Complex Numbers and Quadratic Equations

Question1

Let α, β be the roots of the equation $x^2 - \mathrm{ax} - \mathrm{b} = 0$ with $\mathrm{Im}(\alpha) < \mathrm{Im}(\beta)$. Let $\mathrm{P_n} = \alpha^{\mathrm{n}} - \beta^{\mathrm{n}}$. If $\mathrm{P_3} = -5\sqrt{7}i, \mathrm{P_4} = -3\sqrt{7}i, \mathrm{P_5} = 11\sqrt{7}i$ and $\mathrm{P_6} = 45\sqrt{7}i$, then $\alpha^4 + \beta^4$ is equal to __________.

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Answer: 31

Solution:

We begin with the equations for the roots:

$\alpha + \beta = \mathrm{a}$

$\alpha\beta = -\mathrm{b}$

Given:

$\mathrm{P_6} = \mathrm{aP_5} + \mathrm{bP_4}$

$\mathrm{P_5} = \mathrm{aP_4} + \mathrm{bP_3}$

Using the given values:

For $\mathrm{P_6}$:

$45\sqrt{7}i = \mathrm{a} \times 11\sqrt{7}i + \mathrm{b}(-3\sqrt{7})i$

Simplifying, we obtain:

$45 = 11\mathrm{a} - 3\mathrm{b} \quad \text{(Equation 1)}$

For $\mathrm{P_5}$:

$11\sqrt{7}i = \mathrm{a}(-3\sqrt{7}i) + \mathrm{b}(-5\sqrt{7}i)$

Simplifying, we obtain:

$11 = -3\mathrm{a} - 5\mathrm{b} \quad \text{(Equation 2)}$

Solving these linear equations, we find:

$\mathrm{a} = 3$

$\mathrm{b} = -4$

Now, we calculate $\alpha^4 + \beta^4$ using the relation:

$\alpha^4 + \beta^4 = \sqrt{(\alpha^4 - \beta^4)^2 + 4(\alpha^4\beta^4)}$

From $\mathrm{b} = -4$, we know:

$\alpha\beta = -\mathrm{b} = 4 \quad \Rightarrow \alpha^4\beta^4 = (\alpha\beta)^4 = 4^4 = 256$

Substitute into the relation:

$\alpha^4 + \beta^4 = \sqrt{(-63) + 1024}$

$= \sqrt{961} = 31$

Thus, $\alpha^4 + \beta^4$ is equal to 31.

Question2

Let integers $\mathrm{a}, \mathrm{b} \in [-3, 3]$ be such that $\mathrm{a} + \mathrm{b} \neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-\mathrm{a}}{z+\mathrm{b}}\right| = 1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1, z \in \mathrm{C}$, where ω and ω^2 are the roots of $x^2 + x + 1 = 0$, is equal to _____________ .

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Answer: 10

Solution:

$a, b \in I, -3 \leq a, b \leq 3, a + b \neq 0$

$|z - a| = |z + b|$

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow \begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow z \begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow z \begin{vmatrix} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{vmatrix} = 1$$

$\Rightarrow z^3 = 1$

$\Rightarrow z = \omega, \omega^2, 1$

Now

$|1 - \mathrm{a}| = |1 + \mathrm{b}|$

$\Rightarrow$ 10 pairs

Question3

Let $\mathrm{A} = \{z \in \mathrm{C} : |z - 2 - i| = 3\}, \mathrm{B} = \{z \in \mathrm{C} : \operatorname{Re}(z - iz) = 2\}$ and $\mathrm{S} = \mathrm{A} \cap \mathrm{B}$. Then $\sum_{z \in S} |z|^2$ is equal to _________.

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Answer: 22

Solution:

Let $z = x + iy$
$|z-2-i| = 3 \Rightarrow (x-2)^2 + (y-1)^2 = 3^2$
$\operatorname{Re}(z - iz) = \operatorname{Re}(x + iy - ix + y) = x + y \Rightarrow x + y = 2$
$\Rightarrow A = \{(x, y) : (x-2)^2 + (y-1)^2 = 3^2, x, y \in R\}$,
$B = \{(x, y) : x + y = 2\}$
$\Rightarrow x - 2 = -y \Rightarrow y^2 + (y-1)^2 = 3^2$
$\Rightarrow 2y^2 - 2y - 8 = 0 \Rightarrow y^2 - y - 4 = 0$
$y_1 + y_2 = 1, y_1 y_2 = -4$
$\Rightarrow y_1^2 + y_2^2$
$= (y_1 + y_2)^2 - 2y_1 y_2 = 9$
$\Rightarrow x_1 + x_2 = 4(y_1 + y_2) = 3$,
$x_1 x_2 = (2 - y_1)(2 - y_2) = 4 - 2(y_1 + y_2) + y_1 y_2 = -2$
$\Rightarrow x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = 13$
$\because S = \{(x_1, y_1), (x_2, y_2)\}$
$\Rightarrow \sum_{z \in S} |z|^2 = (x_1^2 + y_1^2) + (x_2^2 + y_2^2) = 22$

Question4

If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{\mathrm{k}=1}^{\mathrm{n}} \left(\alpha^{\mathrm{k}} + \frac{1}{\alpha^{\mathrm{k}}}\right)^2 = 20$, then n is equal to _________.

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Answer: 11

Solution:

α is root of equation $1 + x + x^2 = 0, \alpha = \omega$ or ω^2

$$\left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = \alpha^{2k} + \frac{1}{\alpha^{2k}} + 2 = \omega^k + \frac{1}{\omega^k} + 2$$

$$\Rightarrow \quad \omega^k + \frac{1}{\omega^k} + 2 = \begin{cases} 4, 3 \text{ divides } k \\ 1, 3 \text{ does not divide } k \end{cases}$$

$$\therefore \quad \sum_{k=1}^{n} \left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = 20$$

$\Rightarrow \quad (1+1+4) + (1+1+4) + (1+1+4) + (1+1)$
$= 20$
$\Rightarrow \quad n = 11$

Question5

Let z_1, z_2 and z_3 be three complex numbers on the circle $|z| = 1$ with $\arg(z_1) = \frac{-\pi}{4}, \arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$. If $|z_1 \bar{z}_2 + z_2 \bar{z}_3 + z_3 \bar{z}_1|^2 = \alpha + \beta\sqrt{2}, \alpha, \beta \in Z$, then the value of $\alpha^2 + \beta^2$ is :

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Options:

A. 41

B. 29

C. 24

D. 31

Answer: B

Solution:

To solve the problem, we start with the given information about the complex numbers z_1, z_2, and z_3, which lie on the unit circle $|z| = 1$. Their arguments are as follows:

$\arg(z_1) = -\frac{\pi}{4}$

$\arg(z_2) = 0$

$\arg(z_3) = \frac{\pi}{4}$

Thus, the complex numbers can be represented as:

$z_1 = e^{-i\frac{\pi}{4}} = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$z_2 = e^{i\cdot 0} = 1$

$z_3 = e^{i\frac{\pi}{4}} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

Next, calculate the conjugates needed:

$\bar{z}_2 = 1$

$\bar{z}_3 = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$\bar{z}_1 = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

We need to evaluate:

$z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1$

Calculate each term separately:

$z_1\bar{z}_2 = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) \cdot 1 = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$z_2\bar{z}_3 = 1 \cdot \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$z_3\bar{z}_1 = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) \cdot \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \frac{(1+i)^2}{2} = \frac{1+2i-1}{2} = i$

Sum the evaluated terms:

$z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1 = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) + i$

Simplify:

$= \sqrt{2} + i - \sqrt{2}i = \sqrt{2} + i(1 - \sqrt{2})$

Calculate the modulus squared:

$\left|\sqrt{2} + i(1 - \sqrt{2})\right|^2 = (\sqrt{2})^2 + (1 - \sqrt{2})^2$

$= 2 + (1 - 2\sqrt{2} + 2) = 5 - 2\sqrt{2}$

Thus, the expression $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2$ simplifies as follows:

$\alpha = 5$

$\beta = -2$

Finally, compute $\alpha^2 + \beta^2$:

$\alpha^2 + \beta^2 = 5^2 + (-2)^2 = 25 + 4 = 29$

Therefore, the value of $\alpha^2 + \beta^2$ is 29.

Question6

Let the curve $z(1+i) + \bar{z}(1-i) = 4, z \in C$, divide the region $|z-3| \leq 1$ into two parts of areas α and β. Then $|\alpha - \beta|$ equals :

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Options:

A. $1+\frac{\pi}{3}$

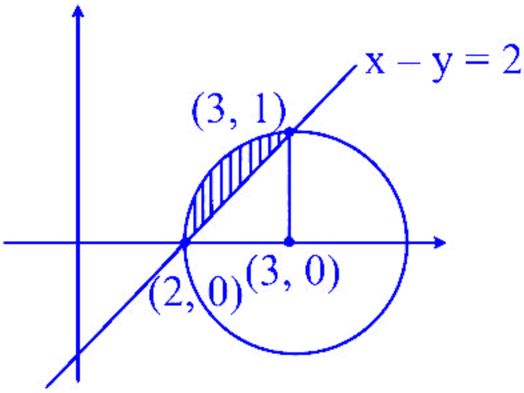
B. $1+\frac{\pi}{6}$

C. $1+\frac{\pi}{2}$

D. $1+\frac{\pi}{4}$

Answer: C

Solution:



Let $z=x+iy$
$(x+iy)(1+i)+(x-iy)(1-i)=4$
$x+ix+iy-y+x-ix-iy-y=4$
$2x-2y=4$
$x-y=2$
$|z-3|\leq 1$
$(x-3)^2+y^2\leq 1$

Area of shaded region $=\frac{\pi\cdot 1^2}{4}-\frac{1}{2}\cdot 1\cdot 1=\frac{\pi}{4}-\frac{1}{2}$
Area of unshaded region inside the circle
$=\frac{3}{4}\pi\cdot 1^2+\frac{1}{2}\cdot 1\cdot 1=\frac{3\pi}{4}+\frac{1}{2}$
$\therefore$ difference of area $=\left(\frac{3\pi}{4}+\frac{1}{2}\right)-\left(\frac{\pi}{4}-\frac{1}{2}\right)$
$=\frac{\pi}{2}+1$

Question7

Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}, z\in C$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points $(0,0), C$ and $(\alpha,0)$ is 11 square units, then α^2 equals:

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Options:

A. $\frac{121}{25}$

B. 100

C. $\frac{81}{25}$

D. 50

Answer: B

Solution:

$$\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}$$
$$\left|\frac{\bar{z}-i}{\bar{z}+\frac{i}{2}}\right|=\frac{2}{3}$$
$$3|x-iy-i|=2\left|x-iy+\frac{i}{2}\right|$$
$$9\left(x^2+(y+1)^2\right)=4\left(x^2+(y-1/3)^2\right)$$
$$9x^2+9y^2+18y+9=4x^2+4y^2-4y+1$$
$$5x^2+5y^2+22y+8=0$$
$$x^2+y^2+\frac{22}{5}y+\frac{8}{5}=0$$
$$\text{centre } \Rightarrow\left(0,-\frac{11}{5}\right)$$
$$\left|\frac{1}{2}\left|\begin{array}{ccc}0 & 0 & 1 \\ 0 & -11/5 & 1 \\ \alpha & 0 & 1\end{array}\right|\right|=11$$
$$\Rightarrow\left(-\frac{11}{5}\alpha\right)^2=(11\times 2)^2$$
$$\Rightarrow \alpha^2=100$$

Question8

The number of complex numbers z, satisfying $|z|=1$ and $\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1$, is :

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Options:

A. 8

B. 10

C. 4

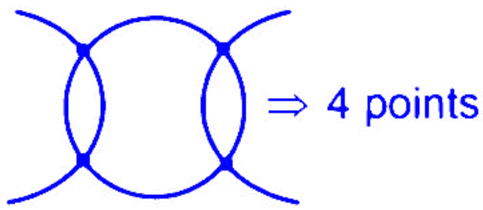
D. 6

Answer: A

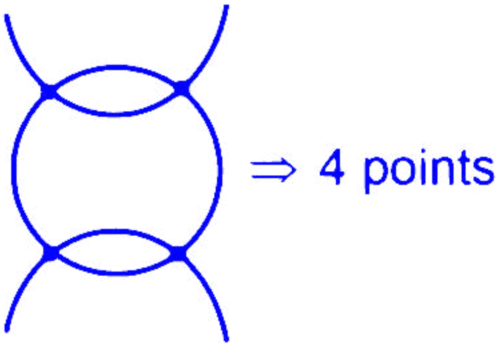
Solution:

$$|z|=1$$
$$\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1$$
$$\Rightarrow\left|z^2+(\bar{z})^2\right|=1$$
$$\text{Let } z=x+iy$$
$$\Rightarrow\left|(x+iy)^2+(x-iy)^2\right|=1$$
$$\Rightarrow\left|2x^2-2y^2\right|=1$$
$$\Rightarrow\left|x^2-y^2\right|=\frac{1}{2}$$
$$\Rightarrow x^2-y^2=\frac{\pm 1}{2}$$
$$\text{and } x^2+y^2=1$$

Case I: $x^2-y^2=\frac{1}{2}$



Case II: $x^2-y^2=-\frac{1}{2}$



Hence, we get 8 complex numbers.

Question9

If α and β are the roots of the equation $2z^2 - 3z - 2i = 0$, where $i = \sqrt{-1}$, then $16 \cdot \operatorname{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right) \cdot \operatorname{lm}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$ is equal to

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Options:

A. 441

B. 312

C. 409

D. 398

Answer: A

Solution:

Sol. $2z^2 - 32 - 2i = 0$

$$2\left(z - \frac{i}{z}\right) = 3$$

$$\alpha - \frac{i}{\alpha} = \frac{3}{2}$$

$$\Rightarrow \alpha^2 - \frac{1}{\alpha^2} - 2i = \frac{9}{4}$$

$$\Rightarrow \alpha^2 - \frac{1}{\alpha^2} - 2i = \frac{9}{4}$$

$$\Rightarrow \frac{9}{4} + 2i = \alpha^2 - \frac{1}{\alpha^2}$$

$$\Rightarrow \frac{81}{16} - 4 + 9i = \alpha^4 + \frac{1}{\alpha^4} - 2$$

$$\Rightarrow \frac{49}{16} + 9i = \alpha^4 + \frac{1}{\alpha^4}$$

Similarly

$$\Rightarrow \frac{49}{16} + 9i = \beta^4 + \frac{1}{\beta^4}$$

$$\Rightarrow \frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}} = \frac{\alpha^{15}\left(\alpha^4+\frac{1}{\alpha^4}\right)+\beta^{15}\left(\beta^4+\frac{1}{\beta^4}\right)}{\alpha^{15}+\beta^{15}}$$

$$= \frac{\left(\alpha^{15}+\beta^{15}\right)\left(\frac{49}{16}+9\mathrm{i}\right)}{\left(\alpha^{15}+\beta^{15}\right)}$$

Real $= \frac{49}{16}$

Im $= 9$

Ans. 441

Question10

Let O be the origin, the point A be $z_1 = \sqrt{3} + 2\sqrt{2}i$, the point $B(z_2)$ be such that $\sqrt{3}\,|z_2| = |z_1|$ and $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$. Then

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Options:

A. area of triangle ABO is $\frac{11}{4}$

B. area of triangle ABO is $\frac{11}{\sqrt{3}}$

C. ABO is a scalene triangle

D. ABO is an obtuse angled isosceles triangle

Answer: D

Solution:

$z_1 = \sqrt{3} + 2\sqrt{2}i \quad \& \frac{|z_2|}{|z_1|} = \frac{1}{\sqrt{3}}$

given $\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{6}$

$$z_2 = \frac{|z_2|}{|z_1|} \cdot z_1 e^{i\left(\frac{\pi}{6}\right)}$$
$$z_2 = \frac{1}{\sqrt{3}} \cdot \frac{(\sqrt{3} + 2\sqrt{2}i)(\sqrt{3} + i)}{2}$$
$$z_2 = \frac{(3 - 2\sqrt{2}) + i(2\sqrt{6} + \sqrt{3})}{2\sqrt{3}}$$

Now,

$z_1 - z_2 = \frac{(3+2\sqrt{2}) + i(2\sqrt{6} - \sqrt{3})}{2\sqrt{3}}$

$|z_1 - z_2| = |z_2| \Rightarrow \Delta$ABO is isosceles with angles $\frac{\pi}{6}, \frac{\pi}{6} \& \frac{2\pi}{3}$

Question11

If $\alpha + i\beta$ and $\gamma + i\delta$ are the roots of $x^2 - (3 - 2i)x - (2i - 2) = 0$, $i = \sqrt{-1}$, then $\alpha\gamma + \beta\delta$ is equal to:

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Options:

A.

2

B.

-6

C.

6

D.

-2

Answer: A

Solution:

$$
\begin{aligned}
& x^2-(3-2i)x-(2i-2)=0 \\
& x=\frac{(3-2i)\pm\sqrt{(3-2i)^2-4(1)(-(2i-2))}}{2(1)} \\
& ==\frac{(3-2i)\pm\sqrt{9-4-12i+8i-8}}{2} \\
& ==\frac{3-2i\pm\sqrt{-3-4i}}{2} \\
& =\frac{3-2i\pm\sqrt{(1)^2+(2i)^2-2(1)(2i)}}{2} \\
& =\frac{3-2\mathrm{i}\pm(1-2\mathrm{i})}{2} \\
& \Rightarrow \frac{3-2\mathrm{i}+1-2\mathrm{i}}{2} \text{ or } \frac{3-2\mathrm{i}-1+2\mathrm{i}}{2} \\
& \Rightarrow 2-2\mathrm{i} \text{ or } 1+0\mathrm{i} \\
& \text{So } \alpha\gamma+\beta\delta=2(1)+(-2)(0)=2
\end{aligned}
$$

Question12

Let $|z_1-8-2i|\leq 1$ and $|z_2-2+6i|\leq 2$, $z_1, z_2\in\mathbb{C}$. Then the minimum value of $|z_1-z_2|$ is :

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Options:

A.

3

B.

10

C.

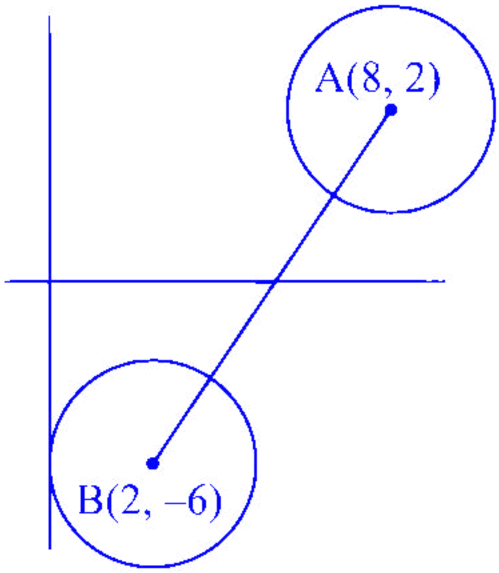
7

D.

13

Answer: C

Solution:



$\because \mathrm{AB} = \sqrt{100} = 10$
$\therefore |\mathrm{Z}_1 - \mathrm{Z}_2|_{\min} = 10 - 2 - 1 = 7$

Question13

Let z be a complex number such that $|z| = 1$. If $\frac{2+\mathrm{k}^2 z}{\mathrm{k}+\bar{z}} = \mathrm{k}z, \mathrm{k} \in \mathbf{R}$, then the maximum distance of $\mathrm{k} + i\mathrm{k}^2$ from the circle $|z - (1 + 2i)| = 1$ is :

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Options:

A. $\sqrt{5} + 1$

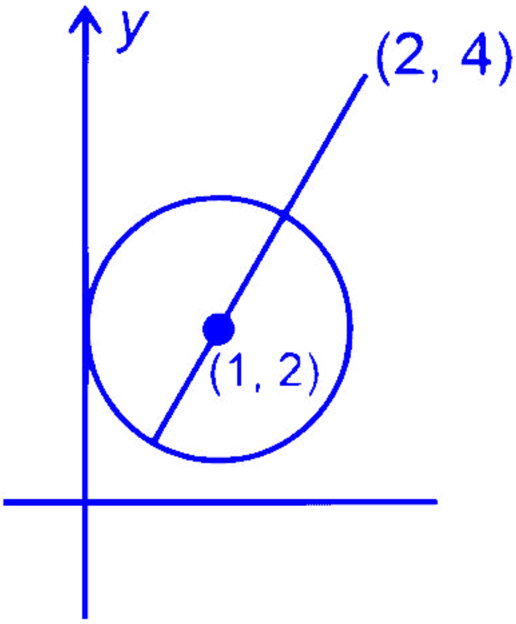
B. 3

C. $\sqrt{3} + 1$

D. 2

Answer: A

Solution:



$\frac{2+k^2z}{k+\bar{z}} = kz$
$\Rightarrow 2+k^2z = k^2z + k\bar{z}$
$\Rightarrow 2+k|z|^2 \quad \left(z\bar{z} = |z|^2, |z| = 1\right)$
$\Rightarrow 2 = k$
$\therefore k + k^2 i = 2 + 4i$
$\therefore$ The maximum distance is
$= \sqrt{(4-2)+(2-1)^2} +$ radius
$= \sqrt{(2)^2+(1)^2} + 1$
$= \sqrt{5} + 1$

Question14

Let $z \in C$ be such that $\frac{z^2+3i}{z-2+i} = 2+3i$. Then the sum of all possible values of z^2 is :

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Options:

A.

$-19+2i$

B. $-19-2i$

C. $19-2i$

D. $19+2i$

Answer: B

Solution:

$\frac{z^2+3i}{z-2+i} = 2+3i$
$z^2+3i = (z-2+i)(2+3i)$
$z^2+3i = 2z-4+2i+3iz-6i-3$
$z^2+3i = (2z-7)+i(3z-4)$
$z^2-(2+3i)z+(7+7i) = 0$
This is a quadratic in z.
$z_1+z_2 = 2+3i$
$z_1+z_2 = 7+7i$
$z_1^2+z_2^2 = (z_1+z_2)^2-2z_1z_2$
$= (2+3i)^2-2(7+7i)$
$= 4-9+12i-14-14i$
$= -19-2i$

Question15

$If\ z_1, z_2, z_3 \in\ are\ the\ vertices\ of\ an\ equilateral\ triangle,\ whose\ centroid\ is\ z_0,\ then\ \sum_{k=1}^{3}(z_k-z_0)^2$

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Options:

A. 0

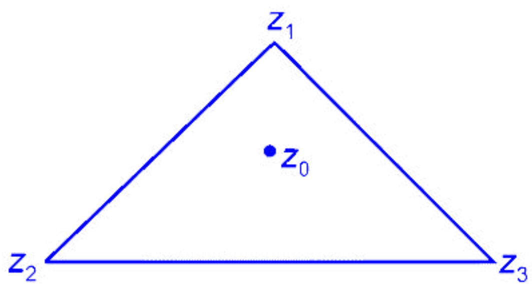
B. 1

C. i

D. -i

Answer: A

Solution:



$z_0 = z_1+z_2+z_3$
$\sum_{k=1}^{3}(z_k-z_0)^2 = (z_1-z_0)^2+(z_2-z_0)^2+(z_3-z_0)^2$

Let z_0 is origin $\Rightarrow z_1, z_2, z_3$ lies on a circle having $|z_0-z_i| = R$

$\therefore z_1 = Re^{i2\pi/3} z_2 = Re^{i4\pi/3} z_3 = Re^{i6\pi/3}$
$\Rightarrow z_1^2+z_2^2+z_3^2 = R^2\left[e^{i4\pi/3}+e^{i8\pi/3}+e^{i12\pi/3}\right]$
$= 0$
$\therefore \sum_{k=1}^{3}(z_k-z_0)^2 = 0$

Question16

Let the product of $\omega_1 = (8+i)\sin\theta+(7+4i)\cos\theta$ and $\omega_2 = (1+8i)\sin\theta+(4+7i)\cos\theta$ be $\alpha+i\beta$, $i = \sqrt{-1}$. Let p and q be the maximum and the minimum values of $\alpha+\beta$ respectively. Then $\mathrm{p}+\mathrm{q}$ is equal to :

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Options:

A. 130

B. 150

C. 160

D. 140

Answer: A

Solution:

$$\begin{aligned}
&\omega_1 = (8\sin\theta + 7\cos\theta) + i(\sin\theta + 4\cos\theta) \\
&\omega_2 = (\sin\theta + 4\cos\theta) + i(8\sin\theta + 7\cos\theta) \\
&\alpha = (8\sin\theta + 7\cos\theta) + (\sin\theta + 4\cos\theta) \\
&\quad -(\sin\theta + 4\cos\theta) + (8\sin\theta + 7\cos\theta) = 0 \\
&\beta = (8\sin\theta + 7\cos\theta)^2 + (\sin\theta + 4\cos\theta)^2
\end{aligned}$$

$$\begin{aligned}
&= 65\sin^2\theta + 65\cos^2\theta + 56\sin 2\theta + 4\sin 2\theta \\
&= 65 + 60\sin 2\theta \\
&(\alpha+\beta)_{\text{max}} = 125 = p \\
&(\alpha+\beta)_{\text{min}} = 5 = q \\
&p + q = 130
\end{aligned}$$

Question17

Among the statements

(S1) : The set $\{z \in \mathbb{C} - \{-i\} : |z| = 1$ and $\frac{z-i}{z+i}$ is purely real $\}$ contains exactly two elements, and

(S2) : The set $\{z \in \mathbb{C} - \{-1\} : |z| = 1$ and $\frac{z-1}{z+1}$ is purely imaginary $\}$ contains infinitely many elements.

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Options:

A. both are incorrect

B. both are correct

C. only (S2) is correct

D. only (S1) is correct

Answer: C

Solution:

$$\begin{aligned}
&\frac{z-i}{z+i} = \frac{\bar{z}+i}{\bar{z}-i} \\
&= z\bar{z} - i\bar{z} - iz - 1 = z\bar{z} + zi + i\bar{z} - 1 \\
&= z + \bar{z} = 0 \\
&= 2x = 0 \\
&= x = 0 \quad \text{(y-axis)}
\end{aligned}$$

$$\begin{aligned}
&|z| = 1 \\
&\therefore \quad z = i \quad (z \neq -i \text{ is given })
\end{aligned}$$

Statement 1 is incorrect

$$\frac{z-i}{z+i}+\frac{\bar{z}-1}{\bar{z}+1}=0$$
$$=z\bar{z}-\bar{z}+z-1+z\bar{z}-z+\bar{z}-1=0$$
$$=z\bar{z}=1$$
$$=|z|=1$$

Statement 2 is correct

Question18

If the locus of z $\in \mathbb{C}$, such that Re$\left(\frac{z-1}{2z+i}\right)+\operatorname{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right)=2$, is a circle of radius r and center (a,b), then $\frac{15ab}{r^2}$ is equal to :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

16

B.

