DeFi's Concentrated Liquidity From Scratch

Lecture 2 of 5

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<Homework Discussion>

<Homework Discussion>

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$$

$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$

$$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$$

$$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$$

$$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$$

<Homework Discussion>

We discussed these in the last lecture

We did not discuss this one, but you should be able to derive it.

$$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)} \qquad \Delta x = -\frac{\Delta y \cdot x}{y + \Delta y} \qquad \Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$$

$$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)} \qquad \Delta y = -\frac{\Delta x \cdot y}{x + \Delta x} \qquad \Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$$

<Homework Discussion>

Inputs:	y, x_0, y_0	x, y	x, x_0, y_0
Δy	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
Δx	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

<Homework Discussion>

Inputs:	y, x_0, y_0	x, y	x, x_0, y_0
Δy	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
Δx	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

In theory:

$$x_0, y_0, x, y, \Delta x, \Delta y \in \mathbb{R}$$

In practice:

$$x_0, y_0, x, y, \Delta x, \Delta y \in \mathbb{Q}$$

<Homework Discussion>

Inputs:	y, x_0, y_0	x, y	x, x_0, y_0
Δy	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
Δx	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

Do these equations always agree?

In practice:

$$x_0, y_0, x, y, \Delta x, \Delta y \in \mathbb{Q}$$

<Homework Discussion>

Inputs:	y, x_0, y_0	x, y	x, x_0, y_0
Δy	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
Δx	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

<Homework Discussion>

inputs	'y_and_invariant'	'x_and_y'	'x_and_invariant'
'Dy'	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
'Dx'	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

<Homework Discussion>

inputs	'y_and_invariant'	'x_and_y'	'x_and_invariant'
'Dy'			
'Dx'			

```
self.swap_functions = {
    ‘Dy_from_Dx' : {
                'x_and_y' : self.calculate_Dy_from_x_y_Dx,
        'x_and_invariant' : self.calculate_Dy_from_x_invariant_Dx,
        'y_and_invariant' : self.calculate_Dy_from_y_invariant_Dx,
    },
    ‘Dx_from_Dy' : {
                'x_and_y' : self.calculate_Dx_from_x_y_Dy,
        'x_and_invariant' : self.calculate_Dx_from_x_invariant_Dy,
        'y_and_invariant' : self.calculate_Dx_from_y_invariant_Dy,

    },
}
```

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```
create_static_trade_plot(
        token_pair = {'x' : 'FOO', 'y' : 'BAR'},
        x_0 = 1234,
        y_0 = 5678,
        trade_actions = [('Dx', +100),
                         ('Dy', +321),
                         ('Dx', -123),
                         ('Dy', -321),
                         ('Dx', +150),
                         ('Dy', +500),
                         ('Dx', -150),
                         ('Dy', -600)],
)
```

Input Information:

```
+--------------+--------------------------------------------------------------------------+
|    Parameter | Value                                                                    |
+--------------+--------------------------------------------------------------------------+
| x_coordinate | 1234.0000000000000000000000000000000000000000000000000000000000000000000 |
| y_coordinate | 5678.0000000000000000000000000000000000000000000000000000000000000000000 |
|    invariant | 7006652.0000000000000000000000000000000000000000000000000000000000000000 |
|           Dx | 100.00000000000000000000000000000000000000000000000000000000000000000000 |
+--------------+--------------------------------------------------------------------------+
```

Output Calculations:

```
+-----------------+-----------------------------------------------------------------------+
|        Function | Output                                                                |
+-----------------+-----------------------------------------------------------------------+
|         x_and_y | -425.6371814092953513863903935998678207397460937500000000000000000000 |
| x_and_invariant | -425.6371814092953513863903935998678207397460937500000000000000000000 |
| y_and_invariant | -425.6371814092953513863903935998678207397460937500000000000000000000 |
+-----------------+-----------------------------------------------------------------------+
```

No discrepancies detected.
Discrepancy check completed.

Input Information:

```
+--------------+---------------------------------------------------------------------------+
|    Parameter | Value                                                                     |
+--------------+---------------------------------------------------------------------------+
| x_coordinate | 1334.00000000000000000000000000000000000000000000000000000000000000000000 |
| y_coordinate | 5252.36281859070459177019074559211730957031250000000000000000000000000000 |
|    invariant | 7006652.00000000000000000000000000000000000000000000000000000000000000000 |
|           Dy | 321.000000000000000000000000000000000000000000000000000000000000000000000 |
+--------------+---------------------------------------------------------------------------+
```

Output Calculations:

```
+-----------------+------------------------------------------------------------------------+
|        Function | Output                                                                 |
+-----------------+------------------------------------------------------------------------+
|         x_and_y | -76.8322490277565179894736502319574356079101562500000000000000000000000 |
| x_and_invariant | -76.8322490277565179894736502319574356079101562500000000000000000000000 |
| y_and_invariant | -76.8322490277565179894736502319574356079101562500000000000000000000000 |
+-----------------+------------------------------------------------------------------------+
No discrepancies detected.
Discrepancy check completed.
```

Input Information:

Parameter	Value
x_coordinate	1257.16775097224353885394521057605743408203125000000000000000000000000000
y_coordinate	5573.36281859070459177019074559211730957031250000000000000000000000000000
invariant	7006652.0000000000000000000000000000000000000000000000000000000000000000
Dx	-123.0000000000000000000000000000000000000000000000000000000000000000000

Output Calculations:

Function	Output
x_and_y	604.428777047314724768511950969696044921875000000000000000000000000000
x_and_invariant	604.428777047314838455349672585725784301757812500000000000000000000000
y_and_invariant	604.428777047314838455349672585725784301757812500000000000000000000000

Selected most positive value: 604.428777047314838
Discrepancy check completed.

Input Information:

Parameter	Value
x_coordinate	1078.31858862879994376271497458219528198242187500000000000000000000000
y_coordinate	6497.75685394584979803767055273056030273437500000000000000000000000000
invariant	7006652.00000000000000000000000000000000000000000000000000000000000000
Dy	-600.000000000000000000000000000000000000000000000000000000000000000000

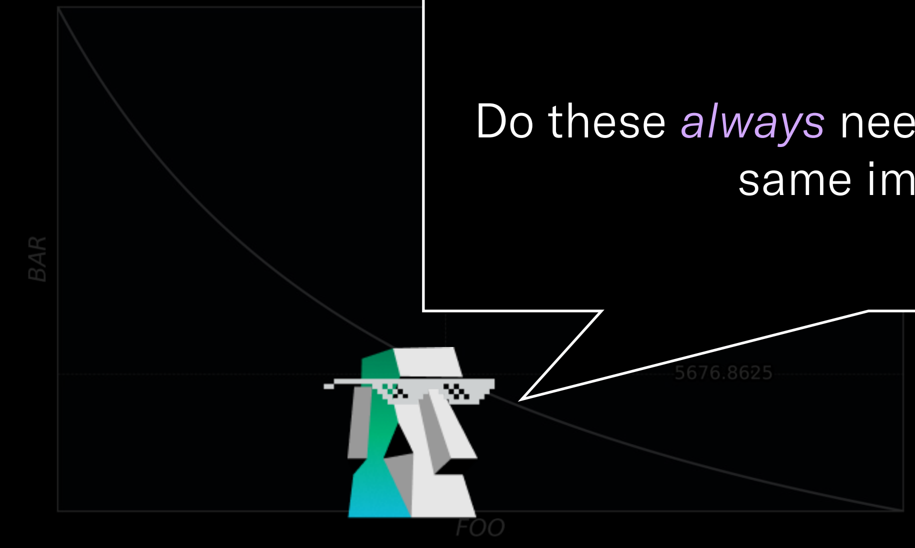
Output Calculations:

Function	Output
x_and_y	109.701225262349609579359821509569883346557617187500000000000000000000
x_and_invariant	109.701225262349595368505106307566165924072265625000000000000000000000
y_and_invariant	109.701225262349595368505106307566165924072265625000000000000000000000

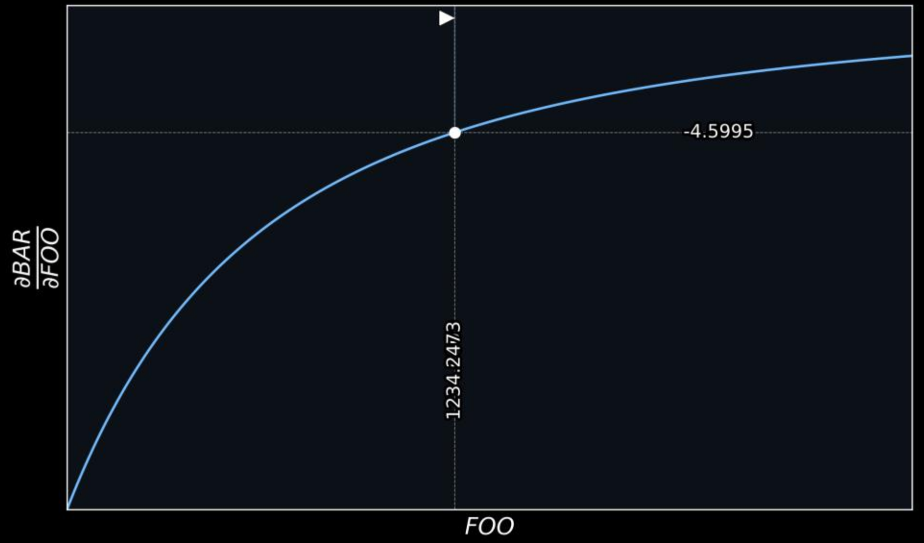
Selected most positive value: 109.701225262349610
Discrepancy check completed.

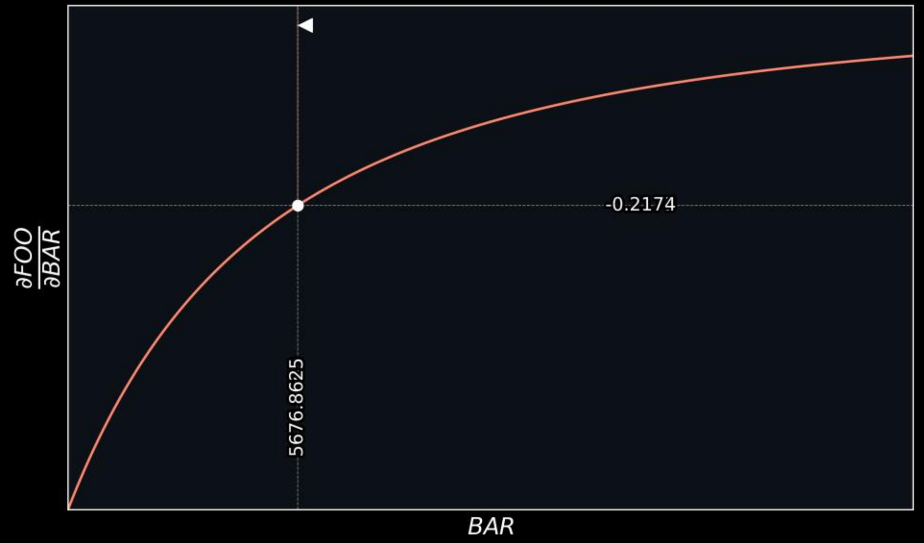
[We will return to this point in Lecture 4]

Do these *always* need to refer to one and the same implicit curve?



arameter	Value
ordinate	1234.00000000
ordinate	1234.24726232
O (ΔFOO)	0.24726232
BAR/∂FOO	-4.60129660
BAR/∂FOO	-4.59945318
BAR/ΔFOO	-4.60037480
Initial BAR coordinate	5678.00000000
Final BAR coordinate	5676.86250067
Change in BAR (ΔBAR)	-1.13749933
Initial marginal rate ∂FOO/∂BAR	-0.21733005
Final marginal rate ∂FOO/∂BAR	-0.21741715
Effective rate ΔFOO/ΔBAR	-0.21737359

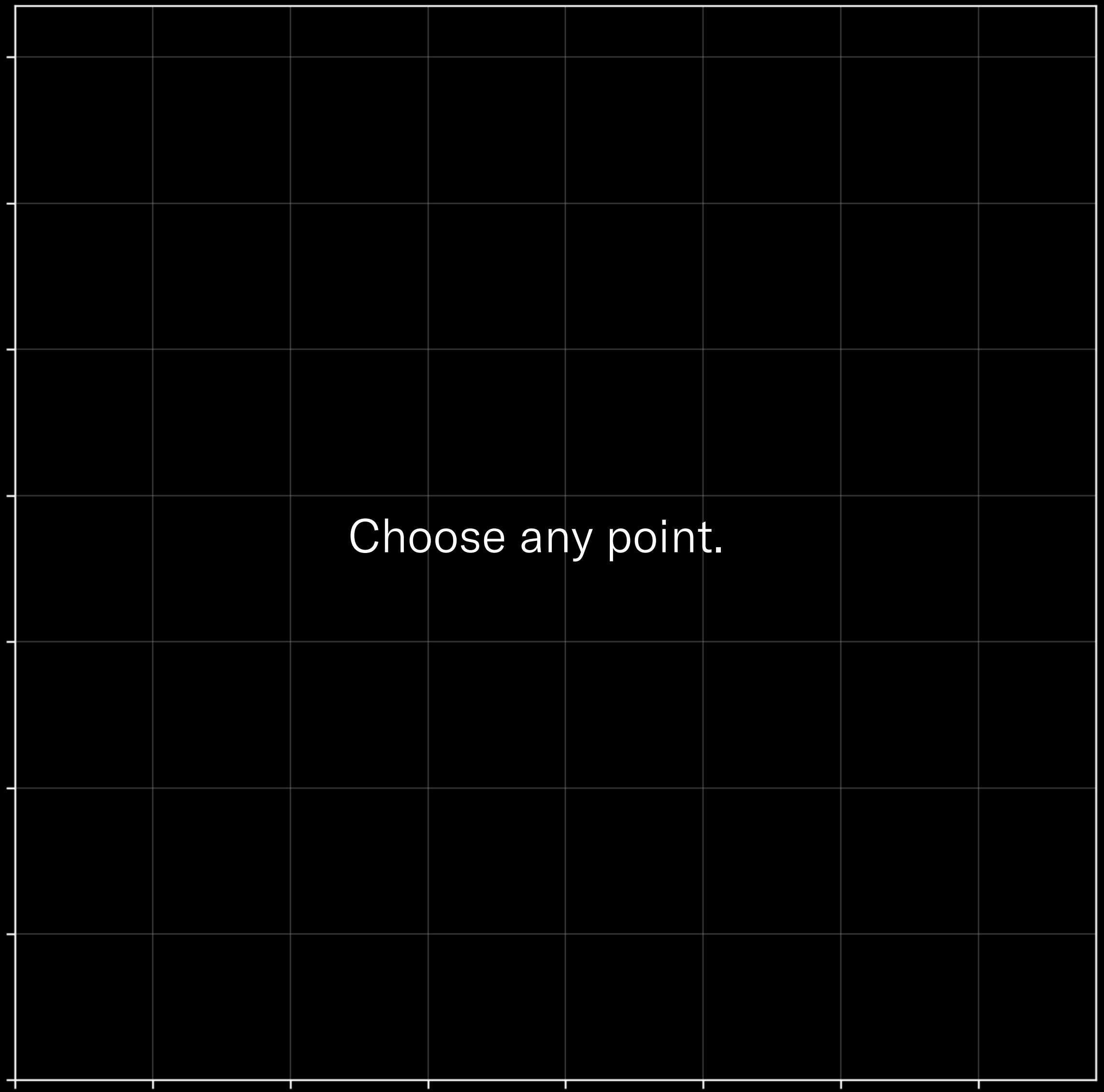




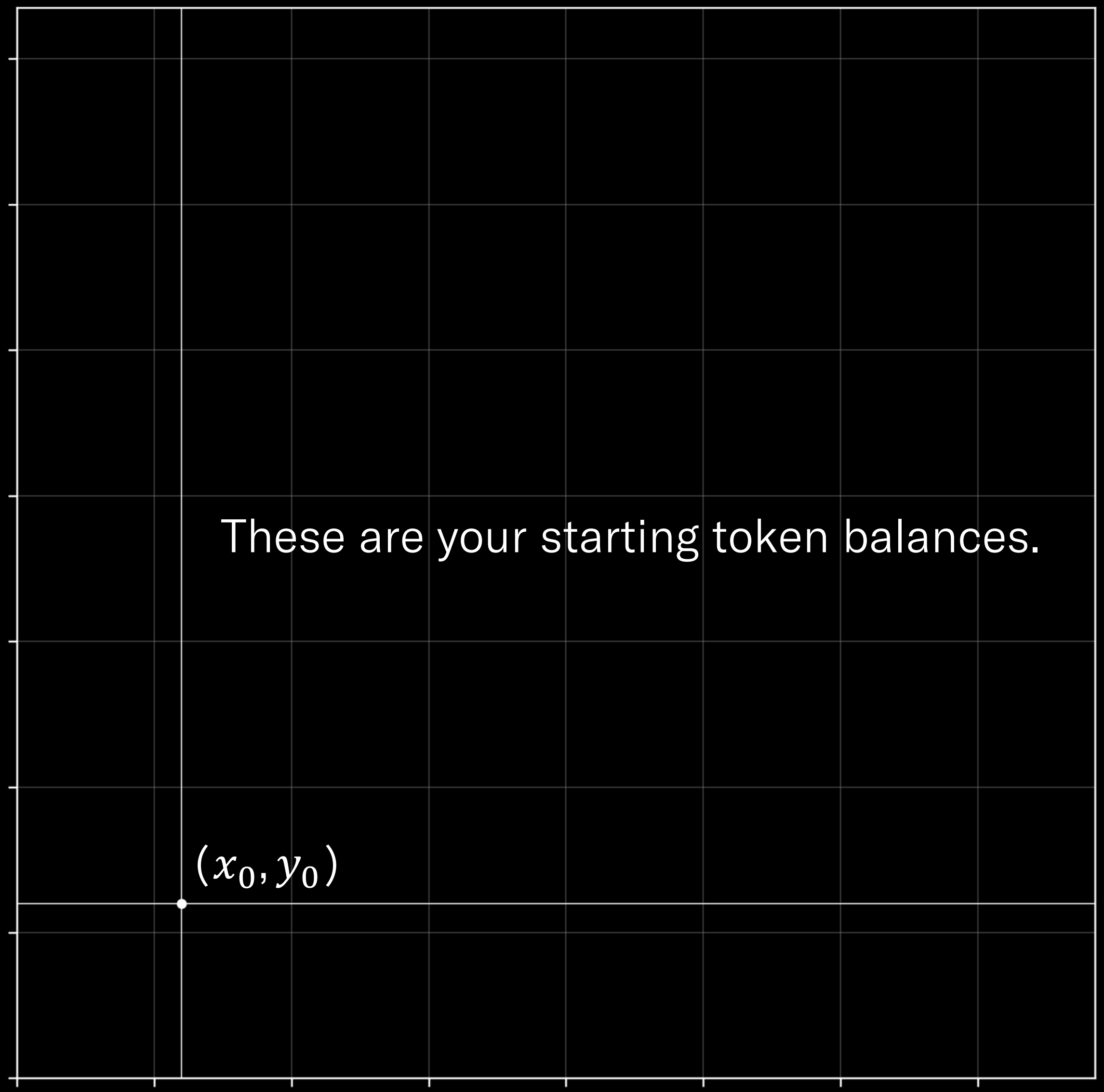
</Homework Discussion>

<Lecture 2>

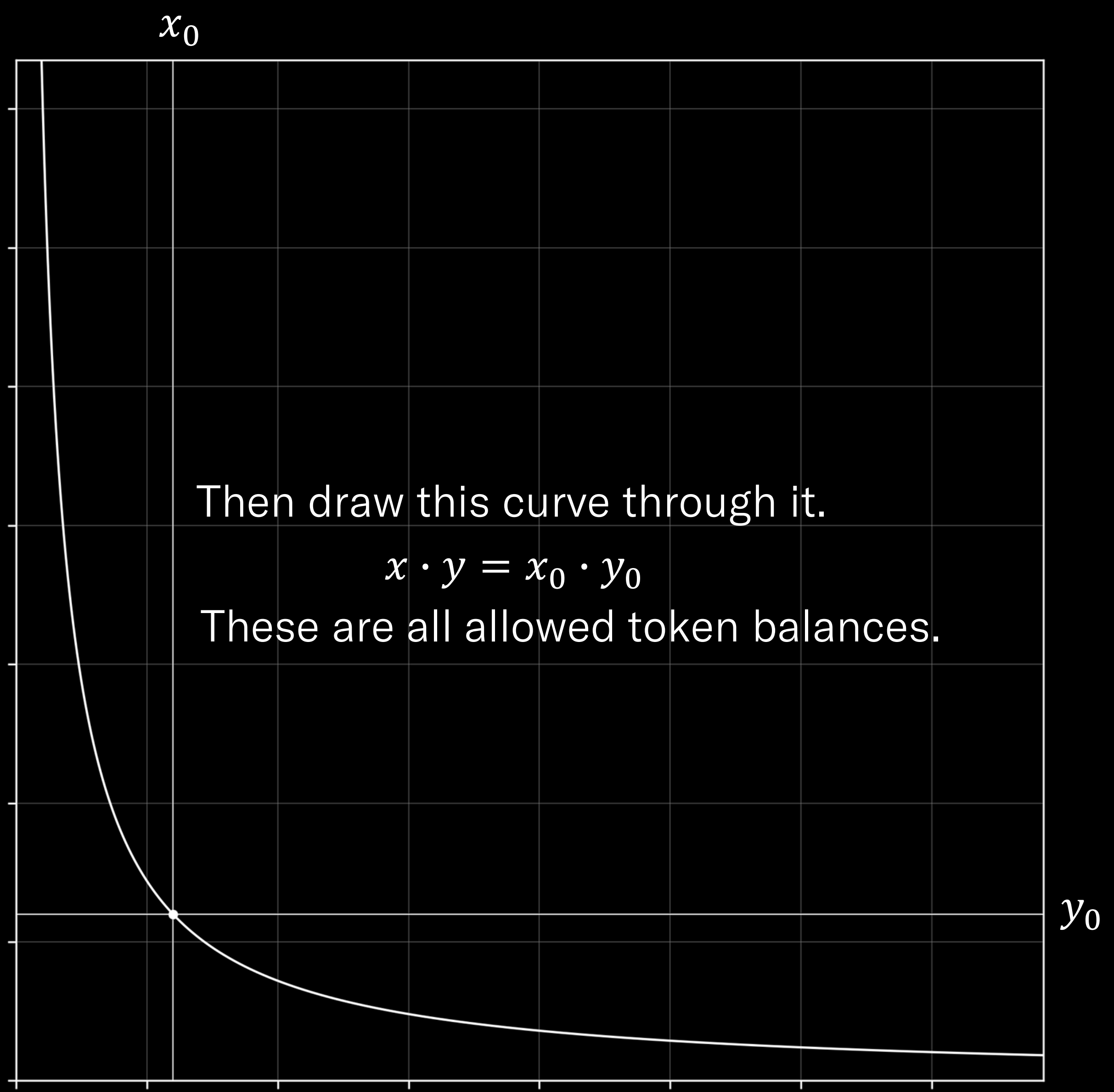
Recall from the first lecture:



Recall from the first lecture:

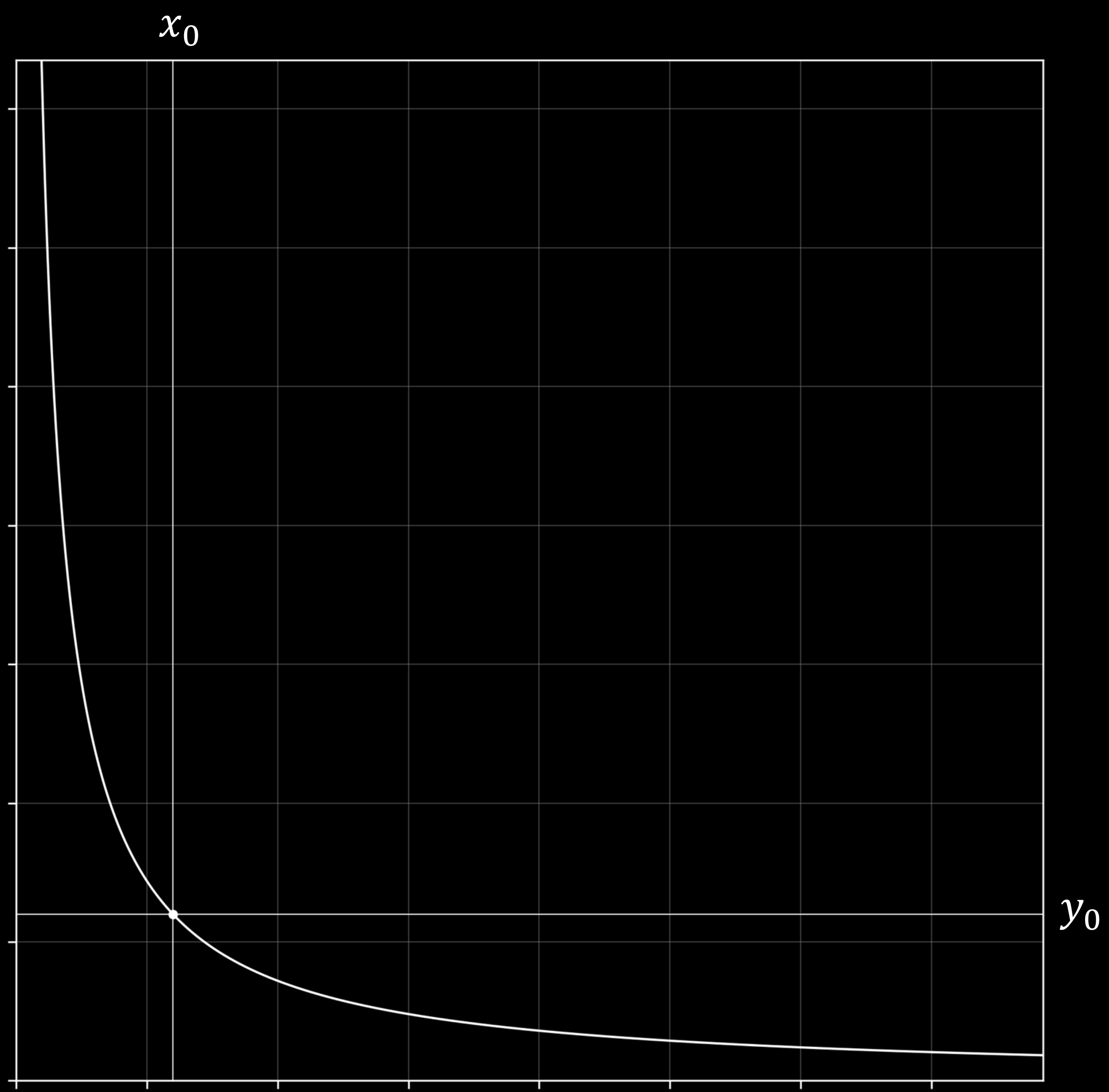


Recall from the first lecture:



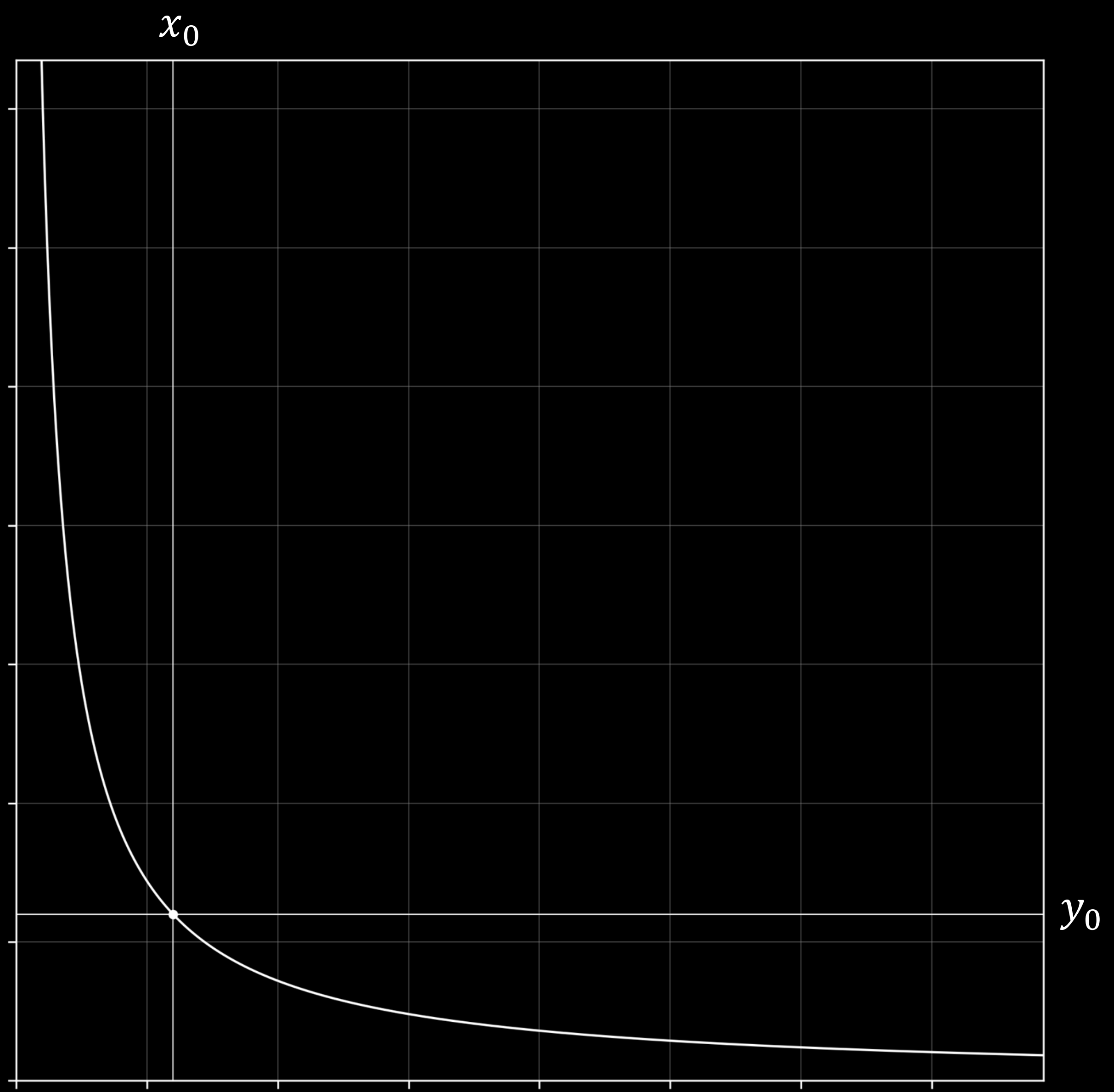
Recall from the first lecture:

$$x \cdot y = x_0 \cdot y_0$$



Recall from the first lecture:

$$x = \frac{x_0 \cdot y_0}{y}$$

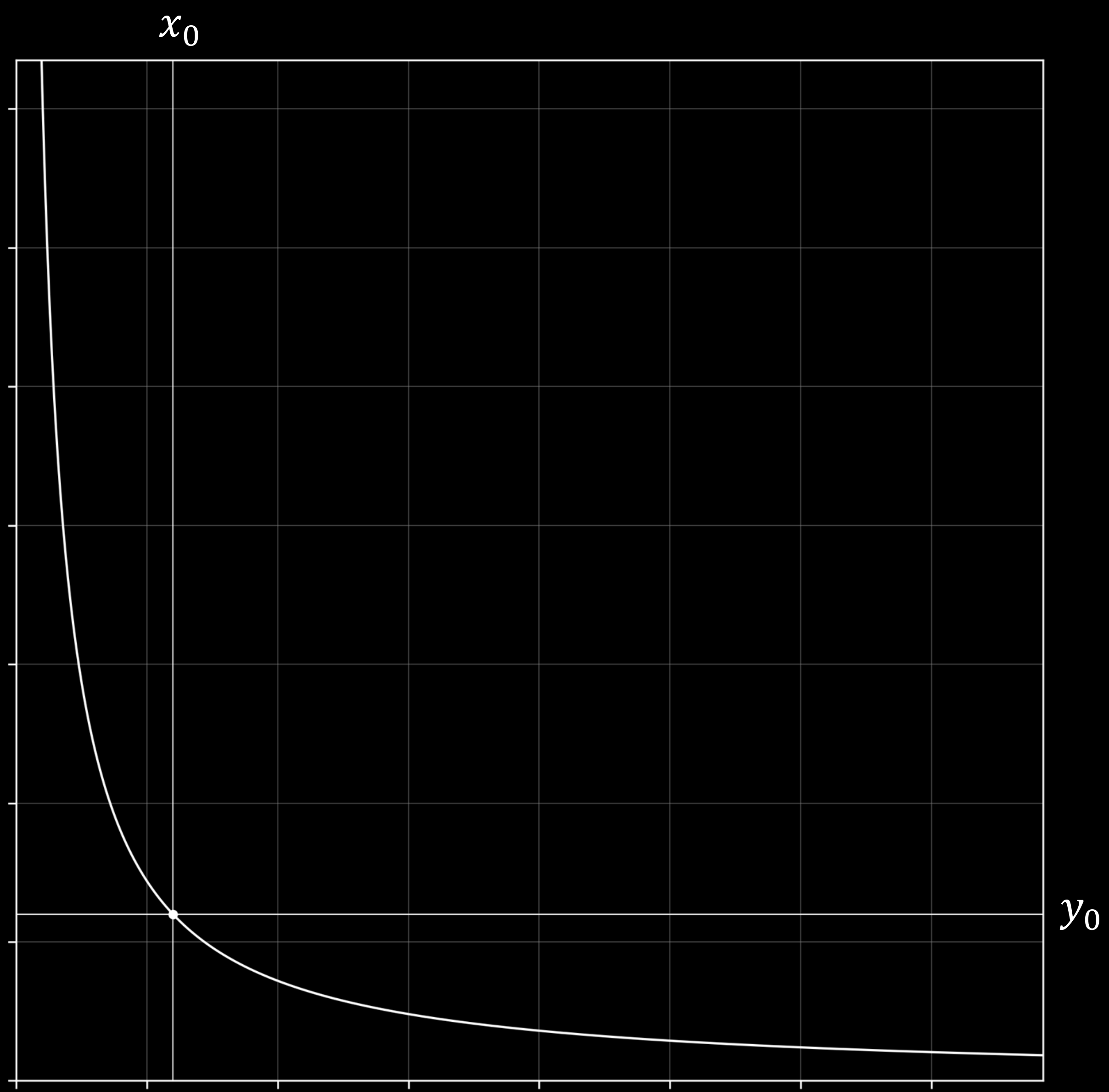


Recall from the first lecture:

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

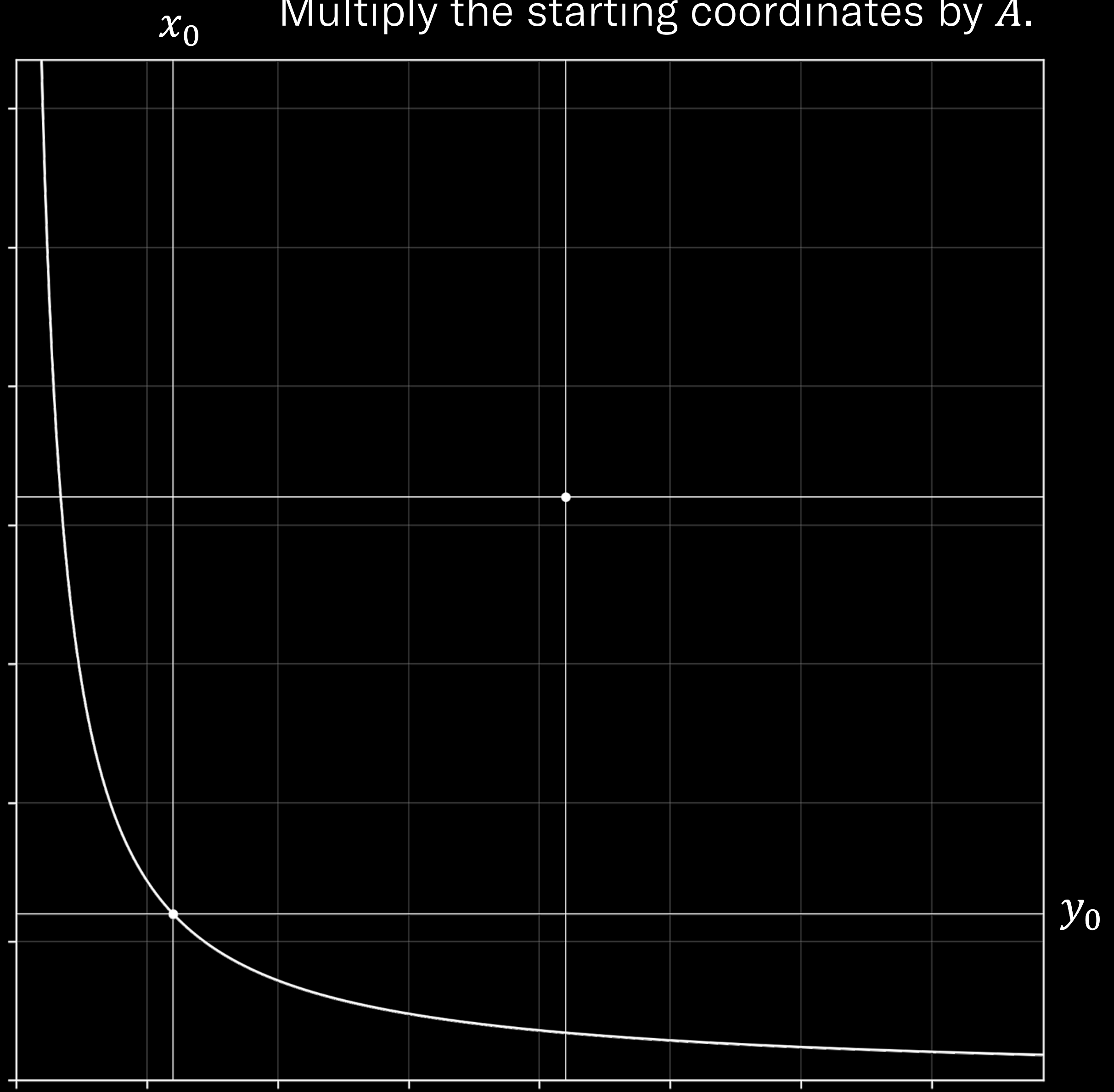


Multiply the starting coordinates by A.

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

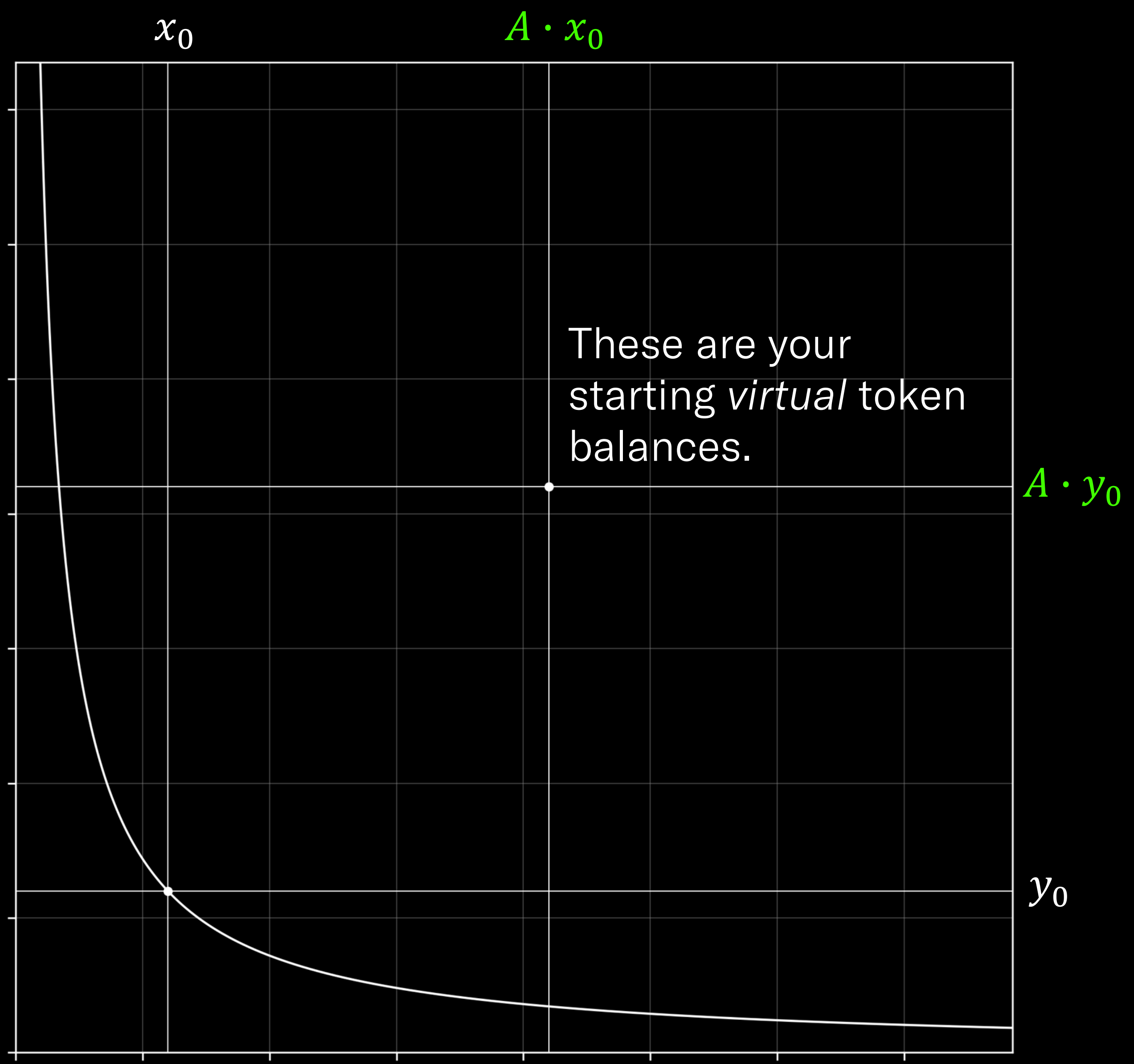
$$y = \frac{x_0 \cdot y_0}{x}$$



$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

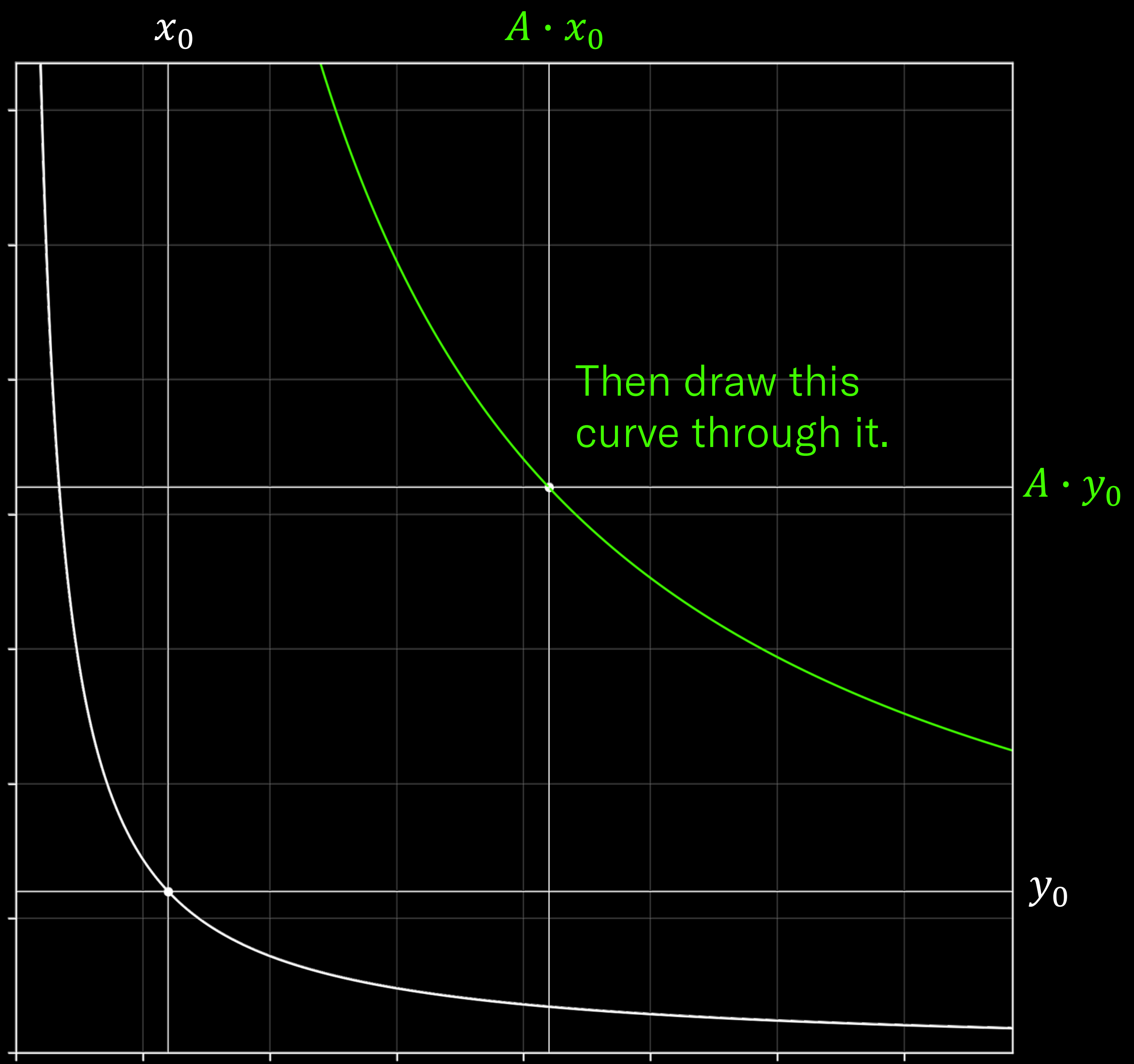
$$y = \frac{x_0 \cdot y_0}{x}$$



$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

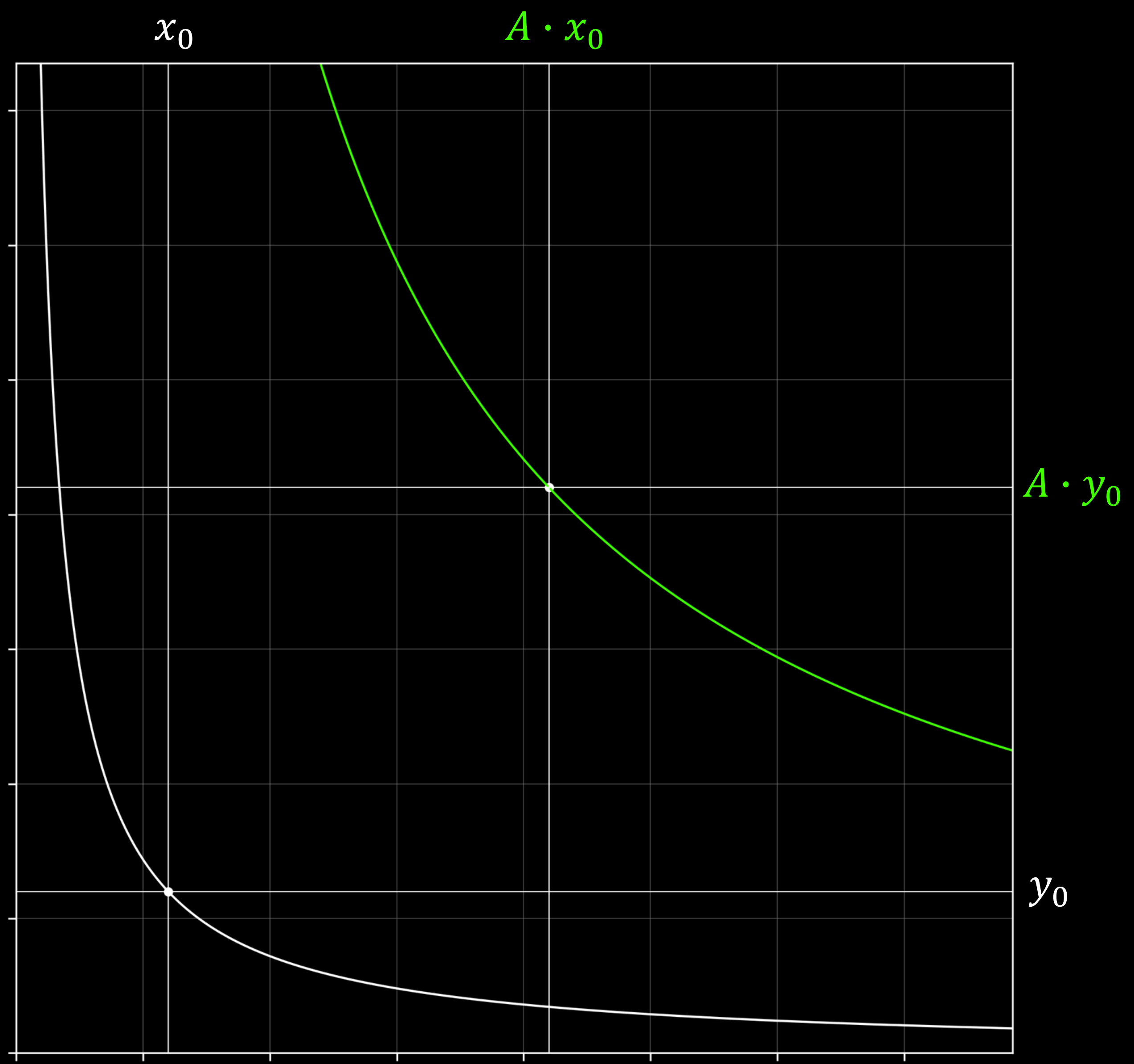


