Trigonometric Functions

Question1

The value of $(\sin 70^{\circ})(\cot 10^{\circ} \cot 70^{\circ}-1)$ is

JEE Main 2025 (Online) 23rd January Morning Shift

Options:

A. 0

B. 2/3

C. 1

D. 3/2

Answer: C

Solution:

$$\sin 70^{\circ}(\cot 10^{\circ} \cot 70^{\circ}-1)$$
$$\Rightarrow \frac{\cos (80^{\circ})}{\sin 10}=1$$

Question2

Let the range of the function $f(x)=6+16 \cos x \cdot \cos \left(\frac{\pi}{3}-x\right) \cdot \cos \left(\frac{\pi}{3}+x\right) \cdot \sin 3x \cdot \cos 6x, x \in \mathbf{R}$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x+4y+12=0$ is :

JEE Main 2025 (Online) 23rd January Evening Shift

Options:

A. 11

B. 10

C. 8

D. 9

Answer: A

Solution:

$$
\begin{aligned}
f(x) &= 6+16\left(\frac{1}{4}\cos 3x\right)\sin 3x\cdot\cos 6x \\
&= 6+4\cos 3x\sin 3x\cos 6x \\
&= 6+\sin 12x
\end{aligned}
$$

Range of $f(x)$ is [5, 7]

$(\alpha,\beta)\equiv(5,7)$

distance $=\left|\frac{15+28+12}{5}\right|=11$

Question3

If $\sum_{r=1}^{13}\left\{\frac{1}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}\right\}=a\sqrt{3}+b, a, b\in Z$, then a^2+b^2 is equal to :

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Options:

A.

10

B.

4

C.

8

D.

2

Answer: C

Solution:

$$\frac{1}{\sin\frac{\pi}{6}}\sum_{r=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)-\left(\frac{\pi}{4}\right)-(r-1)\frac{\pi}{6}\right]}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}$$

$$\frac{1}{\sin\frac{\pi}{6}}\sum_{r=1}^{13}\left(\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\right)$$

$$=2\sqrt{3}-2=\alpha\sqrt{3}+b$$

So $a^2+b^2=8$

Question4

If $\sin x+\sin^2 x=1, x\in\left(0,\frac{\pi}{2}\right)$, then

$(\cos^{12} x+\tan^{12} x)+3(\cos^{10} x+\tan^{10} x+\cos^8 x+\tan^8 x)+(\cos^6 x+\tan^6 x)$ **is equal to:**

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A. 3

B.

4

C.

2

D.

1

Answer: C

Solution:

$$\sin x+\sin^2 x=1$$
$$\Rightarrow \sin x=\cos^2 x\Rightarrow \tan x=\cos x$$
$\therefore$ Given expression
$$=2\cos^{12} x+6\left[\cos^{10} x+\cos^8 x\right]+2\cos^6 x$$
$$=2\left[\sin^6 x+3\sin^5 x+3\sin^4 x+\sin^3 x\right]$$
$$=2\sin^3 x\left[(\sin x+1)^3\right]$$
$$=2\left[\sin^2 x+\sin x\right]^3$$
$$=2$$

Question5

If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of $\frac{27\operatorname{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is

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Options:

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{1}{5}$

D. $\frac{3}{4}$

Answer: A

Solution:

$$
\begin{aligned}
&10\sin^4\theta + 15\cos^4\theta = 6\\
\Rightarrow &10\sin^4\theta + 10\cos^4\theta + 5\cos^4\theta = 6\\
\Rightarrow &10\left[\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta\right] + 5\cos^4\theta = 6\\
\Rightarrow &10 - 20\left(1 - \cos^2\theta\right)\cos^2\theta + 5\cos^4\theta = 6\\
&\text{Let } \cos^2\theta = x\\
&10 - 20\left(x - x^2\right) + 5x^2 = 6\\
\Rightarrow &25x^2 - 20x + 4 = 0\\
&(5x-2)^2 = 0 \Rightarrow x = \frac{2}{5}\\
\Rightarrow &\cos^2\theta = \frac{2}{5} \Rightarrow \sin^2\theta = \frac{3}{5},\\
&\sec^2\theta = \frac{5}{2}, \operatorname{cosec}^2\theta = \frac{5}{3}
\end{aligned}
$$

$$
\begin{aligned}
\frac{27\operatorname{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta} &= \frac{27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3}{16\left(\frac{5}{2}\right)^4}\\
&= \frac{5^3 + 5^3}{5^4} = \frac{2.5^3}{5^4} = \frac{2}{5}
\end{aligned}
$$

Question6

If for $\theta \in \left[-\frac{\pi}{3}, 0\right]$, the points $(x, y) = \left(3\tan\left(\theta + \frac{\pi}{3}\right), 2\tan\left(\theta + \frac{\pi}{6}\right)\right)$ lie on $xy + \alpha x + \beta y + \gamma = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 75

B. 96

C. 80

D. 72

Answer: A

Solution:

$$\text{Let } \phi = \theta + \frac{\pi}{3} \Rightarrow \theta = \phi - \frac{\pi}{6}$$
$$x = 3\tan\left(\theta + \frac{\pi}{3}\right) = 3\tan\left(\theta + \frac{\pi}{6}\right)$$
$$y = 2\tan\phi$$
$$\tan\left(\phi + \frac{\pi}{6}\right) = \frac{\tan\phi + \frac{1}{\sqrt{3}}}{1 - \tan\phi \cdot \frac{1}{\sqrt{3}}}$$
$$\frac{x}{3} = \frac{\frac{y}{2} + \frac{1}{\sqrt{3}}}{1 - \frac{y}{2} \cdot \frac{1}{\sqrt{3}}}$$

$$\Rightarrow \quad x = \frac{3(y\sqrt{3}+2)}{2\sqrt{3}-y}$$
$$xy + \alpha x + \beta y + r = 0$$
$$3\left(\frac{y\sqrt{3}+2}{2\sqrt{3}-y}\right) + \alpha\left(3\frac{(y\sqrt{3}+2)}{(2\sqrt{3}-y)}\right) + \beta y + r = 0$$
$$= (3\sqrt{3}-\beta)y^2 + (6 + 3\sqrt{3}\alpha + 2\sqrt{3}\beta - y)y$$
$$+(6\alpha + 2\sqrt{3}y) = 0$$

For this identity to hold for all θ, coefficients must be 0

$$\therefore \quad \beta = 3\sqrt{3}$$
$$\gamma = -\alpha\sqrt{3}$$
$$6 + 3\sqrt{3}\alpha + (2\sqrt{3})(3\sqrt{3}) + \alpha\sqrt{3} = 0$$
$$\Rightarrow \alpha = -2\sqrt{3}$$
$$\Rightarrow \beta = 6$$
$$\alpha^2 + \beta^2 + \gamma^2 = 75$$

Question7

Let the set of all a ∈ R such that the equation cos2 x + a sin x = 2a − 7 has a solution be [p, q] and

$r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to

[27-Jan-2024 Shift 1]

Answer: 48

Solution:

$\cos 2x + a \cdot \sin x = 2a - 7$

$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$

$\sin x = 2,\ a = 2(\sin x + 2)$

$\Rightarrow a \in [2, 6]$

$p = 2 \quad q = 6$

$r = \tan 9^\circ + \cot 9^\circ - \tan 27 - \cot 27$

$r = \frac{1}{\sin 9 \cdot \cos 9} - \frac{1}{\sin 27 \cdot \cos 27}$

$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right]$

$r = 4$

$p \cdot q \cdot r = 2 \times 6 \times 4 = 48$

Question8

If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $[0, n\pi/2]$, for the least value of $n \in N$ then $\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to :

[27-Jan-2024 Shift 2]

Options:

A.

$\frac{1}{2^{15}}(2^{14} - 14)$

B.

$$\frac{1}{2^{14}}(2^{15}-15)$$

C.

$$1-\frac{15}{2^{13}}$$

D.

$$\frac{1}{2^{13}}(2^{14}-15)$$

Answer: D

Solution:

$$2\tan^2\theta-5\sec\theta-1=0$$

$$\Rightarrow 2\sec^2\theta-5\sec\theta-3=0$$

$$\Rightarrow (2\sec\theta+1)(\sec\theta-3)=0$$

$$\Rightarrow \sec\theta=-\frac{1}{2}, 3$$

$$\Rightarrow \cos\theta=-2, \frac{1}{3}$$

$$\Rightarrow \cos\theta=\frac{1}{3}$$

For 7 solutions $n=13$

$$\text{So, } \sum_{k=1}^{13}\frac{k}{2^k}=S \text{ (say)}$$

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{13}{2^{13}}$$

$$\frac{1}{2}S=\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{12}{2^{13}}+\frac{13}{2^{14}}$$

$$\Rightarrow \frac{S}{2}=\frac{1}{2}\cdot\frac{1-\frac{1}{2^{13}}}{1-\frac{1}{2}}-\frac{13}{2^{14}} \Rightarrow S=2\cdot\left(\frac{2^{13}-1}{2^{13}}\right)-\frac{13}{2^{13}}$$

Question9

If α, $-\frac{\pi}{2}<\alpha<\frac{\pi}{2}$ is the solution of 4cosθ + 5 sinθ = 1, then the value of tanα is

[29-Jan-2024 Shift 1]

Options:

A.

$$\frac{10-\sqrt{10}}{6}$$

B.

$$\frac{10-\sqrt{10}}{12}$$

C.

$$\frac{\sqrt{10}-10}{12}$$

D.

$$\frac{\sqrt{10}-10}{6}$$

Answer: C

Solution:

$4+5\tan\theta = sec\theta$

Squaring : $24\tan^2\theta+40\tan\theta+15=0$

$\tan\theta = \dfrac{-10\pm\sqrt{10}}{12}$

and $\tan\theta = -\left(\dfrac{10+\sqrt{10}}{12}\right)$ is Rejected.

(3) is correct.

Question10

The sum of the solutions x ∈ ℝ of the equation $\dfrac{3\cos 2x+\cos^3 2x}{\cos^6 x-sin^6 x} = x^3-x^2+6$ is

[29-Jan-2024 Shift 2]

Options:

A.

0

B.

1

C.

-1

D.

3

Answer: C

Solution:

$$\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$$

$$\Rightarrow \frac{\cos 2x(3+\cos^2 2x)}{\cos 2x(1-\sin^2 x\cos^2 x)}=x^3-x^2+6$$

$$\Rightarrow \frac{4(3+\cos^2 2x)}{(4-\sin^2 2x)}=x^3-x^2+6$$

$$\Rightarrow \frac{4(3+\cos^2 2x)}{(3+\cos^2 2x)}=x^3-x^2+6$$

$$x^3-x^2+2=0 \Rightarrow (x+1)(x^2-2x+2)=0$$

so, sum of real solutions $=-1$

Question11

If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $[0, n\pi/2]$, $n \in N$, then the roots of the equation

$x^2 + nx + (n - 3) = 0$ belong to :

[30-Jan-2024 Shift 1]

Options:

A.

$(0, \infty)$

B.

$(-\infty, 0)$

C.

$\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

D.

Z

Answer: B

Solution:

$2sin^3x + 2sin\,x \cdot \cos^2x + 4sin\,x - 4 = 0$

$2sin^3x + 2sin\,x \cdot (1 - sin^2x) + 4sin\,x - 4 = 0$

$6sin\,x - 4 = 0$

$sin\,x = \frac{2}{3}$

$n = 5$ (in the given interval)

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{17}}{2}$

$n = 5$ (in the given interval)

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{17}}{2}$

Required interval $(-\infty, 0)$

Question12

The number of solutions of the equation $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

[1-Feb-2024 Shift 2]

Options:

A.

1

B.

3

C.

2

D.

0

Answer: D

Solution:

$4sin^2x - 4\cos^3x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$

$4 - 4\cos^2x - 4\cos^3x + 9 - 4\cos x = 0$

$4\cos^3x + 4\cos^2x + 4\cos x - 13 = 0$

$4\cos^3x + 4\cos^2x + 4\cos x = 13$

L.H.S. ≤ 12 can't be equal to 13.

Question13

For α, β ∈ (0, π/2), let 3sin (α + β) = 2sin (α − β) and a real number k be such that tanα = k tan β. Then the value of k is equal to :

[30-Jan-2024 Shift 2]

Options:

A.

$-\frac{2}{3}$

B.

-5

C.

2/3

D.

5

Answer: B

Solution:

$$3\sin\alpha\cos\beta + 3\sin\beta\cos\alpha$$

$$= 2\sin\alpha\cos\beta - 2\sin\beta\cos\alpha$$

$$5\sin\beta\cos\alpha = -\sin\alpha\cos\beta$$

$$\tan\beta = -\frac{1}{5}\tan\alpha$$

$$\tan\alpha = -5\tan\beta$$

Question14

If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and

$\tan C = (x^{-3}+x^{-2}+x^{-1})^{\frac{1}{2}}$, $0 < A, B, C < \frac{\pi}{2}$, then $A + B$ is equal to :

[1-Feb-2024 Shift 1]

Options:

A.

C

B.

π − C

C.

2π − C

D.

π/2 −C

Answer: A

Solution:

Finding $\tan(A+B)$ we get

$$\Rightarrow \tan(A+B) =$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B} = \frac{\frac{1}{\sqrt{x(x^2+x+1)}}+\frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\frac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B) = \frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)(\sqrt{x})}$$

$$\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)(\sqrt{x})}$$

$$\tan(A+B) = \frac{\sqrt{x^2+x+1}}{x\sqrt{x}} = \tan C$$

$$A+B=C$$

Question15

If m and n respectively are the numbers of positive and negative value of θ in the interval [−π, π] that satisfy the equation $\cos 2\theta \cos \frac{\theta}{2} = \cos 3\theta \cos \frac{9\theta}{2}$, then mn is equal to ________.
[25-Jan-2023 Shift 2]

Answer: 25

Solution:

Solution:

$$\cos 2\theta \cdot \cos\frac{\theta}{2} = \cos 3\theta \cdot \cos\frac{9\theta}{2}$$

$$\Rightarrow 2\cos 2\theta \cdot \cos\frac{\theta}{2} = 2\cos\frac{9\theta}{2} \cdot \cos 3\theta$$

$$\Rightarrow \cos\frac{5\theta}{2}+\cos\frac{3\theta}{2} = \cos\frac{15\theta}{2}+\cos\frac{3\theta}{2}$$

$$\Rightarrow \cos\frac{15\theta}{2} = \cos\frac{5\theta}{2}$$

$$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$$

$$5\theta = 2k\pi \text{ or } 10\theta = 2k\pi$$

$$\theta = \frac{2k\pi}{5}$$

$$\therefore \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$$

$$\therefore m \cdot n = 25$$

Question16

Let $S = \{\theta \in [0, 2\pi) : \tan(\pi \cos\theta) + \tan(\pi \sin\theta) = 0\}$.

Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

[24-Jan-2023 Shift 2]

Answer: 2

Solution:

Solution:

$\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$

$\tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\pi\cos\theta = n\pi - \pi\sin\theta$

$\sin\theta + \cos\theta = n$ where $n \in I$

possible values are $n = 0, 1$ and -1 because

$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$

Now it gives $\theta \in \left\{0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{2}, \pi\right\}$

So $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = 2(0) + 4\left(\frac{1}{2}\right) = 2$

Question17

Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and

$S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to

[29-Jan-2023 Shift 1]

Options:

A. $\frac{11}{8}$

B. $\frac{5}{4}$

C. $\frac{9}{8}$

D. $\frac{3}{2}$

Answer: B

Solution:

Solution:

$$f(\theta)=3\left(\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^4(3x+\theta)\right)-2(1-\sin^2 2\theta)$$

$$S=\left\{\theta\in[0,\pi]:f'(\theta)=-\frac{\sqrt{3}}{2}\right\}$$

$$\Rightarrow f(\theta)=3(\cos^4\theta+\sin^4\theta)-2\cos^2 2\theta$$

$$\Rightarrow f(\theta)=3\left(1-\frac{1}{2}\sin^2 2\theta\right)-2\cos^2 2\theta$$

$$\Rightarrow f(\theta)=3-\frac{3}{2}\sin^2 2\theta-2\cos^2\theta$$

$$=\frac{3}{2}-\frac{1}{2}\cos^2 2\theta=\frac{3}{2}-\frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right)$$

$$f(\theta)=\frac{5}{4}-\frac{\cos 4\theta}{4}$$

$$f'(\theta)=\sin 4\theta$$

$$\Rightarrow f'(\theta)=\sin 4\theta=-\frac{\sqrt{3}}{2}$$

$$\Rightarrow 4\theta=n\pi+(-1)^n\frac{\pi}{3}$$

$$\Rightarrow \theta=\frac{n\pi}{4}+(-1)^n\frac{\pi}{12}$$

$$\Rightarrow \theta=\frac{\pi}{12},\left(\frac{\pi}{4}-\frac{\pi}{12}\right),\left(\frac{\pi}{2}+\frac{\pi}{12}\right),\left(\frac{3\pi}{4}-\frac{\pi}{12}\right)$$

$$\Rightarrow 4\beta=\frac{\pi}{4}+\frac{\pi}{2}+\frac{3\pi}{4}=\frac{3\pi}{2}$$

$$\Rightarrow \beta=\frac{3\pi}{8}\Rightarrow f(\beta)=\frac{5}{4}-\frac{\cos\frac{3\pi}{2}}{4}=\frac{5}{4}$$

Question18

The set of all values of λ for which the equation $\cos^2 2x-2\sin^4 x-2\cos^2 x=\lambda$ [29-Jan-2023 Shift 2]

Options:

A. $[-2,-1]$

B. $\left[-2,-\frac{3}{2}\right]$

C. $\left[-1,-\frac{1}{2}\right]$

D. $\left[-\frac{3}{2},-1\right]$

Answer: D

Solution:

Solution:
$\lambda = \cos^2 2x - 2\sin^4 x - 2\cos^2 x$
convert all in to $\cos x$.
$\lambda = (2\cos^2 x - 1)^2 - 2(1 - \cos^2 x)^2 - 2\cos^2 x$
$= 4\cos^4 x - 4\cos^2 x + 1 - 2(1 - 2\cos^2 x + \cos^4 x) -$
$2\cos^2 x$
$= 2\cos^4 x - 2\cos^2 x + 1 - 2$
$= 2\cos^4 x - 2\cos^2 x - 1$
$= 2\left[\cos^4 x - \cos^2 x - \frac{1}{2}\right]$
$= 2\left[\left(\cos^2 x - \frac{1}{2}\right)^2 - \frac{3}{4}\right]$
$\lambda_{max} = 2\left[\frac{1}{4} - \frac{3}{4}\right] = 2 \times \left(-\frac{2}{4}\right) = -1$ (max Value)
$\lambda_{min} = 2\left[0 - \frac{3}{4}\right] = -\frac{3}{2}$ (MinimumValue)
So, Range $= \left[-\frac{3}{2}, -1\right]$

Question19

If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$ then the value of $\left(a + \frac{1}{a}\right)$ is :
[30-Jan-2023 Shift 1]

Options:

A. 4

B. $4 - 2\sqrt{3}$

C. 2

D. $5 - \frac{3}{2}\sqrt{3}$

Answer: A

Solution:

Solution:
Option (1)
$\tan 15^\circ = 2 - \sqrt{3}$
$\frac{1}{\tan 75^\circ} = \cot 75^\circ = 2 - \sqrt{3}$
$\frac{1}{\tan 105^\circ} = \cot(105^\circ) = -\cot 75^\circ = \sqrt{3} - 2$
$\tan 195^\circ = \tan 15^\circ = 2 - \sqrt{3}$

$\therefore\ 2(2-\sqrt{3}) = 2a \Rightarrow a = 2-\sqrt{3}$
$\Rightarrow\ a + \dfrac{1}{a} = 4$

Question20

If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha+\beta$ is equal to:
[30-Jan-2023 Shift 1]

Options:

A. 3

B. 5

C. 6

D. 4

Answer: D

Solution:

Solution:
$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1$
$\Rightarrow \dfrac{\ln\cos x - \ln\sin x}{\ln\cos x} + 4\dfrac{\ln\sin x - \ln\cos x}{\ln\sin x} = 1$
$\Rightarrow (\ln\sin x)^2 - 4(\ln\sin x)(\ln\cos x) + 4(\ln\cos x)^2 = 1$
$\Rightarrow \ln\sin x = 2\ln\cos x$
$\Rightarrow \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-1+\sqrt{5}}{2}$
$\therefore \alpha+\beta = 4$
Correct option (4)

Question21

The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is _______ :
[6-Apr-2023 shift 2]

Answer: 4

Solution:

Solution:

$(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$

$\frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$

$\frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$

$\frac{2(4)}{\sqrt{5}-1} - \frac{2(4)}{(\sqrt{5}+1)}$

$\frac{8(\sqrt{5}+1)}{4} - \frac{8(\sqrt{5}-1)}{4}$

$2[(\sqrt{5}+1) - (\sqrt{5}-1)]$

$= 4$

Question22

The value of $36(4\cos^2 9^\circ - 1)(4\cos^2 27^\circ - 1)(4\cos^2 81^\circ - 1)(4\cos^2 243^\circ - 1)$ is [8-Apr-2023 shift 2]

Options:

A. 27

B. 54

C. 18

D. 36

Answer: D

Solution:

Solution:

$4\cos^2\theta - 1 = 4(1 - \sin^2\theta) - 1 = 3 - 4\sin^2\theta = \frac{\sin 3\theta}{\sin\theta}$

so given expression can be written as

$36 \times \frac{\sin 27^\circ}{\sin 9^\circ} \times \frac{\sin 81^\circ}{\sin 27^\circ} \times \frac{\sin 243^\circ}{\sin 81^\circ} \times \frac{\sin 729^\circ}{\sin 243^\circ}$

$36 \times \frac{\sin 729^\circ}{\sin 9^\circ} = 36$

Question23

$96 \cos\frac{\pi}{33} \cos\frac{2\pi}{33} \cos\frac{4\pi}{33} \cos\frac{8\pi}{33} \cos\frac{16\pi}{33}$ is equal to : [10-Apr-2023 shift 1]

Options:

A. 4

B. 2

C. 3

D. 1

Answer: C

Solution:

Solution:

$96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{2^2\pi}{33}\cos\frac{2^3\pi}{33}\cos\frac{2^4\pi}{33}$

$\because \cos A\cos 2A\cos 2^2A\ldots\cos 2^{n-1}A=\frac{\sin(2^nA)}{2^n\sin A}$

Here $A=\frac{\pi}{33}, n=5$

$$=\frac{96\sin\left(2^5\frac{\pi}{33}\right)}{2^5\sin\left(\frac{\pi}{33}\right)}$$

$$=\frac{96\sin\left(\frac{32\pi}{33}\right)}{32\sin\left(\frac{\pi}{33}\right)}$$

$$=\frac{3\sin\left(\pi-\frac{\pi}{33}\right)}{\sin\left(\frac{\pi}{33}\right)}=3$$

Question24

Let $S=\left\{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right):9^{1-\tan^2x}+9^{\tan^2x}=10\right\}$ and $b=\sum\limits_{x\in S\tan}^{2}\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta-14)^2$ is equal to
[10-Apr-2023 shift 2]

Options:

A. 16

B. 32

C. 8

D. 64

Answer: B

Solution:

Solution:

Let $9^{\tan^2 x} = P$

$\frac{9}{P} + P = 10$

$P^2 - 10P + 9 = 0$

$(P - 9)(P - 1) = 0$

$P = 1, 9$

$9^{\tan^2 x} = 1, 9^{\tan^2 x} = 9$

$\tan^2 x = 0, \tan^2 x = 1$

$x = 0, \pm\frac{\pi}{4} \quad \therefore x \in \left(-\frac{\pi}{2}, \frac{p}{2}\right)$

$\beta = \tan^2(0) + \tan^2\left(+\frac{\pi}{12}\right) + \tan^2\left(-\frac{\pi}{12}\right)$

$= 0 + 2(\tan 15^\circ)^2$

$2(2 - \sqrt{3})^2$

$2(7 - 4\sqrt{3})$

Than $\frac{1}{6}(14 - 8\sqrt{3} - 14)^2 = 32$

Question25

The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \leq x \leq 4\pi$ is : [25-Jul-2022-Shift-1]

Options:

A. 4

B. 6

C. 8

D. 12

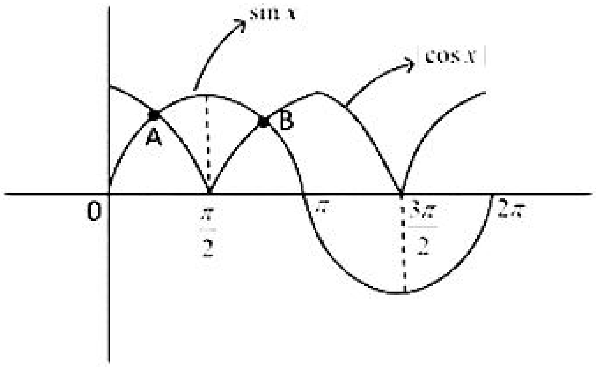
Answer: C

Solution:

Solution:

Period of $|\cos x| = \pi$

And period of $\sin x = 2\pi$



Graph of $\sin x$ and $|\cos x|$ cuts each other at two points A and B in $[0, 2\pi]$
So, in $[-4\pi, 4\pi]$, total 4 similar graph will be present and graph of $\sin x$ and $|\cos x|$ will cut $4 \times 2 = 8$ times.
$\therefore$ Total possible solutions $= 8$

Question26

Let $S = \left\{ \theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16 \right\}$. Then $n(s) + \sum_{\theta \in S} \left(\sec\left(\frac{\pi}{4} + 2\theta \right) \operatorname{cosec}\left(\frac{\pi}{4} + 2\theta \right) \right)$ is equal to:
[26-Jul-2022-Shift-1]

Options:

A. 0

B. −2

C. −4

D. 12

Answer: C

Solution:

Solution:
$S = \left\{ \theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16 \right\}$
Now apply AM $\geq$ GM for $8^{2\sin^2\theta}, 8^{2\cos^2\theta}$

$$\frac{8^{2\sin^2\theta} + 8^{2\cos^2\theta}}{2} \geq \left(8^{2\sin^2\theta + 2\cos^2\theta} \right) \frac{1}{2}$$

$8 \geq 8$
$\Rightarrow 8^{2\sin^2\theta} = 8^{2\cos^2\theta}$
or $\sin^2\theta = \cos^2\theta$
$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$$n(S) + \sum_{\theta \in S} \sec\left(\frac{\pi}{4} + 2\theta \right) \operatorname{cos\,e\,c}\left(\frac{\pi}{4} + 2\theta \right)$$

$$4 + \sum_{\theta \in S} \frac{2}{2\sin\left(\frac{\pi}{4} + 2\theta \right) \cos\left(\frac{\pi}{4} + 2\theta \right)}$$

$= 4 + \sum_{\theta \in S} \frac{2}{\sin\left(\frac{\pi}{2} + 4\theta\right)} = 4 + 2\sum_{\theta \in S} \operatorname{cosec}\left(\frac{\pi}{2} + 4\theta\right)$

$= 4 + 2\left[\operatorname{cosec}\left(\frac{\pi}{2} + \pi\right) \operatorname{cosec}\left(\frac{\pi}{2} + 3\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 5\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 7\pi\right)\right]$

$= 4 + 2\left[-\operatorname{cosec}\frac{\pi}{2} - \cos e\, c\frac{\pi}{2} - \cos e\, c\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2}\right]$

$= 4 - 2(4)$

$= 4 - 8$

$= -4$

Question27

If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to ________.
[26-Jul-2022-Shift-2]

Answer: 3

Solution:

Solution:

Equation (1) $2\sin^2\theta = 1 - 2\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \pm\frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Equation (2) $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow 2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{-1}{2}$

$\Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore$ Common solutions $= \frac{7\pi}{6}; \frac{11\pi}{6}$

Sum of solutions $= \frac{7\pi + 11\pi}{6} = \frac{18\pi}{6} = 3\pi$

$\therefore k = 3$

Question28

Let S = {$\theta \in (0, 2\pi)$: $7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$}. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$, $\theta \in S$, is _______.
[29-Jul-2022-Shift-1]

Answer: 16

Solution:

Solution:

$7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$\Rightarrow 4\left(\frac{1+\cos 2\theta}{2}\right) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$\Rightarrow 2 + 5\cos^2\theta - 2\cos^2 2\theta = 2$

$\Rightarrow \cos 2\theta = 0$ or $\frac{5}{2}$(rejected)

$\Rightarrow \cos 2\theta = 0 = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \tan^2\theta = 1$

$\therefore$ Sum of roots $= 2(\tan^2\theta + \cot^2\theta) = 2 \times 2 = 4$

But as $\tan\theta = \pm 1$ for $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in the interval $(0, 2\pi)$

$\therefore$ Four equations will be formed

Hence sum of roots of all the equations $= 4 \times 4 = 16.$

Question29

$2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to :

[25-Jul-2022-Shift-2]

Options:

A. $\frac{3}{16}$

B. $\frac{1}{16}$

C. $\frac{1}{32}$

D. $\frac{9}{32}$

Answer: B

Solution:

Solution:

$2 \sin \frac{\pi}{22} \sin \frac{3\pi}{22} \sin \frac{5\pi}{22} \sin \frac{7\pi}{22} \sin \frac{9\pi}{22}$

$= 2 \sin\left(\frac{11\pi - 10\pi}{22} \right) \sin\left(\frac{11\pi - 8\pi}{22} \right) \sin\left(\frac{11\pi - 6\pi}{22} \right) \sin\left(\frac{11\pi - 4\pi}{22} \right) \sin\left(\frac{11\pi - 2\pi}{22} \right)$

$= 2 \cos \frac{\pi}{11} \cos \frac{2\pi}{11} \cos \frac{3\pi}{11} \cos \frac{4\pi}{11} \cos \frac{5\pi}{11}$

$= \frac{2 \sin \frac{32\pi}{11}}{2^5 \sin \frac{\pi}{11}}$

$= \frac{1}{16}$

Question30

Let S = $\left\{ \theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right) \sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}} \right\}$. Then [27-Jul-2022-Shift-2]

Options:

A. $S = \left\{ \frac{\pi}{12} \right\}$

B. $S = \left\{ \frac{2\pi}{3} \right\}$

C. $\sum_{\theta \in S} \theta = \frac{\pi}{2}$

D. $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

Answer: C

Solution:

Solution:

$s = \left\{ 0 \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right) \sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}} \right\}.$

$\sum_{m=1}^{9} \frac{1}{\cos\left(\theta + (m-1)\frac{\pi}{6}\right)} \cos\left(\theta + m\frac{\pi}{6}\right)$

$\frac{1}{\sin\left(\frac{\pi}{6}\right)} \sum_{m=1}^{9} \frac{\sin\left[\left(\theta + \frac{m\pi}{6}\right) - \left(\theta + (m-1)\frac{\pi}{6}\right)\right]}{\cos\left(\theta + (m-1)\frac{\pi}{6}\right) \cos\left(\theta + \frac{\pi}{6}\right)}$

$= 2 \sum_{m=1}^{9} \left[\tan\left(\theta + \frac{m\pi}{6}\right) - \tan\left(\theta + (m-1)\frac{\pi}{6}\right) \right]$

Now,

$m = 1 \quad 2\left[\tan\left(\theta + \frac{\pi}{6}\right) - \tan(\theta)\right]$

$m = 2 \quad 2\left[\tan\left(\theta + \frac{2\pi}{6}\right) - \tan\left(\theta + \frac{\pi}{6}\right)\right]$

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$m = 9 \quad 2\left[\tan\left(\theta + \frac{9\pi}{6}\right) - \tan\left(\theta + 8\frac{\pi}{6}\right)\right]$

$\therefore = 2\left[\tan\left(\theta + \frac{3\pi}{2}\right) - \tan\theta\right] = \frac{-8}{\sqrt{3}}$

$= -2[\cot\theta + \tan\theta] = \frac{-8}{\sqrt{3}}$

$= -\frac{2 \times 2}{2\sin\theta\cos\theta} = \frac{-8}{\sqrt{3}}$

$= \frac{1}{\sin 2\theta} = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = \frac{\pi}{3}$

$2\theta = \frac{2\pi}{3}$

$\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{3}$

$\sum\theta_i = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

Question31

Let $S = \left[-\pi, \frac{\pi}{2}\right) - \left\{-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right\}$. Then the number of elements in the set $|A = \{\theta \in S : \tan\theta(1 + \sqrt{5}\tan(2\theta)) = \sqrt{5} - \tan(2\theta)\}$ is _______.
[28-Jul-2022-Shift-2]

Answer: 5

Solution:

Solution:
Let $\tan\alpha = \sqrt{5}$

$\therefore \tan\theta = \frac{\tan\alpha - \tan 2\theta}{1 + \tan\alpha\tan 2\theta}$

$\therefore \tan\theta = \tan(\alpha - 2\theta)$

$\alpha - 2\theta = n\pi + \theta$

$\Rightarrow 3\theta = \alpha - n\pi$

$\Rightarrow \theta = \frac{\alpha}{3} - \frac{n\pi}{3}$; $n \in Z$

If $\theta \in [-\pi, \pi/2]$ then $n = 0, 1, 2, 3, 4$ are acceptable
$\therefore 5$ solutions.

Question32

Let $S = \left\{\theta \in [-\pi, \pi] - \left\{\pm \frac{\pi}{2}\right\} : \sin\theta \tan\theta + \tan\theta = \sin 2\theta\right\}$.
If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal to:
[24-Jun-2022-Shift-1]

Options:

A. $7 + \sqrt{3}$

B. 9

C. $8 + \sqrt{3}$

D. 10

Answer: B

Question33

The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is____
[25-Jun-2022-Shift-1]

Answer: 4

Solution:

Solution:

$$\frac{14}{\sin^2 x} - 2\sin^2 x = 21 - 4(1 - \sin^2 x)$$

Let $\sin^2 x = t$

$\Rightarrow 14 - 2t^2 = 21t - 4t + 4t^2$

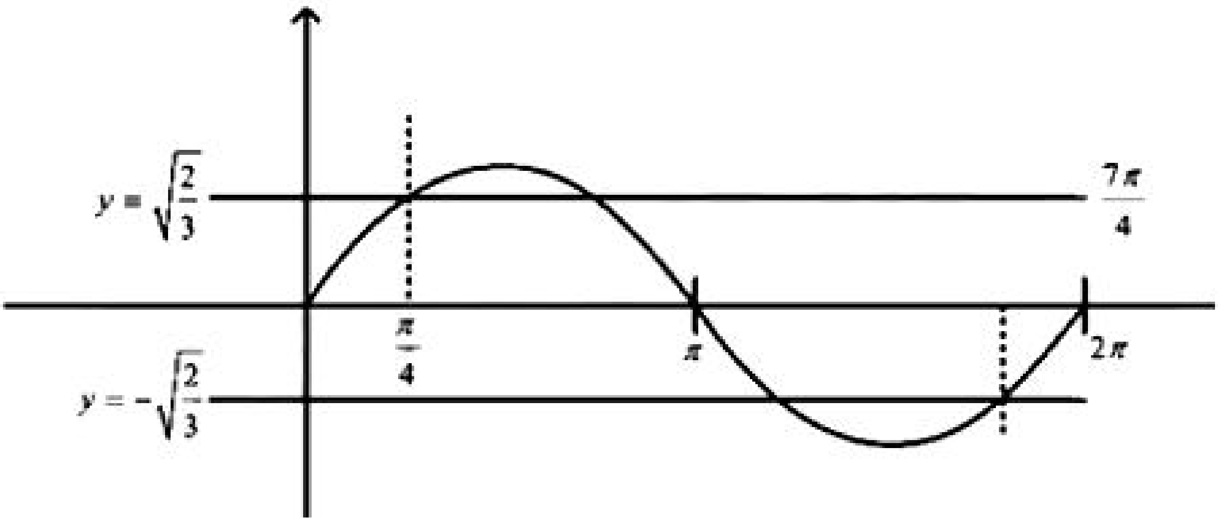
$\Rightarrow 6t^2 + 17t - 14 = 0$

$\Rightarrow 6t^2 + 21t - 4t - 14 = 0$

$\Rightarrow 3t(2t + 7) - 2(2t + 7) = 0$

$\Rightarrow \sin 2x = \frac{2}{3}$ or $-\frac{7}{3}$ (rejected)

$\Rightarrow \sin x = \pm\sqrt{\frac{2}{3}}$



Question34

The number of elements in the set
$S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is
[29-Jun-2022-Shift-1]

Answer: 32

Solution:

Solution:
$3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$
$3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$
$3\cos^2 2\theta + \cos 2\theta = 0$
$\mathrm{Cos}\, 2\theta = 0 \;\; \mathrm{OR} \;\; \cos 2\theta = -1/3$
$\theta \in [-4\pi, 4\pi]$
$2\theta = (2n + 1) \cdot \frac{\pi}{2}$
$\therefore \;\; \theta = \pm\pi/4. \pm 3\pi/4 \ldots\ldots . \pm 15\pi/4$
Similarly $\cos 2\theta = -1/3$ gives 16 solution

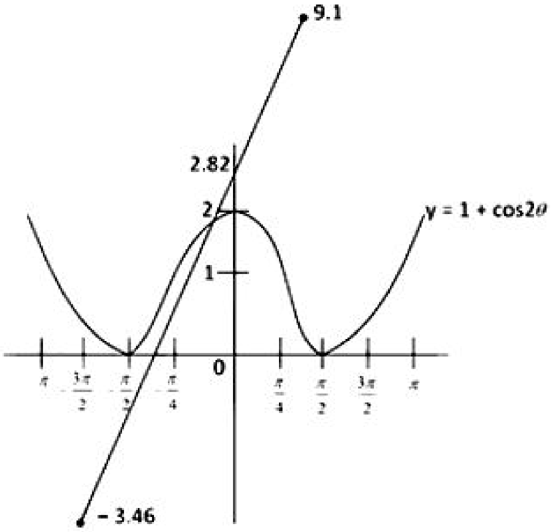
Question35

The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in R is equal to [29-Jun-2022-Shift-1]

Answer: 1

Solution:

Solution:
Given,
$2\theta - \cos^2\theta + \sqrt{2} = 0$
$\Rightarrow 2\theta + \sqrt{2} = \cos^2\theta$
$\Rightarrow 2\theta + \sqrt{2} = \frac{1 + \cos 2\theta}{2}$
$\Rightarrow 4\theta + 2\sqrt{2} = 1 + \cos 2\theta = y \;\; \text{(Assume)}$
$\therefore y = 4\theta + 2\sqrt{2} \;\; \text{and}$
$y = 1 + \cos 2\theta$



For $y = 1 + \cos 2\theta$
when $\theta = 0$, $y = 1 + 1 = 2$
when $\theta = \frac{\pi}{4}$, $y = 1 + \cos\frac{\pi}{2} = 1$
$\theta = \frac{\pi}{2}$, $y = 1 + \cos\pi = 1 - 1 = 0$
For $y = 4\theta + 2\sqrt{2}$
when $\theta = 0$, $y = 2\sqrt{2}$
when $\theta = \frac{\pi}{2}$, $y = 2\pi + 2\sqrt{2}$
$= 2(\pi + \sqrt{2})$
$= 2(3.14 + 1.41)$
$= 2(4.55)$
$= 9.1$
when $\theta = -\frac{\pi}{2}$, $y = -2\pi + 2\sqrt{2}$
$= 2(-\pi + \sqrt{2})$
$= 2(-3.14 + 1.41)$
$= -3.46$
∴ Two graph cut's at only one point so one solution possible.

Question36

The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0, 10) is____
[29-Jun-2022-Shift-2]

Answer: 4

Solution:

Solution:
$\sin x = \cos^2 x, x \in (0, 10)$
$\Rightarrow \sin x = 1 - \sin^2 x$
$\Rightarrow \sin^2 x + \sin x - 1 = 0$

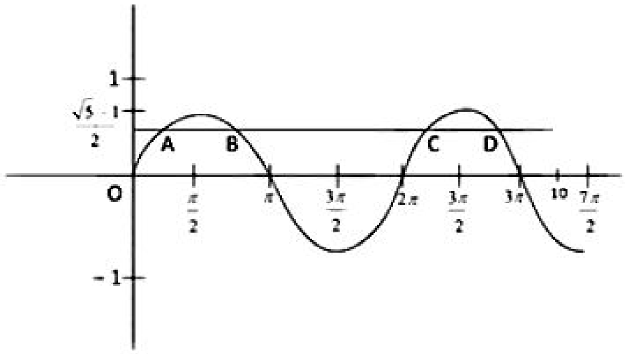
$\therefore \sin x = \frac{-1 \pm \sqrt{1+4}}{2}$

$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{2}$

We know $\sin \in (-1, 1)$

$\therefore \frac{-1-\sqrt{5}}{2}$ can't be a value of $\sin x$

$\therefore \sin x = \frac{\sqrt{5}-1}{2}$



$3\pi = 3 \times 3.14 = 9.42 < 10$

$\frac{7\pi}{2} = \frac{7}{2} \times 3.14 = 10.99 > 10$

$\therefore 10$ will be in between 3π and $\frac{7\pi}{2}$.

There are 4 intersection at A, B, C and D between $\sin x$ graph and $y = \frac{\sqrt{5}-1}{2}$ graph.

$\therefore$ possible solution $= 4$

Question37

The number of solutions of the equation $\cos\left(x + \frac{\pi}{3}\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}\cos^2 2x$, $x \in [-3\pi, 3\pi]$ is : [24-Jun-2022-Shift-2]

Options:

A. 8

B. 5

C. 6

D. 7

Answer: D

Solution:

Solution:

$$\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x,\ x \in [-3\pi, 3\pi]$$

$$\Rightarrow \cos 2x + \cos\frac{2\pi}{3} = \frac{1}{2}\cos^2 2x$$

$$\Rightarrow \cos^2 2x - 2\cos 2x - 1 = 0$$

$$\Rightarrow \cos 2x = 1$$

$$\therefore x = -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$$

$\therefore$ Number of solutions $= 7$

Question38

The value of $2\sin(12^\circ) - \sin(72^\circ)$ is : [25-Jun-2022-Shift-2]

Options:

A. $\frac{\sqrt{5}(1-\sqrt{3})}{4}$

B. $\frac{1-\sqrt{5}}{8}$

C. $\frac{\sqrt{3}(1-\sqrt{5})}{2}$

D. $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Answer: D

Solution:

Solution:

$$2\sin 12^\circ - \sin 72^\circ$$
$$= \sin 12^\circ + (-2\cos 42^\circ \cdot \sin 30^\circ)$$
$$= \sin 12^\circ - \cos 42^\circ$$
$$= \sin 12^\circ - \sin 48^\circ$$
$$= 2\sin 18^\circ \cdot \cos 30^\circ$$
$$= -2\left(\frac{\sqrt{5}-1}{4}\right)\cdot\frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}(1-\sqrt{5})}{4}$$

Question39

If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$, then $16 + \alpha^{-1}$ is equal to____
[26-Jun-2022-Shift-1]

Answer: 80

Solution:

Solution:

$(\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ) \cdot (\sin 10^\circ \cdot \sin 20^\circ \cdot \sin 40^\circ)$

$= \left(\frac{1}{4}\sin 30^\circ\right) \cdot \left[\frac{1}{2}\sin 10^\circ(\cos 20^\circ - \cos 60^\circ)\right]$

$= \frac{1}{16}\left[\sin 10^\circ\left(\cos 20^\circ - \frac{1}{2}\right)\right]$