24

C.

12

D.

18

Answer: D

Solution:

$\operatorname{Re}\left(\frac{z-1}{2z+i}\right)+\operatorname{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right)=2$

Here, $\frac{z-1}{2z+i}=\left(\overline{\frac{\bar{z}-1}{2\bar{z}-i}}\right)=2$

$=\operatorname{Re}\left(\frac{z-1}{2z+i}\right)+\operatorname{Re}\left(\overline{\frac{z-1}{2z+i}}\right)=2$

$=2\operatorname{Re}\left(\frac{z-1}{2z+1}\right)=2\Rightarrow \operatorname{Re}\left(\frac{z-1}{2z+i}\right)=1$

Let $z=x+iy$

$$\operatorname{Re}\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right)=1\Rightarrow \operatorname{Re}\left[\frac{((x-1)+iy)(2x-i(y+1)}{(2x+i(2y+1)(2x-i(2y+1))}\right]=1$$
$$\Rightarrow \frac{2x(x-1)+y(2y+1)}{4x^2+(2y+1)^2}=1$$
$$\Rightarrow 2x^2-2x+2y^2+y=4x^2+4y^2+1+4y$$
$$\Rightarrow 2x^2+2y^2+3y+2x+1=0$$
$$\Rightarrow x^2+y^2+x+\frac{3}{2}y+\frac{1}{2}=0$$
$$\text{centre }=\left(\frac{-1}{2},\frac{-3}{4}\right), r=\sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}}=\frac{\sqrt{5}}{4}$$
$$a=\frac{-1}{2}, b=\frac{-3}{4}, r^2=\frac{5}{16}$$

$15\frac{\mathrm{ab}}{\mathrm{r}^2}=15\times\left(\frac{-1}{2}\right)\times\left(\frac{-3}{4}\right)\times\frac{16}{5}=18$

Question19

Let $A = \left\{\theta \in [0, 2\pi] : 1 + 10\,\mathrm{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0\right\}$. Then $\sum\limits_{\theta \in A} \theta^2$ is equal to

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

$\frac{21}{4}\pi^2$

B.

$6\pi^2$

C.

$\frac{27}{4}\pi^2$

D.

$8\pi^2$

Answer: A

Solution:

$$1 + 10\,\mathrm{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0$$
$$\therefore \mathrm{z} + \overline{\mathrm{z}} = 2\,\mathrm{Re}(\mathrm{z})$$
$$\frac{2\cos\theta + \mathrm{i}\sin\theta}{\cos\theta - 3\mathrm{i}\sin\theta} + \frac{2\cos\theta - \mathrm{i}\sin\theta}{\cos\theta + 3\mathrm{i}\sin\theta} = 2 \times \left(\frac{-1}{10}\right)$$
$$\frac{\left(2\cos^2\theta - 3\sin^2\theta\right) + \left(2\cos^2\theta\right) - \left(3\sin^2\theta\right)}{\cos^2\theta + 9\sin^2\theta} = \frac{-2}{10}$$
$$\Rightarrow \frac{2\cos^2\theta - 3\sin^2\theta}{\cos^2\theta + 9\sin^2\theta} = \frac{-1}{10}$$
$$\Rightarrow 20\cos^2\theta - 30\sin^2\theta = -\cos^2\theta - 9\sin^2\theta$$
$$21\cos^2\theta - 21\sin^2\theta = 0$$
$$\Rightarrow \cos 2\theta = 0$$
$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$
$$\Rightarrow \sum \theta^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} + \frac{25\pi^2}{16} + \frac{49\pi^2}{16} = \frac{84\pi^2}{16} = \frac{21\pi^2}{4}$$

Question20

If S = {z ∈ C : |z − i| = |z + i| = |z − 1|}, then, n(S) is:

[27-Jan-2024 Shift 1]

Options:

A.

1

B.

0

C.

3

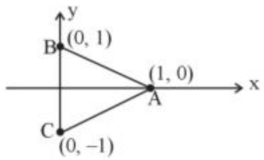
D.

2

Answer: A

Solution:

$|z-i| = |z+i| = |z-1|$



ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.
So n(S) = 1

Question21

If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≥ 0, then 5(3A − 2B − C) is equal to

[27-Jan-2024 Shift 1]

Answer: 5

Solution:

$x^2+x+1=0 \Rightarrow x=\omega, \omega^2=\alpha$

Let $\alpha=\omega$

Now $(1+\alpha)^7=-\omega^{14}=-\omega^2=1+\omega$

$A=1, B=1, C=0$

$\therefore 5(3A-2B-C)=5(3-2-0)=5$

Question22

Let the complex numbers α and 1/α lie on the circles $|z-z_0|^2=4$ and $|z-z_0|^2=16$ respectively, where $z_0 = 1 + i$. Then, the value of $100\,|\alpha|^2$ is.___

[27-Jan-2024 Shift 2]

Answer: 20

Solution:

$$|z-z_0|^2=4$$
$$\Rightarrow(\alpha-z_0)(\bar{\alpha}-\bar{z}_0)=4$$
$$\Rightarrow\alpha\bar{\alpha}-\alpha\bar{z}_0-z_0\bar{\alpha}+|z_0|^2=4$$
$$\Rightarrow|\alpha|^2-\alpha\bar{z}_0-z_0\bar{\alpha}=2\ \ldots\ldots\ldots\ldots(1)$$
$$|z-z_0|^2=16$$
$$\Rightarrow\left(\frac{1}{\alpha}-z_0\right)\left(\frac{1}{\bar{\alpha}}-\bar{z}_0\right)=16$$
$$\Rightarrow(1-\bar{\alpha}z_0)(1-\alpha\bar{z}_0)=16|\alpha|^2$$
$$\Rightarrow1-\bar{\alpha}z_0-\alpha\bar{z}_0+|\alpha|^2|z_0|^2=16|\alpha|^2$$
$$\Rightarrow1-\bar{\alpha}z_0-\alpha\bar{z}_0=14|\alpha|^2\ \ldots\ldots\ldots\ldots(2)$$

From (1) and (2)

$$\Rightarrow5|\alpha|^2=1$$
$$\Rightarrow100|\alpha|^2=20$$

Question23

If α, β are the roots of the equation, $x^2-x-1=0$ and $S_n=2023\alpha^n+2024\beta^n$, then

[27-Jan-2024 Shift 2]

Options:

A.

$2S_{12}=S_{11}+S_{10}$

B.

$S_{12}=S_{11}+S_{10}$

C.

$2S_{11}=S_{12}+S_{10}$

D.

$S_{11}=S_{10}+S_{12}$

Answer: B

Solution:

$$x^2-x-1=0$$
$$S_n=2023\alpha^n+2024\beta^n$$
$$S_{n-1}+S_{n-2}=2023\alpha^{n-1}+2024\beta^{n-1}+2023\alpha^{n-2}+2024\beta^{n-2}$$
$$=2023\alpha^{n-2}[1+\alpha]+2024\beta^{n-2}[1+\beta]$$
$$=2023\alpha^{n-2}[\alpha^2]+2024\beta^{n-2}[\beta^2]$$
$$=2023\alpha^n+2024\beta^n$$
$$S_{n-1}+S_{n-2}=S_n$$

Put n = 12

$$S_{11}+S_{10}=S_{12}$$

Question24

If $z = \frac{1}{2} - 2i$, is such that $|z+1| = \alpha z + \beta(1+i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in \mathrm{R}$, then $\alpha + \beta$ is equal to

[29-Jan-2024 Shift 1]

Options:

A.

-4

B.

3

C.

2

D.

-1

Answer: B

Question25

Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with Im(α) > Im(β). Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to

[29-Jan-2024 Shift 1]

Answer: 13

Solution:

$\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$

$= \alpha^4(\alpha - 2) + \alpha^4 - 5\alpha^2 + (\beta - 2)^2$

$= \alpha^5 - \alpha^4 - 5\alpha^2 + \beta^2 - 4\beta + 4$

$= \alpha^3(\alpha - 2) - \alpha^4 - 5\alpha^2 + \beta - 2 - 4\beta + 4$

$= -2\alpha^3 - 5\alpha^2 - 3\beta + 2$

$= -2\alpha(\alpha - 2) - 5\alpha^2 - 3\beta + 2$

$= -7\alpha^2 + 4\alpha - 3\beta + 2$

$= -7(\alpha - 2) + 4\alpha - 3\beta + 2$

$= -3\alpha - 3\beta + 16 = -3(1) + 16 = 13$

Question26

Let r and θ respectively be the modulus and amplitude of the complex number z = 2 − i (2 tan 5π/8), then (r,θ) is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$\left(2\ sec\ \frac{3\pi}{8}, \frac{3\pi}{8}\right)$

B.

$\left(2\ sec\ \frac{3\pi}{8}, \frac{5\pi}{8}\right)$

C.

$\left(2\ sec\ \frac{5\pi}{8}, \frac{3\pi}{8}\right)$

D.

$\left(2\ sec\ \frac{11\pi}{8}, \frac{11\pi}{8}\right)$

Answer: A

Solution:

$z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy(\text{ let })$

$r = \sqrt{x^2+y^2}\ \&\ \theta = \tan^{-1}\frac{y}{x}$

$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$

$= \left|2\ sec\ \frac{5\pi}{8}\right| = \left|2\ sec\left(\pi - \frac{3\pi}{8}\right)\right|$

$= 2\ sec\ \frac{3\pi}{8}$

$\&\ \theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$.

$= \tan^{-1}\left(\tan^2\left(\pi - \frac{5\pi}{8}\right)\right)$

$= \frac{3\pi}{8}$

Question27

Let α,β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that Im(α) > Im(β). Let a,b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib), i = \sqrt{-1}.$ **Then n + a + b is equal to____**

[29-Jan-2024 Shift 2]

Answer: 49

Solution:

$x^2 - \sqrt{6}x + 6 = 0$

$x = \frac{\sqrt{6} \pm i\sqrt{6}}{2} = \frac{\sqrt{6}}{2}(1 \pm i)$

$\alpha = \sqrt{3}\left(e^{i\frac{\pi}{4}}\right), \beta = \sqrt{3}\left(e^{-i\frac{\pi}{4}}\right)$

$\therefore \frac{\alpha^{99}}{\beta} + \alpha^{98} = \alpha^{98}\left(\frac{\alpha}{\beta} + 1\right)$

$= \frac{\alpha^{98}(\alpha + \beta)}{\beta} = 3^{49}\left(e^{i99\frac{\pi}{4}}\right) \times \sqrt{2}$

$= 3^{49}(-1 + i)$

$= 3^{n}(a + ib)$

$\therefore n = 49, a = -1, b = 1$

$\therefore n + a + b = 49 - 1 + 1 = 49$

Question28

Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in \mathbb{N}\}$.
Then $\sum_{(x, y) \in C} (x + y)$ is equal to___

[29-Jan-2024 Shift 2]

Answer: 46

Solution:

$x^2 - 2^y = 2023$

$\Rightarrow x = 45, y = 1$

$\sum_{(x, y) \in C} (x + y) = 46.$

Question29

If $z = x + iy, xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ is equal to :

[30-Jan-2024 Shift 1]

Options:

A.

9

B.

1

C.

4

D.

1/4

Answer: B

Solution:

$z^2 = -i\bar{z}$

$|z^2| = |i\bar{z}|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z| - 1) = 0$

$|z| = 0$ (not acceptable)

$\therefore |z| = 1$

$\therefore |z|^2 = 1$

Question30

If z is a complex number, then the number of common roots of the equation $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to :

[30-Jan-2024 Shift 2]

Options:

A.

1

B.

2

C.

0

D.

3

Answer: B

Solution:

$z^{1985} + z^{100} + 1 = 0$ & $z^3 + 2z^2 + 2z + 1 = 0$

$(z+1)(z^2 - z + 1) + 2z(z+1) = 0$

$(z+1)(z^2 + z + 1) = 0$

$\Rightarrow z = -1, \quad z = w, w^2$

Now putting $z = -1$ not satisfy

Now put $z = w$

$\Rightarrow w^{1985} + w^{100} + 1$

$\Rightarrow w^2 + w + 1 = 0$

Also, $z = w^2$

$\Rightarrow w^{3970} + w^{200} + 1$

$\Rightarrow w + w^2 + 1 = 0$

Two common root

Question31

If α denotes the number of solutions of |1 − i| x = 2x and β = $\left(\frac{|z|}{\arg(z)}\right)$, where $z = \frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right)$, $i = \sqrt{-1}$, then **the distance of the point (α, β) from the line 4x − 3y = 7 is____**

[31-Jan-2024 Shift 1]

Answer: 3

Solution:

$(\sqrt{2})^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$

$z = \frac{\pi}{4}(1+i)^4\left[\frac{\sqrt{\pi}-\pi i-i-\sqrt{\pi}}{\pi+1} + \frac{\sqrt{\pi}-i-\pi i-\sqrt{\pi}}{1+\pi}\right]$

$= -\frac{\pi i}{2}(1+4i+6i^2+4i^3+1)$

$= 2\pi i$

$\beta = \frac{2\pi}{\frac{\pi}{2}} = 4$

Distance from $(1, 4)$ to $4x-3y=7$

Will be $\frac{15}{5} = 3$

Question32

Let z_1 and z_2 be two complex number such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $|z_1^4 + z_2^4|$ equals-

[31-Jan-2024 Shift 2]

Options:

A.

$30\sqrt{3}$

B.

75

C.

$15\sqrt{15}$

D.

$25\sqrt{3}$

Answer: B

Solution:

$z_1 + z_2 = 5$

$z_1^3 + z_2^3 = 20 + 15i$

$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$

$z_1^3 + z_2^3 = 125 - 3z_1 \cdot z_2(5)$

$\Rightarrow 20 + 15i = 125 - 15z_1z_2$

$\Rightarrow 3z_1z_2 = 25 - 4 - 3i$

$\Rightarrow 3z_1z_2 = 21 - 3i$

$\Rightarrow z_1 \cdot z_2 = 7 - i$

$\Rightarrow (z_1 + z_2)^2 = 25$

$\Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i)$

$\Rightarrow 11 + 2i$

$(z_1^2 + z_2^2)^2 = 121 - 4 + 44i$

$\Rightarrow z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$

$\Rightarrow z_1^4 + z_2^4 = 117 + 44i - 2(49 - 1 - 14i)$

$\Rightarrow |z_1^4 + z_2^4| = 75$

Question33

Let α, β ∈ N be roots of equation $x^2 - 70x + \lambda = 0$, where λ/2, λ/3 ∉ N. If λ assumes the minimum possible value, then

$\frac{(\sqrt{\alpha - 1} + \sqrt{\beta - 1})(\lambda + 35)}{|\alpha - \beta|}$ is equal to :

[30-Jan-2024 Shift 1]

Answer: 60

Solution:

$x^2 - 70x + \lambda = 0$

$\alpha + \beta = 70$

$\alpha\beta = \lambda$

$\therefore \alpha(70 - \alpha) = \lambda$

Since, 2 and 3 does not divide λ

$\therefore \alpha = 5, \beta = 65, \lambda = 325$

By putting value of α, β, λ we get the required value 60 .

Question34

The number of real solutions of the equation $x(x^2 + 3x| + 5x - 1| + 6x - 2|) = 0$ is

[30-Jan-2024 Shift 2]

Answer: 1

Solution:

$x=0$ and $x^2+3x\,|+5x-1|+6x-2\,|\ =0$

Here all terms are $+ve$ except at $x=0$

So there is no value of x

Satisfies this equation

Only solution $x=0$

No of solution 1 .

Question35

Let S be the set of positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}<0,\ \forall x \in \mathbb{R}.$

Then, the number of elements in S is :

[31-Jan-2024 Shift 1]

Options:

A.

1

B.

0

C.

∞

D.

3

Answer: B

Solution:

$ax^2+2(a+1)x+9a+4<0\ \forall x \in R$

$\therefore a<0$

Question36

For $0 < c < b < a$, let $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

[31-Jan-2024 Shift 1]

Options:

A.

Both (I) and (II) are true

B.

Neither (I) nor (II) is true

C.

Only (II) is true

D.

Only (I) is true

Answer: A

Solution:

$f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$

$f(x) = a+b-2c+b+c-2a+c+a-2b = 0$

$f(1) = 0$

$\therefore \alpha \cdot 1 = \frac{c+a-2b}{a+b-2c}$

$\alpha = \frac{c+a-2b}{a+b-2c}$

If $-1 < \alpha < 0$

$-1 < \frac{c+a-2b}{a+b-2c} < 0$

$b+c < 2a$ and $b > \frac{a+c}{2}$

therefore, b cannot be G.M. between a and c.

If, $0 < \alpha < 1$

$0 < \frac{c+a-2b}{a+b-2c} < 1$

$b > c$ and $b < \frac{a+c}{2}$

Therefore, b may be the G.M. between a and c.

Question37

The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

[31-Jan-2024 Shift 2]

Options:

A.

2

B.

more than 2

C.

1

D.

0

Answer: D

Solution:

Take $e^{\sin x} = t(t > 0)$

$\Rightarrow t - \frac{2}{t} = 2$

$\Rightarrow \frac{t^2 - 2}{t} = 2$

$\Rightarrow t^2 - 2t - 2 = 0$

$\Rightarrow t^2 - 2t + 1 = 3$

$\Rightarrow (t - 1)^2 = 3$

$\Rightarrow t = 1 \pm \sqrt{3}$

$\Rightarrow t = 1 \pm 1.73$

$\Rightarrow t = 2.73$ or -0.73 (rejected as $t > 0$)

$\Rightarrow e^{\sin x} = 2.73$

$\Rightarrow \log_e e^{\sin x} = \log_e 2.73$

$\Rightarrow \sin x = \log_e 2.73 > 1$

So no solution.

Question38

Let a,b,c be the length of three sides of a triangle satisfying the condition $(a^2 + b^2)x^2 - 2b(a + c). x + (b^2 + c^2) = 0$. If the set of all possible values of x is the interval (α, β), then $12(\alpha^2 + \beta^2)$ is equal to____

[31-Jan-2024 Shift 2]

Answer: 36

Solution:

$(a^2+b^2)x^2 - 2b(a+c)x + b^2 + c^2 = 0$

$\Rightarrow a^2x^2 - 2abx + b^2 + b^2x^2 - 2bcx + c^2 = 0$

$\Rightarrow (ax-b)^2 + (bx-c)^2 = 0$

$\Rightarrow ax - b = 0, \quad bx - c = 0$

$\Rightarrow a + b > c \quad b + c > a \quad c + a > b$

$a + ax > bx \quad ax + bx > a \quad ax^2 + a > ax$

$a + ax > ax^2 \quad ax + ax^2 > a \quad x^2 - x + 1 > 0$

$x^2 - x - 1 < 0 \quad x^2 + x - 1 > 0 \quad$ always true

$\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}$

$x < \frac{-1-\sqrt{5}}{2}, \text{ or } x > \frac{-1+\sqrt{5}}{2}$

$\Rightarrow \frac{\sqrt{5}-1}{2} < x < \frac{\sqrt{5}+1}{2}$

$\Rightarrow \alpha = \frac{\sqrt{5}-1}{2}, \beta = \frac{\sqrt{5}+1}{2}$

$12(\alpha^2+\beta^2) = 12\left(\frac{(\sqrt{5}-1)^2+(\sqrt{5}+1)^2}{4}\right) = 36$

Question39

Let S = {z ∈ C : |z − 1| = 1 and (√2 − 1) (z + z)−i(z − z) = 2√2}. Let z_1, z_2 ∈ S be such that
$|z_1| = \max_{z \in S} |z|$ and $z_2 = \min_{z \in S} |z|$. Then $|\sqrt{2}z_1 - z_2|^2$ equals :

[1-Feb-2024 Shift 1]

Options:

A.

1

B.

4

C.

3

D.

2

Answer: D

Solution:

Let $Z = x + iy$

Then $(x-1)^2 + y^2 = 1 \rightarrow (1)$

$(\sqrt{2}-1)(2x) - i(2iy) = 2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)x + y = \sqrt{2} \rightarrow (2)$

Solving (1) & (2) we get

Either $x = 1$ or $x = \dfrac{1}{2-\sqrt{2}} \rightarrow (3)$

On solving (3) with (2) we get

For $x = 1 \Rightarrow y = 1 \Rightarrow Z_2 = 1 + i$

& for

$x = \dfrac{1}{2-\sqrt{2}} \Rightarrow y = \sqrt{2} - \dfrac{1}{\sqrt{2}} \Rightarrow Z_1 = \left(1 + \dfrac{1}{\sqrt{2}}\right) + \dfrac{i}{\sqrt{2}}$

Now

$|\sqrt{2}z_1 - z_2|^2$

$= \left|\left(\dfrac{1}{\sqrt{2}} + 1\right)\sqrt{2} + i - (1+i)\right|^2$

$= (\sqrt{2})^2$

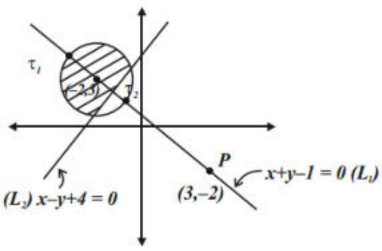
$= 2$

Question40

Let $P = \{z \in C : |z + 2 - 3i| \leq 1\}$ and $Q = \{z \in C : z(1+i) + \bar{z}(1-i) \leq -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2z|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals______ *[1-Feb-2024 Shift 1]*

Answer: 8

Solution:



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through $P(L_1)$ and z_2 is intersection of line L_1 & L_2

Circle : $(x+2)^2 + (y-3)^2 = 1$

$L_1 : x + y - 1 = 0$

$L_2 : x - y + 4 = 0$

On solving circle &L_1 we get

$z_1 : \left(-2 - \dfrac{1}{\sqrt{2}}, 3 + \dfrac{1}{\sqrt{2}}\right)$

On solving L_1 and z_2 is intersection of line L_1&L_2 we get $z_2 : \left(\dfrac{-3}{2}, \dfrac{5}{2}\right)$

$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 17$

$= 31 + 5\sqrt{2}$

So $\alpha = 31$

$\beta = 5$

$\alpha + \beta = 36$

Question41

If z is a complex number such that |z| ≥ 1, then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is:

[1-Feb-2024 Shift 2]

Options:

A.

5/2

B.

2

C.

3/2

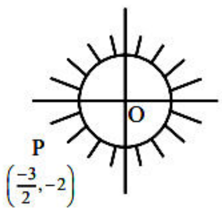
D.

None of above

Answer: D

Solution:

$|z| \geq 1$



Min. value of $\left|z+\frac{3}{2}+2i\right|$ is actually zero.

Question42

Let $S=\{x \in R:(\sqrt{3}+\sqrt{2})^{x}+(\sqrt{3}-\sqrt{2})^{x}=10\}$

Then the number of elements in S is :

[1-Feb-2024 Shift 1]

Options:

A.

4

B.

0

C.

2

D.

1

Answer: C

Solution:

$$(\sqrt{3}+\sqrt{2})^x+(\sqrt{3}-\sqrt{2})^x=10$$

Let $(\sqrt{3}+\sqrt{2})^x=t$

$$t+\frac{1}{t}=10$$

$$t^2-10t+1=0$$

$$t=\frac{10\pm\sqrt{100-4}}{2}=5\pm2\sqrt{6}$$

$$(\sqrt{3}+\sqrt{2})^x=(\sqrt{3}\pm\sqrt{2})^2$$

$x=2$ or $x=-2$

Number of solutions $=2$

Question43

Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p,q and r be the consecutive terms of a non-constant G.P and $1/\alpha + 1/\beta + = 3/4$, then the value of $(\alpha - \beta)^2$ is :

[1-Feb-2024 Shift 2]

Options:

A.

80/9

B.

9

C.

20/3

D.