$$x \cdot y = A \cdot x_0 \cdot A \cdot y_0$$

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

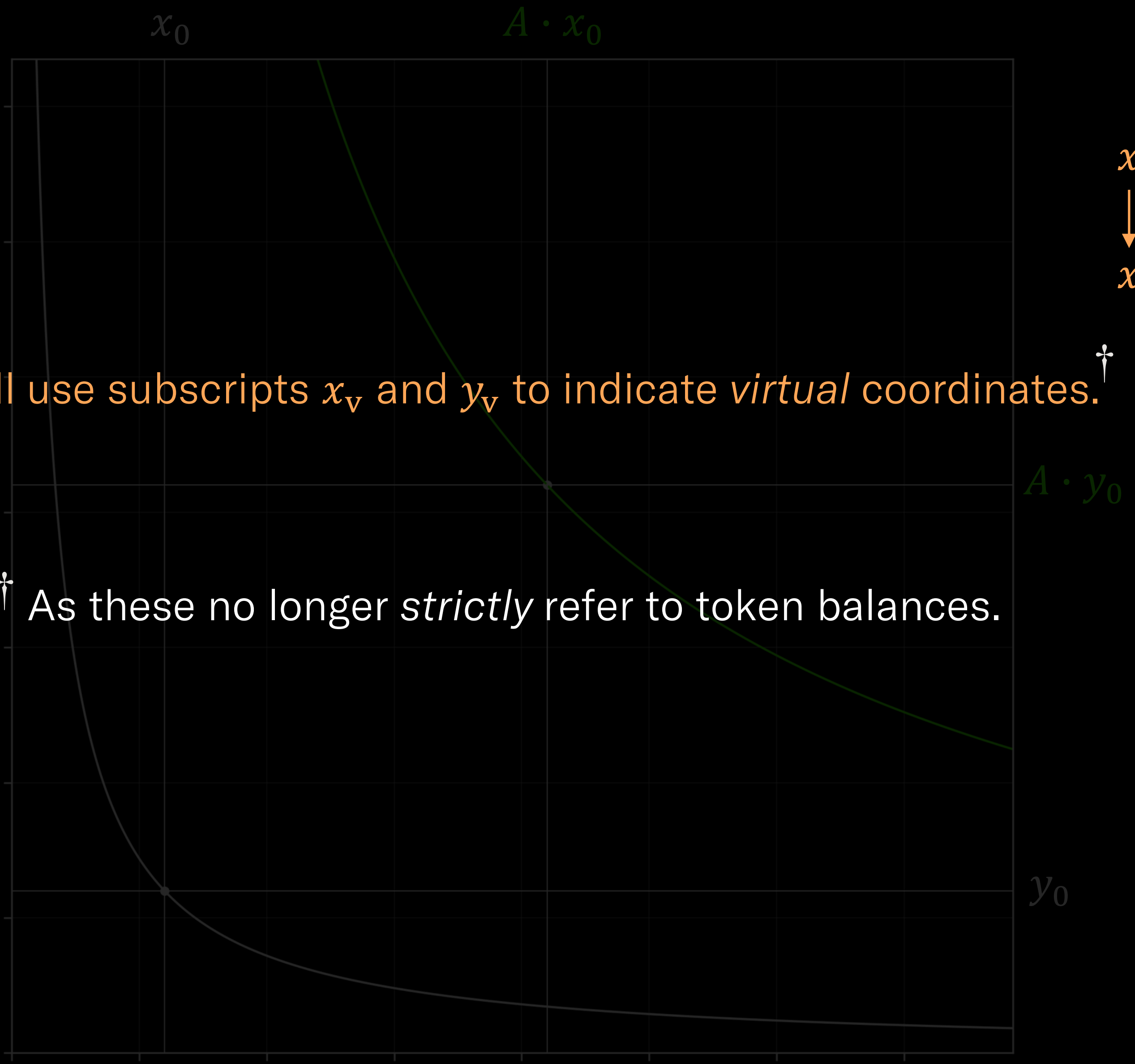


$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$



$$\underset{\downarrow}{x} \quad \underset{\downarrow}{y}$$

$$x_{\mathrm{v}} \cdot y_{\mathrm{v}} = A^2 \cdot x_0 \cdot y_0$$

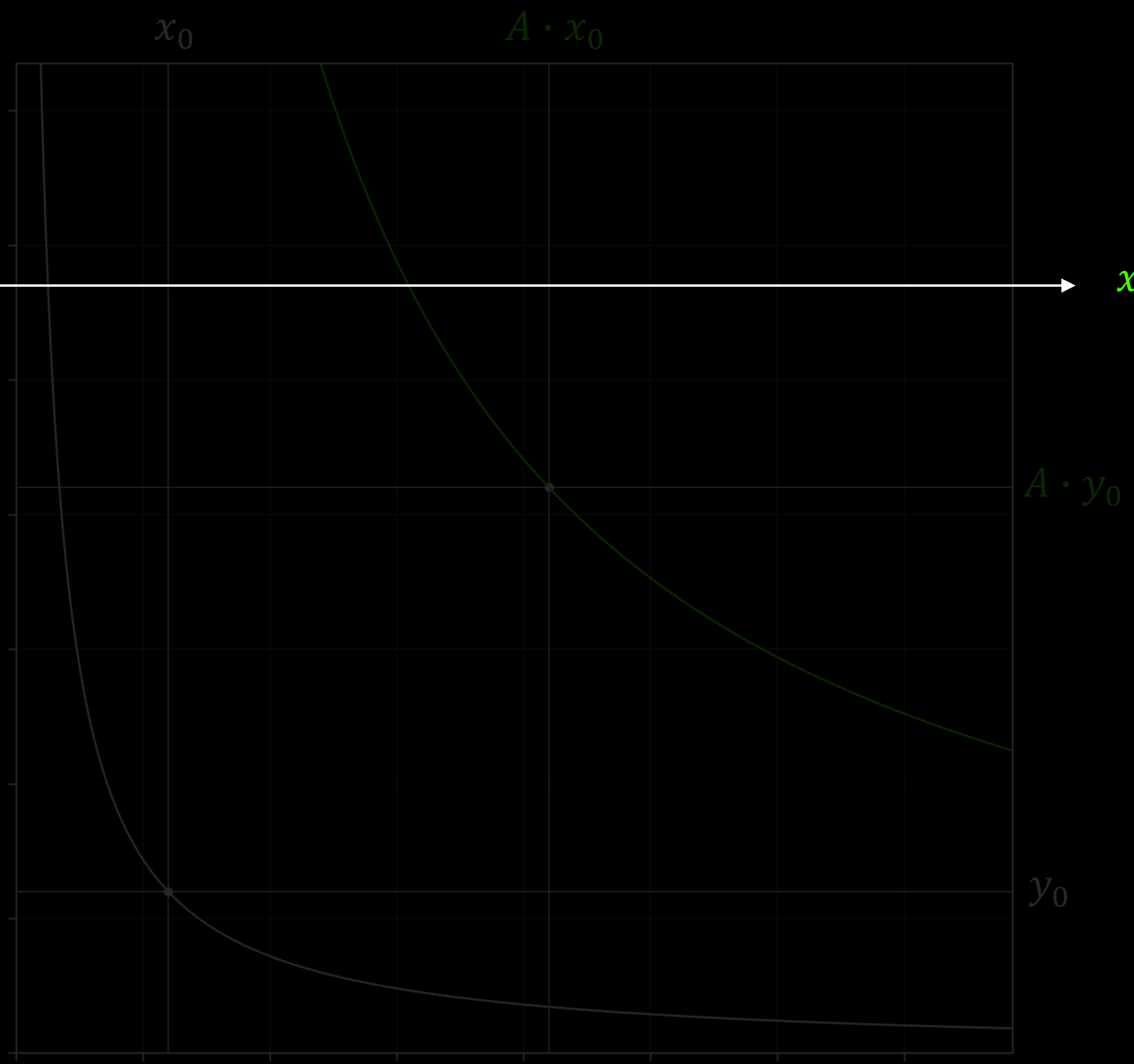
I will use subscripts x_{v} and y_{v} to indicate *virtual* coordinates.†

† As these no longer *strictly* refer to token balances.

$x \cdot y = x_0 \cdot y_0$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

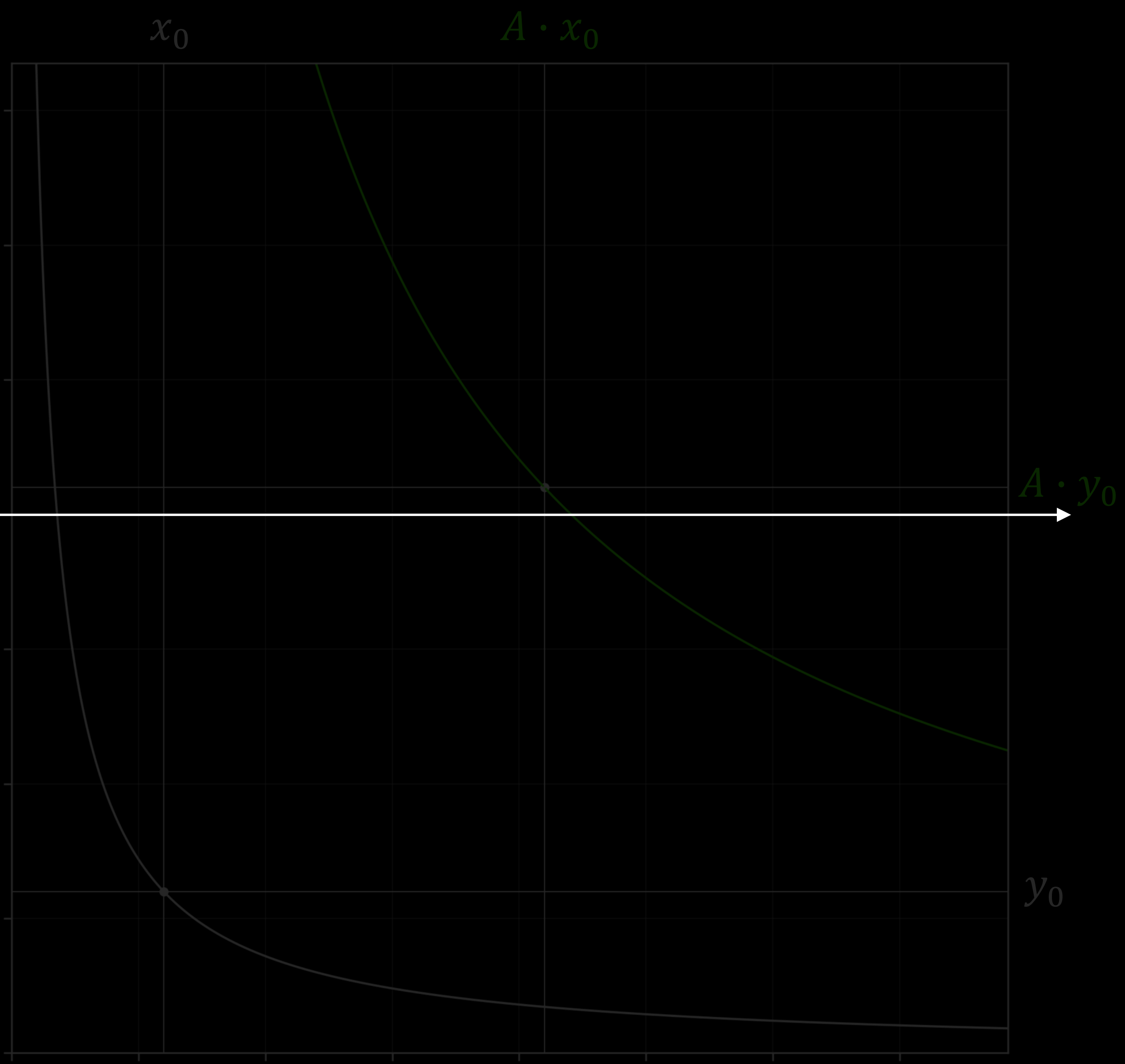


$x_v \cdot y_v = A^2 \cdot x_0 \cdot y_0$

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$

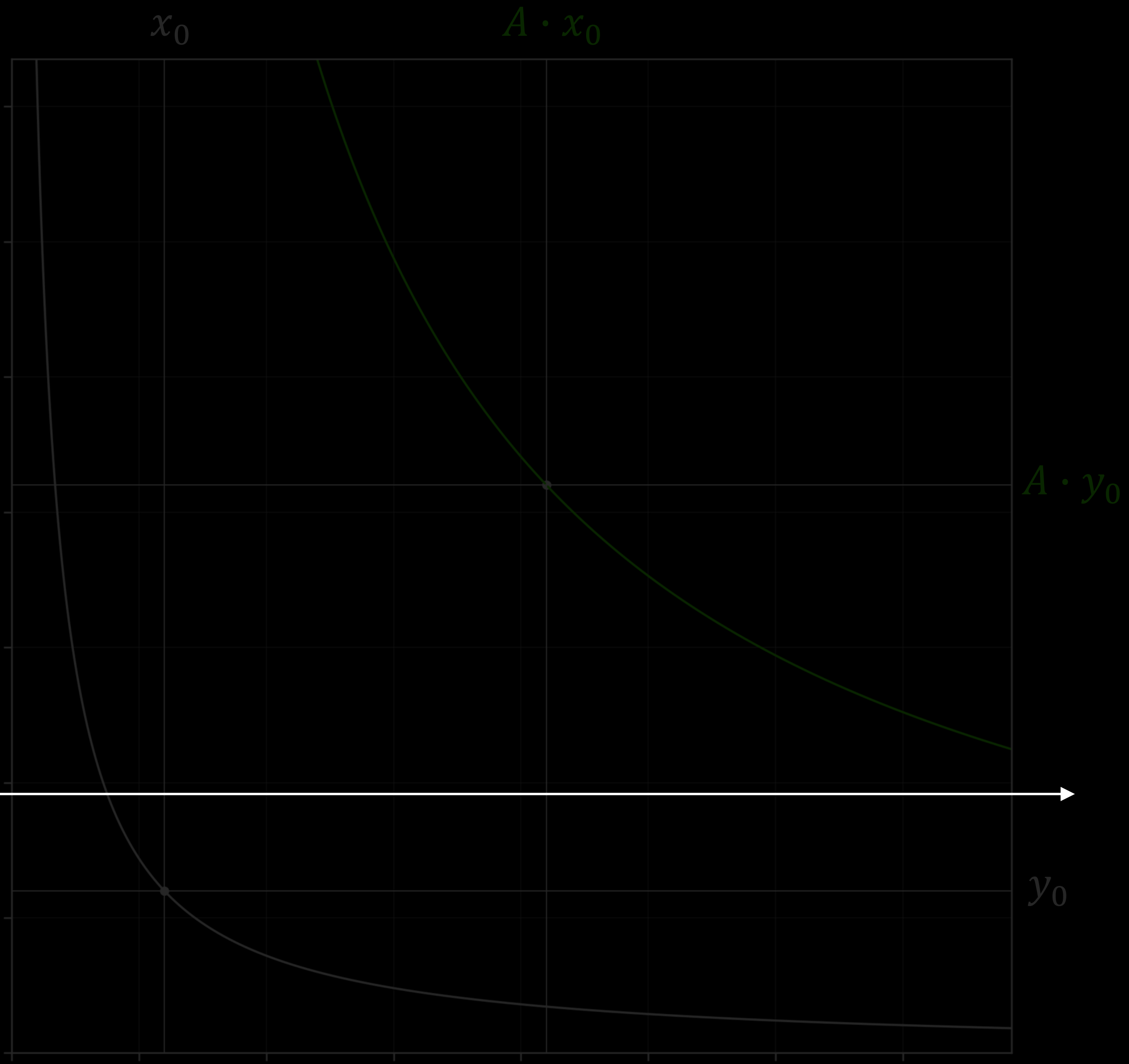


$$x_v = \frac{A^2 \cdot x_0 \cdot y_0}{y_v}$$

$$x \cdot y = x_0 \cdot y_0$$

$$x = \frac{x_0 \cdot y_0}{y}$$

$$y = \frac{x_0 \cdot y_0}{x}$$



$$y_\mathrm{v} = \frac{A^2 \cdot x_0 \cdot y_0}{x_\mathrm{v}}$$

$$\boxed{x \cdot y = x_0 \cdot y_0}$$

Inputs:	y, x_0, y_0	x, y	x, x_0, y_0
Δy	$\Delta x = -\frac{\Delta y \cdot x_0 \cdot y_0}{y \cdot (y + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x^2}{\Delta y \cdot x + x_0 \cdot y_0}$
Δx	$\Delta y = -\frac{\Delta x \cdot y^2}{\Delta x \cdot y + x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y}{x + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot x_0 \cdot y_0}{x \cdot (x + \Delta x)}$

$$\boxed{x_\text{v} \cdot y_\text{v} = A^2 \cdot x_0 \cdot y_0}$$

Inputs:	y_v, A, x_0, y_0	x_v, y_v	x_v, A, x_0, y_0
Δy	$\Delta x = -\dfrac{\Delta y \cdot A^2 \cdot x_0 \cdot y_0}{y_\text{v} \cdot (y_\text{v} + \Delta y)}$	$\Delta x = -\dfrac{\Delta y \cdot x_\text{v}}{y_\text{v} + \Delta y}$	$\Delta x = -\dfrac{\Delta y \cdot x_\text{v}^2}{\Delta y \cdot x_\text{v} + A^2 \cdot x_0 \cdot y_0}$
Δx	$\Delta y = -\dfrac{\Delta x \cdot y_\text{v}^2}{\Delta x \cdot y_\text{v} + A^2 \cdot x_0 \cdot y_0}$	$\Delta y = -\dfrac{\Delta x \cdot y_\text{v}}{x_\text{v} + \Delta x}$	$\Delta y = -\dfrac{\Delta x \cdot A^2 \cdot x_0 \cdot y_0}{x_\text{v} \cdot (x_\text{v} + \Delta x)}$

$$\boxed{x_v \cdot y_v = A^2 \cdot x_0 \cdot y_0}$$

The only difference here is that the new invariant is larger by a factor of A^2.

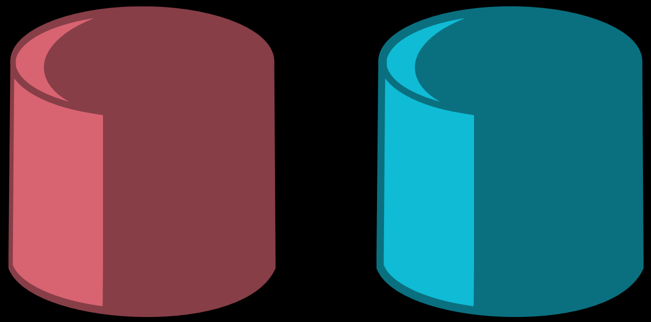
Inputs:	y_v, A, x_0, y_0	x_v, y_v	x_v, A, x_0, y_0
Δy	$\Delta x = -\dfrac{\Delta y \cdot A^2 \cdot x_0 \cdot y_0}{y_v \cdot (y_v + \Delta y)}$	$\Delta x = -\dfrac{\Delta y \cdot x_v}{y_v + \Delta y}$	$\Delta x = -\dfrac{\Delta y \cdot x_v^2}{\Delta y \cdot x_v + A^2 \cdot x_0 \cdot y_0}$
Δx	$\Delta y = -\dfrac{\Delta x \cdot y_v^2}{\Delta x \cdot y_v + A^2 \cdot x_0 \cdot y_0}$	$\Delta y = -\dfrac{\Delta x \cdot y_v}{x_v + \Delta x}$	$\Delta y = -\dfrac{\Delta x \cdot A^2 \cdot x_0 \cdot y_0}{x_v \cdot (x_v + \Delta x)}$

$$\boxed{x_\mathrm{v} \cdot y_\mathrm{v} = A^2 \cdot x_0 \cdot y_0}$$

Inputs:	$y_\mathrm{v}, A, x_0, y_0$	$x_\mathrm{v}, y_\mathrm{v}$	$x_\mathrm{v}, A, x_0, y_0$
Δy	$\Delta x = -\frac{\Delta y \cdot A^2 \cdot x_0 \cdot y_0}{y_\mathrm{v} \cdot (y_\mathrm{v} + \Delta y)}$	$\Delta x = -\frac{\Delta y \cdot x_\mathrm{v}}{y_\mathrm{v} + \Delta y}$	$\Delta x = -\frac{\Delta y \cdot x_\mathrm{v}^2}{\Delta y \cdot x_\mathrm{v} + A^2 \cdot x_0 \cdot y_0}$
Δx	$\Delta y = -\frac{\Delta x \cdot y_\mathrm{v}^2}{\Delta x \cdot y_\mathrm{v} + A^2 \cdot x_0 \cdot y_0}$	$\Delta y = -\frac{\Delta x \cdot y_\mathrm{v}}{x_\mathrm{v} + \Delta x}$	$\Delta y = -\frac{\Delta x \cdot A^2 \cdot x_0 \cdot y_0}{x_\mathrm{v} \cdot (x_\mathrm{v} + \Delta x)}$

What does it mean?

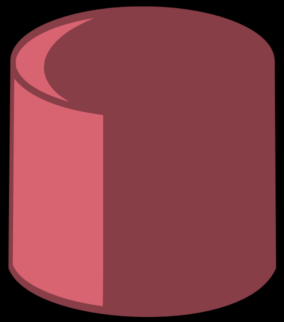
Imagine there are two smart contract systems (i.e. “liquidity pools”), using the same pricing algorithms we are studying here.



Both contain the tokens FOO, and BAR.

One of them has 100 tokens in each balance.