$= \frac{1}{32}[2\sin 10^\circ \cdot \cos 20^\circ - \sin 10^\circ]$

$= \frac{1}{32}[\sin 30^\circ - \sin 10^\circ - \sin 10^\circ]$

$= \frac{1}{64} - \frac{1}{64}\sin 10^\circ$

Clearly, $\alpha = \frac{1}{64}$

Hence $16 + \alpha^{-1} = 80$

Question40

$16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ)$ is equal to :
[26-Jun-2022-Shift-2]

Options:

A. $\sqrt{3}$

B. $2\sqrt{3}$

C. 3

D. $4\sqrt{3}$

Answer: B

Solution:

Solution:

$$16 \sin 20^{\circ} \cdot \sin 40^{\circ} \cdot \sin 80^{\circ}$$

$$= 4 \sin 60^{\circ} \{\because 4 \sin \theta \cdot \sin(60^{\circ} - \theta) \cdot \sin(60^{\circ} + \theta) = \sin 3\theta\}$$

$$= 2\sqrt{3}$$

Question41

The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to: [27-Jun-2022-Shift-1]

Options:

A. -1

B. $-\frac{1}{2}$

C. $-\frac{1}{3}$

D. $-\frac{1}{4}$

Answer: B

Solution:

Solution:

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = \frac{\sin 3\left(\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \cos\frac{\left(\frac{2\pi}{7} + \frac{6\pi}{7}\right)}{2}$$

$$= \frac{\sin\left(\frac{3\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}$$

$$= \frac{2 \sin\frac{4\pi}{7} \cos\frac{4\pi}{7}}{2 \sin\frac{\pi}{7}}$$

$$= \frac{\sin\left(\frac{3\pi}{7}\right)}{2 \sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{2 \sin\frac{\pi}{7}} = \frac{-1}{2}$$

Question42

$\alpha = \sin 36^{\circ}$ is a root of which of the following equation?
[27-Jun-2022-Shift-2]

Options:

A. $16x^4 - 10x^2 - 5 = 0$

B. $16x^4 + 20x^2 - 5 = 0$

C. $16x^4 - 20x^2 + 5 = 0$

D. $16x^4 - 10x^2 + 5 = 0$

Answer: C

Solution:

Solution:
$\alpha = \sin 36^{\circ} = x$ (say)
$\therefore x = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$
$\Rightarrow 16x^2 = 10 - 2\sqrt{5}$
$\Rightarrow (8x^2 - 5)^2 = 5$
$\Rightarrow 16x^4 - 80x^2 + 20 = 0$
$\therefore 4x^4 - 20x^2 + 5 = 0$

Question43

If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are :
[28-Jun-2022-Shift-2]

Options:

A. $-\frac{1}{7}$ and IV $^{\text{th}}$ quadrant

B. 7 and I $^{\text{st}}$ quadrant

C. -7 and I V $^{\text{th}}$ quadrant

D. $\frac{1}{7}$ and I $^{\text{st}}$ quadrant

Answer: A

Solution:

Solution:

$\because \cot\alpha = 1, \quad \alpha \in \left(\pi, \frac{3\pi}{2}\right)$

then $\tan\alpha = 1$

and $\sec\beta = -\frac{5}{3}, \quad \beta \in \left(\frac{\pi}{2}, \pi\right)$

then $\tan\beta = -\frac{4}{3}$

$\therefore \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

$= \dfrac{1 - \frac{4}{3}}{1 + \frac{4}{3}}$

$= -\frac{1}{7}$

$\alpha + \beta \in \left(\frac{3\pi}{2}, 2\pi\right)$ i.e. fourth quadrant

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Question44

cosec 18° is a root of the equation [31 Aug 2021 Shift 1]

Options:

A. $x^2 + 2x - 4 = 0$

B. $4x^2 + 2x - 1 = 0$

C. $x^2 - 2x + 4 = 0$

D. $x^2 - 2x - 4 = 0$

Answer: D

Solution:

Solution:

$\operatorname{cosec} 18^\circ = \dfrac{1}{\sin 18^\circ} = \dfrac{4}{\sqrt{5} - 1} = \sqrt{5} + 1$

If $x = \sqrt{5} + 1$, then

$(x-1)^2 = 5$

$\Rightarrow x^2 - 2x - 4 = 0$

Question45

The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right) \text{ is}$$

[26 Aug 2021 Shift 2]

Options:

A. $\frac{1}{4\sqrt{2}}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{8\sqrt{2}}$

Answer: C

Solution:

Solution:

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$= 2\sin\frac{\pi}{8}\sin\frac{2\pi}{8}\sin\frac{3\pi}{8}\sin\left(\pi-\frac{3\pi}{8}\right)\sin\left(\pi-\frac{2\pi}{8}\right)\sin\left(\pi-\frac{\pi}{8}\right)$

$= 2\sin\frac{\pi}{8}\sin\frac{2\pi}{8}\sin\frac{3\pi}{8}\sin\frac{3\pi}{8}\sin\frac{2\pi}{8}\sin\frac{\pi}{8}$

$= 2\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{2\pi}{8}\right)\sin^2\left(\frac{3\pi}{8}\right)2\sin^2\frac{\pi}{8}\left(\frac{1}{\sqrt{2}}\right)^2\sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$= 2\sin^2\frac{\pi}{8}\times\frac{1}{2}\times\cos^2\frac{\pi}{8} = \sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)$

$= \frac{1}{4}\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)^2\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{8}$

Question46

If $\sin\theta + \cos\theta = \frac{1}{2}$, then16 $(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to:

[27 Jul 2021 Shift 1]

Options:

A. 23

B. −27

C. −23

D. 27

Answer: C

Solution:

Solution:

$\sin\theta + \cos\theta = \frac{1}{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$

$\sin 2\theta = -\frac{3}{4}$

Now :

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$= 1 - 2\left(-\frac{3}{4}\right)^2$

$= 1 - 2 \times \frac{9}{16} = -\frac{1}{8}$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$= (3 - 4\sin^2 2\theta) . \sin 2\theta$

$= \left[3 - 4\left(\frac{9}{16}\right)\right] . \left(-\frac{3}{4}\right)$

$\Rightarrow \left[\frac{3}{4}\right] \times \left(-\frac{3}{4}\right) = -\frac{9}{16}$

$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$

$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$

Question47

The value of $\cot\frac{\pi}{24}$ is:
[25 Jul 2021 Shift 2]

Options:

A. $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$

B. $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

C. $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$

D. $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

Answer: B

Solution:

Solution:

$$\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$\theta = \frac{\pi}{24}$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$$

$$= \sqrt{6}+\sqrt{2}+\sqrt{3}+2$$

Question48

If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\mathrm{cosec}^6\alpha$ is equal to
[18 Mar 2021 Shift 2]

Options:

A. 350

B. 500

C. 400

D. 250

Answer: D

Solution:

Solution:
Given, $15\sin^4\alpha + 10\cos^4\alpha = 6$
$\Rightarrow\ 15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$
$\Rightarrow\ 15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^4\alpha + \cos^4\alpha + 2\sin^2\alpha\cos^2\alpha)$
$\Rightarrow 9\sin^4\alpha + 4\cos^4\alpha - 12\sin^2\alpha\cos^2\alpha = 0$
$\Rightarrow\ (3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$
$\Rightarrow\ 3\sin^2\alpha - 2\cos^2\alpha = 0$
$\Rightarrow 3\sin^2\alpha = 2\cos^2\alpha$
$\Rightarrow \tan^2\alpha = \frac{2}{3}$
$\therefore\ \cot^2\alpha = 3/2$
Now,
$27\sec^6\alpha + 8\mathrm{cosec}^6\alpha = 27(\sec^2\alpha)^3 + 8(\mathrm{cosec}^2\alpha)^3$

$= 27(1 + \tan^2\alpha)^3 + 8(+\cot^2\alpha)^3$
$= 27\left(1 + \frac{2}{3}\right)^3 + 8\left(1 + \frac{3}{2}\right)^3 = 250$

Question49

If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$, then the value of n is equal to
[16 Mar 2021 Shift 1]

Options:

A. 20

B. 12

C. 9

D. 16

Answer: B

Solution:

Solution:
$\log_{10} \sin x + \log_{10} \cos x = -1, x \in (0, \pi/2)$
$\log_{10}(\sin x \cos x) = -1$
$\Rightarrow \sin x \cos x = 10^{-1} = 1/10$
$\log_{10}(\sin x + \cos x) = 1/2(\log_{10} n - 1), n > 0$
$2\log_{10}(\sin x + \cos x) = (\log_{10} n - \log_{10} 10)$
$\Rightarrow \log_{10}(\sin x + \cos x)^2 = \log_{10}(n/10)$
$\Rightarrow (\sin x + \cos x)^2 = n/10$
$\Rightarrow \sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{n}{10}$
$\Rightarrow 1 + 2(1/10) = n/10 \Rightarrow 12/10 = n/10$
$\therefore n = 12$

Question50

If $0 < a, b < 1$ and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of

$$(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + \ldots \text{ is}$$

[26 Feb 2021 Shift 2]

Options:

A. $\log_e 2$

B. $e^2 - 1$

C. e

D. $\log_e\left(\frac{e}{2}\right)$

Answer: A

Solution:

Solution:

Given, $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{a+b}{1-ab}\right] = \frac{\pi}{4}$

$\Rightarrow \frac{a+b}{1-ab} = \tan\frac{\pi}{4} = 1$

$\Rightarrow a+b = 1-ab$

$\Rightarrow (1+a)(1+b) = 2 \ldots$ (i)

Now, $(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots$

$= \left[a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \ldots\right] + \left[b - \frac{b^2}{2} + \frac{b^3}{3} - \frac{b^4}{4} + \ldots\right]$

$= \log_e(1+a) + \log_e(1+b)$ [Using expansion of $\log_e(1+x)$]

$= \log_e(1+a)(1+b)$ [$\because \log a + \log b = \log ab$]

$= \log_e 2$ [use Eq. (i)]

Question51

All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in [25 Feb 2021 Shift 1]

Options:

A. $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

B. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

C. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

D. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup$

Answer: D

Solution:

Solution:
$\sin 2\theta + \tan 2\theta > 0$
$\Rightarrow \sin 2\theta + \dfrac{\sin 2\theta}{\cos 2\theta} > 0$
$\Rightarrow \sin 2\theta \left(1 + \dfrac{1}{\cos 2\theta}\right) > 0$
$\Rightarrow \sin 2\theta \left(\dfrac{\cos 2\theta + 1}{\cos 2\theta}\right) > 0$
$\Rightarrow \tan 2\theta (2\cos^2\theta) > 0$
$\cos 2\theta \neq 0$
$\Rightarrow 1 - 2\sin^2\theta \neq 0$
$\sin\theta \neq \pm \dfrac{1}{\sqrt{2}}$
Now, $\tan 2\theta(1 + \cos 2\theta) > 0$
$\Rightarrow \tan 2\theta > 0$ as $1 + \cos 2\theta > 0$
$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$
$\therefore \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$
Since, $\sin\theta \neq \pm \dfrac{1}{\sqrt{2}}$]

Question52

If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to [25 Feb 2021 Shift 2]

Options:

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1-\sqrt{3}}{2}$

D. $\frac{1+\sqrt{3}}{2}$

Answer: D

Solution:

Solution:
Given, $\cos x + \cos y - \cos(x + y) = \dfrac{3}{2}$
$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - \left[2\cos^2\left(\frac{x+y}{2}\right) - 1\right] = \frac{3}{2}$ [Use formula,
$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

$\cos 2x = 2\cos^2 x - 1$]

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right)$

$= \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 4\cos^2\left(\frac{x+y}{2}\right)$

$= \frac{1}{2} \times 2 = 1 = \cos^2\frac{(x-y)}{2} + \sin^2\frac{(x-y)}{2}$

$\Rightarrow \left[\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right)\right]^2 + \sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0$ and $\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right) = 0$

$\Rightarrow x = y$ and $\cos 0 - 2\cos x = 0$

Gives, $\cos x = \frac{1}{2} = \cos y$

$\therefore \sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1+\sqrt{3}}{2}$

Question53

If n is the number of solutions of the equation

$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right) = 1, x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is
[1 Sep 2021 Shift 2]

Options:

A. $\left(3, \frac{13\pi}{9}\right)$

B. $\left(2, \frac{2\pi}{3}\right)$

C. $\left(2, \frac{8\pi}{9}\right)$

D. $\left(3, \frac{5\pi}{3}\right)$

Answer: A

Solution:

Solution:

$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)$

$\Rightarrow 2\cos x\left(2\cos(2x) - 2\cos\left(\frac{\pi}{2}\right) - 1\right) = 1$

$\Rightarrow 2\cos x(4\cos^2 x - 3) = 1$
$\Rightarrow \cos 3x = \frac{1}{2}$
$\Rightarrow 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$
$\Rightarrow x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$
Number of solutions = n = 3
Sum of solutions $= S = \frac{13\pi}{9}$

Question54

The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \le x \le \frac{\pi}{4}$ is [31 Aug 2021 Shift 2]

Options:

A. 3

B. 1

C. 0

D. 2

Answer: B

Solution:

Solution:
$32^{\tan^2 x} + 32^{\sec^2 x} = 81$
$\Rightarrow 32^{\tan^2 x} + 32^{1 + \tan^2 x} = 81$
$\Rightarrow 33 \times 32^{\tan^2 x} = 81$
$\Rightarrow 32^{\tan^2 x} = \frac{27}{11}$
$\Rightarrow \tan^2 x = \ln_{32}\left(\frac{27}{11}\right)$
$\tan x = \sqrt{\ln_{32}\left(\frac{27}{11}\right)} \in (0, 1)$
$\Rightarrow$ One solution in $\left[0, \frac{\pi}{4}\right]$

Question55

Section B : Numerical Type Questions
Let S be the sum of all solutions (in radians) of the equation

$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then, $\frac{85}{\pi}$ is equal to
[27 Aug 2021 Shift 2]

Answer: 56

Solution:

Solution:
$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$
$\Rightarrow 1 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$
$\Rightarrow 2 - \sin^2 2\theta - \sin 2\theta = 0$
$\Rightarrow \sin^2 2\theta + \sin 2\theta - 2 = 0$
$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$
$\Rightarrow \sin 2\theta = 1,\ 2\theta \in [0, 8\pi]$
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$
Sum of solutions $S = \frac{28\pi}{4}$
Then, $\frac{8S}{\pi} = \frac{8}{\pi} \times \frac{28\pi}{4} = 56$

Question56

The sum of solutions of the equation $\frac{\cos x}{1 + \sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is
[26 Aug 2021 Shift 1]

Options:

A. $-\frac{11\pi}{30}$

B. $\frac{\pi}{10}$

C. $-\frac{7\pi}{30}$

D. $-\frac{\pi}{15}$

Answer: A

Solution:

Solution:

We have, $\frac{\cos x}{1}=\left|\tan 2x\right|$

$$\Rightarrow \frac{\cos^2\left(\frac{x}{2}\right)-\sin^2\left(\frac{x}{2}\right)}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}=\left|\tan 2x\right|$$

$$\Rightarrow \frac{\left[\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right]\left[\cos\frac{x}{2}+\sin\frac{x}{2}\right]}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}=\left|\tan 2x\right|$$

$$\Rightarrow \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}=\left|\tan 2x\right|$$

$$\Rightarrow \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=\left|\tan 2x\right|$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\tan^2 2x$$

$$\Rightarrow 2x=n\pi\pm\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\Rightarrow 2x=n\pi+\frac{\pi}{4}-\frac{x}{2}$$

or $2x=n\pi-\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$$\Rightarrow \frac{5x}{2}=\left(n+\frac{1}{4}\right)\pi$$

or $\frac{3x}{2}=\left(n-\frac{1}{4}\right)\pi$

$$\Rightarrow \frac{-\pi}{2}<\frac{2}{5}\left(n+\frac{1}{4}\right)\pi<\frac{\pi}{2}$$

or $\frac{-\pi}{2}<\frac{2}{3}\left(n-\frac{1}{4}\right)\pi<\frac{\pi}{2}$

$$\Rightarrow -\frac{5}{4}<n+\frac{1}{4}<\frac{5}{4}$$

or $\frac{-3}{4}<n-\frac{1}{4}<\frac{3}{4}$

$$\Rightarrow \frac{-6}{4}<n<1$$

or $\frac{-1}{2}<n<1$

$\Rightarrow n=-1, 0$
or n = 0

When $n=-1,\ x=\left(\frac{-3}{10}\right)\pi$

or when $n=0,\ x=-\frac{\pi}{6}$

$n=0,\ x=\left(\frac{1}{10}\right)\pi$

$\therefore$ Required sum $=\left(\frac{-3}{10}\right)\pi+\left(\frac{1}{10}\right)\pi+\left(\frac{-1}{6}\right)\pi=\left(\frac{-11}{30}\right)\pi$

Question57

The number of solutions of $\sin^7 x+\cos^7 x=1$, $x\in[0, 4\pi]$ is equal to [22 Jul 2021 Shift 2]

Options:

A. 11

B. 7

C. 5

D. 9

Answer: C

Solution:

Solution:
$\sin^7 x \leq \sin^2 x \leq \lambda 1$ ....(1)
and $\cos^7 x \leq \cos^2 x \leq 1$ ....(2)
also $\sin^2 x + \cos^2 x = 1$
⇒ equality must hold for (1) & (2)
$\Rightarrow \sin^7 x = \sin^2 x$ & $\cos^7 = \cos^2 x$
$\Rightarrow \sin x = 0$ & $\cos x = 1$
or
$\cos x = 0$ & $\sin x = 1$
$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$
⇒5 solutions

Question58

Let $\alpha = \max\limits_{x \in R}\{8^{2\sin 3x} . 4^{4\cos 3x}\}$ andbeta $= \min\limits_{x \in R}\{8^{2\sin 3x} . 4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$,then the value of $c - b$ is equal to :
[27 Jul 2021 Shift 2]

Options:

A. 42

B. 47

C. 43

D. 50

Answer: A

Solution:

Solution:
$\alpha = \max\{8^{2\sin 3x} . 4^{4\cos 3x}\}$
$= \max\{2^{6\sin 3x} . 2^{8\cos 3x}\}$

$= \max\{2^{6\sin 3x + 8\cos 3x}\}$
and $\beta = \min\{8^{2\sin 3x} . 4^{4\cos 3x}\} = \min\{2^{6\sin 3x + 8\cos 3x}\}$
Now range of $6\sin 3x + 8\cos 3x$
$= [-\sqrt{6^2+8^2}, +\sqrt{6^2+8^2}] = [-10, 10]$
$\alpha = 2^{10}$ & $\beta = 2^{-10}$
So, $\alpha^{1/5} = 2^2 = 4$
$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$
quadratic $8x^2 + bx + c = 0$, $c - b =$
$8 \times$ [(product of roots] + (sum of roots)
$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4}\right] = 8 \times \left[\frac{21}{4}\right] = 42$

Question59

The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is
[18 Mar 2021 Shift 1]

Answer: 1

Solution:

Solution:
Given, $|\cot x| = \cot x + \frac{1}{\sin x}$....(i)
If $\cot x > 0$, then $|\cot x| = \cot x$
From Eq. (i), $\cot x = \cot x + \frac{1}{\sin x}$
$\Rightarrow \frac{1}{\sin x} = 0$ (not possible)
If $\cot x < 0$, then $|\cot x| = -\cot x$
From Eq. (ii), $-\cot x = \cot x + \frac{1}{\sin x}$
$\Rightarrow 2\cot x + \frac{1}{\sin x} = 0 \Rightarrow 2\cos x = -1$
$\Rightarrow x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$
Here, $x = \frac{4\pi}{3}$ rejected because $\frac{4\pi}{3} \in$ third quadrant and in third quadrant $\cot x$ is positive. Since, we considered $\cot x < 0$. $\therefore x = 2\pi/3$ is the only one solution.

Question60

The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is
[17 Mar 2021 Shift 2]

Options:

A. 3

B. 4

C. 2

D. 5

Answer: A

Solution:

Solution:

Given, $x + 2\tan x = \frac{\pi}{2}$

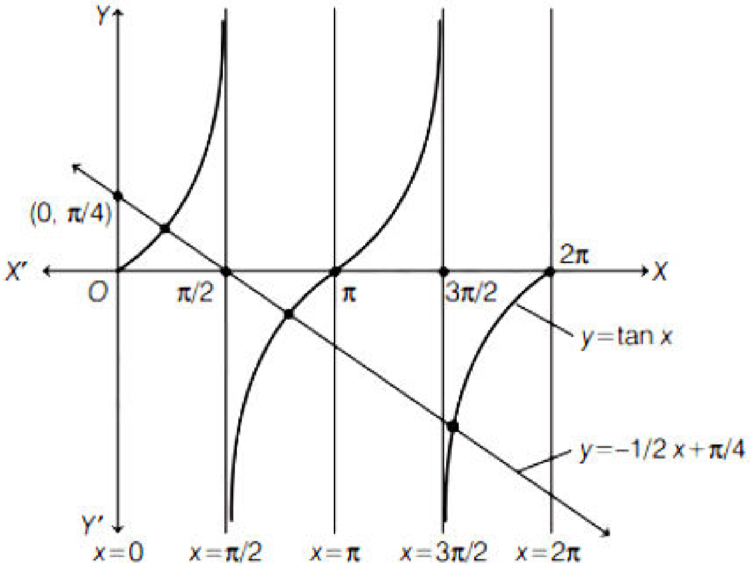
$\Rightarrow\ 2\tan x = \frac{\pi}{2} - x \Rightarrow \tan x = \frac{\pi}{4} - \frac{x}{2}$

$\Rightarrow\ \tan x = \left(-\frac{1}{2}\right)x + \frac{\pi}{4}$...(i)

Approach In this type of problem solving, graphical approach is best because we have to find only number of solutions, not the solution (i.e. not the value(s) of x).

Concept To find the number of solution(s) for Eq. (i), first of all, let $y = \tan x \ldots$ (ii) and $y = \left(\frac{-1}{2}\right)x + \frac{\pi}{4} \ldots$ (iii) and then draw the graph of Eqs. (ii) and (iii).

Now, total number of solution(s) = Total number of point(s) of intersection of the graph (ii) and (iii).



$y = -\frac{1}{2}x + \frac{\pi}{4}$ intersects $y = \tan x$ at three distinct points in $[0, 2\pi]$.

$\therefore$ Total number of solutions $= 3$

Question61

The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to
[16 Mar 2021 Shift 1]

Options:

A. 3

B. 4

C. 8

D. 2

Answer: B

Solution:

Solution:
Given, $81^{\sin^2 x} + 81^{\cos^2 x} = 30$
$\Rightarrow\ 81^{\sin^2 x} + 81^{(1 - \sin^2 x)} = 30$
$\Rightarrow\ 81^{\sin^2 x} + \dfrac{81}{81^{\sin^2 x}} = 30$
Let $81^{\sin^2 x} = y$.
$\therefore\ y + \dfrac{81}{y} = 30$
$\Rightarrow\ y^2 - 30y + 81 = 0$
$\Rightarrow\ (y - 27)(y - 3) = 0$
$\Rightarrow\ y = 3$ or $y = 27$
$81^{\sin^2 x} = 3$ or $81^{\sin^2 x} = 27$
$3^{4\sin^2 x} = 3$ or $3^{4\sin^2 x} = 3^3$
$\Rightarrow\ 4\sin^2 x = 1$ or $4\sin^2 x = 3$
$\Rightarrow\ \sin^2 x = 1/4$ or $\sin^2 x = 3/4$
$\Rightarrow\ \sin^2 x = \sin^2(\pi/6)$ or $\sin^2 x = \sin^2(\pi/3)$
$\Rightarrow\ x = n\pi \pm \pi/6$ or $x = n\pi \pm \pi/3$
From $[0, \pi]$,
$x = \pi/6, 5\pi/6$ or $x = \pi/3, 2\pi/3$
Hence, the total number of solutions $= 4$

Question62

If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ....
[26 Feb 2021 Shift 1]

Answer: 1

Solution:

Solution:
Given, $\sqrt{3}\cos^2 x = (\sqrt{3} - 1)\cos x + 1,\ x \in [0, \pi/2]$
Let $\cos x = t$, then
$\sqrt{3}t^2 = (\sqrt{3} - 1)t + 1$
$\Rightarrow \sqrt{3}t^2 - \sqrt{3}t + t - 1 = 0$
$\Rightarrow (\sqrt{3}t^2 - \sqrt{3}t) + (t - 1) = 0$
$\Rightarrow \sqrt{3}t(t - 1) + 1(t - 1) = 0$
$\Rightarrow (t - 1)(\sqrt{3}t + 1) = 0$
This gives $t = 1$ and $t = \frac{-1}{\sqrt{3}}$
Put, $t = \cos x$, then
$\cos x = 1$ and $\cos x = \frac{-1}{\sqrt{3}}$
$\cos x = -1/\sqrt{3}$ is rejected as $x \in [0, \pi/2]$
$\cos x = -1/\sqrt{3}$ is rejected as $x \in [0, \pi$
$\therefore \cos x = 1$
Since, $x \in \left[0, \frac{\pi}{2}\right]$, then $\cos x = \cos 0$
This gives $x = 0$ is only solution.
Therefore, number of solution when $x \in [0, \pi/2]$ is 1 .

Question63

The number of integral values of k for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is .... [26 Feb 2021 Shift 1]

Answer: 11

Solution:

Solution:
Given, $3\sin x + 4\cos x = k + 1 \ldots$ (i)
Multiply and divide LHS of Eq. (i) by $\sqrt{3^2 + 4^2} = 5$
i.e. $5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right) = k + 1$
$\Rightarrow 5(\cos\alpha \sin x + \sin\alpha \cos x) = k + 1$
[Let $\cos\alpha = 3/5$ then $\sin\alpha = \sqrt{1 - (3/5)^2} = \frac{4}{5}$]
$\Rightarrow 5\sin(x + \alpha) = k + 1$ [Use $\sin(a + b) = \sin a \cos b + \cos a \sin b$]
$\Rightarrow \sin(x + \alpha) = \frac{k + 1}{5}$
Let $x + \alpha = \theta$
Then, $\sin\theta = \frac{k + 1}{5}$
$\because\ -1 \le \sin\theta \le 1$
$\Rightarrow\ -1 \le \frac{k + 1}{5} \le 1$
$\Rightarrow\ -5 \le k + 1 \le 5$
$\Rightarrow\ -6 \le k \le 4$

$\therefore$ Possible integral values of k are $-6, -5, -4, -3, -2, -1, 0, 1, 2,$ 3 and 4. i.e.
Total 11 integral values of k are possible for which Eq. (i) has solution.

Question64

The value of $\cos^3\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \cdot \sin\left(\frac{3\pi}{8}\right)$ is

[Jan. 9, 2020 (I)]

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:

$\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$

$= 4\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]$

$\left[\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]$;

$-3\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)\right]$;

$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right]$

$= \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}\Big]$.

Question65

If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then :

[Sep. 05, 2020 (II)]

Options:

A. $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

B. $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

C. $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

D. $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

Answer: D

Solution:

Solution:

$L + M = 1 - 2\sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ....(i)

and $L - M = -\cos\frac{\pi}{8}$ ....(ii)

From equations (i) and (ii), $L = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$ and $M = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

Question66

If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$ $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to ________

[Jan. 8, 2020 (II)]

Answer: 1

Solution:

Solution:

$\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos^2\beta}{2}} = \frac{1}{10}$

$\Rightarrow \frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\therefore\ \tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$

$\tan\beta = \frac{1}{3}$

$\therefore\ \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$

$\tan(\alpha+2\beta) = \dfrac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}$

$$= \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$

Question67

The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1,2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x+y=1$ is: [Sep. 02, 2020 (II)]

Options:

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

C. $\left(0, \frac{3\pi}{4}\right)$

D. $\left(0, \frac{\pi}{4}\right)$

Answer: A

Solution:

Solution:
Let $f(x, y) = x+y-1$
Given (1,2) and $(\sin\theta, \cos\theta)$ are lies on same side.
$\therefore f(1, 2)\cdot f(\sin\theta, \cos\theta) > 0$
$\Rightarrow 2[\sin\theta+\cos\theta-1] > 0$
$\Rightarrow \sin\theta+\cos\theta| > 1 \Rightarrow \sin\left(\theta+\frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$
$\Rightarrow \theta+\frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

Question68

If the equation $\cos^4\theta+\sin^4\theta+\lambda = 0$ has real solutions for θ, then λ lies in the interval : [Sep. 02, 2020 (II)]

Options:

A. $\left(-\frac{5}{4},-1\right)$

B. $\left[-1,-\frac{1}{2}\right]$

C. $\left(-\frac{1}{2},-\frac{1}{4}\right]$

D. $\left[-\frac{3}{2},-\frac{5}{4}\right]$

Answer: B

Solution:

Solution:

Given equation is $\cos^4(\theta)+\sin^4(\theta)+\lambda=0$

$\Rightarrow\ \lambda=-[\cos^4(\theta)+\sin^4(\theta)]$

$\Rightarrow\lambda=-[(\cos^2(\theta)+\sin^2(\theta))^2-2\cos^2(\theta)\sin^2(\theta)]$

$[\because a^2+b^2=(a+b)^2-2ab]$

$\Rightarrow\lambda=-(1)^2+\frac{1}{2}(2\cos(\theta)\sin(\theta))^2$

$[\because\cos^2(\theta)+\sin^2(\theta)=1]$

$\Rightarrow\lambda=\frac{1}{2}\sin^2(2\theta)-1$

We know that $-1\le\sin(x)\le1$

Therefore $0\le\sin^2(x)\le1$

So $\frac{1}{2}\sin^2(2\theta)\in\left[0,\ \frac{1}{2}\right]$

for $\frac{1}{2}\sin^2(2\theta)=0\Rightarrow\lambda=0-1=-1$

for $\frac{1}{2}\sin^2(2\theta)=\frac{1}{2}\Rightarrow\lambda=\frac{1}{2}-1=\frac{-1}{2}$

$\therefore\lambda\in\left[-1,-\frac{1}{2}\right]$

Hence, Option(B) i.e. $\left[-1,\ \frac{-1}{2}\right]$ is correct

----

Question69

The number of distinct solutions of the equation, $\log_{1/2}\sin x\,|\ =2-\log_{1/2}|\cos x|$ in the interval $[0,2\pi]$, is _______
[Jan. 9, 2020 (I)]

Answer: 8

Solution:

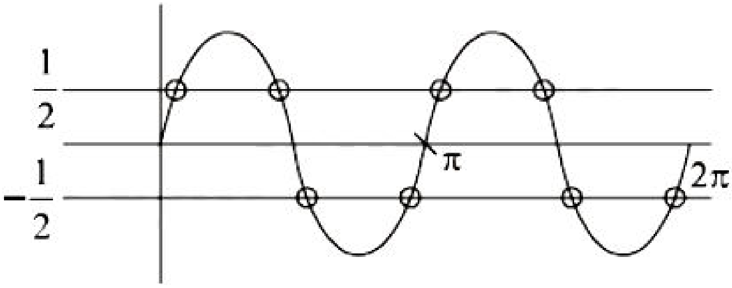
Solution:

$\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x$

$\Rightarrow \log_{1/2}|\sin x \cos x| = 2$

$\Rightarrow \left|\sin x \cos x\right| = \frac{1}{4}$

$\Rightarrow \sin 2x = \pm\frac{1}{2}$



Hence, total number of solutions = 8.

