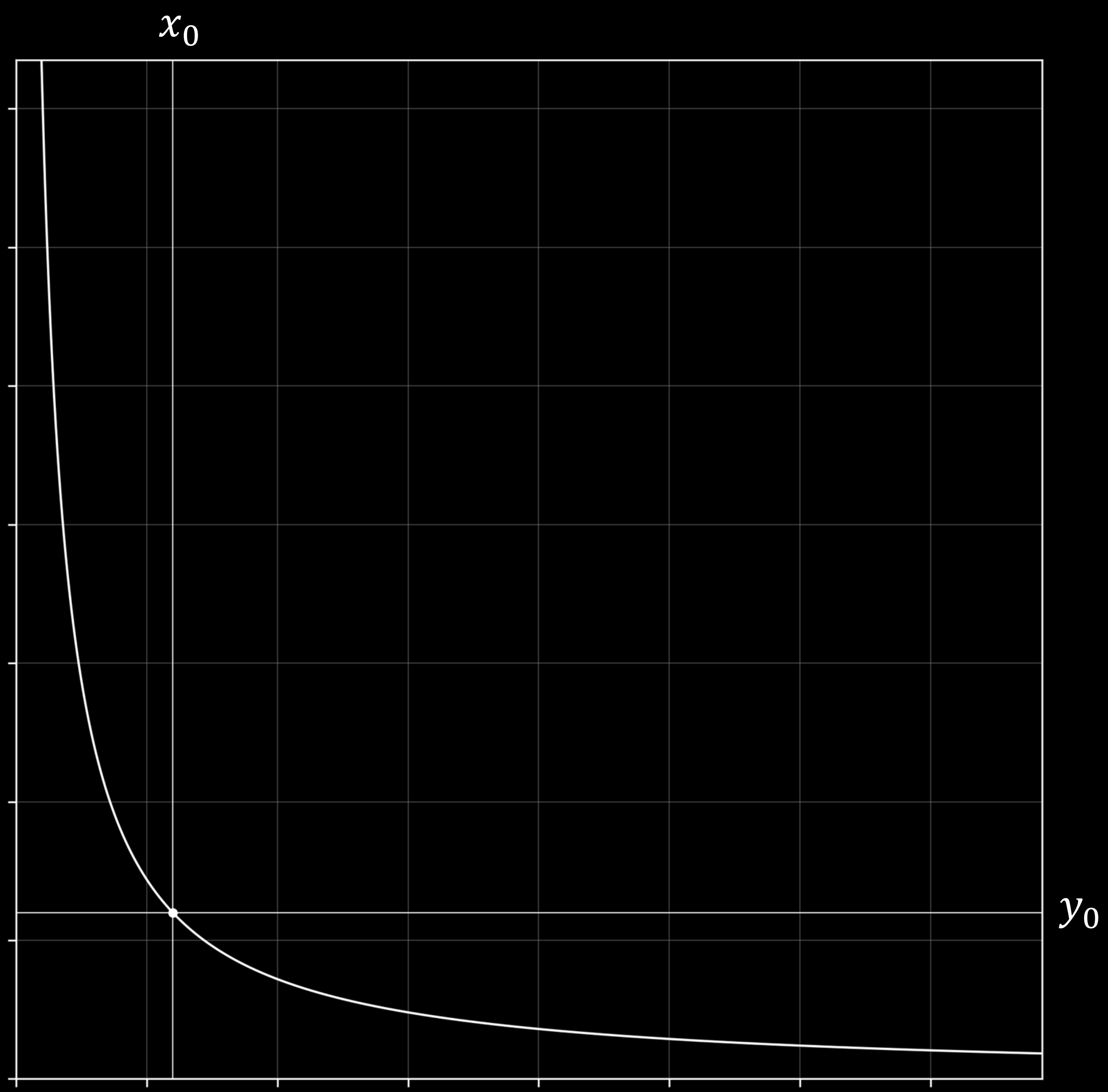

In theory:
$x, y \in \mathbb{R}^+$
Token balance
In practice:
$x, y \in \mathbb{Q}^+$
Token balance
0
0


Choose any point.

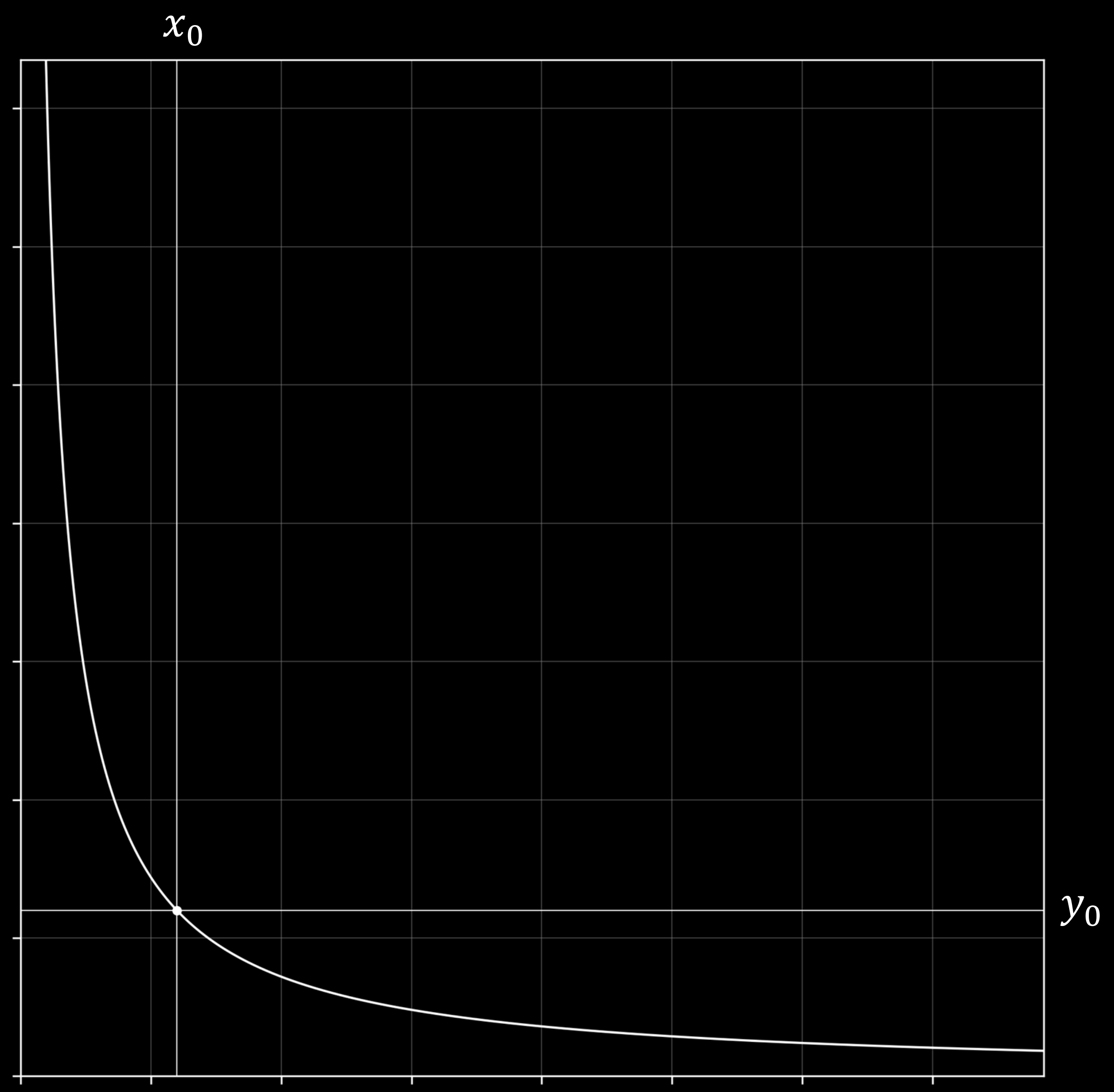

These are your starting token balances.
(x_0, y_0)


x_0
Then draw this curve through it.
$x \cdot y = x_0 \cdot y_0$
These are all allowed token balances.
y_0

$$x \cdot y = x_0 \cdot y_0$$



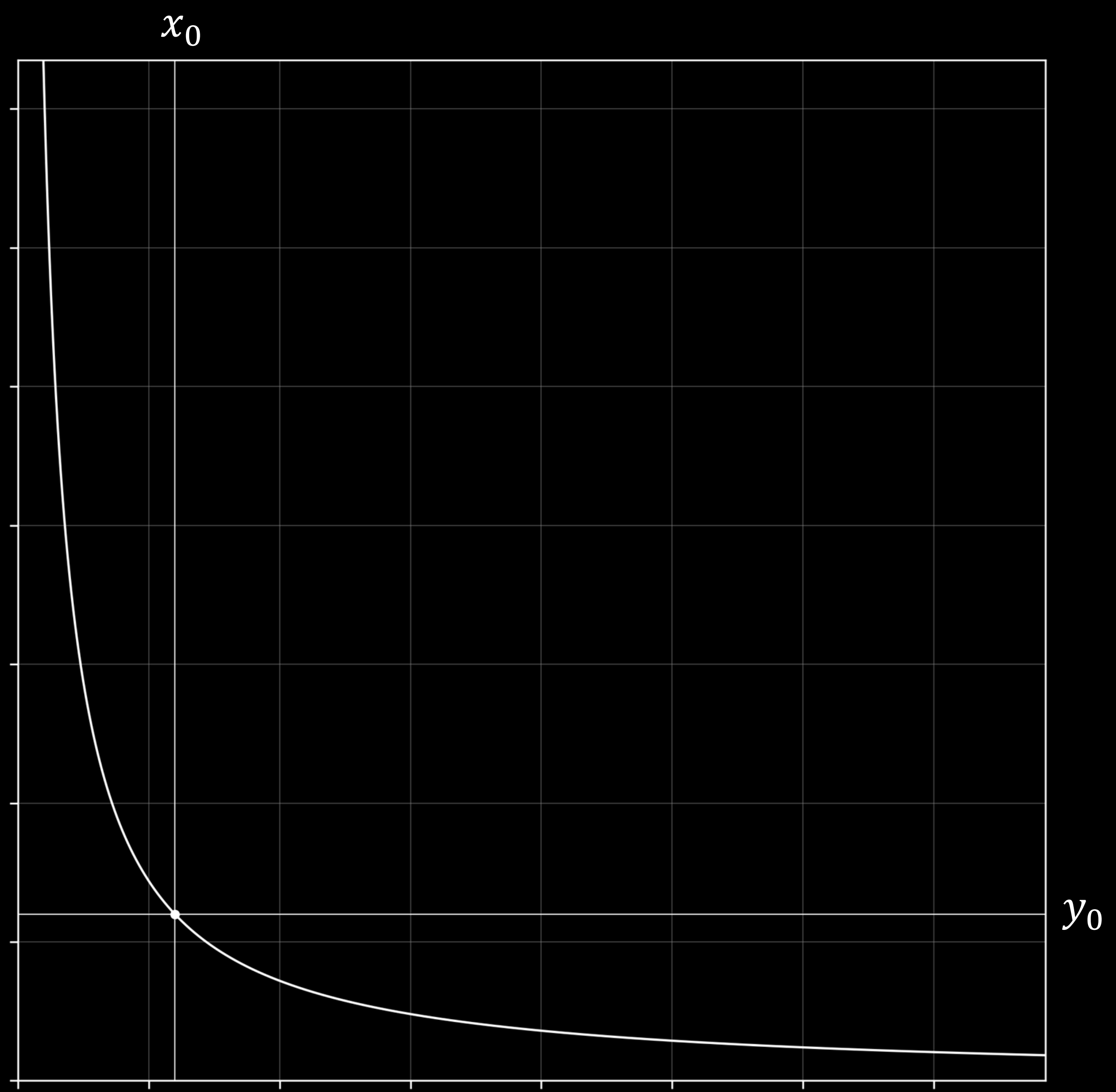
$$x = \frac{x_0 \cdot y_0}{y}$$



$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

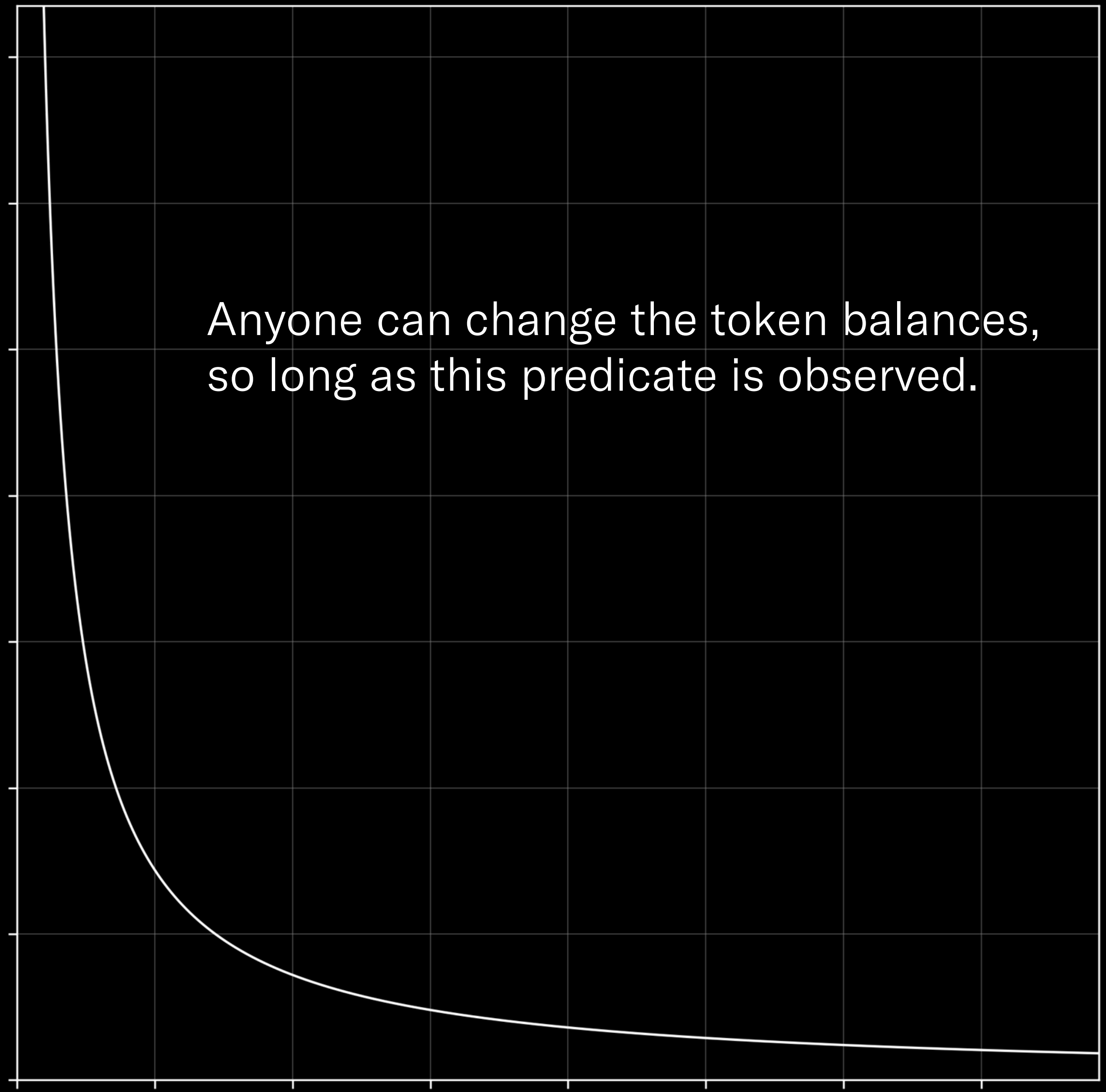
$$y = \frac{x_0 \cdot y_0}{x}$$



$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

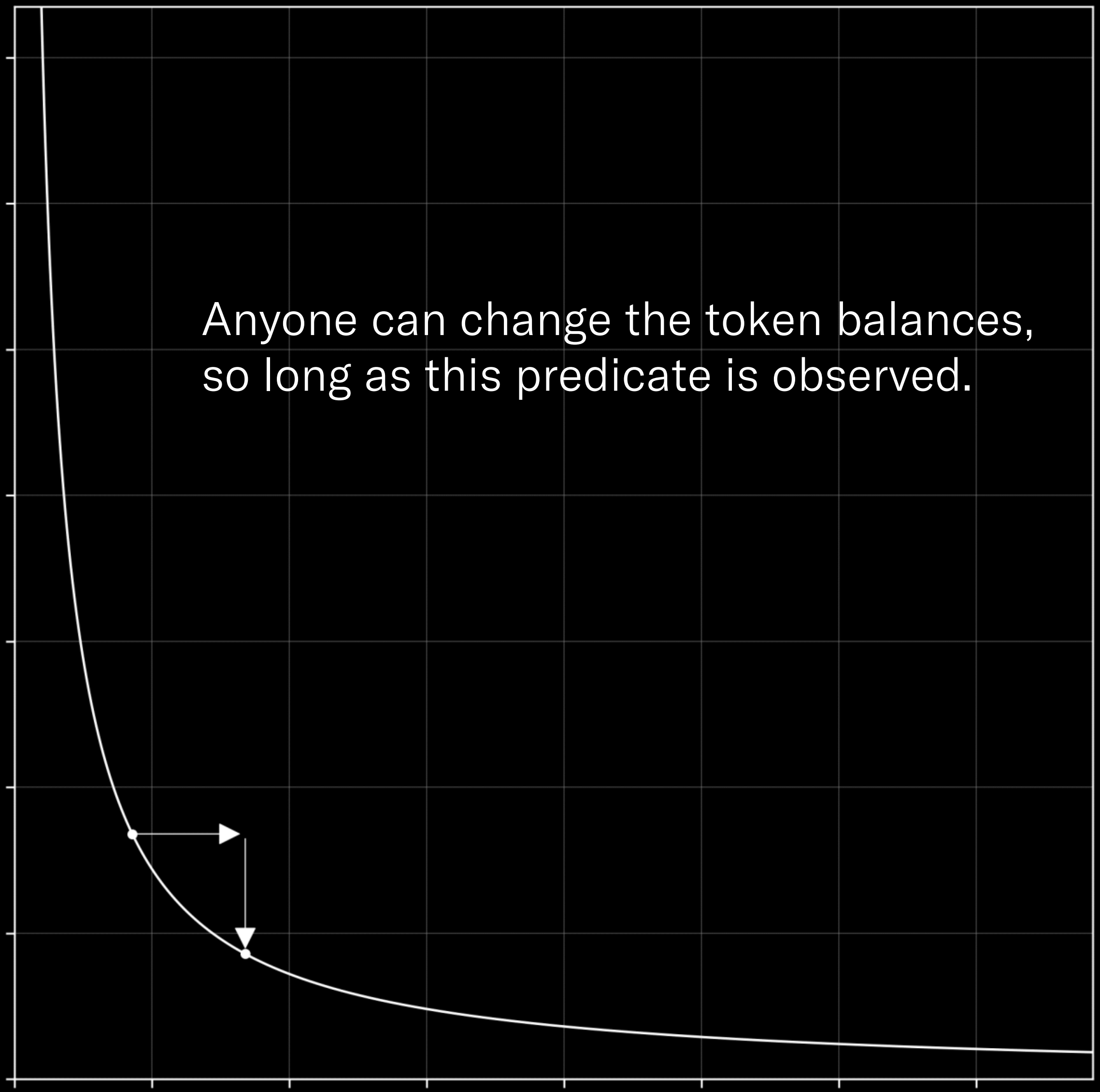


This is what a token swap looks like.