FOO	BAR
100	100



And the other has 10 × more of both.

FOO	BAR
1,000	1,000

One of them has 100 tokens in each balance.

And the other has 10 × more of both.

FOO	BAR
100	100

$$\frac{\partial \mathrm{FOO}}{\partial \mathrm{BAR}} = -\frac{100}{100} = -1$$

FOO	BAR
1,000	1,000

$$\frac{\partial \mathrm{FOO}}{\partial \mathrm{BAR}} = -\frac{1{,}000}{1{,}000} = -1$$

Both systems are quoting the same marginal rate.

But - consider the rate for a non-zero swap amount.

$$\Delta \mathrm{FOO} = +25$$

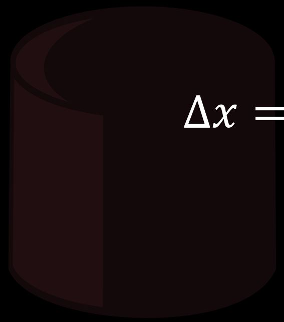
One of them has 100 tokens in each balance.

And the other has 10 × more of both.

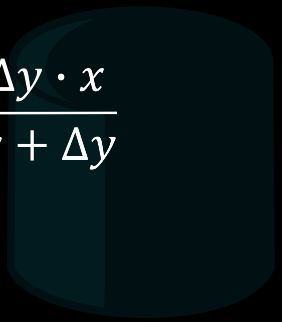
$$\Delta \mathrm{FOO} = +25$$

FOO	BAR
100	100

FOO	BAR
1,000	1,000



$$\Delta x = -\frac{\Delta y \cdot x}{y + \Delta y}$$



One of them has 100 tokens in each balance.

FOO	BAR
100	100

And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta\mathrm{FOO} = +25$$

$$\Delta\mathrm{BAR} = -\frac{\Delta\mathrm{FOO} \cdot \mathrm{BAR}}{\mathrm{FOO} + \Delta\mathrm{FOO}}$$

One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta\mathrm{BAR} = -\frac{\Delta\mathrm{FOO} \cdot \mathrm{BAR}}{\mathrm{FOO} + \Delta\mathrm{FOO}}$$

$$\Delta\mathrm{FOO} = +25$$

And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta\mathrm{BAR} = -\frac{\Delta\mathrm{FOO} \cdot \mathrm{BAR}}{\mathrm{FOO} + \Delta\mathrm{FOO}}$$

One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta \mathrm{BAR} = -\frac{25 \cdot \mathrm{BAR}}{\mathrm{FOO} + 25}$$

And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta \mathrm{BAR} = -\frac{25 \cdot \mathrm{BAR}}{\mathrm{FOO} + 25}$$

One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta\text{BAR} = -\frac{25 \cdot 100}{100 + 25}$$

And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta\text{BAR} = -\frac{25 \cdot 1{,}000}{1{,}000 + 25}$$

One of them has 100 tokens in each balance.

FOO	BAR
100	100

$$\Delta\text{BAR} = -\frac{2{,}500}{125}$$

And the other has 10 × more of both.

FOO	BAR
1,000	1,000

$$\Delta\text{BAR} = -\frac{25{,}000}{1{,}025}$$

One of them has 100 tokens in each balance.

FOO	BAR
100	100

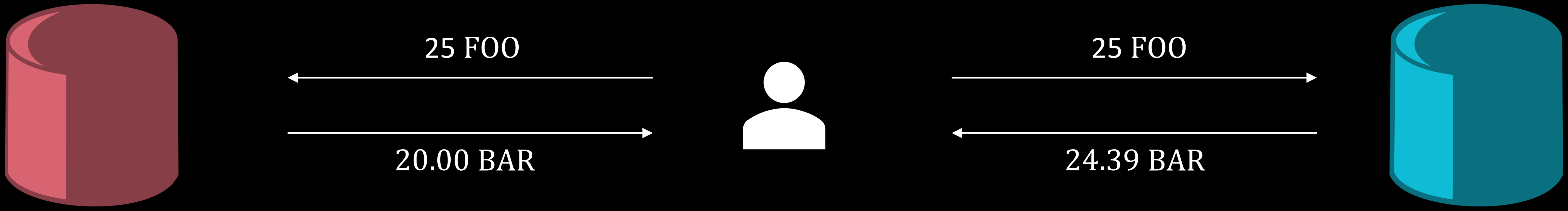
$$\Delta\mathrm{BAR} = -\frac{2{,}500}{125} = -20$$

And the other has 10 × more of both.

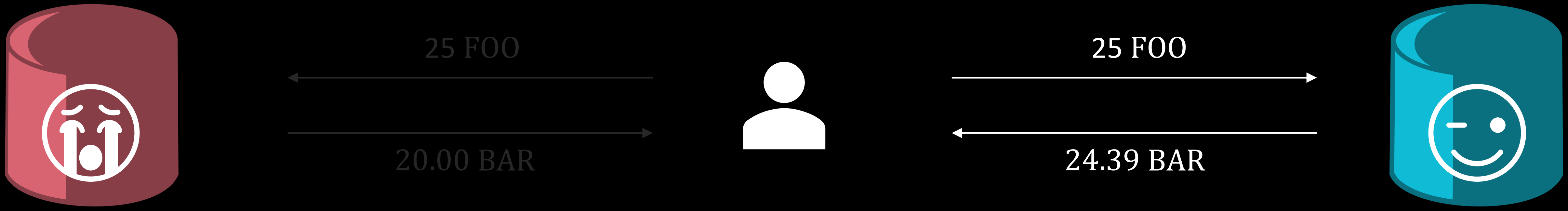
FOO	BAR
1,000	1,000

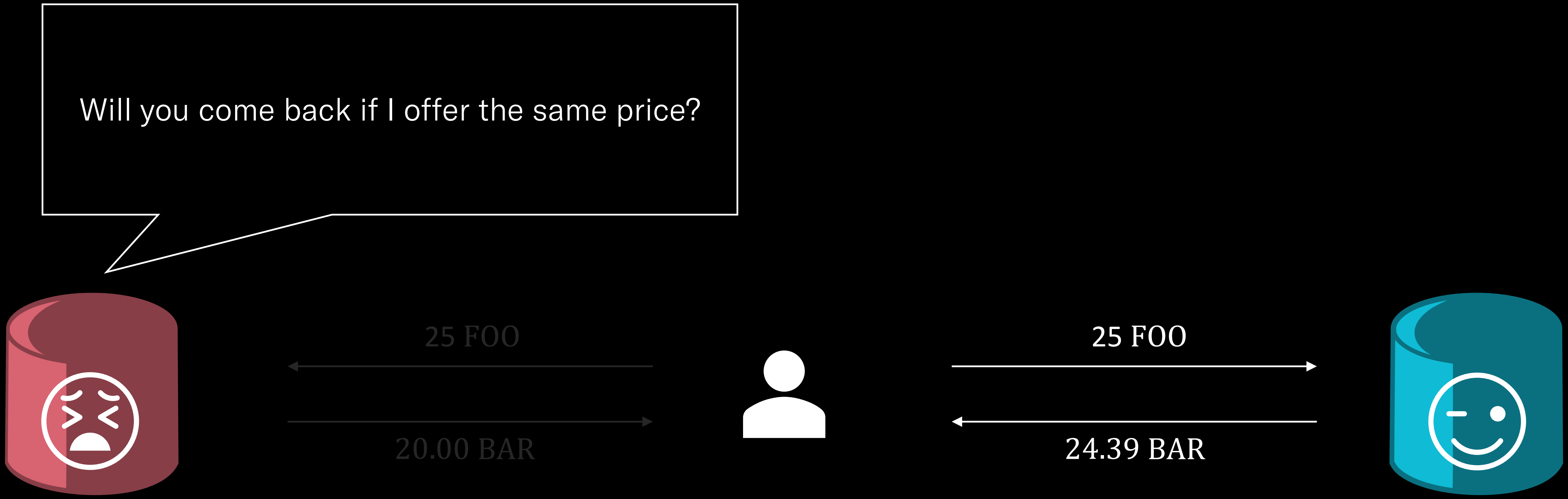
$$\Delta\mathrm{BAR} = -\frac{25{,}000}{1{,}025} = -24.39$$

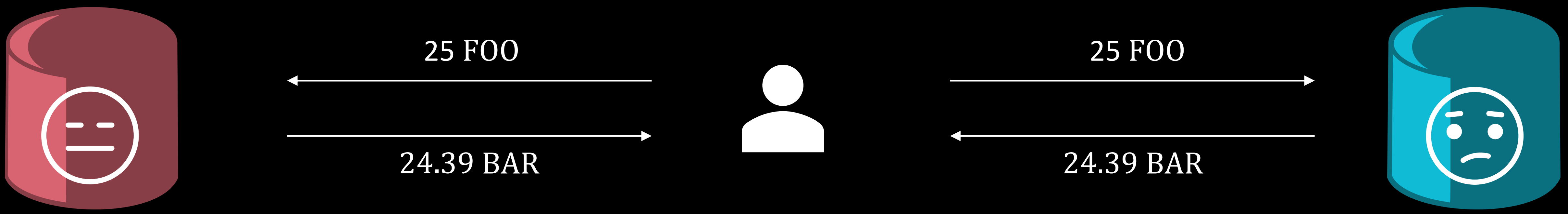
Which offer is better for the consumer?



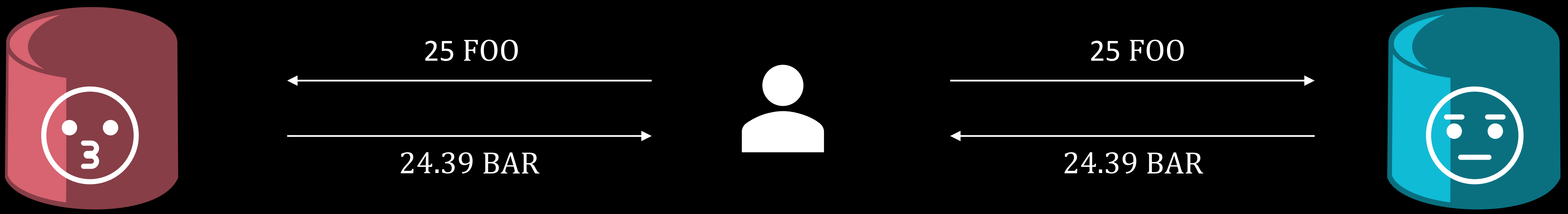
Which offer is better for the consumer?

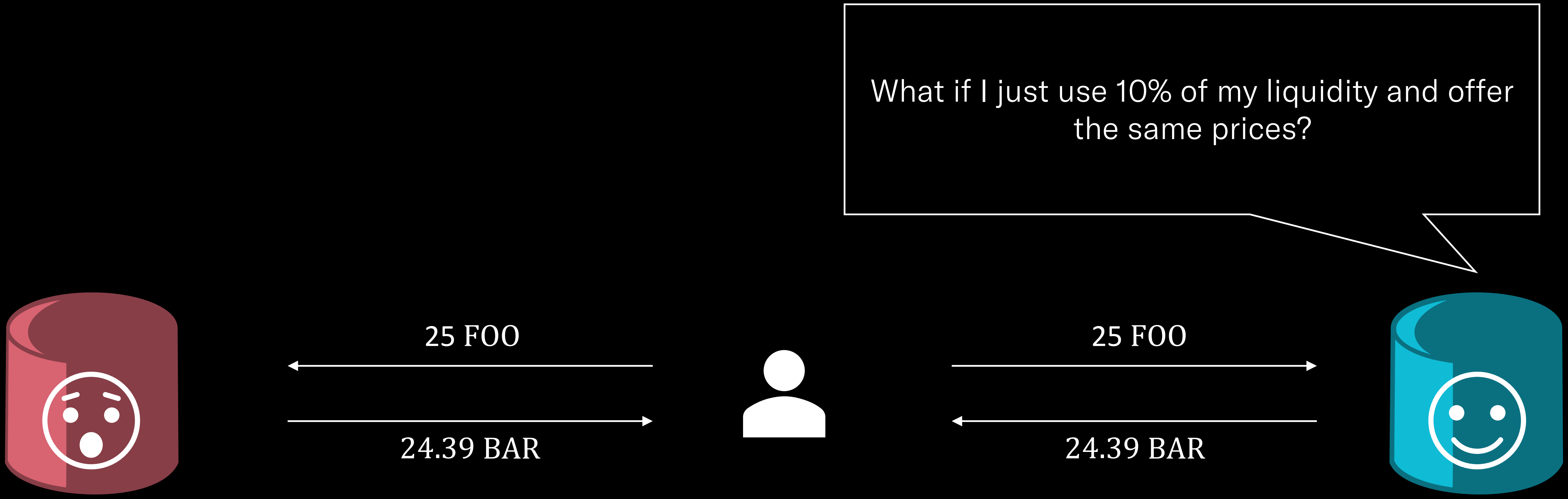


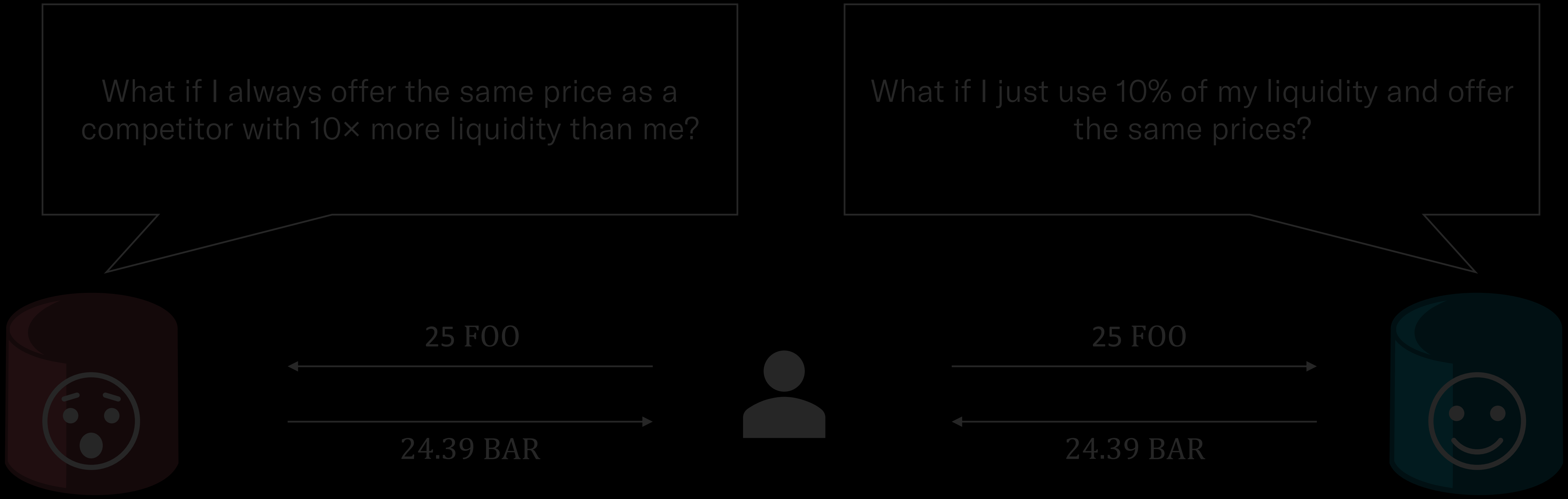

Will you come back if I offer the same price?
25 FOO
20.00 BAR
25 FOO
24.39 BAR


25 FOO
24.39 BAR
25 FOO
24.39 BAR

What if I always offer the same price as a competitor with 10× more liquidity than me?




What if I just use 10% of my liquidity and offer the same prices?
25 FOO
24.39 BAR
25 FOO
24.39 BAR



The short version is this:

We can re-work the pricing algorithm to allow *any* liquidity pool to emulate an *arbitrary size*.

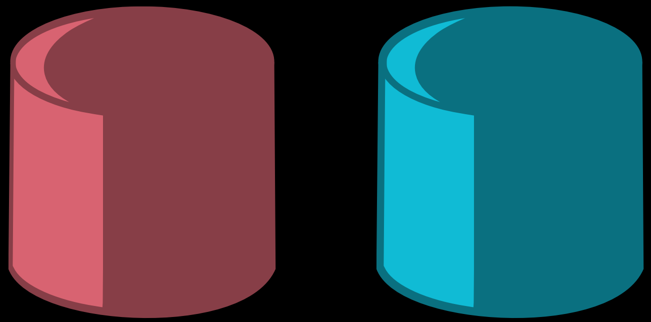
What's the catch?

REAL

VIRTUAL

“Amplification Coefficient” or “Capital Efficiency Term”

x	y	$A = 10$	x_{v}	y_{v}
100	100	→	1,000	1,000



$\Delta x = +25.00$ $\Delta y = -24.39$

REAL

VIRTUAL

x	y
100	100

$A = 10$

x_{v}	y_{v}
1,000	1,000

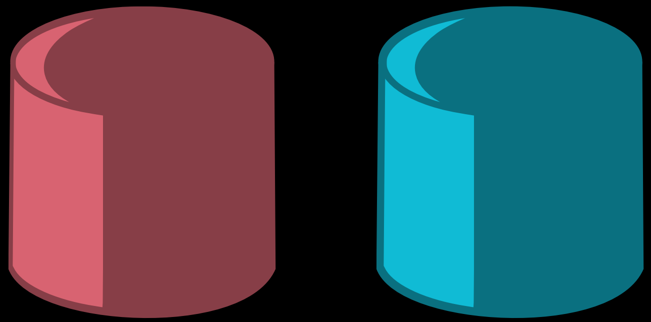
$\Delta x = +25.00$ $\Delta y = -24.39$

The trade is calculated on the virtual side.

REAL

x	y
100	100

$$\xrightarrow{A = 10}$$



VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61

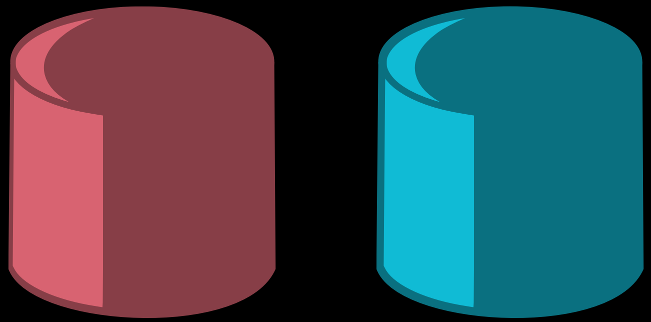
$\Delta x = +25.00$ $\Delta y = -24.39$

REAL

x	y
100	100
125	75.61

$\Delta x = +25.00$ $\Delta y = -24.39$

$$A = 10$$



VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61