Question70

For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:
[Jan. 9, 2019 (I)]

Options:

A. $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

B. $13 - 4\cos^6\theta$

C. $13 - 4\cos^2\theta + 6\cos^4\theta$

D. $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

Answer: B

Solution:

Solution:

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - 2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6$

$- 12\sin\theta\cos\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$

$= 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$= 9 + 12\cos^2\theta - 12\cos^4\theta + 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 9 + 4 - 4\cos^6\theta$

$= 13 - 4\cos^6\theta$

Question71

The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (inm) of the foot of the tower from the point A is: [April 12, 2019 (II)]

Options:

A. $15(3+\sqrt{3})$

B. $15(5-\sqrt{3})$

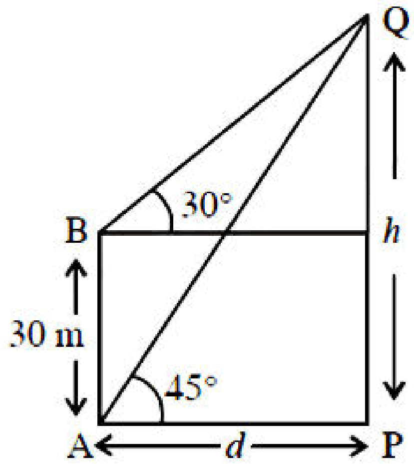
C. $15(3-\sqrt{3})$

D. $15(1+\sqrt{3})$

Answer: A

Solution:

Solution:
Let the height of the tower be h and distance of the foot of the tower from the point A is d. By the diagram,



$\tan 45^{\circ}=\frac{h}{d}=1$

$h=d$ ......(i)

$\tan 30^{\circ}=\frac{h-30}{d}$

$\sqrt{3}(h-30)=d$ ....(ii)

Put the value of h
from (i) to (ii),

$\sqrt{3}d=d+30\sqrt{3}$

$d=\frac{30\sqrt{3}}{\sqrt{3}-1}=15\sqrt{3}(\sqrt{3}+1)=15(3+\sqrt{3})$

Question72

The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is
[April 9, 2019 (II)]

Options:

A. $\frac{3}{4} + \cos 20^\circ$

B. .3/4

C. $\frac{3}{2}(1 + \cos 20^\circ)$

D. $\frac{3}{2}$

Answer: B

Solution:

Solution:
$\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$
$= \left(\frac{1 + \cos 20^\circ}{2} \right) + \left(\frac{1 + \cos 100^\circ}{2} \right) - \frac{1}{2}(2 \cos 10^\circ \cos 50^\circ)$
$= 1 + \frac{1}{2}(\cos 20^\circ + \cos 100^\circ) - \frac{1}{2}[\cos 60^\circ + \cos 40^\circ]$
$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20^\circ + \cos 100^\circ - \cos 40^\circ]$
$= \frac{3}{4} + \frac{1}{2}[2 \cos 60^\circ \times \cos 40^\circ - \cos 40^\circ]$
$= \frac{3}{4}$

Question73

Two poles standing on a horizontal ground are of heights 5m and 10m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is:
[April. 09, 2019 (II)]

Options:

A. $5(2 + \sqrt{3})$

B. $5(\sqrt{3} + 1)$

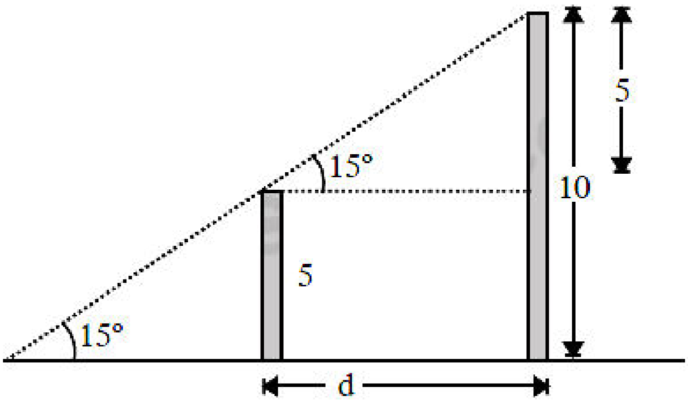
C. $\frac{5}{2}(2 + \sqrt{3})$

D. $10(\sqrt{3}-1)$

Answer: A

Solution:

Solution:



By the diagram.

$\tan 15^{\circ}=\frac{5}{d} \Rightarrow d \quad=\frac{5}{\tan 15^{\circ}}=\frac{5(\sqrt{3}+1)}{\sqrt{3}-1}$

$=\frac{5(4+2\sqrt{3})}{2}=5(2+\sqrt{3})$

Question74

The value of $\sin 10^{\circ} \sin 30^{\circ} \sin 50^{\circ} \sin 70^{\circ}$ is: [April. 09, 2019 (II)]

Options:

A. $\frac{1}{16}$

B. $\frac{1}{32}$

C. $\frac{1}{18}$

D. $\frac{1}{36}$

Answer: A

Solution:

Solution:

$\because \sin(60^{\circ}+A)\cdot\sin(60^{\circ}-A)\sin A=\frac{1}{4}\sin 3A$

$\therefore \sin 10^{\circ}\sin 50^{\circ}\sin 70^{\circ}=\sin 10^{\circ}\sin(60^{\circ}-10^{\circ})$

$\sin(60^\circ + 10^\circ) = \frac{1}{4}\sin 30^\circ$

$\Rightarrow \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{4}\sin^2 30 = \frac{1}{16}$

Question75

If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to : [April 8, 2019 (I)]

Options:

A. $\frac{63}{52}$

B. $\frac{63}{16}$

C. $\frac{21}{16}$

D. $\frac{33}{52}$

Answer: B

Solution:

Solution:
$\because \alpha+\beta$ and $\alpha-\beta$ both are acute angles.

$\cos(\alpha+\beta) = \frac{3}{5}$, then $\sin(\alpha+\beta) = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\tan(\alpha+\beta) = \frac{4}{3}$

And $\sin(\alpha-\beta) = \frac{5}{13}$, then

$\cos(\alpha-\beta) = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$

$\Rightarrow \tan(\alpha-\beta) = \frac{5}{12}$

Now, $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta))$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)} = \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$$

Question76

If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to : [Jan. 12, 2019 (II)]

Options:

A. 0

B. -1

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: D

Solution:

Solution:
∵ The given equation is
$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha \cdot \cos\beta,\ \alpha, \beta \in [0, \pi]$
Then, by A.M., G.M. ineqality;
A.M. ≥ G.M.
$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq (\sin^4\alpha \cdot 4\cos^4\beta \cdot 1 \cdot 1)^{\frac{1}{4}}$
$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2}\sin\alpha \cdot |\cos\beta|$
Inequality still holds when $\cos\beta < 0$ but L.H.S. is positive than $\cos\beta > 0$, then
$L.H.S. = R.H.S$
$\therefore\ \sin^4\alpha = 1$ and $\cos^4\beta = \frac{1}{4}$
$\Rightarrow\ \alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$
$\therefore\ \cos(\alpha + \beta) - \cos(\alpha - \beta)$
$= \cos\left(\frac{\pi}{2} + \beta\right) - \cos\left(\frac{\pi}{2} - \beta\right)$
$= -\sin\beta - \sin\beta = -2\sin\frac{\pi}{4} = -\sqrt{2}$

Question77

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to :
[Jan. 11, 2019 (I)]

Options:

A. $\frac{1}{12}$

B. $\frac{1}{4}$

C. $\frac{-1}{12}$

D. $\frac{5}{12}$

Answer: A

Solution:

Solution:

$f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x]$

$= \frac{1}{4}\left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2}\right]$

$= \frac{1}{4}\left[1 - \frac{(\sin 2x)^2}{2}\right]$

$f_6(x) = \frac{1}{6}[\sin^6 x + \cos^6 x]$

$= \frac{1}{6}\left[\left(\sin^2 x + (\cos^2 x) - \frac{3}{4}(\sin^2 x)^2\right]\right.$

$= \frac{1}{6}\left[1 - \frac{3}{4}(\sin 2x)^2\right]$

Now $f_4(x) - f_{(6)}(x) = \frac{1}{4} - \frac{1}{6} - \frac{(\sin 2x)^2}{8} + \frac{1}{8}(\sin 2x)^2$

$= \frac{1}{12}$

Question78

The value of
$\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$
[Jan. 10, 2019 (II)]

Options:

A. $\frac{1}{512}$

B. $\frac{1}{1024}$

C. $\frac{1}{256}$

D. $\frac{1}{2}$

Answer: A

Solution:

Solution:

$A = \cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$

$= \frac{1}{2}\left(\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^9}\sin\frac{\pi}{2^9}\right)$

$= \frac{1}{2^8}\left(\cos\frac{\pi}{2^2}\cdot\sin\frac{\pi}{2^2}\right) = \frac{1}{2^9}\sin\frac{\pi}{2}$

$= \frac{1}{512}$

Question79

The number of solutions of the equation $1+\sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is : [April 12, 2019 (I)]

Options:

A. 3

B. 5

C. 7

D. 4

Answer: C

Solution:

Solution:
Consider equation, $1+\sin^4 x = \cos^2 3x$
L.H.S. $= 1+\sin^4 x$ and $R\cdot H\ .S. = \cos^2 3x$
$\because L.H\ .S. \geq 1$ and $R.H\ .S. \leq 1 \Rightarrow L.H\ .S. = R.H\ .S. = 1$
$\sin^4 x = 0$, and $\cos^2 3x = 1$
$\Rightarrow\ \sin x = 0$ and $(4\cos^2 x - 3)^2\cos^2 x = 1$
$\Rightarrow\ \sin x = 0$ and $\cos^2 x = 1 \Rightarrow x = 0, \pm\pi, \pm 2\pi$
Hence, total number of solutions is 5.

Question80

Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to
[April 12, 2019 (II)]

Options:

A. R

B. [1,4]

C. [3,7]

D. [2,6]

Answer: D

Solution:

Solution:
Given equation is, $\cos 2x + \alpha \sin x = 2\alpha - 7$ $1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$
$2\sin^2 x - \alpha \sin x + (2\alpha - 8) = 0$
$\Rightarrow \sin x = \dfrac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha + 8)}}{4}$
$\Rightarrow \sin x = \dfrac{\alpha \pm (\alpha - 8)}{4} \Rightarrow \sin x = \dfrac{\alpha - 4}{4}$
$[\sin x = 2(\text{ rejected })]$
$\because$ equation has solution, then $\dfrac{\alpha - 4}{4} \in [-1, 1]$
$\Rightarrow \alpha \in [2, 6]$

Question81

If [x] denotes the greatest integer ≤x, then the system of linear equations $[\sin \theta]x + [-\cos \theta]y = 0$ $[\cot \theta]x + y = 0$ [April 12, 2019 (II)]

Options:

A. have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

B. has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

C. has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

D. have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

Answer: A

Solution:

Solution:
According to the question, there are two cases.
Case $1 : \theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
In this interval, $[\sin \theta] = 0$, $[-\cos \theta] = 0$ and $[\cot \theta] = -1$ Then the system of equations will be ;
$0 \cdot x + 0 \cdot y = 0$ and $-x + y = 0$

Which have infinitely many solutions.

Case $2 : \theta \in \left(\pi, \frac{7\pi}{6}\right)$

In this interval, $[\sin\theta] = -1$ and $[-\cos\theta] = 0$

Then the system of equations will be ;

$-x + 0 \cdot y = 0$ and $[\cot\theta]x + y = 0$

Clearly, $x = 0$ and $y = 0$ which has unique solution.