8

Answer: A

Solution:

$px^2 + qx - r = 0$

$p = A, q = AR, r = AR^2$

$Ax^2 + ARx \ \ AR^2 = 0$

$x^2 + Rx - R^2 = 0$

$\because \ \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$

$\therefore \ \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{4} \Rightarrow \frac{-R}{-R^2} = \frac{3}{4} \Rightarrow R = \frac{4}{3}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = R^2 - 4(-R^2) = 5\left(\frac{16}{9}\right)$

$= 80/9$

Question44

Let p, q $\in \mathbb{R}$ and $(1-\sqrt{3}i)^{200} = 2^{199}(p+iq)$, $i = \sqrt{-1}$ Then $p+q+q^2$ and $p-q+q^2$ are roots of the equation.
[24-Jan-2023 Shift 1]

Options:

A. $x^2 + 4x - 1 = 0$

B. $x^2 - 4x + 1 = 0$

C. $x^2 + 4x + 1 = 0$

D. $x^2 - 4x - 1 = 0$

Answer: B

Solution:

Solution:
$(1-\sqrt{3}i)^{200} = 2^{199}(p+iq)$
$2^{200}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^{200} = 2^{199}(p+iq)$
$2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = p + iq$
$p = -1, q = -\sqrt{3}$
$\alpha = p + q + q^2 = 2 - \sqrt{3}$
$\beta = p - q + q^2 = 2 + \sqrt{3}$
$= 4$
$= 1$
equation $x^2 - 4x + 1 = 0$

Question45

The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

[24-Jan-2023 Shift 2]

Options:

A. $\frac{-1}{2}(1 - i\sqrt{3})$

B. $\frac{1}{2}(1-i\sqrt{3})$

C. $\frac{-1}{2}(\sqrt{3}-i)$

D. $\frac{1}{2}(\sqrt{3}+i)$

Answer: C

Solution:

Solution:

Let $\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}=z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3=\left(\frac{1+z}{1+\frac{1}{z}}\right)^3=z^3$$

$$\Rightarrow\left(i\left(\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}\right)\right)^3$$

$$=-i\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)=-i\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow\frac{-1}{2}(\sqrt{3}-i).$$

Question46

Let $z_1=2+3i$ and $z_2=3+4i$. The set
$S=\left\{z\in C:\left|z-z_1\right|^2-\left|z-z_2\right|^2=\left|z_1-z_2\right|^2\right\}$
represents a
[25-Jan-2023 Shift 1]

Options:

A. straight line with sum of its intercepts on the coordinate axes equals 14

B. hyperbola with the length of the transverse axis 7

C. straight line with the sum of its intercepts on the coordinate axes equals −18

D. hyperbola with eccentricity 2

Answer: A

Solution:

Solution:

$$((x-2)^2+(y-3)^2)-((x-3)^2-(y-4)^2)=1+1$$
$$\Rightarrow x+y=7$$

Question47

Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right|=2, z\neq-i$. Then z lies on the circle of radius 2 and centre
[25-Jan-2023 Shift 2]

Options:

A. (2, 0)

B. (0, 0)

C. (0, 2)

D. $(0, -2)$

Answer: D

Solution:

Solution:
$(z-2i)(\bar{z}+2i)=4(z+i)(\bar{z}-i)$
$z\bar{z}+4+2i(z-\bar{z})=4(z\bar{z}+1+i(\bar{z}-z))$
$3z\bar{z}-6i(z-\bar{z})=0$
$x^2+y^2-2i(2iy)=0$
$x^2+y^2+4y=0$

Question48

For two non-zero complex number z_1 and z_2, if $\operatorname{Re}(z_1z_2)=0$ and $\operatorname{Re}(z_1+z_2)=0$, then which of the following are possible ?
(A) $\operatorname{Im}(z_1)>0$ and $\operatorname{Im}(z_2)>0$
(B) $\operatorname{Im}(z_1)<0$ and $\operatorname{Im}(z_2)>0$
(C) $\operatorname{Im}(z_1)>0$ and $\operatorname{Im}(z_2)<0$
(D) $\operatorname{Im}(z_1)<0$ and $\operatorname{Im}(z_2)<0$
Choose the correct answer from the options given below :
[29-Jan-2023 Shift 1]

Options:

A. B and D

B. B and C

C. A and B

D. A and C

Answer: B

Solution:

Solution:
$z_1=x_1+iy_1$
$z_2=x_2+y_2$
$\operatorname{Re}(z_1z_2)=x_1x_2-y_1y_2=0$
$\operatorname{Re}(z_1+z_2)=x_1+x_2=0$
$x_1\&x_2$ are of opposite sign
$y_1\&y_2$ are of opposite sign

Question49

Let $\alpha=8-14i$, $A=\left\{z\in C:\ \frac{\alpha z-\overline{\alpha z}}{z^2-(\bar{z})^2-112i}=1\right\}$ and $B=\{z\in C: |z+3i|=4\}$
Then $\sum_{z\in A\cap B}(\operatorname{Re}z-\operatorname{Im}z)$ is equal to _______.
[29-Jan-2023 Shift 2]

Answer: 14

Solution:

Solution:
$\alpha = 8 - 14i$
$z = x + iy$
$a\bar{z} = (8x + 14y) + i(-14x + 8y)$
$z + \bar{z} = 2x \quad z - \bar{z} = 2iy$
Set A: $\frac{2i(-14x + 8y)}{i(4xy - 112)} = 1$
$(x - 4)(y + 7) = 0$
$x = 4 \quad \text{or} \quad y = -7$
Set B: $x^2 + (y + 3)^2 = 16$
when $x = 4 \;\; y = -3$
when $y = -7 \;\; x = 0$
$\therefore A \cap B = \{4 - 3i, 0 - 7i\}$
So, $\sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z) = 4 - (-3) + (0 - (-7)) = 14$

Question50

Let $z = 1 + i$ and $z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$. Then $\frac{12}{\pi}$ arg(z_1) is equal to _______.

[30-Jan-2023 Shift 1]

Answer: 9

Solution:

Solution:
$z_1 = \frac{1 + i}{\bar{z}(1 - z) + \frac{1}{z}}$
$z_1 = \frac{1 + i(1 - i)}{(1 - i)(1 - 1 - i) + \frac{1}{1 + i}}$
$= \frac{1 + i - i^2}{(1 - i)(-i) + \frac{1 - i}{2}}$
$= \frac{2 + i}{-3i - 1} = \frac{4 + 2i}{-3i - 1}$
$= \frac{-(4 + 2i)(3i - 1)}{(3i)^2 - (1)^2}$
$\operatorname{Arg}(z_1) = \frac{3\pi}{4}$
$\therefore \frac{12}{\pi} \arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

Question51

For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then
[31-Jan-2023 Shift 1]

Options:

A. the curves C_1 and C_2 intersect at 4 points

B. the curves C_1 lies inside C_2

C. the curves C_1 and C_2 intersect at 2 points

D. the curves C_2 lies inside C_1

Answer: A

Solution:

Solution:
Let $w=z+\frac{1}{z}=4e^{i\theta}+\frac{1}{4}e^{-i\theta}$
$\Rightarrow w=\frac{17}{4}\cos\theta+i\frac{15}{4}\sin\theta$
So locus of w is ellipse $\frac{x^2}{\left(\frac{17}{4}\right)^2}+\frac{y^2}{\left(\frac{15}{4}\right)^2}=1$
Locus of z is circle $x^2+y^2=16$
So intersect at 4 points

Question52

The complex number $z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to:

[31-Jan-2023 Shift 2]

Options:

A. $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

B. $\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}$

C. $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

D. $\sqrt{2}i\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$

Answer: A

Solution:

Solution:
$Z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}=\frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}$
$=\frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}\times\frac{\frac{1}{2}-\sqrt{\frac{3}{2}}i}{\frac{1}{2}-\sqrt{3/2}i}=\frac{\sqrt{3}-1}{2}+\frac{\sqrt{3}+1}{2}i$
Apply polar form,
$r\cos\theta=\frac{\sqrt{3}-1}{2}$
$r\sin\theta=\frac{\sqrt{3}+1}{2}$
Now, $\tan\theta=\frac{\sqrt{3}+1}{\sqrt{3}-1}$
So, $\theta=\frac{5\pi}{12}$

Question53

Let α be a root of the equation $(a-c)x^2+(b-a)x+(c-b)=0$ where a, b, c are distinct real numbers such that the matrix

$$\begin{bmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{bmatrix}$$

is singular. Then the value of $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ is
[24-Jan-2023 Shift 1]

Options:

A. 6

B. 3

C. 9

D. 12

Answer: B

Solution:

Solution:

$$\Delta = 0 = \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$$

$\Rightarrow \alpha^2(c-b) - \alpha(c-a) + (b-a) = 0$

It is singular when $\alpha = 1$

$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$

$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$

$= 3\,\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

Question54

Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set S = $\{x + \lambda : x$ is an integer solution of E$\}$ is
[24-Jan-2023 Shift 1]

Answer: 5

Solution:

Solution:

$|x|^2 - 2|x| + |\lambda - 3| = 0$

$|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$

$(|x| - 1)^2 + |\lambda - 3| = 1$

At $\lambda = 3$, $x = 0$ and 2 ,

at $\lambda = 4$ or 2 , then

$x = 1$ or -1

So maximum value of $x + \lambda = 5$

Question55

The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

[24-Jan-2023 Shift 2]

Options:

A. 4

B. 0

C. 3

D. 2

Answer: B

Solution:

Solution:

$$3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$$

$$3\left[\left(x+\frac{1}{x}\right)^2-2\right]-2\left(x+\frac{1}{x}\right)+5=0$$

Let $x+\frac{1}{x}=t$

$$3t^2-2t-1=0$$
$$3t^2-3t+t-1=0$$
$$3t(t-1)+1(t-1)=0$$
$$(t-1)(3t+1)=0$$
$$t=1,-\frac{1}{3}$$
$$x+\frac{1}{x}=1,-\frac{1}{3}\Rightarrow \text{ No solution.}$$

Question56

Let

$S=\left\{\alpha : \log_2(9^{2\alpha-4}+13)-\log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4}+1\right)=2\right\}$.

Then the maximum value of β for which the equation $x^2-2\left(\sum_{\alpha\in s}\alpha\right)^2x+\sum_{\alpha\in s}(\alpha+1)^2\beta=0$ has real roots, is ______.

[25-Jan-2023 Shift 1]

Answer: 25

Solution:

Solution:

$$\log_2(9^{2\alpha-4}+13)-\log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4}+1\right)=2$$

$$\Rightarrow \frac{9^{2\alpha-4}+13}{\frac{5}{2}3^{2\alpha-4}+1}=4$$

$\sum_{\alpha\in S}\alpha=5$ and $\sum_{\alpha\in S}(\alpha+1)^2=25$

$$\Rightarrow x^2-50x+25\beta=0 \text{ has real roots}$$
$$\Rightarrow \beta\le 25$$
$$\Rightarrow \beta_{max}=25$$

Question57

Let a ∈ R and let α, β be the roots of the equation $x^2+60^{\frac{1}{4}}x+a=0$. If $\alpha^4+\beta^4=-30$, then the product of all possible values of a is ________.

[25-Jan-2023 Shift 2]

Answer: 45

Solution:

Solution:

$\alpha+\beta=-60^{\frac{1}{4}}$ & $\alpha\beta=a$
Given $\alpha^4+\beta^4=-30$
$\Rightarrow(\alpha^2+\beta^2)^2-2\alpha^2\beta^2=-30$
$\Rightarrow\{(\alpha+\beta)^2-2\alpha\beta\}^2-2a^2=-30$
$\Rightarrow\left\{60^{\frac{1}{2}}-2a\right\}^2-2a^2=-30$
$\Rightarrow 60+4a^2-4a\times 60^{\frac{1}{2}}-2a^2=-30$
$\Rightarrow 2a^2-4.60^{\frac{1}{2}}a+90=0$
Product $=\frac{90}{2}=45$

Question58

Let $\lambda\neq 0$ be a real number. Let α, β be the roots of the equation $14x^2-31x+3\lambda=0$ and α, γ be the roots of the equation $35x^2-53x+4\lambda=0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation :
[29-Jan-2023 Shift 1]

Options:

A. $7x^2+245x-250=0$

B. $7x^2-245x+250=0$

C. $49x^2-245x+250=0$

D. $49x^2+245x+250=0$

Answer: C

Solution:

Solution:
$14x^2-31x+3\lambda=0$
$\alpha+\beta=\frac{31}{14}\ldots(1)$ and $\alpha\beta=\frac{3\lambda}{14}$
$35x^2-53x+4\lambda=0$
$\alpha+\gamma=\frac{53}{35}\ldots(3)$ and $\alpha\gamma=\frac{4\lambda}{35}\ldots$
$\frac{(2)}{(4)}\Rightarrow\frac{\beta}{\gamma}=\frac{3\times 35}{4\times 14}=\frac{15}{8}\Rightarrow\beta=\frac{15}{8}\gamma$
$(1)-(3)\Rightarrow\beta-\gamma=\frac{31}{14}-\frac{53}{35}=\frac{155-106}{70}=\frac{7}{10}$
$\frac{15}{8}\gamma-\gamma=\frac{7}{10}\Rightarrow\gamma=\frac{4}{5}$
$\Rightarrow\beta=\frac{15}{8}\times\frac{4}{5}=\frac{3}{2}$
$\Rightarrow\alpha=\frac{31}{14}-\beta=\frac{31}{14}-\frac{3}{2}=\frac{5}{7}$
$\Rightarrow\lambda=\frac{14}{3}\alpha\beta=\frac{14}{3}\times\frac{5}{7}\times\frac{3}{2}=5$
so, sum of roots $\frac{3\alpha}{\beta}+\frac{4\alpha}{\gamma}=\left(\frac{3\alpha\gamma+4\alpha\beta}{\beta\gamma}\right)$
$=\frac{\left(3\times\frac{4\lambda}{35}+4\times\frac{3\lambda}{14}\right)}{\beta\gamma}=\frac{12\lambda(14+35)}{14\times 35\beta\gamma}$
$=\frac{49\times 12\times 5}{490\times\frac{3}{2}\times\frac{4}{5}}=5$
Product of roots
$=\frac{3\alpha}{\beta}\times\frac{4\alpha}{\gamma}=\frac{12\alpha^2}{\beta\gamma}=\frac{12\times\frac{25}{49}}{\frac{3}{2}\times\frac{4}{5}}=\frac{250}{49}$
So, required equation is $x^2-5x+\frac{250}{49}=0$
$\Rightarrow 49x^2-245x+250=0$

Question59

If the value of real number a > 0 for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real roots is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ________.
[30-Jan-2023 Shift 2]

Answer: 13

Solution:

Solution:
Two equations have common root
$\therefore (4a)(26a) = (-6)^2 = 36$
$\Rightarrow a^2 = \frac{9}{26} \quad \therefore a = \frac{3}{\sqrt{26}} \Rightarrow \beta = 13$

Question60

The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is:
[31-Jan-2023 Shift 1]

Options:

A. 0

B. 1

C. 3

D. 2

Answer: B

Solution:

Solution:
$\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)}$
$= \sqrt{4\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right)}$
$\Rightarrow \sqrt{x-3} = 0 \Rightarrow x = 3$ which is in domain
$\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$
$2\sqrt{(x-1)(x+3)} = 2x - 4$
$x^2 + 2x - 3 = x^2 - 4x + 4$
$6x = 7$
$x = 7/6$ or

Question61

The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0, x \in R$ has:
[31-Jan-2023 Shift 2]

Options:

A. two solutions and both are negative

B. no solution

C. four solutions two of which are negative

D. two solutions and only one of them is negative

Answer: A

Solution:

Solution:
$e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0$
Let $e^{x} = t$
Now, $t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$
Dividing equation by t^2,
$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$
$t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$
$\left(t - \frac{1}{t}\right)^2 + 2 + 8\left(t - \frac{1}{t}\right) + 13 = 0$
Let $t - \frac{1}{t} = z$
$z^2 + 8z + 15 = 0$
$(z+3)(z+5) = 0$
$z = -3$ or $z = -5$
So, $t - \frac{1}{t} = -3$ or $t - \frac{1}{t} = -5$
$t^2 + 3t - 1 = 0$ or $t^2 + 5t - 1 = 0$
$t = \frac{-3 \pm \sqrt{13}}{2}$ or $t = \frac{-5 \pm \sqrt{29}}{2}$
as $t = e^{x}$ so t must be positive,
$t = \frac{\sqrt{13} - 3}{2}$ or $\frac{\sqrt{29} - 5}{2}$
So, $x = \ln\left(\frac{\sqrt{13} - 3}{2}\right)$ or $x = \ln\left(\frac{\sqrt{29} - 5}{2}\right)$
Hence two solution and both are negative.

Question62

If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then $3(\alpha + \beta + \gamma)$ is equal to [1-Feb-2023 Shift 1]

Options:

A. 11

B. 9

C. 10

D. 12

Answer: D

Solution:

Solution:
$\sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$
$= x^2 + y^2 - 4x + 4 = 4x^2 + 4y^2 - 24x + 36$
$= 3x^2 + 3y^2 - 20x + 32 = 0$
$= x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$
$= (\alpha, \beta) = \left(\frac{10}{3}, 0\right)$
$\gamma = \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$
$3(\alpha, \beta, \gamma) = 3\left(\frac{10}{3} + \frac{2}{3}\right)$
$= 12$

Question63

Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a+ib$ lies on the circle $|z-1| = |2z|$, then a possible value of $\frac{1+[a]}{4b}$, where [t] is greatest integer function, is :
[1-Feb-2023 Shift 2]

Options:

A. $-\frac{1}{2}$

B. -1

C. 1

D. $\frac{1}{2}$

E. 0

Answer: E

Solution:

Solution:

$\left|\frac{1+ai}{b+i}\right| = 1$

$|1+ia| = |b+i|$

$a^2+1 = b^2+1 \Rightarrow a = \pm b \Rightarrow b = -a \quad \text{as } ab < 0$

$(a+ib)$ lies on $|z-1| = |2z|$

$|a+ib-1| = 2|a+ib|$

$(a-1)^2+b^2 = 4(a^2+b^2)$

$(a-1)^2 = a^2 = 4(2a^2)$

$1-2a = 6a^2 \Rightarrow 6a^2+2a-1 = 0$

$a = \frac{-2\pm\sqrt{28}}{12} = \frac{-1\pm\sqrt{7}}{6}$

$a = \frac{\sqrt{7}-1}{6}$ and $b = \frac{1-\sqrt{7}}{6}$

$[a] = 0$

$\therefore \frac{1+[a]}{4b} = \frac{6}{4(1-\sqrt{7})} = -\left(\frac{1+\sqrt{7}}{4}\right)$

Similarly when $a = \frac{-1-\sqrt{7}}{6}$ and $b = \frac{1+\sqrt{7}}{6}$ then $[a] = -1$

$\therefore \frac{1+[a]}{4b} = \frac{1-1}{4\times\frac{1+\sqrt{7}}{6}} = 0$

Question64

Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then
[1-Feb-2023 Shift 2]

Options:

A. the number of favourable cases of the event $(A \cup B) \cap C$ is 6

B. A and B are mutually exclusive

C. The number of favourable cases of the events A, B and C are 15,6 and 6 respectively

D. B and C are independent

Answer: A

Solution:

Solution:

A : no. on 1^{st} die $<$ no. on 2^{nd} die

A : no. on 1^{st} die $=$ even & no. of 2^{nd} die $=$ odd

C : no. on 1^{st} die $=$ odd & no. on 2^{nd} die $=$ even

$n(A) = 5 + 4 + 3 + 2 + 1 = 15$

$n(B) = 9$

$n(C) = 9$

$n((A \cup B) \cap C) = (A \cap C) \cup (B \cap C)$

$= (3 + 2 + 1) + 0 = 6.$

Question65

Let
$S = \{ x : x \in \mathbb{R}.\ \text{and}\ (\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10 \}.$
Then n(S) is equal to
[1-Feb-2023 Shift 1]

Options:

A. 2

B. 4

C. 6

D. 0

Answer: B

Solution:

Solution:

Sol. Let $(\sqrt{3} + \sqrt{2})^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$\Rightarrow\ t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow\ (\sqrt{3} + \sqrt{2})^{x^2-4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow\ x^2 - 4 = 2, -2 \quad \text{or} \quad x^2 = 6, 2$

$\Rightarrow\ x = \pm\sqrt{2}, \pm\sqrt{6}$

Question66

Let a $\neq$ b be two-zero real numbers. Then the number of elements in the set
$X = \{ z \in C : \text{Re}(az^2 + bz) = a.\ \text{and}\ \text{Re}(bz^2 + az) = b \}$ is equal to :
[6-Apr-2023 shift 2]

Options:

A. 0

B. 2

C. 1

D. 3

Answer: A

Solution:

Solution:

(1) Bonus

$\because z + \bar{z} = 2\,\text{Re}(z) \quad \text{If}\ z = x + iy$

$\Rightarrow z+\bar{z}=2x$
$z^2+(\bar{z})^2=2(x^2-y^2)$
$(az^2+bz)+(a\bar{z}^2+b\bar{z})=2a \ldots (1)$
$(bz^2+az)+(b\bar{z}^2+a\bar{z})=2b \ldots (2)$
add (1) and (2)
$(a+b)z^2+(a+b)z+(a+b)\bar{z}^2+(a+b)\bar{z}=2(a+b)$
$(a+b)[z^2+z+(\bar{z})^2+\bar{z}]=2(a+b)$
sub. (1) and (2)
$(a-b)[z^2-z+\bar{z}^2-\bar{z}]=2(a-b) \ldots (3)$
$z^2+\bar{z}^2-z-\bar{z}=2 \ldots (4)$
Case I: If $a+b\neq 0$
From (3) & (4)
$2x+2(x^2-y^2)=2 \Rightarrow x^2-y^2+x=1 \ldots (5)$
$2(x^2-y^2)-2x=2 \Rightarrow x^2-y^2-x=1 \ldots (6)$
$(5)-(6)$
$2x=0\Rightarrow x=0$
from (5) $y^2=-1 \Rightarrow$ not possbible
$\therefore$ Ans $=0$
Case II: If $a+b=0$ then infinite number of solution.
So, the set X have infinite number of elements.

Question67

For α, β, z ∈ C and λ > 1, if $\sqrt{\lambda-1}$ is the radius of the circle $|z-\alpha|^2+|z-\beta|^2=2\lambda$, then $|\alpha-\beta|$ is equal to _______ :
[6-Apr-2023 shift 2]

Answer: 2

Solution:

Solution:
$|z-z_1|^2+|z-z_2|^2=|z_1-z_2|^2$
$z_1=\alpha, z_2=\beta$
$|\alpha-\beta|^2=2\lambda$
$|\alpha-\beta|=\sqrt{2\lambda}$
$2r=\sqrt{2\lambda}$
$2\sqrt{\lambda-1}=\sqrt{2\lambda}$
$\Rightarrow 4(\lambda-1)=2\lambda$
$\lambda=2$
$|\alpha-\beta|=2$

Question68

If for $z=\alpha+i\beta$, $|z+2|=z+4(1+i)$, then $\alpha+\beta$ and $\alpha\beta$ are the roots of the equation
[8-Apr-2023 shift 1]

Options:

A. $x^2+3x-4=0$

B. $x^2+7x+12=0$

C. $x^2+x-12=0$

D. $x^2+2x-3=0$

Answer: B

Solution:

Solution:
$|z+2| = |\alpha + i\beta + 2|$
$= \alpha + i\beta + 4 + 4i$
$\sqrt{(\alpha+2)^2+\beta^2} = (\alpha+4) + i(\beta+4) \quad \beta+4=0$
$(\alpha+2)^2 + 16 = (\alpha+4)^2$
$\alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$
$4 = 4\alpha$
$\alpha = 1$
$\alpha = 1, \beta = -4$
$\alpha + \beta = -3, \alpha\beta = -4$
Sum of roots $= -7$
Product of roots $= 12$
$x^2 + 7x + 12 = 0$

Question69

Let A = $\left\{\theta \in (0, 2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta}\right.$. is purely imaginary $\}$. Then the sum of the elements in A is. [8-Apr-2023 shift 2]

Options:

A. π

B. 3π

C. 4π

D. 2π

Answer: C

Solution:

Solution:
$z = \frac{1+2i\sin\theta}{1-i\sin\theta} \times \frac{1+i\sin\theta}{1+i\sin\theta}$
$z = \frac{1-2\sin^2\theta + i(3\sin\theta)}{1+\sin^2\theta}$
$\text{Re}(z) = 0$
$\frac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$
$\sin\theta = \frac{\pm 1}{\sqrt{2}}$
$A = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
sum $= 4\pi$(Option 3)

Question70

Let the complex number z = x+ iy be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to : [10-Apr-2023 shift 1]

Options:

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: D

Solution:

Solution:

$z = x + iy$

$\frac{(2z-3i)}{2z+i} =$ purely imaginary

Means $\operatorname{Re}\left(\frac{2z-3i}{2z+i}\right) = 0$

$\Rightarrow \frac{(2z-3i)}{(2z+i)} = \frac{2(x+iy)-3i}{2(x+iy)+i}$

$= \frac{2x+2yi-3i}{2x+i2y+i}$

$= \frac{2x+i(2y-3)}{2x+i(2y+1)} \times \frac{2x-i(2y+1)}{2x-i(2y+1)}$

$\operatorname{Re}\left[\frac{2z-3i}{2z+i}\right] = \frac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2} = 0$

$\Rightarrow 4x^2+(2y-3)(2y+1) = 0$

$\Rightarrow 4x^2+[4y^2+2y-6y-3] = 0$

$\because x+y^2 = 0 \Rightarrow x = -y^2$

$\Rightarrow 4(-y^2)^2+4y^2-4y-3 = 0$

$\Rightarrow 4y^4+4y^2-4y = 3$

$\Rightarrow y^4+y^2-y = \frac{3}{4}$

Therefore, correct answer is option (4).

Question71

Let S = $\left\{z = x+iy : \frac{2z-3i}{4z+2i}\right.$. is a real number }. Then which of the following is NOT correct? [10-Apr-2023 shift 2]

Options:

A. $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

B. $(x, y) = \left(0, -\frac{1}{2}\right)$

C. $x = 0$

D. $y + x^2 + y^2 \neq -\frac{1}{4}$

Answer: B

Solution:

Solution:

$\frac{2z-3i}{4z+2i} \in R$

$\frac{2(x+iy)-3i}{4(x+iy)+2i} = \frac{2x+(2y-3)i}{4x+(4y+2)i} \times \frac{4x-(4y+2)i}{4x-(4y+2)i}$

$4x(2y-3)-2x(4y+2) = 0$

$x = 0 \quad y \neq -\frac{1}{2}$

Ans. $= 2$

Question72

Let w_1 be the point obtained by the rotation of $z_1 = 5+4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3+5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to : [11-Apr-2023 shift 1]

Options:

A. $\pi - \tan^{-1} \frac{8}{9}$

B. $-\pi + \tan^{-1} \frac{8}{9}$

C. $\pi - \tan^{-1} \frac{33}{5}$

D. $-\pi + \tan^{-1} \frac{33}{5}$

Answer: A

Solution:

Solution:
$W_1 = z_1 i = (5 + 4i)i = -4 + 5i \dots$ (i)
$W_1 = z_2(-i) = (3 + 5i)(-i) = 5 - 3i \dots$ (2)
$W_1 - W_2 = -9 + 8i$
Principal argument $= \pi - \tan^{-1}\left(\frac{8}{9}\right)$

Question73

For $a \in C$, let $A = \{z \in C : Re(a + \overline{z}) > Im(\overline{a} + z)\}$ and $B = \{z \in C : Re(a + \overline{z}) < Im(\overline{a} + z)\}$. The among the two statements:
(S1) : If Re(a), Im(a) > 0, then the set A contains all the real numbers
(S2) : If Re(a), Im(a) < 0, then the set B contains all the real numbers,
[11-Apr-2023 shift 2]

Options:

A. only (S1) is true

B. both are false

C. only (S2) is true

D. both are true

Answer: B

Solution:

Solution:
Let $a = x_1 + iy_1 z = x + iy$
Now $Re(a + \overline{z}) > Im(\overline{a} + z)$
$\therefore x_1 + x > -y_1 + y$
$x_1 = 2, y_1 = 10, x = -12, y = 0$
Given inequality is not valid for these values.
S1 is false.
Now $Re(a + \overline{z}) < Im(\overline{a} + z)$
$x_1 + x < -y_1 + y$
$x_1 = -2, y_1 = -10, x = 12, y = 0$
Given inequality is not valid for these values. S2 is false.