REAL

x	y
100	100
125	75.61

$\Delta x = +25.00$ $\Delta y = -24.39$

$A = 10$

VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61

Then, the trade amounts are applied verbatim to the real token balances.

REAL

x	y
100	100
125	75.61

$$A = 10$$

VIRTUAL

x_{v}	y_{v}
1,000	1,000
1,025	975.61

$$100 \cdot 100 = 10{,}000$$

$$125 \cdot 75.61 = 9451.22$$

REAL

VIRTUAL

x	y
100	100
125	75.61

$$\xrightarrow{A = 10}$$

x_v	y_v
1,000	1,000
1,025	975.61

$$10{,}000 \neq 9451.22$$

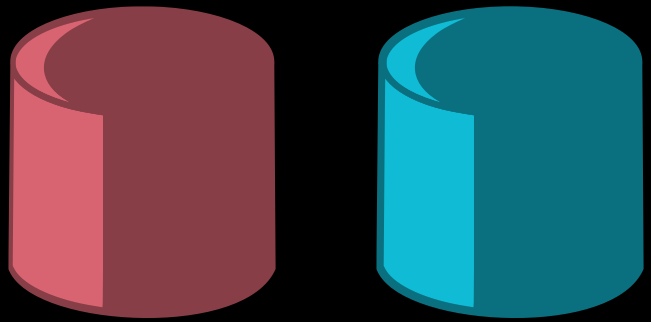
Note that this means the general invariant no longer applies.

(We need to determine a new one.)

REAL

x	y
100	100
125	75.61

$$A = 10$$



VIRTUAL

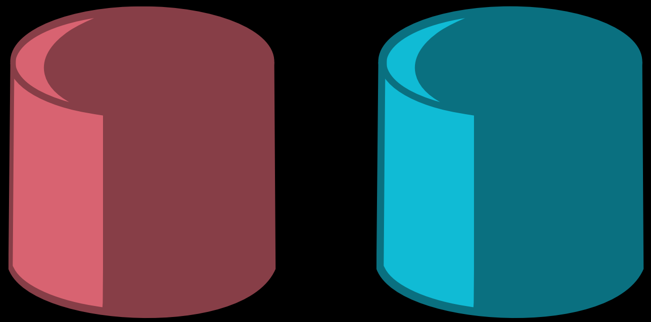
x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00
$\Delta x = +86.11$	$\Delta y = -75.61$

REAL

x	y
100	100
125	75.61
211.11	0

$\Delta x = +86.11$ $\quad \Delta y = -75.61$

$$\xrightarrow{A = 10}$$



VIRTUAL

x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00

REAL			VIRTUAL	
x	y	$A = 10$	x_{v}	y_{v}
100	100		1,000	1,000
125	75.61		1,025	975.61
211.11	0		1,111.11	900.00
$\Delta x = +86.11$	$\Delta y = -75.61$			

Since the general invariant is no longer applicable, token balances can be driven to zero.

REAL

VIRTUAL

$A = 10$

x	y
100	100
125	75.61
211.11	0

x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00

$$\frac{\partial y}{\partial x} = -\frac{900.00}{1{,}111.11} = -0.81$$

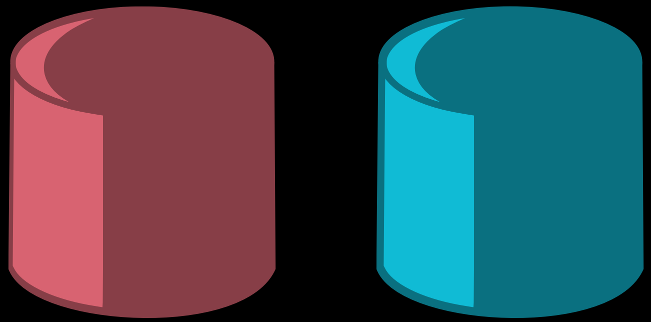
REAL

VIRTUAL

x	y
100	100
125	75.61
211.11	0

$$\xrightarrow{A = 10}$$

x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00



$$\frac{\partial y}{\partial x} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0

$A = 10$

VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61
1,111.11	900.00

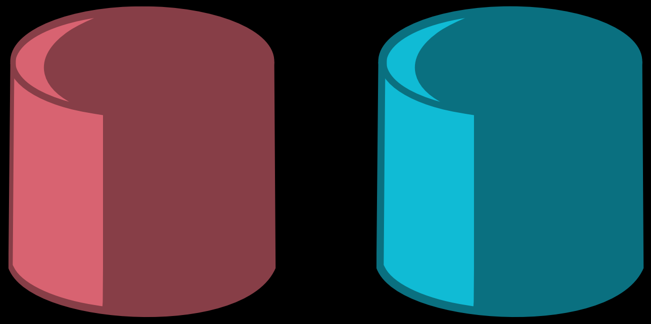
$$\frac{\partial y}{\partial x} = -0.81$$

As the x balance is now zero, marginal rates beyond this value are impossible to reach.

REAL	
x	y
100	100
125	75.61
211.11	0

$$\xrightarrow{A = 10}$$

VIRTUAL	
x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00

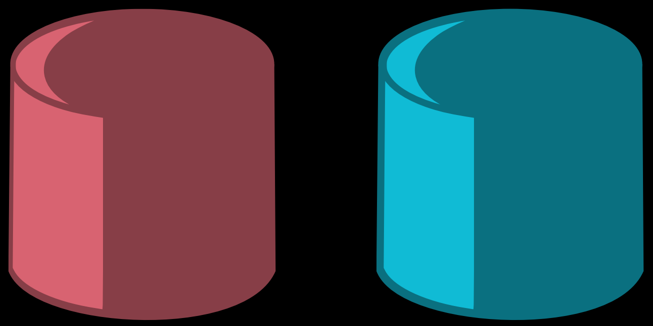


$$\frac{\partial y}{\partial x} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0

$$A = 10$$



VIRTUAL

x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

$\Delta x = -211.11$ $\Delta y = +211.11$

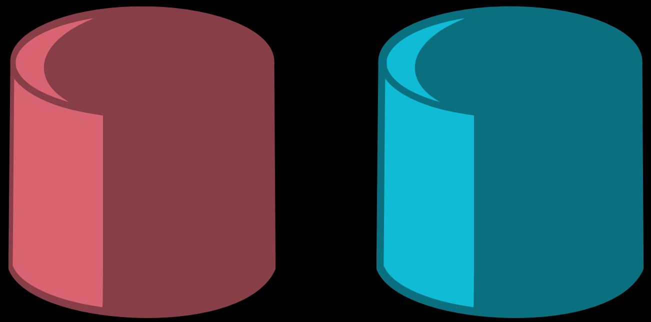
$$\frac{\partial y}{\partial x} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0
0	211.11

$\Delta x = -211.11$ $\Delta y = +211.11$

$A = 10$



VIRTUAL

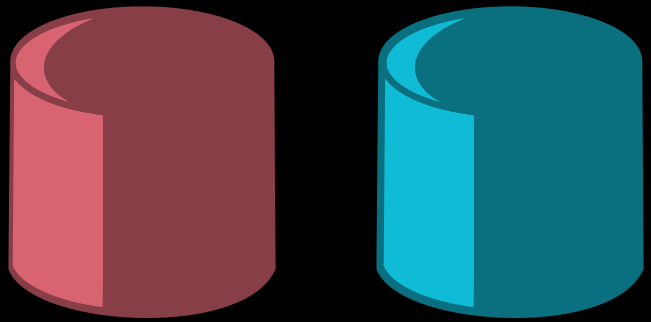
x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0
0	211.11

$A = 10$



VIRTUAL

x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

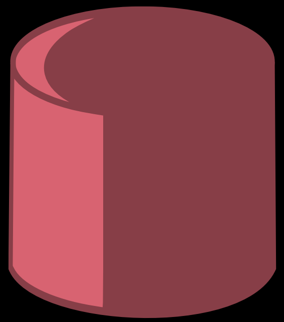
$$\frac{\partial y}{\partial x} = -\frac{1{,}111.11}{900.00} = -1.23$$

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0
0	211.11

$A = 10$



VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

$$\frac{\partial y}{\partial x} = -1.23$$

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0
0	211.11

$A = 10$

VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

$$\frac{\partial y}{\partial x} = -1.23$$

$$\frac{\partial y}{\partial x} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0
0	211.11

$\xrightarrow{A = 10}$

VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

The amplified pool can only service the market between these price bounds.

$$\boxed{\frac{\partial y}{\partial x} = -1.23 \qquad \frac{\partial y}{\partial x} = -0.81}$$

REAL

x	y
100	100
125	75.61
211.11	0
0	211.11

$$\xrightarrow{A = 10}$$

VIRTUAL

x_v	y_v
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

$$\frac{\partial y}{\partial x} = -1.23 \qquad \frac{\partial y}{\partial x} = -0.81$$

$$-P_{\text{high}} = -1.23 \qquad -P_{\text{low}} = -0.81$$