Question82

Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is: [April 9, 2019 (I)]

Options:

A. $\frac{13\pi}{6}$

B. $\frac{5\pi}{3}$

C. 2π

D. π

Answer: C

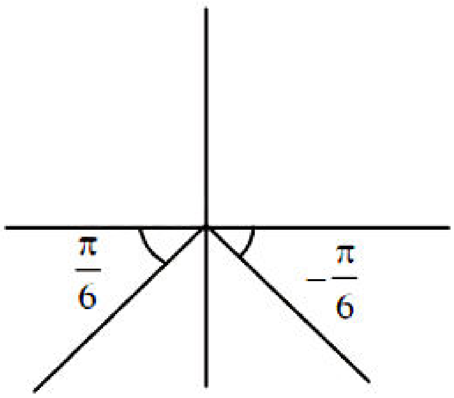
Solution:

Solution:

$2\cos^2\theta + 3\sin\theta = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$ or $\sin\theta = 2 \rightarrow$ Not possibe



The required sum of all solutions in $[-2\pi, 2\pi]$ is

$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$

Question83

If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is:
[Jan. 09, 2019 (II)]

Options:

A. 3

B. 1

C. 4

D. 2

Answer: D

Solution:

Solution:
$\sin x - \sin 2x + \sin 3x = 0$
$\Rightarrow \sin x - 2\sin x \cdot \cos x + 3\sin x - 4\sin^3 x = 0$
$\Rightarrow 4\sin x - 4\sin^3 x - 2\sin x \cdot \cos x = 0$
$\Rightarrow 2\sin x(1 - \sin^2 x) - \sin x \cdot \cos x = 0$
$\Rightarrow 2\sin x \cdot \cos^2 x - \sin x \cdot \cos x = 0$
$\Rightarrow \sin x \cdot \cos x(2\cos x - 1) = 0$
$\therefore \sin x = 0, \cos x = 0, \cos x = \frac{1}{2}$
$\therefore x = 0, \frac{\pi}{3} \quad \because x \in \left[0, \frac{\pi}{2}\right)$

Question84

The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is
[Online April 15, 2018]

Options:

A. 3

B. 4

C. 2

D. 1

Answer: D

Solution:

Solution:
$\sin 3x = \cos 2x$
$\Rightarrow 3\sin x - 4\sin^3 x = 1 - 2\sin^2 x$
$\Rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$
$\Rightarrow \sin x = 1, \ \dfrac{-2 \pm 2\sqrt{5}}{8}$
In the interval $\left(\frac{\pi}{2}, \pi\right)$, $\sin x = \dfrac{-2 + 2\sqrt{5}}{8}$
So, there is only one solution.

Question85

If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) - 1$ in $[0, \pi]$ is $k\pi$ then k is equal to : [2018]

Options:

A. $\frac{13}{9}$

B. $\frac{8}{9}$

C. $\frac{20}{9}$

D. $\frac{2}{3}$

Answer: A

Solution:

Solution:
$\because 8\cos x\left(\cos^2\frac{\pi}{6} - \sin^2 x - \frac{1}{2}\right) = 1$
$\Rightarrow \ 8\cos x\left(\frac{3}{4} - \frac{1}{2} - \sin^2 x\right) = 1$
$\Rightarrow \ 8\cos x\left(\frac{1}{4} - (1 - \cos^2 x)\right) = 1$
$\Rightarrow 8\cos x\left(\frac{1}{4} - 1 + \cos^2 x\right) = 1$
$\Rightarrow \ 8\cos x\left(\cos^2 x - \frac{3}{4}\right) = 1$
$\Rightarrow 8\left(\dfrac{4\cos^3 x - 3\cos x}{4}\right) = 1$
$\Rightarrow \ 2(4\cos^3 x - 3\cos x) = 1$
$\Rightarrow \ 2\cos 3x = 1 \Rightarrow \cos 3x = \frac{1}{2}$
$\therefore \ 3x = 2n\pi \pm \frac{\pi}{3}, n \in 1$

$\Rightarrow \ x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

In $x \in [0, \pi] : x = \frac{\pi}{9}, \frac{2\pi}{3} + \frac{\pi}{9}, \frac{2\pi}{3} - \frac{\pi}{9}$, only

Sum of all the solutions of the equation

$= \left(\frac{1}{9} + \frac{2}{3} + \frac{1}{9} + \frac{2}{3} - \frac{1}{9} \right)\pi = \frac{13}{9}\pi$

Question86

If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is [2017]

Options:

A. $-\frac{7}{9}$

B. $-\frac{3}{5}$

C. $\frac{1}{3}$

D. $\frac{2}{9}$

Answer: A

Solution:

Solution:
We have

$5\tan^2 x - 5\cos^2 x = 2(2\cos^2 x - 1) + 9$

$\Rightarrow 5\tan^2 x - 5\cos^2 x = 4\cos^2 x - 2 + 9$

$\Rightarrow 5\tan^2 x = 9\cos^2 x + 7$

$\Rightarrow 5(\sec^2 x - 1) = 9\cos^2 x + 7$

Let $\cos^2 x = t$

$\Rightarrow \frac{5}{t} - 9t - 12 = 0$

$\Rightarrow 9t^2 + 12t - 5 = 0$

$\Rightarrow 9t^2 + 15t - 3t - 5 = 0$

$\Rightarrow (3t - 1)(3t + 5) = 0$

$\Rightarrow t = \frac{1}{3}$ as $t \neq -\frac{5}{3}$

$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{3} \right) - 1 = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\frac{1}{3} \right)^2 - 1 = -\frac{7}{9}$

Question87

If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x-2\cos^4 x$, $x \in R$, then $M-m$ is equal to :
[Online April 9, 2016]

Options:

A. $\frac{9}{4}$

B. $\frac{15}{4}$

C. $\frac{7}{4}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:

$4+\frac{1}{2}\sin^2 2x-2\cos^4 x$

$4+2(1-\cos^2 x)\cos^2 x-2\cos^4 x$

$-4\left\{\cos^4 x-\frac{\cos^2 x}{2}-1+\frac{1}{16}-\frac{1}{16}\right\}$

$-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}$

$0 \leq \cos^2 x \leq 1$

$-\frac{1}{4} \leq \cos^2 x-\frac{1}{4} \leq \frac{3}{4}$

$0 \leq \left(\cos^2 x-\frac{1}{4}\right)^2 \leq \frac{9}{16}$

$-\frac{17}{16} \leq \left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16} \leq \frac{9}{16}-\frac{17}{16}$

$\frac{17}{4} \geq -4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\} \geq \frac{1}{2}$

$M=\frac{17}{4}$

$m=\frac{1}{2}$

$M-m=\frac{17}{4}-\frac{2}{4}=\frac{15}{4}$

Question88

If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$ is:
[2016]

Options:

A. 7

B. 9

C. 3

D. 5

Answer: A

Solution:

Solution:

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$\Rightarrow 2\cos 2x \cos x + 2\cos 3x \cos x = 0$

$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right) = 0$

$\cos x = 0, \cos\frac{5x}{2} = 0, \cos\frac{x}{2} = 0$

$x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

Question89

The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is [Online April 9, 2016]

Options:

A. 2

B. 6

C. 4

D. 8

Answer: D

Solution:

Solution:

$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$

$\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x} = \pm 1$

$\sqrt{2\sin^4 x + 18\cos^2 x} = \pm 1 + \sqrt{2\cos^4 x + 18\sin^2 x}$

by squaring both the sides we will get 8 solutions

Question90

If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta\ \frac{1}{2}$ and θ is the the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to :
[Online April 11, 2015]

Options:

A. $\frac{3}{5}$

B. $\frac{7}{5}$

C. $\frac{4}{5}$

D. $\frac{8}{5}$

Answer: B

Solution:

Solution:

Let $\cos\alpha + \cos\beta = \frac{3}{2}$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{3}{2}$ ....(i)

and $\sin\alpha + \sin\beta = \frac{1}{2}$

$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{1}{2}$ ....(ii)

On dividing (ii) by (i),

we get $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{3}$

Given : $\theta = \frac{\alpha+\beta}{2} \Rightarrow 2\theta = \alpha+\beta$

Consider $\sin 2\theta + \cos 2\theta = \sin(\alpha+\beta) + \cos(\alpha+\beta) = \dfrac{\frac{2}{3}}{1+\frac{1}{9}} + \dfrac{1-\frac{1}{9}}{1+\frac{1}{9}} = \frac{6}{10} + \frac{8}{10} = \frac{7}{5}$

Question91

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$ Then $f_4(x) - f_6(x)$ equals
[2014]

Options:

A. $\frac{1}{4}$

B. $\frac{1}{12}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

Answer: B

Solution:

Solution:
(b) Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ Consider $f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x)$
$- \frac{1}{6}(\sin^6 x + \cos^6 x)$
$= \frac{1}{4}[1 - 2\sin^2 x\cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x\cos^2 x]$
$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Question92

If $2\cos\theta + \sin\theta = 1\left(\theta \neq \frac{\pi}{2}\right)$ then $7\cos\theta + 6\sin\theta$ is equal to: [Online April 11, 2014]

Options:

A. $\frac{1}{2}$

B. 2

C. $\frac{11}{2}$

D. $\frac{46}{5}$

Answer: D

Solution:

Solution:
Given $2\cos\theta + \sin\theta = 1$
Squaring both sides,
we get
$(2\cos\theta + \sin\theta)^2 = 1^2$
$\Rightarrow 4\cos^2\theta + \sin^2\theta + 4\sin\theta\cos\theta = 1$
$\Rightarrow 3\cos^2\theta + (\cos^2\theta + \sin^2\theta) + 4\sin\theta\cos\theta = 1$
$\Rightarrow 3\cos^2\theta + \text{not}\,x + 4\sin\theta\cos\theta = \text{not}$
$\Rightarrow 3\cos^2\theta + 4\sin\theta\cos\theta = 0$

$\Rightarrow \cos\theta(3\cos\theta + 4\sin\theta) = 0$
$\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow 3\cos\theta = -4\sin\theta$
$\Rightarrow \frac{-3}{4} = \tan\theta = \sqrt{\sec^2\theta - 1} = \frac{-3}{4}$

$\left(\because \tan\theta = \sqrt{\sec^2\theta - 1}\right)$

$\Rightarrow \sec^2\theta - 1 = \left(\frac{-3}{4}\right)^2 = \frac{9}{16}$

$\Rightarrow \sec^2\theta = \frac{9}{16} + 1 = \frac{25}{16} \Rightarrow \sec\theta = \frac{5}{4}$

or $\cos\theta = \frac{4}{5}$ ......(1)

Now, $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$

$\sin^2\theta + \frac{4}{5} = 1 \Rightarrow \sin^2\theta = 1 - \frac{16}{25} = \frac{9}{25}$

$\sin\theta = \pm\frac{3}{5}$ ... (2)

Taking $\left(\sin\theta = +\frac{3}{5}\right)$ because $\left(\sin\theta = -\frac{3}{5}\right)$ cannot satisfy the given equation.
Therefore; $7\cos\theta + 6\sin\theta$
$= 7 \times \frac{4}{5} + 6 \times \frac{3}{5} = \frac{28}{5} + \frac{18}{5} = \frac{46}{5}$

Question93

If cosecθ = $\frac{p+q}{p-q}$ $(p \neq q \neq 0)$, then $\left|\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right|$ is equal to: [Online April 9, 2014]

Options:

A. $\sqrt{\frac{p}{q}}$

B. $\sqrt{\frac{q}{p}}$

C. $\sqrt{pq}$

D. pq

Answer: B

Solution:

Solution:
$\text{cosec}\theta = \frac{p+q}{p-q}, \sin\theta = \frac{p-q}{p+q}$
$\cos\theta = \pm\sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{p-q}{p+q}\right)^2} = \frac{2\sqrt{pq}}{(p+q)}$

$\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right| = \left|\frac{\cot\frac{\pi}{4}\cot\frac{\theta}{2}-1}{\cot\frac{\pi}{4}+\cot\frac{\theta}{2}}\right| = \left|\frac{\cot\frac{\theta}{2}-1}{\cot\frac{\theta}{2}+1}\right|$

$\left|\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right|$

On rationalizing denominator, we get

$\left|\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\right|$

$=\left|\frac{\cos\theta}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right|$

$=\left|\frac{\cos\theta}{1+\sin\theta}\right|=\left|\frac{2\sqrt{pq}/(p+q)}{1+\frac{(p-q)}{p+q}}\right| = \frac{\sqrt{pq}}{p} = \sqrt{\frac{q}{p}}$

Question94

The number of values of α in [0, 2π] for which $2\sin^3\alpha-7\sin^2\alpha+7\sin\alpha=2$, is: [Online April 9, 2014]

Options:

A. 6

B. 4

C. 3

D. 1

Answer: C

Solution:

Solution:

$2\sin^3\alpha-7\sin^2\alpha+7\sin\alpha-2=0$

$\Rightarrow 2\sin^2\alpha(\sin\alpha-1)-5\sin\alpha(\sin\alpha-1)$
$+2(\sin\alpha-1)=0$

$\Rightarrow(\sin\alpha-1)(2\sin^2\alpha-5\sin\alpha+2)=0$

$\Rightarrow\sin\alpha-1=0$ or $2\sin^2\alpha-5\sin\alpha+2=0$

$\sin\alpha=1$ or $\sin\alpha=\frac{5\pm\sqrt{25-16}}{4}=\frac{5\pm3}{4}$

$\alpha=\frac{\pi}{2}$ or $\sin\alpha=\frac{1}{2}, 2$

Now, $\sin\alpha\neq2$

for, $\sin\alpha=\frac{1}{2}$

$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$

There are three values of α between $[0, 2\pi]$

Question95

The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as: [2013]

Options:

A. $\sin A \cos A + 1$

B. $\sec A \operatorname{cosec} A + 1$

C. $\tan A + \cot A$

D. $\sec A + \operatorname{cosec} A$

Answer: B

Solution:

Solution:
Given expression can be written as

$$\frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$\left(\begin{array}{l} \because \tan A = \frac{\sin A}{\cos A} \text{ and} \\ \qquad \cot A = \frac{\cos A}{\sin A} \end{array} \right)$$

$$= \frac{1}{\sin A - \cos A} \left\{ \frac{\sin^3 A - \cos^3 A}{\cos A \sin A} \right\}$$

$$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= \frac{\sin^2 A + \sin A \cos A + \cos^2 A}{\sin A \cos A}$$

=1+sec A cosec A

Question96

Let A = {θ : sin(θ) = tan(θ)} and B = {θ : cos(θ) = 1} be two sets. Then : [Online April 25, 2013]

Options:

A. $A = B$

B. $A \subseteq B$

C. $B \subseteq A$

D. $A \subset B$ and $B - A \neq \varphi$

Answer: B

Solution:

Solution:
Let $A = \{\theta : \sin\theta = \tan\theta\}$
and $B = \{\theta : \cos\theta = 1\}$
Now, $A = \left\{\theta : \sin\theta = \frac{\sin\theta}{\cos\theta}\right\}$
$= \{\theta : \sin\theta(\cos\theta - 1) = 0\}$
$= \{\theta = 0, \pi, 2\pi, 3\pi, \ldots..\}$
For $B : \cos\theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, \ldots\ldots$
This shows that A is not contained in B. i.e. $A \text{ not} \subset B$. but $B \subset A$

Question97

The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is:
[Online April 23, 2013]

Options:

A. 3

B. 5

C. 4

D. 6

Answer: A

Solution:

Solution:
$\sin 2x - 2\cos x + 4\sin x = 4$
$\Rightarrow 2\sin x \cdot \cos x - 2\cos x + 4\sin x - 4 = 0$
$\Rightarrow (\sin x - 1)(\cos x - 2) = 0$
$\because \cos x - 2 \neq 0, \quad \therefore \sin x = 1$
$\therefore x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

Question98

Statement-1: The number of common solutions of the trigonometric equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is two.
Statement- 2 : The number of solutions of the equation, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, \pi]$ is two.
[Online April 22, 2013]

Options:

A. Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement- 1 .