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

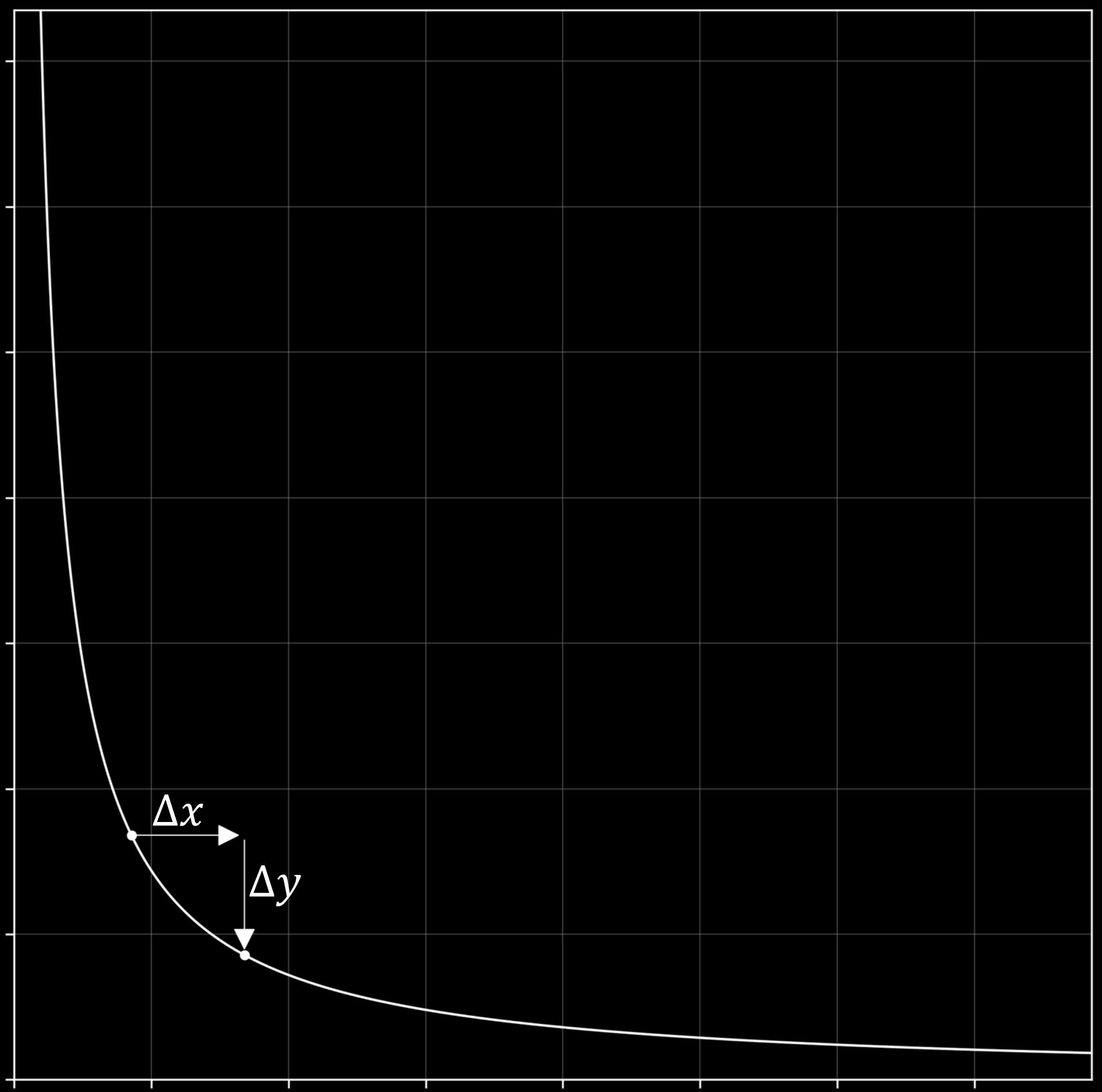


This is what a token swap looks like.

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

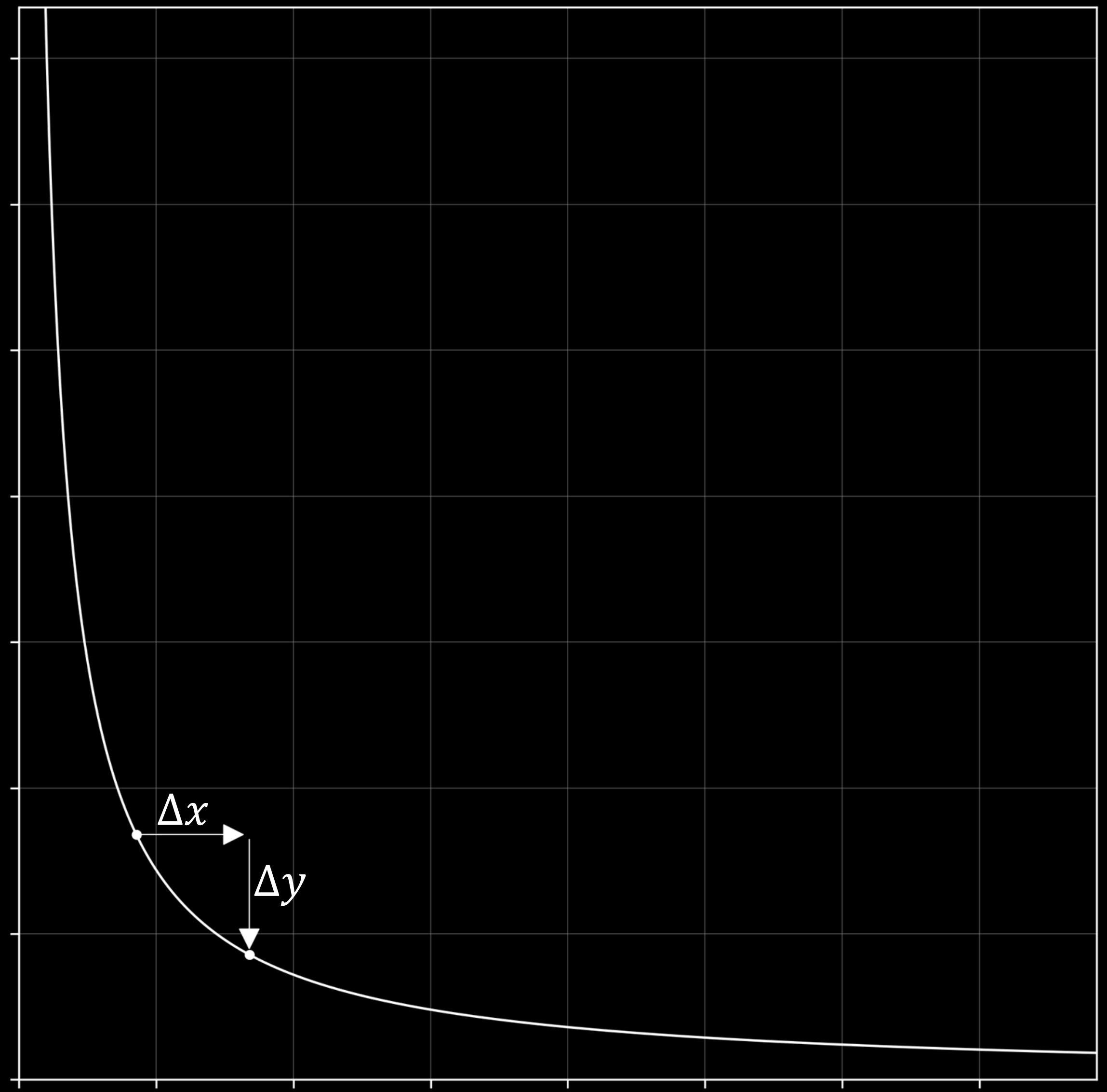


This is what a token swap looks like.

$$(x + \Delta x) \cdot (y + \Delta y) = x_0 \cdot y_0$$

$$x + \Delta x = \frac{x_0 \cdot y_0}{y + \Delta y}$$

$$y + \Delta y = \frac{x_0 \cdot y_0}{x + \Delta x}$$



$$x \cdot y = x_0 \cdot y_0$$

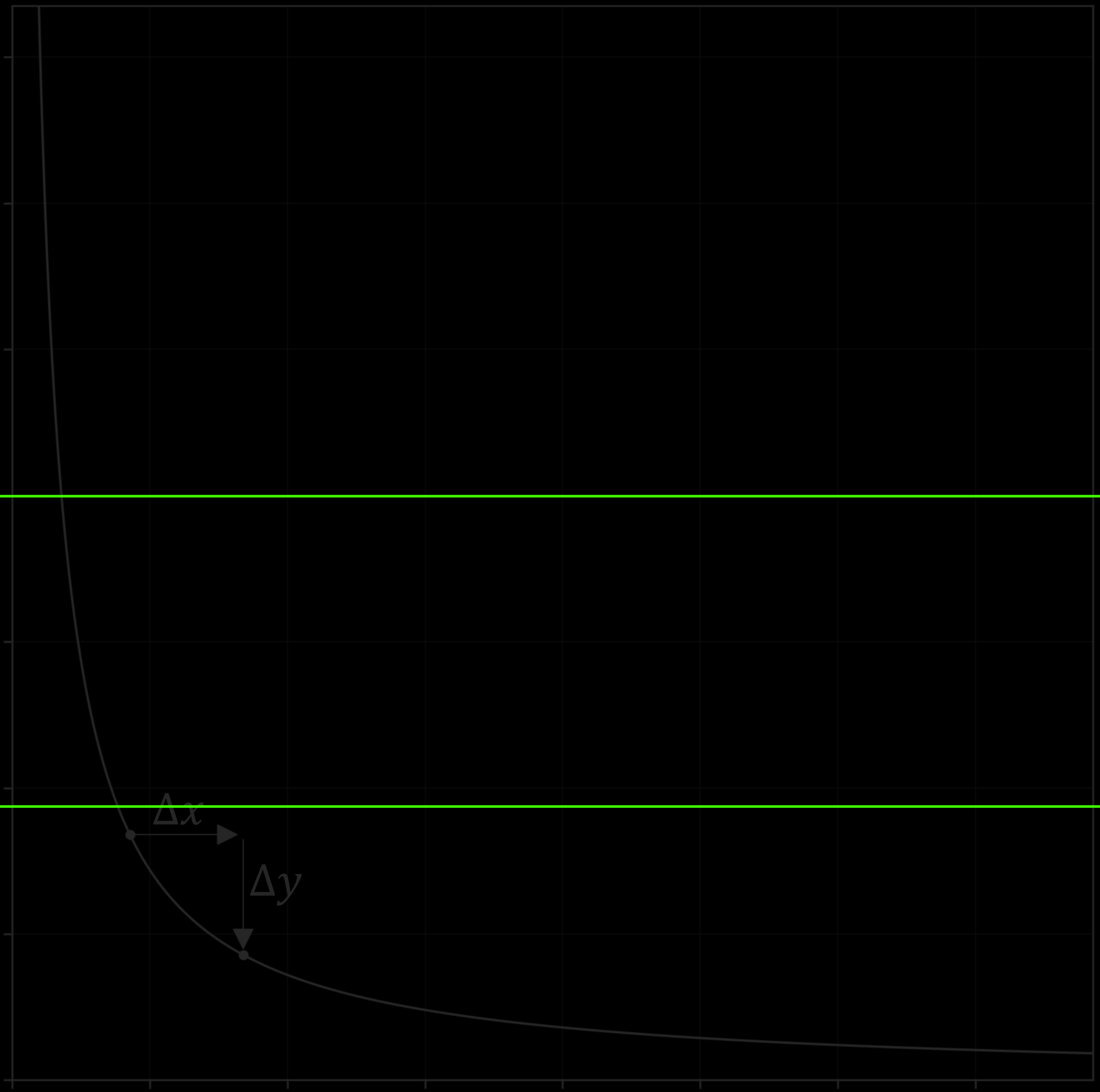
$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

$$x \cdot y = x_0 \cdot y_0$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - x$$