Question74

Let $S = \left\{ z \in C - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in R \right\}$. If $\alpha - \frac{13}{11}i \in S$, $a \in R - \{0\}$, then $242\alpha^2$ is equal to _______.
[11-Apr-2023 shift 2]

Answer: 1680

Solution:

Solution:

$\left(\frac{z^2+8iz-15}{z^2-3iz-2}\right) \in R$

$\Rightarrow 1+\frac{(11iz-13)}{(z^2-3iz-2)} \in R$

Put $Z = \alpha - \frac{13}{11}i$

$\Rightarrow (z^2-3iz-2)$ is imaginary

Put $z = x+iy$

$\Rightarrow (x^2-y^2+2xyi-3ix+3y-2) \in$ Imaginary

$\Rightarrow \text{Re}(x^2-y^2+3y-2+(2xy-3x)i) = 0$

$\Rightarrow x^2-y^2+3y-2 = 0$

$x^2 = y^2-3y+2$

$x^2 = (y-1)(y-2) \therefore z = \alpha - \frac{13}{11}i$

Put $x = \alpha,\ y = \frac{-13}{11}$

$\alpha^2 = \left(\frac{-13}{11}-11\right)\left(\frac{-13}{11}-2\right)$

$\alpha^2 = \frac{(24\times 35)}{121}$

$242\alpha^2 = 48\times 35 = 1680$

Question75

Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius $r = 1$. Let $z_1 = 1+i$ and the complex number z_2 be outside the circle C such that $|z_1-z_0||z_2-z_0| = 1$. If $z_0 \cdot z_1$ and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

[12-Apr-2023 shift 1]

Options:

A. $\frac{7}{2}$

B. $\frac{13}{2}$

C. $\frac{5}{2}$

D. $\frac{3}{2}$

Answer: C

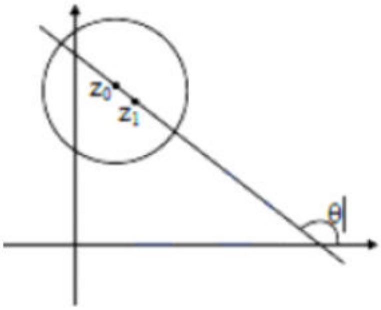
Solution:

Solution:

$|z_1-z_0| = \left|\frac{1-i}{2}\right| = \frac{1}{2}$

$\Rightarrow |z_2-z_0| = \sqrt{2}$: centre $\left(\frac{1}{2}, \frac{3}{2}\right)$

$z_0\left(\frac{1}{2}, \frac{3}{2}\right)$ and $z_1(1, 1)$



$\tan\theta = -1 \Rightarrow \theta = 135^\circ$

$z_2\left(\frac{1}{2}+\sqrt{2}\cos 135^\circ,\ \frac{3}{2}+\sqrt{2}\sin 135^\circ\right)$

or

$\left(\frac{1}{2}-\sqrt{2}\cos 135^\circ,\ \frac{3}{2}-\sqrt{2}\sin 135^\circ\right)$

$\Rightarrow z_2\left(-\frac{1}{2},\ \frac{5}{2}\right)$ or $z_2\left(\frac{3}{2},\ \frac{1}{2}\right)$

$\Rightarrow |z_3|^2 = \frac{26}{4},\ \frac{5}{2}$

$\Rightarrow |z_2|_{min}{}^2 = \frac{5}{2}$

Question76

Let $S = \{z \in C : \bar{z} = i(z^2 + Re(\bar{z}))\}$. Then $\sum_{z \in S} |z|^2$ is equal to

[13-Apr-2023 shift 2]

Options:

A. 4

B. $\frac{7}{2}$

C. 3

D. $\frac{5}{2}$

Answer: A

Solution:

Solution:
Let $z = x + iy$

$\bar{z} = i(z^2 + Re(\bar{z}))$

$\Rightarrow x - iy = i(x^2 - y^2 + 2ixy + x)$

$\Rightarrow x - iy = -2xy + i(x^2 - y^2 + x)$

$x + 2xy = 0$ and $x^2 - y^2 + x + y = 0$

$x(1 + 2y) = 0$ and $x^2 - y^2 + x + y = 0$

If $x = 0$ then $-y^2 + y = 0$

$\Rightarrow y = 1, 0$

If $y = \frac{-1}{2}$ then $x^2 - \frac{1}{4} + x - \frac{1}{2} = 0$

$\Rightarrow x = -\frac{3}{2}, \frac{1}{2}$

$= \left\{0 + i0,\ 0 + i,\ -\frac{3}{2} - \frac{1}{2}i,\ \frac{1}{2} - \frac{1}{2}i\right\}$

$\sum_{Z \in S|Z|}^{2=0+1+} \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 4$

Question77

If the set $\left\{Re\left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5z}\right) : z \in C, Re(z) = 3\right\}$ is equal to the interval $(\alpha, \beta]$, then $24(\beta - \alpha)$ is equal to

[15-Apr-2023 shift 1]

Options:

A. 36

B. 27

C. 30

D. 42

Answer: C

Solution:

Solution:

Let $z_1 = \left(\dfrac{z - \bar{z} + z\bar{z}}{2 - 3z + 5\bar{z}} \right)$

Let $z = 3 + iy$

$\bar{z} = 3 - iy$

$$z_1 = \frac{2iy + (9 + y^2)}{2 - 3(3 + iy) + 5(3 - iy)}$$

$$= \frac{9 + y^2 + i(2y)}{8 - 8iy}$$

$$= \frac{(9 + y^2) + i(2y)}{8(1 - iy)}$$

$$\mathrm{Re}(z_1) = \frac{(9 + y^2) - 2y^2}{8(1 + y^2)}$$

$$= \frac{9 - y^2}{8(1 + y^2)}$$

$$= \frac{1}{8}\left[\frac{10 - (1 + y^2)}{(1 + y^2)} \right]$$

$$= \frac{1}{8}\left[\frac{10}{(1 + y^2)} - 1 \right]$$

$1 + y^2 \in [1, \infty]$

$\dfrac{1}{1 + y^2} \in (0, 1]$

$\dfrac{10}{1 + y^2} \in (0, 10]$

$\dfrac{10}{1 + y^2} - 1 \in (-1, 9]$

$\mathrm{Re}(z_1) \in \left(\dfrac{-1}{8}, \dfrac{9}{8} \right]$

$\alpha = \dfrac{-1}{8}, \beta = \dfrac{9}{8}$

$24(\beta - \alpha) = 24\left(\dfrac{9}{8} + \dfrac{1}{8} \right) = 30$

Question78

The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is : [6-Apr-2023 shift 1]

Options:

A. $11 - \sqrt{3}$

B. $9 - \sqrt{3}$

C. $9 + \sqrt{3}$

D. $11 + \sqrt{3}$

Answer: C

Solution:

Solution:

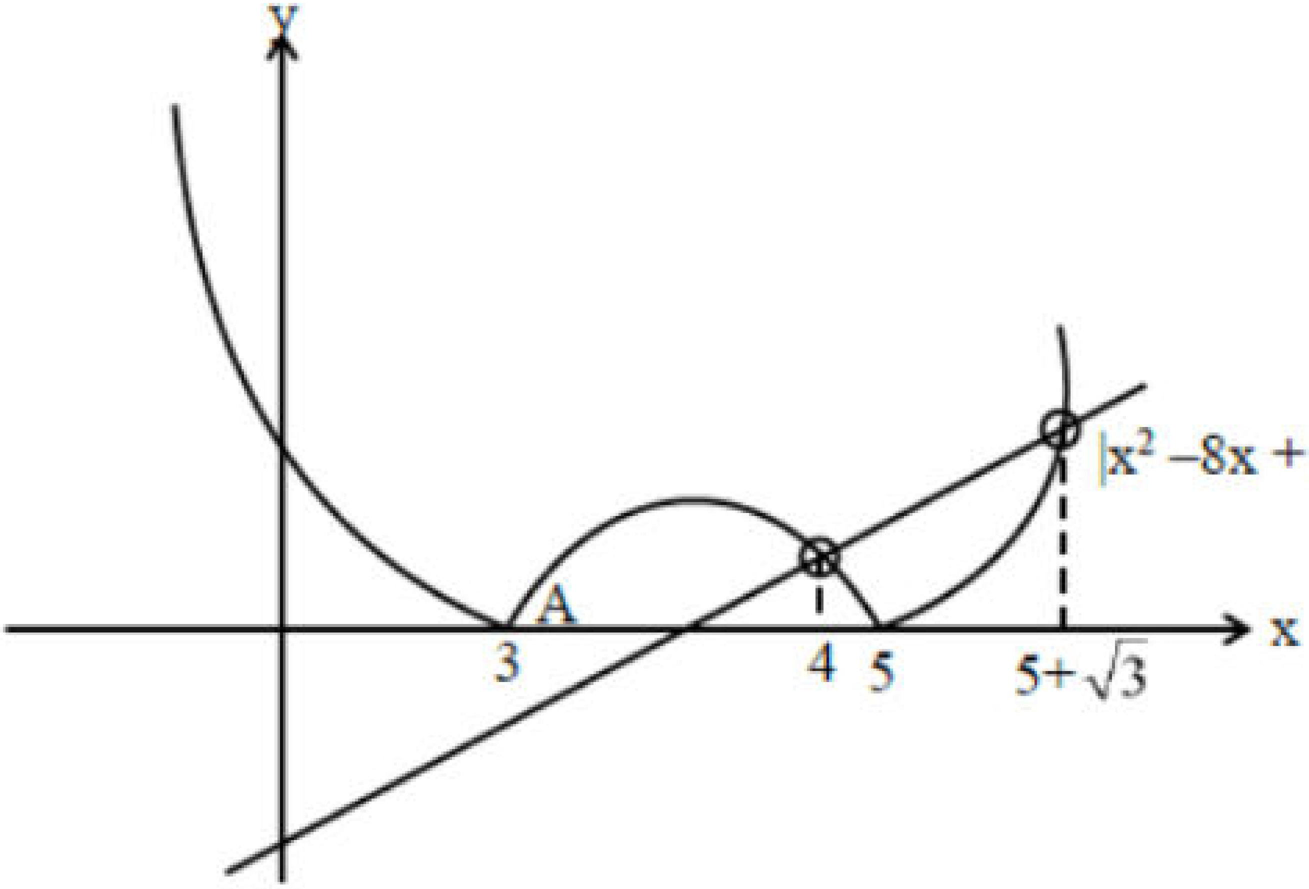
$|x^2 - 8x + 15| = 2x - 7$

$x^2 - 8x + 15 = 2x - 7 \;\; \& \;\; x^2 - 8x + 15 = 7 - 2x$

$x^2 - 10x + 22 = 0$ & $x^2 - 6x + 8 = 0$

$x_1 = 5 + \sqrt{3}$ $x_2 = 5 - \sqrt{3}$ (reject) $x_3 = 4$ $x_4 = 2$ (reject)

Sum of of roots is $= 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$



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Question79

Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to
[8-Apr-2023 shift 1]

Options:

A. $\frac{155}{8}$

B. 21

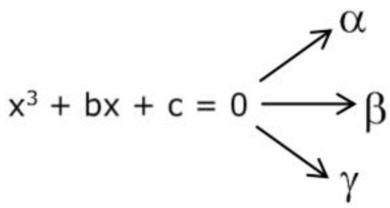
C. 19

D. $\frac{169}{8}$

Answer: C

Solution:

Solution:



$\beta\gamma = 1$
$\alpha = -1$
Put $\alpha = -1$
$-1 - b + c = 0$
$c - b = 1$
also
$\alpha \cdot \beta \cdot \gamma = -c$
$-1 = -c \Rightarrow c = 1$
$\therefore b = 0$
$x^3 + 1 = 0$
$\alpha = -1, \beta = -w, \gamma = -w^2$
$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$
$0 + 2 + 3 + 6 + 8 = 19$

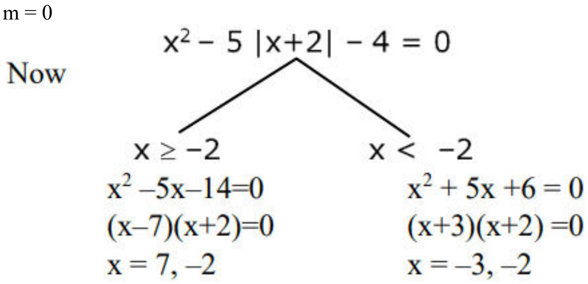
Question80

Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where [x] denotes the greatest integer leq x. Then $m^2 + mn + n^2$ is equal to
[8-Apr-2023 shift 2]

Answer: 9

Solution:

Solution:
$x^2 - 12x + [x] + 31 = 0$
$\{x\} = x^2 - 11x + 31$
$0 \le x^2 - 11x + 31 < 1$
$x^2 - 11x + 30n < 0$
$x \in (5, 6)$
so $[x] = 5$
$x^2 - 12x + 5 + 31 = 0$
$x^2 - 12x + 36 = 0$
$x = 6$ but $x \in (5, 6)$
so $x \in \varphi$
$m = 0$



$x = \{7, -2, -3\}$
$n = 3$
$m^2 + mn + n^2 = n^2 = 9$

Question81

If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to ________.

[11-Apr-2023 shift 1]

Answer: 51

Solution:

Solution:
By newton's theorem

$S_{n+2} - 7S_{n+1} - S_n = 0$

$S_{21} - 7S_{20} - S_{19} = 0$

$S_{20} - 7S_{19} - S_{18} = 0$

$S_{19} - 7S_{18} - S_{17} = 0$

$\frac{S_{21} + S_{17}}{S_{19}} = \frac{S_{21} + (S_{19} - 7S_{18})}{S_{19}}$

$= \frac{50S_{19} + (S_{21} - 7S_{20})}{S_{19}}$

$= 51 \cdot \frac{S_{19}}{S_{19}} = 51$

Question82

The number of points where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1,\ x \in R$ cuts x-axis, is equal to ________

[11-Apr-2023 shift 2]

Answer: 2

Solution:

Solution:
Let $e^{2x} = t$

$\Rightarrow t^4 - t^3 - 3t^2 - t + 1 = 0$

$\Rightarrow t_2 + \frac{1}{t_2} - \left(t + \frac{1}{t}\right) - 3 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$

$\Rightarrow t + \frac{1}{t} = \frac{1 + \sqrt{21}}{2}$

Two real values of t.

Question83

Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to

[12-Apr-2023 shift 1]

Options:

A. 9

B. 729

C. 72

D. 81

Answer: D

Solution:

Solution:

$\alpha, \beta = \frac{-\sqrt{6} \pm \sqrt{6-12}}{2} = \frac{-\sqrt{6} \pm \sqrt{6}i}{2}$

$= \sqrt{3e^{\pm\frac{3\pi i}{4}}}$

Required expression

$\frac{(\sqrt{3})^{23}\left(2\cos\frac{69\pi}{4}\right) + (\sqrt{3})^{14}\left(2\cos\frac{42\pi}{4}\right)}{(\sqrt{3})^{15}\left(2\cos\frac{45\pi}{4}\right) + (\sqrt{3})^{10}\left(2\cos\frac{30\pi}{4}\right)}$

$(\sqrt{3})^8 = 81$

Question84

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$, Then $\alpha^{14} + \beta^{14}$ is equal to [13-Apr-2023 shift 2]

Options:

A. $-128\sqrt{2}$

B. $-64\sqrt{2}$

C. -128

D. -64

Answer: C

Solution:

Solution:

$x^2 - \sqrt{2}x + 2 = 0$

$x = \frac{\sqrt{2} \pm \sqrt{-6}}{2}$

$= \sqrt{2}\left(\frac{1 \pm i\sqrt{3}}{2}\right)$

$= -\sqrt{2}\omega, -\sqrt{2}\omega^2$

$\Rightarrow \alpha - \sqrt{2}\omega, \beta = -\sqrt{2}\omega^2$

$\alpha^{14} + \beta^{14} = 2^7(\omega^{14} + \omega^{28}) = 2^7(\omega^2 + \omega) = -128$

Question85

The number of real roots of the equation $x|x| - 5|x+2| + 6 = 0$, is [15-Apr-2023 shift 1]

Options:

A. 5

B. 6

C. 4

D. 3

Answer: D

Solution:

Solution:
$x|x|-5|x+2|+6=0$
$C-1:-x\in[0,\infty]$
$x^2-5x-4=0$
$x=\frac{5\pm\sqrt{25+16}}{2}=\frac{5+\sqrt{41}}{2}$
$x=\frac{5\pm\sqrt{41}}{2}$
$C-2:-:-x\in[-2,0)$
$-x^2-5x-4=0$
$x^2+5x+4=0$
$x=-1,-4$
$x=-1$
$C-3:x\in[-\infty,-2)$
$-x^2+5x+16=0$
$x^2-5x-16=0$
$x=\frac{5\pm\sqrt{25+64}}{2}$
$x=\frac{5\pm\sqrt{89}}{2}$
$x=\frac{5-\sqrt{89}}{2}$

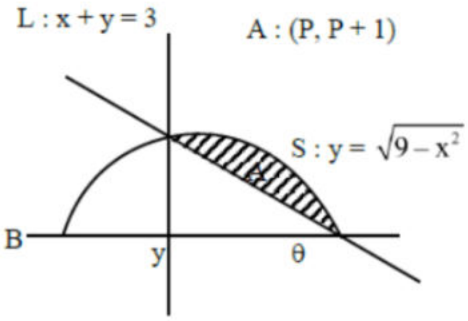
Question86

Let the point (p, p + 1) lie inside the region E $=\{(x,y):3-x\le y\le\sqrt{9-x^2},0\le x\le3\}$. If the set of all values of p is the interval (a, b), then b^2+b-a^2 is equal to _______.
[6-Apr-2023 shift 1]

Answer: 3

Solution:

Solution:
$3-x\le y\le\sqrt{9-x^2};0\le x\le3$



$L(A)>0\Rightarrow P+P+1-3>0\Rightarrow P>1$ . . . (1)
$S(A)<0\Rightarrow P+1-\sqrt{9-P^2}<0$
$\Rightarrow P+1<\sqrt{9-P^2}$
$\Rightarrow P+2P+1<9-P^2$
$\Rightarrow 2P^2+2P-8<0$
$\Rightarrow P^2+P-4<0$
$\Rightarrow P\in\left(\frac{-1-\sqrt{17}}{2},\frac{-1+\sqrt{17}}{2}\right)$ . . . (2)
(1) $\cap$(2)$P\in\left(1,\frac{\sqrt{17}-1}{2}\right)\equiv(a,b)$
$b^2+b-a^2=4-1=3$

Question87

Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_c c}$. Then $6a + 5bc$ is equal to _______.
[10-Apr-2023 shift 1]

Answer: 8

Solution:

Solution:
$(2a)^{\ln a} = (bc)^{\ln b}$ $2a > 0, bc > 0$
$\ln a(\ln 2 + \ln a) = \ln b(\ln b + \ln c)$
$\ln 2 \cdot \ln b = \ln c \cdot \ln a$
$\ln 2 = \alpha, \ln a = x, \ln b = y, \ln c = z$
$\alpha y = xz$
$x(\alpha + x) = y(y + z)$
$\alpha = \frac{xz}{y}$
$x\left(\frac{xz}{y} + x\right) = y(y + z)$
$x^2(z + y) = y^2(y + z)$
$y + z = 0$ or $x^2 = y^2 \Rightarrow x = -y$
$bc = 1$ or $ab = 1$
$bc = 1$ or $ab = 1$

(1) if $bc = 1 \Rightarrow (2a)^{\ln a} = 1$ $\to a = 1$, $\to a = 1/2$

$(a, b, c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right), \lambda \neq 1, 2, \frac{1}{2}$
then
$6a + 5bc = 3 + 5 = 8$
(II) $(a, b, c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right), \lambda \neq 1, 2, \frac{1}{2}$
In this situation infinite answer are possible
So, Bonus.

Question88

The number of integral solutions x of $\log_{\left(x + \frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

[11-Apr-2023 shift 1]

Options:

A. 5

B. 7

C. 8

D. 6

Answer: D

Solution:

Solution:
$\log_{x + \frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$
Feasible region: $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$
And $x + \frac{7}{2} \neq 1 \Rightarrow x \neq \frac{-5}{2}$
Taking intersection: $x \in \left(\frac{-7}{2}, \infty\right) - \left\{-\frac{5}{2}, \frac{3}{2}, 7\right\}$
Now $\log_a b \geq 0$ if $a > 1$ and $b \geq 1$

$a \in (0, 1)$ and $b \in (0, 1)$

$C-I; x+\frac{7}{2}>1$ and $\left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$x>-\frac{5}{2}; (2x-3)^2-(x-7)^2 \leq 0$

$(2x-3+x-7)(2x-3-x+7) \leq 0$

$(3x-10)(x+4) \leq 0$

$x \in \left[-4, \frac{10}{3}\right]$

Intersection: $x \in \left(\frac{-5}{2}, \frac{10}{3}\right]$

$C-II: x+\frac{7}{2} \in (0, 1)$ and $\left(\frac{x-7}{2x-3}\right)^2 \in (0, 1)$

$0<x+\frac{7}{2}<1; \left(\frac{x-7}{2x-3}\right)^2<1$

$-\frac{7}{2}<x<\frac{-5}{2}; (x-7)^2<(2x-3)^2$

$x \in (-\infty, -4) \cup \left(\frac{10}{3}, \infty\right)$

No common values of x.
Hence intersection with feasible region

We get $x \in \left(\frac{-5}{2}, \frac{10}{3}\right]-\left\{\frac{3}{2}\right\}$

Integral value of x are $\{-2, -1, 0, 1, 2, 3\}$
No. of integral values $=6$

Question89

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2+\lambda x-1=0$ is 15 , then $6(\alpha^3+\beta^3)^2$ is equal to:
[24-Jun-2022-Shift-1]

Options:

A. 18

B. 24

C. 36

D. 96

Answer: B

Solution:

Solution:

$3x^2+\lambda x-1=0$

Given, two roots are α and β.

$\therefore$ Sum of roots $=\alpha+\beta=\frac{-\lambda}{3}$

And product of roots $=\alpha\beta=\frac{-1}{3}$

Given that,

Sum of square of reciprocal of roots α and β is 15 .

$$\therefore \frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = 15$$

$$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = 15$$

$$\Rightarrow \frac{\frac{\lambda^2}{9} + 2 \times \frac{1}{3}}{\frac{1}{9}} = 15$$

$$\Rightarrow \frac{\frac{\lambda^2 + 6}{9}}{\frac{1}{9}} = 15$$

$$\Rightarrow \lambda^2 + 6 = 15$$

$$\Rightarrow \lambda^2 = 9$$

Now, $6(\alpha^3 + \beta^3)^2$

$$= 6\{(\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)\}^2$$

$$= 6(\alpha+\beta)^2[(\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta]^2$$

$$= 6\left(\frac{-\lambda}{3}\right)^2\left[\left(\frac{-\lambda}{3}\right)^2 - 3 \cdot \frac{-1}{3}\right]^2$$

$$= 6 \times \frac{\lambda^2}{9} \times \left[\frac{\lambda^2}{9} + 1\right]$$

$$= 6 \times \frac{9}{9} \times \left[\frac{9}{9} + 1\right]^2$$

$$= 6 \times (2)^2$$

$$= 6 \times 4 = 24$$

Question90

Let $S = \{z \in C : |z - 3| \le 1$ and $z(4 + 3i) + \overline{z}(4 - 3i) \le 24\}$. If $\alpha + i\beta$ is the point in S which is closest to 4i, then $25(\alpha + \beta)$ is equal to____
[24-Jun-2022-Shift-2]

Answer: 80

Solution:

Solution:

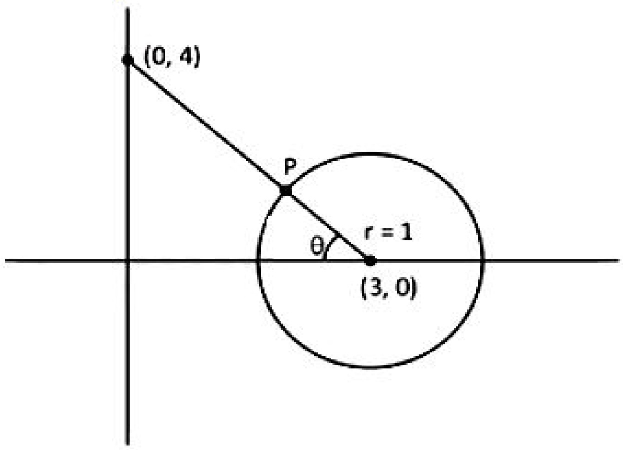
Here $|z - 3| < 1$

$\Rightarrow (x - 3)^2 + y^2 < 1$

and $z = (4 + 3i) + \overline{z}(4 - 3i) \le 24$

$\Rightarrow 4x - 3y \le 12$

$\tan\theta = \frac{4}{3}$



$\therefore$ Coordinate of $P = (3 - \cos\theta, \sin\theta)$

$= \left(3 - \frac{3}{5}, \frac{4}{5}\right)$

$\therefore \alpha + i\beta = \frac{12}{5} + \frac{4}{5}i$

$\therefore 25(\alpha + \beta) = 80$

Question91

Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to:
[25-Jun-2022-Shift-1]

Options:

A. $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)-\pi$

B. $\tan^{-1}\left(\frac{24}{7}\right)-\pi$

C. $\tan^{-1}(3)-\pi$

D. $\tan^{-1}\left(\frac{3}{4}\right)-\pi$

Answer: B

Solution:

Solution:

$z_1=3+4i,\ z_2=4+3i$ and $z_3=5i$

Clearly, $C\equiv x^2+y^2=25$

Let $z(x,y)$

$\Rightarrow\left(\frac{y-4}{x-3}\right)\left(\frac{2}{-4}\right)=-1$

$\Rightarrow y=2x-2\equiv L$

$\therefore$z is intersection of C&L

$\Rightarrow z\equiv\left(\frac{-7}{5},\frac{-24}{5}\right)$

$\therefore \operatorname{Arg}(z)=-\pi+\tan^{-1}\left(\frac{24}{7}\right)$

Question92

Let z_1 and z_2 be two complex numbers such that $\bar{z}_1=i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right)=\pi$. Then

[25-Jun-2022-Shift-2]

Options:

A. $\arg z_2=\frac{\pi}{4}$

B. $\arg z_2=-\frac{3\pi}{4}$

C. $\arg z_1=\frac{\pi}{4}$

D. $\arg z_1=-\frac{3\pi}{4}$

Answer: C

Solution:

Solution:

$\because \frac{z_1}{z_2} = -i \Rightarrow z_1 = -iz_2$

$\Rightarrow \arg(z_1) = -\frac{\pi}{2} + \arg(z_2) \ldots\ldots$ (i)