REAL

x	y
100	100
125	75.61
211.11	0
0	211.11

$A = 10$

VIRTUAL

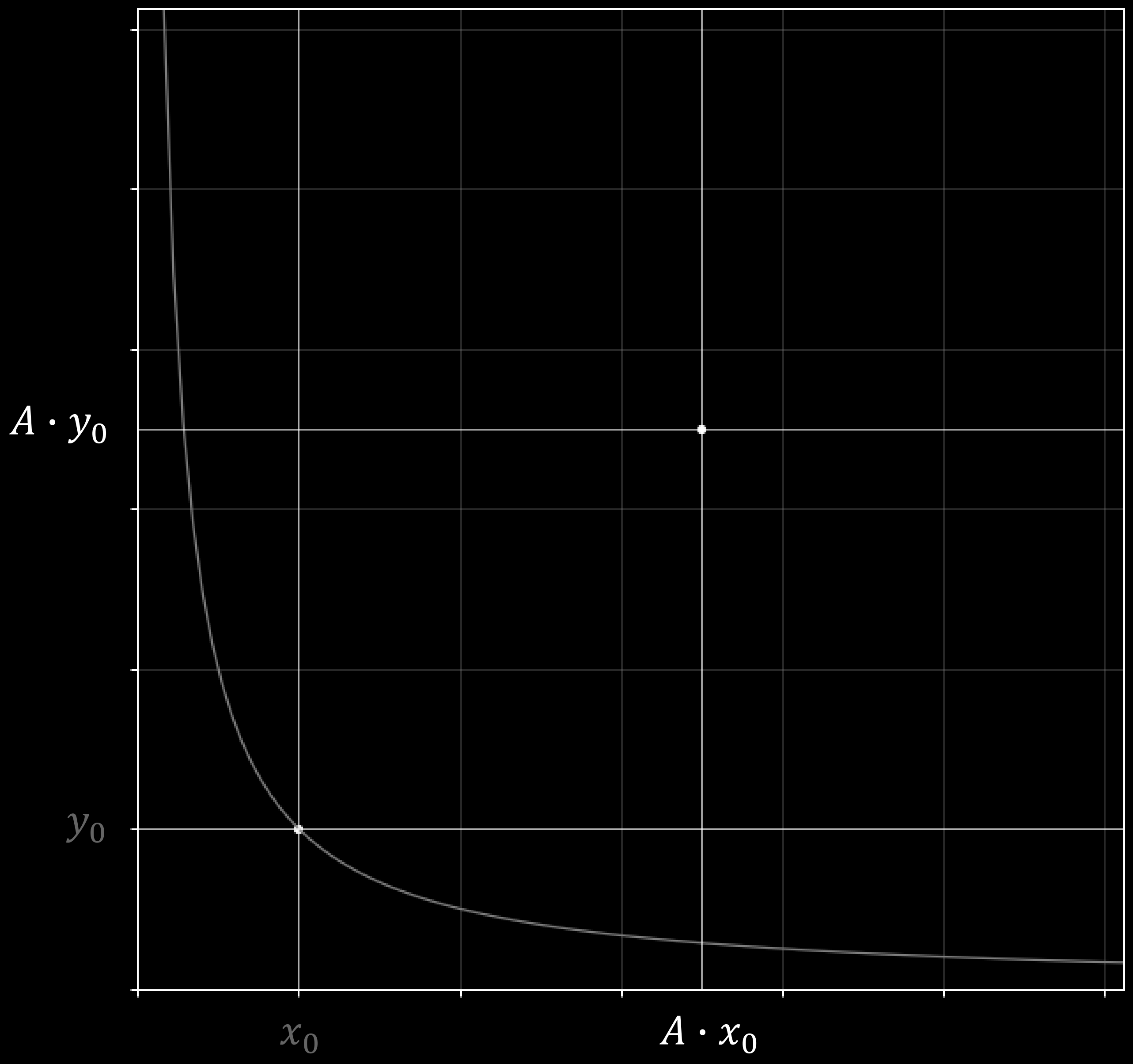
x_{v}	y_{v}
1,000	1,000
1,025	975.61
1,111.11	900.00
900.00	1,111.11

$$P_{\mathrm{high}} = 1.23 \qquad P_{\mathrm{low}} = 0.81$$

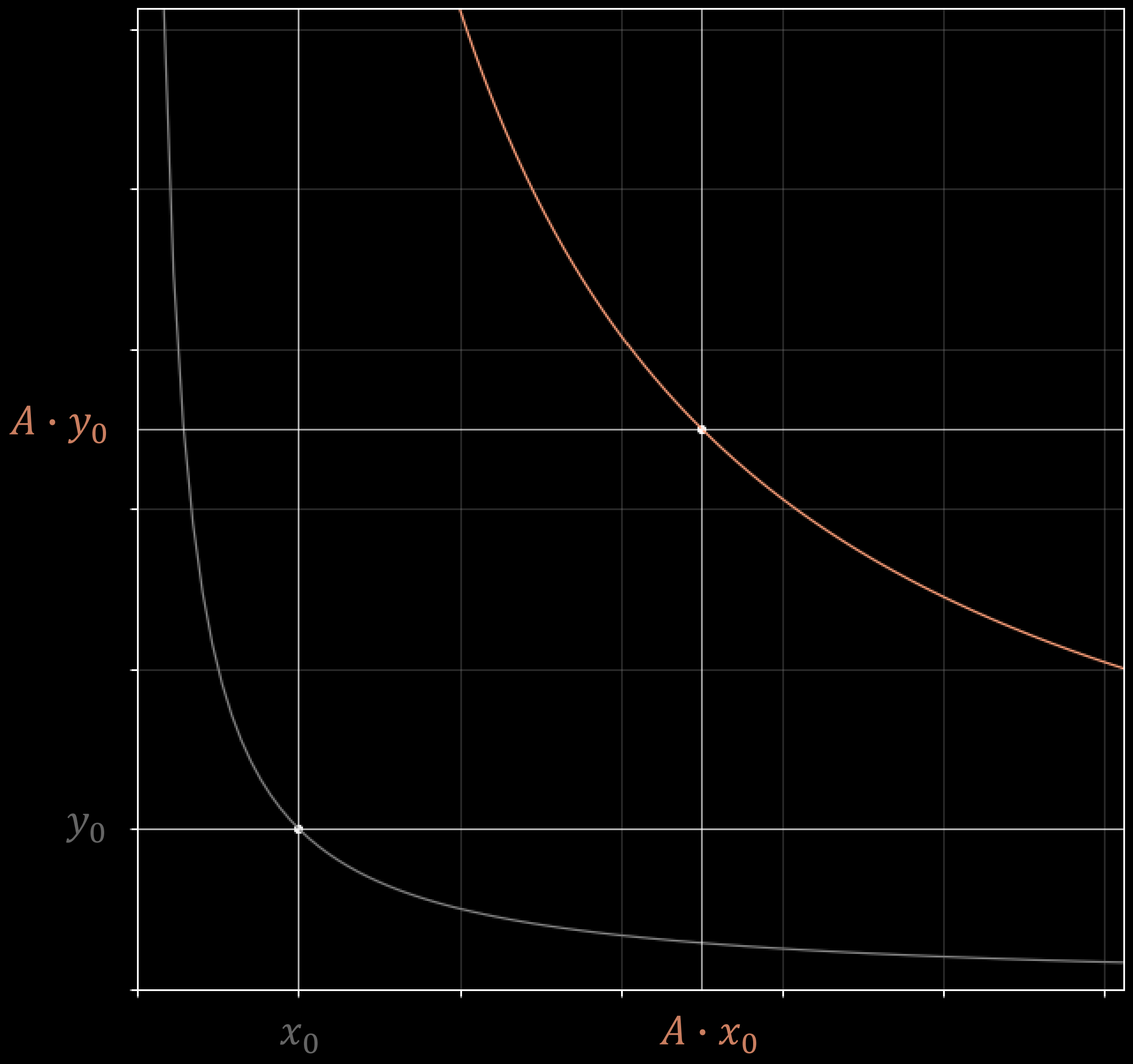
For this course, we will adopt this convention.

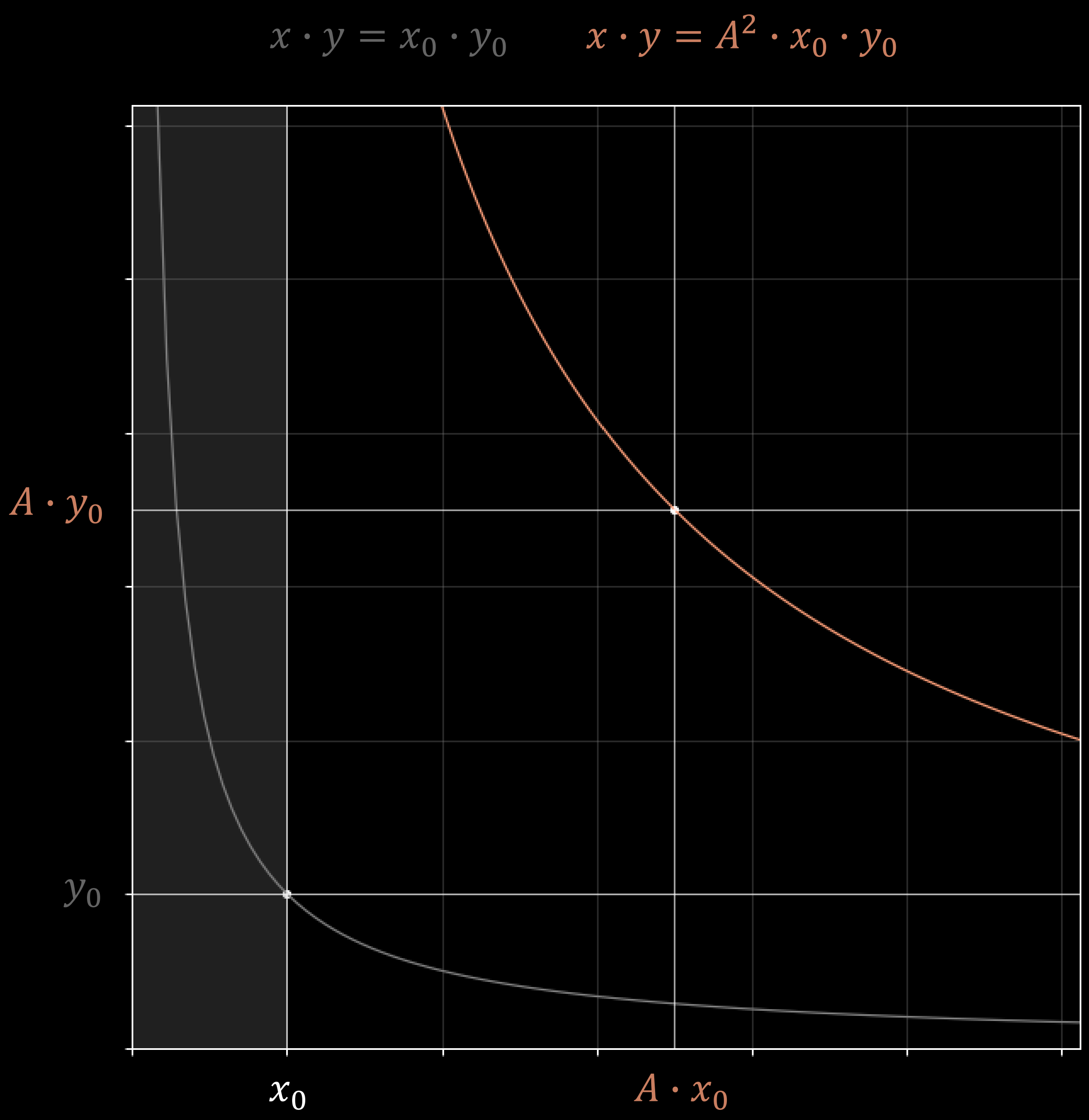
What is the geometric and algebraic intuition?

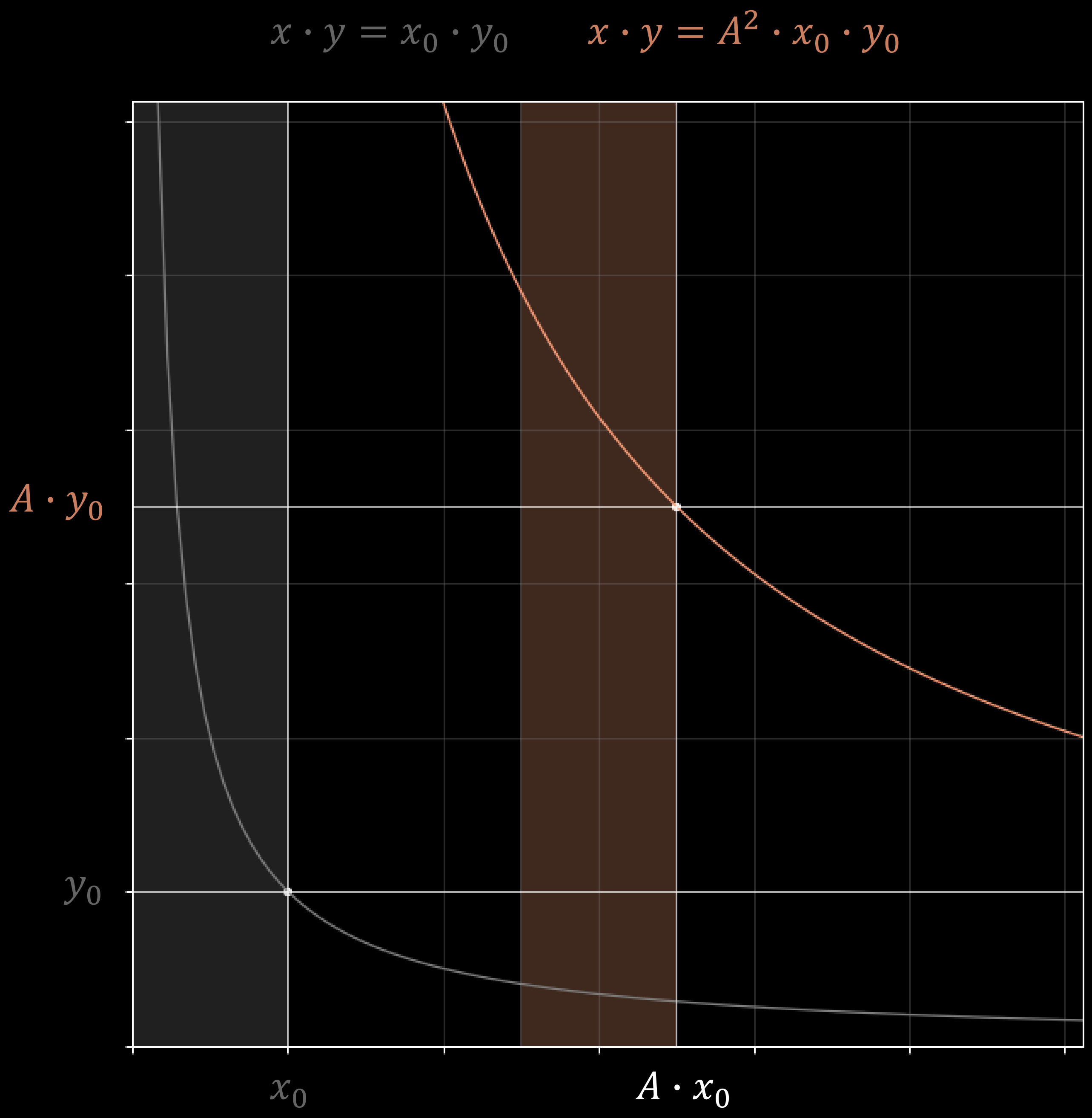
$$x \cdot y = x_0 \cdot y_0$$



$x \cdot y = x_0 \cdot y_0$ $\quad$ $x \cdot y = A^2 \cdot x_0 \cdot y_0$

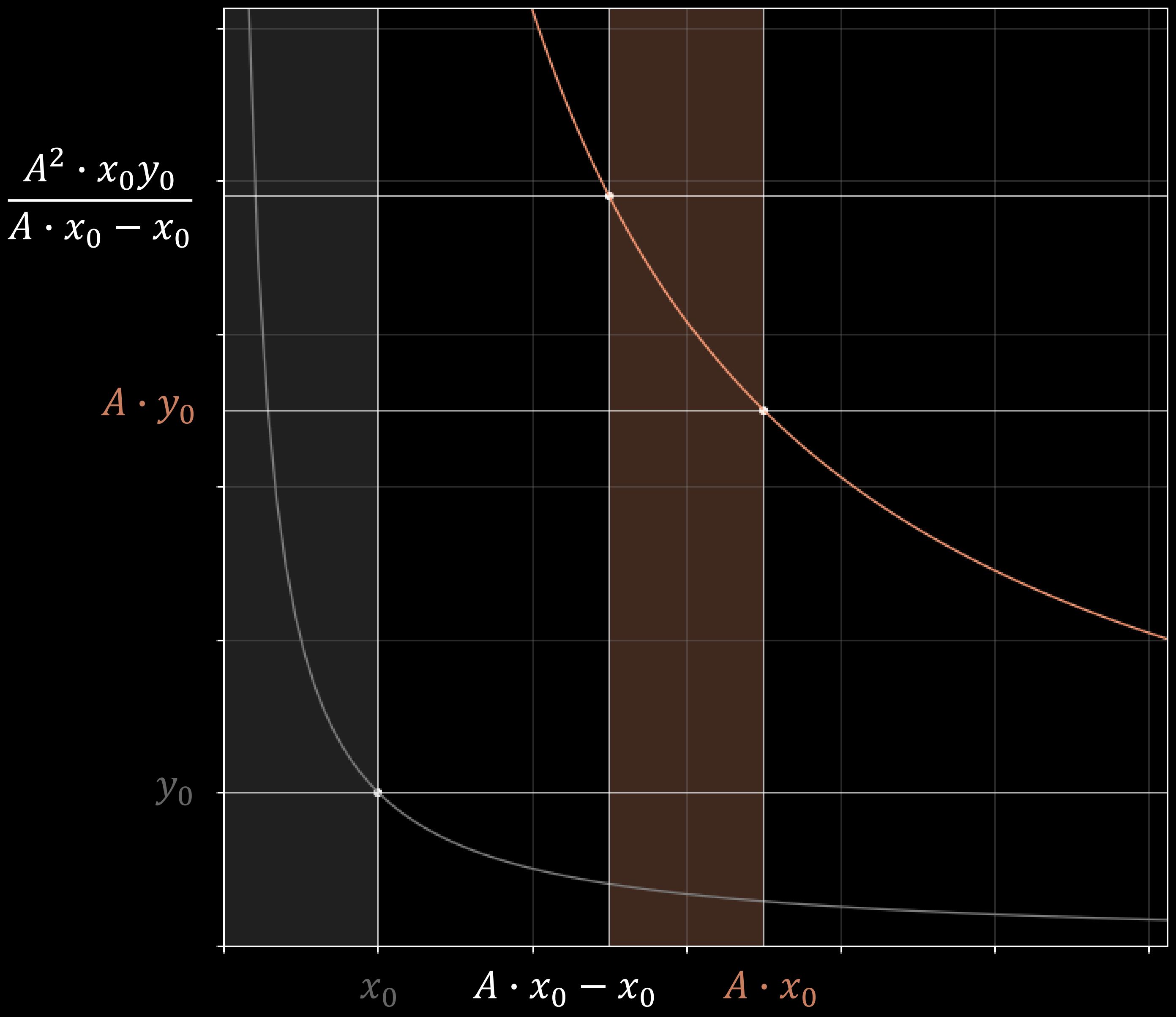


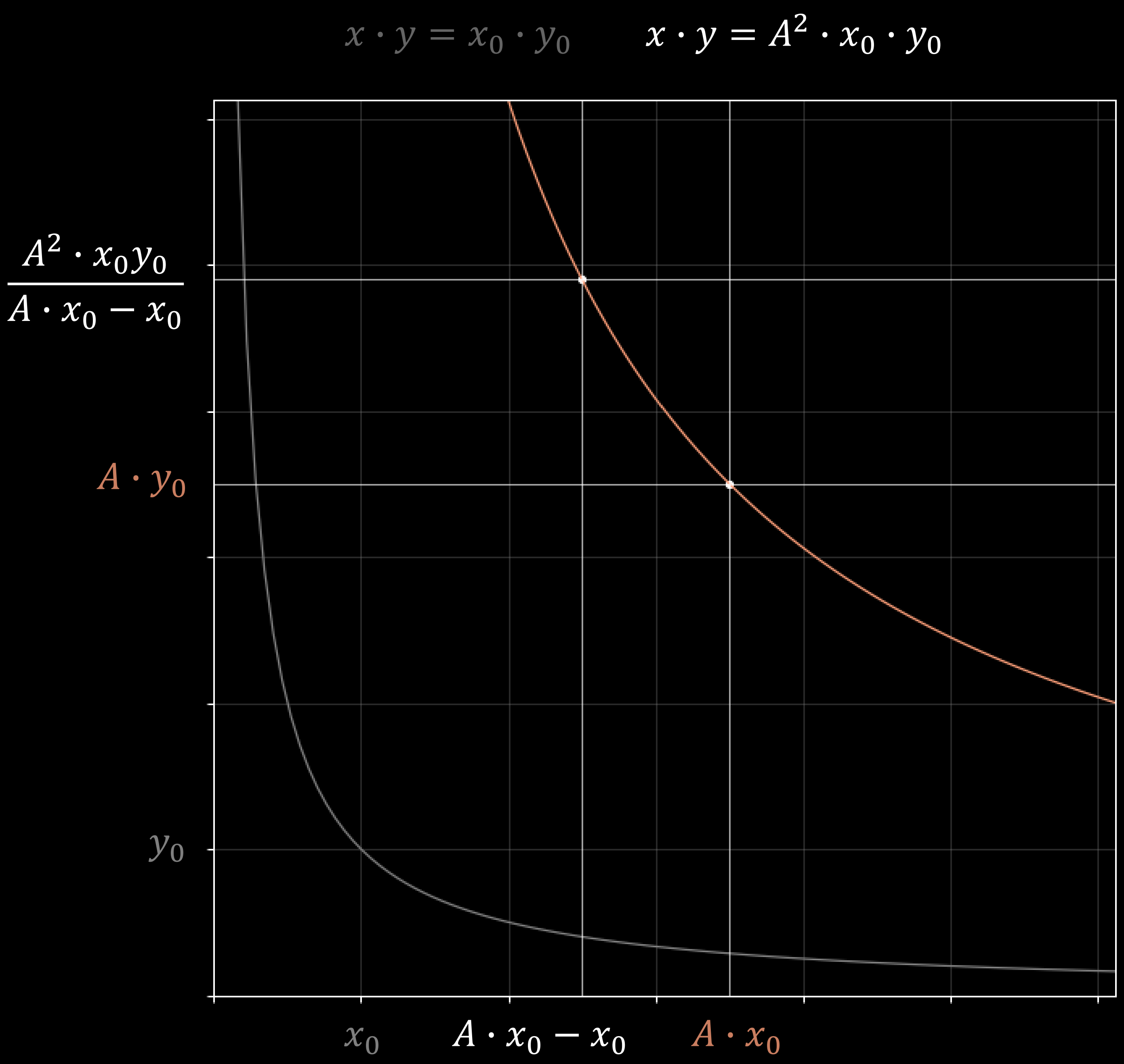

$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$A \cdot y_0$
y_0
x_0
$A \cdot x_0$



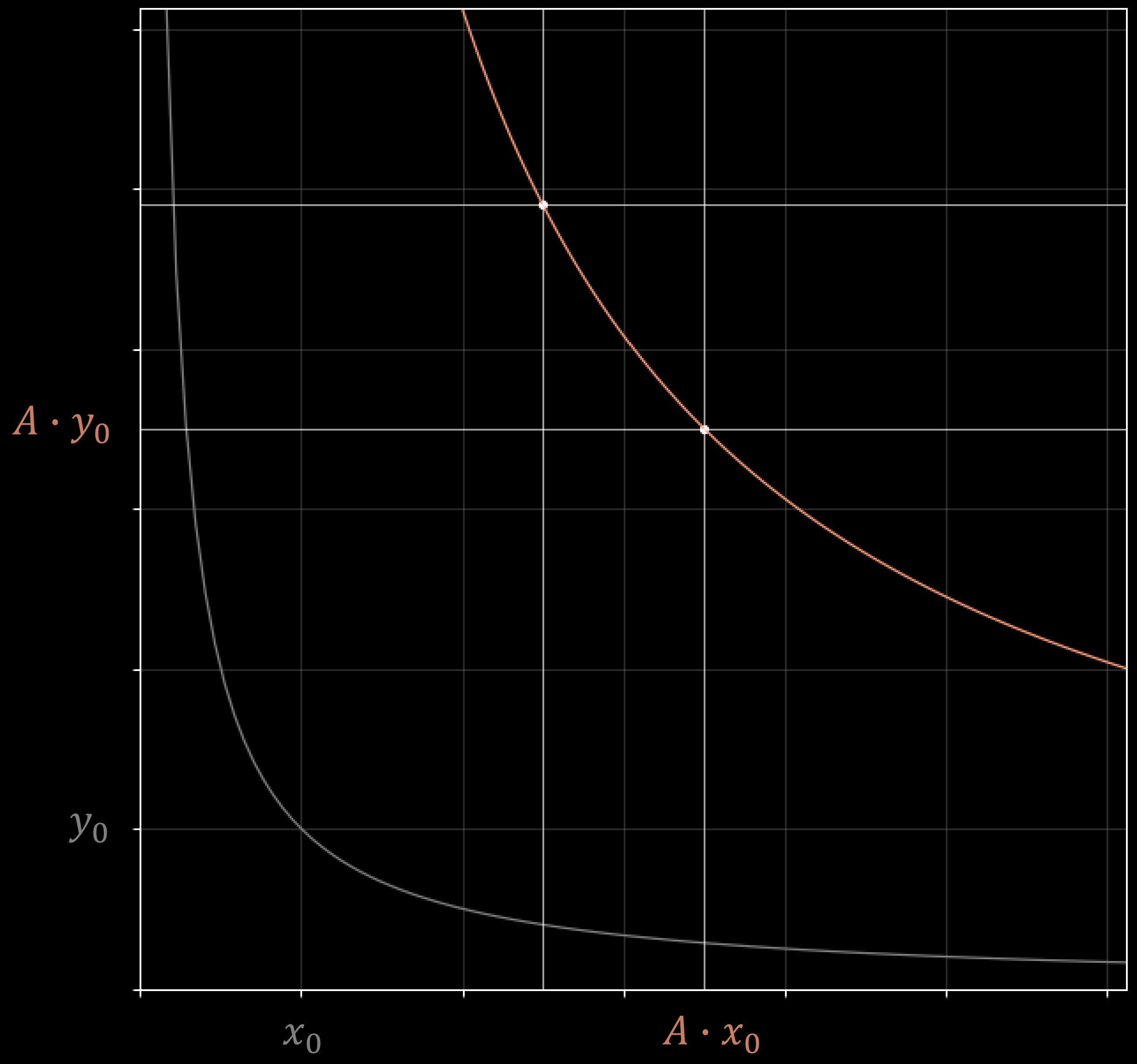
$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$A \cdot y_0$
y_0
x_0
$A \cdot x_0$

$x \cdot y = x_0 \cdot y_0$ $\quad$ $x \cdot y = A^2 \cdot x_0 \cdot y_0$



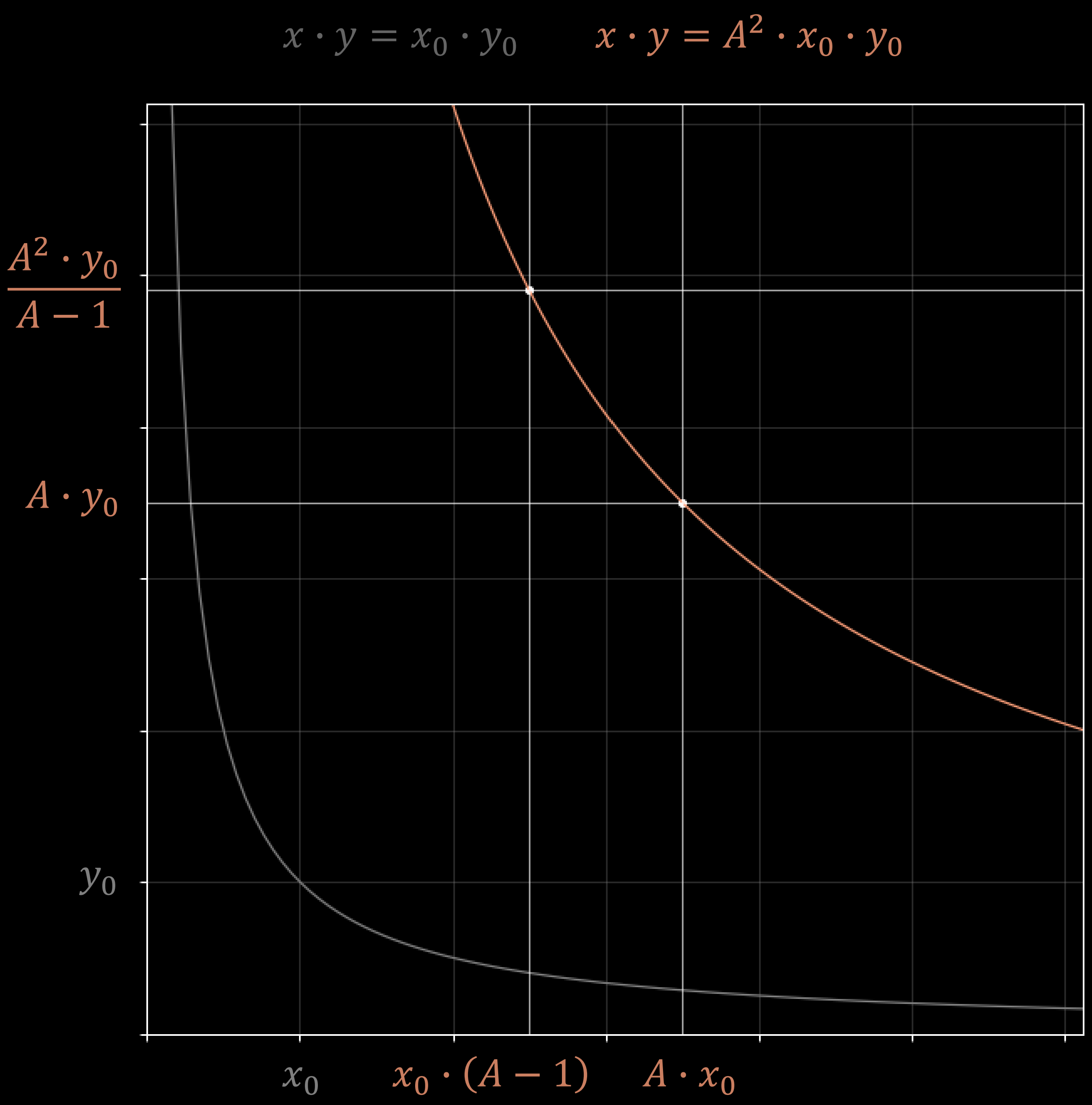

$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$\frac{A^2 \cdot x_0 y_0}{A \cdot x_0 - x_0}$
$A \cdot y_0$
y_0
x_0
$A \cdot x_0 - x_0$
$A \cdot x_0$

$x \cdot y = x_0 \cdot y_0$ $\quad$ $x \cdot y = A^2 \cdot x_0 \cdot y_0$

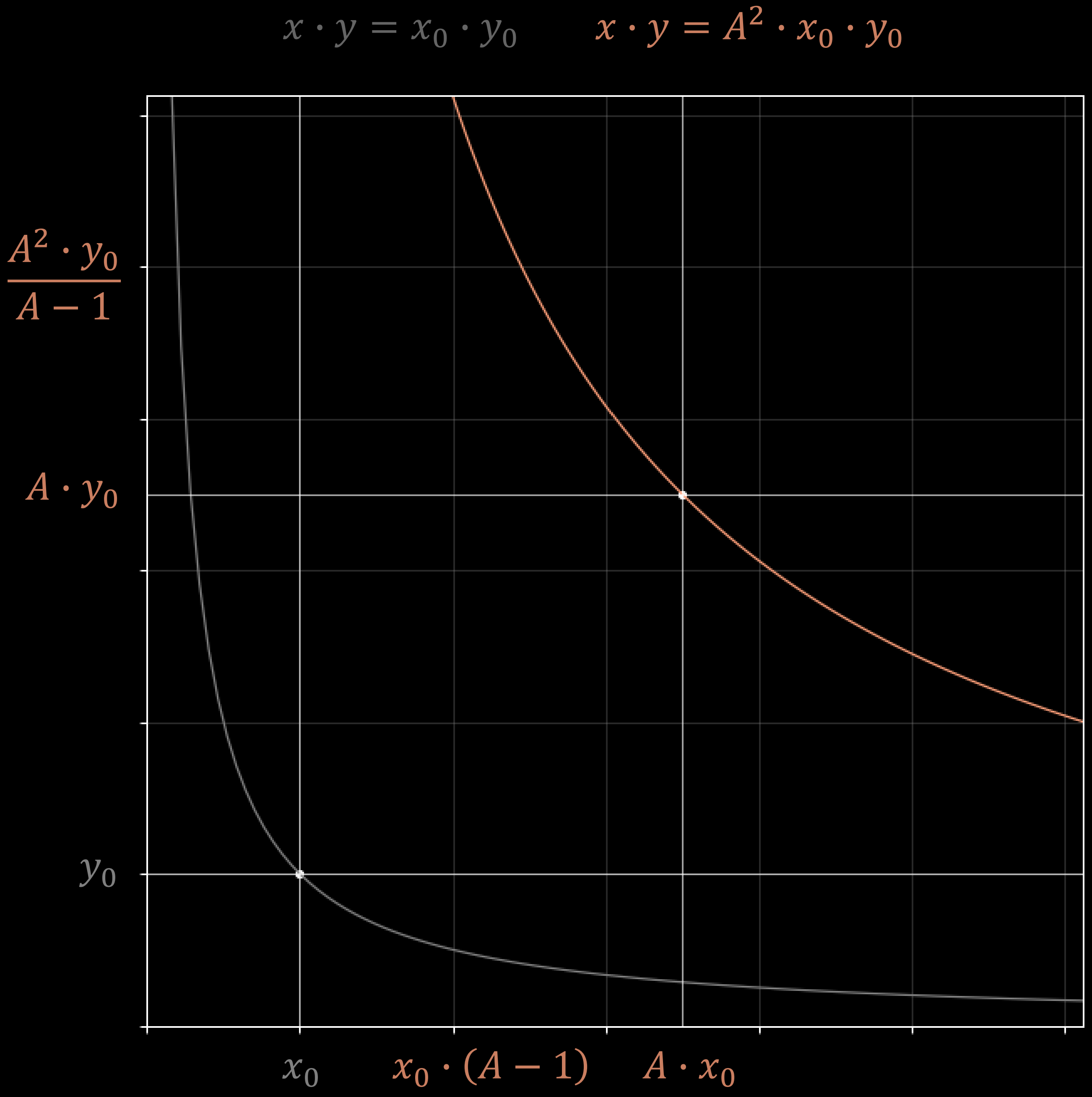


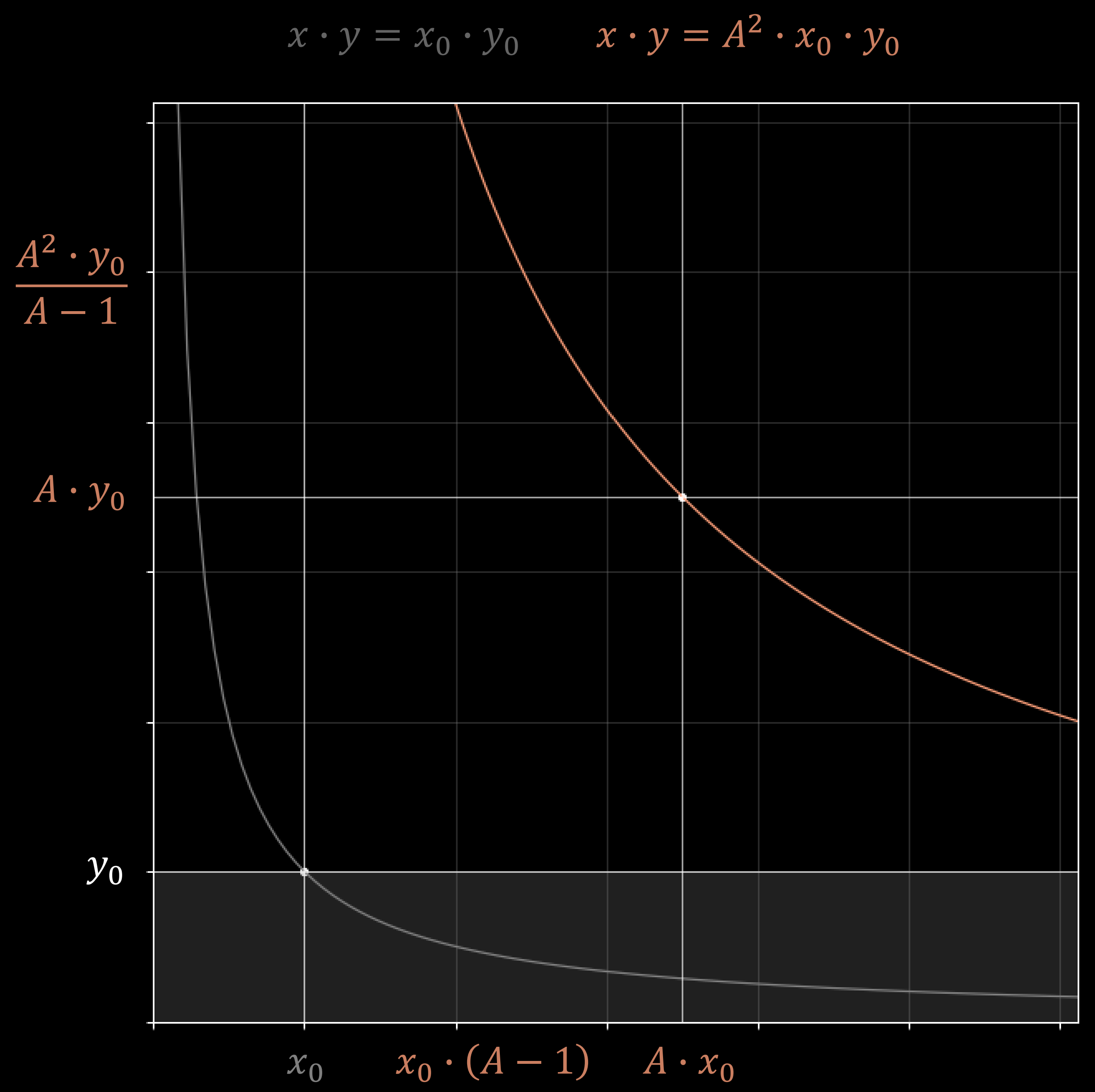
$$\frac{A^2 \cdot x_0 y_0}{A \cdot x_0 - x_0} = \frac{A^2 \cdot y_0}{A - 1}$$

$$A \cdot x_0 - x_0 = x_0 \cdot (A - 1)$$

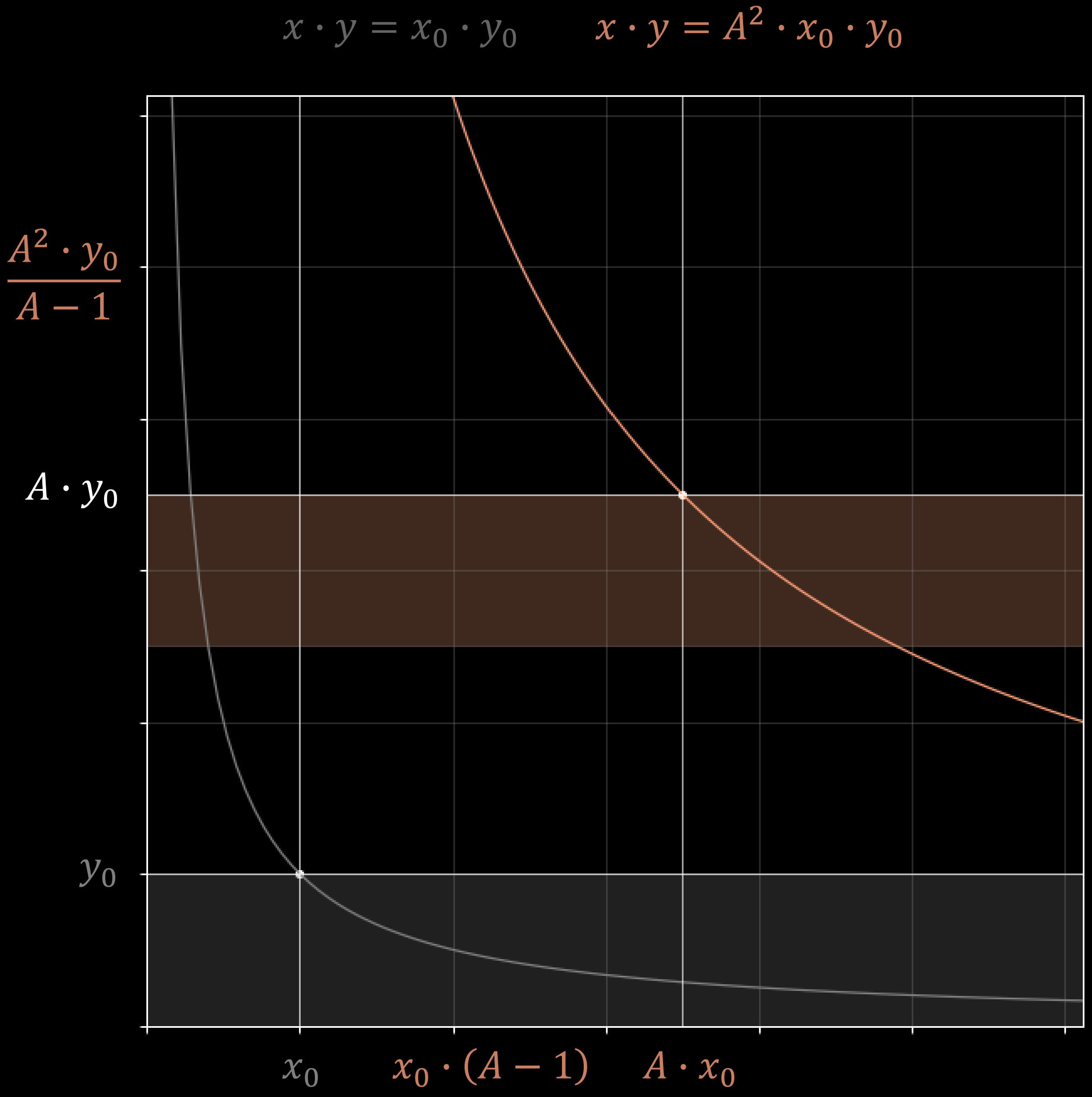


$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$\frac{A^2 \cdot y_0}{A - 1}$
$A \cdot y_0$
y_0
x_0
$x_0 \cdot (A - 1)$
$A \cdot x_0$


$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$\frac{A^2 \cdot y_0}{A - 1}$
$A \cdot y_0$
y_0
x_0
$x_0 \cdot (A - 1)$
$A \cdot x_0$

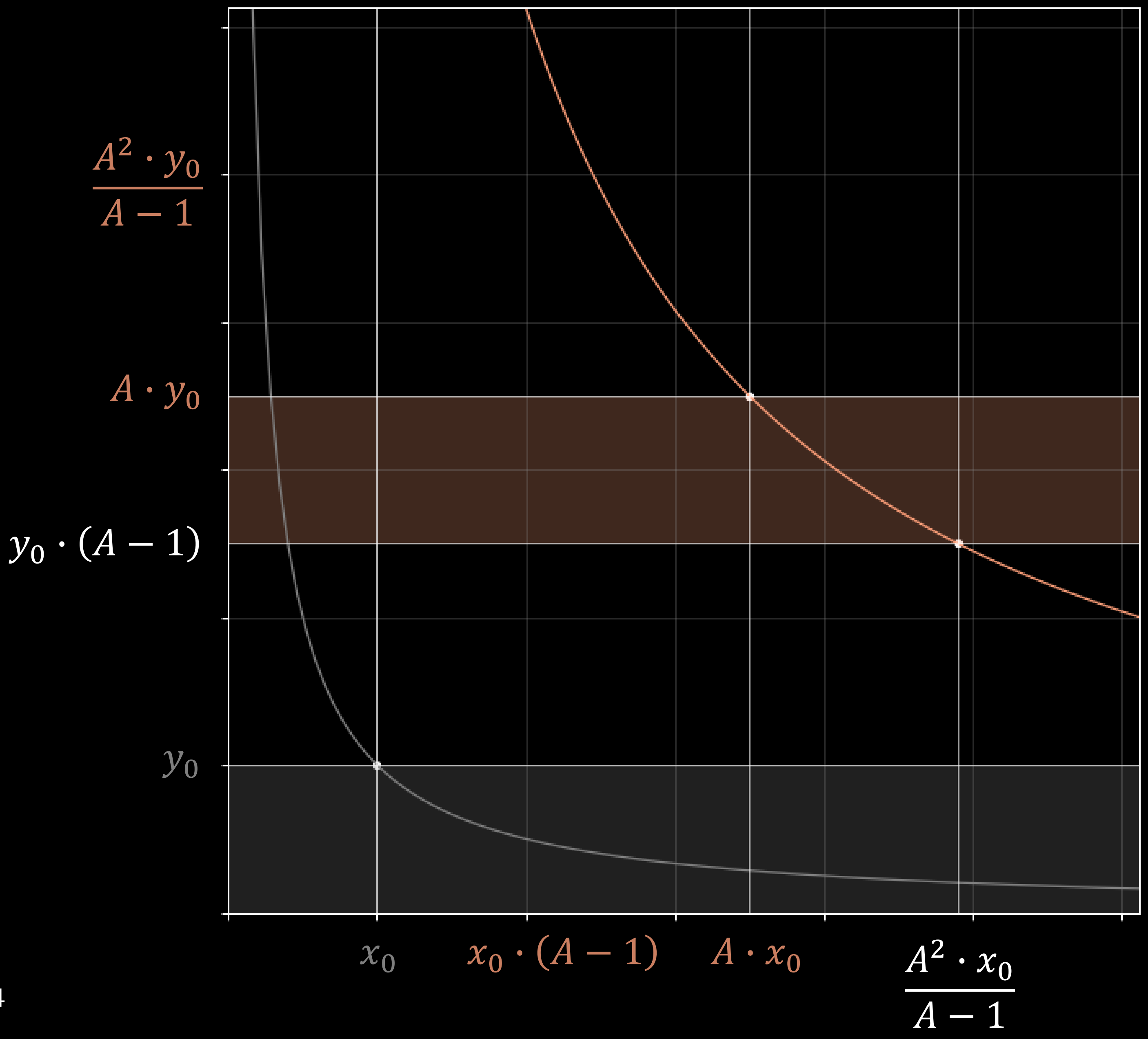


$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$\frac{A^2 \cdot y_0}{A - 1}$
$A \cdot y_0$
y_0
x_0
$x_0 \cdot (A - 1)$
$A \cdot x_0$

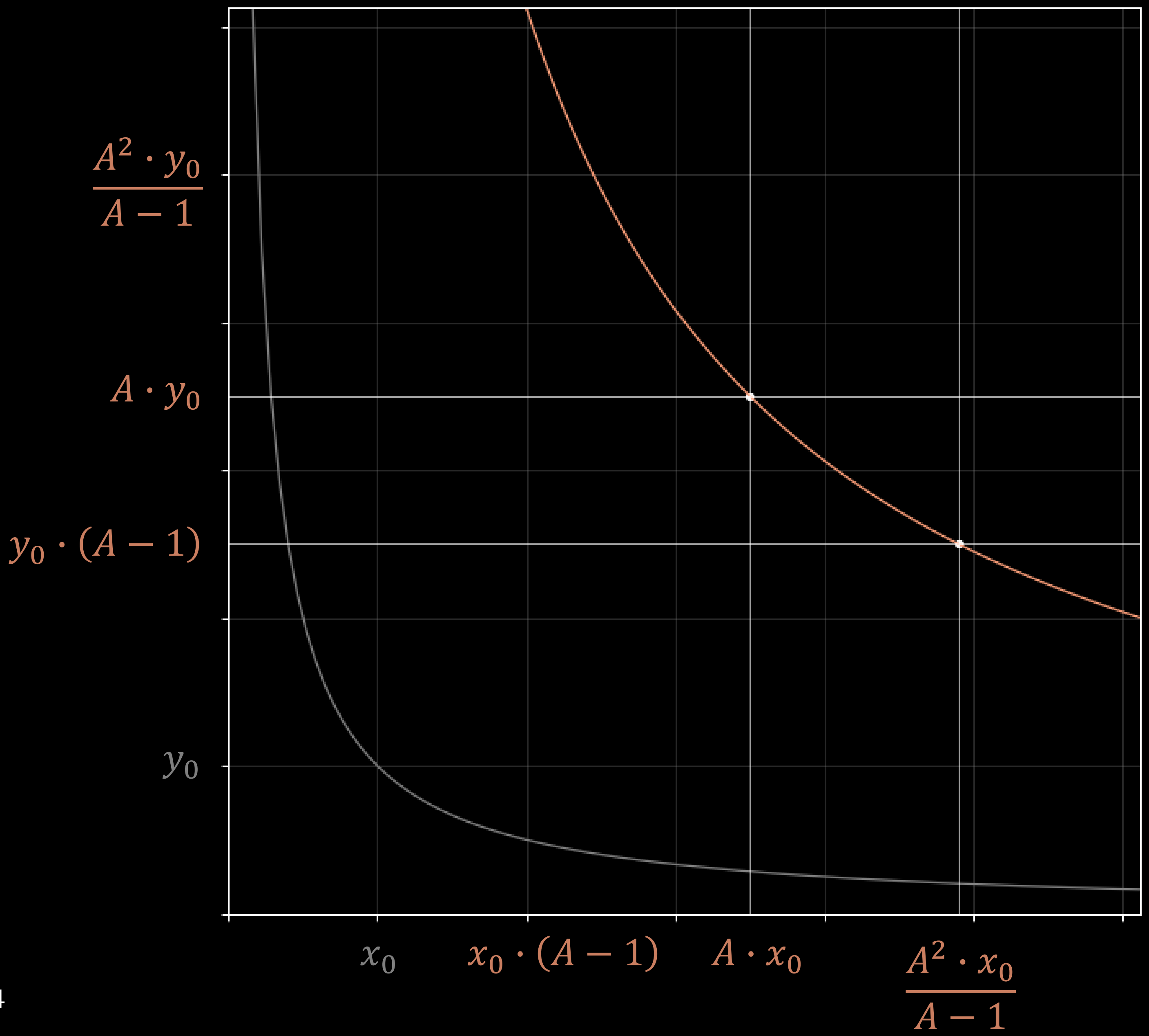


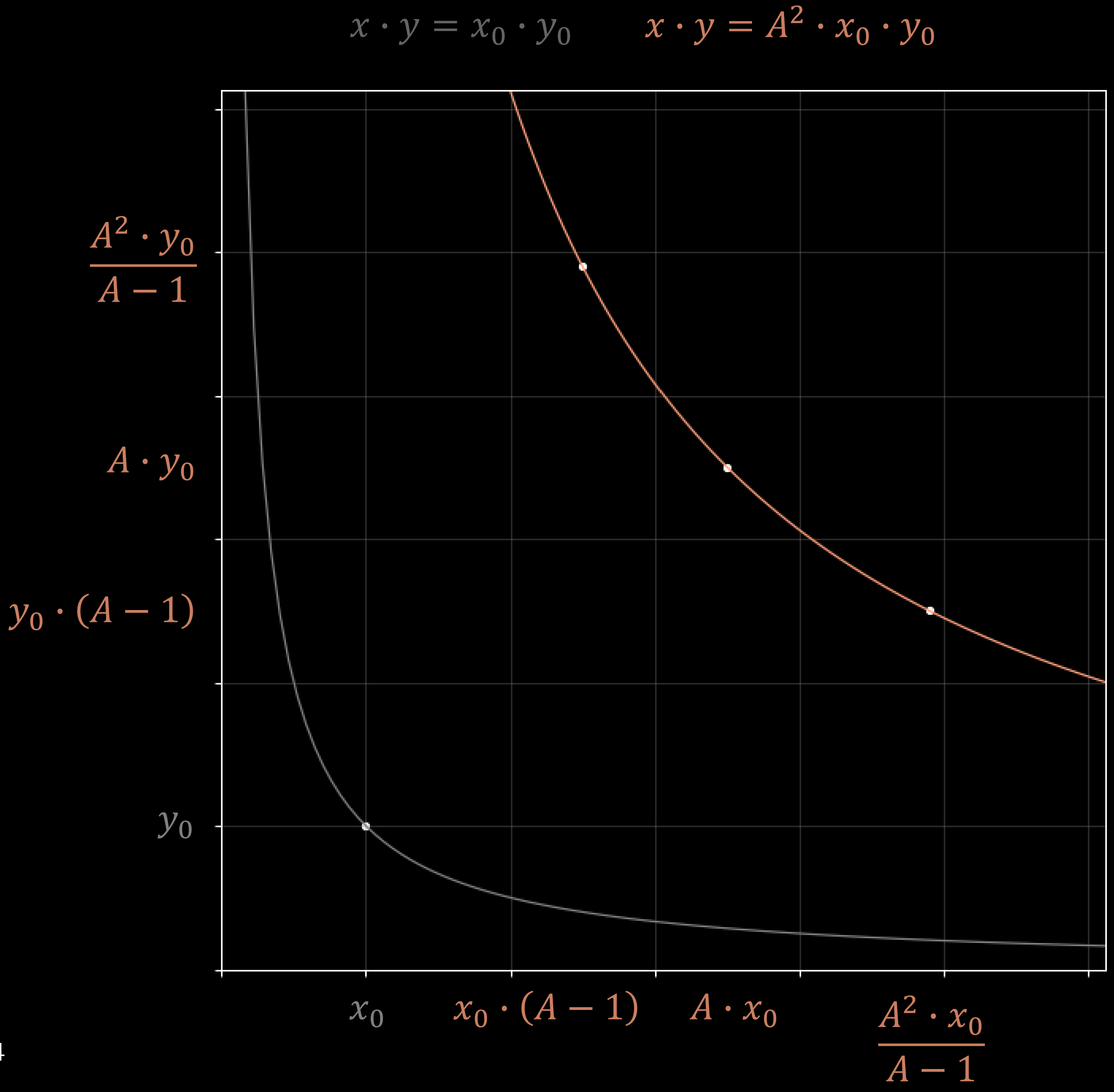
$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$\frac{A^2 \cdot y_0}{A - 1}$
$A \cdot y_0$
y_0
x_0
$x_0 \cdot (A - 1)$
$A \cdot x_0$

$x \cdot y = x_0 \cdot y_0$ $\quad$ $x \cdot y = A^2 \cdot x_0 \cdot y_0$

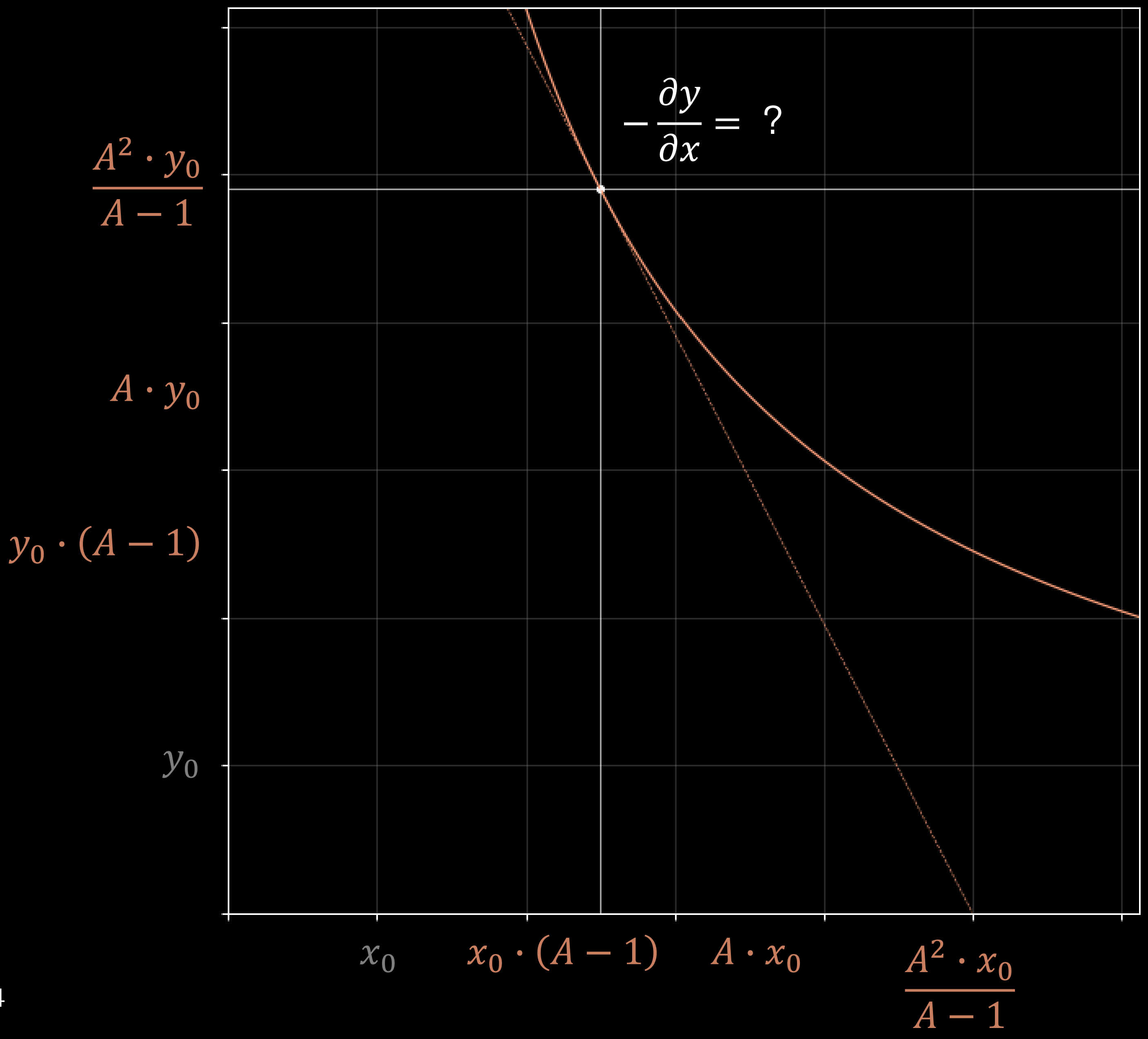


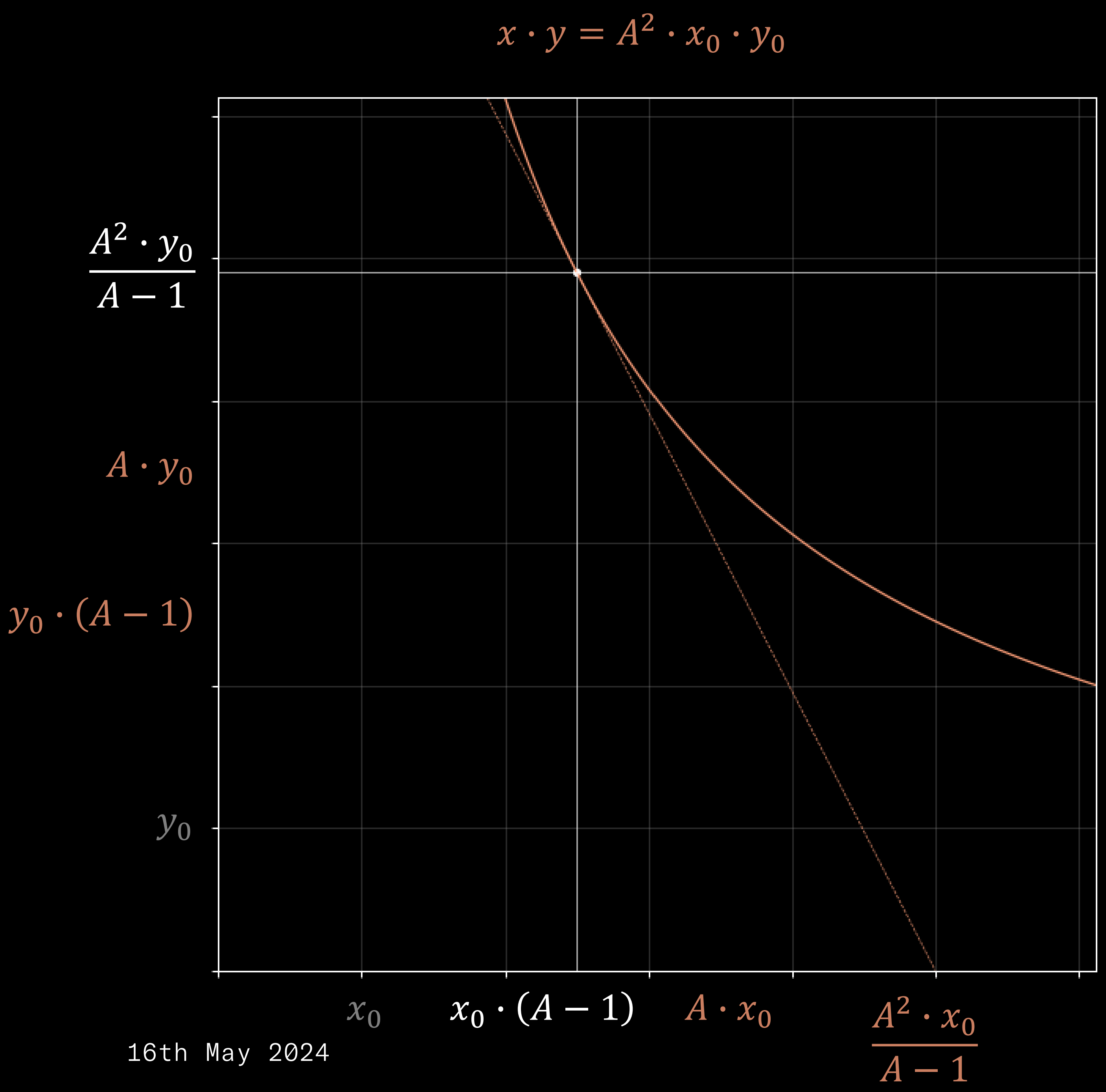
$x \cdot y = x_0 \cdot y_0$ $\quad$ $x \cdot y = A^2 \cdot x_0 \cdot y_0$




$x \cdot y = x_0 \cdot y_0$
$x \cdot y = A^2 \cdot x_0 \cdot y_0$
$\frac{A^2 \cdot y_0}{A - 1}$
$A \cdot y_0$
$y_0 \cdot (A - 1)$
y_0
x_0
$x_0 \cdot (A - 1)$
$A \cdot x_0$
$\frac{A^2 \cdot x_0}{A - 1}$

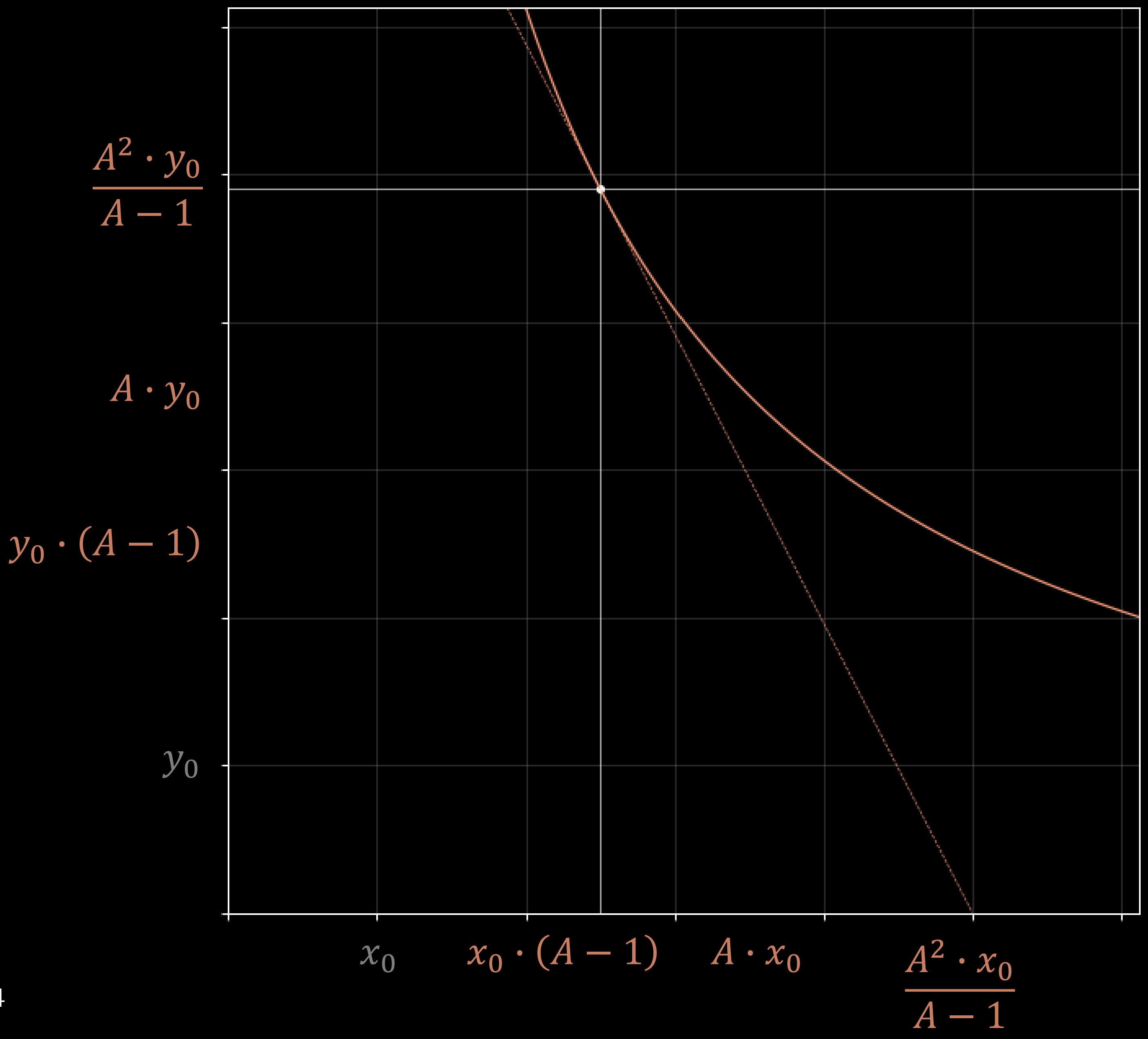
$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$





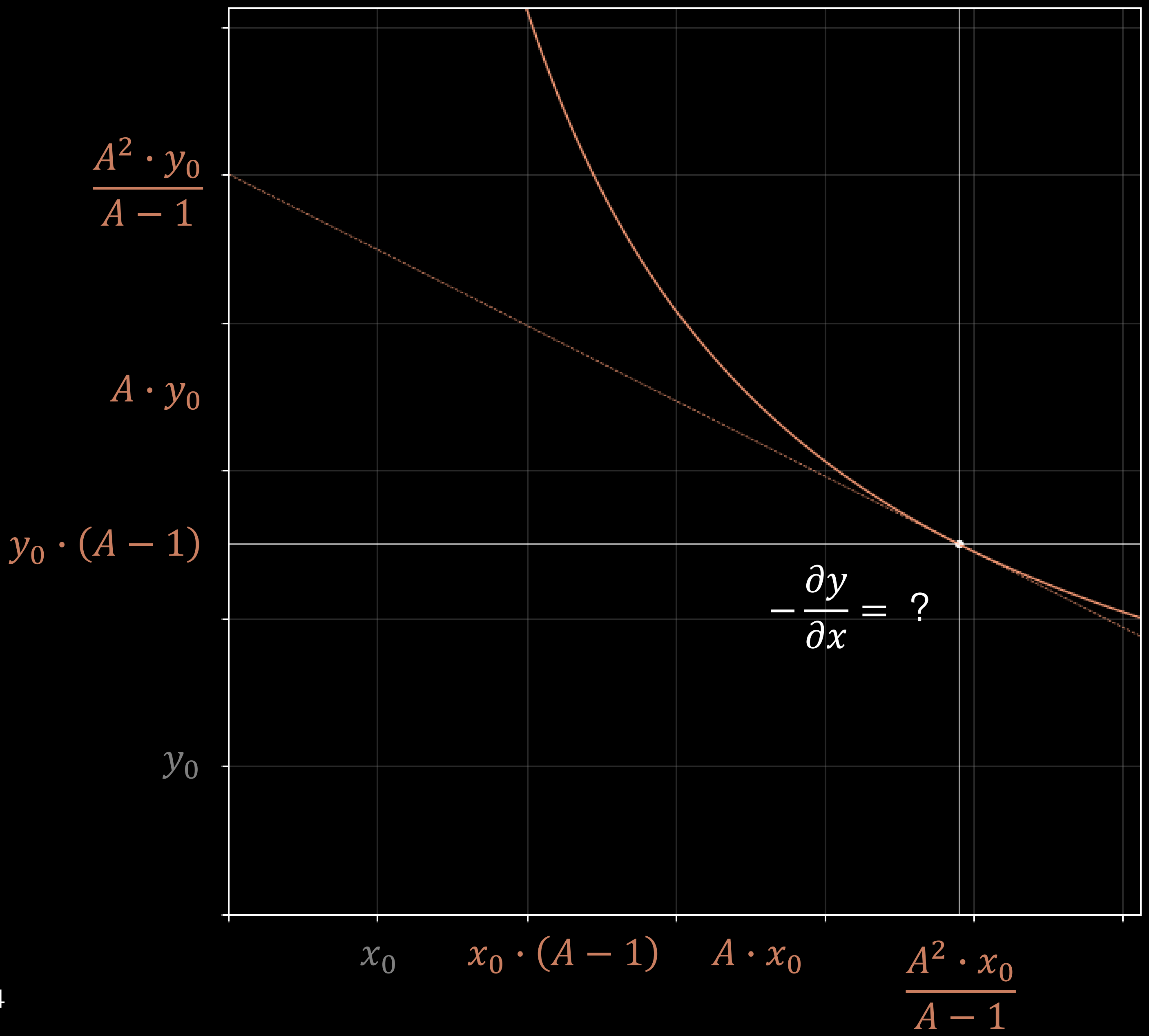
$$-\frac{\partial y}{\partial x} = \frac{\frac{A^2 \cdot y_0}{A-1}}{x_0 \cdot (A-1)} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



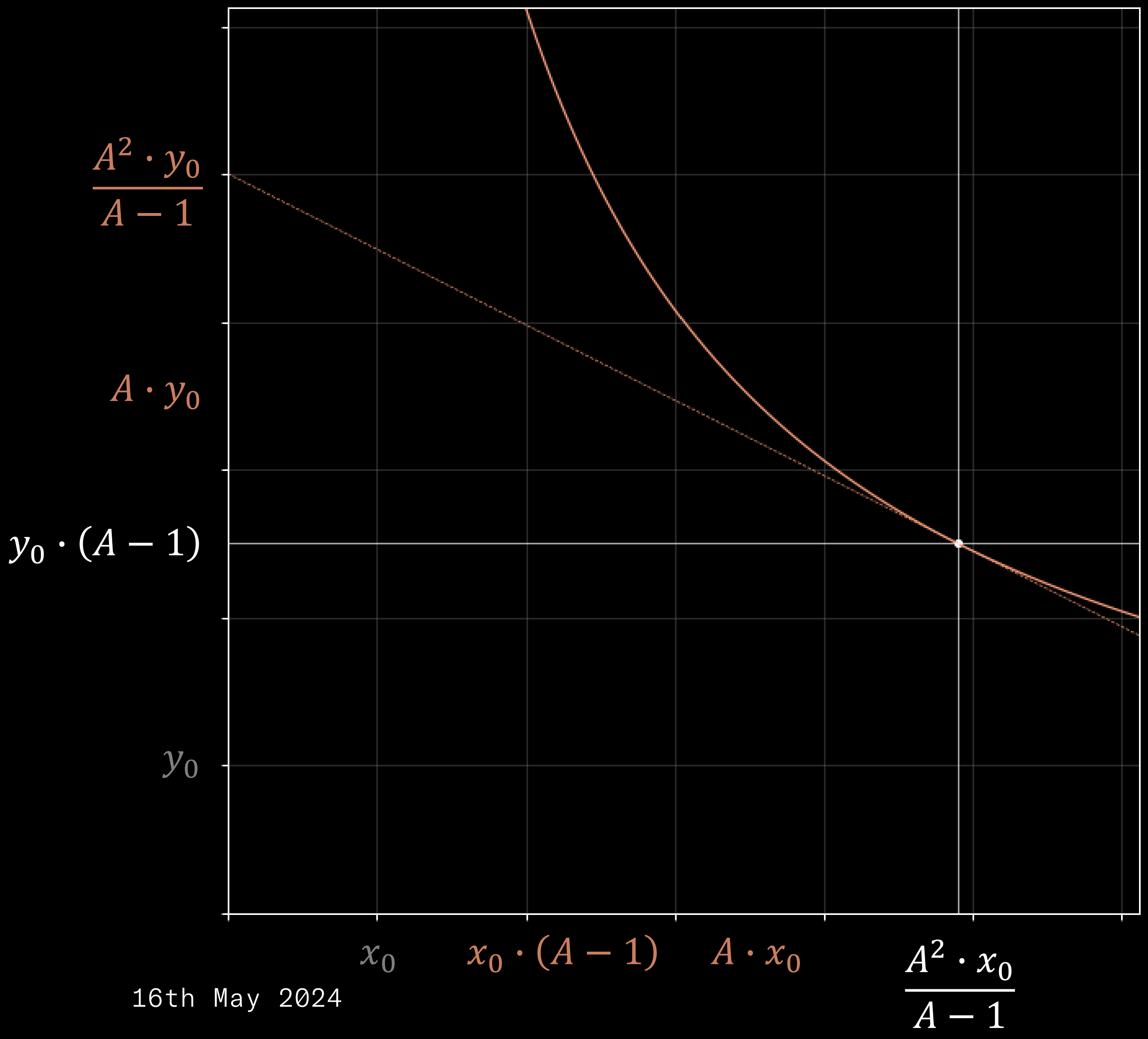
$$-\frac{\partial y}{\partial x} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$-\frac{\partial y}{\partial x} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

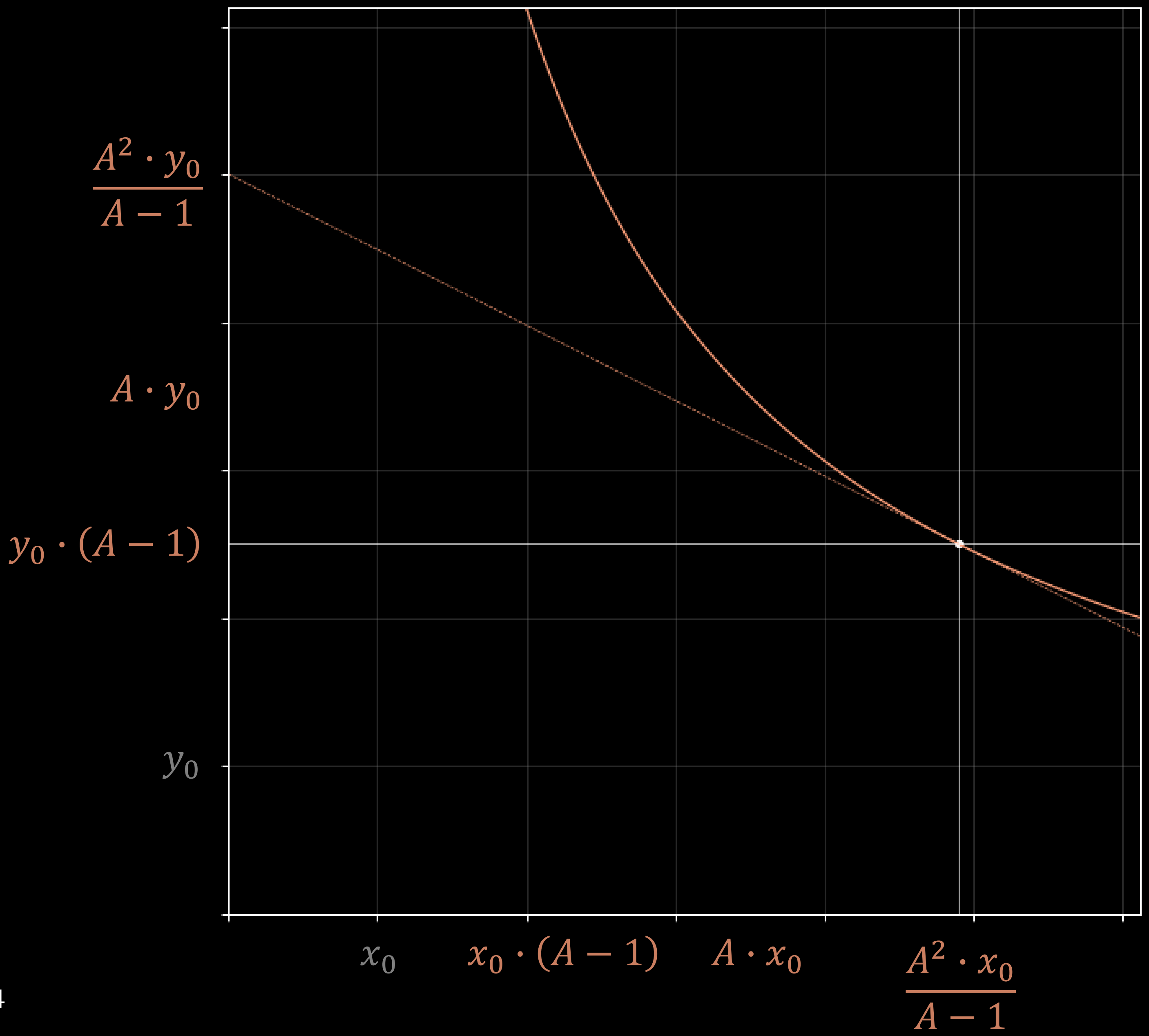
$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



$$-\frac{\partial y}{\partial x} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$-\frac{\partial y}{\partial x} = \frac{y_0 \cdot (A-1)}{\frac{A^2 \cdot x_0}{A-1}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$



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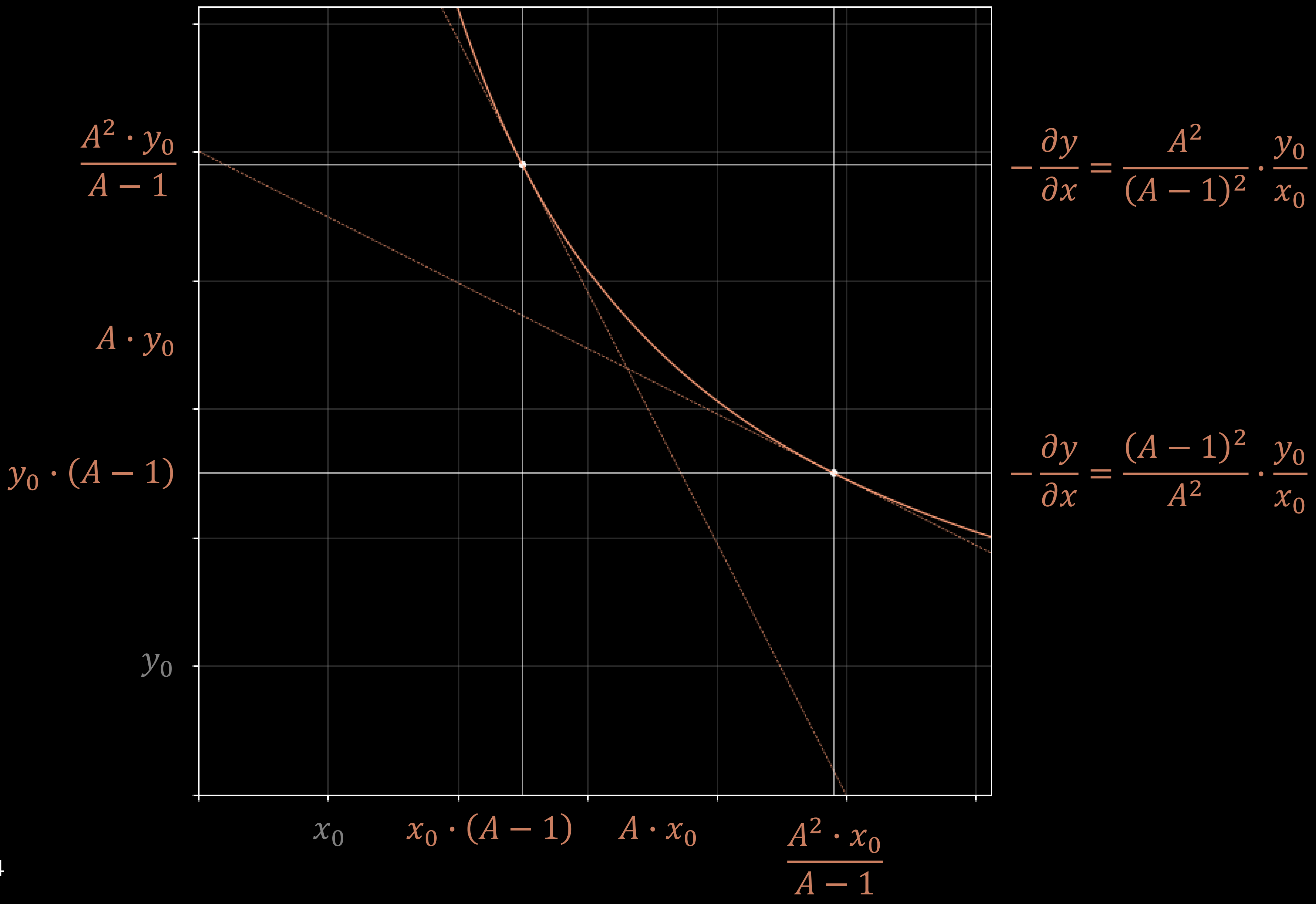

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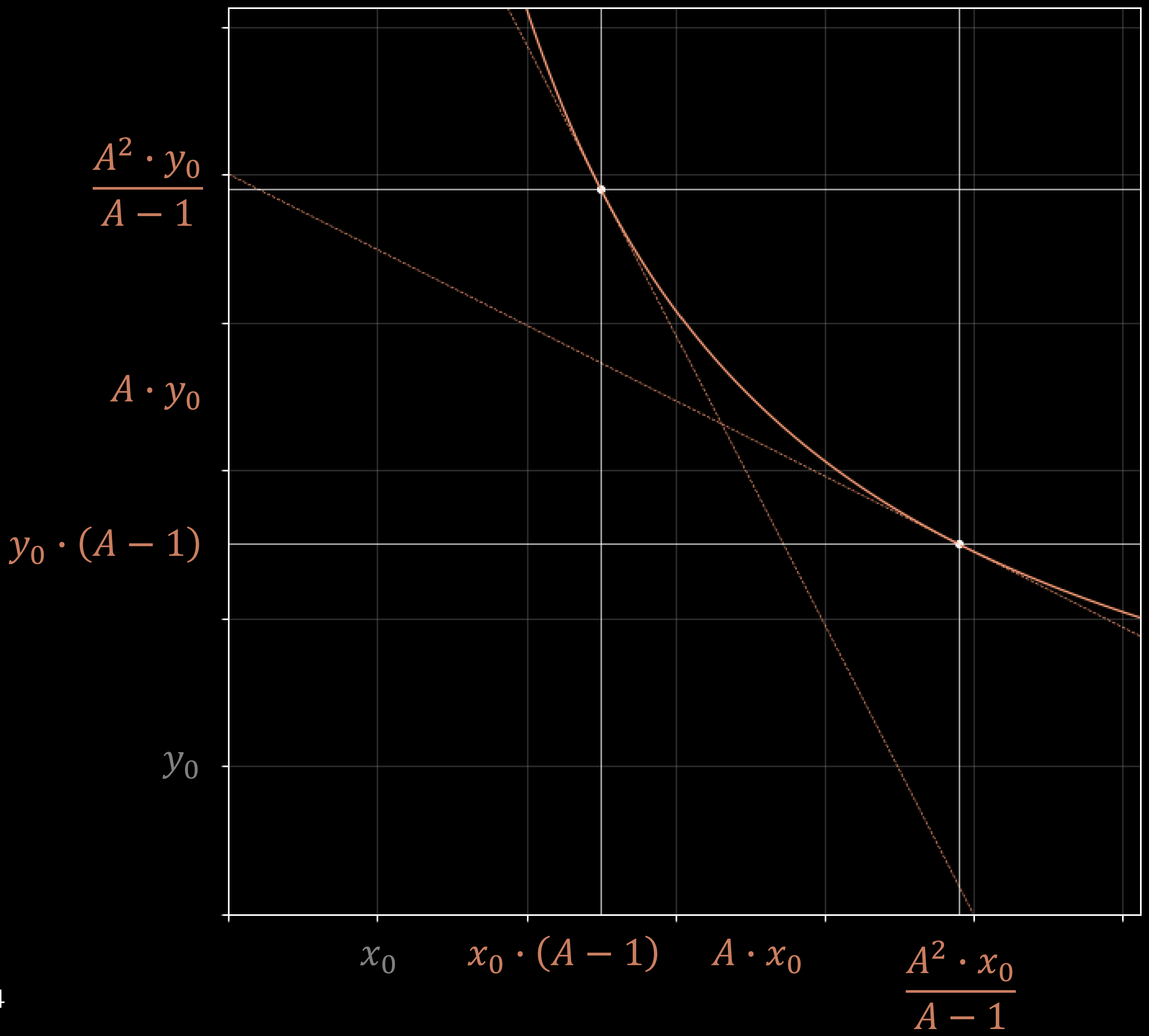

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$

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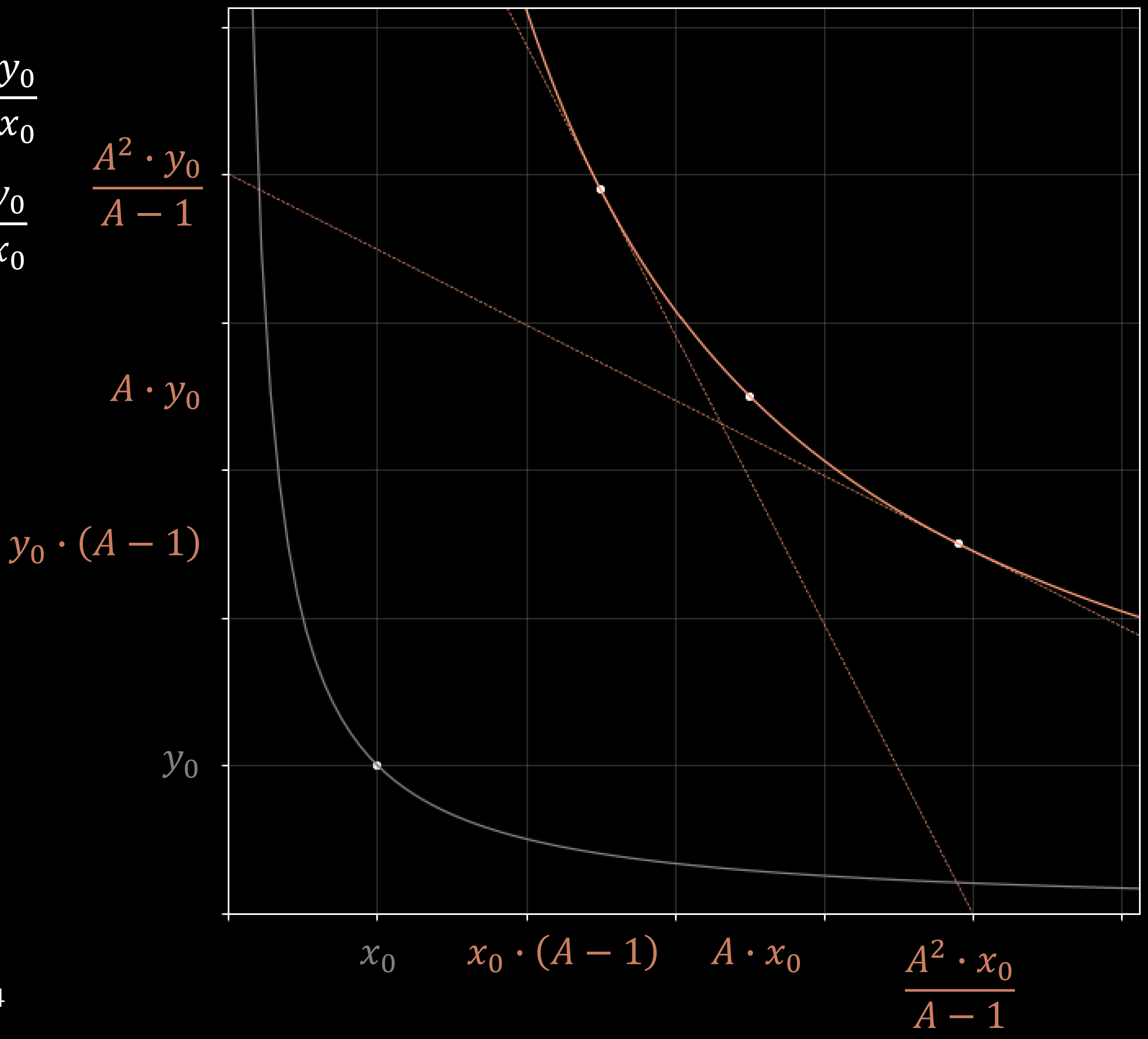

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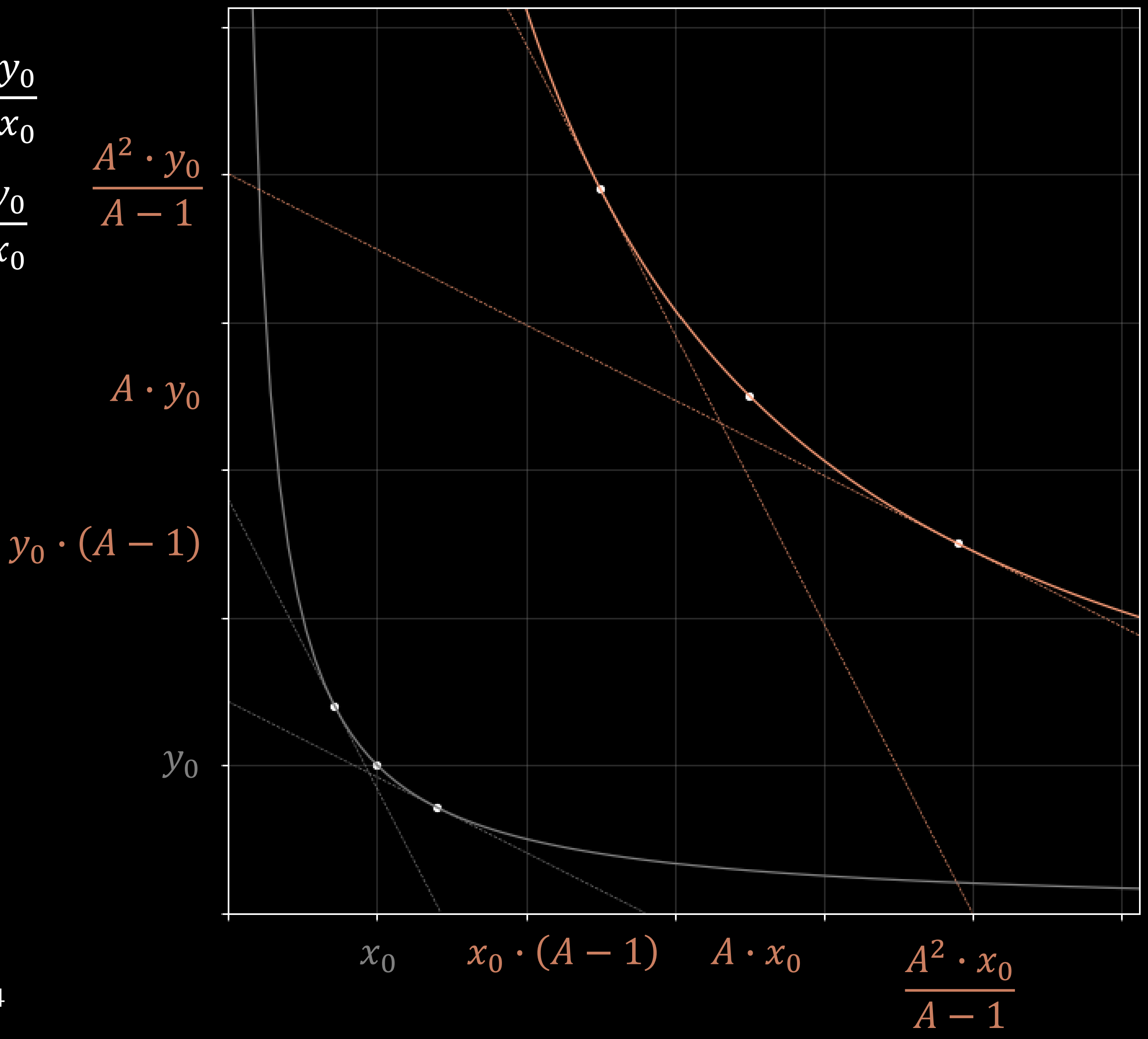