$$x = \frac{x_0 \cdot y_0}{y}$$

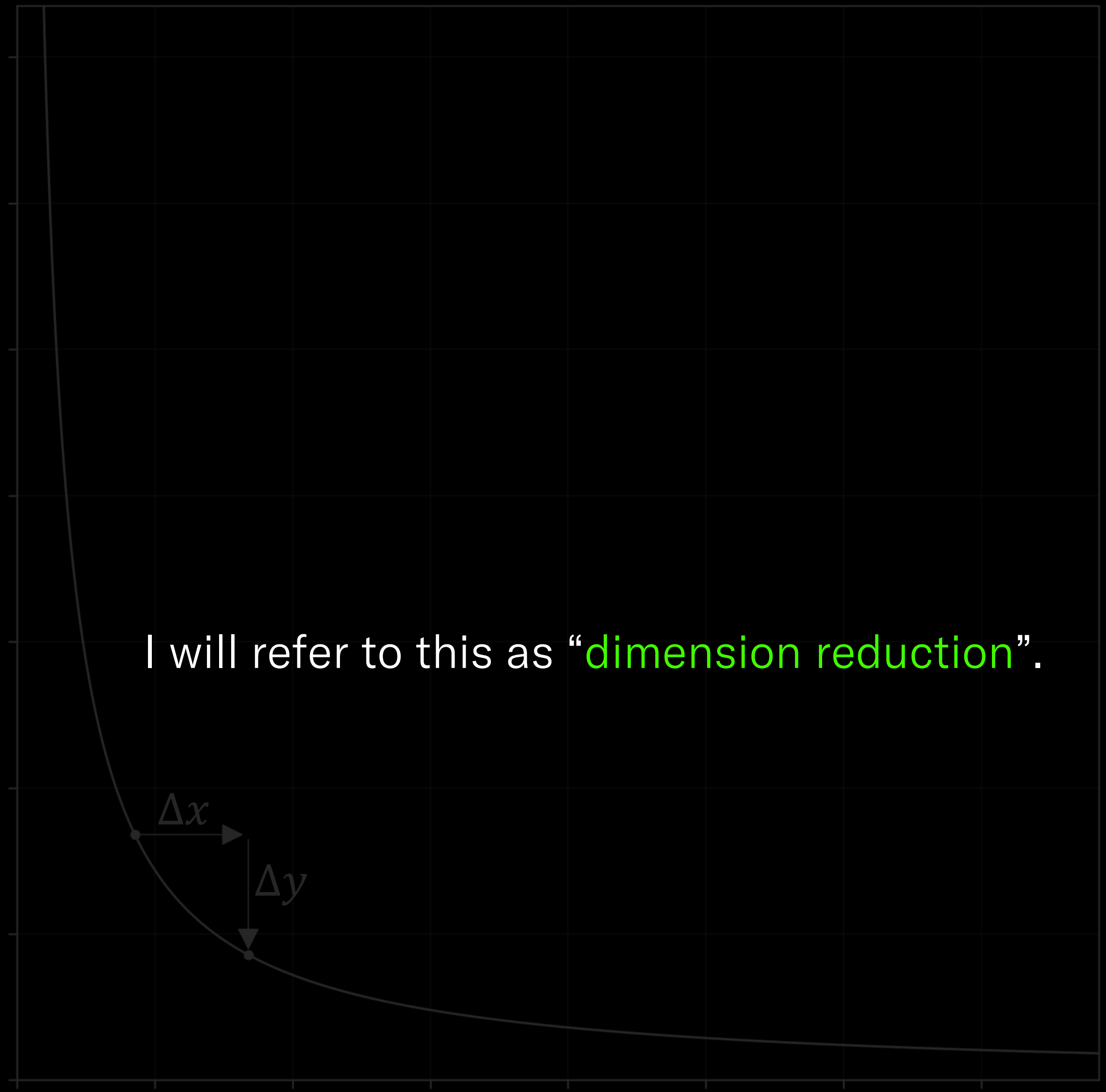


$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - y$$

$$y = \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

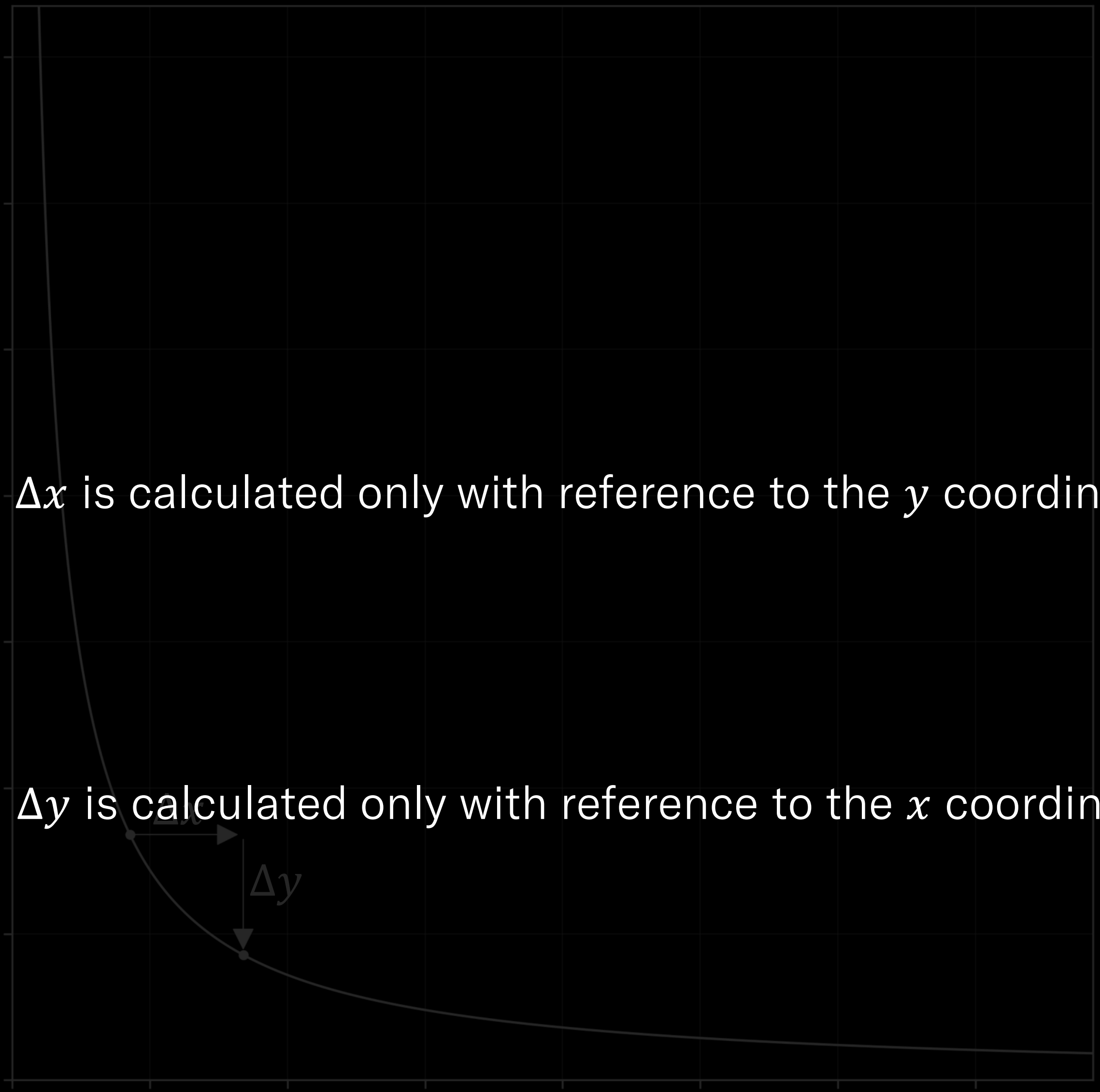
$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$



I will refer to this as "dimension reduction".

$$x \cdot y = x_0 \cdot y_0$$

$$x \cdot y = x_0 \cdot y_0$$



$$\boxed{\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}}$$

Δx is calculated only with reference to the y coordinate.

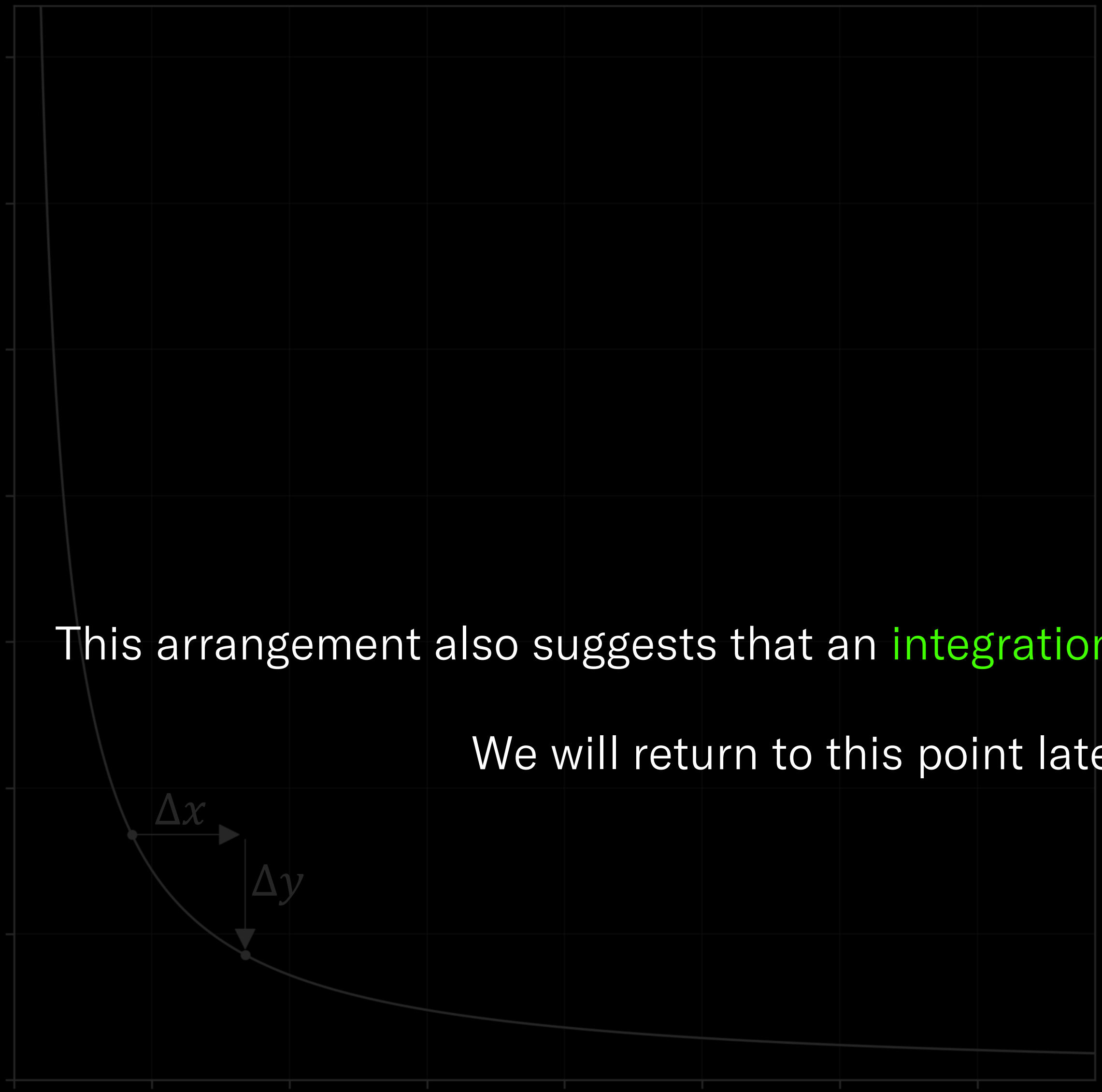
$$\boxed{\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}}$$

Δy is calculated only with reference to the x coordinate.

$$x \cdot y = x_0 \cdot y_0$$

$$\boxed{\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}}$$

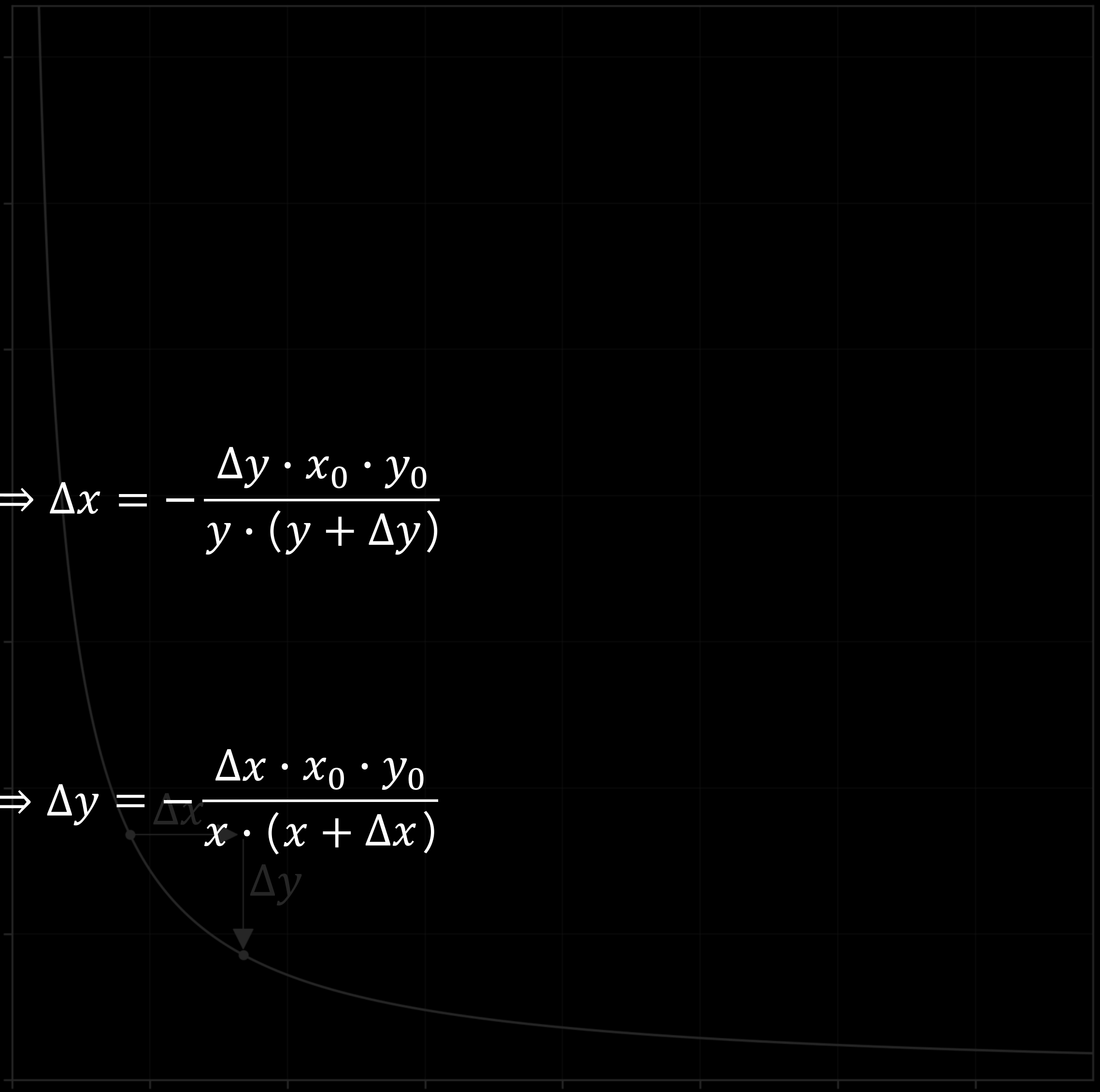
$$\boxed{\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}}$$



This arrangement also suggests that an integration has been performed.

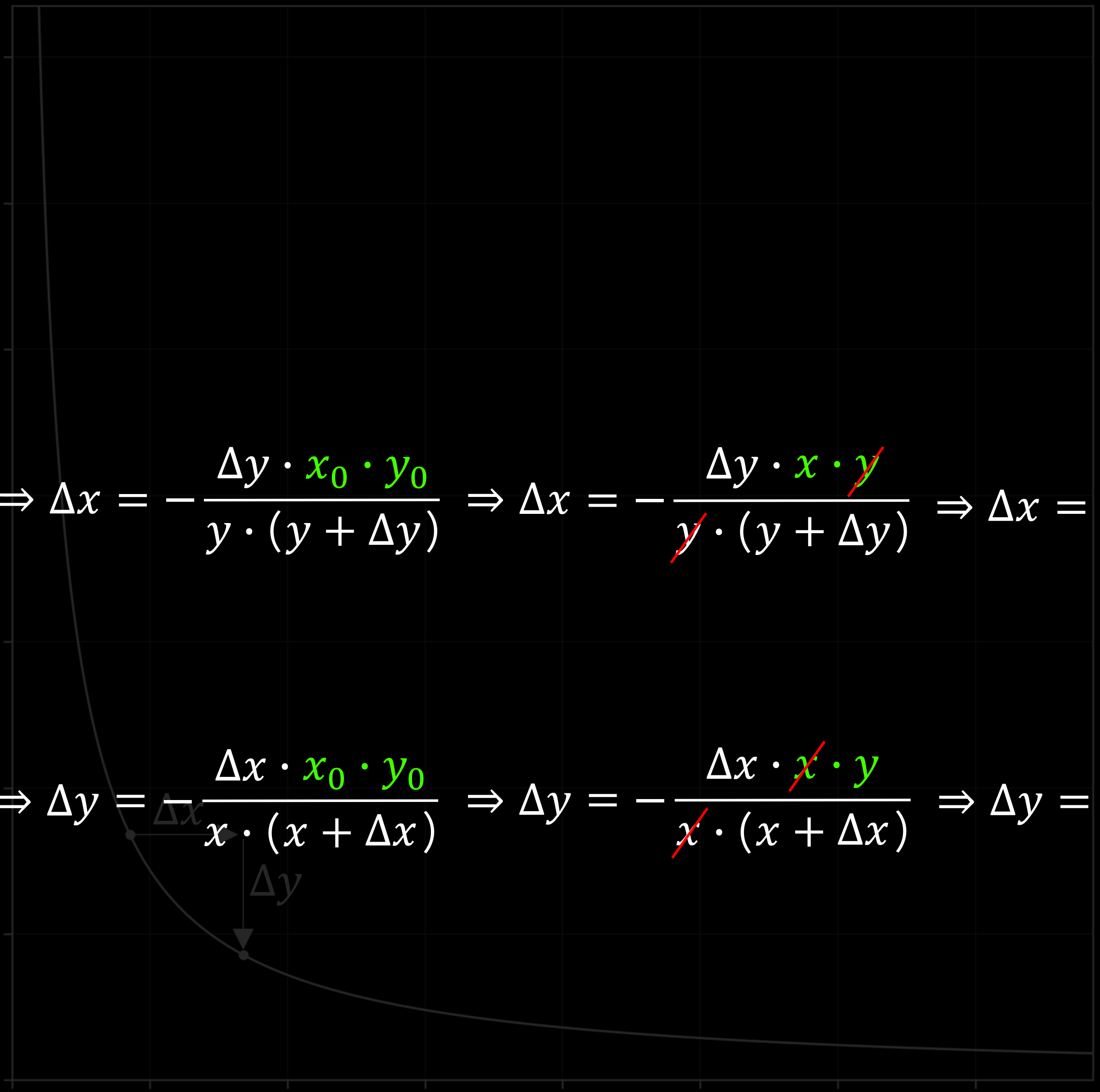
We will return to this point later.

$$x \cdot y = x_0 \cdot y_0$$



$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y} \Rightarrow \Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x} \Rightarrow \Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