Also $\arg(z_1) - \arg(\bar{z}_2) = \pi$

$\Rightarrow \arg(z_1) + \arg(z_2) = \pi \ldots.$ (ii)

From (i) and (ii), we get $\arg(z_1) = \frac{\pi}{4}$ and $\arg(z_2) = \frac{3\pi}{4}$

Question93

Let $A = \left\{z \in C : \left|\frac{z+1}{z-1}\right| < 1\right\}$ and $B = \left\{z \in C : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}\right\}$. Then $A \cap B$ is :
[26-Jun-2022-Shift-1]

Options:

A. a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

B. a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

C. an empty

D. a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Answer: B

Solution:

Solution:

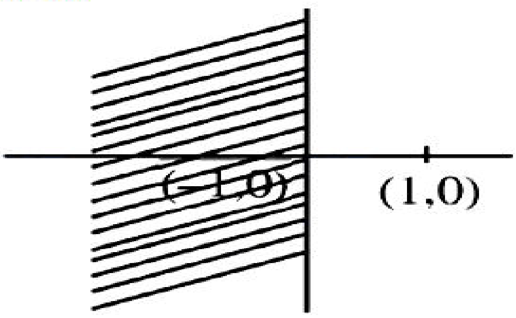
Set A

$\Rightarrow \left|\frac{z+1}{z-1}\right| < 1$

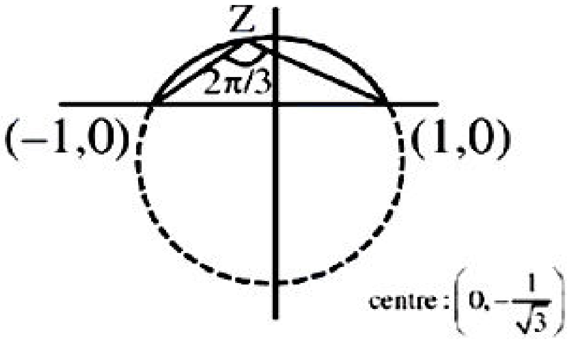
$\Rightarrow |z+1| < |z-1|$

$\Rightarrow (x+1)^2 + y^2 < (x-1)^2 + y^2$

$\Rightarrow x < 0$



Set B



$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{2\pi}{3}$

$\Rightarrow x^2 + y^2 + \frac{2y}{\sqrt{3}} - 1 = 0$

$A \cap B$

$\Rightarrow$ Centre $\left(0, -\frac{1}{\sqrt{3}}\right)$

Question94

If $z^2 + z + 1 = 0$, $z \in C$, then

$\left| \sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n}\right)^2 \right|$ **is equal to____**

[26-Jun-2022-Shift-2]

Answer: 2

Solution:

Solution:

$\because z^2+z+1=0$

$\Rightarrow \omega$ or ω^2

$\because \left|\sum_{n=1}^{15}\left(z^n+(-1)^n\frac{1}{z^n}\right)^2\right|$

$=\left|\sum_{n=1}^{15} z^{2n}+\sum_{n=1}^{15} z^{-2n}+2\cdot\sum_{n=1}^{15}(-1)^n\right|$

$=|0+0-2|$

$=2$

Question95

The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z}=iz^2$ is : [27-Jun-2022-Shift-1]

Options:

A. $\frac{3\sqrt{3}}{4}$

B. $\frac{3\sqrt{3}}{2}$

C. $\frac{3}{2}$

D. $\frac{3}{4}$

Answer: A

Solution:

Solution:
$\bar{z}=iz^2$
Let $z=x+iy$
$x-iy=i(x^2-y^2+2xiy)$
$x-iy=i(x^2-y^2)-2xy$
$\therefore x=-2yx$ or $x^2-y^2=-y$
$x=0$ or $y=-\frac{1}{2}$
Case - I
$x=0$
$-y^2=-y$
$y=0,1$
Case - II
$y=-\frac{1}{2}$
$\Rightarrow x^2-\frac{1}{4}=\frac{1}{2}\Rightarrow x=\pm\frac{\sqrt{3}}{2}$
$x=\left\{0, i, \frac{\sqrt{3}}{2}-\frac{i}{2}, \frac{-\sqrt{3}}{2}-\frac{i}{2}\right\}$

$$\text{Area of polygon} = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 1 \end{vmatrix}$$

$=\frac{1}{2}\left|-\sqrt{3}-\frac{\sqrt{3}}{2}\right|=\frac{3\sqrt{3}}{4}$

Question96

The number of points of intersection of $|z-(4+3i)|=2$ and $|z|+|z-4|=6, z \in C$, is [27-Jun-2022-Shift-2]

Options:

A. 0

B. 1

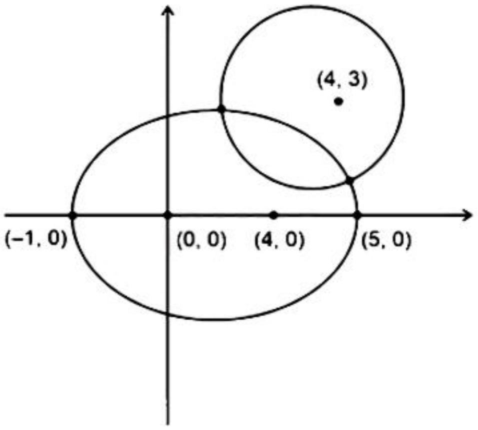
C. 2

D. 3

Answer: C

Solution:

Solution:
$C_1 : |z-(4+3i)| = 2$ and $C_2 : |z|+|z-4| = 6, z \in C$
C_1 represents a circle with centre $(4, 3)$ and radius 2 and C_2 represents a ellipse with focii at $(0, 0)$ and $(4, 0)$ and length of major axis $=6$, and length of semi-major axis $2\sqrt{5}$ and $(4, 2)$ lies inside the both C_1 and C_2 and $(4, 3)$ lies outside the C_2



$\therefore$ number of intersection points $=2$

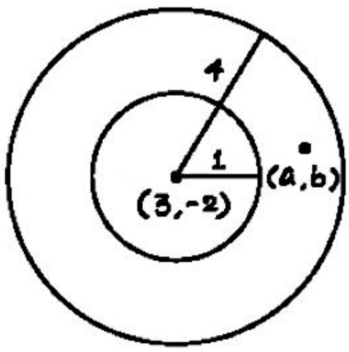
Question97

The number of elements in the set $\{z = a + ib \in C : a, b \in Z \text{ and } 1 < |z-3+2i| < 4\}$ is___ [28-Jun-2022-Shift-1]

Answer: 40

Solution:

Solution:
$1 < |Z-3+2i| < 4$



$1 < (a-3)^2 + (b+2)^2 < 16$
$(0, \pm 2), (\pm 2, 0), (\pm 1, \pm 2), (\pm 2, \pm 1)$
$(\pm 2, \pm 3), (3\pm, \pm 2), (\pm 1, \pm 1), (2\pm, \pm 2)$

$(\pm 3, 0)$, $(0, \pm 3)$, $(\pm 3 \pm 1)$, $(\pm 1, \pm 3)$
Total 40 points

Question98

Sum of squares of modulus of all the complex numbers z satisfying $\overline{z} = iz^2 + z^2 - z$ is equal to_____ [28-Jun-2022-Shift-2]

Answer: 2

Solution:

Solution:
Let $z = x + iy$
So $2x = (1 + i)(x^2 - y^2 + 2xyi)$
$\Rightarrow 2x = x^2 - y^2 - 2xy$
(i) and
$x^2 - y^2 + 2xy = 0$
From (i) and (ii) we get
$x = 0 \text{ or } y = -\frac{1}{2}$
When $x = 0$ we get $y = 0$
When $y = -\frac{1}{2}$ we get $x^2 - x - \frac{1}{4} = 0$
$\Rightarrow x = \frac{-1 \pm \sqrt{2}}{2}$
So there will be total 3 possible values of z, which are 0, $\left(\frac{-1+\sqrt{2}}{2}\right) - \frac{1}{2}i$ and $\left(\frac{-1-\sqrt{2}}{2}\right) - \frac{1}{2}i$
Sum of squares of modulus
$= 0 + \left(\frac{\sqrt{2}-1}{2}\right)^2 + \frac{1}{4} + \left(\frac{\sqrt{2}+1}{2}\right)^2 = + \frac{1}{4}$
$= 2$

Question99

Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to: [29-Jun-2022-Shift-1]

Options:

A. 50

B. 250

C. 1250

D. 1500

Answer: A

Solution:

Solution:
Given equation,
$x^2 + (2i - 1) = 0$
$\Rightarrow x^2 = 1 - 2i$
Let α and β are the two roots of the equation.
As, we know roots of a equation satisfy the equation so
$\alpha^2 = 1 - 2i$
and $\beta^2 = 1 - 2i$
$\therefore \alpha^2 = \beta^2 = 1 - 2i$
$\therefore \left|\alpha^2\right| = \sqrt{1^2 + (-2)^2} = \sqrt{15}$

Now, $\left|\alpha^8+\beta^8\right|$

$\left|\alpha^8+\alpha^8\right|$

$=2\left|\alpha^8\right|$

$=2\left|\alpha^2\right|^4$

$=2(\sqrt{5})^4$

$=2\times 25$

$=50$

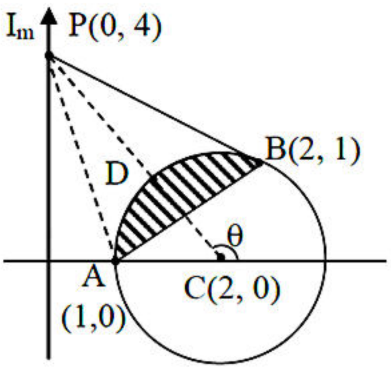
Question100

Let $S=\{z\in C:|z-2|\le 1, z(1+i)+\bar{z}(1-i)\le 2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1\in S$ and $z_2\in S$. If $5(z_1|^2+z_2|^2)=\alpha+\beta\sqrt{5}$, where α and β are integers, then the value of $\alpha+\beta$ is equal to_____
[29-Jun-2022-Shift-1]

Answer: 26

Solution:

Solution:
$|z-2|\le 1$



$(x-2)^2+y^2\le 1$
&
$z(1+i)+\bar{z}(1-i)\le 2$
Put $z=x+iy$
$\therefore x-y\le 1......\ (2)$
$PA=\sqrt{17}, PB=\sqrt{13}$
Maximum is PA & Minimum is PD
Let $D(2+\cos\theta, 0+\sin\theta)$
$\therefore m_{cp}=\tan\theta=-2$
$\cos\theta=-\frac{1}{\sqrt{5}}, \sin\theta=\frac{2}{\sqrt{5}}$

$\therefore D\left(2-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\Rightarrow z_1=\left(2-\frac{1}{\sqrt{5}}\right)+\frac{2i}{\sqrt{5}}$

$\left|z_1\right|=\frac{25-4\sqrt{5}}{5\&}z_2=1$

$\therefore\left|z_2\right|^2=1$

$\therefore 5\left(\left|z_1\right|^2+\left|z_2\right|^2\right)=30-4\sqrt{5}$

$\therefore\alpha=30$
$\beta=-4$
$\therefore\alpha+\beta=26$

Question101

Let arg(z) represent the principal argument of the complex number z. Then, $|z| = 3$ and $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$ intersect
[29-Jun-2022-Shift-2]

Options:

A. exactly at one point.

B. exactly at two points.

C. nowhere.

D. at infinitely many points.

Answer: C

Solution:

Solution:
Let $z = x + iy$
$\therefore |z| = \sqrt{x^2 + y^2}$
Given, $|z| = 3$
$\therefore \sqrt{x^2 + y^2} = 3$
$\Rightarrow x^2 + y^2 = 9 = 3^2$
This represent a circle with center at $(0, 0)$ and radius $= 3$
Now, given

$$\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$$

$$\Rightarrow \arg(x+iy-1) - \arg(x+iy+1) = \frac{\pi}{4}$$

$$\Rightarrow \arg(x-1+iy) - \arg(x+1+iy) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{\pi}{4}$$

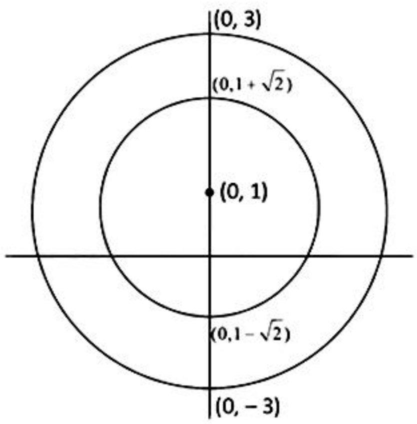
$$\Rightarrow \tan^{-1}\left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \frac{y}{x-1} \times \frac{y}{x+1}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{xy+y-xy+y}{x^2-1}}{\frac{x^2-1+y^2}{x^2-1}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{xy+y-xy+y}{x^2-1+y^2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{2y}{x^2-1+y^2} = \tan\left(\frac{\pi}{4}\right)$$

$\Rightarrow 2y = x^2 + y^2 - 1$
$\Rightarrow x^2 + y^2 - 2y - 1 = 0$
$\Rightarrow x^2 + (y-1)^2 = (\sqrt{2})^2$
This represent a circle with center at $(0, 1)$ and radius $\sqrt{2}$.



From diagram you can see both the circles do not cut anywhere.

Question102

The sum of all the real roots of the equation $(e^{2x}-4)(6e^{2x}-5e^{x}+1)=0$ is
[24-Jun-2022-Shift-2]

Options:

A. $\log_e 3$

B. $-\log_e 3$

C. $\log_e 6$

D. $-\log_e 6$

Answer: B

Solution:

Solution:

$(e^{2x}-4)(6e^{2x}-5e^{x}+1)=0$

Let $e^{x}=t$

$\therefore (t^2-4)(6t^2-5t+1)=0$

$\Rightarrow (t^2-4)(2t-1)(3t-1)=0$

$\therefore t=2,-2,\ \frac{1}{2},\ \frac{1}{3}$

$\therefore e^{x}=2\Rightarrow x=\ln 2$

$e^{x}=-2$(not possible)

$e^{x}=\frac{1}{2}\Rightarrow x=-\ln 2$

$e^{x}=\frac{1}{3}\Rightarrow x=-\ln 3$

$\therefore$ Sum of all real roots

$=\ln 2-\ln 2-\ln 3$

$=-\ln 3$

Question103

For a natural number n, let $\alpha_n=19^{n}-12^{n}$. Then, the value of $\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}$ is___
[25-Jun-2022-Shift-1]

Answer: 4

Solution:

Solution:

$\alpha_n = 19^n - 12^n$

Let equation of roots $12 \& 19$ i.e.

$x^2 - 31x + 228 = 0$

$\Rightarrow (31 - x) = \frac{228}{x}$ (where x can be 19 or 12)

$\therefore \frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} = \frac{31(19^9 - 12^9) - (19^{10} - 12^{10})}{57(19^8 - 12^8)}$

$= \frac{19^9(31 - 19) - 12^9(31 - 12)}{57(19^8 - 12^8)}$

$= \frac{228(19^8 - 12^8)}{57(19^8 - 12^8)} = 4$

Question104

Let a, b ∈ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to :
[25-Jun-2022-Shift-2]

Options:

A. 37

B. 58

C. 68

D. 92

Answer: B

Solution:

Solution:
$ax^2 - 2bx + 15 = 0$ has repeated root so $b^2 = 15a$ and $\alpha = \frac{15}{b}$
$\because \alpha$ is a root of $x^2 - 2bx + 21 = 0$
So $\frac{225}{b^2} = 9 \Rightarrow b^2 = 25$
Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4b^2 - 42 = 100 - 42 = 58$

Question105

The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is ____
[26-Jun-2022-Shift-1]

Answer: 36

Solution:

Solution:

$x^4 - 3x^3 - x^2 - x^2 + 3x + 1 = 0$

$(x^2 - 1)(x^2 - 3x - 1) = 0$

Let the root of $x^2 - 3x - 1 = 0$ be α and β and other two roots of given equation are 1 and -1

So sum of cubes of roots

$= 1^3 + (-1)^3 + \alpha^3 + \beta^3$

$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (3)^3 - 3(-1)(3)$

$= 36$

Question106

If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to____
[27-Jun-2022-Shift-1]

Answer: 45

Solution:

Solution:
Let $e^x = t$ then equation reduces to
$t^2 - 11t - \frac{45}{t} + \frac{81}{2} = 0$
$\Rightarrow 2t^3 - 22t^2 + 81t - 45 = 0.....$ (i)
if roots of $e^{2xt} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ are α, β, γ then roots of (i) will be $e^{\alpha_1}e^{\alpha_2}e^{\alpha3}$ using product of roots
$e^{\alpha_1 + \alpha_2 + \alpha_3} = 45$
$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 = \ln 45 \Rightarrow p = 45$

Question107

Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\lambda\sqrt{3} = 0$, $\lambda > 0$. If $\beta + \gamma = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to___
[27-Jun-2022-Shift-2]

Answer: 98

Solution:

Solution:
$\because \alpha, \beta$ are roots of $x^2 - 4\lambda x + 5 = 0$
$\therefore \alpha + \beta = 4\lambda$ and $\alpha\beta = 5$
Also, α, γ are roots of
$x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\sqrt{3}\lambda = 0, \lambda > 0$

$\therefore \alpha+\gamma=3\sqrt{2}+2\sqrt{3},\ \alpha\gamma=7+3\sqrt{3}\lambda$
$\because \alpha$ is common root
$\therefore \alpha^2-4\lambda\alpha+5=0$
and $\alpha^2-(3\sqrt{2}+2\sqrt{3})\alpha+7+3\sqrt{3}\lambda=0$
From (i) - (ii) : we get $\alpha=\dfrac{2+3\sqrt{3}\lambda}{3\sqrt{2}+2\sqrt{3}-4\lambda}$
$\because \beta+\gamma=3\sqrt{2}$
$\therefore 4\lambda+3\sqrt{2}+2\sqrt{3}-2\alpha=3\sqrt{2}$
$\Rightarrow 3\sqrt{2}=4\lambda+3\sqrt{2}+2\sqrt{3}-\dfrac{4+6\sqrt{3}\lambda}{3\sqrt{2}+2\sqrt{3}-4\lambda}$
$\Rightarrow 8\lambda^2+3(\sqrt{3}+2\sqrt{2})\lambda-4-3\sqrt{6}=0$
$\therefore \lambda=\dfrac{6\sqrt{2}-3\sqrt{2}\pm\sqrt{9(11-4\sqrt{6})+32(4+3\sqrt{6})}}{16}$
$\therefore \lambda=\sqrt{2}$
$\therefore (\alpha+2\beta+\gamma)^2=(\alpha+\beta+\beta+\gamma)^2$
$=(4\sqrt{2}+3\sqrt{2})^2$
$=(7\sqrt{2})^2=98$

Question108

The number of real solutions of the equation $e^{4x}+4e^{3x}-58e^{2x}+4e^x+1=0$ is_____
[28-Jun-2022-Shift-1]

Answer: 2

Solution:

Solution:
Dividing by e^{2x}
$e^{2x}+4e^x-58+4e^{-x}+e^{-2x}=0$
$\Rightarrow (e^x+e^{-x})^2+4(e^x+e^{-x})-60=0$
Let $e^x+e^{-x}=t\in[2,\infty)$
$\Rightarrow t^2+4t-60=0$
$\Rightarrow t=6$ is only possible solution
$e^x+e^{-x}=6\Rightarrow e^{2x}-6e^x+1=0$
Let $e^x=p$
$p^2-6p+1=0$
$\Rightarrow p=\dfrac{3+\sqrt{5}}{2}$ or $\dfrac{3-\sqrt{5}}{2}$
So $x=\ln\left(\dfrac{3+\sqrt{5}}{2}\right)$ or $\ln\left(\dfrac{3-\sqrt{5}}{2}\right)$

Question109

Let $f(x)$ be a quadratic polynomial such that $f(-2)+f(3)=0$. If one of the roots of $f(x)=0$ is -1, then the sum of the roots of $f(x)=0$ is equal to :
[28-Jun-2022-Shift-2]

Options:

A. $\frac{11}{3}$

B. $\frac{7}{3}$

C. $\frac{13}{3}$

D. $\frac{14}{3}$

Answer: A

Solution:

Solution:
$\because x=-1$ be the roots of $f(x)=0$
$\therefore$ Let $f(x)=A(x+1)(x-1)$...... (i)
Now, $f(-2)+f(3)=0$
$\Rightarrow A[-1(-2-b)+4(3-b)]=0$
$b=\frac{14}{3}$
$\therefore$ Second root of $f(x)=0$ will be $\frac{14}{3}$
$\therefore$ Sum of roots $=\frac{14}{3}-1=\frac{11}{3}$

Question110

Let α be a root of the equation $1+x^2+x^4=0$. Then, the value of $\alpha^{1011}+\alpha^{2022}-\alpha^{3033}$ is equal to : [29-Jun-2022-Shift-2]

Options:

A. 1

B. α

C. $1+\alpha$

D. $1+2\alpha$

Answer: A

Solution:

Solution:
Given, α is a root of the equation $1+x^2+x^4=0$
$\therefore\alpha$ will satisfy the equation.
$\therefore 1+\alpha^2+\alpha^4=0$
$\alpha^2=\frac{-1\pm\sqrt{1-4}}{2}$
$=\frac{-1\pm\sqrt{3}i}{2}$
$\therefore\alpha^2=\omega\mathrm{ar}\omega^2$
Now,
$\alpha^{1011}+\alpha^{2022}-\alpha^{3033}$
$=\alpha\cdot(\alpha^2)^{505}+(\alpha^2)^{1011}-\alpha\cdot(\alpha^2)^{1516}$
$=\alpha(\omega)^{505}+(\omega)^{1011}-\alpha\cdot(\omega)^{1516}$
$=\alpha\cdot(\omega^3)^{168}\cdot\omega+(\omega^3)^{337}-\alpha\cdot(\omega^3)^{505}\cdot\omega$
$=\alpha\omega+1-\alpha\omega$
$=1$

Question111

Let x, y > 0. If $x^3y^2=2^{15}$, then the least value of $3x+2y$ is [24-Jun-2022-Shift-2]

Options:

A. 30

B. 32

C. 36

D. 40

Answer: D

Solution:

Solution:

$x, y > 0$ and $x^3y^2 = 2^{15}$

Now, $3x + 2y = (x + x + x) + (y + y)$

So, by $A \cdot M \geq$ G.M inequality

$\frac{3x+2y}{5} \geq \sqrt[5]{x^3 \cdot y^2}$

$\therefore 3x + 2y \geq 5\sqrt[5]{2^{15}} \geq 40$

$\therefore$ Least value of $3x + 4y = 40$

Question112

Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$ is equal to____
[26-Jun-2022-Shift-2]

Answer: 4

Solution:

Solution:

$\because p + q = 3 \ldots\ldots$ (i)

and $p^4 + q^4 = 369 \ldots\ldots$ (ii)

$\{(p+q)^2 - 2pq\}^2 - 2p^2q^2 = 369$

or $(9 - 2pq)^2 - 2(pq)^2 = 369$

or $(pq)^2 - 18pq - 144 = 0$

$\therefore pq = -6$ or 24

But $pq = 24$ is not possible

$\therefore pq = -6$

Hence, $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2} = \left(\frac{pq}{p+q}\right)^2 = (-2)^2 = 4$

Question113

If α, β, γ, δ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to :
[25-Jul-2022-Shift-1]

Options:

A. −4

B. −1

C. 1

D. 4

Answer: B

Solution:

Solution:
When, $x^5 = 1$
then $x^5 - 1 = 0$
$\Rightarrow (x-1)(x^4 + x^3 + x^2 + x + 1) = 0$
Given, $x^4 + x^3 + x^2 + x + 1 = 0$ has roots α, β, γ and 8 .
$\therefore$ Roots of $x^5 - 1 = 0$ are $1, \alpha, \beta, \gamma$ and 8
We know, Sum of p^{th} power of n^{th} roots of unity $= 0$. (If p is not multiple of n) or n (If p is multiple of n)
$\therefore$ Here, Sum of p^{th} power of n^{th} roots of unity
Here, $p = 2021$, which is not multiple of 5 .
$\therefore 1^{2021} + \alpha^{2021} + \beta^{2021} + \gamma^{2021} + 8^{2021} = 0$
$\Rightarrow \alpha^{2021} + \beta^{2021} + \gamma^{2021} + 8^{2021} = -1$

Question114

For $n \in N$, let $S_n = \left\{ z \in C : \left| z - 3 + 2i \right| = \frac{n}{4} \right\}$ and $T_n = \left\{ z \in C : \left| z - 2 + 3i \right| = \frac{1}{n} \right\}$. Then the number of elements in the set $\{n \in N : S_n \cap T_n = \varphi\}$ is : [25-Jul-2022-Shift-1]

Options:

A. 0

B. 2

C. 3

D. 4

Answer: D

Solution:

Solution:
$S_n = \left\{ z \in C : \left| z - 3 + 2i \right| = \frac{n}{4} \right\}$ represents a circle with centre $C_1(3, -2)$ and radius $r_1 = \frac{n}{4}$
Similarly T_n represents circle with centre $C_2(2, -3)$ and radius $r_2 = \frac{1}{n}$
As $S_n \cap T_n = \varphi$
$C_1C_2 > r_1 + r_2$ OR $C_1C_2 < | r_1 - r_2 |$
$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$ OR $\sqrt{2} < \left| \frac{n}{4} - \frac{1}{n} \right|$
$n = 1, 2, 3, 4$ n may take infinite values

Question115

For $z \in C$ if the minimum value of $(|z - 3\sqrt{2}| + | z - p\sqrt{2}i|)$ is $5\sqrt{2}$, then a value Question: of p is [25-Jul-2022-Shift-2]

Options:

A. 3

B. $\frac{7}{2}$

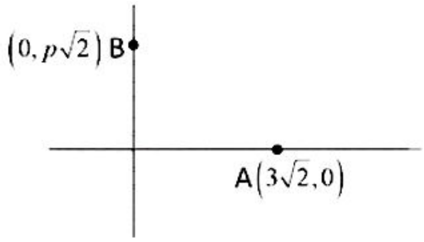
C. 4

D. $\frac{9}{2}$

Answer: C

Solution:

Solution:



It is sum of distance of z from $(3\sqrt{2}, 0)$ and $(0, p\sqrt{2})$ For minimising, z should lie on AB and $AB = 5\sqrt{2}$
$(AB)^2 = 18 + 2p^2$
$p = \pm 4$

Question116

Let 0 be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true? [26-Jul-2022-Shift-1]