B. Statement-1 is true; Statement- 2 is true; Statement-2 is not a correct explanation for statement-1.

C. Statement-1 is false; Statement- 2 is true.

D. Statement- 1 is true; Statement- 2 is false.

Answer: B

Solution:

Solution:
$2\sin^2\theta - \cos 2\theta = 0$
$\Rightarrow 2\sin^2\theta - (1 - 2\sin^2\theta) = 0$
$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$
$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\frac{1}{2}$
$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \theta \in [0, 2\pi]$
$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
Now $2\cos^2\theta - 3\sin\theta = 0$
$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$
$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$
$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$
$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$
$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$
Now $2\cos^2\theta - 3\sin\theta = 0$
$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$
$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$
$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$
$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$
$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$
$\Rightarrow \sin\theta = \frac{1}{2}, -2$
But $\sin\theta = -2$, is not possible $\therefore$ $\sin\theta = \frac{1}{2}, -2 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$
Hence, there are two common solution, there each of the statement- 1 and 2 are true but statement- 2 is not a correct explanation for statement-1.

Question99

The value of $\cos 255^{\circ} + \sin 195^{\circ}$ is
[Online May 26, 2012]

Options:

A. $\frac{\sqrt{3}-1}{2\sqrt{2}}$

B. $\frac{\sqrt{3}-1}{\sqrt{2}}$

C. $-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$

D. $\frac{\sqrt{3}+1}{\sqrt{2}}$

Answer: C

Solution:

Solution:
Consider $\cos 255^{\circ} + \sin 195^{\circ}$
$= \cos(270^{\circ} - 15^{\circ}) + \sin(180^{\circ} + 15^{\circ})$
$= -\sin 15^{\circ} - \sin 15^{\circ}$
$= -2\sin 15^{\circ} = -2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = -\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$

Question100

Let $f(x) = \sin x$, $g(x) = x$
Statement 1: $f(x) \leq g(x)$ for x in $(0, \infty)$
Statement 2 : $f(x) \leq 1$ for x in $(0, \infty)$ but $g(x) \to \infty$ as $x \to \infty$.
[Online May 7, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1 .

C. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1 .

D. Statement 1 is false, Statement 2 is true.

Answer: C

Solution:

Solution:

Let $f(x) = \sin x$ and $g(x) = x$
Statement-1: $f(x) \le g(x)\ \forall x \in (0, \infty)$
i.e., $\sin x \le x\ \forall x \in (0, \infty)$
which is true
Statement- 2 : $f(x) \le 1\ \forall x \in (0, \infty)$
i.e., $\sin x \le 1\ \forall x \in (0, \infty)$
It is true and
$g(x) = x \to \infty$ as $x \to \infty$ also true.

Question101

The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has: [2012]

Options:

A. infinite number of real roots

B. no real roots

C. exactly one real root

D. exactly four real roots

Answer: B

Solution:

Solution:
Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$
Put $e^{\sin x} = t$ in the given equation, we get
$t^2 - 4t - 1 = 0$
$\Rightarrow\ t = \dfrac{4 \pm \sqrt{16+4}}{2} = \dfrac{4 \pm \sqrt{20}}{2}$
$= \dfrac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$
$\Rightarrow\ e^{\sin x} = 2 \pm \sqrt{5} (\because t = e^{\sin x})$
$\Rightarrow e^{\sin x} = 2 - \sqrt{5}$ and $e^{\sin x} = 2 + \sqrt{5}$
$\Rightarrow\ e^{\sin x} = 2 - \sqrt{5} < 0$
and $\sin x = \ln(2+\sqrt{5}) > 1$
So, rejected.
Hence, given equation has no solution.
$\therefore$ The equation has no real roots.

Question102

If $A = \sin^2 x + \cos^4 x$, then for all real x : [2011]

Options:

A. $\frac{13}{16} \le A \le 1$

B. $1 \le A \le 2$

C. $\frac{3}{4} \le A \le \frac{13}{16}$

D. $\frac{3}{4} \le A \le 1$

Answer: D

Solution:

Solution:

$A = \sin^2 x + \cos^4 x$

$= \sin^2 x + \cos^2 x(1 - \sin^2 x)$

$= \sin^2 x + \cos^2 x - \frac{1}{4}(2 \sin x \cdot \cos x)^2$

$= 1 - \frac{1}{4}\sin^2(2x)$

$\because -1 \le \sin 2x \le 1$

$\Rightarrow 0 \le \sin^2(2x) \le 1$

$\Rightarrow 0 \ge -\frac{1}{4}\sin^2(2x) \ge -\frac{1}{4}$

$\Rightarrow 1 \ge 1 - \frac{1}{4}\sin^2(2x) \ge 1 - \frac{1}{4}$

$\Rightarrow 1 \ge A \ge \frac{3}{4}$

Question103

The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are [2011RS]

Options:

A. $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

B. $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

C. $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

D. $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

Answer: D

Solution:

Solution:

$\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0$
$\sin 4\theta(1 + 2\cos 3\theta) = 0$
$\sin 4\theta = 0$

or $\cos 3\theta = -\frac{1}{2}$

$4\theta = n\pi;\ n \in I$

or $3\theta = 2n\pi \pm \frac{2\pi}{3},\ n \in I$

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$ $[\because \theta, \in (0, \pi)]$

Question104

Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$ where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ [2010]

Options:

A. $\frac{56}{33}$

B. $\frac{19}{12}$

C. $\frac{20}{7}$

D. $\frac{25}{16}$

Answer: A

Solution:

Solution:

$\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$

$\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

$\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{56}{33}$$

Question105

Let A and B denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

I f $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$, then :
[2009]

Options:

A. A is false and B is true

B. both A and B are true

C. both A and B are false

D. A is true and B is false

Answer: B

Solution:

Solution:
Given that
$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$
$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3=0$
$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]$
$+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta$
$+\sin^2\gamma+\cos^2\alpha=0$
$\Rightarrow\ [\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta$
$+2\sin\beta\sin\gamma+2\sin\gamma\sin\alpha]+[\cos^2\alpha+\cos^2\beta$
$+\cos^2\gamma+2\cos\alpha\cos\beta+2\cos\beta\cos\gamma$
$+2\cos\gamma\cos\alpha]=0$
$[\because\cos(A-B)=\cos A\cdot\cos B+\sin A\cdot\sin B]$
$\Rightarrow\ [\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$
$\Rightarrow\ \sin\alpha+\sin\beta+\sin\gamma=0$ and $\cos\alpha+\cos\beta+\cos\gamma=0$
$\therefore$ A and B both are true.

Question106

If p and q are positive real numbers such that $p^2+q^2=1$ then the maximum value of $(p+q)$ is
[2007]

Options:

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. 2.

Answer: C

Solution:

Solution:
Given that $p^2 + q^2 = 1$
$\therefore p = \cos\theta$ and $q = \sin\theta$ satisfy the given equation
Then $p + q = \cos\theta + \sin\theta$
We know that
$-\sqrt{a^2 + b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2 + b^2}$
$\therefore -\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2}$
Hence max. value of $p + q$ is $\sqrt{2}$

Question107

A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is [2006]

Options:

A. $\frac{3}{2}x^2$

B. $\sqrt{\frac{x^3}{8}}$

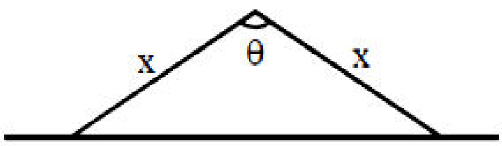
C. $\frac{1}{2}x^2$

D. πx^2

Answer: C

Solution:

Solution:
Area $= \frac{1}{2}x^2 \sin\theta$



Maximum value of $\sin\theta$ is 1 at $\theta = \frac{\pi}{2}$

$A_{max} = \frac{1}{2}x^2$

Question108

If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is
[2006]

Options:

A. $\frac{(1-\sqrt{7})}{4}$

B. $\frac{(4-\sqrt{7})}{3}$

C. $-\frac{(4+\sqrt{7})}{3}$

D. $\frac{(1+\sqrt{7})}{4}$

Answer: C

Solution:

Solution:

$\cos x + \sin x = \frac{1}{2} \Rightarrow 1 + \sin 2x = \frac{1}{4}$

$\Rightarrow \sin 2x = -\frac{3}{4}$

$\therefore \pi < 2x < 2\pi$

$\Rightarrow \frac{\pi}{2} < x \leq \pi$ ....(i)

$\frac{2\tan x}{1+\tan^2 x} = -\frac{3}{4}$

$\Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$

$\therefore \tan x = \frac{-8 \pm \sqrt{64-36}}{6} = -\frac{-4 \pm \sqrt{7}}{3}$

for $\frac{\pi}{2} < x < \pi$, $\tan x < 0$

$\therefore \tan x = \frac{-4-\sqrt{7}}{3}$

Question109

The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is
[2006]

Options:

A. 4

B. 6

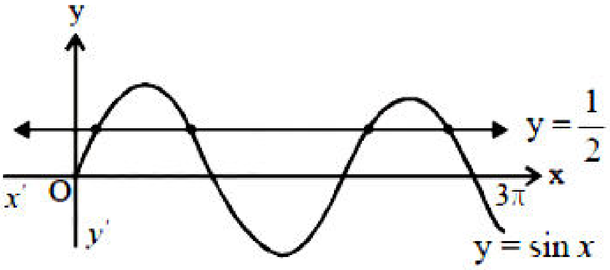
C. 1

D. 2

Answer: A

Solution:

Solution:



$2\sin^2 x + 5\sin x - 3 = 0$
$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$
$\Rightarrow \sin x = \frac{1}{2}$ and $\sin x \neq -3$
$\therefore$ In $[0, 3\pi]$, x has 4 values.

Question110

If u = $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by [2004]

Options:

A. $(a - b)^2$

B. $2\sqrt{a^2 + b^2}$

C. $(a + b)^2$

D. $2(a^2 + b^2)$

Answer: A

Solution:

Solution:
$u^2 = a^2 + b^2 + 2\sqrt{\left(a^4 + b^4\right)\cos^2\theta\sin^2\theta; + a^2b^2(\cos^4\theta + \sin^4\theta)}$. ....(1)
Now, $(a^4 + b^4)\cos^2\theta\sin^2\theta + a^2b^2(\cos^4\theta + \sin^4\theta)$
$= (a^4 + b^4)\cos^2\theta\sin^2\theta + a^2b^2(1 - 2\cos^2\theta\sin^2\theta)$
$= (a^4 + b^4 - 2a^2b^2)\cos^2\theta\sin^2\theta + a^2b^2$

$$= (a^2 - b^2)^2 \cdot \frac{\sin^2 2\theta}{4} + a^2b^2 \text{ ....(2)}$$

$$\because 0 \le \sin^2 2\theta \le 1$$

$$\Rightarrow 0 \le (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} \le \frac{(a^2 - b^2)^2}{4}$$

$$\Rightarrow a^2b^2 \le (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} + a^2b^2$$

$$\le (a^2 - b^2)^2 \cdot \frac{1}{4} + a^2b^2 \text{ ....(3)}$$

From (1)

$$a^2 + b^2 + 2\sqrt{a^2b^2} \le u^2 \le a^2 + b^2 + \frac{2}{2}\sqrt{(a^2 + b^2)^2}$$

$$(a + b)^2 \le u^2 \le 2(a^2 + b^2)$$

∴ Max. value − Min. value

$$= 2(a^2 + b^2) - (a + b^2) = (a - b)^2$$

Question111

Let α, β be such that $\pi < \alpha - \beta < 3\pi$.
If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha - \beta}{2}$ [2004]

Options:

A. $\frac{-6}{65}$

B. $\frac{3}{\sqrt{130}}$

C. $\frac{6}{65}$

D. $-\frac{3}{\sqrt{130}}$

Answer: D

Solution:

Solution:

$$\pi < \alpha - \beta < 3\pi$$

$$\Rightarrow \frac{\pi}{2} < \frac{\alpha - \beta}{2} < \frac{3\pi}{2} \Rightarrow \cos\frac{\alpha - \beta}{2} < 0 \text{ ......(1)}$$

$$\sin\alpha + \sin\beta = -\frac{21}{65}$$

$$\Rightarrow 2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2} = -\frac{21}{65} \text{ ....(2)}$$

$$\cos\alpha + \cos\beta = -\frac{27}{65}$$

$$\Rightarrow 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2} = -\frac{27}{65} \text{ ......(3)}$$

Squaring and adding (2) and (3) , we get

$4\cos^2\frac{\alpha-\beta}{2}=\frac{(21)^2+(27)^2}{(65)^2}=\frac{1170}{65\times65}$

$\therefore\cos^2\frac{\alpha-\beta}{2}=\frac{9}{130}\Rightarrow\cos\frac{\alpha-\beta}{2}=-\frac{3}{\sqrt{130}}$ [from(1)]

Question112

The function $f(x)=\log\left(x+\sqrt{x^2+1}\right)$, is [2003]

Options:

A. neither an even nor an odd function

B. an even function

C. an odd function

D. a periodic function.

Answer: C

Solution:

Solution:

Given $f(x)=\log\left(x+\sqrt{x^2+1}\right)$

$f(-x)=\log\left\{-x+\sqrt{x^2+1}\right\}=\log\left\{\frac{x^2-x^2+1}{x+\sqrt{x^2+1}}\right\}$

$=-\log\left(x+\sqrt{x^2+1}\right)=-f(x)$

$\Rightarrow f(x)$ is an odd function.

Question113

The period of $\sin^2\theta$ is [2002]

Options:

A. π^2

B. π

C. 2π

D. $\pi/2$

Answer: B

Solution:

Solution:
We know that $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$;
Since period of $\cos 2\theta = \dfrac{2\pi}{2} = \pi$
Hence period of $\sin^2\theta$ is also π.

Question114

Which one is not periodic?
[2002]

Options:

A. $|\sin 3x| + \sin^2 x$

B. $\cos\sqrt{x} + \cos^2 x$

C. $\cos 4x + \tan^2 x$

D. $\cos 2x + \sin x$

Answer: B

Solution:

Solution:
we know that $\cos\sqrt{x}$ is non periodic
$\therefore \cos\sqrt{x} + \cos^2 x$ can not be periodic.

Question115

The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is
[2002]

Options:

A. 2

B. 3

C. 0

D. 1

Answer: B

Solution:

Solution:

$\because \tan x + \sec x = 2\cos x$

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1.$

$\Rightarrow x = 30^\circ, 150^\circ, 270^\circ$

Number of solution $= 3$