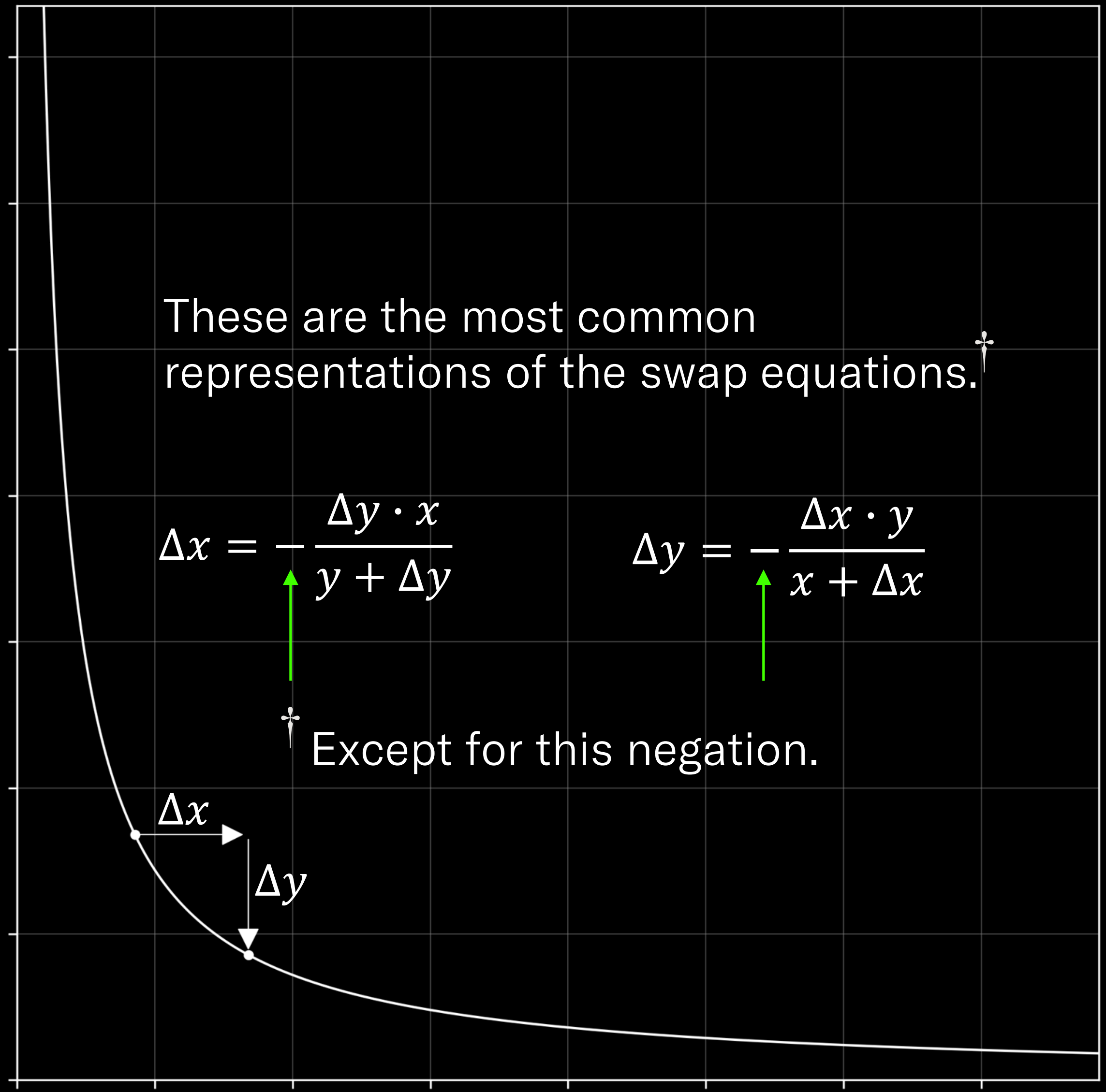


$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y} \Rightarrow \Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)} \Rightarrow \Delta x = -\frac{\Delta y \cdot x \cdot \cancel{y}}{\cancel{y} \cdot (y + \Delta y)} \Rightarrow \Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x} \Rightarrow \Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)} \Rightarrow \Delta y = -\frac{\Delta x \cdot \cancel{x} \cdot y}{\cancel{x} \cdot (x + \Delta x)} \Rightarrow \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$



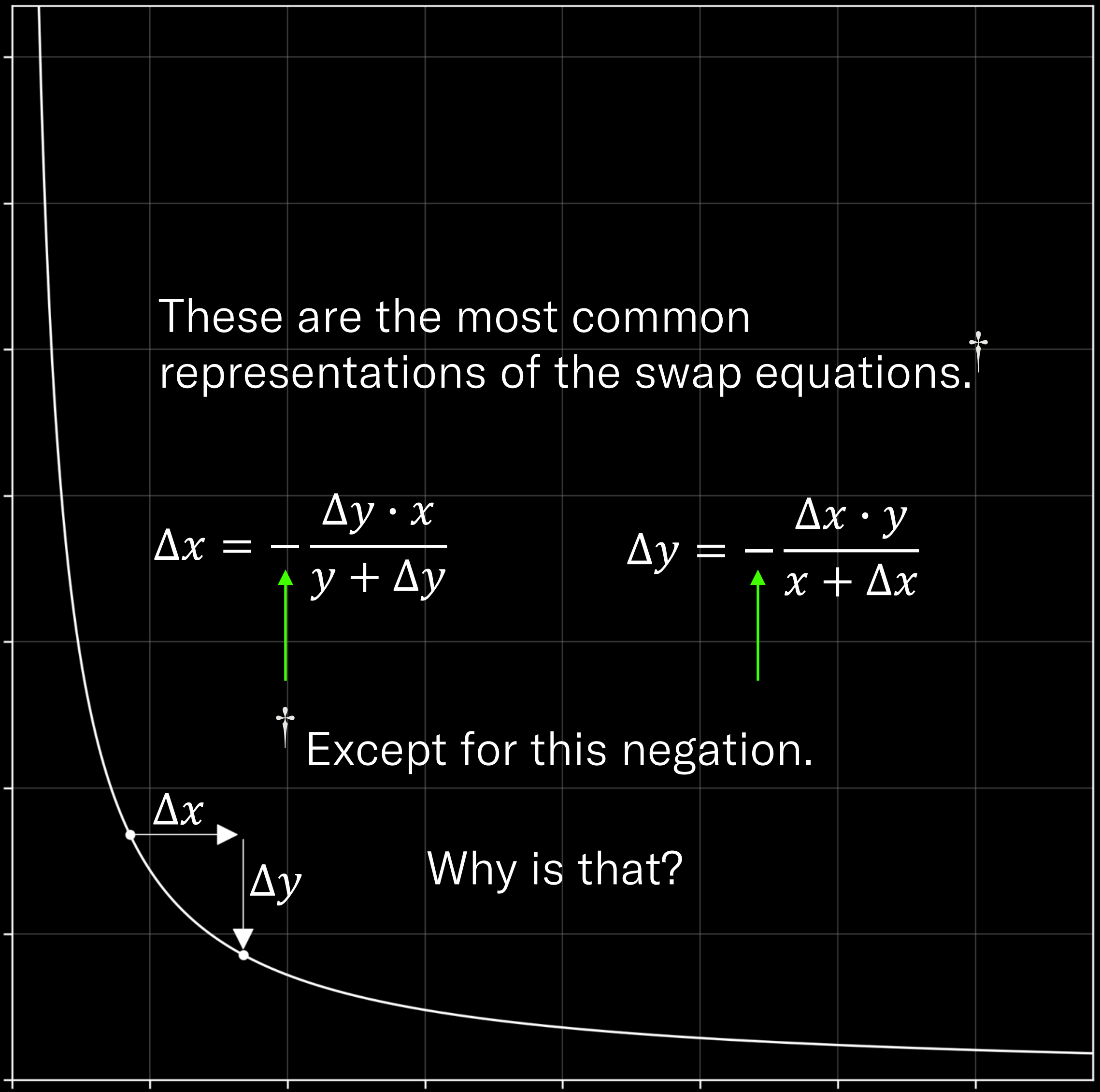
These are the most common representations of the swap equations.†

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y} \qquad \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

† Except for this negation.

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

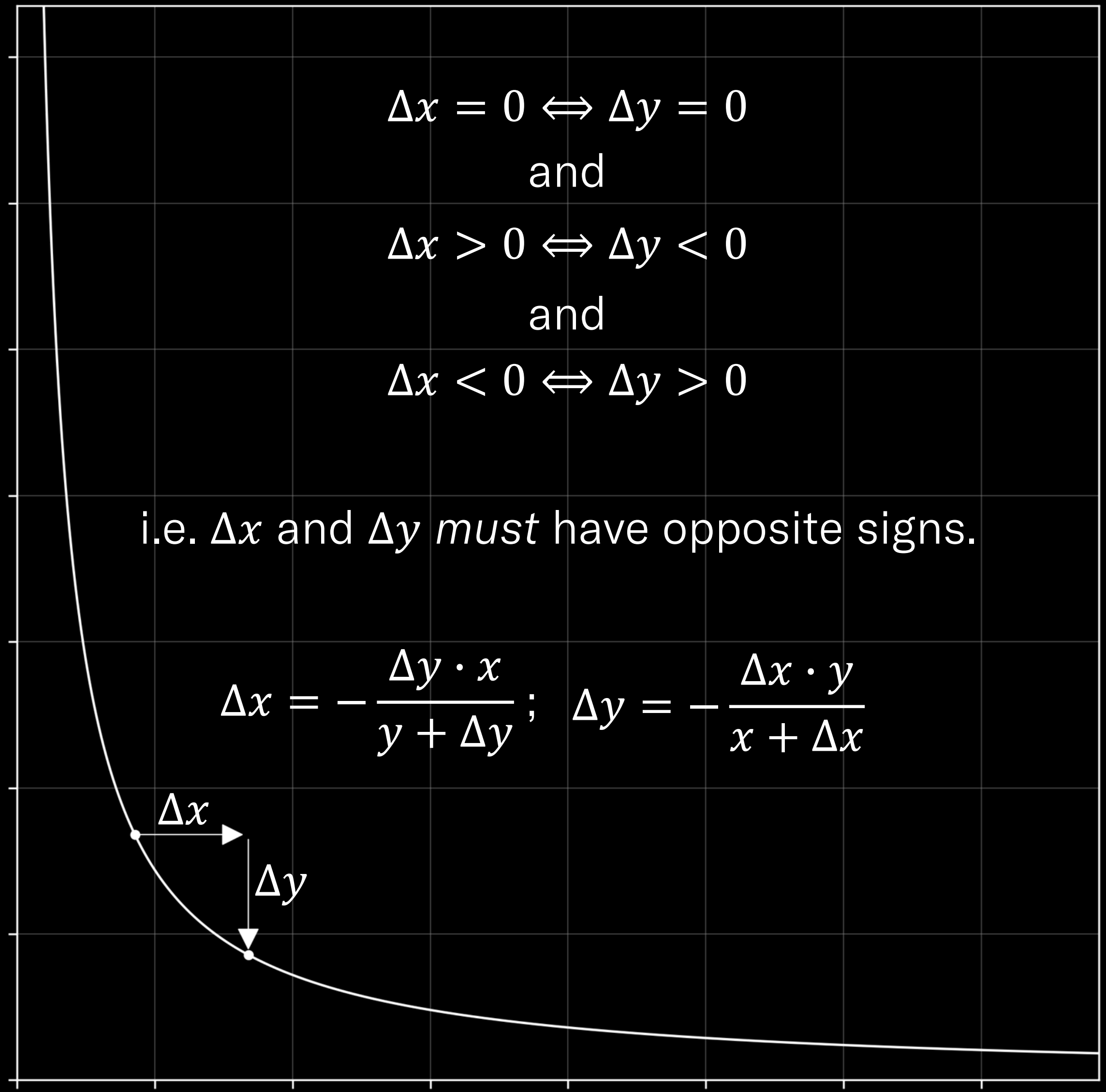


These are the most common representations of the swap equations.†

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y} \qquad \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

† Except for this negation.

Why is that?



$$\Delta x = 0 \Leftrightarrow \Delta y = 0$$

and

$$\Delta x > 0 \Leftrightarrow \Delta y < 0$$

and

$$\Delta x < 0 \Leftrightarrow \Delta y > 0$$

i.e. Δx and Δy *must* have opposite signs.

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}\,; \quad \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$$\Delta x = 0 \Leftrightarrow \Delta y = 0$$

and

$$\Delta x > 0 \Leftrightarrow \Delta y < 0$$

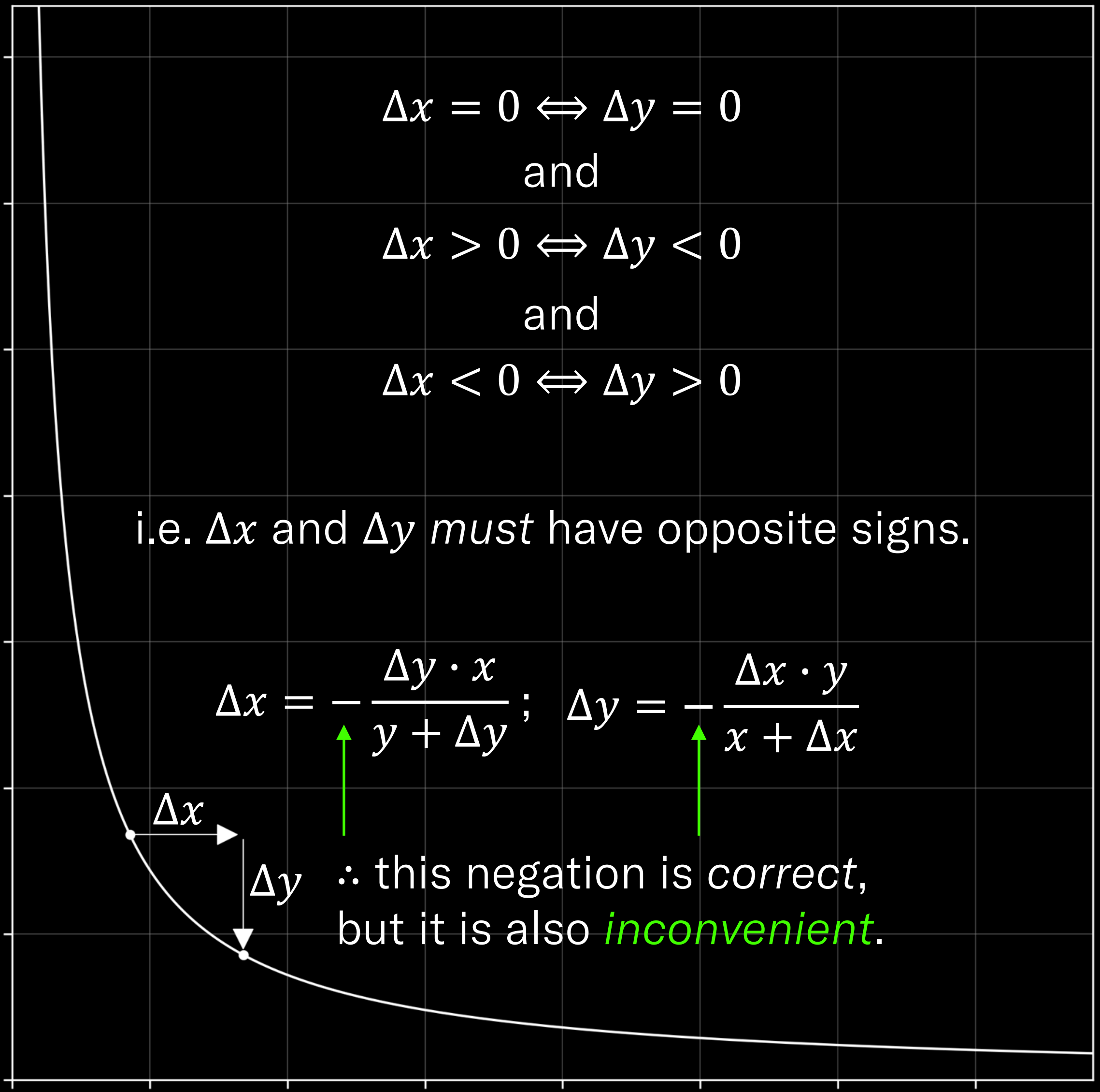
and

$$\Delta x < 0 \Leftrightarrow \Delta y > 0$$

i.e. Δx and Δy *must* have opposite signs.

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}\,; \quad \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$\therefore$ this negation is *correct*,
but it is also *inconvenient*.



This is what 32 bits looks like.

00110000110101100010001110111011

The largest unsigned integer it can represent is $2^{32} - 1 = 4{,}294{,}967{,}295$.

Most Significant Bit (MSB) → 00110000110101100010001110111011 -50%

The largest signed integer it can represent is $2^{31} - 1 = 2{,}147{,}483{,}647$.

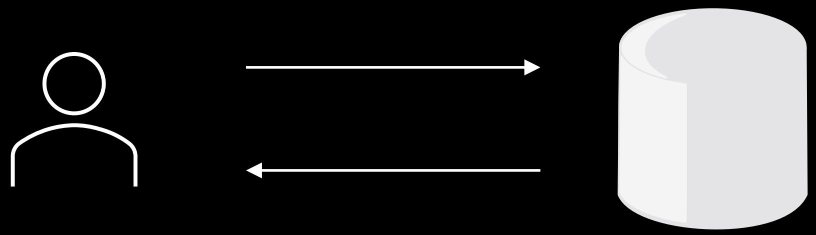
0 = non-negative
1 = negative

Unsigned integers are better, provided you know something about context.

I often refer to this as adopting a “moving frame of reference”.

$\Delta x < 0$

$\Delta y > 0$



$\Delta x > 0$

$\Delta y < 0$

Theory ≠ Implementation

These are uncommon.