Options:

A. $\arg z_2 = \pi - \tan^{-1} 3$

B. $\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$

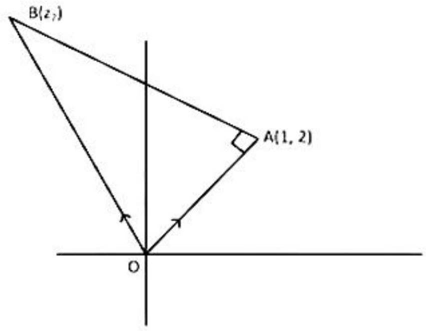
C. $z_2| = \sqrt{10}$

D. $2z_1 - z_2| = 5$

Answer: D

Solution:

Solution:



$\frac{z_2 - 0}{(1 + 2i) - 0} = \frac{|OB|}{|OA|} e^{\frac{i\pi}{4}}$
$\Rightarrow \frac{z_2}{1 + 2i} = \sqrt{2} e^{\frac{i\pi}{4}}$
OR $z_2 = (1 + 2i)(1 + i)$
$= -1 + 3i$
$\arg z_2 = \pi - \tan^{-1} 3$
$|z_2| = \sqrt{10}$
$z_1 - 2z_2 = (1 + 2i) + 2 - 6i = 3 - 4i$
$\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$
$|2z_1 - z_2| = |2 + 4i + 1 - 3i| = |3 + i| = \sqrt{10}$

Question117

If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then
[26-Jul-2022-Shift-2]

Options:

A. $x + 2y - 4 = 0$

B. $x^2 + y - 4 = 0$

C. $x + 2y + 4 = 0$

D. $x^2 - y + 3 = 0$

Answer: C

Solution:

Solution:
$|z - i| = |z + 5i|$
So, z lies on $\perp^r$ bisector of $(0, 1)$ and $(0, -5)$
i.e., line $y = -2$
as $|z| = 2$
$\Rightarrow z = -2i$
$x = 0$ and $y = -2$
so, $x + 2y + 4 = 0$

Question118

Let the minimum value v_0 of $v = |z|^2 + |z - 3|^2 + |z - 6i|^2$, $z \in C$ is attained at $z = z_0$. Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. 1000

B. 1024

C. 1105

D. 1196

Answer: A

Solution:

Solution:
Let $z = x + iy$
$v = x^2 + y^2 + (x - 3)^2 + y^2 + x^2 + (y - 6)^2$
$= (3x^2 - 6x + 9) + (3y^2 - 12y + 36)$
$= 3(x^2 + y^2 - 2x - 4y + 15)$
$= 3[(x - 1)^2 + (y - 2)^2 + 10]$
v_{min} at $z = 1 + 2i = z_0$ and $v_0 = 30$
so $|2(1 + 2i)^2 - (1 - 2i)^3 + 3|^2 + 900$
$= |2(-3 + 4i) - (1 - 8i^3 - 6i(1 - 2i) + .3|^2 + 900.$
$= |-6 + 8i - (1 + 8i - 6i - 12) + 3|^2 + 900$
$= |8 + 6i|^2 + 900$

$= |8 + 6i|^2 + 900$
$= 1000$

Question119

Let $S = \{z \in C : z^2 + \bar{z} = 0\}$. Then $\sum\limits_{z \in S} (\mathrm{Re}(z) + \mathrm{Im}(z))$ is equal to ________.
[27-Jul-2022-Shift-1]

Answer: 0

Solution:

Solution:
$\because z^2 + \bar{z} = 0$
Let $z = x + iy$
$\therefore x^2 - y^2 + 2ixy + x - iy = 0$
$(x^2 - y^2 + x) + i(2xy - y) = 0$
$\therefore x^2 + y^2 = 0$ and $(2x - 1)y = 0$
if $x = +\frac{1}{2}$ then $y = \pm\frac{\sqrt{3}}{2}$
And if $y = 0$ then $x = 0, -1$
$\therefore z = 0 + 0i, -1 + 0i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$
$\therefore \sum (R_e(z) + m(z)) = 0$

Question120

Let S be the set of all (α, β), $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum\limits_{(\alpha,\beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}}\right)$ is equal to :
[27-Jul-2022-Shift-2]

Options:

A. 3

B. 3i

C. 1

D. $2 - i$

Answer: C

Solution:

Solution:
$\because \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary
$\therefore \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} + \frac{1 + i\sin\alpha}{1 - 2i\sin\alpha} = 0$
$\Rightarrow 1 - 2\sin^2\alpha = 0$
$\therefore \alpha = \frac{5\pi}{4}, \frac{7\pi}{4}$
and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real
$\frac{1 + i\cos\beta}{1 - 2i\cos\beta} - \frac{1 - i\cos\beta}{1 + 2i\cos\beta} = 0$
$\Rightarrow \cos\beta = 0$
$\therefore \beta = \frac{3\pi}{2}$
$\therefore S = \left\{ \left(\frac{5\pi}{2}, \frac{3\pi}{2}\right), \left(\frac{7\pi}{4}, \frac{3\pi}{2}\right) \right\}$
$Z_{\alpha\beta} = 1 - i$ and $Z_{\alpha\beta} = -1 - i$
$\therefore \sum\limits_{(\alpha,\beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{i\overline{Z}_{\alpha\beta}}\right) = i(-2i) + \frac{1}{i}\left[\frac{1}{1 + i} + \frac{1}{-1 + i}\right]$
$= 2 + \frac{1}{i}\frac{2i}{-2} = 1$

Question121

Let $S_1 = \left\{ z_1 \in C : z_1 - 3 \right| = \frac{1}{2} \right\}$ and $S_2 = \{z_2 \in C : z_2 - |z_2 + 1|| = z_2 + |z_2 - 1||\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $z_2 - z_1|$ is :
[28-Jul-2022-Shift-1]

Options:

A. 0

B. $\frac{1}{2}$

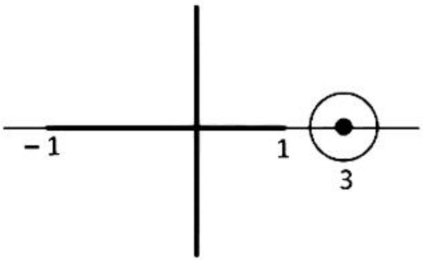
C. $\frac{3}{2}$

D. $\frac{5}{2}$

Answer: C

Solution:

Solution:
$\because |Z_2 + |Z_2 - 1||^2 = |Z_2 - |Z_2 + 1||^2$
$\Rightarrow (Z_2 + |Z_2 - 1|)(\overline{Z}_2 + |Z_2 - 1|) = (Z_2 - |Z_2 + 1|)(\overline{Z}_2 - |Z_2 + 1|)$
$\Rightarrow Z_2(|Z_2 - 1| + |Z_2 + 1|) + \overline{Z}_2(|Z_2 - 1| + |Z_2 + 1|) = |Z_2 + 1|^2 - |Z_2 - 1|^2$
$\Rightarrow (Z_2 + \overline{Z}_2)(|Z_2 + 1| + |Z_2 - 1|) = 2(Z_2 + \overline{Z}_2)$
$\Rightarrow$ Either $Z_2 + \overline{Z}_2 = 0$ or $|Z_2 + 1| + |Z_2 - 1| = 2$
So, Z_2 lies on imaginary axis or on real axis within $[-1, 1]$
Also $|Z_1 - 3| = \frac{1}{2} \Rightarrow Z_1$ lies on the circle having center 3 and radius $\frac{1}{2}$.



Clearly $|Z_1 - Z_2|_{min} = \frac{3}{2}$

Question122

Let $z = a + ib$, $b \neq 0$ be complex numbers satisfying $z^2 = \overline{z} \cdot 2^{1-|z|}$. Then the least value of $n \in N$, such that $z^n = (z+1)^n$, is equal to ________.
[28-Jul-2022-Shift-2]

Answer: 6

Solution:

Solution:
$\because z^2 = \overline{z} \cdot 2^{1-|z|}$...... (1)
$\Rightarrow |z|^2 = |\overline{z}| \cdot 2^{1-|z|}$
$\Rightarrow |z| = 2^{1-|z|}$
$\because b \neq 0 \Rightarrow |z| \neq 0$
$\therefore |z| = 1$...... (2)
$\because z = a + ib$ then $\sqrt{a^2 + b^2} = 1$...... (3)

Now again from equation (1), equation (2), equation (3) we get :
$a^2 - b^2 + i2ab = (a - ib)2^0$
$\therefore a^2 - b^2 = a$ and $2ab = -b$
$\therefore a = -\frac{1}{2}$ and $b = \pm\frac{\sqrt{3}}{2}$
$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ or $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
$z^n = (z + 1)^n \Rightarrow \left(\frac{z+1}{z}\right)^n = 1$
$\left(1 + \frac{1}{z}\right)^n = 1$
$\left(\frac{1+\sqrt{3}i}{2}\right) = 1$, then minimum value of n is 6 .

Question123

If $z = 2 + 3i$, then $z^5 + \left(\bar{z}\right)^5$ is equal to :

[29-Jul-2022-Shift-1]

Options:

A. 244

B. 224

C. 245

D. 265

Answer: A

Solution:

Solution:
$z = (2 + 3i)$
$\Rightarrow z^5 = (2 + 3i)((2 + 3i)^2)^2$
$= (2 + 3i)(-5 + 12i)^2$
$= (2 + 3i)(-119 - 120i)$
$= -238 - 240i - 357i + 360$
$= 122 - 597i$
$\bar{z}^5 = 122 + 597i$
$z^5 + \bar{z}^5 = 244$

Question124

If $z \neq 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of $|z|$ is:

[29-Jul-2022-Shift-2]

Options:

A. $\sqrt{2}$

B. 1

C. $\sqrt{2} - 1$

D. $\sqrt{2} + 1$

Answer: D

Solution:

Solution:

We know,

$||z_1|-|z_2|| \le |z_1+z_2| \le |z_1|+|z_2|$

$\therefore \left||z|-\frac{1}{|z|}\right| \le \left|z-\frac{1}{z}\right|$

$\Rightarrow \left||z|-\frac{1}{|z|}\right| \le 2 \left[\text{ Given } \left|z-\frac{1}{z}\right|=2\right]$

$\Rightarrow \left|\frac{|z|^2-1}{|z|}\right| \le 2$

$\Rightarrow -2 \le \frac{|z|^2-1}{|z|} \le 2$

$\therefore \frac{|z|^2-1}{|z|} \le 2$

$\Rightarrow |z|^2-1 \le 2|z|$

$\Rightarrow |z|^2-2|z|-1 \le 0$

$\Rightarrow |z|^2-2|z|+1-2 \le 0$

$\Rightarrow (|z|-1)^2-2 \le 0$

$\Rightarrow -\sqrt{2} \le |z|-1 \le \sqrt{2}$

$\Rightarrow 1-\sqrt{2} \le |z| \le 1+\sqrt{2} \ldots\ldots (1)$

or

$-2 \le \frac{|z|^2-1}{|z|}$

$\Rightarrow |z|^2-1 \le -2|z|$

$\Rightarrow |z|^2+2|z|-1 \le 0$

$\Rightarrow |z|^2+2|z|+1-2 \le 0$

$\Rightarrow (|z|+1)^2-2 \le 0$

$\Rightarrow -\sqrt{2} \le |z|+1 \le +\sqrt{2}$

$\Rightarrow -\sqrt{2}-1 \le |z| \le \sqrt{2}-1 \ldots\ldots$ (2)

From (1) and (2) we get,

Maximum value of $|z|=\sqrt{2}+1$ and minimum value of $|z|=-\sqrt{2}-1$

Question125

Let $S=\{z=x+iy: |z-1+i| \ge z|, |z|<2, |z+i|=|z-1|\}$. Then the set of all values of x, for which $u=2x+iy \in S$ for some $y \in \mathbb{R}$, is
[29-Jul-2022-Shift-2]

Options:

A. $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$

B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

C. $\left(-\sqrt{2}, \frac{1}{2}\right]$

D. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

Answer: B

Solution:

Solution:

Question126

If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2+\alpha x+\beta>0$, for all $x \in R$, is :
[25-Jul-2022-Shift-1]

Options:

A. $\frac{17}{36}$

B. $\frac{4}{9}$

C. $\frac{1}{2}$

D. $\frac{19}{36}$

Answer: A

Solution:

Solution:
For $x^2 + \alpha x + \beta > 0 \ \forall x \in R$ to hold, we should have $\alpha^2 - 4\beta < 0$
If $\alpha = 1$, β can be $1, 2, 3, 4, 5, 6$ i.e., 6 choices
If $\alpha = 2$, β can be $2, 3, 4, 5, 6$ i.e., 5 choices
If $\alpha = 3$, β can be $3, 4, 5, 6$ i.e., 4 choices
If $\alpha = 4$, β can be 5 or 6 i.e., 2 choices
If $\alpha = 6$, No possible value for β i.e., 0 choices
Hence total favourable outcomes
$= 6 + 5 + 4 + 2 + 0 + 0$
$= 17$
Total possible choices for α and $\beta = 6 \times 6 = 36$
Required probability $= \frac{17}{36}$

Question127

Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to
[25-Jul-2022-Shift-1]

Answer: 38

Solution:

Solution:
$\because$ Roots of $2ax^2 - 8ax + 1 = 0$ are $\frac{1}{p}$ and $\frac{1}{r}$ and roots of $6bx^2 + 12bx + 1 = 0$ are $\frac{1}{q}$ and $\frac{1}{s}$
Let $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ as $\alpha - 3\beta, \alpha - \beta, \alpha + \beta, \alpha + 3\beta$
So sum of roots $2\alpha - 2\beta = 4$ and $2\alpha + 2\beta = -2$
Clearly $\alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$
Now product of roots, $\frac{1}{p} \cdot \frac{1}{r} = \frac{1}{2a} = -5 \Rightarrow \frac{1}{a} = -10$
and $\frac{1}{q} \cdot \frac{1}{x} = \frac{1}{6b} = -8 \Rightarrow \frac{1}{b} = -48$
So, $\frac{1}{a} - \frac{1}{b} = 38$

Question128

If for some p, q, r $\in$ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2 + r^2}{p^2}$ is equal to ________.
[26-Jul-2022-Shift-1]

Answer: 272

Solution:

Solution:

$$(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$$

$\therefore \alpha + \beta > 0$ and $\alpha\beta > 0$

Also, it has a common root with $x^2 + 2x - 8 = 0$

$\therefore$ The common root between above two equations is 4 .

$\Rightarrow 16(p^2 + q^2) - 8q(p + r) + q^2 + r^2 = 0$

$\Rightarrow (16p^2 - 8pq + q^2) + (16q^2 - 8qr + r^2) = 0$

$\Rightarrow (4p - q)^2 + (4q - r)^2 = 0$

$\Rightarrow q = 4p$ and $r = 16p$

$\therefore \dfrac{q^2 + r^2}{p^2} = \dfrac{16p^2 + 256p^2}{p^2} = 272$

Question129

The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is ____________. [26-Jul-2022-Shift-1]

Answer: 3

Solution:

Solution:

$x^8 - x^7 - x^6 + x^5 + 3x^4 - 4x^3 - 2x^2 + 4x - 1 = 0$

$\Rightarrow x^7(x - 1) - x^5(x - 1) + 3x^3(x - 1) - x(x^2 - 1) + 2x(1 - x) + (x - 1) = 0$

$\Rightarrow (x - 1)(x^7 - x^5 + 3x^3 - x(x + 1) - 2x + 1) = 0$

$\Rightarrow (x - 1)(x^7 - x^5 + 3x^3 - x^2 - 3x + 1) = 0$

$\Rightarrow (x - 1)(x^5(x^2 - 1) + 3x(x^2 - 1) - 1(x^2 - 1)) = 0$

$\Rightarrow (x - 1)(x^2 - 1)(x^5 + 3x - 1) = 0$

$\therefore x = \pm 1$ are roots of above equation and $x^5 + 3x - 1$ is a monotonic term hence vanishs at exactly one value of x other than 1 or -1.

$\therefore 3$ real roots.

Question130

The minimum value of the sum of the squares of the roots of $x^2 + (3 - a)x + 1 = 2a$ is: [26-Jul-2022-Shift-2]

Options:

A. 4

B. 5

C. 6

D. 8

Answer: C

Solution:

Solution:

$$x^2 + (3 - a)x + 1 = 2a \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$\alpha + \beta = a - 3,\ \alpha\beta = 1 - 2a$
$\Rightarrow \alpha^2 + \beta^2 = (a - 3)^2 - 2(1 - 2a)$
$= a^2 - 6a + 9 - 2 + 4a$
$= a^2 - 2a + 7$
$= (a - 1)^2 + 6$
So, $\alpha^2 + \beta^2 \geq 6$

Question131

Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a + b - c)$ is __________.
[26-Jul-2022-Shift-2]

Options:

A. 12

B. 13

C. 14

D. 16

Answer: A

Solution:

Solution:
Abscissae of PQ are roots of $x^2 - 4x - 6 = 0$
Ordinates of PQ are roots of $y^2 + 2y - 7 = 0$
and PQ is diameter
$\Rightarrow$ Equation of circle is
$x^2 + y^2 - 4x + 2y - 13 = 0$
But, given $x^2 + y^2 + 2ax + 2by + c = 0$
By comparison $a = -2,\ b = 1,\ c = -13$
$\Rightarrow a + b - c = -2 + 1 + 13 = 12$

Question132

If α, β are the roots of the equation $x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right) + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$ then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :
[27-Jul-2022-Shift-2]

Options:

A. $3x^2 - 20x - 12 = 0$

B. $3x^2 - 10x - 4 = 0$

C. $3x^2 - 10x + 2 = 0$

D. $3x^2 - 20x + 16 = 0$

Answer: B

Solution:

Solution:

$3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}} = 3^{\sqrt{\log_3 5}} - \left(3^{\log_3 5}\right)^{\sqrt{\log_5 3}}$

$3^{(\log_3 5)\frac{1}{3}} - 5^{(\log_5 3)\frac{2}{3}} = 5^{(\log_5 3)\frac{2}{3}} - 5^{(\log_5 3)\frac{2}{3}} = 0$

Note: In the given equation ' x ' is missing.

So

$x^2 - 5x + 3(-1) = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$

$\alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta}$

$= 5 - \frac{5}{3} = \frac{10}{3}$

$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = 2 + \alpha\beta + \frac{1}{\alpha\beta} = 2 - 3 - \frac{1}{3} = \frac{-4}{3}$

So Equation must be option (B).

Question133

The sum of all real values of x for which $\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} - \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$ is equal to ________.
[28-Jul-2022-Shift-1]

Answer: 6

Solution:

Solution:

$\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$

$\Rightarrow \frac{3x^2 - 9x + 17}{5x^2 - 7x + 19} = \frac{x^2 + 3x + 10}{3x^2 + 5x + 12}$

$\frac{-2x^2 - 2x - 2}{5x^2 - 7x + 19} = \frac{-2x^2 - 2x - 2}{3x^2 + 5x + 12}$

Either $x^2 + x + 1 = 0$ or No real roots $\Rightarrow 5x^2 - 7x + 19 = 3x^2 + 5x + 12$

$2x^2 - 12x + 7 = 0$

sum of roots $= 6$

Question134

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1$, $\frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :
[28-Jul-2022-Shift-2]

Options:

A. non-real complex numbers

B. real and both negative

C. real and both positive

D. real and exactly one of them is positive

Answer: B

Solution:

Solution:

$\alpha+\beta=\sqrt{2}, \alpha\beta=\sqrt{6}$
$\frac{1}{\alpha^2}+1+\frac{1}{\beta^2}+1=2+\frac{\alpha^2+\beta^2}{6}$
$=2+\frac{2-2\sqrt{6}}{6}=-a$
$\left(\frac{1}{\alpha^2}+1\right)\left(\frac{1}{\beta^2}+1\right)=1+\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\alpha^2\beta^2}$
$=\frac{7}{6}+\frac{2-2\sqrt{6}}{6}=b$
$\Rightarrow a+b=\frac{-5}{6}$
So, equation is $x^2+\frac{17x}{6}+\frac{7}{6}=0$
OR $6x^2+17x+7=0$
Both roots of equation are - ve and distinct

Question135

Let $f(x)=ax^2+bx+c$ be such that $f(1)=3$, $f(-2)=\lambda$ and $f(3)=4$. If $f(0)+f(1)+f(-2)+f(3)=14$, then λ is equal to :
[28-Jul-2022-Shift-2]

Options:

A. -4

B. $\frac{13}{2}$

C. $\frac{23}{2}$

D. 4

Answer: D

Solution:

Solution:
$f(1)=a+b+c=3.....$ (i)
$f(3)=9a+3b+c=4....$ (ii)
$f(0)+f(1)+f(-2)+f(3)=14$
OR $c+3+(4a-2b+c)+4=14$
OR $4a-2b+2c=7....$ (iii)
From (i) and (ii) $8a+2b=1....$ (iv)
From (iii) $-(2)\times$ (i)
$\Rightarrow 2a-4b=1.....(v)$
From (iv) and (v) $a=\frac{1}{6}, b=\frac{-1}{6}$ and $c=3$
$f(-2)=4a-2b+c$
$=\frac{4}{6}+\frac{2}{6}+3=4$

Question136

Let $\alpha, \beta(\alpha>\beta)$ be the roots of the quadratic equation $x^2-x-4=0$. If $P_n=\alpha^n-\beta^n, n\in N$, then $\frac{P_{15}P_{16}-P_{14}P_{16}-P_{15}^2+P_{14}P_{15}}{P_{13}P_{14}}$ is equal to
[29-Jul-2022-Shift-2]

Answer: 16

Solution:

Solution:

α and β are the roots of the quadratic equation $x^2 - x - 4 = 0$.
$\therefore \alpha$ and β are satisfy the given equation.
$\alpha^2 - \alpha - 4 = 0$
$\Rightarrow \alpha^{n+1} - \alpha^n - 4\alpha^{n-1} = 0 \ldots\ldots (i)$
and $\beta^2 - \beta - 4 = 0$
$\Rightarrow \beta^{n+1} - \beta^n - 4\beta^{n-1} = 0 \quad \cdots\cdots (2)$Substituting (2) from (1), we get,
$(\alpha^{n+1} - \beta^{n+1}) - (\alpha^n - \beta^n) - 4(\alpha^{n-1} - \beta^{n-1}) = 0$
$\Rightarrow P_{n+1} - P_n - 4P_{n-1} = 0$
$\Rightarrow P_{n+1} = P_n + 4P_{n-1}$
$\Rightarrow P_{n+1} - P_n = 4P_{n-1}$
For $n = 14, P_{15} - P_{14} = 4P_{13}$
For $n = 15, P_{16} - P_{15} = 4P_{14}$
Now, $\dfrac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^{\ 2} + P_{14}P_{15}}{P_{13}P_{14}}$
$= \dfrac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$
$= \dfrac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}}$
$= \dfrac{(4P_{13})(4P_{14})}{P_{13}P_{14}}$
$= 16$

Question137

Let $S = \left\{ x \in [-6, 3] - \{-2, 2\} : \frac{|x+3| - 1}{|x| - 2} \geq 0 \right\}$ and $T = \{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\}$
Then the number of elements in $S \cap T$ is :
[28-Jul-2022-Shift-2]

Options:

A. 7

B. 5

C. 4

D. 3

Answer: D

Solution:

Solution:
$|x^2| - 7|x| + 9 \leq 0$
$\Rightarrow |x| \in \left[\frac{7 - \sqrt{13}}{2}, \frac{7 + \sqrt{13}}{2} \right]$
As $x \in Z$
So, x can be $\pm 2, \pm 3, \pm 4, \pm 5$
Out of these values of x,
$x = 3, -4, -5$
satisfy S as well
$n(S \cap T) = 3$

Question138

Let $i = \sqrt{-1}$. If $\frac{(-1 + i\sqrt{3})^{21}}{(1 - i)^{24}} + \frac{(1 + i\sqrt{3})^{21}}{(1 + i)^{24}} = k$ and $n = [|k|]$ be the greatest integral part of $|k|$. Then, $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$ is equal to
[2021, 24 Feb. Shift-II]

Answer: 310

Solution:

Solution:
Given, $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$
$\because\ -1+i\sqrt{3}=2e^{i2\pi/3}$
$1+i\sqrt{3}=2e^{i\pi/3}$
$1-i=\sqrt{2}e^{-i\pi/4}$
$1+i=\sqrt{2}e^{i\pi/4}$
Now, $\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{(\sqrt{2}e^{-i\pi/4})^{24}}+\frac{(2e^{i\pi/3})^{21}}{(\sqrt{2}e^{i\pi/4})^{24}}$
$=\frac{2^{21}\cdot e^{i14\pi}}{2^{12}\cdot e^{-i6\pi}}+\frac{2^{21}\cdot e^{i7\pi}}{2^{12}\cdot e^{i6\pi}}$
$=2^9\cdot e^{i20\pi}+2^9\cdot e^{i\pi}$
$=2^9(1)+2^9(-1)$
$\Rightarrow 2^9-2^9=0=k$ (given)
$\therefore\ n=[|k|]=[101]=0$
Now, $\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)\ \ [\because n=0]$
$=[5^2+6^2+7^2+8^2+9^2+10^2]$
$-[5+6+7+8+9+10]$
$=[(1^2+2^2+3^2+\ldots+10^2)-$
$(1^2+2^2+\ldots+4^2)]-[(1+2+3+\ldots+10)$
$-(1+2+3+4)]$
$=\left[\frac{10\times11\times21}{6}-\frac{4\times5\times9}{6}\right]-\left[\frac{10\times11}{2}-\frac{4\times5}{2}\right]$
$=(385-30)-(55-10)$
$=385-45=310$

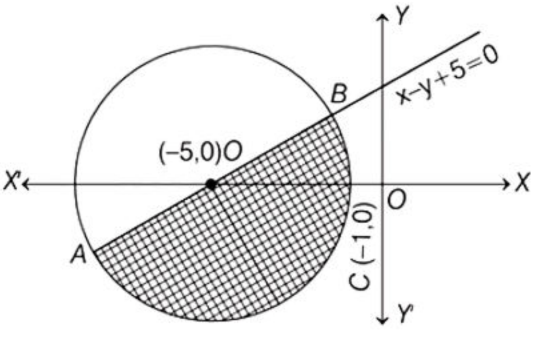
Question139

Let z be those complex numbers which satisfy $|z+5|\le 4$ and $z(1+i)+\bar{z}(1-i)\ge -10$, $i=\sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is
[2021, 26 Feb. Shift-II]