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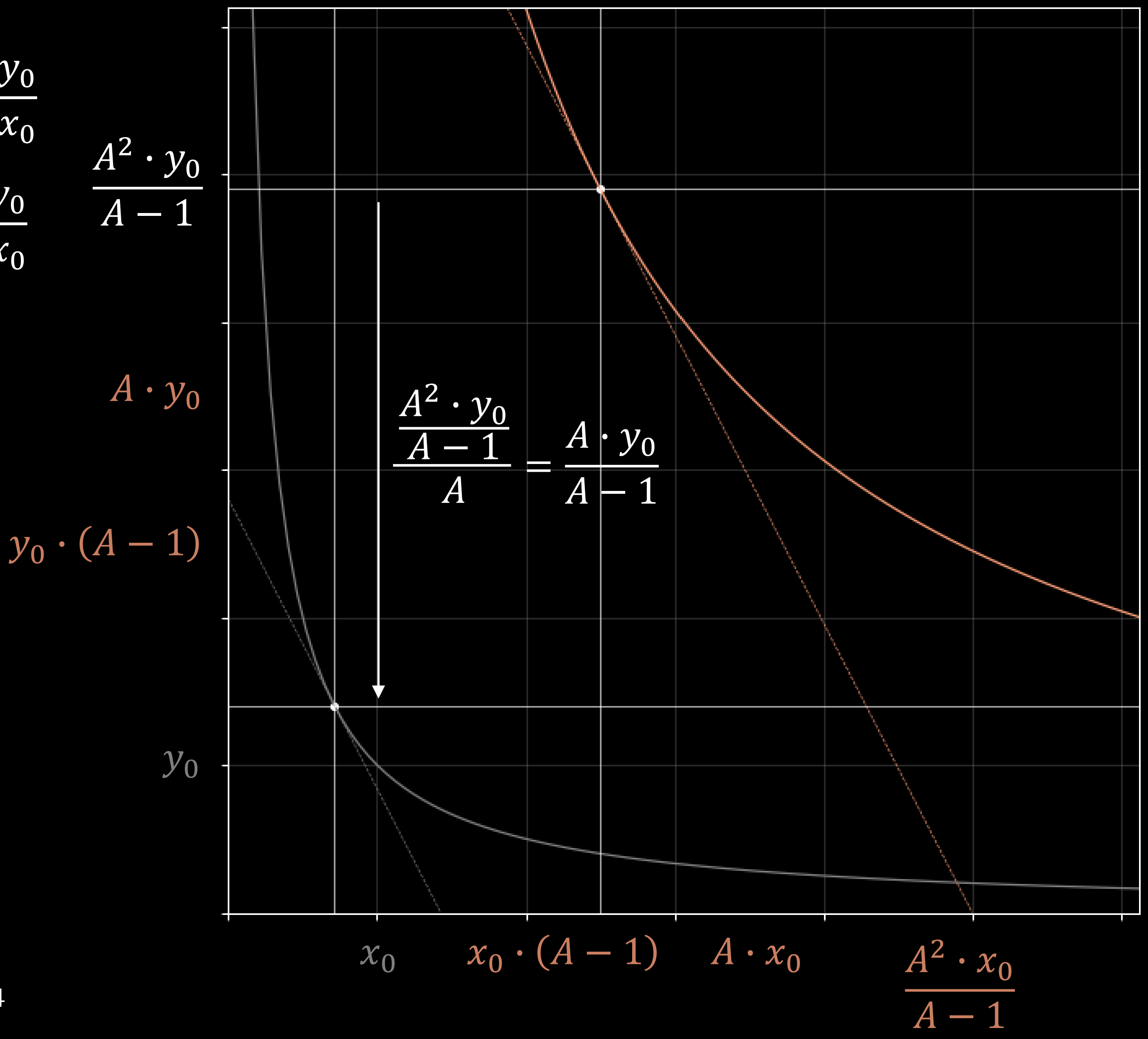

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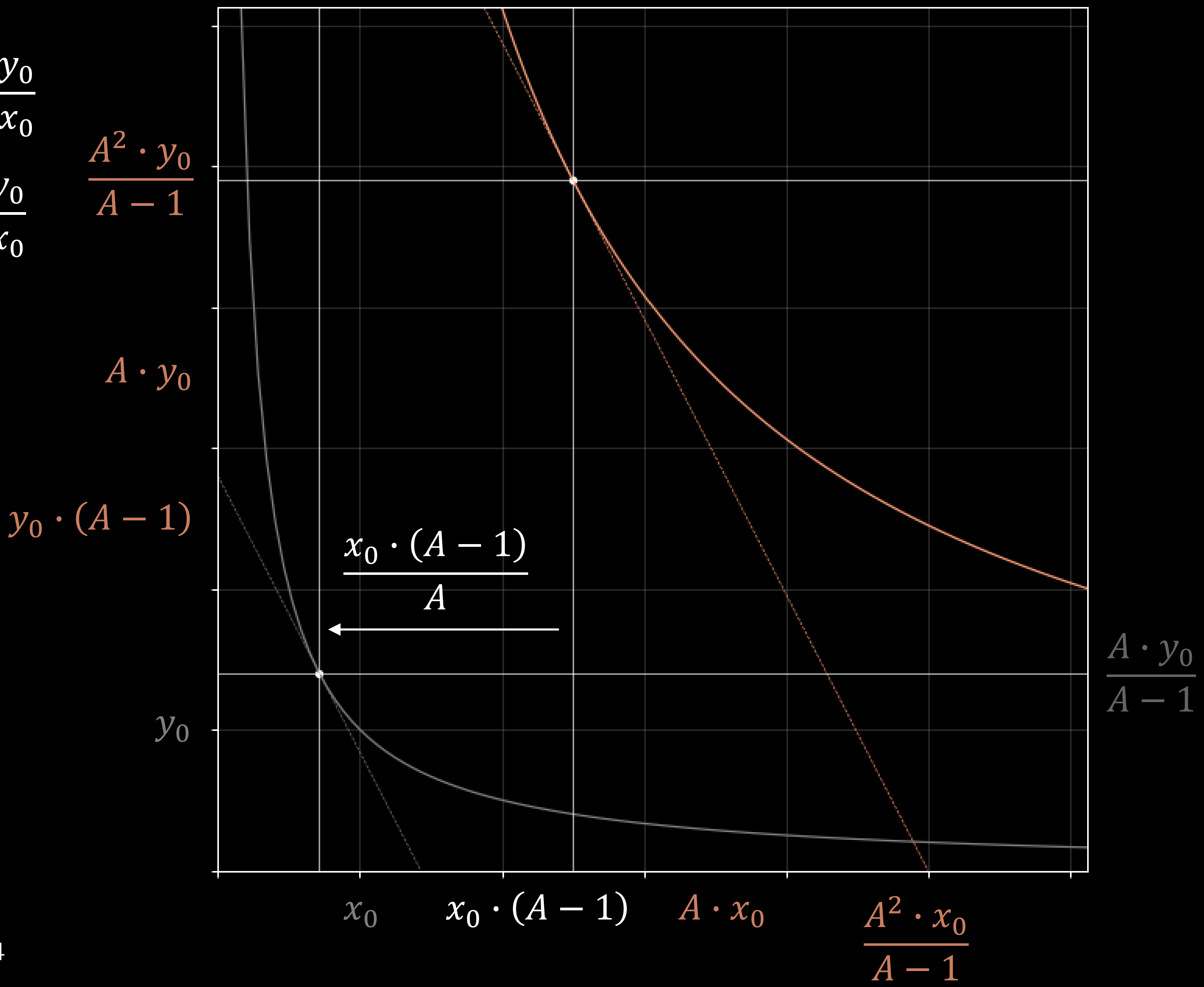

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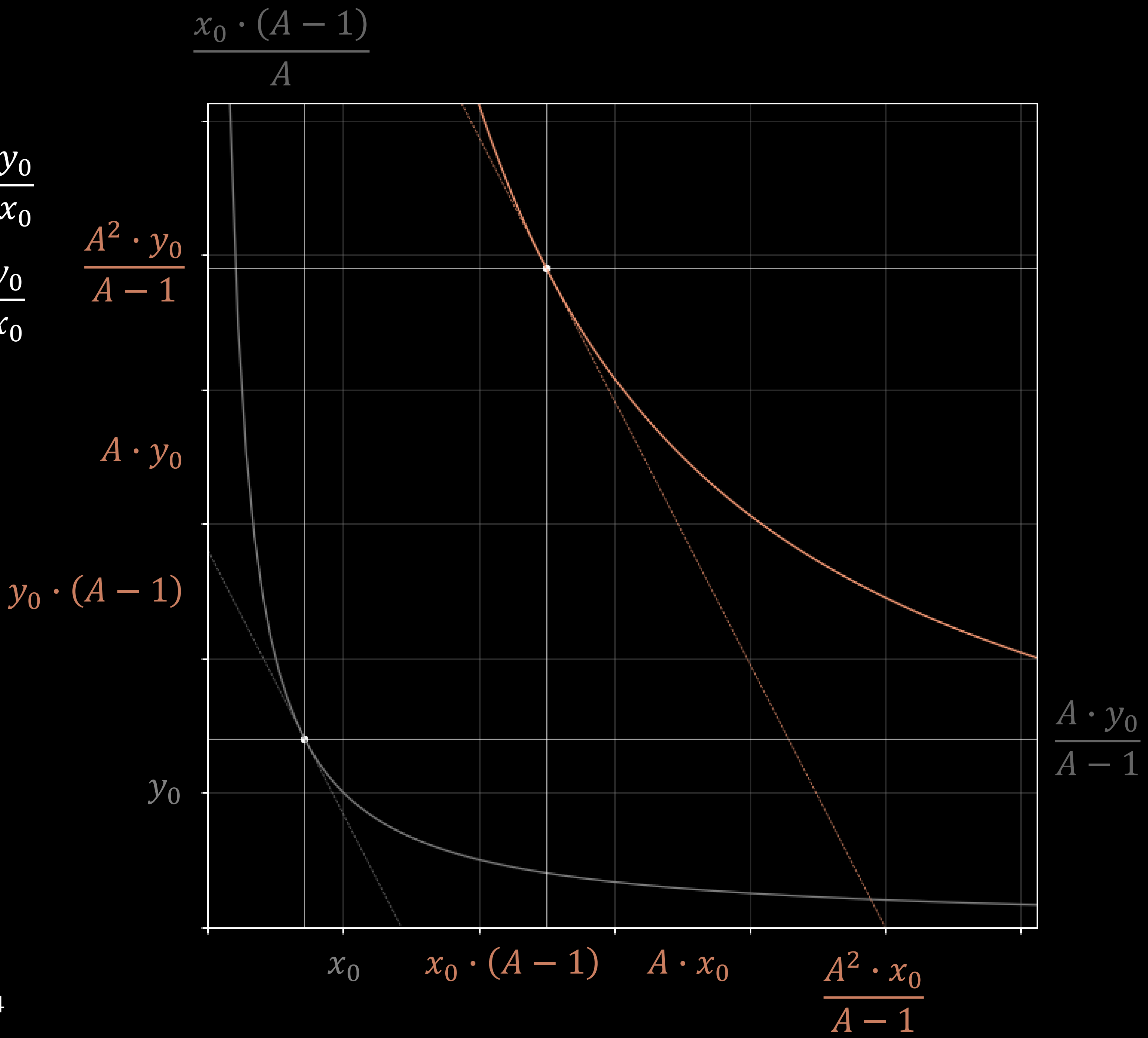

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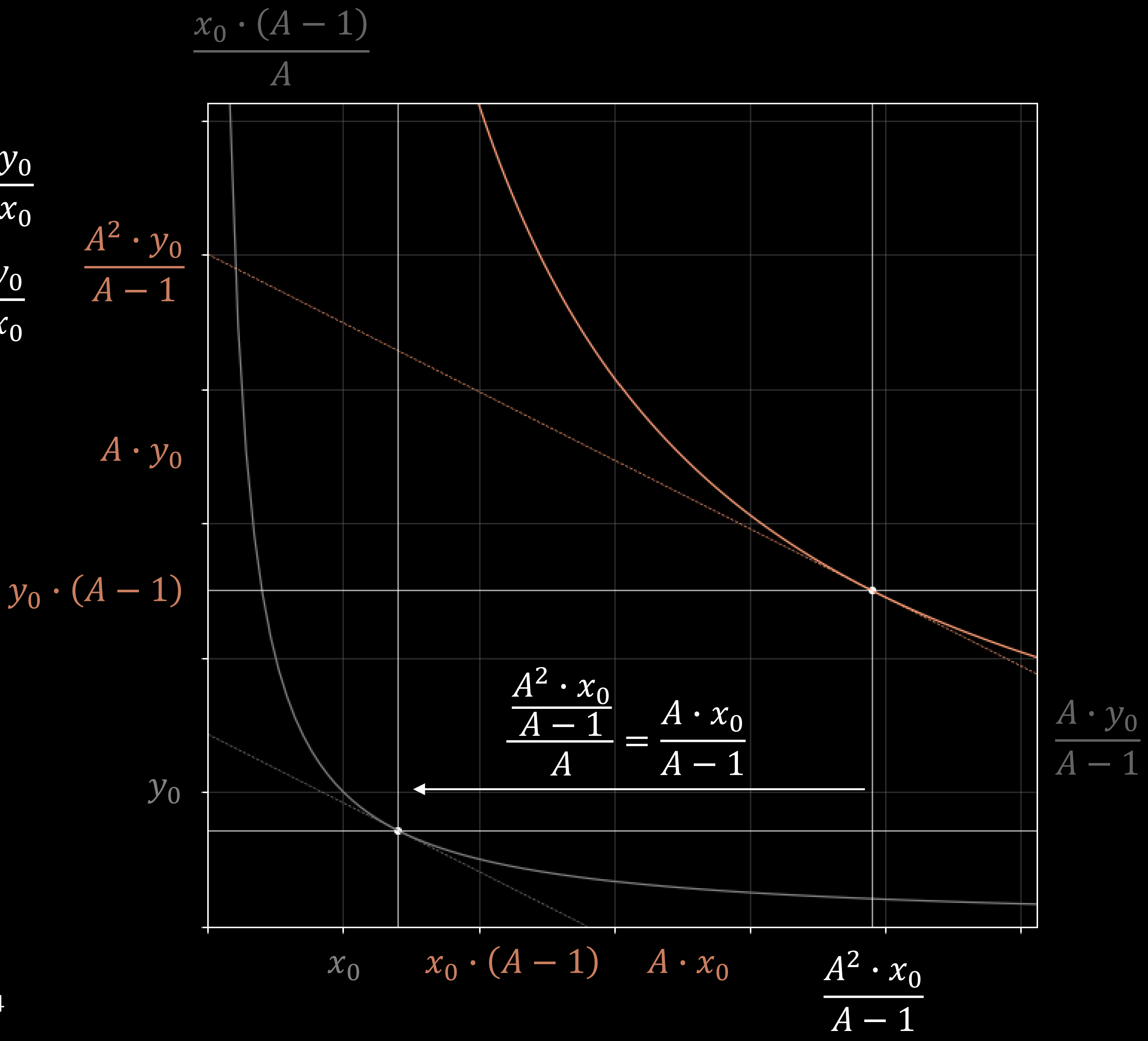

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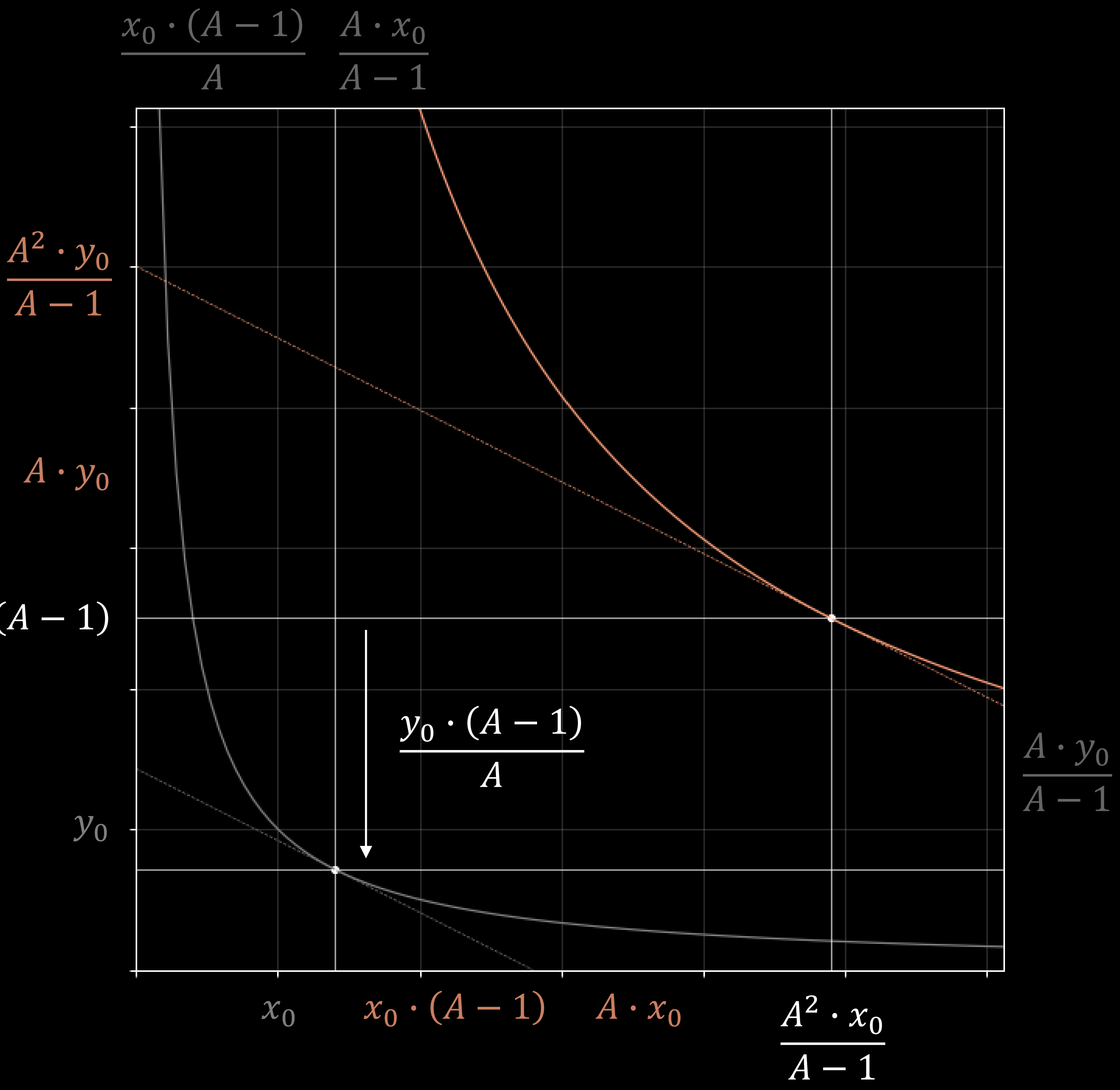

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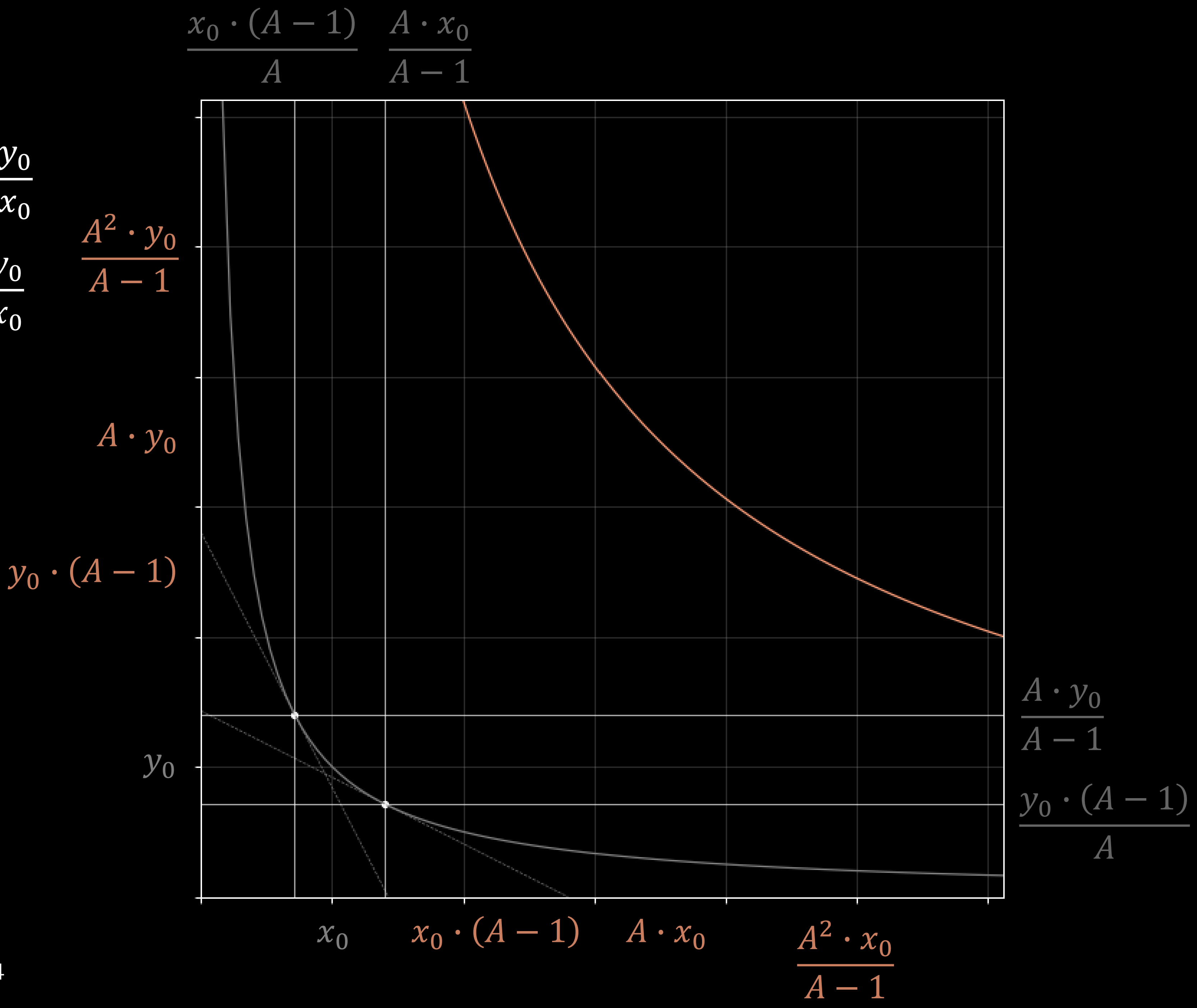

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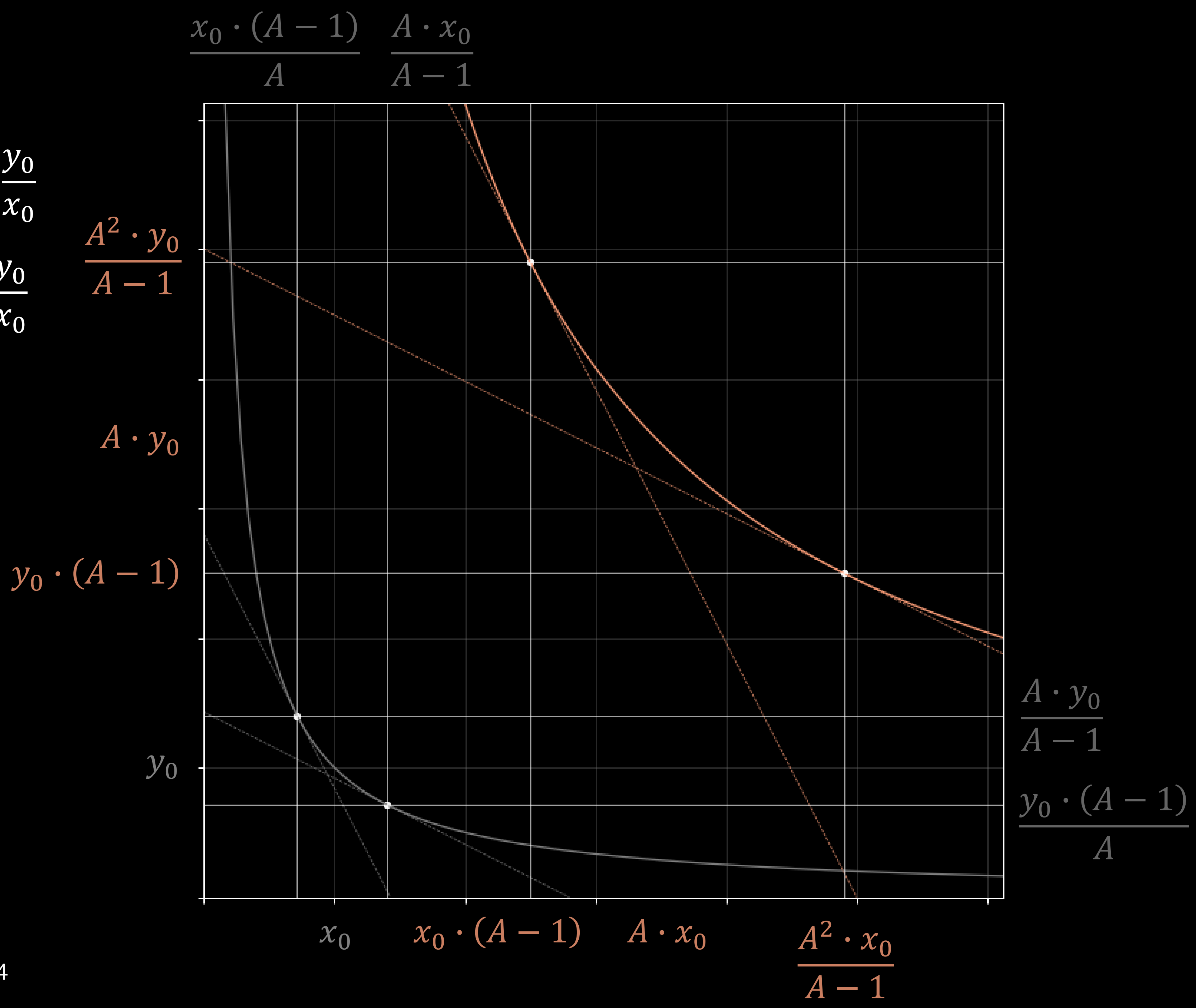

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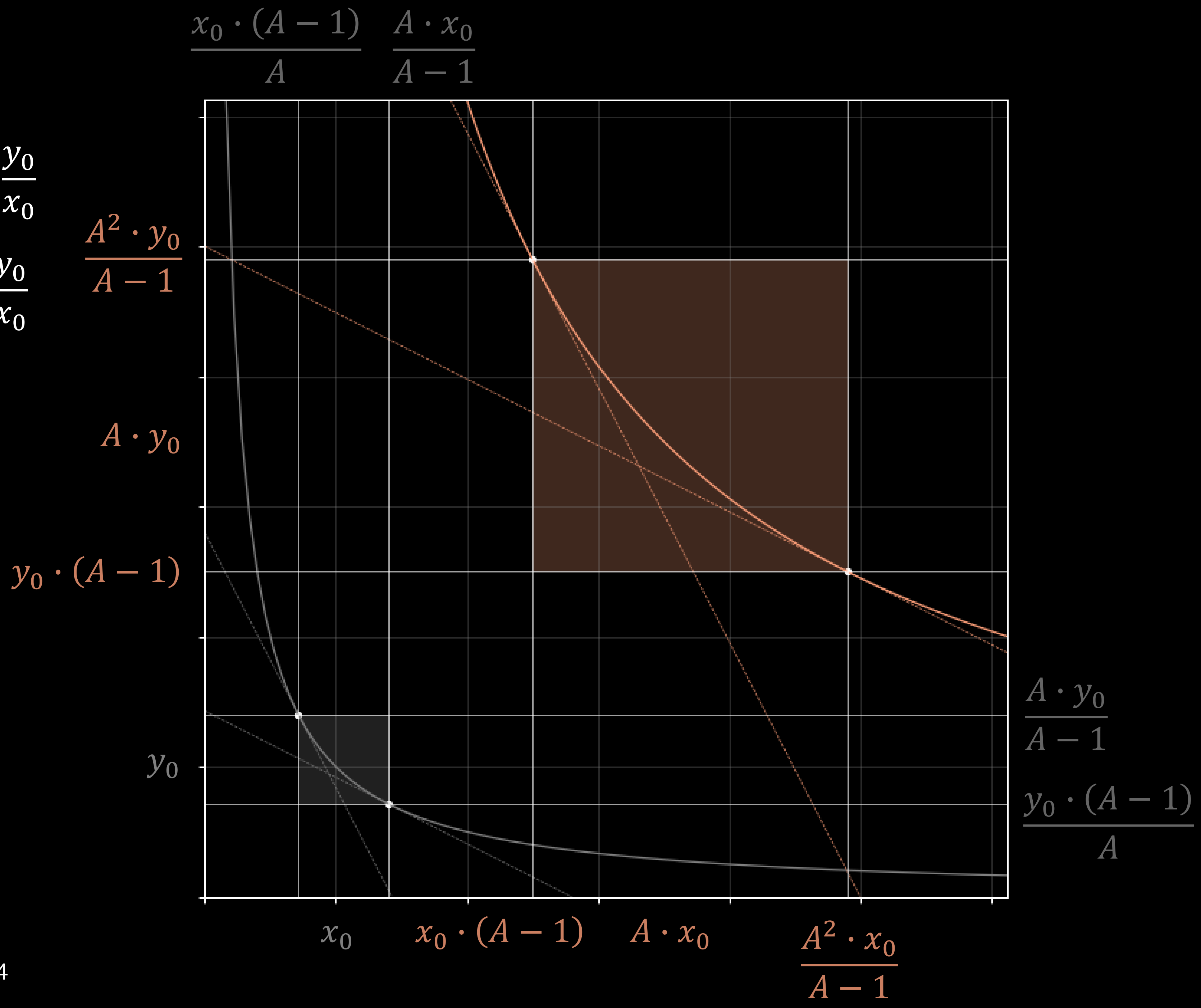

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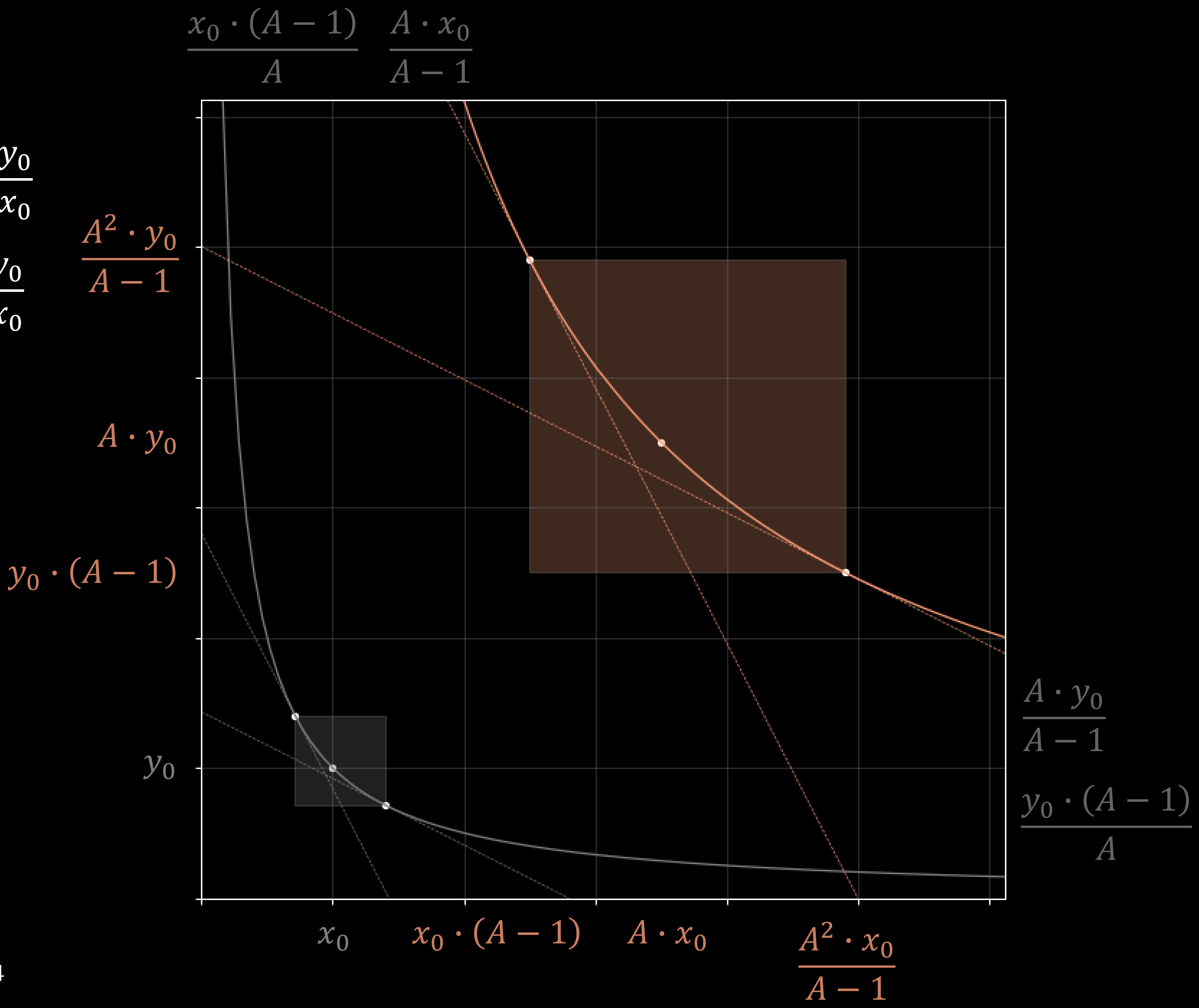

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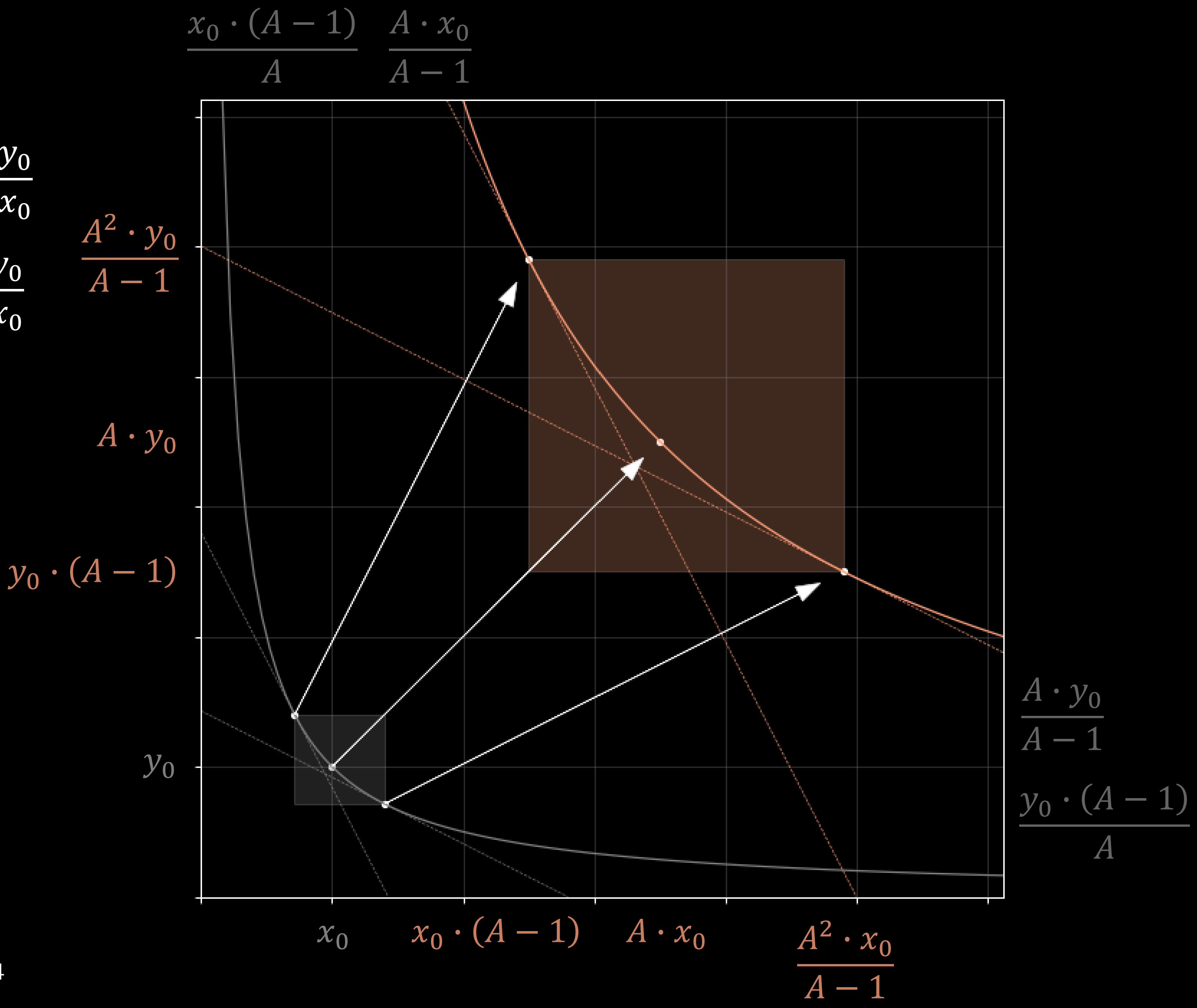

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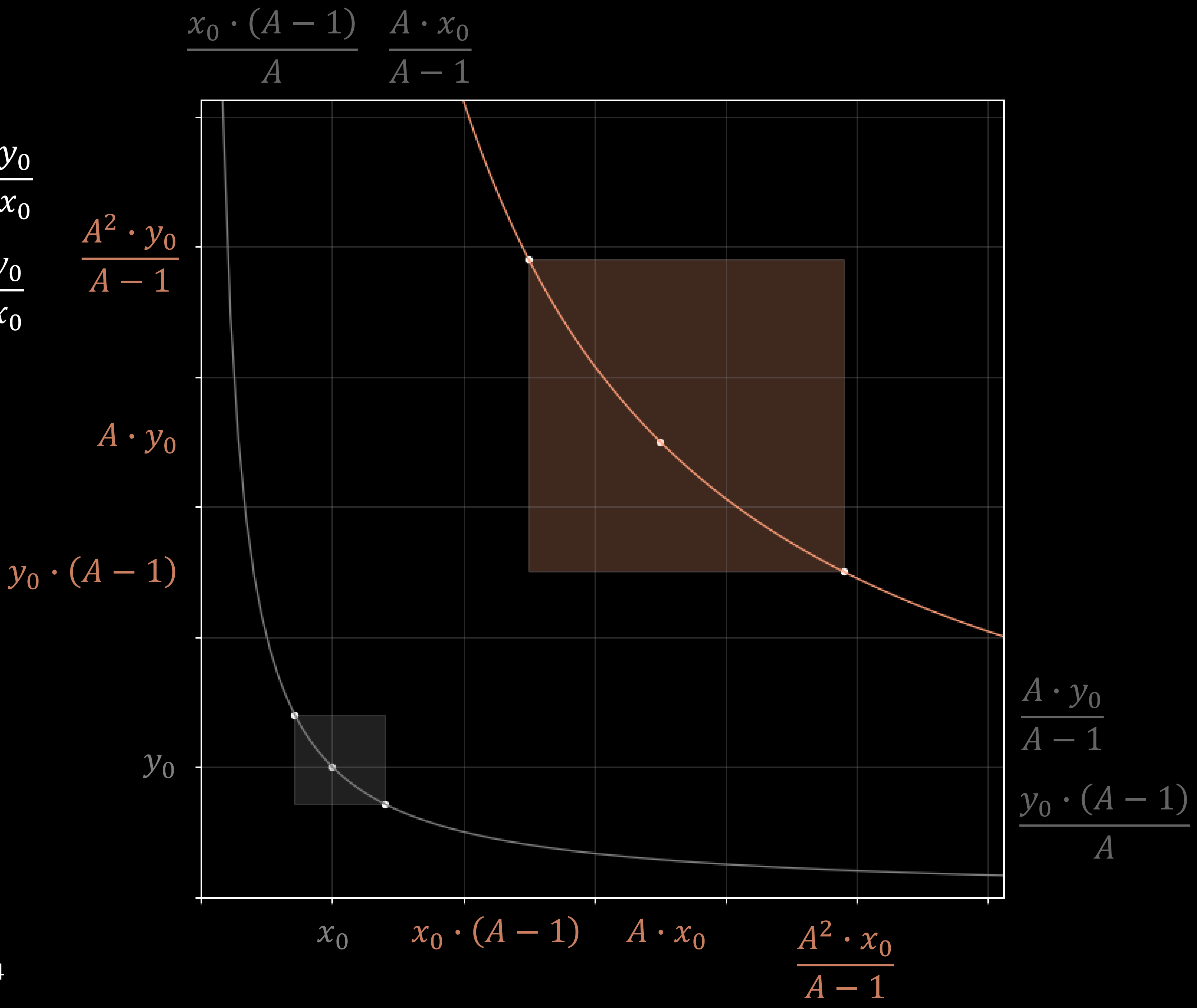

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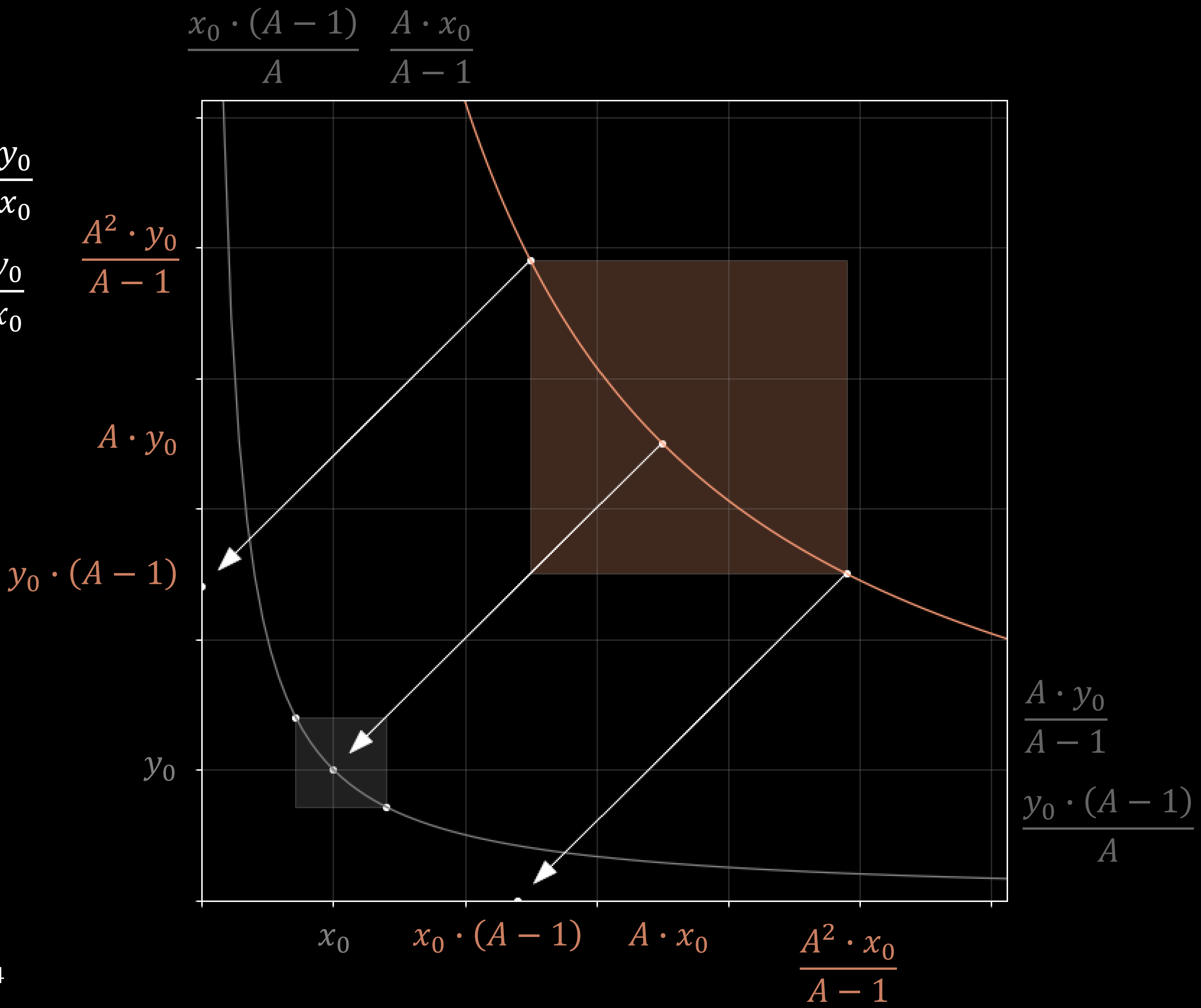

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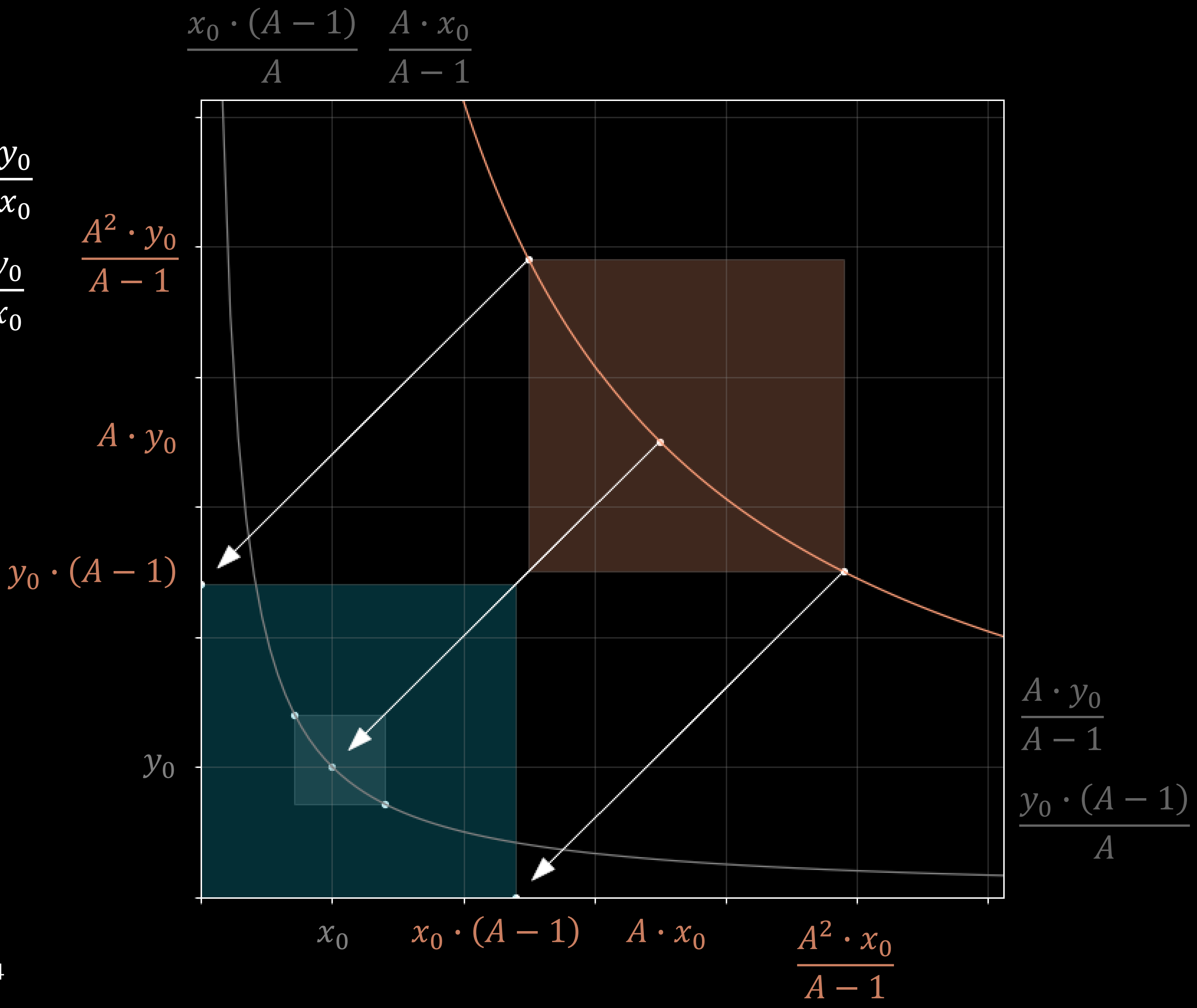

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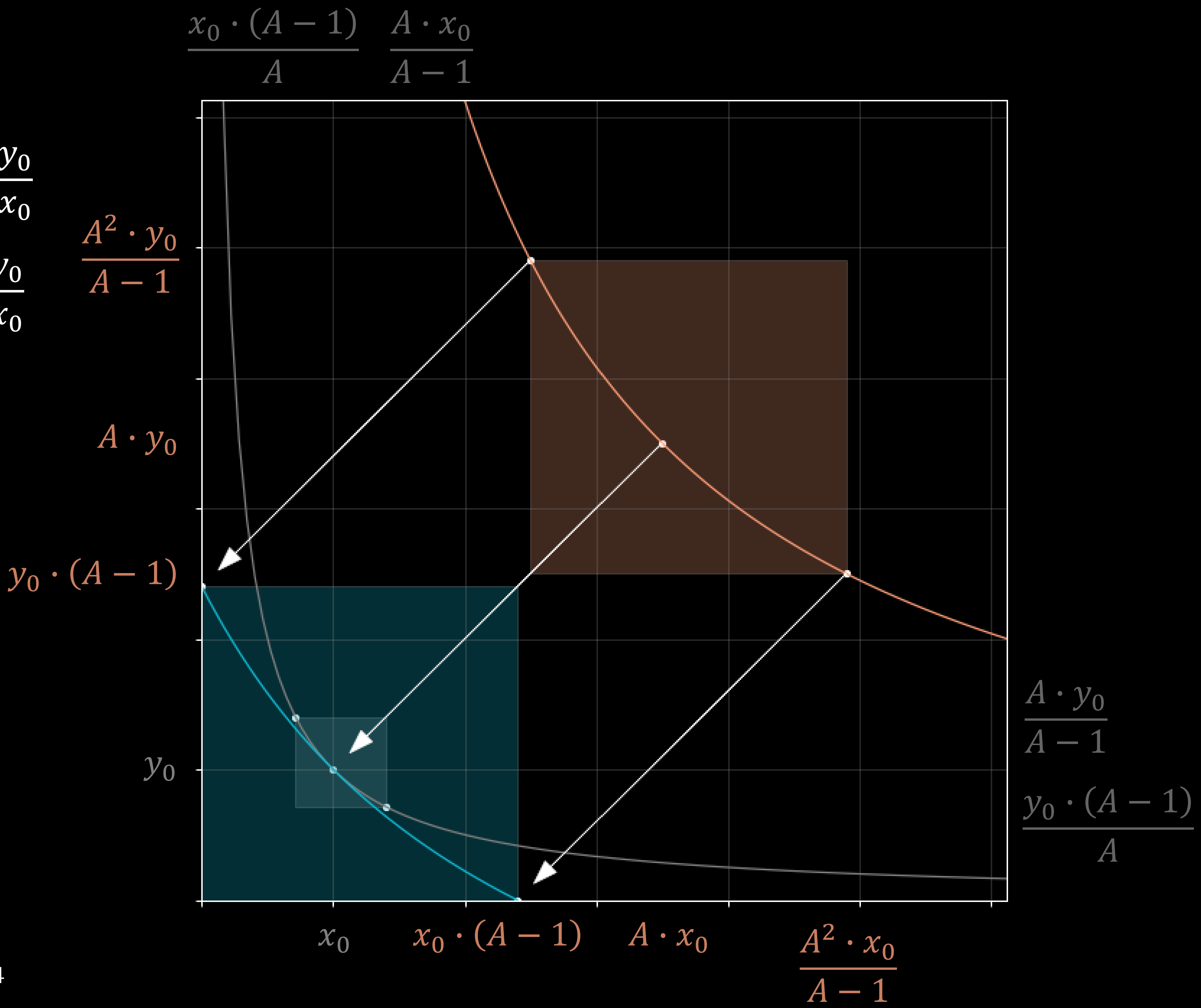

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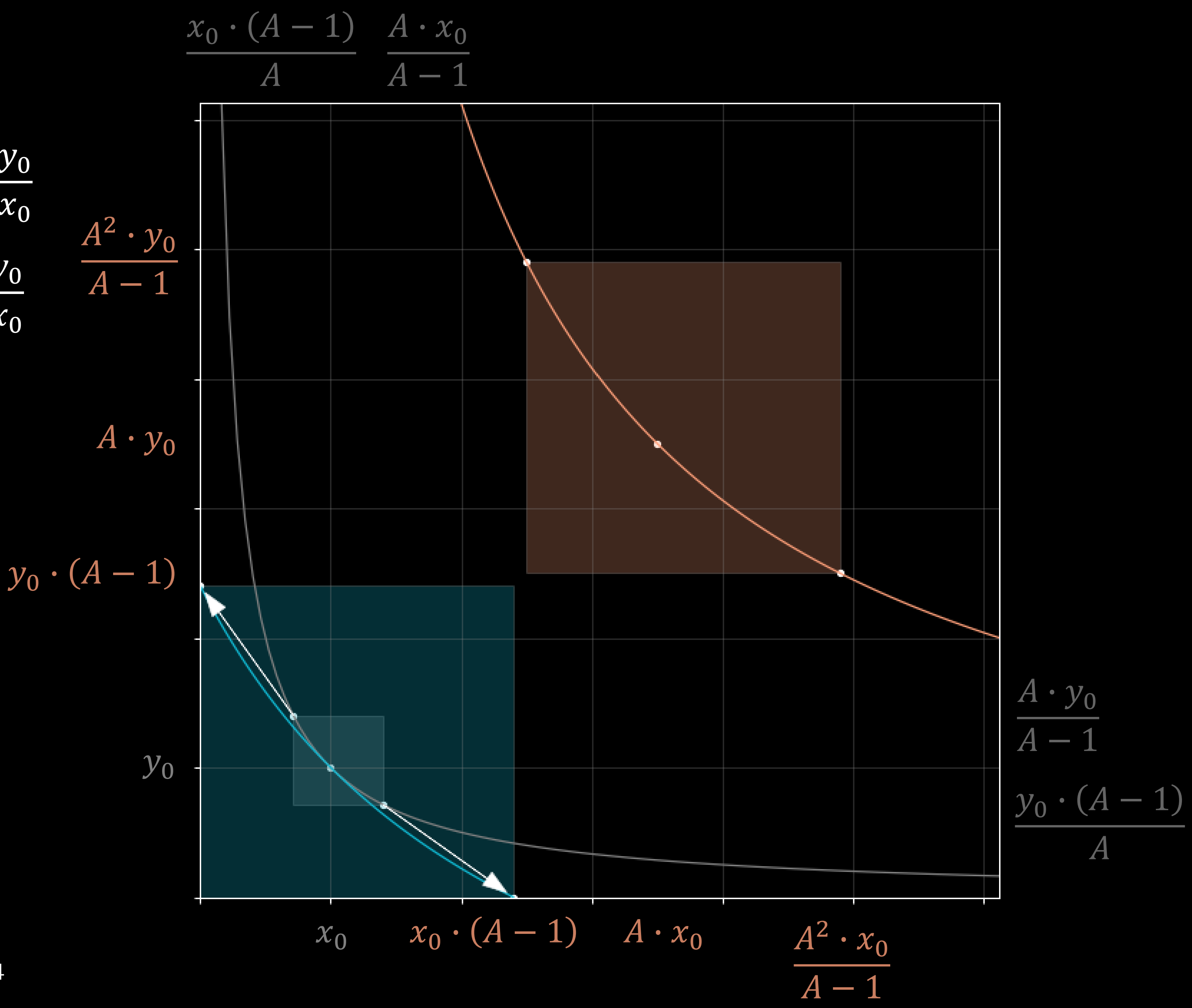

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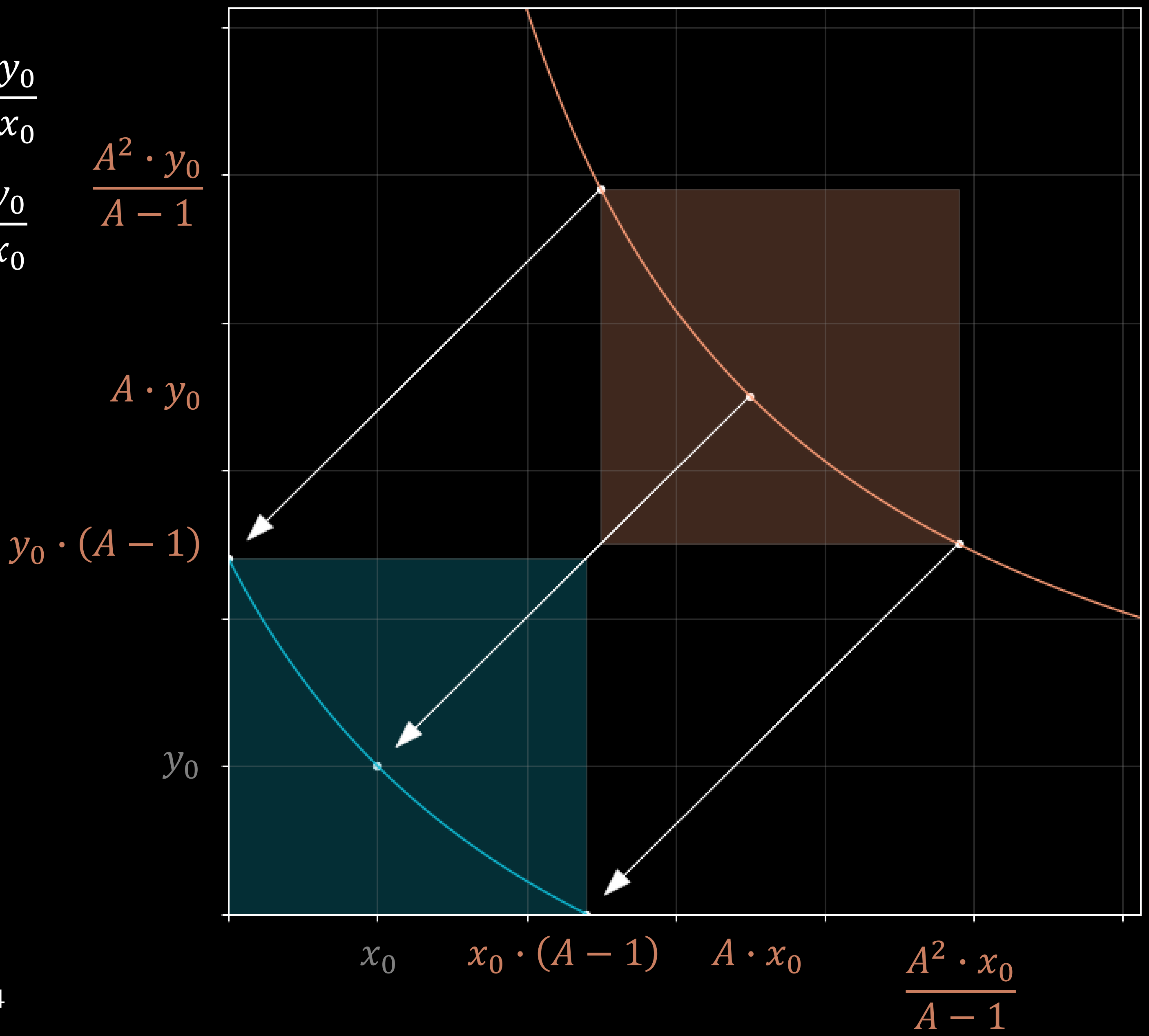

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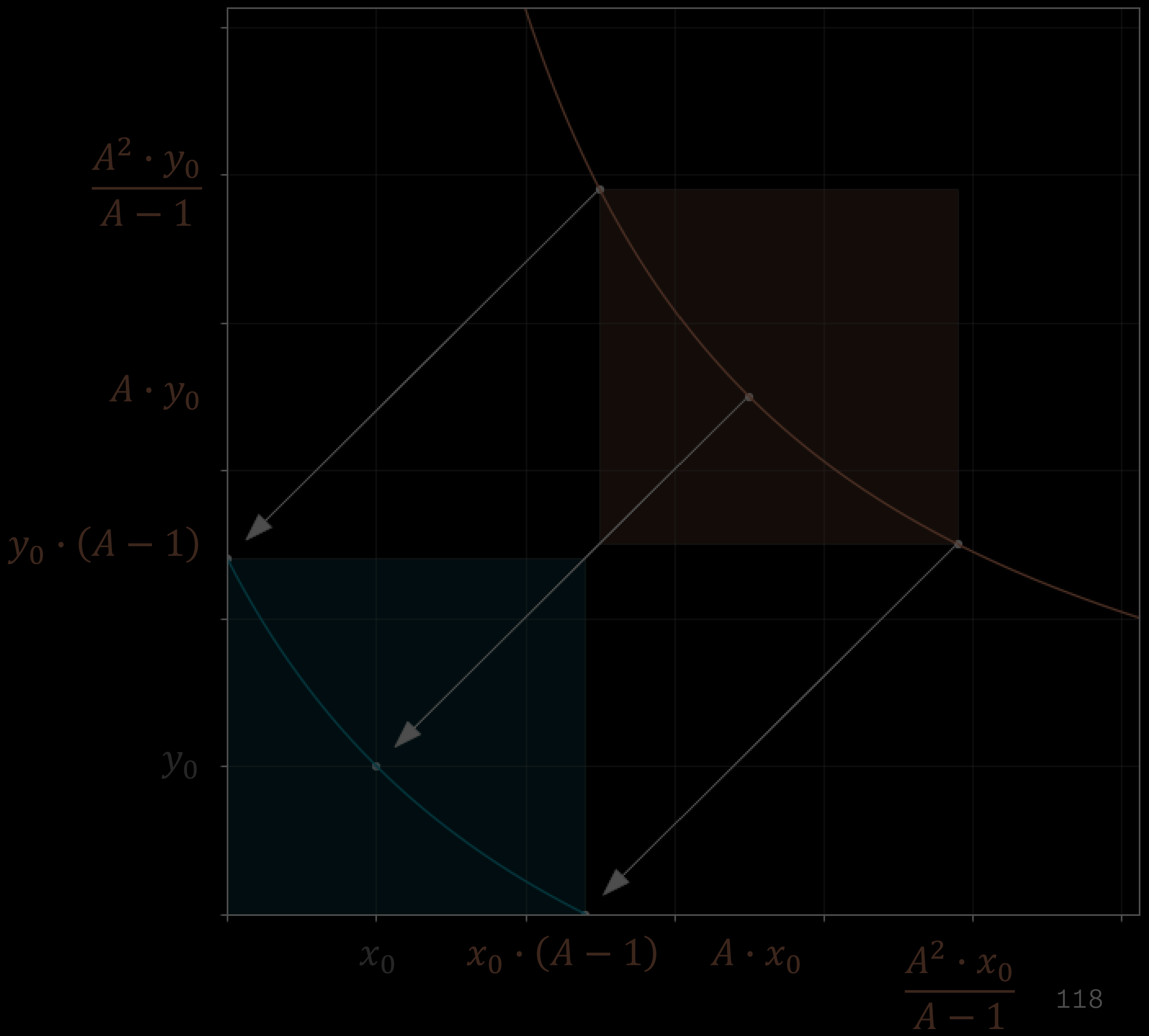
$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$\downarrow$$

$$(x+h) \cdot (y+v) = A^2 \cdot x_0 \cdot y_0$$

$h =$ horizontal shift

$v =$ vertical shift



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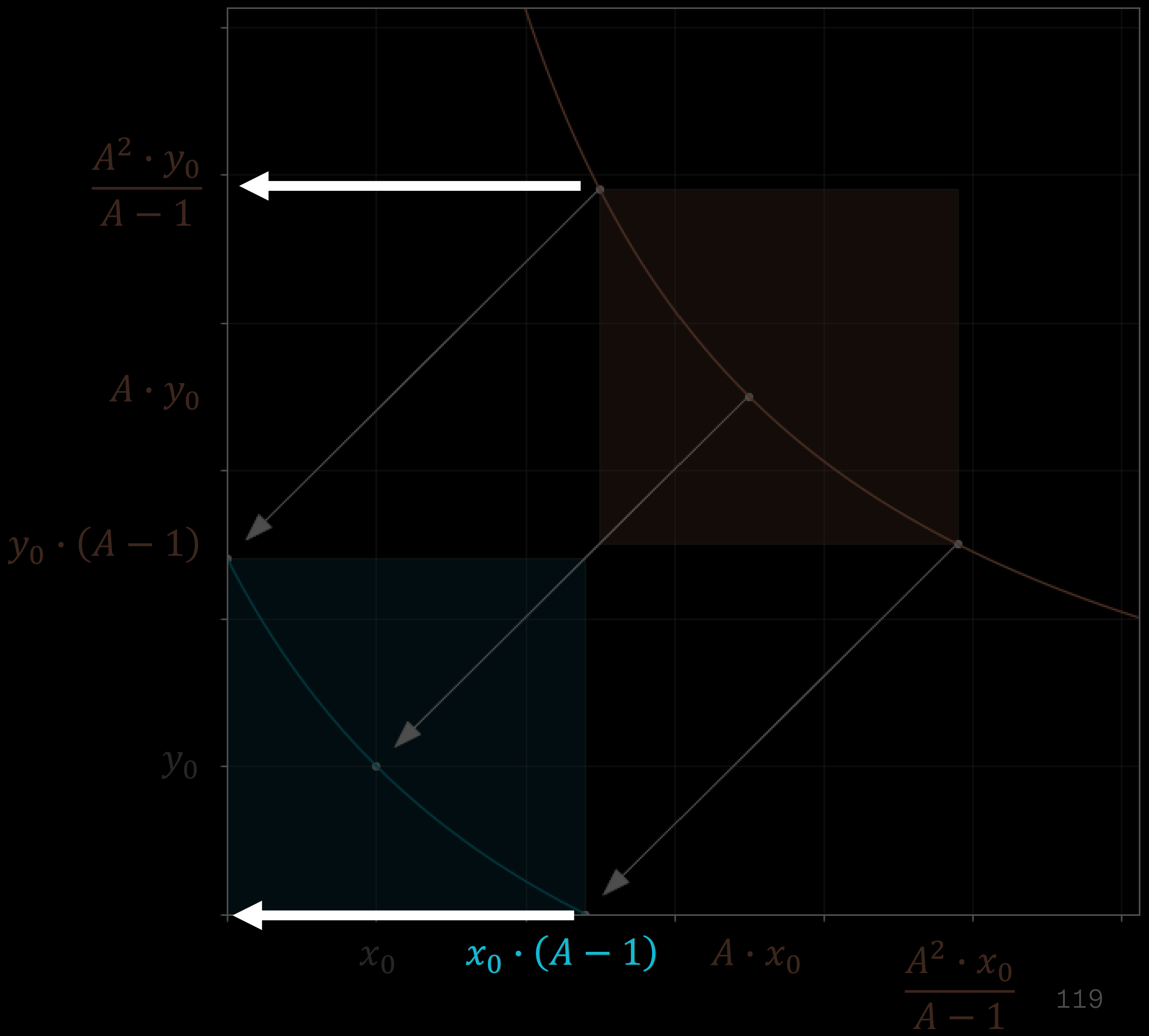

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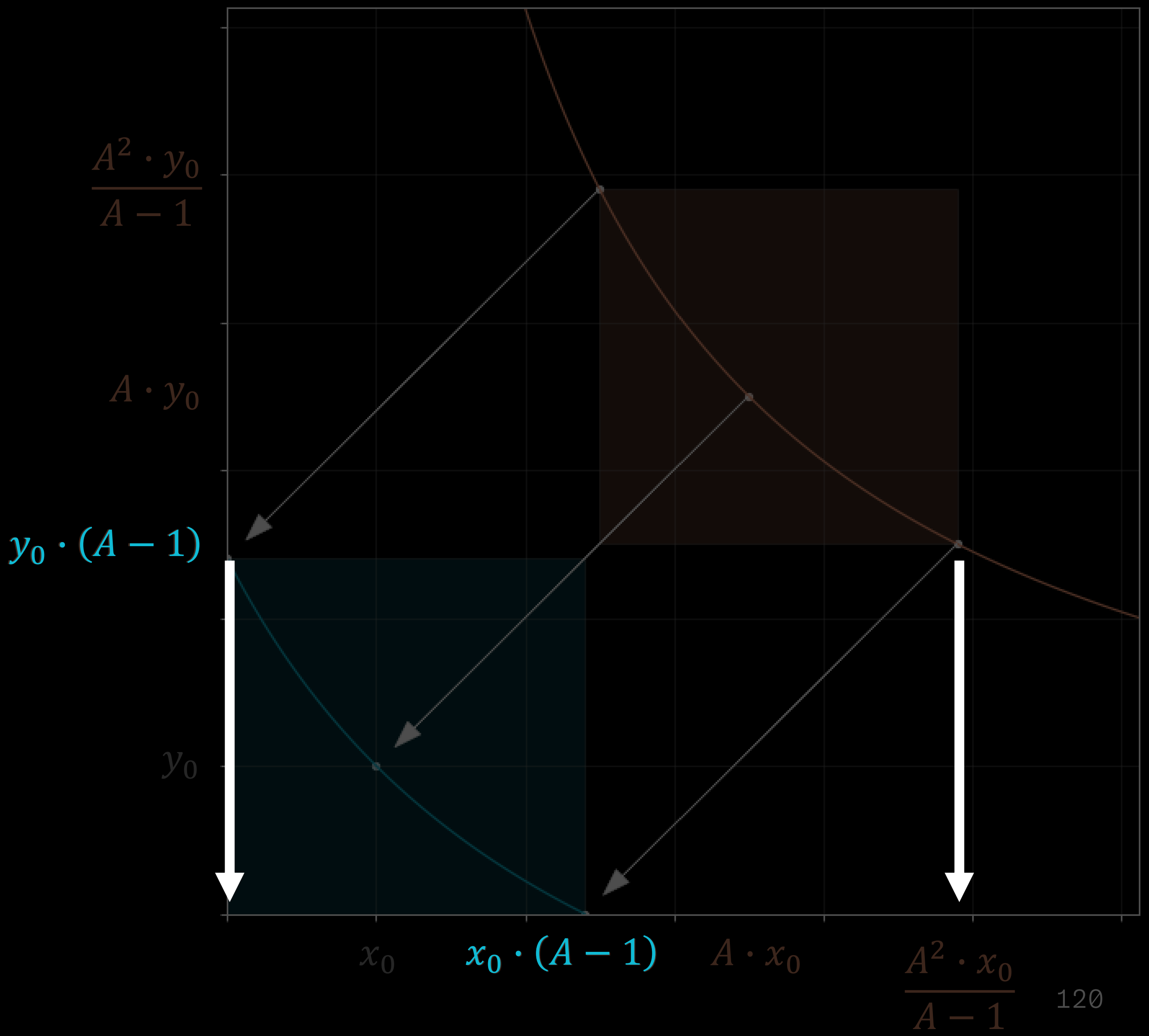

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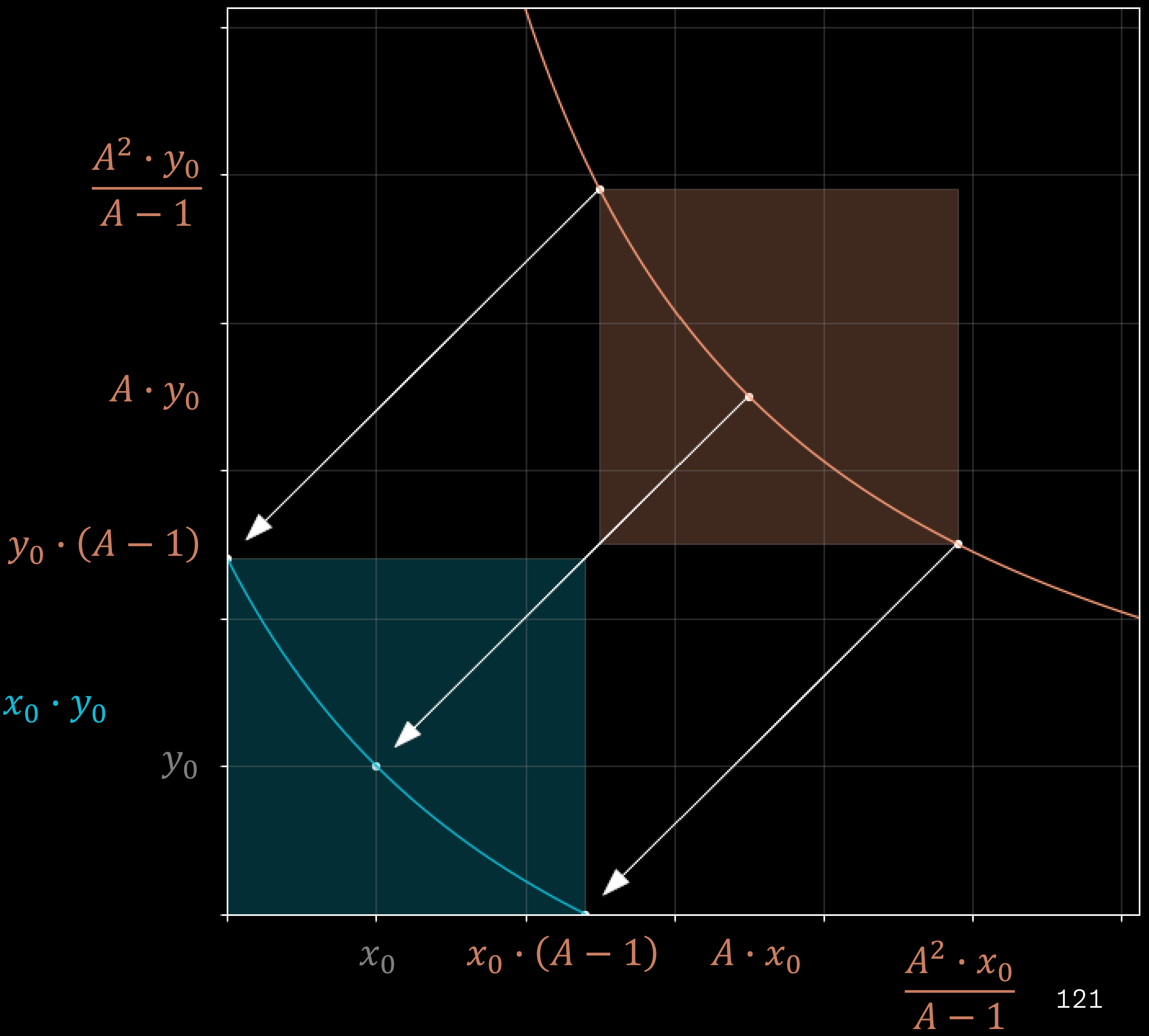
$$\big(x + x_0 \cdot (A-1)\big) \cdot \big(y + y_0 \cdot (A-1)\big) = A^2 \cdot x_0 \cdot y_0$$



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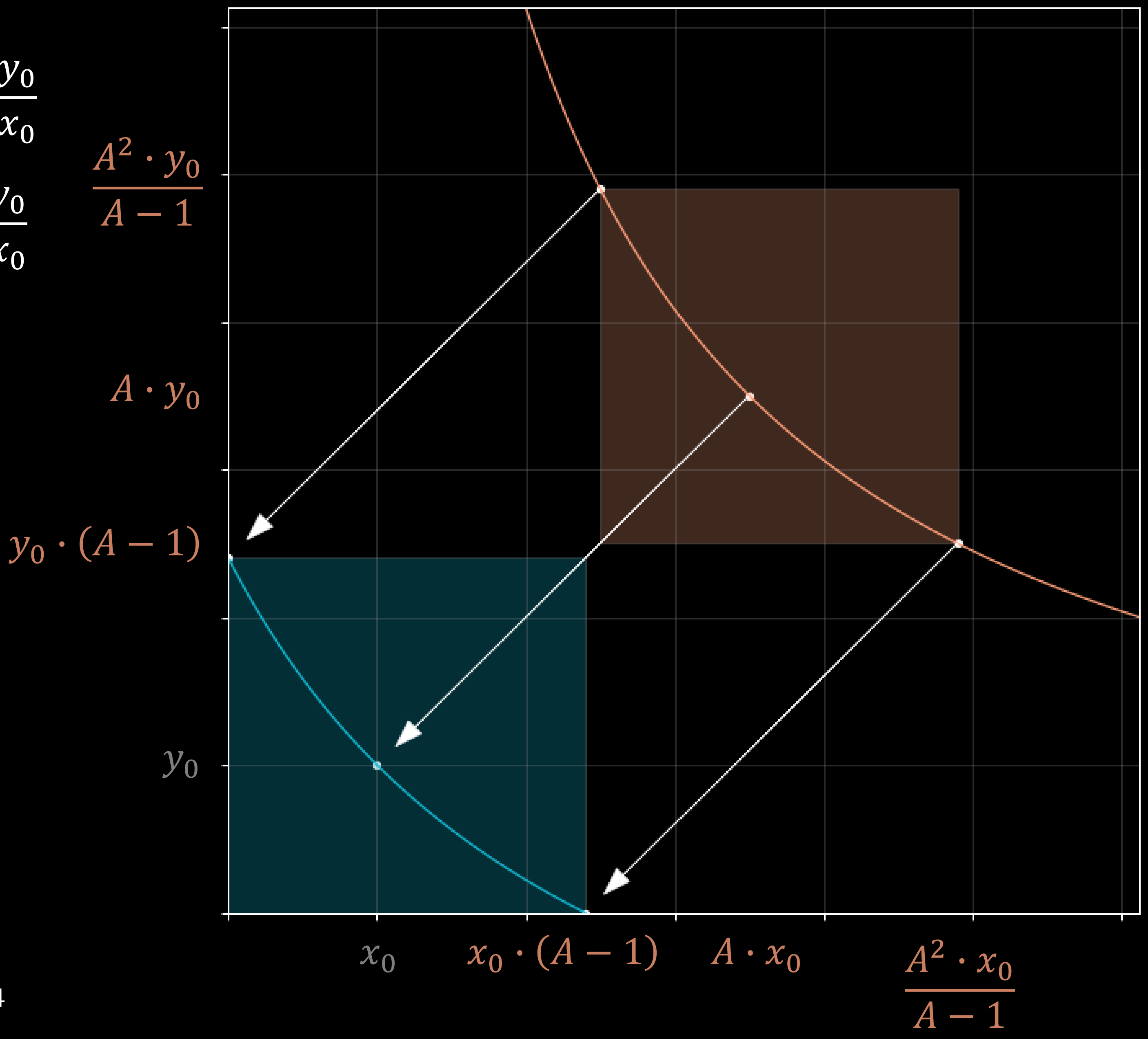

$$\left(x + x_0 \cdot (A-1)\right) \cdot \left(y + y_0 \cdot (A-1)\right) = A^2 \cdot x_0 \cdot y_0 \Big|_{x=0}$$

$$x \cdot y = A^2 \cdot x_0 \cdot y_0$$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

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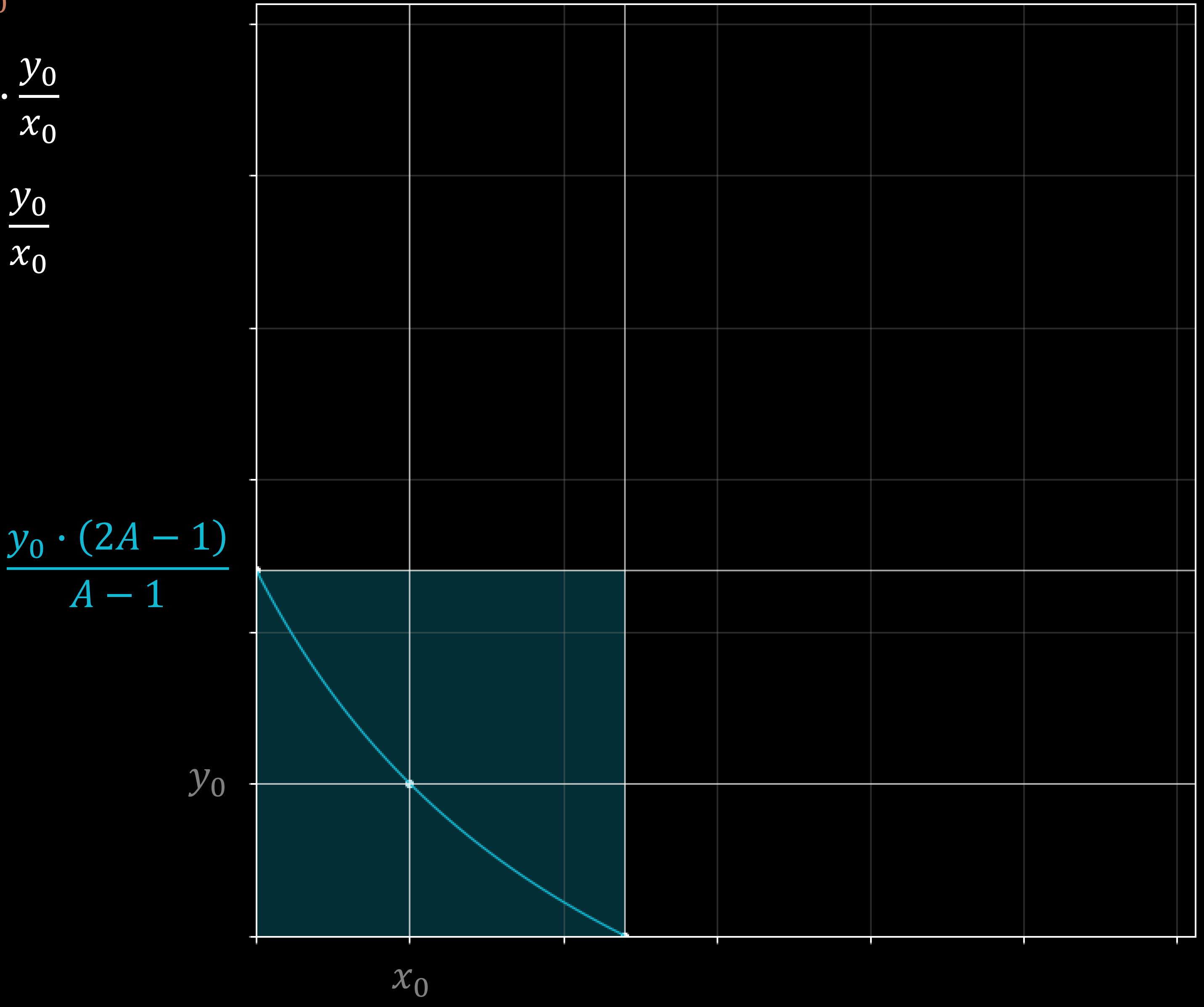

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$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

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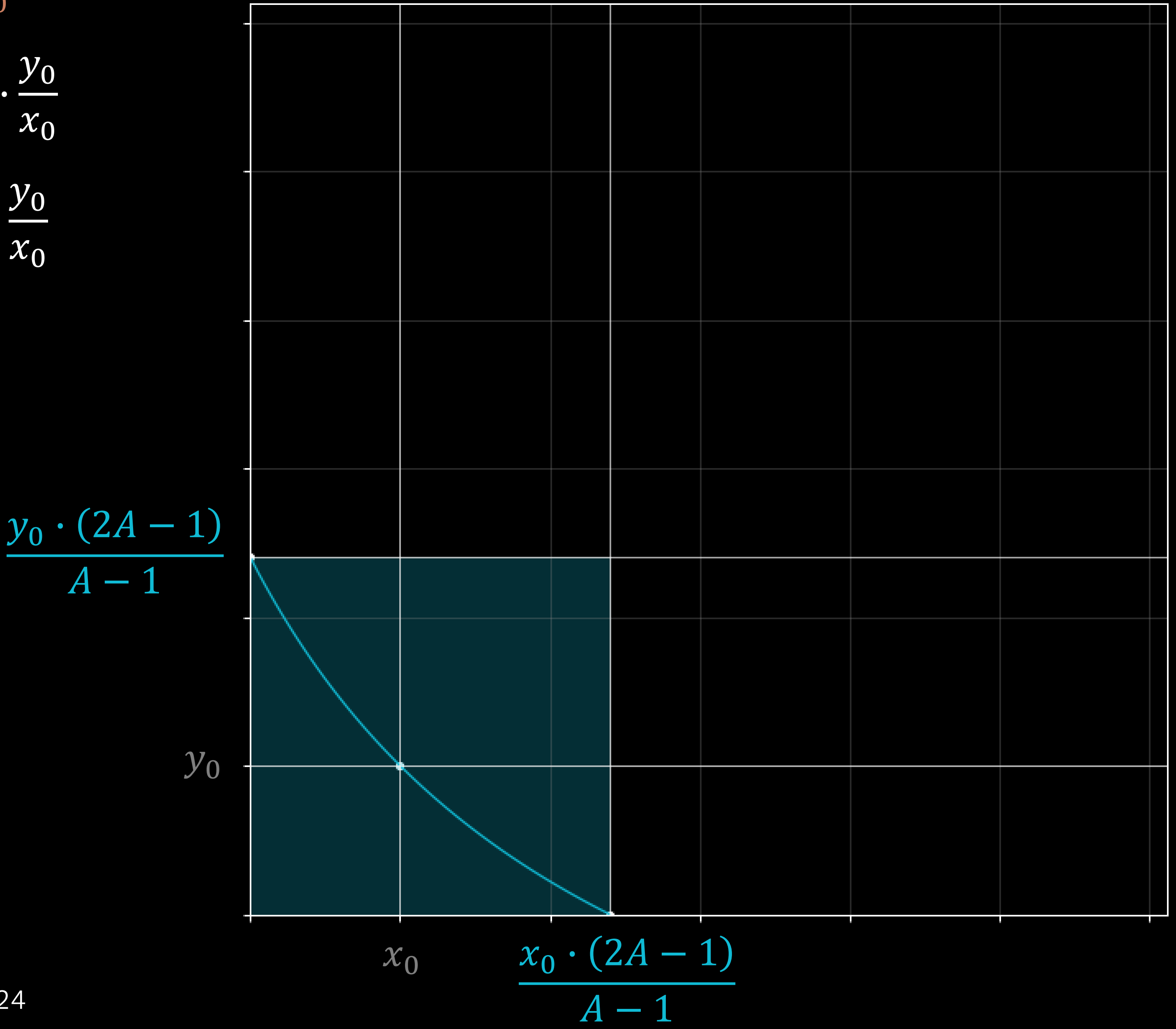
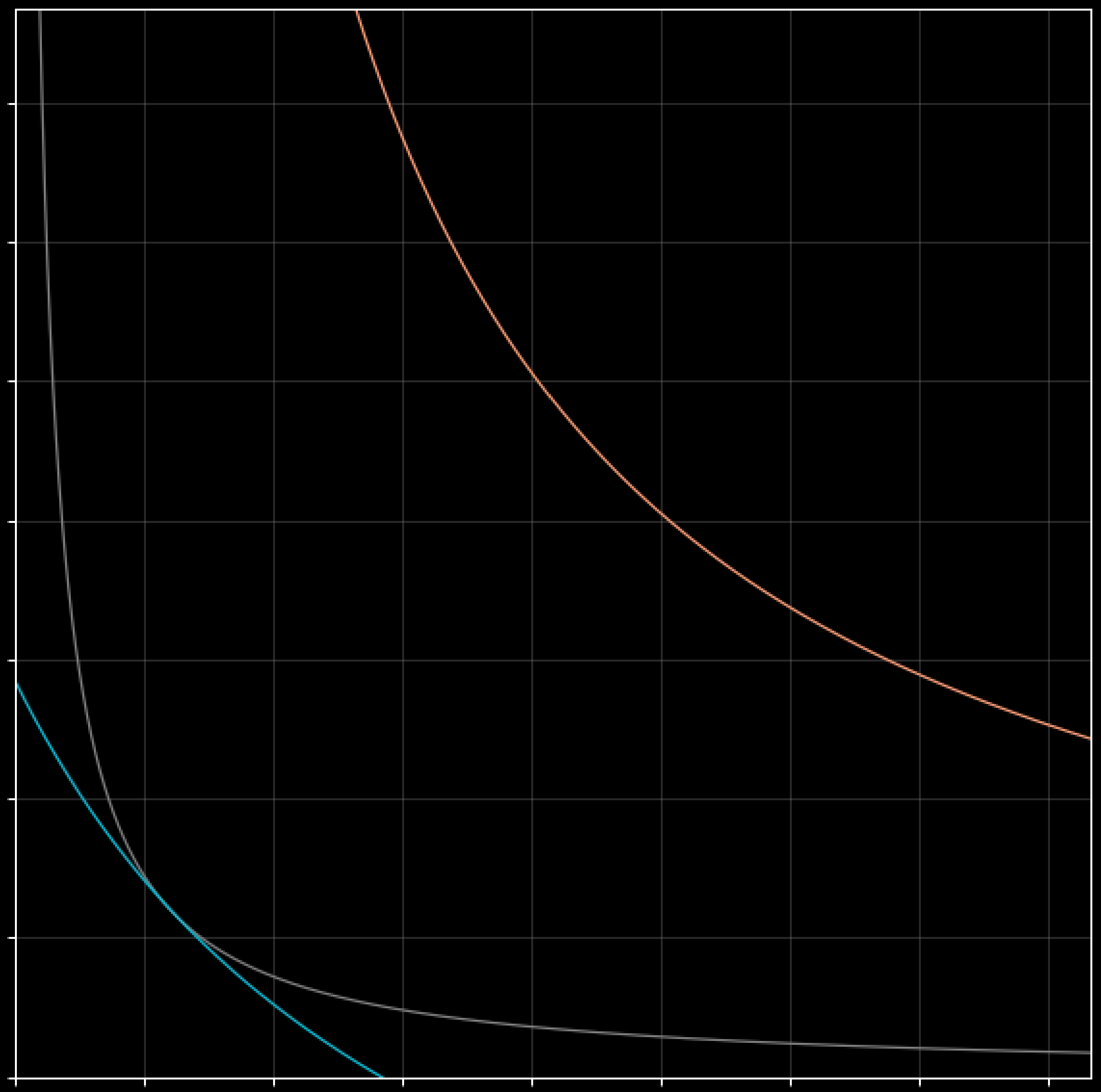
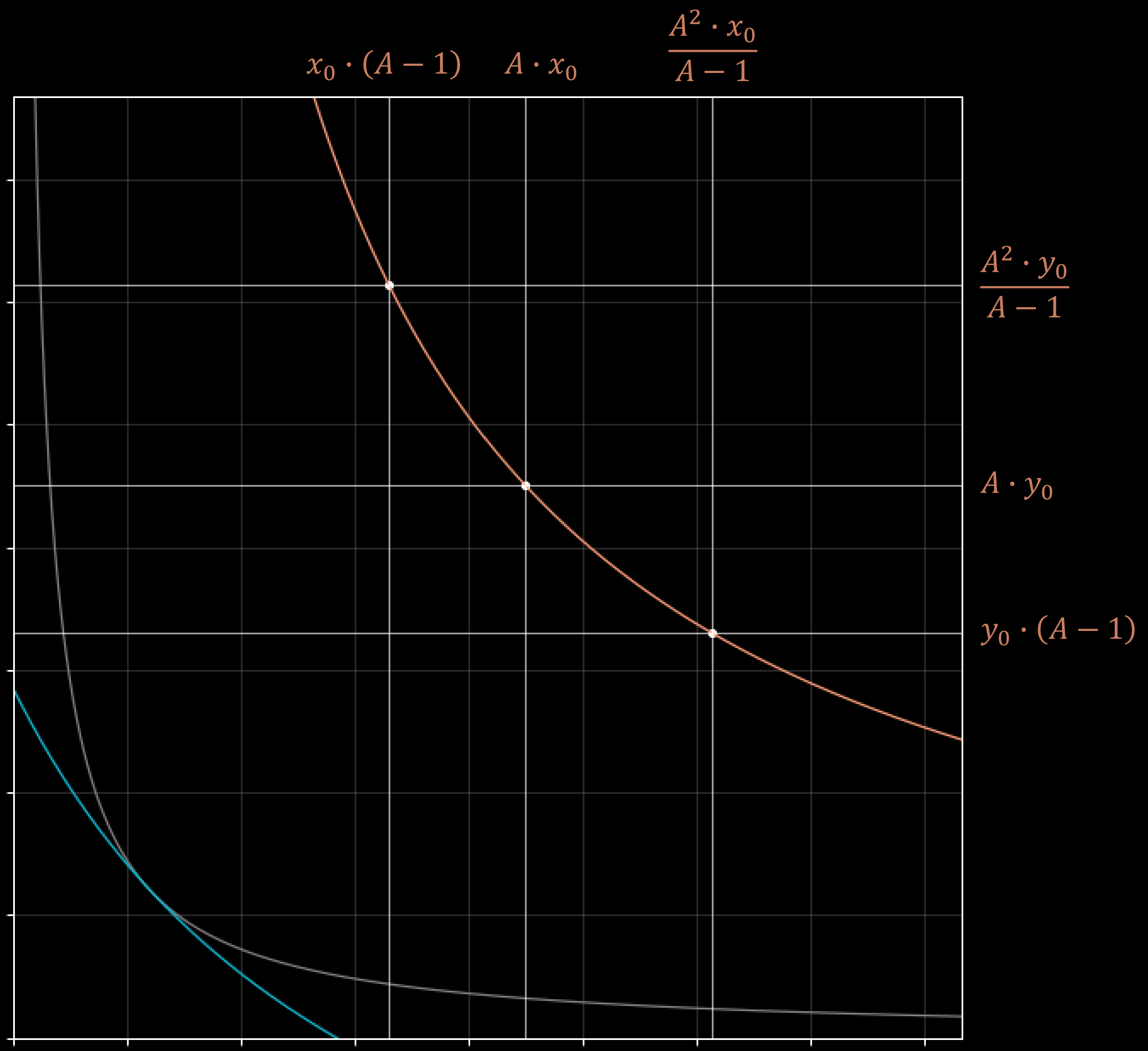
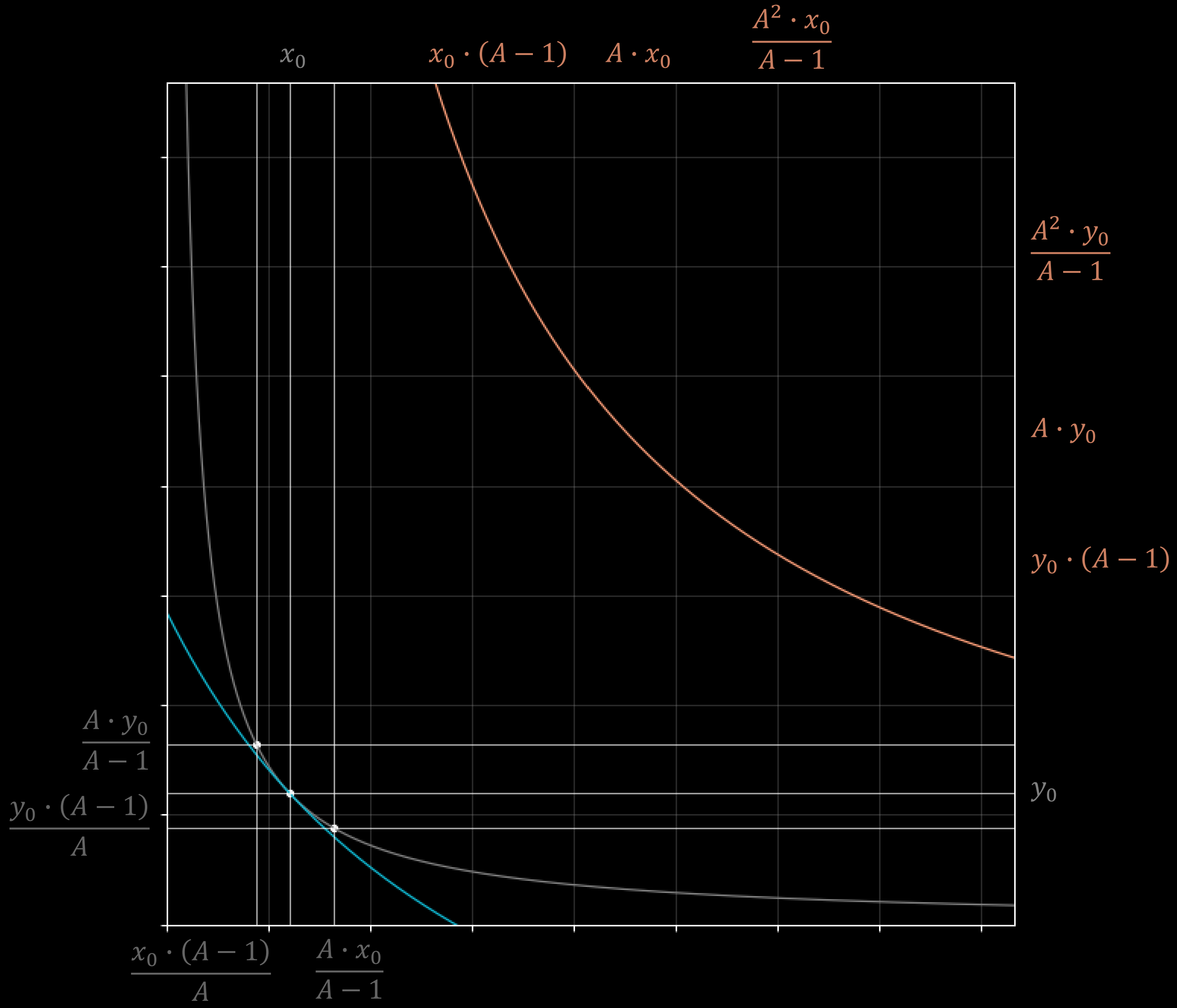
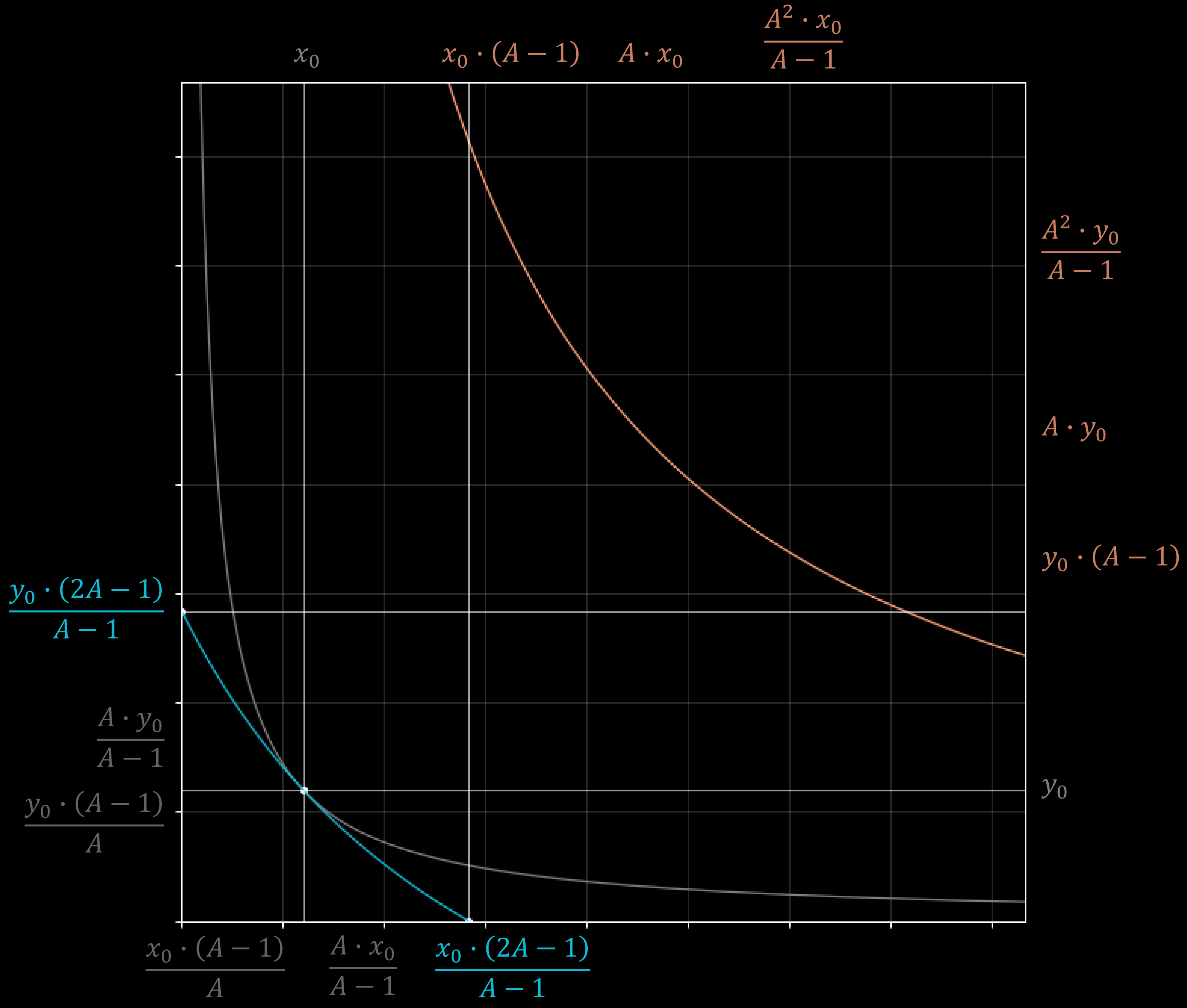






$x_0 \cdot (A-1)$
$A \cdot x_0$
$\frac{A^2 \cdot x_0}{A-1}$
$\frac{A^2 \cdot y_0}{A-1}$
$A \cdot y_0$
$y_0 \cdot (A-1)$


x_0
$x_0 \cdot (A-1)$
$A \cdot x_0$
$\frac{A^2 \cdot x_0}{A-1}$
$\frac{A^2 \cdot y_0}{A-1}$
$A \cdot y_0$
$y_0 \cdot (A-1)$
y_0
$\frac{A \cdot y_0}{A-1}$
$\frac{y_0 \cdot (A-1)}{A}$
$\frac{x_0 \cdot (A-1)}{A}$
$\frac{A \cdot x_0}{A-1}$


x_0
$x_0 \cdot (A-1)$
$A \cdot x_0$
$\frac{A^2 \cdot x_0}{A-1}$
$\frac{A^2 \cdot y_0}{A-1}$
$A \cdot y_0$
$y_0 \cdot (A-1)$
y_0
$\frac{y_0 \cdot (2A-1)}{A-1}$
$\frac{A \cdot y_0}{A-1}$
$\frac{y_0 \cdot (A-1)}{A}$
$\frac{x_0 \cdot (A-1)}{A}$
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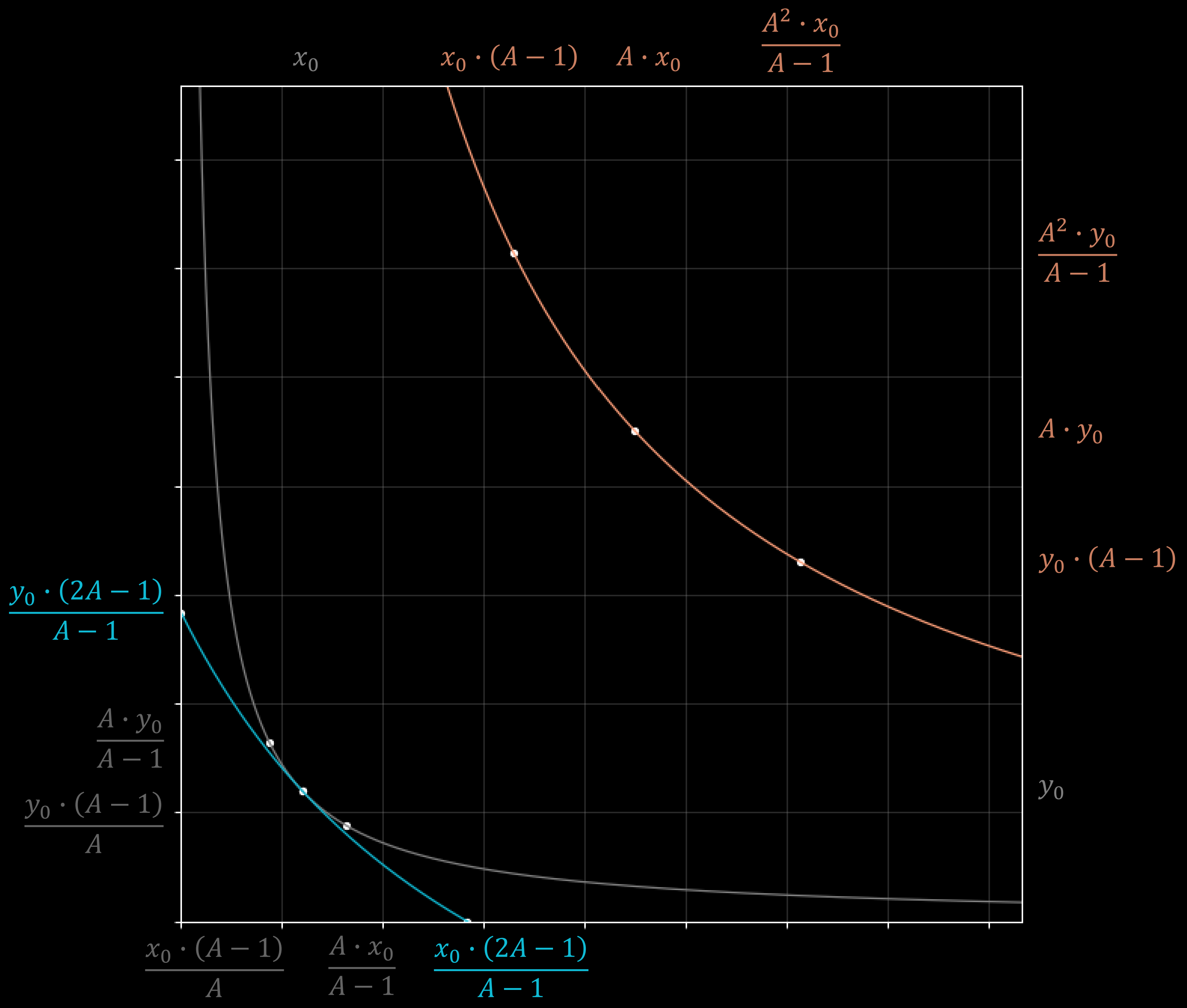
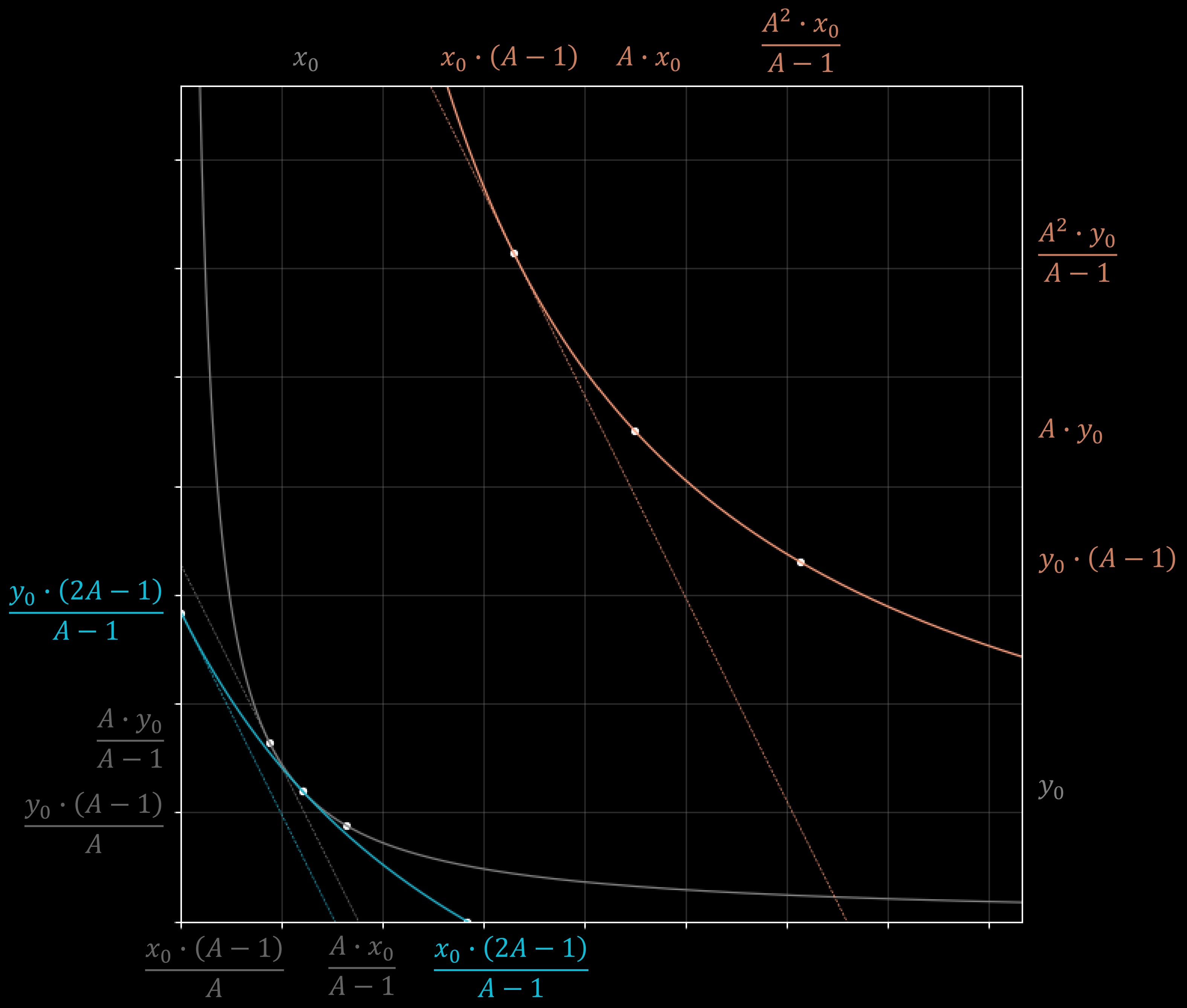
$\frac{x_0 \cdot (2A-1)}{A-1}$

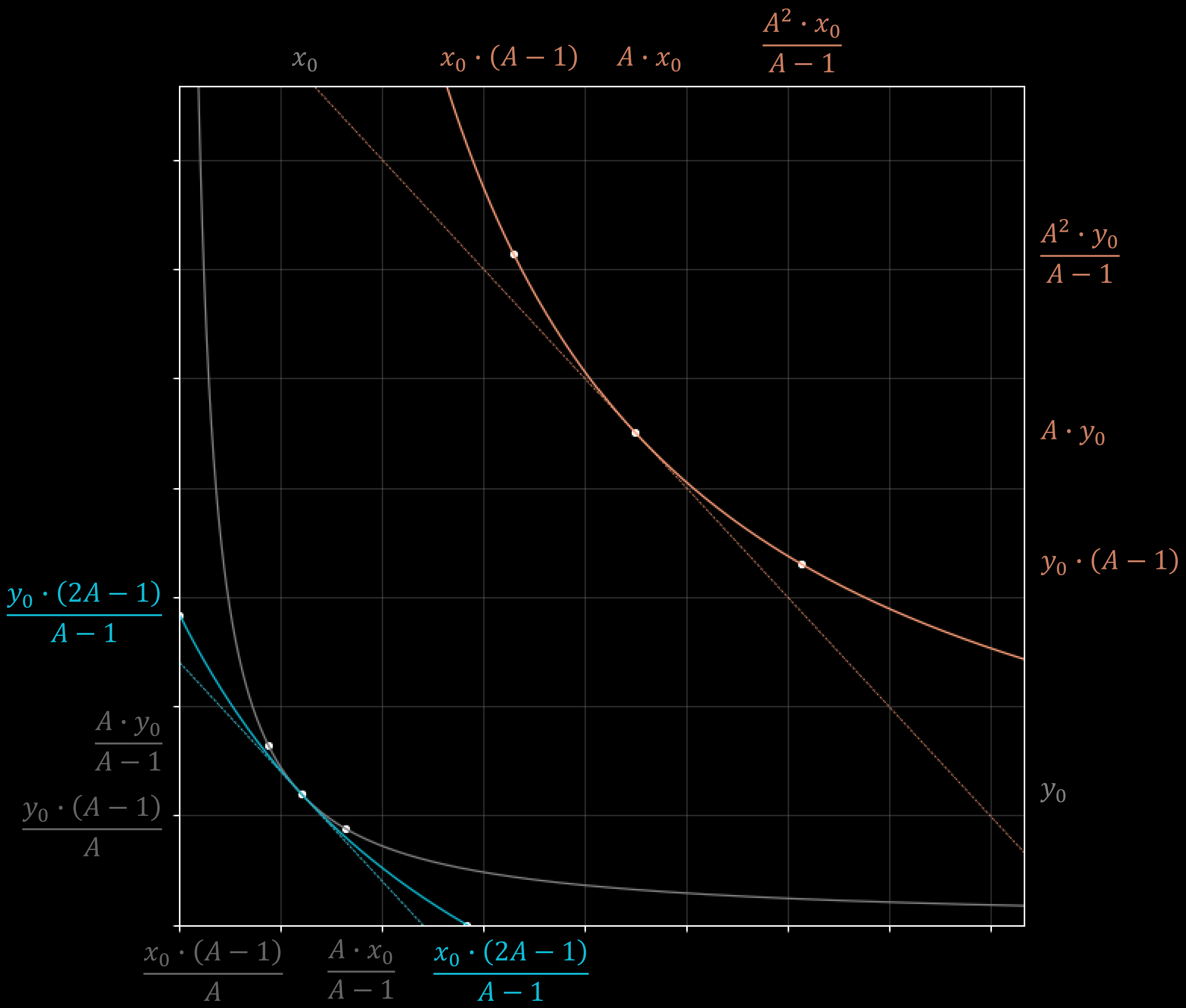


x_0
$x_0 \cdot (A-1)$
$A \cdot x_0$
$\frac{A^2 \cdot x_0}{A-1}$
$\frac{A^2 \cdot y_0}{A-1}$