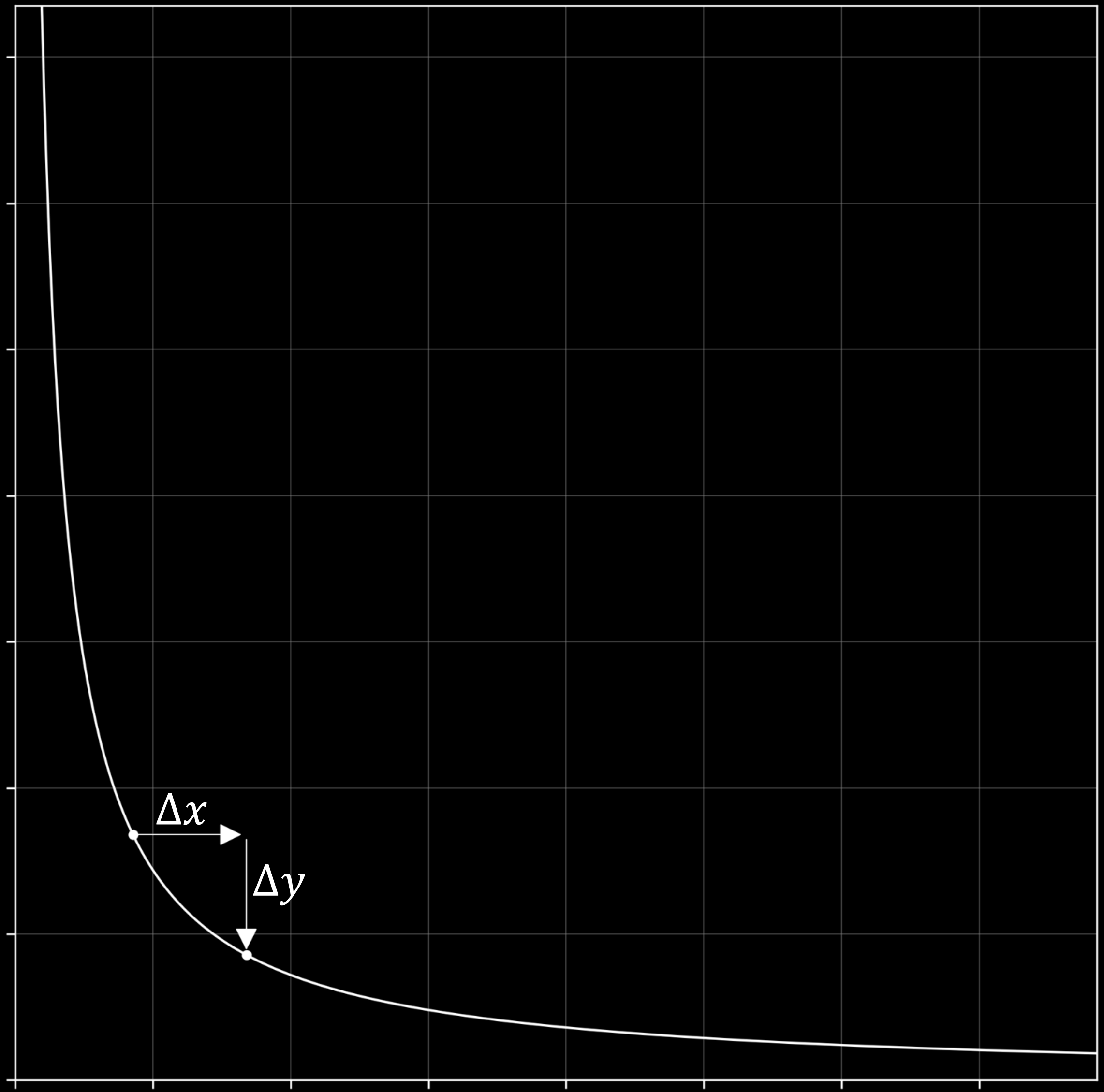
$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

They are also more pedagogically useful.



These are common.

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

They are also “nice”.

These expressions allow for Δx to be calculated given Δy, or Δy to be calculated given Δx.

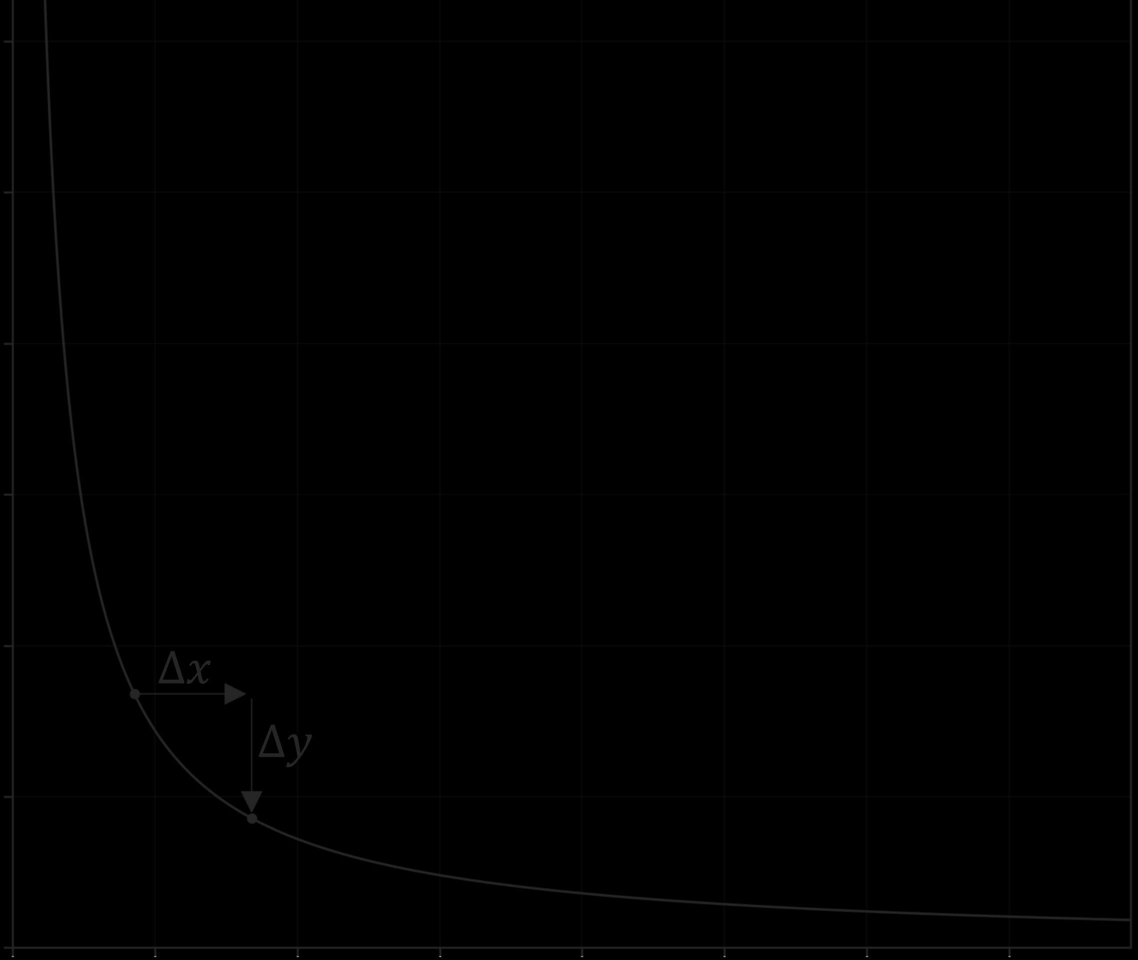
$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y} \qquad \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$



These are the *effective* price equations; $\Delta x/\Delta y$ and $\Delta y/\Delta x$ are rates of exchange for all non-zero Δx and Δy.

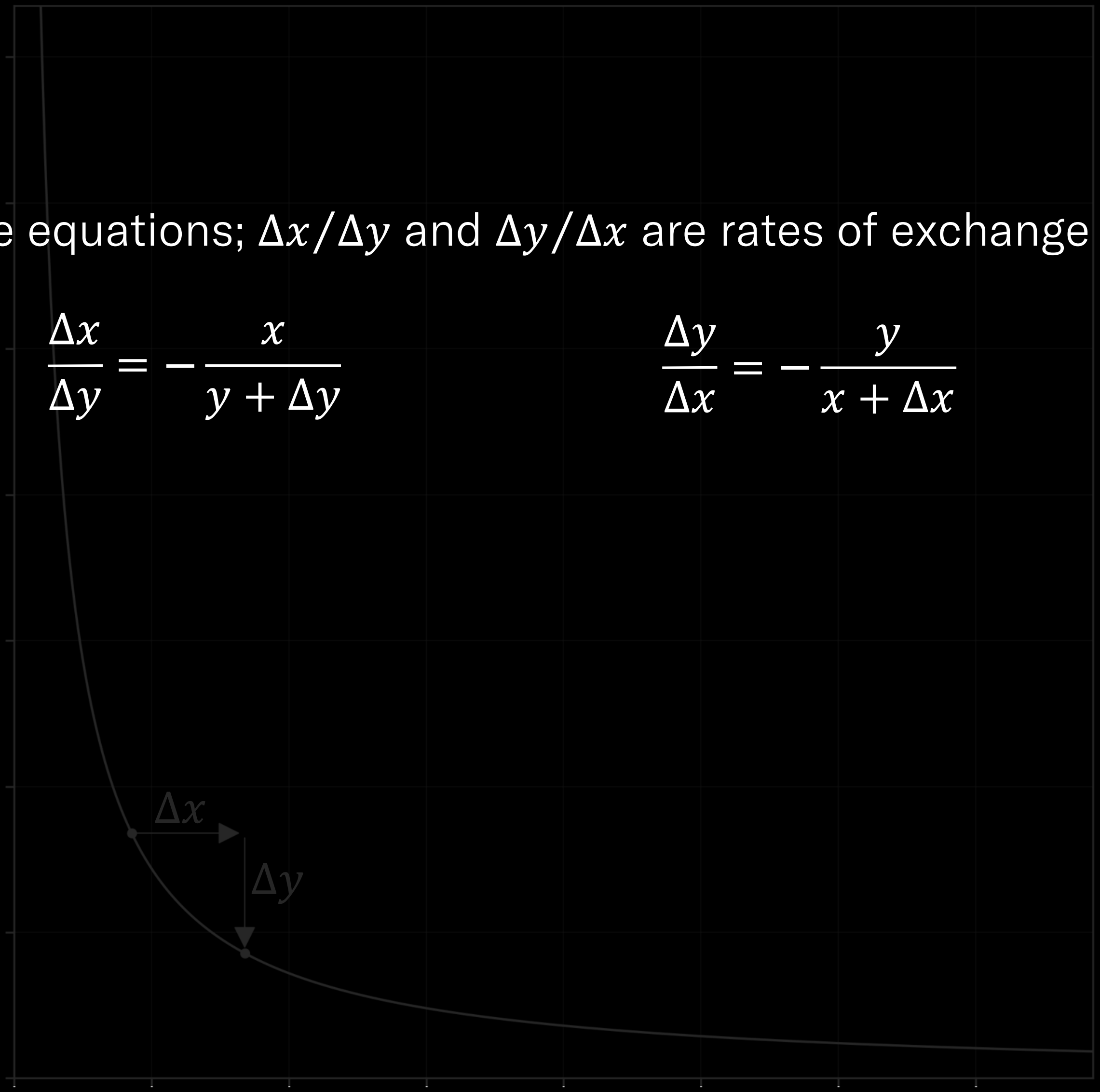
$$\frac{\Delta x}{\Delta y} = -\frac{x}{y + \Delta y} \qquad \frac{\Delta y}{\Delta x} = -\frac{y}{x + \Delta x}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

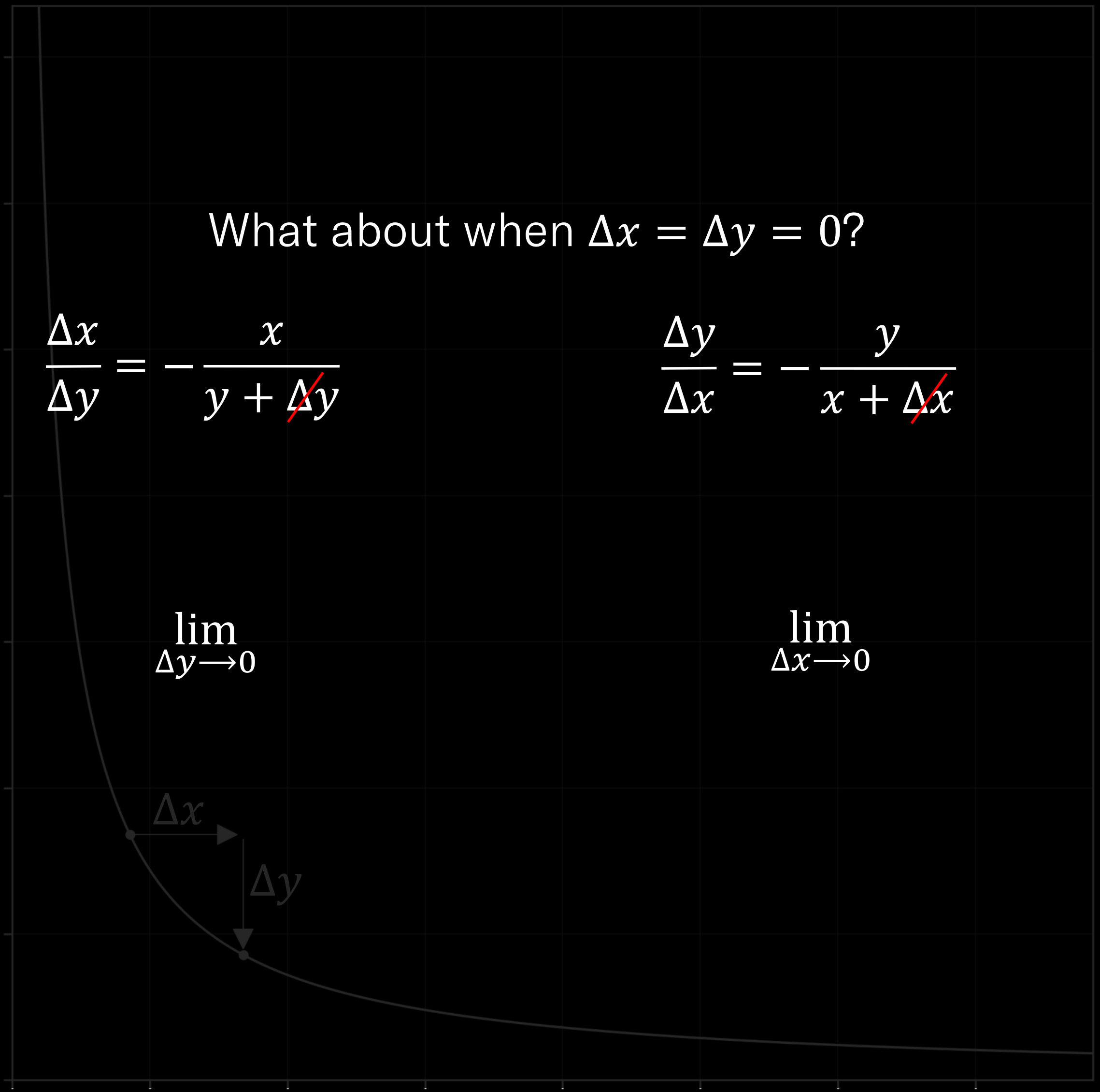


$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$



What about when $\Delta x = \Delta y = 0$?

$$\frac{\Delta x}{\Delta y} = -\frac{x}{y + \cancel{\Delta y}} \qquad \frac{\Delta y}{\Delta x} = -\frac{y}{x + \cancel{\Delta x}}$$