Answer: 48

Solution:

Solution:
Given, $|z+5|\le 4$, which is equation of circle.
$|z+5|\le 4$
$\Rightarrow\ (x+5)^2+y^2\le 16$
and $z(1+i)+\bar{z}(1-i)\ge -10$
$\Rightarrow\ (z+\bar{z})+i(z-\bar{z})\ge -10$
$\Rightarrow\ x-y+5\ge 0$
From Eqs. (i) and (ii), region bounded by inequalities are



Now, $|z+1|^2 = |z-(-1)|^2$
Maximum value of $|z+1|^2$ will be equal to $(AC)^2$.
Now, $(x+5)^2 + y^2 = 16$
and $x - y + 5 = 0$
Given, $y = \pm 2\sqrt{2}$
and $x = \pm 2\sqrt{2} - 5$
∴ Coordinates are
$cA(-2\sqrt{2}-5, -2\sqrt{2})$
$B(2\sqrt{2}-5, 2\sqrt{2})$
$C(-1, 0)$
Then,
$AC^2 = (2\sqrt{2}+4)^2 + (2\sqrt{2})^2$
$= 32 + 16\sqrt{2}$
Given, that maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$
$\Rightarrow \alpha + \beta\sqrt{2} = 32 + 16\sqrt{2}$
$\Rightarrow \alpha = 32, \beta = 16$
$\therefore \alpha + \beta = 32 + 16 = 48$

Question140

Let the lines $(2-i)z = (2+i)\bar{z}$ and $(2+i)z + (i-2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is
[2021, 25 Feb. Shift-1]

Options:

A. $\frac{3}{\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $3\sqrt{2}$

D. $\frac{3}{2\sqrt{2}}$

Answer: D

Solution:

Solution:
Given, $(2-i)z = (2+i)\bar{z}$
Let $z = x + iy$, then $\bar{z} = x - iy$
$\Rightarrow (2-i)(x+iy) = (2+i)(x-iy)$
$\Rightarrow 2x - ix + 2iy + y = 2x + ix - 2iy + y$
$\Rightarrow 2ix - 4iy = 0$
$\therefore$ Equation of line $L_1 \Rightarrow x - 2y = 0 \quad \cdots\cdots (i)$
Also, $(2+i)z + (i-2)\bar{z} - 4i = 0$
$\Rightarrow (2+i)(x+iy) + (i-2)(x-iy) - 4i = 0$
$\Rightarrow 2x + ix + 2iy - y + ix - 2x + y$
$+2iy - 4i = 0$
$\Rightarrow 2ix + 4iy - 4i = 0$
∴ Equation of line $L_2 \Rightarrow x + 2y - 2 = 0$... (ii)
From Eqs. (i) and (ii),
$4y = 2$ or $y = 1/2$ and $x = 1$
Hence, centre $= (1, 1/2)$
Equation of third line
$L_3 \Rightarrow iz + \bar{z} + 1 + i = 0$
$\Rightarrow i(x+iy) + (x-iy) + 1 + i = 0$
$\Rightarrow ix - y + x - iy + 1 + i = 0$
$\Rightarrow (x - y + 1) + i(x - y + 1) = 0$
∴ Radius = Distance of point $(1, 1/2)$ to the line $x - y + 1 = 0$

$$\therefore r = \frac{\left|1 - \frac{1}{2} + 1\right|}{\sqrt{1^2 + 1^2}} = \frac{3}{2\sqrt{2}}$$

Question141

Let α and β be two real numbers, such that $\alpha+\beta=1$ and $\alpha\beta=-1$. Let $p_n=(\alpha)^n+(\beta)^n$, $p_{n-1}=11$ and $p_{n+1}=29$, for some integer $n\geq 1$. Then, the value of $p_n^{\ 2}$ is
[2021, 26 Feb. Shift-III]

Answer: 324

Solution:

Solution:
Given that, $\alpha+\beta=1$, $\alpha\beta=-1$
Let α, β be roots of quadratic equation, then the quadratic equation be
$x^2-x-1=0$
Now, $\alpha^2-\alpha-1=0$
$\Rightarrow \alpha^2=\alpha+1 \quad \cdots\cdots(i)$
Similarly, $\beta^2=\beta+1 \quad \cdots\cdots(ii)$
Multiply α^{n-1} in Eq. (i), we get
$\alpha^{n+1}=\alpha^n+\alpha^{n-1} \quad \cdots\cdots(iii)$
Multiply β^{n-1} in Eq. (ii), we get
$\beta^{n+1}=\beta^n+\beta^{n-1} \quad \cdots\cdots(iv)$
Add Eqs. (iii) and (iv), we get
$\alpha^{n+1}+\beta^{n+1}=(\alpha^n+\beta^n)+(\alpha^{n-1}+\beta^{n-1})$
$p_{n+1}=p_n+p_{n-1}$
$29=p_n+11$
$\Rightarrow P_n=18$
$p_n^{\ 2}=(18)^2=324$

Question142

The number of solutions of the equation $\log_4(x-1)=\log_2(x-3)$ is
[2021, 26 Feb. Shift-1]

Answer: 1

Solution:

Solution:
$\log_4(x-1)=\log_2(x-3)$ (given)
$\Rightarrow \log_{2^2}(x-1)=\log_2(x-3)$
Using property of logarithm,
$\log_{b^c}a=\frac{1}{c}\log_b a$
$\Rightarrow \frac{1}{2}\log_2(x-1)=\log_2(x-3)$
$\Rightarrow \log_2(x-1)=2\log_2(x-3)$
$\Rightarrow \log_2(x-1)=\log_2(x-3)^2$
On comparing, $x-1=(x-3)^2$
or $x-1=x^2+9-6x$
$\Rightarrow x^2-7x+10=0$
$\Rightarrow x^2-5x-2x+10=0$
$\Rightarrow (x-5)(x-2)=0$
$\Rightarrow x=2,5$
$x=2$(rejected) as $x>1$
$\therefore x=5$ is only solution i.e. number of solution is 1.

Question143

Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is [2021, 25 Feb. Shift-II]

Options:

A. 4

B. 3

C. 2

D. 1

Answer: C

Solution:

Solution:
We have, $x^2 - 6x - 2 = 0$
Given, α and β are roots of above quadratic equation, then
$\alpha^2 - 6\alpha - 2 \;\; = 0$
$\beta^2 - 6\beta - 2 \;\; = 0$
Also, given $a_n = \alpha^n - \beta^n$, then

$$\frac{a_{10} - 2a_8}{3a_9}$$
$$= \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$$
$$= \frac{\alpha^{10} - 2\alpha^8 - \beta^{10} + 2\beta^8}{3(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)}$$

[from Eqs. (i) and (ii) $\alpha^2 - 2 = 6\alpha$, $\beta^2 - 2 = 6\beta$]

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)}$$
$$= \frac{6\alpha^9 - 6\beta^9}{3(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)}$$
$$= 2$$

Question144

If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to [2021, 25 Feb. Shift-II]

Options:

A. 3

B. −3

C. 7

D. -7

Answer: D

Solution:

Solution:
Given, root of $z^2 + \alpha z + \beta = 0$ is $1 - 2i$.
Since, it is quadratic equation and one root is complex in nature, its another root is complex conjugate.
∴ Two roots are $1 - 2i$ and $1 + 2i$.
Now, sum of roots $= -\frac{\alpha}{1} = -\alpha$
$= (1 - 2i) + (1 + 2i) = 2$
Gives, $\alpha = -2$

Product of roots $= \frac{\beta}{1} = \beta$
$= (1 - 2i)(1 + 2i) = 1 + 4 = 5$
Gives, $\beta = 5$
$\therefore\ \alpha - \beta = -2 - 5 = -7$

Question145

The integer ' k ', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is [2021, 25 Feb. Shift-1]

Options:

A. 3

B. 2

C. 0

D. 4

Answer: A

Solution:

Solution:
Given, $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$
Here, $a > 0$
$\therefore D < 0$
$\Rightarrow\ [2(3k - 1)]^2 - 4(8k^2 - 7) < 0$
$\Rightarrow\ 4(9k^2 + 1 - 6k) - 4(8k^2 - 7) < 0$
$\Rightarrow\ k^2 - 6k + 8 < 0$
$\Rightarrow\ (k - 4)(k - 2) < 0$
$\therefore$


$k \in (2, 4)$
$\therefore$ Required integer, $k = 3$

Question146

The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is [2021, 26 Feb. Shift-1]

Answer: 3

Solution:

Solution:
Given, $x^3 - 2x^2 + 2x - 1 = 0$
i.e. $(x^3 - 1) - (2x^2 - 2x) = 0$
$\Rightarrow (x - 1)(x^2 + x + 1) - 2x(x - 1) = 0$
$\Rightarrow\ (x - 1)(x^2 + x + 1 - 2x) = 0$
$\Rightarrow\ (x - 1)(x^2 - x + 1) = 0$
$\therefore x = 1$ and $x = \frac{-(-1) \pm \sqrt{1 - 4}}{2} = \frac{1 \pm \sqrt{3}i}{2}$
$\therefore$ Roots are $1, -\omega_1 - \omega^2$.
Then, sum of 162^{th} power of the roots
$= (1)^{162} + (-\omega)^{162} + (-\omega^2)^{162}$
$= 1 + \omega^{162} + \omega^{324}$
$= 1 + (\omega^3)^{54} + (\omega^3)^{108}$

$= 1 + (1)^{54} + (1)^{108}$ $[\because \omega^3 = 1]$
$= 1 + 1 + 1 = 3$

Question147

Let a, b, c be in an arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is [2021, 24 Feb. Shift-II]

Options:

A. $\frac{71}{256}$

B. $\frac{69}{256}$

C. $-\frac{69}{256}$

D. $-\frac{71}{256}$

Answer: D

Solution:

Solution:
Given, a, b, c are in AP.
(a, c), (2, b), (a, b) are vertices of triangle.
Centroid $= \left(\frac{10}{3}, \frac{7}{3}\right)$
α and β are the roots of equation $ax^2 + bx + 1 = 0$
$\because$a, b, c are in AP.
$\therefore\ 2b = a + c$
Centroid $= \left(\frac{a+2+a}{3}, \frac{c+b+b}{3}\right)$
$= \left(\frac{2a+2}{3}, \frac{c+2b}{3}\right) = \left(\frac{10}{3}, \frac{7}{3}\right)$
$\Rightarrow\ \frac{2a+2}{3} = \frac{10}{3}$ and $\frac{c+2b}{3} = \frac{7}{3}$
$\Rightarrow\ a = 4$
$\Rightarrow\ c + a + c = 7$ $[\because 2b = a + c]$
$\Rightarrow\ 2c = 7 - 4$ $[\because a = 4]$
$c = 3/2$
Also, $2b = a + c = 4 + \frac{3}{2}$
$\Rightarrow\ b = 11/4$
Now, α and β are roots of $ax^2 + bx + 1 = 0$
$\therefore\ \alpha + \beta = \frac{-b}{a} = \frac{-11/4}{4}$
$\Rightarrow\ \alpha + \beta = \frac{-11}{16}$
$\Rightarrow\ \alpha\beta = \frac{1}{a} = \frac{1}{4}$
$\Rightarrow\ \alpha\beta = \frac{1}{4}$

Now, $\alpha^2 \& + \beta^2 - \alpha\beta$
$= (\alpha + \beta)^2 - 3\alpha\beta$
$= \left(\frac{-11}{16}\right)^2 - 3 \times \frac{1}{4}$
$= \frac{121 - 192}{256} = \frac{-71}{256}$

Question148

The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is
[2021,24 Feb. Shift-II]

Answer: 2

Solution:

Solution:

Given, equation $(x+1)^2 + |(x-5)| = \frac{27}{4}$

Case I For $x \geq 5$

$11 \Rightarrow (x+1)^2 + (x-5) = \frac{27}{4}$

$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$

$\Rightarrow 4x^2 + 12x - 43 = 0$

$\therefore x = \frac{-12 \pm \sqrt{144+688}}{8}$

$= \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$

$x = \frac{-3 \pm 7.2}{8}$

$x = \frac{-3+7.2}{8}, \frac{-3-7.2}{8}$

Both the values are less than 5.

∴ No solution from here.

Case II $x < 5$

$\Rightarrow (x+1)^2 + (x-5) = \frac{27}{4}$

$\Rightarrow x^2 + x - 6 - \frac{27}{4} = 0$

$\Rightarrow 4x^2 + 4x - 3 = 0$

$\Rightarrow x = \frac{-4 \pm \sqrt{16+48}}{8}$

$= \frac{-4 \pm 8}{8}$

$\Rightarrow x = \frac{-12}{8}, \frac{4}{8}$, both are less than 5.

∴ These values must be the solution. Hence, here 2 real roots are possible.

Question149

If the least and the largest real values of α, for which the equation $z + \alpha|z-1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to
[2021,24 Feb. Shift-I]

Answer: 10

Solution:

Solution:
Given, $\alpha_{least} = p$
$\alpha_{max} = q$
Equation given is $z + \alpha | z - 1 | + 2i = 0$;
$z \in C$ and $i = \sqrt{-1}$
Let $z = x + iy$
Then, $z + \alpha | z - 1 | + 2i = 0$
$\Rightarrow x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$
$\Rightarrow \left(x + \alpha\sqrt{(x-1)^2 + y^2}\right) + i(y + 2) = 0$
$\therefore y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$
$y = -2$ and $x^2 = \alpha^2(x^2 + 1 - 2x + y^2)$
$x^2 = \alpha^2(x^2 - 2x + 5)$ $(\because y = -2)$
$\Rightarrow \alpha^2 = \dfrac{x^2}{x^2 - 2x + 5}$
$\therefore \alpha^2 \in \left[0, \frac{5}{4}\right]$
$\therefore \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$
Now, $4(p^2 + q^2) = 4[(\alpha_{least})^2 + (\alpha_{max})^2]$
$= 4\left[\left(-\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2\right]$
$= 4 \times \left[\frac{5}{4} + \frac{5}{4}\right] = 10$

Question150

Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation:
[24-Feb-2021 Shift 1]

Options:

A. $x^2 - 2x + 2 = 0$

B. $x^2 - 2x + 8 = 0$

C. $x^2 - 2x + 136 = 0$

D. $x^2 - 2x + 16 = 0$

Answer: D

Solution:

Solution:
We have
$(p^2 + q^2)^2 - 2p^2q^2 = 272$
$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$
$(4 - 2pq)^2 - 2p^2q^2 = 272$
$16 - 16pq + 2p^2q^2 = 272$
$(pq)^2 - 8pq - 128 = 0$
$pq = \dfrac{8 \pm 24}{2} = 16, -8$
$\therefore pq = 16$ $(\because p, q > 0)$

$\therefore$ Required equation :
$x^2 - (2)x + 16 = 0$

Question151

If the equation $a|z|^2 + \overline{\bar{\alpha}z + \alpha\bar{z}} + d = 0$ represents a circle, wherea,d are real constants, then which of the following condition is correct?

[2021, 18 March Shift-I]

Options:

A. $|\alpha|^2 - ad \neq 0$

B. $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

C. $|\alpha|^2 - ad \geq 0$ and $a \in R$

D. $\alpha = 0$, a, d $\in R^+$

Answer: B

Solution:

Solution:
Given, $a|z|^2 + \overline{\bar{\alpha}z + \alpha\bar{z}} + d = 0$
$\Rightarrow a|z|^2 + \alpha\bar{z} + \bar{\alpha}z + d = 0$ ...(i)
Putting $z = x + iy$ and $\alpha = p + iq$ in Eq. (i),
we get
$a(x^2 + y^2) + (p + iq)(x - iy) + (p - iq)$
$\Rightarrow (x + iy) + d = 0$
$a(x^2 + y^2) + px + qy - ipy + iqx + px + qy \quad - iqx + ipy + d = 0$
$\Rightarrow a(x^2 + y^2) + 2px + 2qy + d = 0$
$\Rightarrow x^2 + y^2 + \left(\frac{2p}{a}\right)x + \left(\frac{2q}{a}\right)y + \frac{d}{a} = 0$ be a cricle
If $a \neq 0$ and $r^2 = \left(\frac{p^2}{a^2} + \frac{q^2}{a^2} - \frac{d}{a}\right) > 0$If $a \neq 0$ and $r^2 = \left(\frac{p^2}{a^2} + \frac{q^2}{a^2} - \frac{d}{a}\right) > 0$
$\Rightarrow p^2 + q^2 - ad > 0$
$\Rightarrow |\alpha|^2 - ad > 0$
and $a \in R - \{0\}$

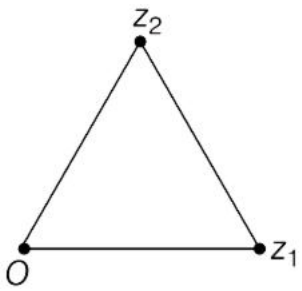
Question152

Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of |a| is [2021, 18 March Shift-I]

Answer: 6

Solution:

Solution:
Given, z_1, z_2 are the roots of
$z^2 + az + 12 = 0$
$\therefore z_1 + z_2 = \frac{-a}{1} = -a$
and $z_1 z_2 = \frac{12}{1} = 12$
Now, z_1, z_2 and origin forms an equilateral triangle.



1 c $\therefore\ z_1^2 + z_2^2 + 0^2 = z_1 z_2 + 0 + 0$
$\Rightarrow z_1^2 + z_1^2 = z_1 z_2$
$\Rightarrow z_1^2 + z_2^2 + 2z_1 z_2 = z_1 z_2 + 2z_1 z_2$
$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$
$\Rightarrow (-a)^2 = 3 \times (12)$
$\Rightarrow a^2 = 36 \Rightarrow |a|^2 = 36$
$\Rightarrow |a| = \pm 6$
But $|a| \geq 0$
$\therefore\ |a| = 6$

Question153

Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area
of the triangle with vertices origin, z and w, is equal to
[2021, 18 March Shift-III]

Options:

A. 4

B. $\frac{1}{2}$

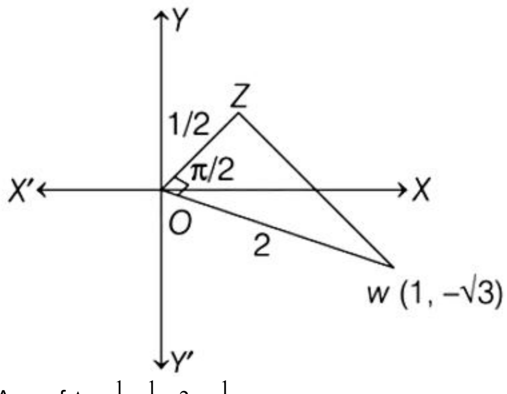
C. $\frac{1}{4}$

D. 2

Answer: B

Solution:

Solution:
Given, $w = 1 - \sqrt{3}i$
$\Rightarrow\ |w| = \sqrt{(1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$
and $|zw| = 1 \Rightarrow |z||w| = 1$
$\Rightarrow\ |z| = \frac{1}{|w|} = \frac{1}{2}$



$\therefore$ Area of $\Delta = \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$

Question154

Let S_1, S_2 and S_3 be three sets defined as
$S_1 = \{z \in C : |z - 1| \leq \sqrt{2}\}$
$S_2 = \{z \in C : \mathrm{Re}[(1 - i)z] \geq 1\}$
$S_3 = \{z \in C : \mathrm{Im}(z) \leq 1\}$

Then, the set $S_1 \cap S_2 \cap S_3$
[2021, 17 March Shift-II]

Options:

A. is a singleton

B. has exactly two elements

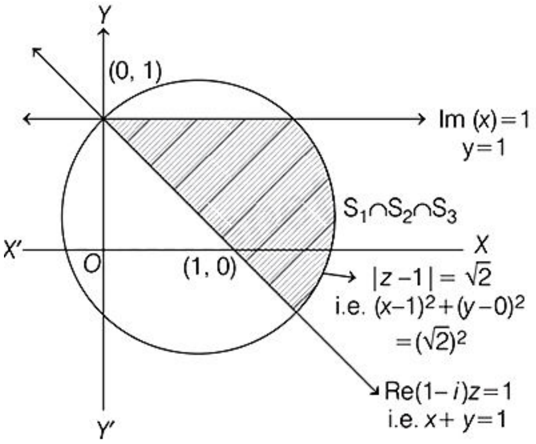
C. has infinitely many elements

D. has exactly three elements

Answer: C

Solution:

Solution:
For $|z-1| \leq \sqrt{2}, \ldots$ (i)
z lies on and inside the circle of radius $\sqrt{2}$ units and centre $(1, 0)$.
For S_2 , let $z = x + iy$
Now $(1-i)(z) = (1-i)(x+iy)$
$= x + iy - ix + y = (x+y) + i(y-x)$
$\therefore \operatorname{Re}[(1-i)z] = (x+y)$, which is greater than or equal to one.
i.e., $x + y \geq 1 \quad \cdots\cdots$ (ii)
Also, for S_3
Let $z = x + iy$
$\therefore I_m(z) = y$, which is less than or equal to one.
i.e., $y \leq 1 \quad \cdots\cdots$ (iii)
Concept Draw the graph of Eqs. (i), (ii) and (iii) and then select the common region bounded by Eqs. (i), (ii) and (iii) for $S_1 \cap S_2 \cap S_3$.



$\therefore S_1 \cap S_2 \cap S_3$ has infinitely many elements.

Question155

The area of the triangle with vertices A(z), B(iz) and C(z + iz) is
[2021, 17 March Shift-I]

Options:

A. 1

B. $\frac{1}{2}|z|^2$

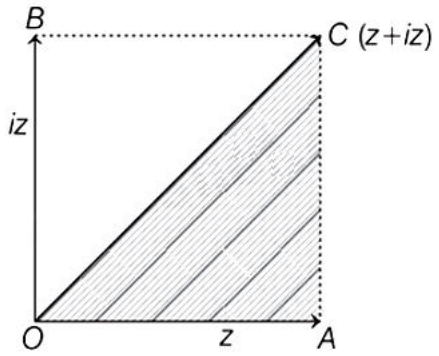
C. $\frac{1}{2}$

D. $\frac{1}{2}|z+iz|^2$

Answer: B

Solution:

Solution:
Area of triangle whose vertices are $A(z), B(iz), C(z+iz)$



Area of the triangle

$= \frac{1}{2}|z|\,|iz| = \frac{1}{2}|z|^2$

Question156

The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\infty}}}}$ is

[2021, 17 March Shift-I]

Options:

A. $2+\frac{2}{5}\sqrt{30}$

B. $2+\frac{4}{\sqrt{5}}\sqrt{30}$

C. $4+\frac{4}{\sqrt{5}}\sqrt{30}$

D. $5+\frac{2}{5}\sqrt{30}$

Answer: A

Solution:

Solution:

Let $x=4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\infty}}}}$

$x=4+\cfrac{1}{5+\cfrac{1}{x}}$

$\Rightarrow\ (x-4)(5x+1)=x$

$\Rightarrow\ 5x^2-19x-4=x$

$\Rightarrow\ 5x^2-20x-4=0$

$\Rightarrow\ x=\frac{20\pm\sqrt{400+80}}{10}$

$\Rightarrow\ x=\frac{20\pm\sqrt{480}}{10}$

$\Rightarrow\ x=2\pm\sqrt{\frac{480}{100}}$

$=2\pm\frac{2}{5}\sqrt{30}$

$\because$ x nless 0

So, $x=2+\frac{2}{5}\sqrt{30}$

Question157

The number of elements in the set $\{x \in R : (|x| - 3) \mid x + 4 \mid = 6\}$ is equal to [2021, 16 March Shift-1]

Options:

A. 3

B. 2

C. 4

D. 1

Answer: B

Solution:

Solution:
Given, set $= \{x \in R : (|x| - 3) \mid x + 4 \mid = 6\}$
As, we already know

$|x| = \begin{cases} x_1 & x \geq 0 \\ -x_1 & x < 0. \end{cases}$ and

$|x + 4| = \begin{cases} x + 4 & x \geq -4 \\ -(x + 4) & x < -4. \end{cases}$

Case I
$x < -4$
$r(-x - 3)(-x - 4) = 6$
$(x + 3)(x + 4) = 6$
$x^2 + 7x + 12 = 6$
$x^2 + 7x + 6 = 0$
$(x + 6)(x + 1) = 0$
$x = -6$ or $x = -1$
We will reject $x = -1$ as, $-1 > -4$
$\therefore$ When $x < -4$, $x = -6$ is the solution.
Case II
$-4 \leq x < 0$
$(-x - 3)(x + 4) = 6$
$\Rightarrow -(x + 3)(x + 4) = 6$
$\Rightarrow -(x^2 + 7x + 12) = 6$
$\Rightarrow x^2 + 7x + 18 = 0$
As, the discriminant of this quadratic equation is $D = 7^2 - 4 \cdot 18 = 49 - 72 = -23$
$\because D = -23$ and $D < 0$
So, no real roots and as per the question, $x \in R$.
No solution when $-4 \leq x < 0$.