$A \cdot y_0$
$y_0 \cdot (A-1)$
y_0
$\frac{y_0 \cdot (2A-1)}{A-1}$
$\frac{A \cdot y_0}{A-1}$
$\frac{y_0 \cdot (A-1)}{A}$
$\frac{x_0 \cdot (A-1)}{A}$
$\frac{A \cdot x_0}{A-1}$
$\frac{x_0 \cdot (2A-1)}{A-1}$

$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

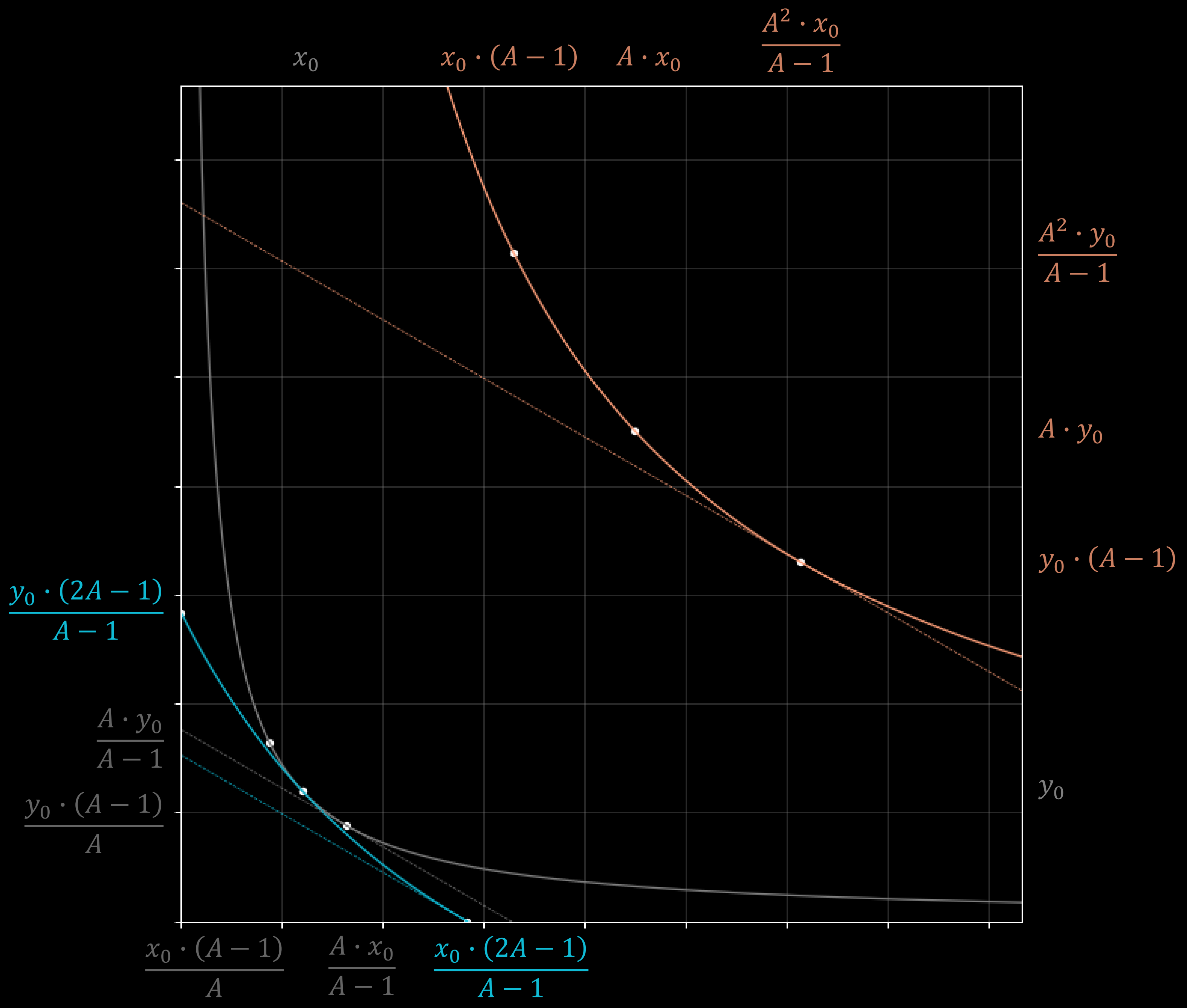


$$P_0 = \frac{y_0}{x_0}$$



x_0
$x_0 \cdot (A-1)$
$A \cdot x_0$
$\frac{A^2 \cdot x_0}{A-1}$
$\frac{A^2 \cdot y_0}{A-1}$
$A \cdot y_0$
$y_0 \cdot (A-1)$
y_0
$\frac{y_0 \cdot (2A-1)}{A-1}$
$\frac{A \cdot y_0}{A-1}$
$\frac{y_0 \cdot (A-1)}{A}$
$\frac{x_0 \cdot (A-1)}{A}$
$\frac{A \cdot x_0}{A-1}$
$\frac{x_0 \cdot (2A-1)}{A-1}$

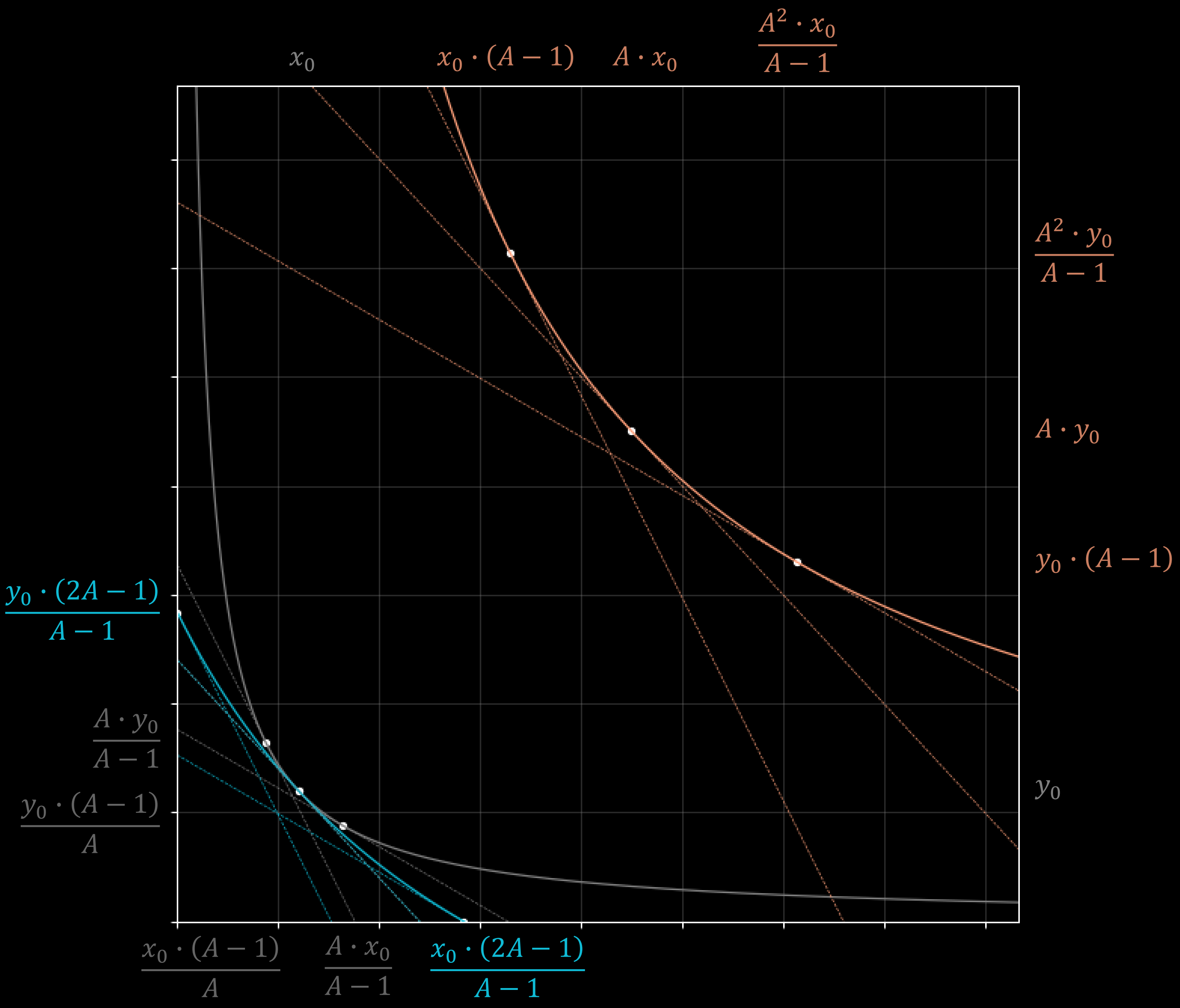
$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



$$P_{\text{high}} = \frac{A^2}{(A-1)^2} \cdot \frac{y_0}{x_0}$$

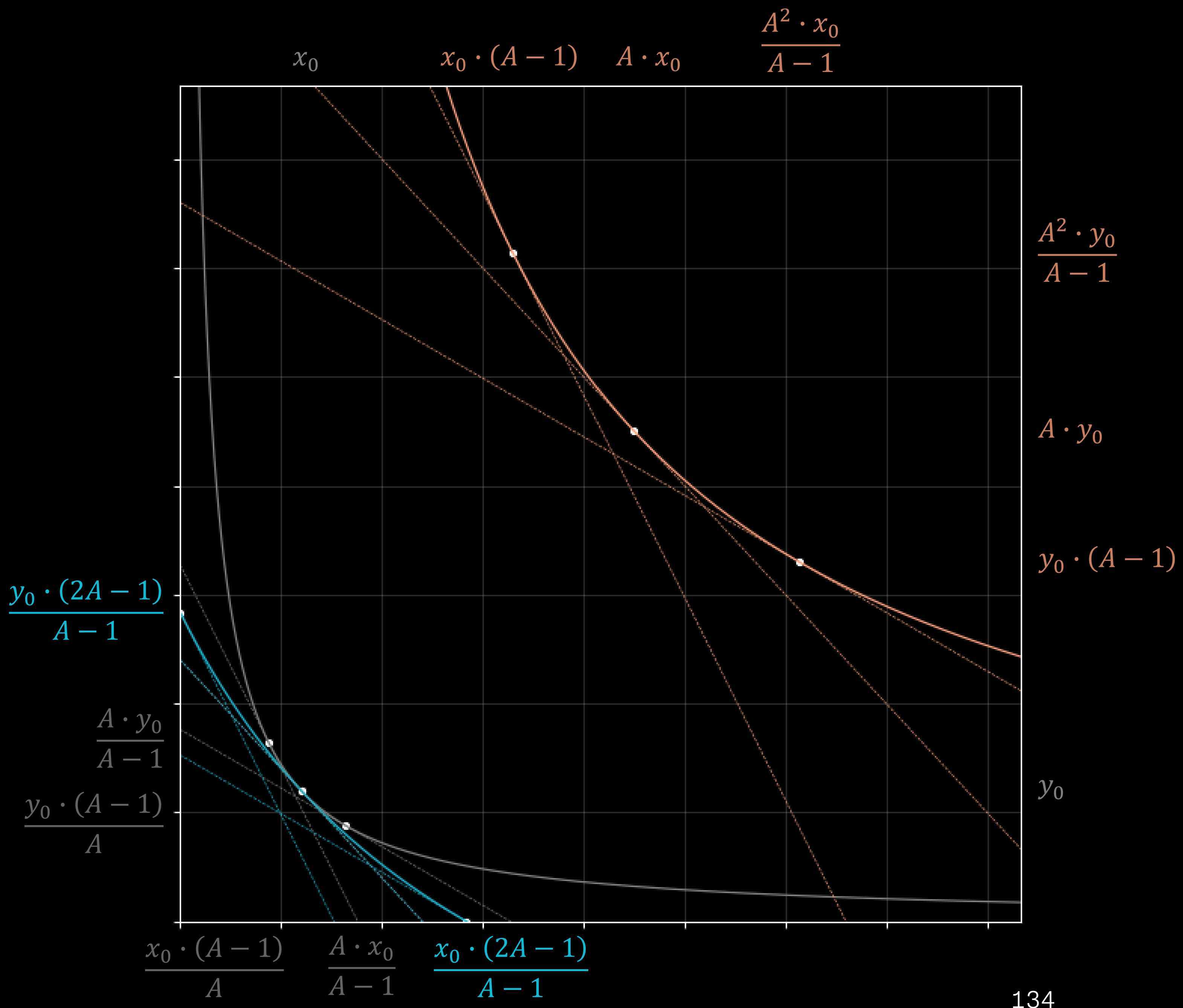
$$P_0 = \frac{y_0}{x_0}$$

$$P_{\text{low}} = \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0}$$



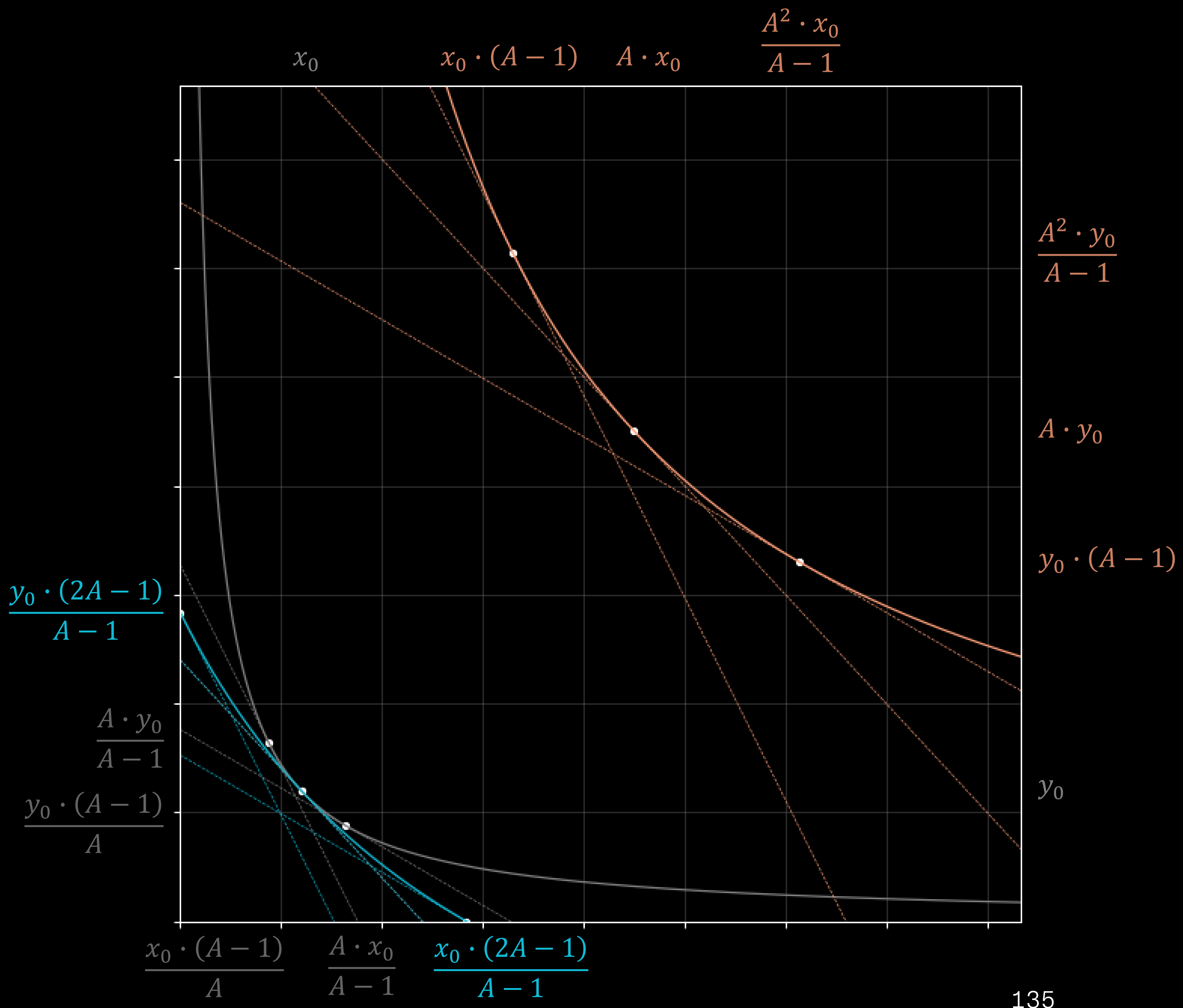
$$P_{\text{high}} \cdot P_{\text{low}} = \frac{A^2}{(A-1)^2} \cdot \frac{(A-1)^2}{A^2} \cdot \frac{y_0}{x_0} \cdot \frac{y_0}{x_0}$$

$$P_0 = \frac{y_0}{x_0}$$



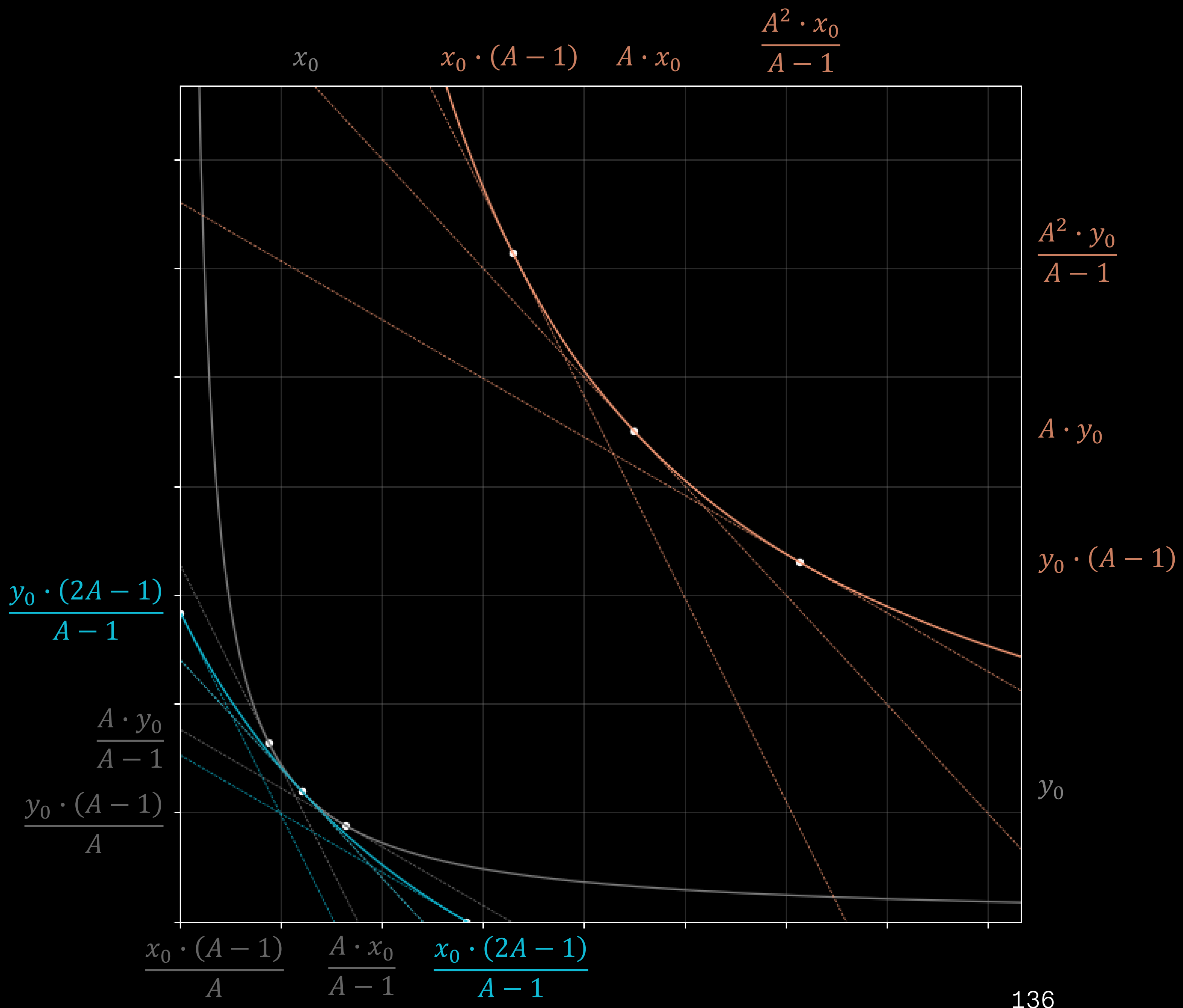
$$P_{\text{high}} \cdot P_{\text{low}} = \frac{A^2}{\cancel{(A-1)^2}} \cdot \frac{\cancel{(A-1)^2}}{A^2} \cdot \frac{y_0}{x_0} \cdot \frac{y_0}{x_0}$$

$$P_0 = \frac{y_0}{x_0}$$



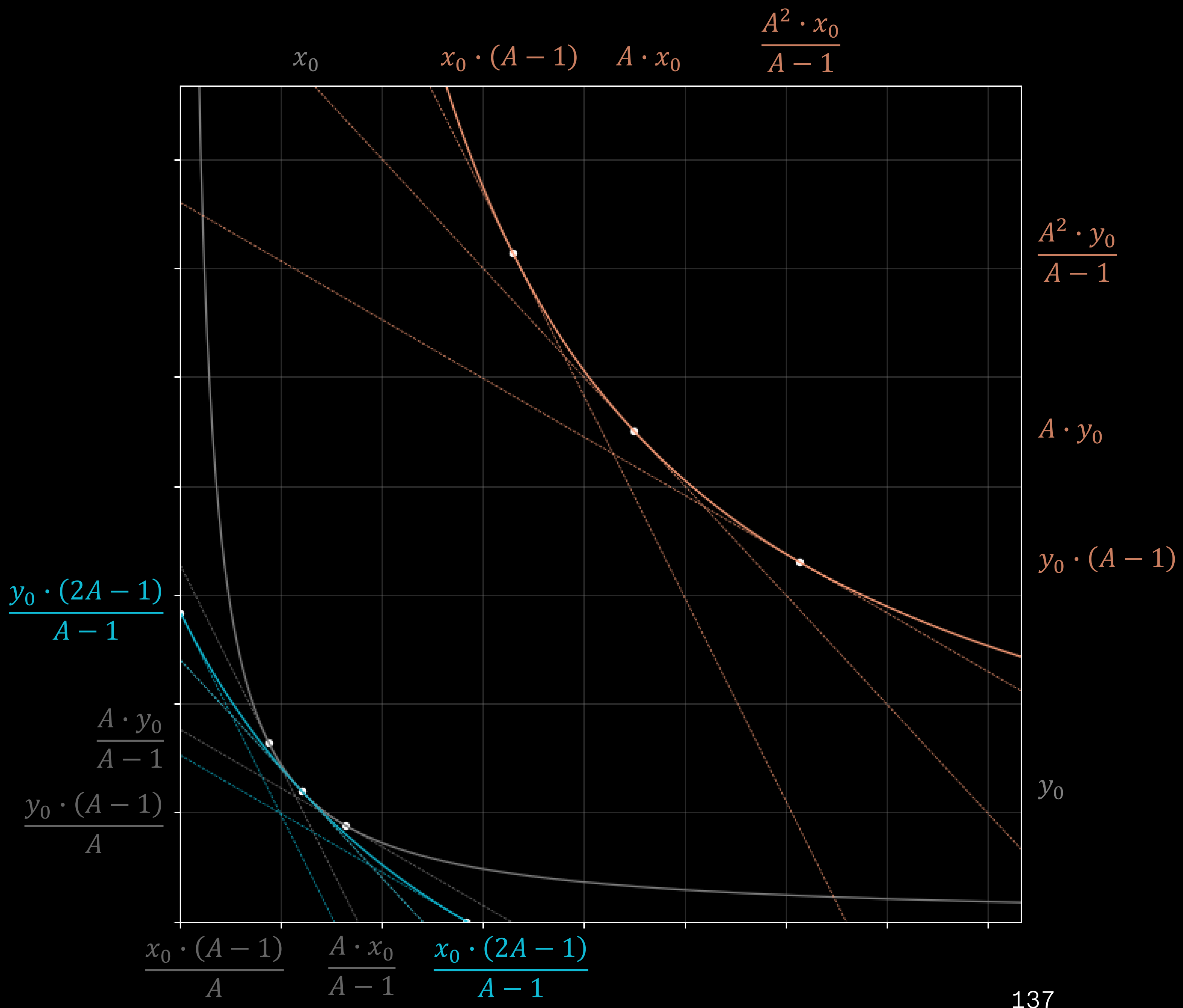
$$P_{\text{high}} \cdot P_{\text{low}} = \frac{y_0}{x_0} \cdot \frac{y_0}{x_0}$$

$$P_0 = \frac{y_0}{x_0}$$

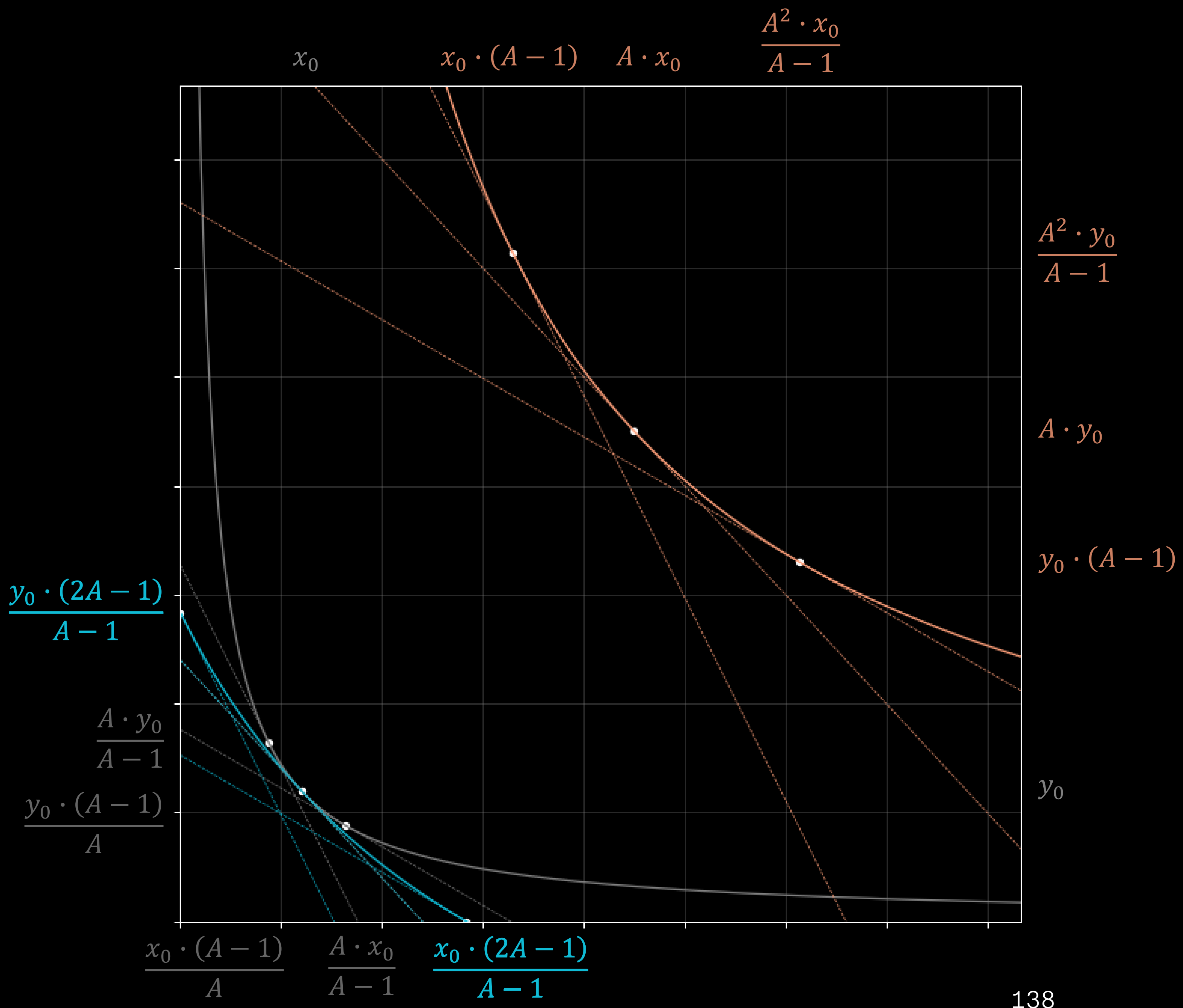


$$\sqrt{P_{\text{high}} \cdot P_{\text{low}}} = \frac{y_0}{x_0}$$

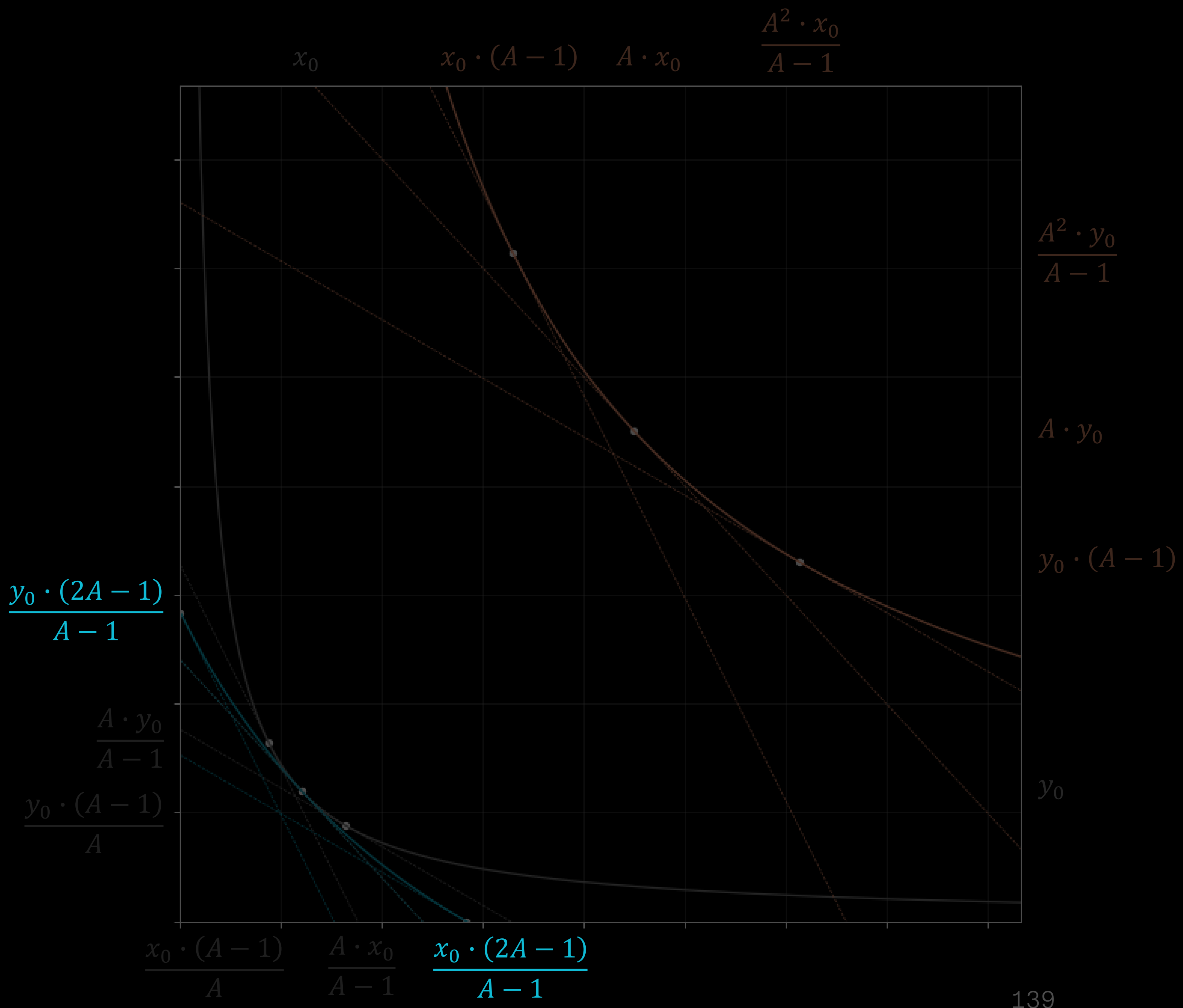
$$P_0 = \frac{y_0}{x_0}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

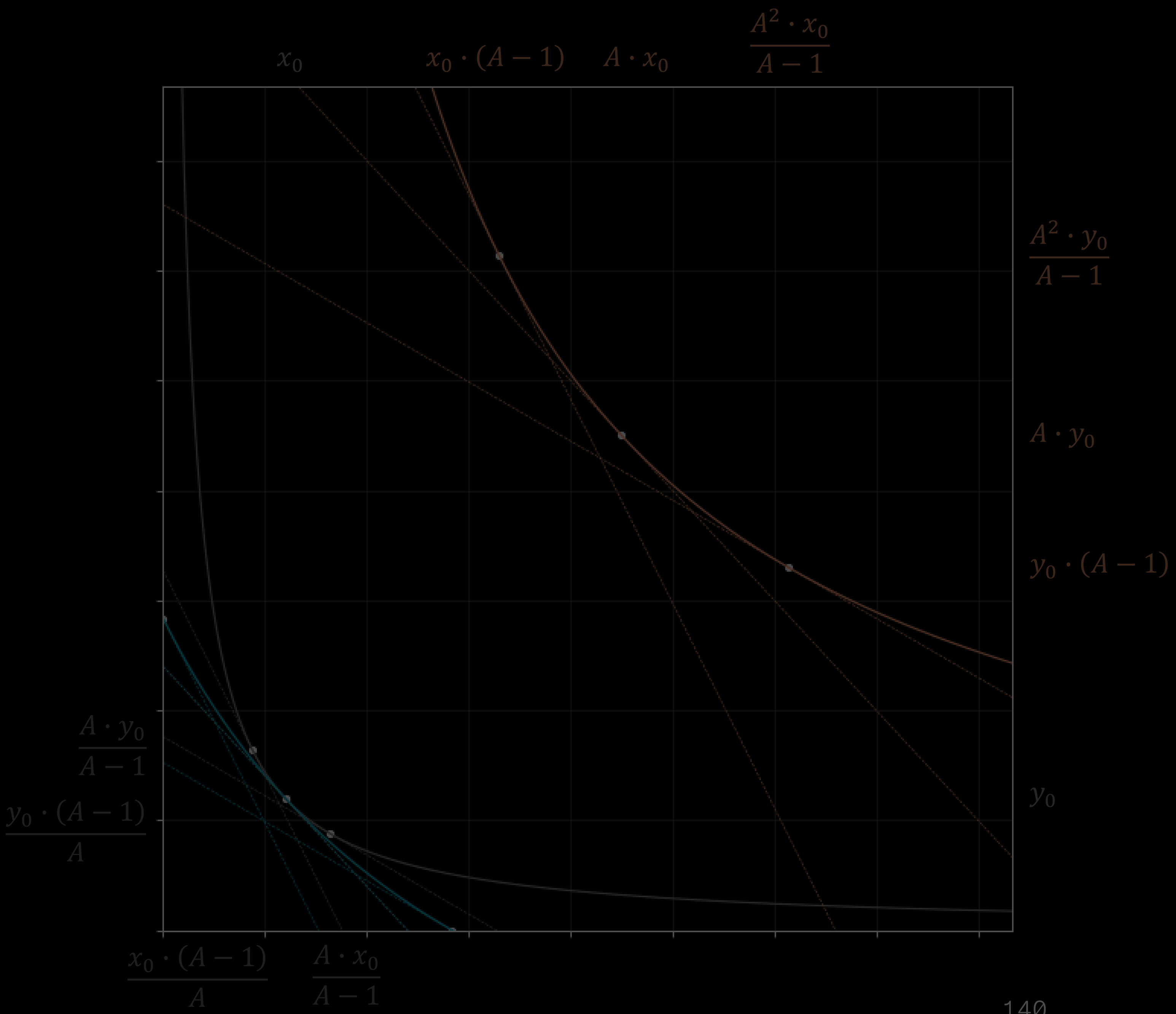


$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$



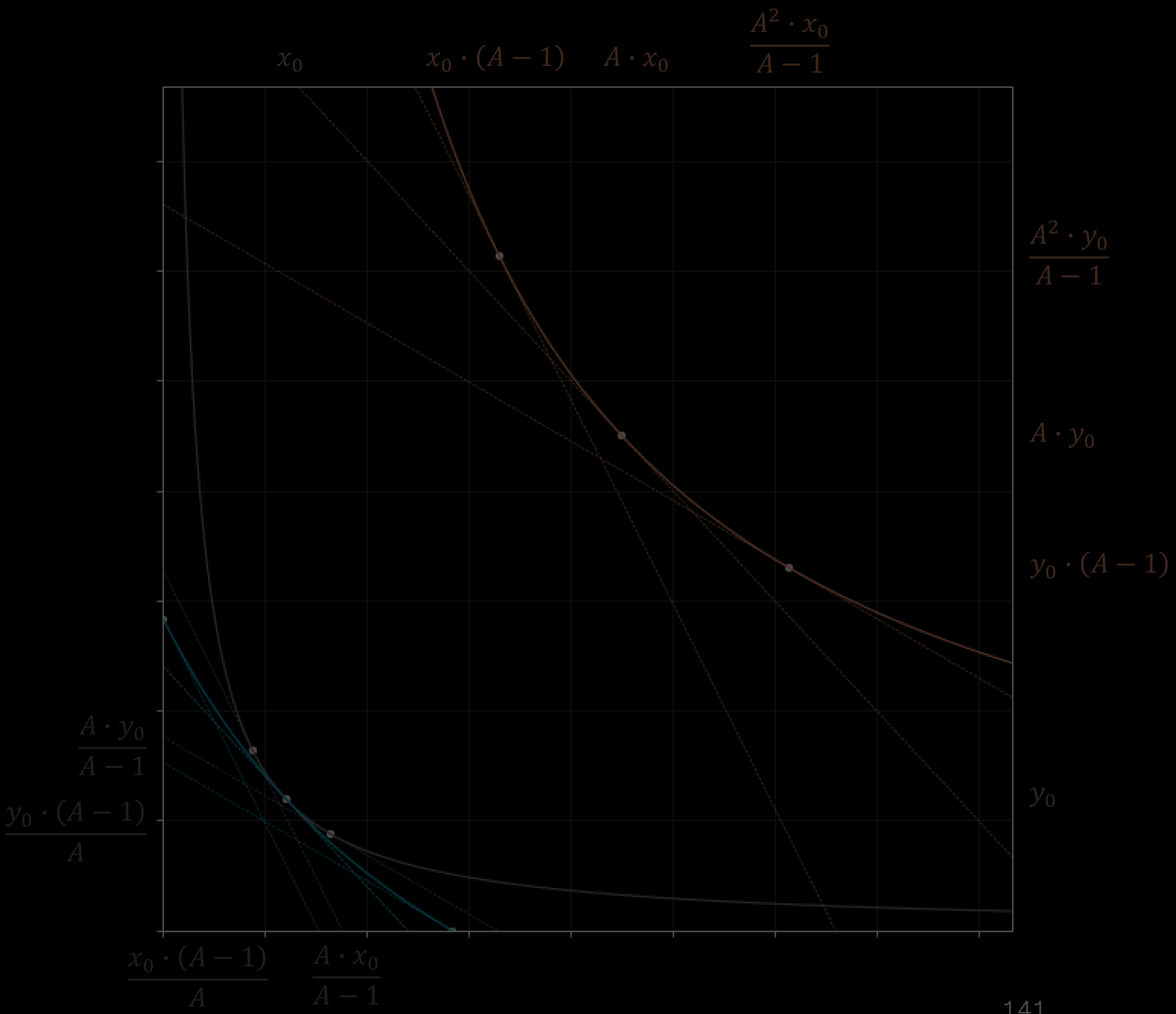
$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

$$\frac{y_0 \cdot (2A-1)(A-1)}{x_0 \cdot (2A-1)(A-1)}$$



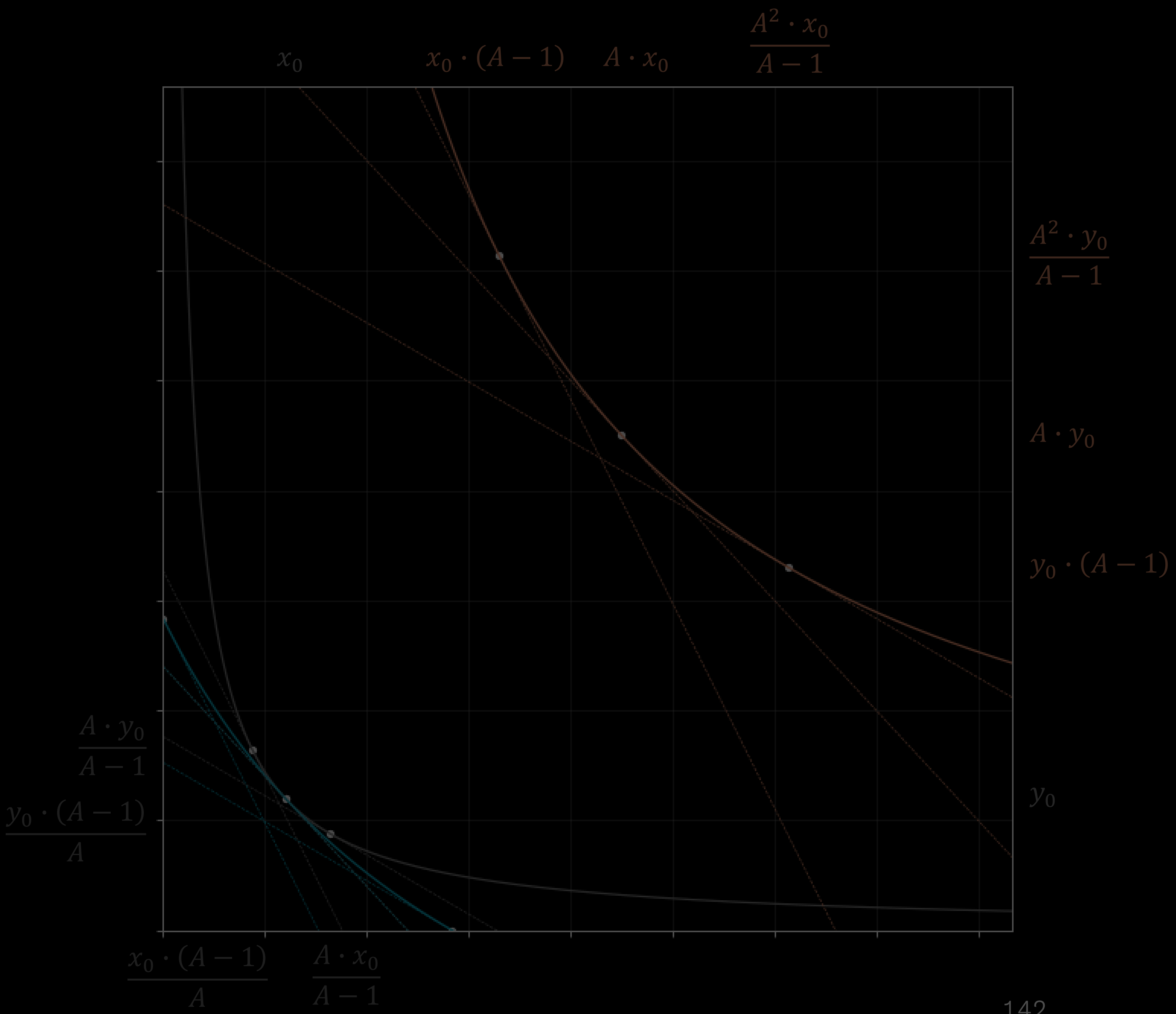
$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

$$\cancel{\frac{y_0 \cdot (2A-1)(A-1)}{x_0 \cdot (2A-1)(A-1)}}$$

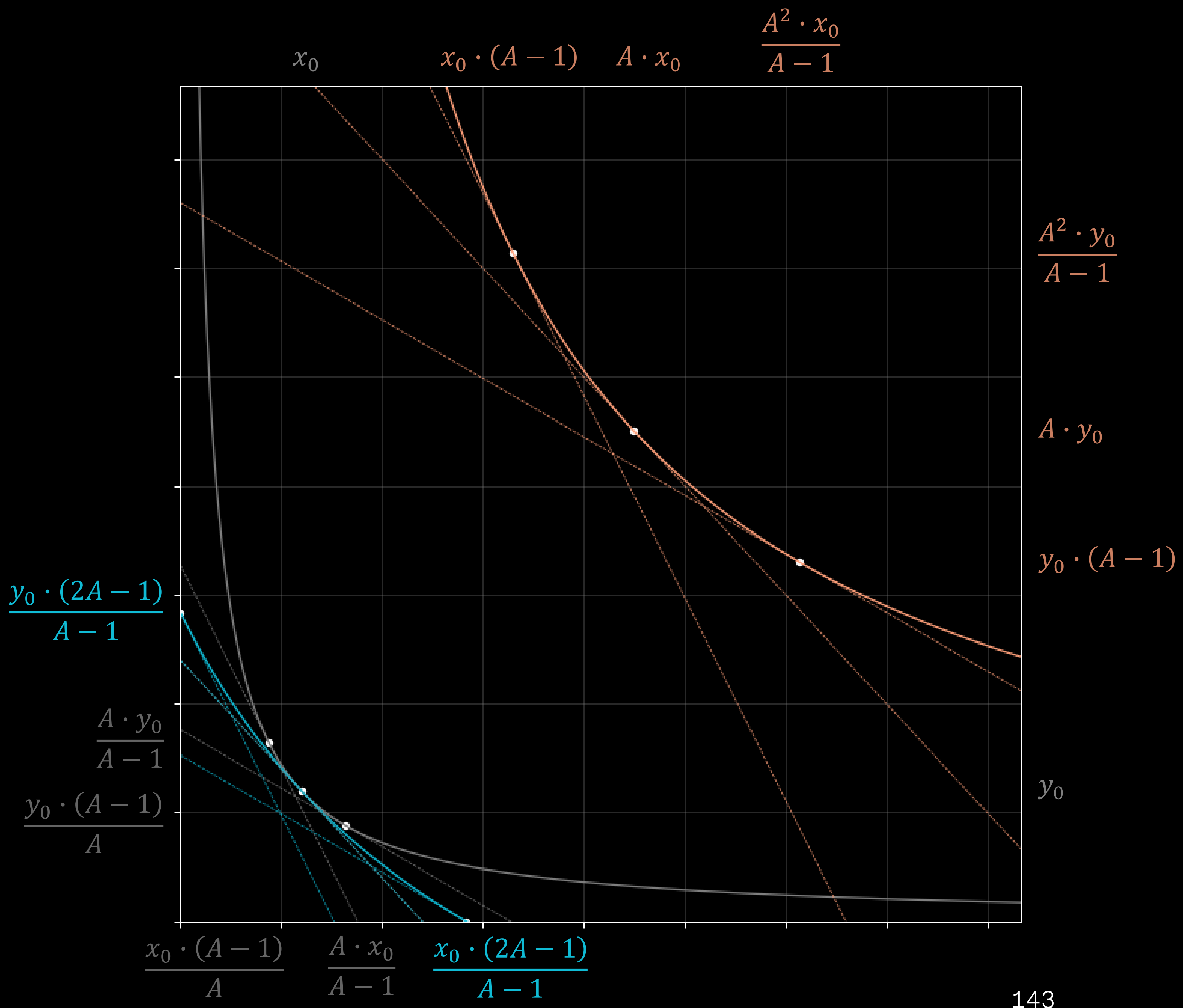


$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

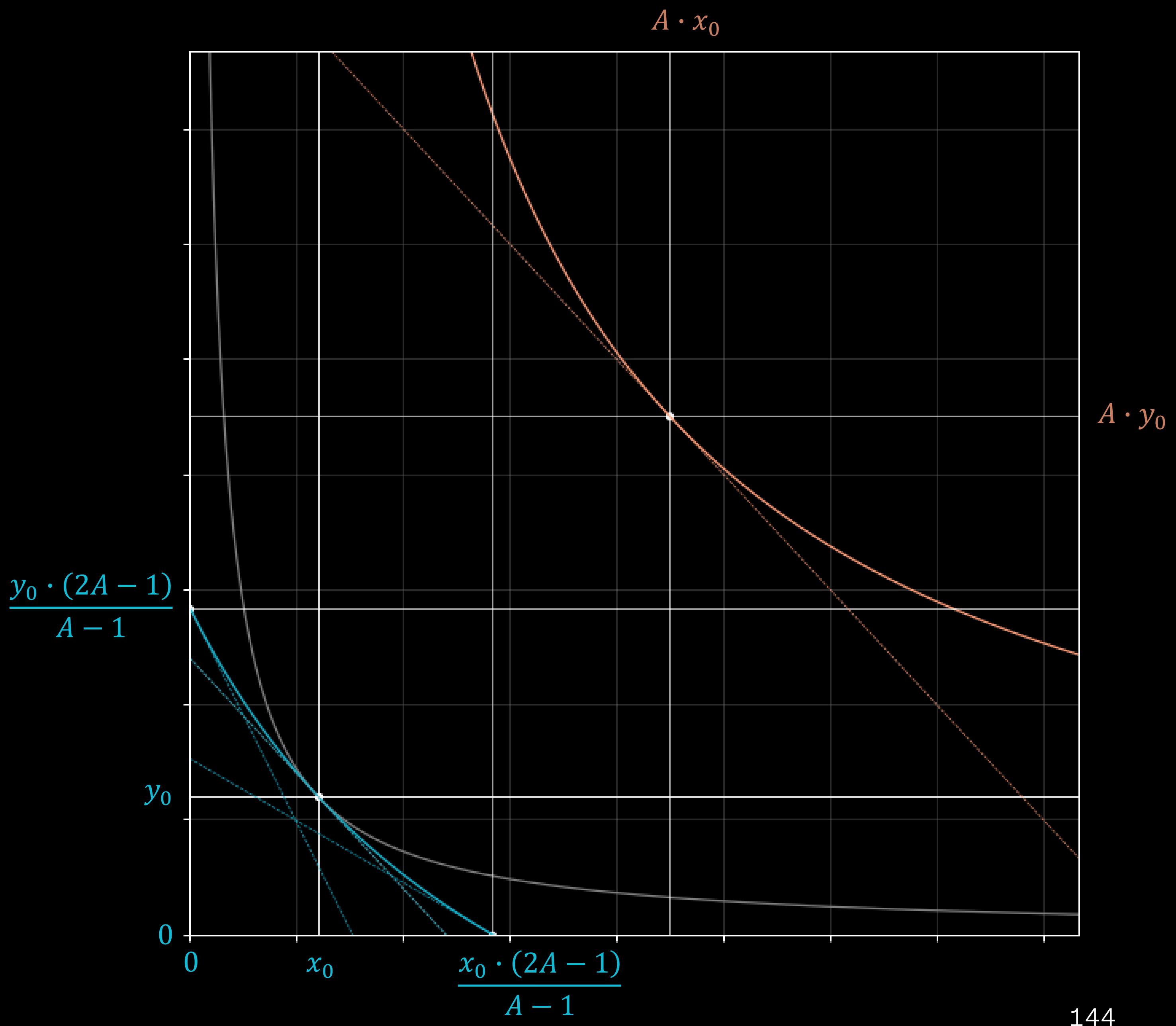
$\frac{y_0}{x_0}$



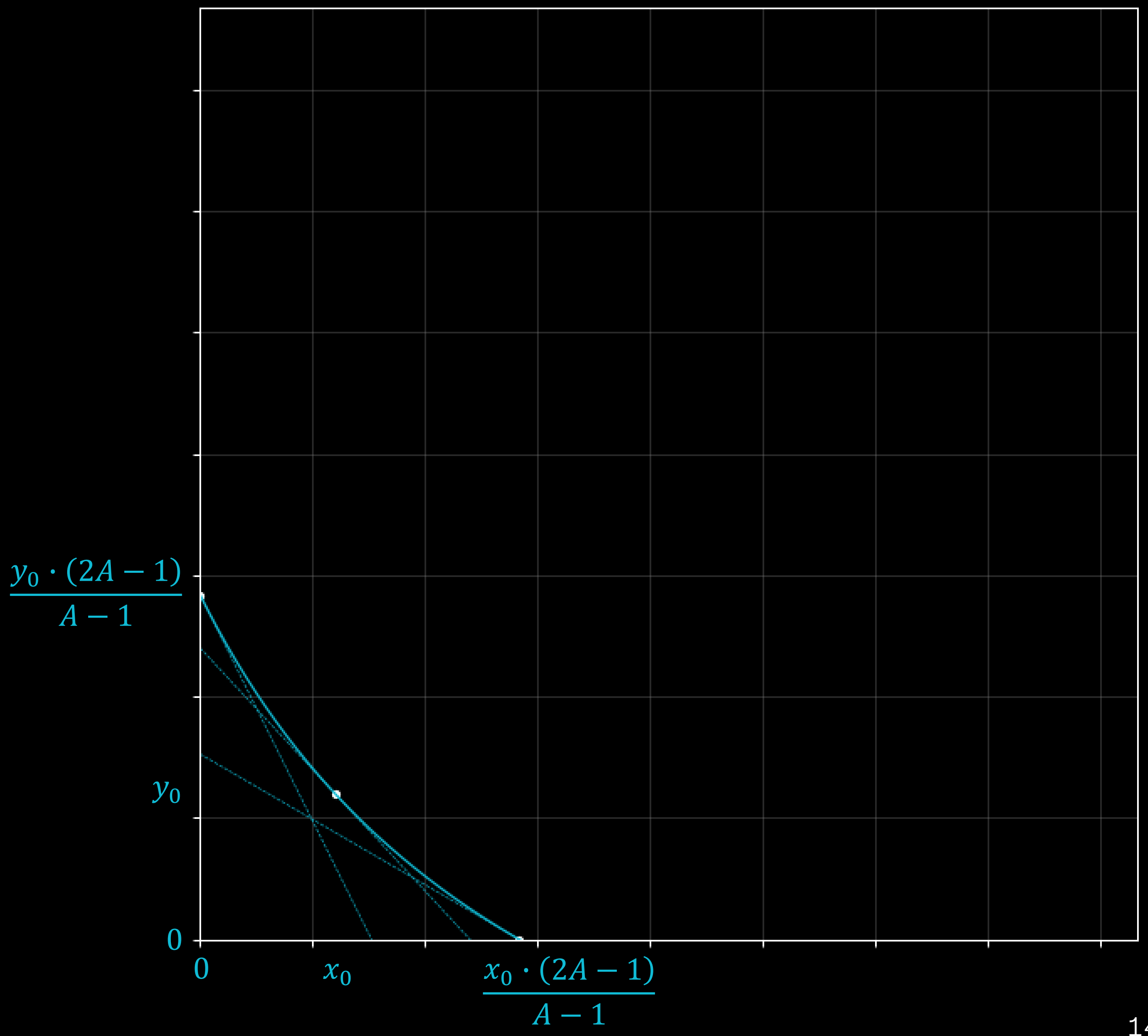
$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$



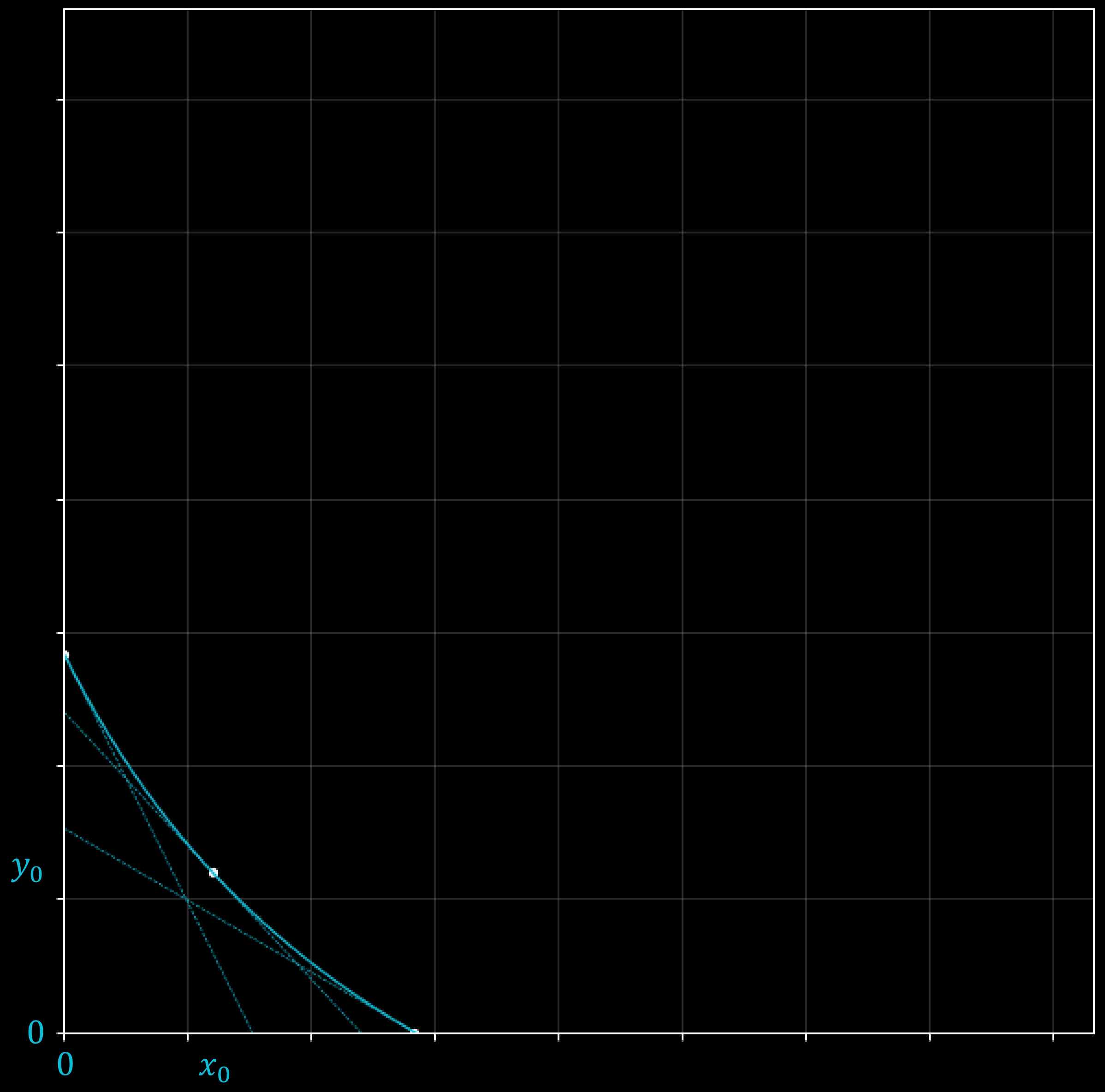
$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}}$$

$$y_{\text{int}} = \frac{y_0 \cdot (2A - 1)}{A - 1}$$

$$x_{\text{int}} = \frac{x_0 \cdot (2A - 1)}{A - 1}$$



$$P_0 = \frac{y_0}{x_0} = \sqrt{P_{\text{high}} \cdot P_{\text{low}}} = \frac{y_{\text{int}}}{x_{\text{int}}}$$

Homework, 16th May

Show that: $$\frac{x_0 \cdot P_{\text{high}} - y_0}{\frac{y_0}{P_{\text{low}}} - x_0} = P_0 = \frac{y_0}{x_0} = \sqrt{P_{high} \cdot P_{\text{low}}} = \frac{y_{\text{int}}}{x_{\text{int}}}$$

Homework, 16th May

Applying what you have learned from the last homework assignment, draw *precise* bonding curve and price curve pairs for each of these invariant functions.

$$x \cdot y = x_0 \cdot y_0$$

$$x_\mathrm{v} \cdot y_\mathrm{v} = A^2 \cdot x_0 \cdot y_0$$

$$\big(x + x_0 \cdot (A - 1)\big) \cdot \big(y + y_0 \cdot (A - 1)\big) = A^2 \cdot x_0 \cdot y_0$$

- Label all axes, annotate everything, provide as much detail as possible.
- Perform a token swap on *all three invariant functions* while forcing at least one of the Δx or Δy values to be consistent.
- Use arrows to indicate the direction of the swap on both the implicit (aka "bonding") curve, and the price curve integration.

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DeFi's Concentrated Liquidity From Scratch

Lecture 2 of 5

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