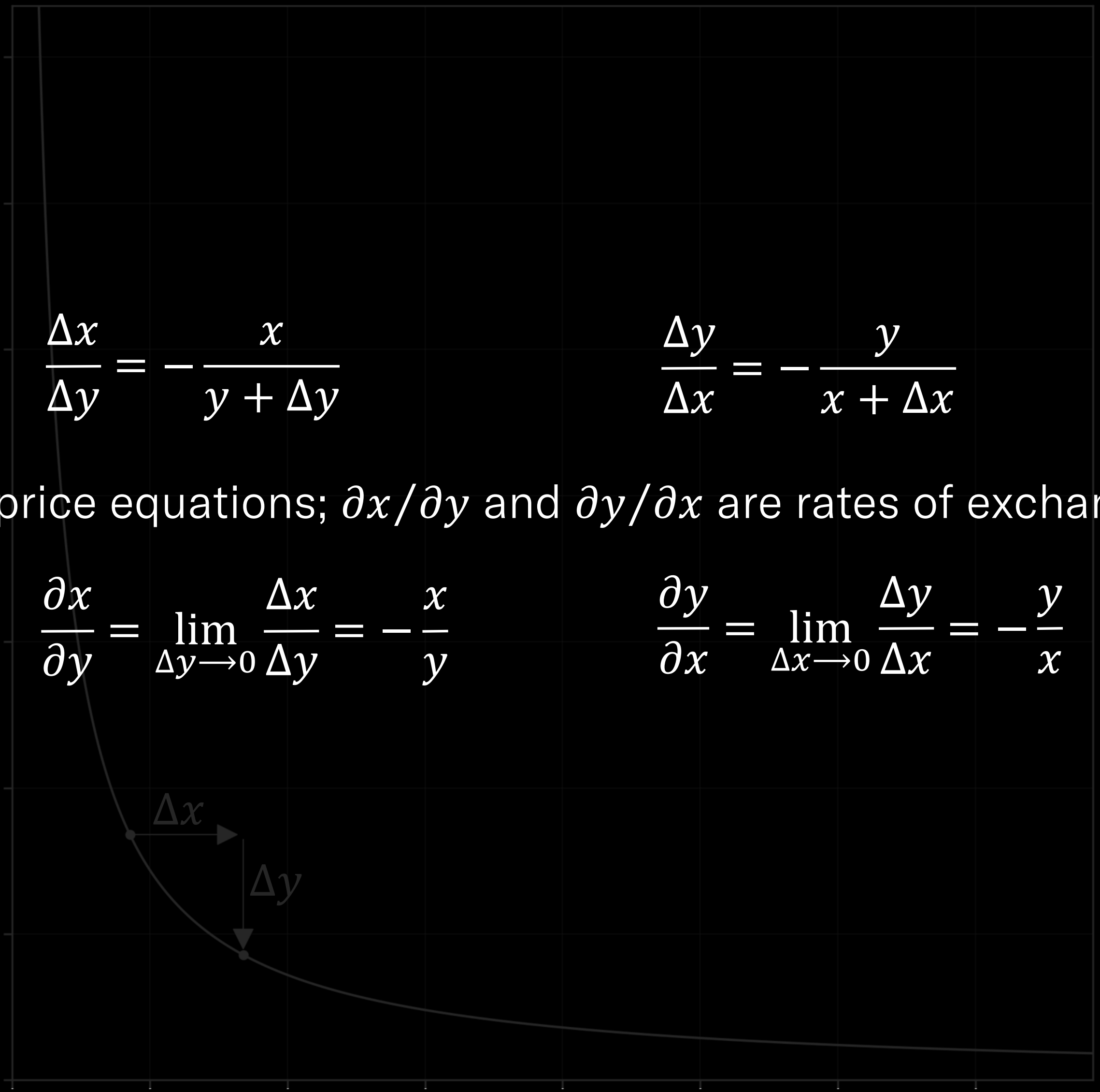
$$\lim_{\Delta y \longrightarrow 0} \qquad \lim_{\Delta x \longrightarrow 0}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$



$$\frac{\Delta x}{\Delta y} = -\frac{x}{y + \Delta y} \qquad \frac{\Delta y}{\Delta x} = -\frac{y}{x + \Delta x}$$

These are the *marginal* price equations; $\partial x / \partial y$ and $\partial y / \partial x$ are rates of exchange when $\Delta x = \Delta y = 0$.

$$\frac{\partial x}{\partial y} = \lim_{\Delta y \longrightarrow 0} \frac{\Delta x}{\Delta y} = -\frac{x}{y} \qquad \frac{\partial y}{\partial x} = \lim_{\Delta x \longrightarrow 0} \frac{\Delta y}{\Delta x} = -\frac{y}{x}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

using implicit differentiation

$$x \cdot y = x_0 \cdot y_0$$

$$\frac{\partial}{\partial x}(x \cdot y) = \frac{\partial}{\partial x}(x_0 \cdot y_0)$$

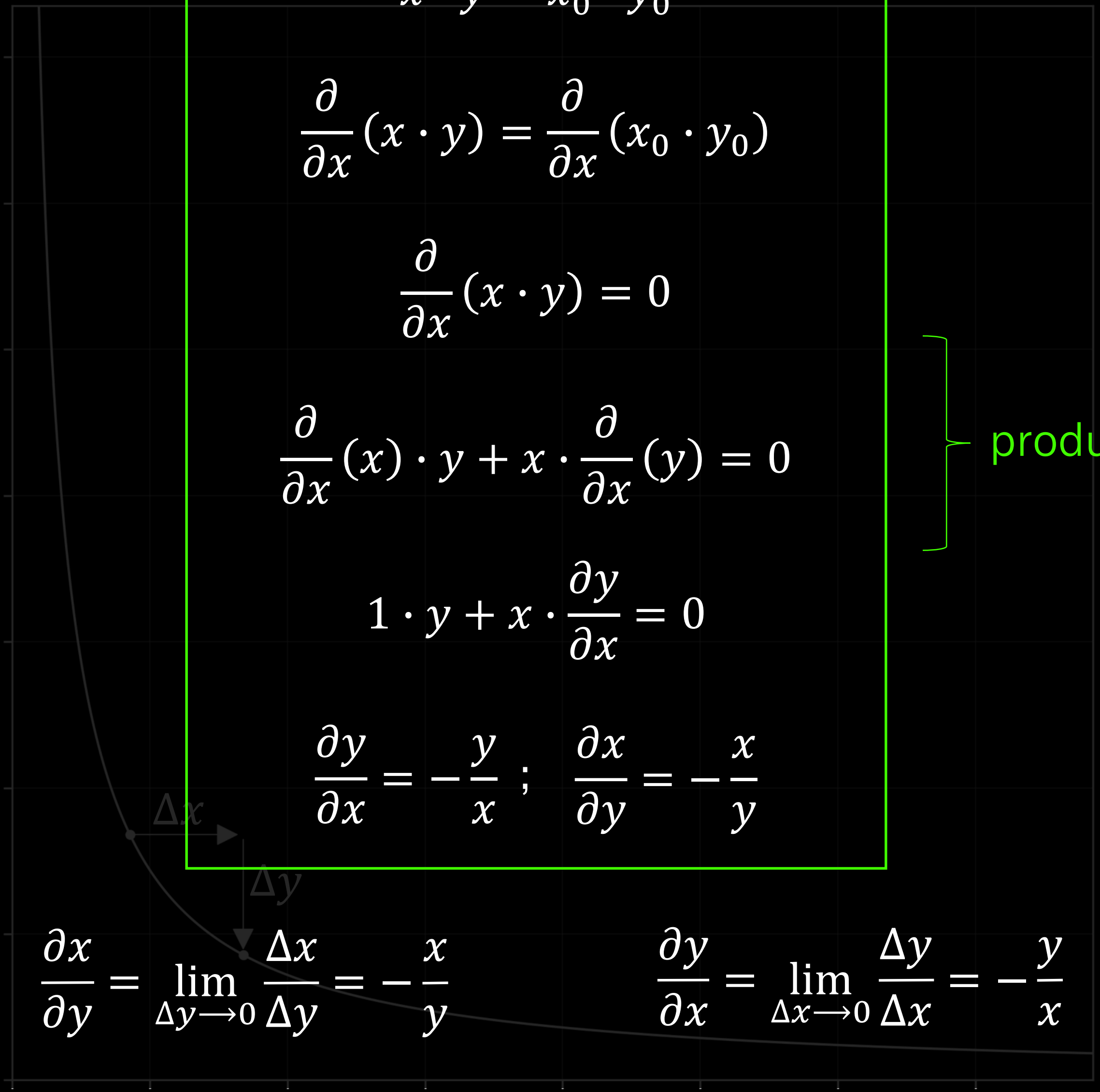
$$\frac{\partial}{\partial x}(x \cdot y) = 0$$

$$\frac{\partial}{\partial x}(x) \cdot y + x \cdot \frac{\partial}{\partial x}(y) = 0$$

product rule

$$1 \cdot y + x \cdot \frac{\partial y}{\partial x} = 0$$

$$\frac{\partial y}{\partial x} = -\frac{y}{x} \; ; \quad \frac{\partial x}{\partial y} = -\frac{x}{y}$$



$$\frac{\partial x}{\partial y} = \lim_{\Delta y \longrightarrow 0} \frac{\Delta x}{\Delta y} = -\frac{x}{y} \qquad \frac{\partial y}{\partial x} = \lim_{\Delta x \longrightarrow 0} \frac{\Delta y}{\Delta x} = -\frac{y}{x}$$

These expressions allow for Δx to be calculated given Δy, or Δy to be calculated given Δx.

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y} \qquad \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

These are the *effective* price equations; $\Delta x / \Delta y$ and $\Delta y / \Delta x$ are rates of exchange for all non-zero Δx and Δy.

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These are the *marginal* price equations; $\partial x / \partial y$ and $\partial y / \partial x$ are rates of exchange when $\Delta x = \Delta y = 0$.

$$\frac{\partial x}{\partial y} = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = -\frac{x}{y} \qquad \frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\frac{y}{x}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

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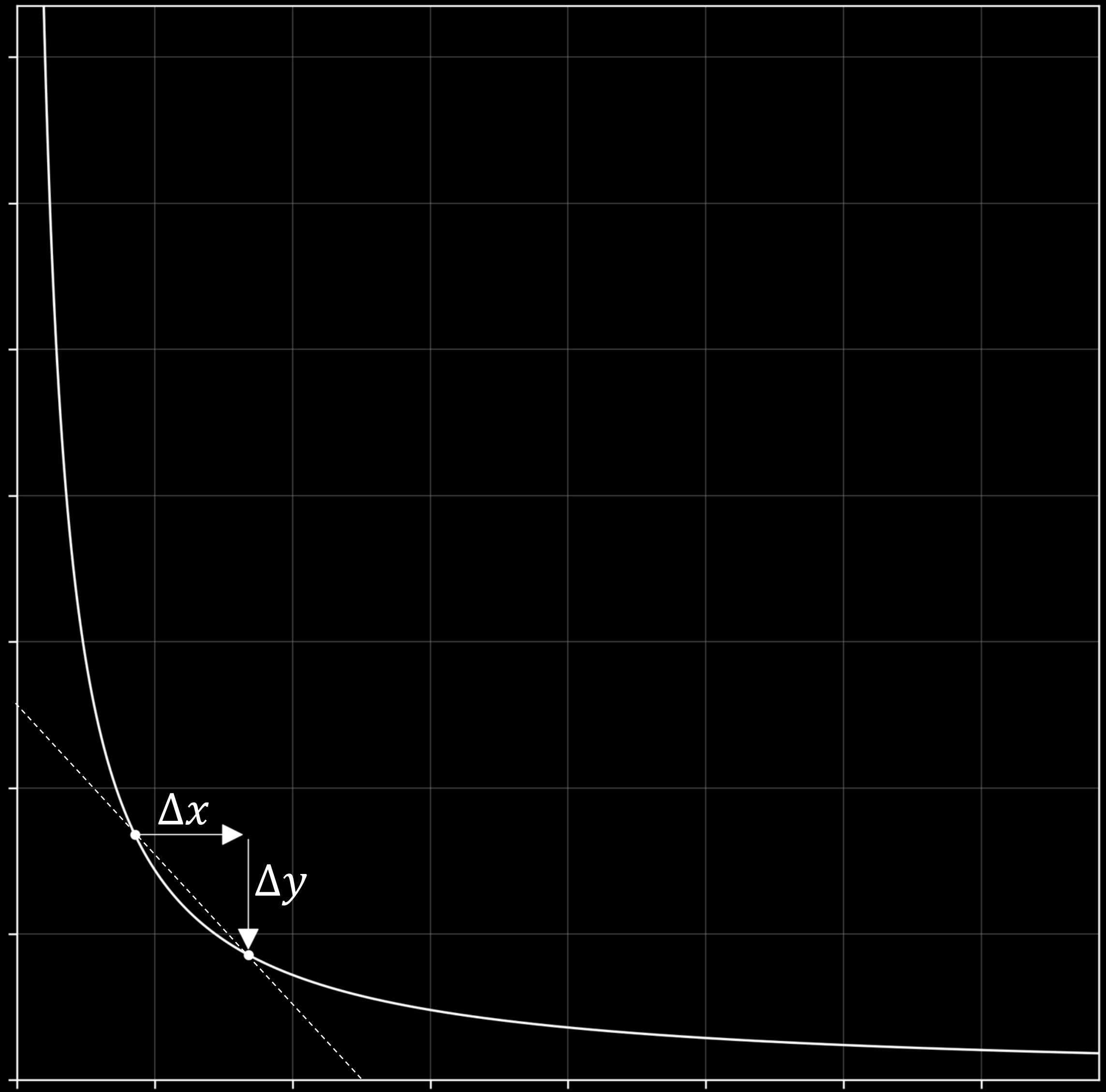


effective exchange rate

$$\frac{\Delta x}{\Delta y} = -\frac{x}{y + \Delta y}$$

$$\frac{\Delta y}{\Delta x} = -\frac{y}{x + \Delta x}$$

marginal exchange rate

$$\frac{\partial x}{\partial y} = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = -\frac{x}{y}$$

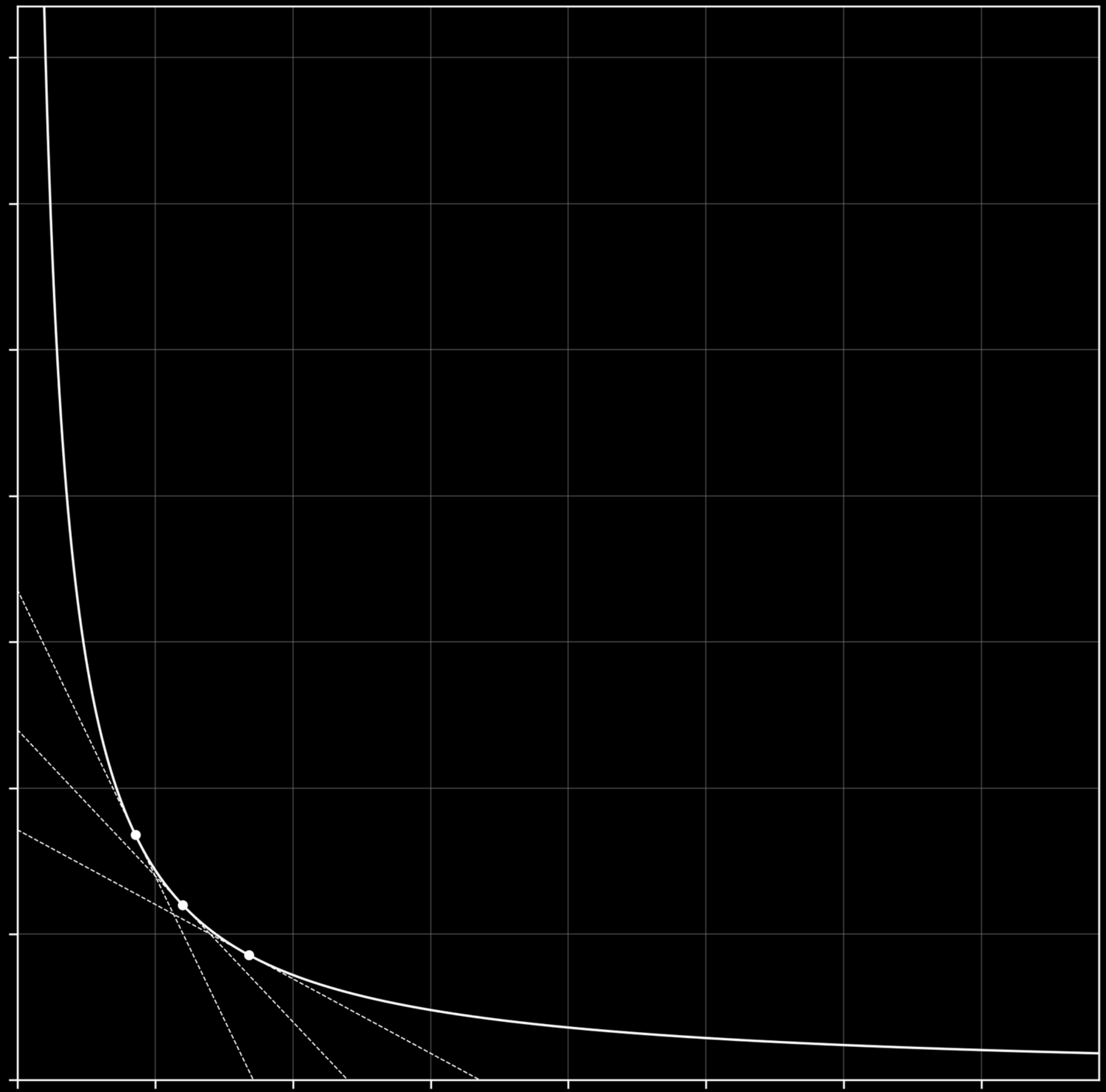
$$\frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\frac{y}{x}$$

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$



effective exchange rate

$$\frac{\Delta x}{\Delta y} = -\frac{x}{y + \Delta y}$$

$$\frac{\Delta y}{\Delta x} = -\frac{y}{x + \Delta x}$$

marginal exchange rate

$$\frac{\partial x}{\partial y} = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = -\frac{x}{y}$$

$$\frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\frac{y}{x}$$

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

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$$\lim_{\Delta y \longrightarrow 0}$$

$$\lim_{\Delta x \longrightarrow 0}$$

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using the power rule:

$$x = \frac{x_0 \cdot y_0}{y} = (x_0 \cdot y_0) \cdot y^{-1}$$

$$\frac{\partial x}{\partial y} = -1 \cdot (x_0 \cdot y_0) \cdot y^{-2}$$

$$\frac{\partial x}{\partial y} = -\frac{x_0 \cdot y_0}{y^2}$$

$$y = \frac{x_0 \cdot y_0}{x} = (x_0 \cdot y_0) \cdot x^{-1}$$

$$\frac{\partial y}{\partial x} = -1 \cdot (x_0 \cdot y_0) \cdot x^{-2}$$

$$\frac{\partial y}{\partial x} = -\frac{x_0 \cdot y_0}{x^2}$$

$$\frac{\partial x}{\partial y} = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = -\frac{x_0 \cdot y_0}{y^2} \qquad \frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\frac{x_0 \cdot y_0}{x^2}$$

evaluating the definite integral:

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\left[\frac{x_0 \cdot y_0}{y}\right]_y^{y+\Delta y} = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\int_y^{y+\Delta y} \frac{dx}{dy} \cdot dy = -\int_y^{y+\Delta y} \frac{x_0 \cdot y_0}{y^2} \cdot dy$$

$$\frac{\partial x}{\partial y} = -\frac{x_0 \cdot y_0}{y^2}$$

$$\left[\frac{x_0 \cdot y_0}{x}\right]_x^{x+\Delta x} = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

$$\int_x^{x+\Delta x} \frac{dy}{dx} \cdot dx = -\int_x^{x+\Delta x} \frac{x_0 \cdot y_0}{x^2} \cdot dx$$

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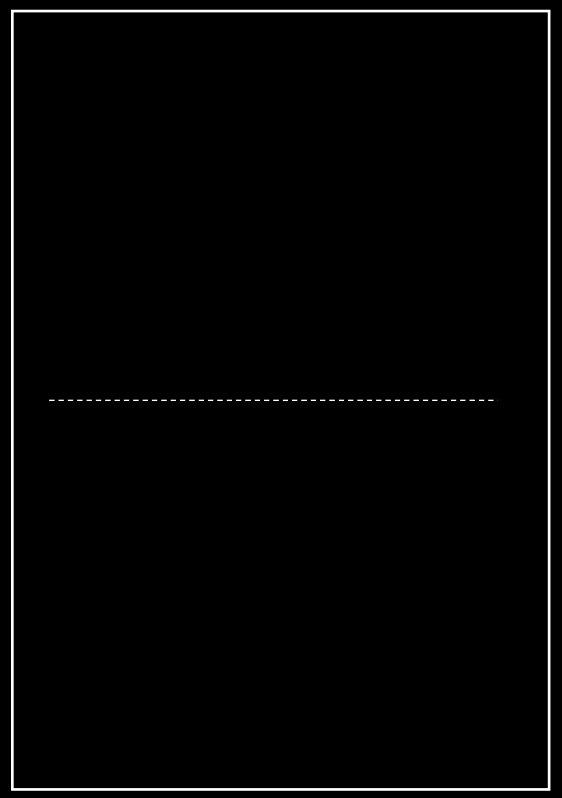

$$\left[\frac{x_0 \cdot y_0}{y}\right]_y^{y+\Delta y} \Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

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This is the definite integral of this (evaluated over some interval).

Because of course it is!

swap formula

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

$$\Delta y = \frac{x_0 \cdot y_0}{x + \Delta x} - \frac{x_0 \cdot y_0}{x}$$

effective rate

$$\frac{\Delta x}{\Delta y} = -\frac{x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\frac{\Delta y}{\Delta x} = -\frac{x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

marginal rate

$$\frac{\partial x}{\partial y} = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = -\frac{x_0 \cdot y_0}{y^2}$$

$$\frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\frac{x_0 \cdot y_0}{x^2}$$

This all refers to one and the same thing.

swap formula

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

effective rate

$$\frac{\Delta x}{\Delta y} = -\frac{x}{y + \Delta y}$$

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$$\frac{\partial x}{\partial y} = \lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = -\frac{x}{y}$$

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Which means this is the definite integral of this (evaluated over some interval).

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

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swap formula

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marginal rate

This is not as easy to integrate because x and y are *dependent on each other* (i.e. nothing is constant).

ಠ_ಠ

$$\frac{\partial y}{\partial x} = -\frac{y}{x}$$

This is how to do it.

First, separate the variables.

$$\frac{1}{\partial x} \cdot \frac{\partial y}{1} = -\frac{1}{x} \cdot \frac{y}{1}$$

First, separate the variables.

$$\frac{1}{y} \cdot \partial y = -\frac{1}{x} \cdot \partial x$$

Then integrate both sides.

$$\int_{y}^{y+\Delta y} \frac{1}{y} \cdot \partial y = -\int_{x}^{x+\Delta x} \frac{1}{x} \cdot \partial x$$

Then integrate both sides.

$$[\ln(y)]_{y}^{y+\Delta y} = -[\ln(x)]_{x}^{x+\Delta x}$$

Exploit the properties of logarithms.

$$\ln\left(y + \Delta y\right) - \ln\left(y\right) = \ln(x) - \ln(x + \Delta x)$$

Exploit the properties of logarithms.

$$\cancel{\ln}\left(\frac{y + \Delta y}{y}\right) = \cancel{\ln}\left(\frac{x}{x + \Delta x}\right)$$

$$\frac{y + \Delta y}{y} = \frac{x}{x + \Delta x}$$

$$y + \Delta y = y \cdot \frac{x}{x + \Delta x}$$

$$\Delta y = y \cdot \frac{x}{x + \Delta x} - y$$

$$\Delta y = y \cdot \left(\frac{x}{x + \Delta x} - 1 \right)$$

$$\Delta y = y \cdot \left(\frac{x}{x + \Delta x} - \frac{x + \Delta x}{x + \Delta x} \right)$$

$$\Delta y = y \cdot \left(\frac{x - (x + \Delta x)}{x + \Delta x} \right)$$

$$\Delta y = y \cdot \left(\frac{\cancel{x} - \cancel{x} - \Delta x}{x + \Delta x} \right)$$

$$\Delta y = y \cdot \left(\frac{-\Delta x}{x + \Delta x} \right)$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$$\therefore \quad \frac{\partial y}{\partial x} = -\frac{y}{x} \iff \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x} \quad \blacksquare$$

swap formula

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

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marginal rate

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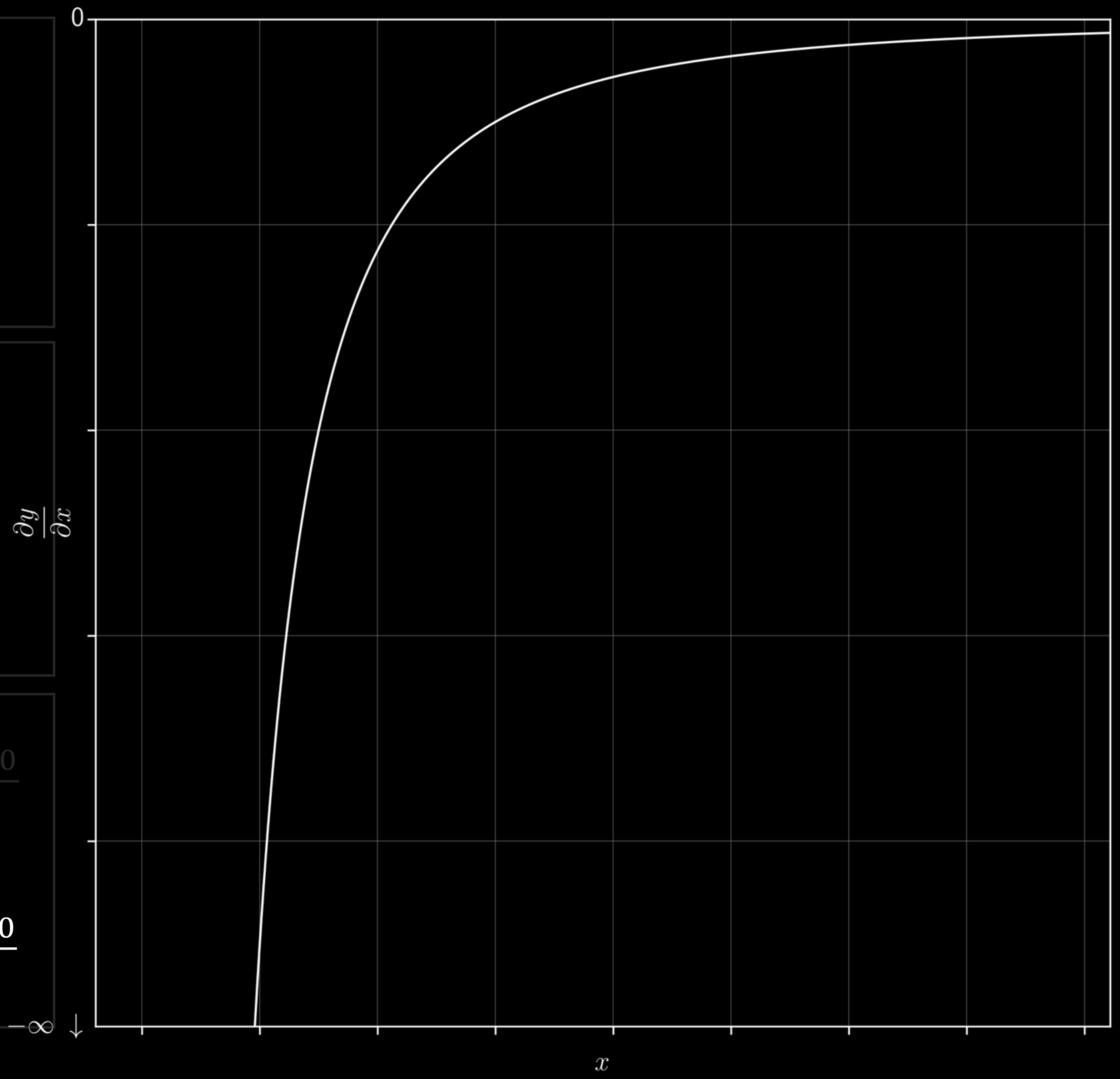

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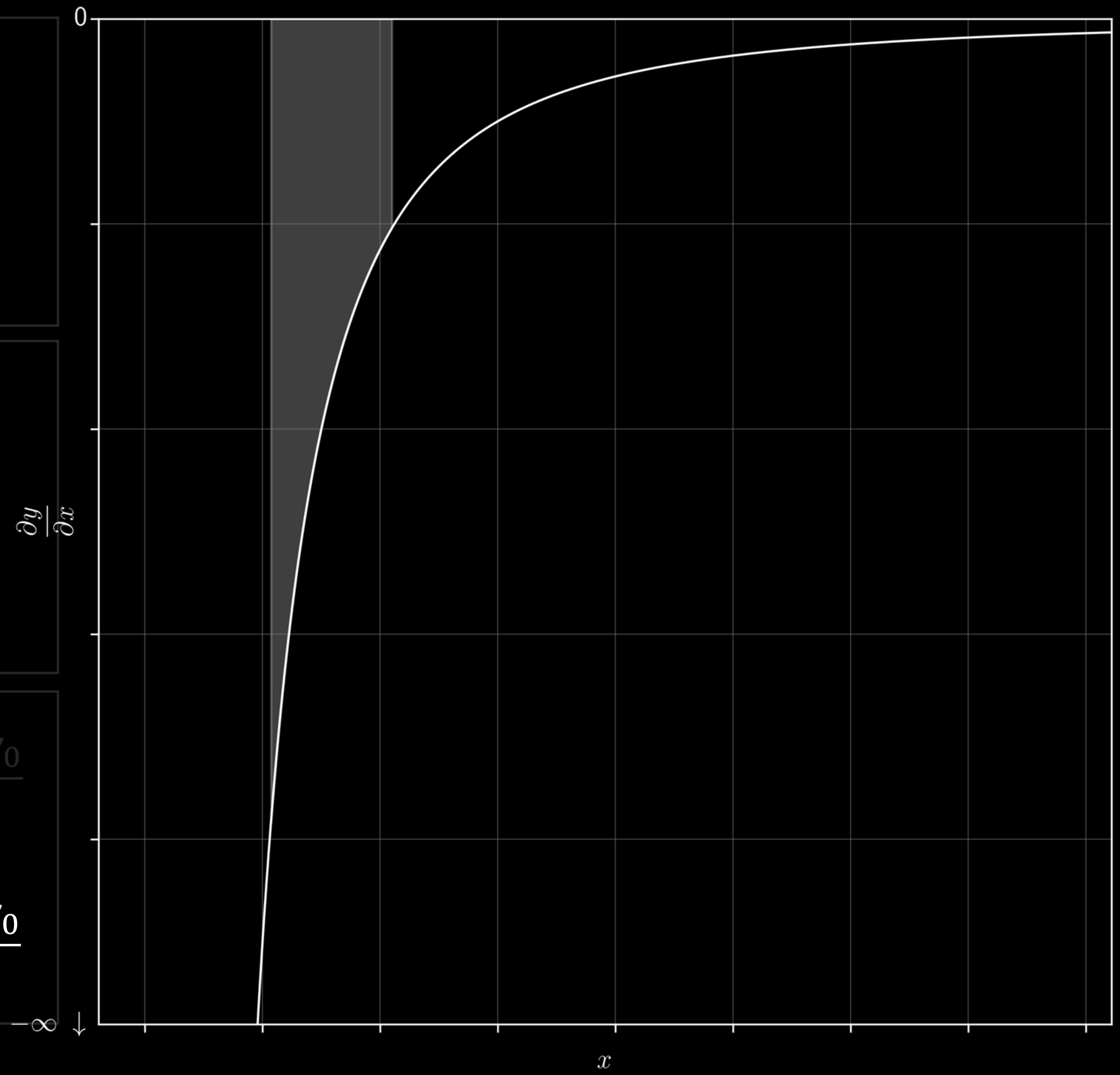

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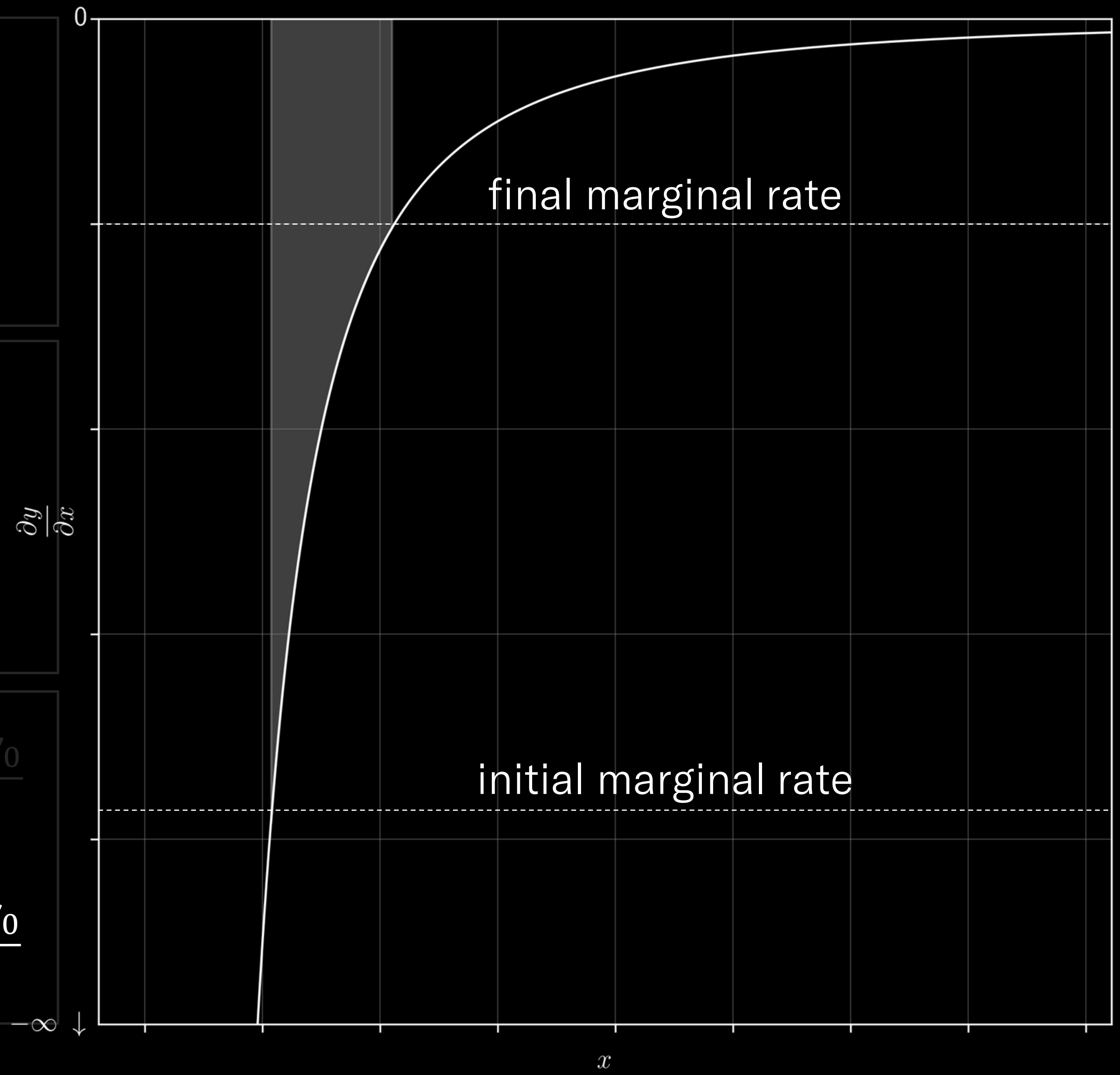

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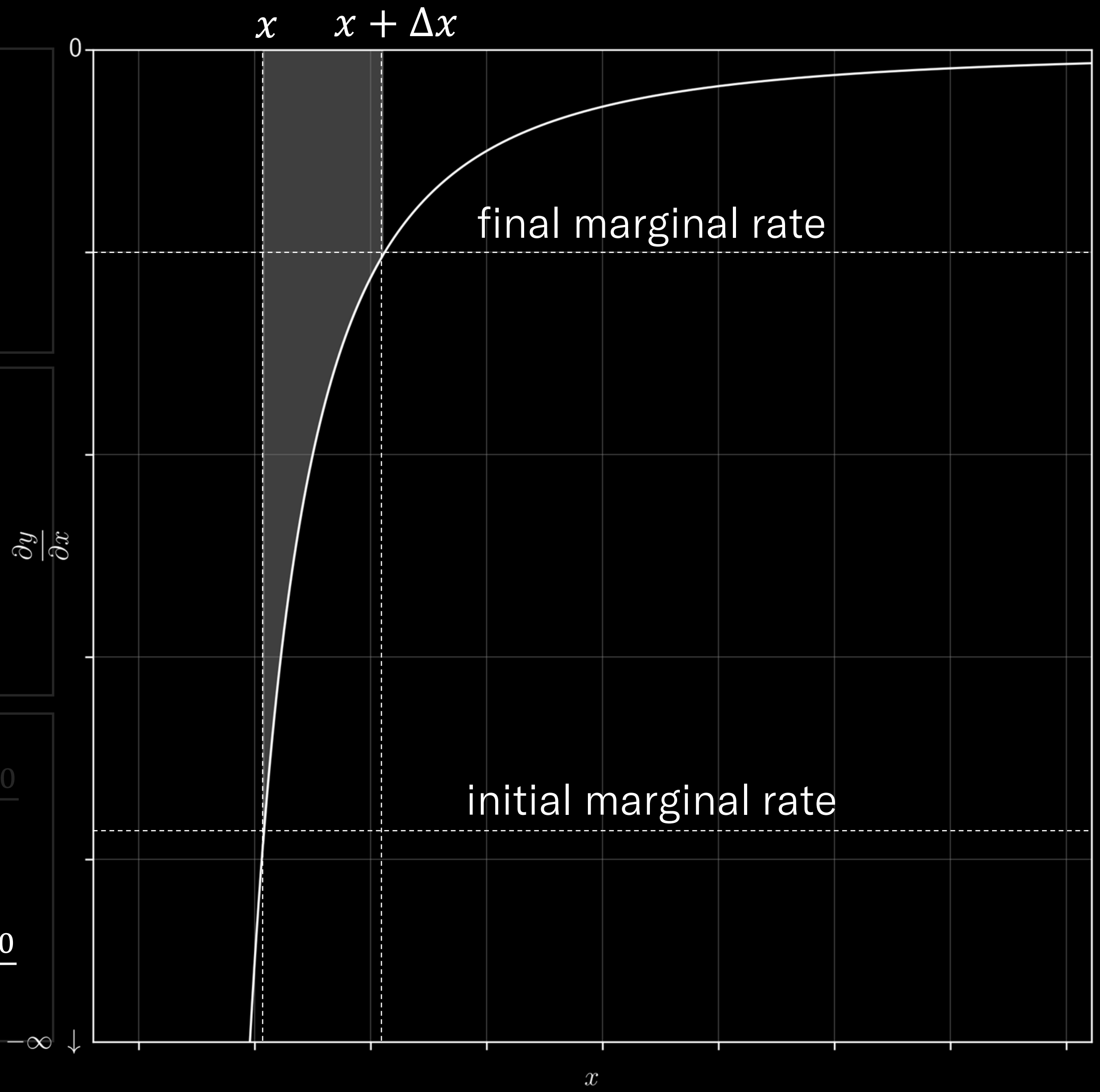

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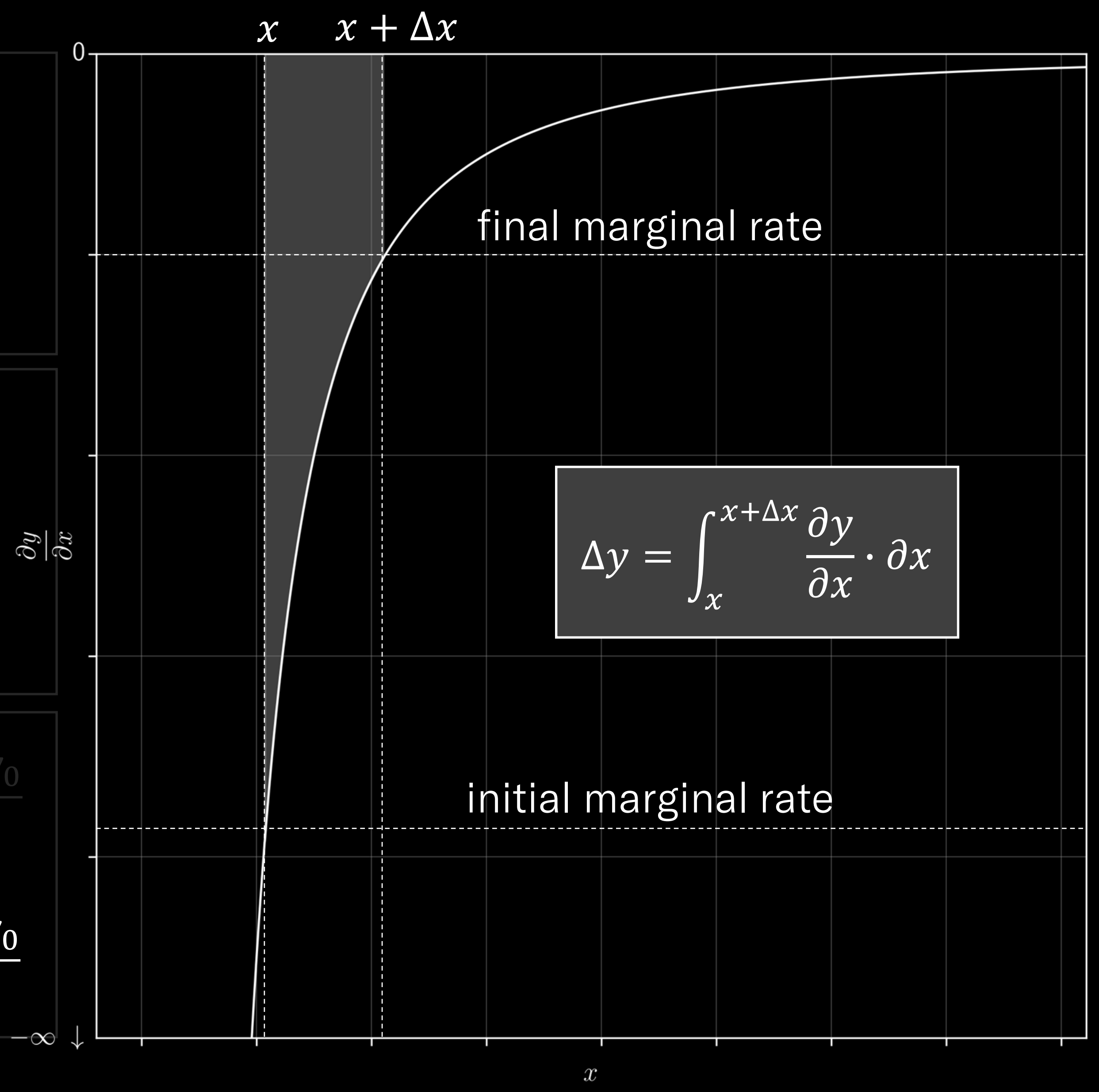

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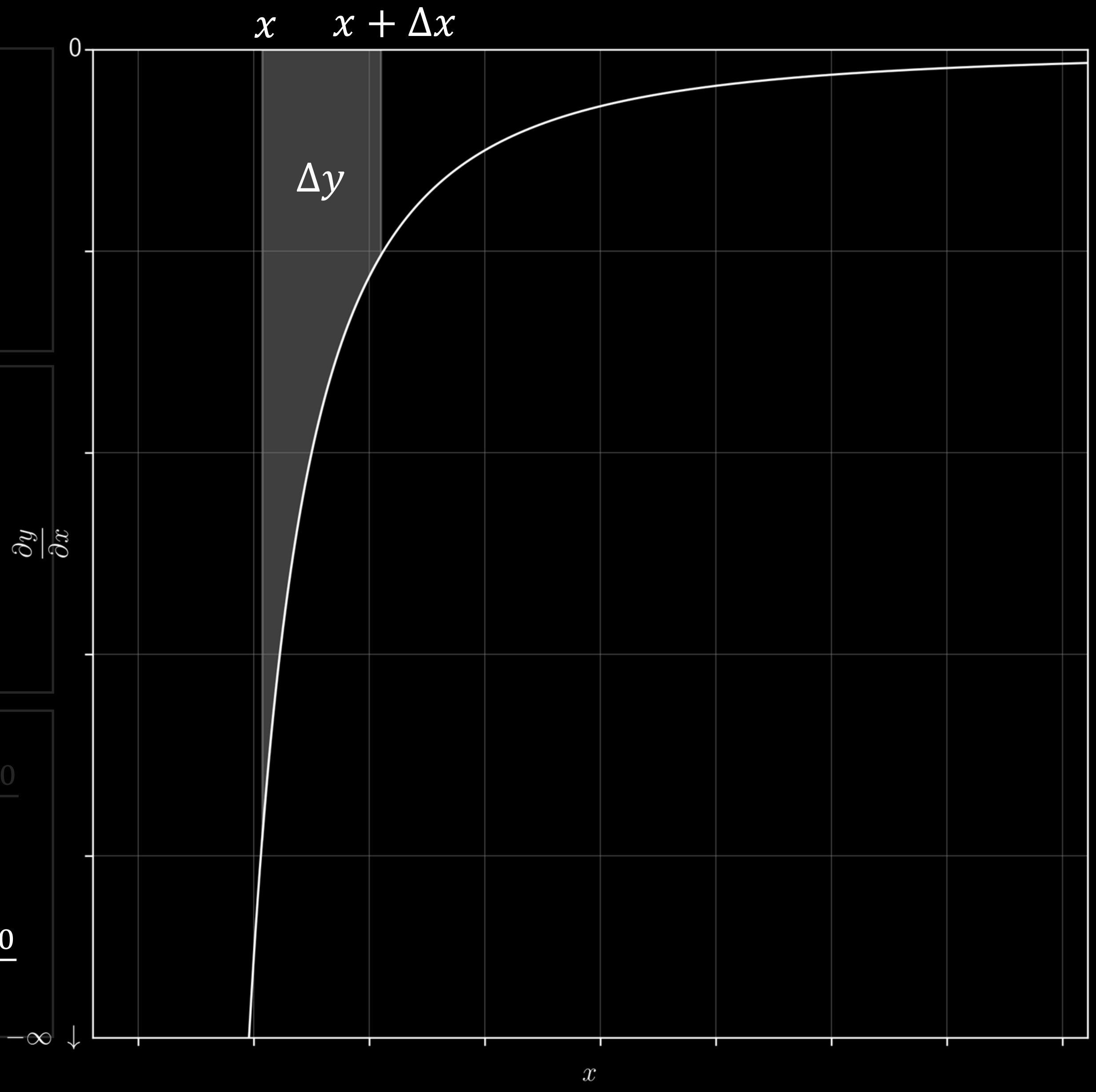

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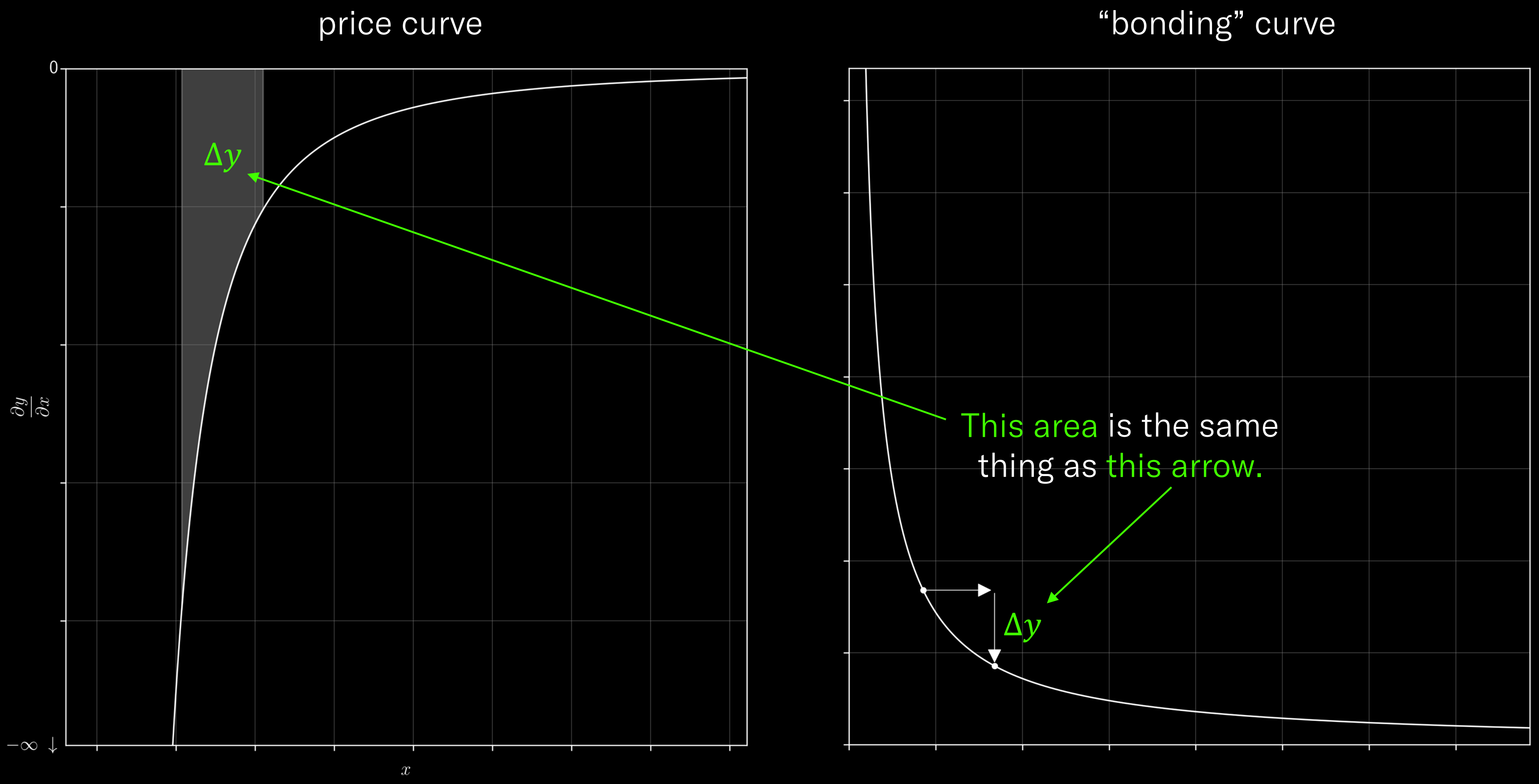

price curve
“bonding” curve
0
Δy
$\frac{\partial y}{\partial x}$
$-\infty \downarrow$
x
This area is the same thing as this arrow.
Δy

swap formula

$$\Delta x = \frac{x_0 \cdot y_0}{y + \Delta y} - \frac{x_0 \cdot y_0}{y}$$

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The rest of this lecture series will assume you have a firm grip on these concepts.

swap formula

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

effective rate

$$\frac{\Delta x}{\Delta y} = -\frac{x}{y + \Delta y}$$

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Homework, 9^{th} May

Draw a precise price curve & bonding curve pair, depicting the same token swap on both.

- Label all axes (do as I say, not as I do).
- Annotate the price curve with the initial and final marginal prices.
- Annotate the price curve with the initial and final token balance (for whatever dimension you choose to keep).
- Annotate the bonding curve with the initial and final token balances.
- Annotate the bonding curve with arrows to indicate the direction of the swap.
- Include an appropriate caption using the following template:
  - “These graphs depict a token swap performed for a system initially comprising [x] TKNX and [y] TKNY, where the TKNX balance is [increased or decreased] by [Δx] tokens and the TKNY balance is [increased or decreased] by [Δy] TKNY tokens. The initial marginal rate is [$\partial y / \partial x$ or $\partial x / \partial y$], the final marginal rate is [$\partial y / \partial x$ or $\partial x / \partial y$], and the effective rate of exchange for the swap is [$\Delta y / \Delta x$ or $\Delta x / \Delta y$].”
- Use whatever software you like (e.g. Excel, MatPlotLib, Desmos), and send the working file (e.g. .xlsx, .py, link to Desmos) to me (mark@bancor.network) with “TE Academy Lecture 1 Homework” in the subject.

MB_Richardson

MBRichardson87

mrichardson87

mark@bancor.network

DeFi's Concentrated Liquidity From Scratch

Lecture 1 of 5

Mark. B. Richardson, Ph.D.
Project Lead, Bancor