Case III
$x \geq 0$
$(|x| - 3) \mid x + 4 \mid = 6$
$\Rightarrow (x - 3)(x + 4) = 6$
$\Rightarrow x^2 + x - 12 = 6$
$\Rightarrow x^2 + x - 18 = 0$
$x = \dfrac{-1 \pm \sqrt{1 + 72}}{2} = \dfrac{-1 \pm \sqrt{73}}{2}$
We will reject $x = \dfrac{-1 - \sqrt{73}}{2}$ as $\dfrac{-1 - \sqrt{73}}{2} < 0$ and here, $x \geq 0$.
So, $x = \dfrac{-1 + \sqrt{73}}{2}$, when $x \geq 0$.
$\therefore x = -6$ and $x = \dfrac{-1 + \sqrt{73}}{2}$
are the two solutions which belong to the set.
Hence, number of solutions $= 2$

Question158

Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients, such that $\int_0^1 P(x) d x = 1$ and P(x) leaves remainder 5 when it is divided by $(x - 2)$. Then, the value of $g(b + c)$ is
[2021, 16 March Shift-II]

Options:

A. 9

B. 15

C. 7

D. 11

Answer: C

Solution:

Solution:
$P(x) = x^2 + bx + c$
$\Rightarrow \int_0^1 (x^2 + bx + c) d x = 1$
$\Rightarrow \left[\frac{x^3}{3} + \frac{bx^2}{2} + cx \right]_0^1 = 1$
$\Rightarrow \frac{1}{3} + \frac{b}{2} + c = 1$
$\Rightarrow b + 2c = 4/3 \quad \cdots\cdots (i)$
And, $P(x) = (x - 2) \cdot Q(x) + 5$
When, $x = 2$
$P(2) = 5$
$4 + 2b + c = 5$
$c = 1 - 2b \quad \cdots\cdots (ii)$
Putting $c = 1 - 2b$ in Eq. (i),
$b + 2(1 - 2b) = 4/3$
$\Rightarrow -3b + 2 = 4/3$
$\Rightarrow b = 2/9$
$\therefore c = 1 - 4/9 = 5/9$
$9(b + c) = 9\left(\frac{2}{9} + \frac{5}{9} \right) = 7$

Question159

Let z and w be two complex numbers, such that $w = z\bar{z} - 2z + 2$, $\left| \frac{z + i}{z - 3i} \right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to
[2021, 16 March Shift-1]

Answer: 4

Solution:

Solution:
Given, $w = z\bar{z} - 2z + z$
$\left| \frac{z + i}{z - 3i} \right| = 1$
$\Rightarrow |z + i| = |z - 3i|$
Let $z = x + iy$
$\Rightarrow |x + i(y + 1)| = |x + i(y - 3)|$
$\Rightarrow x^2 + (y + 1)^2 = x^2 + (y - 3)^2$
$\Rightarrow 2y + 1 = -6y + 9$
$\therefore y = 1$
Now, $w = z\bar{z} - 2z + 2$
$w = |z|^2 - 2z + 2$
$\Rightarrow w = x^2 + y^2 - 2(x + iy) + 2$

$\Rightarrow \ w = (x^2 + y^2 - 2x + 2) + i(-2y)$
$\Rightarrow \ w = (x^2 + 1 - 2x + 2) + i(-2)$
$w = (x-1)^2 + 2 - 2i$
Re(w) has minimum value.
So, $(x-1)^2 + 2$ is minimum when $x = 1$
$\therefore \ w = 2 - 2i$
$= 2(1 - i)$
$= 2\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$
$w = 2\sqrt{2}e^{-i\pi/4}$
Now, $w^n = (2\sqrt{2})^n e^{\frac{-in\pi}{4}}$
$= (2\sqrt{2})^n\left[\cos\left(\frac{n\pi}{4}\right) - i\sin\left(\frac{n\pi}{4}\right)\right]$
This has to be zero for w^n to be real.
So, $\sin\left(\frac{n\pi}{4}\right) = 0$
$\Rightarrow \ \frac{n\pi}{4} = 0, \pi, 2\pi, 3\pi\ldots$
$\Rightarrow \ n = 0, 4, 8, 12\ldots$
The minimum value of n is $4 (n \in N)$.

Question160

The least value of |z|, where z is a complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{\left||z|+1\right|} log_{e^2}\right) \geq log_{\sqrt{2}}|5\sqrt{7} + 9i|,$$

$i = \sqrt{-1}$, is equal to:

[2021, 16 March Shift-II]

Options:

A. 3

B. $\sqrt{5}$

C. 2

D. 8

Answer: A

Solution:

Solution:
$\exp\left[\frac{(|z|+3)(|z|-1)}{(|z|+1)} \times \log_e 2\right] \geq \log_{\sqrt{2}}|5\sqrt{7} + 9i|$
$\exp\left[\frac{(|z|+3)(|z|-1)}{(|z|+1)} \times \log_e 2\right] \geq \log_{\sqrt{2}}16$
$\Rightarrow \ \log_{\sqrt{2}}|5\sqrt{7} + 9i|$
$\Rightarrow \ \frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$
$\Rightarrow \ | z2^3$
$\Rightarrow \ | z | +1)$
$\Rightarrow \ (|z| - 3)(|z| + 2) \geq 0$
$\Rightarrow \ | z | = 3$

Question161

If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then P(1) is equal to
[2021, 18 March Shift-II]

Answer: 0

Solution:

Solution:
Method (1)
Given , $P(x) = f(x^3) + xg(x^3) \quad \cdots\cdots (i)$
$\therefore\ P(1) = f(1) + g(1) \quad \cdots\cdots (ii)$
Given, $P(x)$ is divisible by $(x^2 + x + 1)$.
$\therefore\ P(x) = Q(x) . (x^2 + x + 1)$
As, we know that ω and ω^2 are non-real cube roots of unity and this is also root of $x^2 + x + 1 = 0$
$\therefore\ P(\omega) = P(\omega^2) = 0$
As, we know that ω and ω^2 are non-real cube roots of unity and this is also root of $x^2 + x + 1 = 0$
$\therefore\ P(\omega) = P(\omega^2) = 0\ \ldots\ (iii)$
From Eq. (i),
$P(\omega) = f(\omega^3) + \omega[g(\omega)^3] = 0$[from Eq. (iii)]
$\Rightarrow f(1) + \omega g(1) \ = 0 \ldots\ (iv)$
and $P(\omega^2) \ = 0$ [from Eq. (iii)]
$\Rightarrow f(\omega^6) + \omega^2 \cdot g(\omega^6) = 0$
$\Rightarrow\ f(1) + \omega^2 g(1) = 0 \quad \cdots\cdots (v)$
Now, adding Eqs. (iv) and (v), we get
$2f(1) + (\omega + \omega^2)g(1) = 0$
$\Rightarrow\ 2f(1) - 1g(1) = 0\ (\because 1 + \omega + \omega^2 = 0)$
$\Rightarrow\ 2f(1) = g(1)$ ... (vi)
Subtracting Eq. (iv) from Eq. (v), we get
$0 + (\omega - \omega^2)g(1) = 0$
$\Rightarrow\ g(1) = 0$
$f(1) = \frac{g(1)}{2} = \frac{0}{2}$ [from Eq. (vi)]
$\Rightarrow\ f(1) = 0$
From Eq. (ii), $P(1) = f(1) + g(1) = 0 + 0 = 0$
Method (2)
$\because\ P(\omega) = 0$
$\Rightarrow\ f(1) + \omega g(1) = 0$
$\Rightarrow\ f(1) + \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)g(1) = 0$
$\Rightarrow\left(f(1) - \frac{g(1)}{2}\right) + i\left(\frac{\sqrt{3}}{2}g(1)\right) = 0$
On comparing real and imaginary parts from both sides, we have
$11\ f(1) - \frac{g(1)}{2} = 0, \quad \frac{\sqrt{3}}{2}g(1) = 0$
$\Rightarrow f(1) = \frac{g(1)}{2}, \quad \Rightarrow g(1) = 0$
$\therefore f(1) = \frac{0}{2} = 0$
$\therefore P(1) = f(1) + g(1) = 0 + 0 = 0$

Question162

The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$

is equal to
[2021, 18 March shift-I]

Options:

A. $1.5 + \sqrt{3}$

B. $2 + \sqrt{3}$

C. $3 + 2\sqrt{3}$

D. $4+\sqrt{3}$

Answer: A

Solution:

Solution:

Let $x=3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$

So, $x=3+\frac{1}{4+\frac{1}{x}}=3+\frac{1}{\frac{4x+1}{x}}=3+\frac{x}{4x+1}$

$\Rightarrow\ (x-3)=\frac{x}{4x+1}$

$\Rightarrow\ (4x+1)(x-3)=x$

11 $\Rightarrow 4x^2-12x-3=0$

$\Rightarrow x=\frac{3\pm2\sqrt{3}}{2}$

$\Rightarrow x=\frac{3}{2}\pm\sqrt{3}=1.5\pm\sqrt{3}$

But from above, $x>0$

$\therefore$ Only positive value of x is accepted

$\therefore x=1.5+\sqrt{3}$

Question163

Let C be the set of all complex numbers. Let
$S_1=\{z\in C\mid |z-3-2i|^2=8\}$,
$S_2=\{z\in C\mid Re(z)\geq 5\}$ and
$S_3=\{z\in C\mid |z-\bar{z}|\geq 8$.
Then, the number of elements in $S_1\cap S_2\cap S_3$ is equal to
[2021, 27 July Shift-1]

Options:

A. 1

B. 0

C. 2

D. Infinite

Answer: A

Solution:

Solution:

$S_1: |z-3-2i|^2=8$

$\Rightarrow\ |(x+iy)-(3+2i)|^2=8$

$\Rightarrow\ |(x-3)+i(y-2)|^2=8$

$\Rightarrow\ (x-3)^2+(y-2)^2=8$

$S_2: Re(z)\geq5$

$x\geq5$

$S_3: |z-\bar{z}|\geq8$

$|(x+iy)-(x-iy)|\geq8$

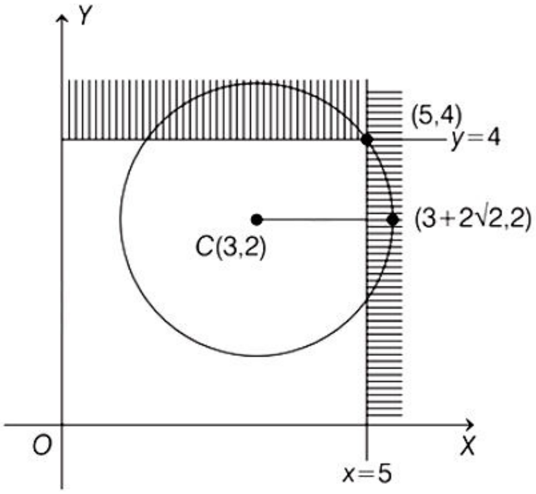
$\Rightarrow\ 2y\geq8$

$\Rightarrow\ y\geq4$

$S_1: (x-3)^2+(y-2)^2=8$

$S_2: x\geq5$

$S_3: y\geq4$



Circle passes through $(5, 4)$ as shown in the figure.
$\Rightarrow$ There is exactly one point $(5, 4)$ in $S_1 \cap S_2 \cap S_3$.

Question164

The point P(a, b) undergoes the following three transformations successively
(A) Reflection about the line y = x.
(B) Translation through 2 units along the positive direction of X-axis.
(C) Rotation through angle $\frac{\pi}{4}$
about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a+b$ is equal to

[2021, 27 July Shift-II]

Options:

A. 13

B. 9

C. 5

D. 7

Answer: B

Solution:

Solution:
The image of $P(a, b)$ along $y=x$ is $Q(b, a)$. Translating it 2 units along the positive direction of X-axis, it becomes $R(b+2, a)$. Then, rotation through $\frac{\pi}{4}$ about the origin in the anticlockwise direction, the final position of the point P is
$\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$.
Now, applying rotational theorem,
$-\frac{1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=[(b+2)+ai] . \left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$
$\Rightarrow -\frac{1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=\left(\frac{b+2}{\sqrt{2}}-\frac{a}{\sqrt{2}}\right)+i\left(\frac{b+2}{\sqrt{2}}+\frac{a}{\sqrt{2}}\right)$
$\Rightarrow -\frac{1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=\left(\frac{b-a+2}{\sqrt{2}}\right)+i\left(\frac{a+b+2}{\sqrt{2}}\right)$
11 So, $\frac{b-a+2}{\sqrt{2}}=-\frac{1}{\sqrt{2}}$
$\Rightarrow b-a=-3 \quad \cdots\cdots (i)$
and $\frac{a+b+2}{\sqrt{2}}=\frac{7}{\sqrt{2}}$
$\Rightarrow a+b=5 \quad \cdots\cdots (ii)$
Adding Eqs. (i) and (ii),
$2b=2 \Rightarrow b=1$

Substitute the value of b in Eq. (ii), $a = 4$
Now, $2a + b = 2 \times 4 + 1 = 9$

Question165

Let C be the set of all complex numbers.
Let $S_1 = \{z \in C : |z - 2| \le 1\}$ and $S_2 = \{z \in C : z(1 + i) + \bar{z}(1 - i) \ge 4\}$.
Then, the maximum value of $z - \frac{5}{2}|^2$ for $z \in S_1 \cap S_2$ is equal to
[2021, 27 July Shift-II]

Options:

A. $\frac{3 + 2\sqrt{2}}{4}$

B. $\frac{5 + 2\sqrt{2}}{2}$

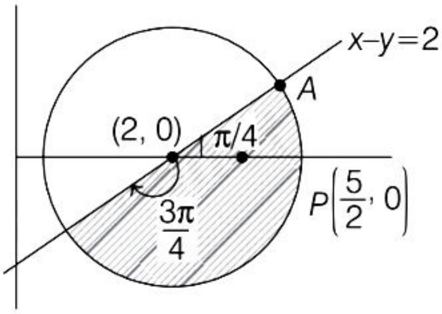
C. $\frac{3 + 2\sqrt{2}}{2}$

D. $\frac{5 + 2\sqrt{2}}{4}$

Answer: D

Solution:

Solution:
Let $S_1 = \{z \in C : |z - 2| \le 1\}$
and $S_2 = \{z \in C : z(1 + i) + \bar{z}(1 - i) \ge 4\}$
Now $|z - 2| \le 1$
Let $z = x + iy$
$\Rightarrow |x + iy - 2| \le 1$
$\Rightarrow (x - 2)^2 + y^2 \le 1$
Also, $z(1 + i) + \bar{z}(1 - i) \ge 4$
$\Rightarrow (x + iy)(1 + i) + (x - iy)(1 - i) \ge 4$
$\Rightarrow 2x - 2y \ge 4$
$\Rightarrow x - y \ge 2$



Let point on circle be $A(2 + \cos\theta, \sin\theta)$,
$1\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$(AP)^2 = \left(2 + \cos\theta - \frac{5}{2}\right)^2 + \sin^2\theta$

$\Rightarrow (AP)^2 = \cos^2\theta + \frac{1}{4} - \cos\theta + \sin^2\theta$

$\Rightarrow (AP)^2 = \frac{5}{4} - \cos\theta$

For $(AP)^2$ to be maximum, $\theta = -\frac{3\pi}{4}$

$\Rightarrow (AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}}$

$\Rightarrow (AP)^2 = \frac{5 + 2\sqrt{2}}{4}$

Question166

Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then, $\alpha^8 + \beta^8$ is equal to [2021, 27 July Shift-I]

Options:

A. 10

B. 50

C. 100

D. 160

Answer: B

Solution:

Solution:

$x^2 + (20)^{\frac{1}{4}} \cdot x + (5)^{\frac{1}{2}} = 0$.

roots α & β.

$\alpha + \beta = -(20)^{\frac{1}{4}}$

$\alpha\beta = (5)^{\frac{1}{2}}$.

$\alpha^8 + \beta^8 = (\alpha^4)^2 + (\beta^4)^2$

$= (\alpha^4 - \beta^4)^2 + 2(\alpha\beta)^4. \quad \cdots\cdots (i)$

$\Rightarrow (\alpha + \beta)^2 = (\alpha^2 + \beta^2) + 2\alpha\beta.$

$\Rightarrow (20)^{\frac{1}{2}} = (\alpha^2 + \beta^2) + 2 \cdot 5^{\frac{1}{2}}$

$\Rightarrow 2 \cdot (5)^{\frac{1}{2}} = (\alpha^2 + \beta^2) + 2 \cdot 5^{\frac{1}{2}}$

$\Rightarrow 0 = (\alpha^2 + \beta^2)$

From eqn (1)

$\alpha^8 + p^8 = ((\alpha^2 + p^2) \cdot (\alpha^2 - \beta^2))^2 + 2 \cdot (5)^{1/2}$

$= 0 + 2 \times 5^2$

$= 2 \times 25$

$= 50$ (Ans)

Question167

The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to .... [2021, 27 July Shift-II]

Answer: 2

Solution:

Solution:

Given equation,

$e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$

Let $e^x = t > 0$

$t^4 - t^3 - 4t^2 - t + 1 = 0$

$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$

$\Rightarrow t^2 + \frac{1}{t^2} + 2 - \left(t + \frac{1}{t}\right) - 6 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 6 = 0$

Let $\alpha = t + \frac{1}{t} \geq 2$

$1c \Rightarrow \alpha^2 - \alpha - 6 = 0$

$\Rightarrow \alpha^2 - 3\alpha + 2\alpha - 6 = 0$

$\Rightarrow \alpha(\alpha - 3) + 2(\alpha - 3) = 0$

$\Rightarrow (\alpha - 3)(\alpha + 2) = 0$

$\Rightarrow \alpha = 3$ or $\alpha = -2$ (not possible)

$\Rightarrow t + \frac{1}{t} = 3$
$\Rightarrow t^2 - 3t + 1 = 0$
$\therefore$ The number of real roots $= 2$

Question168

The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is [2021, 25 July Shift-1]

Options:

A. 2

B. 4

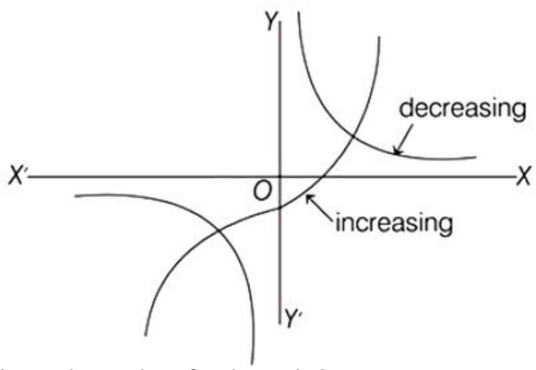
C. 6

D. 1

Answer: A

Solution:

Solution:
$e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 \quad = 0$
$\Rightarrow (e^{3x} - 1)^2 - e^x(e^{3x} - 1) \quad = 12e^{2x}$
$\Rightarrow (e^{3x} - 1)(e^{3x} - e^x - 1) \quad = 12e^{2x}$
$\Rightarrow (e^{3x} - 1)(e^x - e^{-x} - e^{-2x}) \quad = 12$
$\Rightarrow e^x - e^{-x} - e^{-2x} \quad = \frac{12}{e^{3x} - 1}$



Hence, the number of real roots is 2.

Question169

If α, β are roots of the equation. $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^{\ 2}}\right)$ is equal to [2021, 25 July Shift-1]

Answer: 1

Solution:

Solution:
$x^2+5\sqrt{2}x+10 \;=0$
$P_n=\alpha^n-\beta^n$
$\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}{}^2}=\frac{P_{17}(5\sqrt{2}P_{19}+P_{20})}{P_{18}(5\sqrt{2}P_{18}+P_{19})}$
$\Rightarrow\; x^{18}(x^2+5\sqrt{2}x+16)=0$
$\Rightarrow\; x^{20}+5\sqrt{2}x^{19}+x^{18}=0$
$(\alpha^{20}-\beta^{20})+5\sqrt{2}(\alpha^{19}-\beta^{19})+(\alpha^{18}-\beta^{18})=0$
$P_{20}+5\sqrt{2}P_{19}+P_{18}=0$
Similarly,
$P_{19}+5\sqrt{2}P_{18}+P_{17}=0$
So, $\frac{P_{17}(5\sqrt{2}P_{19}+P_{20})}{P_{18}(5\sqrt{2}P_{18}+P_{19})}=\frac{P_{17}(-P_{18})}{P_{18}(-P_{17})}=1$

Question170

The number of real solutions of the equation $x^2-|x|-12=0$ is [2021, 25 July Shift-II]

Options:

A. 2

B. 3

C. 1

D. 4

Answer: A

Solution:

Solution:
Given equation,
$cx^2-|x|-12=0$
$\Rightarrow\quad |x^2|-|x|-12=0$
$\Rightarrow |x|^2-4|x|+3|x|-12=0$
$\Rightarrow\; (|x|-4)(|x|+3)=0$
So $\;|x|-4=0$ or $|x|+3=0$
$|x|=4$ or $|x|=-3$ (not possible)
$x=\pm4$
Hence, the number of real solutions $=2$

Question171

Let [x] denote the greatest integer less than or equal to x. Then, the values of $x\in R$ satisfying the equation $[e^x]^2+[e^x+1]-3=0$ lie in the interval [2021, 22 July Shift-II]

Options:

A. $\left[0,\ \frac{1}{e}\right)$

B. $[\log_e 2, \log_e 3)$

C. $[1, e)$

D. $[0, \log_e 2)$

Answer: D

Solution:

Solution:
$[e^x]^2 + [e^x + 1] - 3 = 0$
$\Rightarrow\ [e^x]^2 + [e^x] + 1 - 3 = 0$
$\Rightarrow\ [e^x]^2 + [e^x] - 2 = 0$
$\Rightarrow\ ([e^x] - 1)([e^x] + 2) = 0$
$[e^x] = 1$ or $[e^x] = -2$
Not possible as $e^x > 0$.
$\Rightarrow\ [e^x] = 1$
$\Rightarrow\ 1 \le e^x < 2$
$\Rightarrow\ 0 \le x < \log_e 2$

Question172

If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to
[2021, 20 July Shift-1]

Options:

A. 56×3^{25}

B. 56×3^{24}

C. 52×3^{24}

D. 28×3^{25}

Answer: C

Solution:

Solution:
$x^2 + 3^{\frac{1}{4}}x + 3^{\frac{1}{2}} = 0$
$\therefore\ x = \dfrac{-3^{1/4} \pm \sqrt{3^{1/2} - 4 \cdot 3^{1/2}}}{2}$
$= \dfrac{3^{1/4}(-1 \pm \sqrt{3}i)}{2}$
$= 3^{1/4}\left(\dfrac{-1 + \sqrt{3}i}{2}\right) \text{or} 3^{1/4}\left(\dfrac{-1 - \sqrt{3}i}{2}\right)$
$= 3^{1/4}\omega$ or $3^{1/4}\omega^2$

$= 3^{1/4}\omega$ or $3^{1/4}\omega^2$
Now, $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$
$= \alpha^{108} - \alpha^{96} + \beta^{108} - \beta^{96}$
$= (\alpha^{108} + \beta^{108}) - (\alpha^{96} + \beta^{96})$
$= \{(3^{1/4}\omega\}^{108} + (3^{1/4}\omega^2)^{108}\}$

$- \{(3^{1/4}\omega)^{96} + (3^{1/4}\omega^2)^{96}\}$
$= 3^{27}(\omega^{108} + \omega^{216}) - 3^{24}(\omega^{96} + \omega^{192})$
$= 3^{27}(2) - 3^{24}(2) = 3^{24}(54) - 3^{24}(2)$
$= 3^{24}(52) = 52 \times 3^{24}$

Question173

The number of solutions of the equation
$\log_{(x+1)}(2x^2 + 7x + 5) +$

$\log_{(2x+5)}(x+1)^2 - 4 = 0$
$x > 0$, is
[2021, 20 July Shift-II]

Answer: 1

Solution:

Solution:
$\log_{(x+1)}(2x^2+7x+5)$
$+\log_{(2x+5)}(x+1)^2 - 4 = 0$
$= \log_{(x+1)}\{(2x+5)(x+1)\}$
$+2\log_{(2x+5)}(x+1) - 4 = 0$
$= \log_{(x+1)}(2x+5) + \log_{(x+1)}(x+1)$
$+2\log_{(2x+5)}(x+1) - 4 = 0$
$= \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) - 3 = 0$
$[\because \log_a a = 1]$
$= \log_{(x+1)}(2x+5) + 2\ \dfrac{\log_{(x+1)}(x+1)}{\log_{(x+1)}(2x+5)} = 3$
Let $\log_{(x+1)}(2x+5) = t$
$t + \dfrac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$
$(t-1))t-2) = 0$
$\Rightarrow\ t = 1, t = 2$
$\Rightarrow \log_{(x+1)}(2x+5) = 1$ and
$\log_{(x+1)}(2x+5) = 2$
$2x+5 = (x+1)$
and $2x+5 = (x+1)^2$
$x = -4$
and $2x+5 = x^2+1+2x$
i.e., $\ x^2 = 4$
$\Rightarrow\ x = +2, -2$

Given, $x > 0$
$x = -4, x = -2$ are discarde(d)
$\therefore\ x = 2$ is only solution.

Question174

If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0,\ \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to
[2021, 27 July Shift-11]

Answer: 1

Solution:

Solution:
We have,
$1z = \dfrac{3+2i\cos\theta}{1-3i\cos\theta} = \dfrac{3+2i\cos\theta}{1-3i\cos\theta} \times \dfrac{1+3i\cos\theta}{1+3i\cos\theta}$
$= \dfrac{(3-6\cos^2\theta) + i(9\cos\theta + 2\cos\theta)}{1+9\cos^2\theta}$
$z = \dfrac{(3-6\cos^2\theta) + (11\cos\theta)i}{1+9\cos^2\theta}$
Given, Re(z) =0
$\Rightarrow\ \dfrac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$

$\Rightarrow\ 3-6\cos^2\theta\ =0$
$\Rightarrow\ \cos^2\theta\ =\frac{1}{\sqrt{2}}$
$\Rightarrow\ \theta\ =\frac{\pi}{4}\left\{\theta\in\left(0,\frac{\pi}{2}\right)\right\}$
$\Rightarrow\ \ =\frac{1}{2}+\frac{1}{2}=1$
Hence, $\sin^2 3\theta+\cos^2\theta$
$=\sin^2\frac{3\pi}{4}+\cos^2\frac{\pi}{4}=\frac{1}{2}+\frac{1}{2}=1$

Question175

Let n denote the number of solutions of the equation $z^2+3\bar{z}=0$, where z is a complex number. Then, the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is equal to
[2021, 22 July Shift-11]

Options:

A. 1

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. 2

Answer: B

Solution:

Solution:
$z^2+3z=0$
$z=x+iy$
$\Rightarrow(x^2-y^2)+i(2xy)+3(x-iy)=0$
$\Rightarrow(x^2-y^2+3x)+i(2xy-3y)=0$
$$\begin{cases} x^2-y^2+3x=0 \\ y(2x-3)=0. \end{cases}$$
$y=0$ or $x=\frac{3}{2}$
If $y=0$,
$\Rightarrow\ x(x+3)=0$
$\Rightarrow\ x=0,-3$
$\Rightarrow$ So, $(0,0)$ and $(-3,0)$ are solutions, when $y=0$.
When $x=\frac{3}{2},\ \ \frac{9}{4}-y^2+\frac{9}{2}=0\Rightarrow y^2=\frac{27}{4}$
$\Rightarrow\ y=\pm\frac{3\sqrt{3}}{2}$
$\therefore\left(\frac{3}{2},\frac{3\sqrt{3}}{2}\right)$ and $\left(\frac{3}{2},-\frac{3\sqrt{3}}{2}\right)$
There are 4 solutions.
$\sum_{k=0}^{\infty}\left(\frac{1}{n^k}\right)\ =1+\frac{1}{4}+\frac{1}{4^2}+\ldots\infty$
$=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$

Question176

If the real part of the complex number
$(1-\cos\theta+2i\sin\theta)^{-1}$ is $\frac{1}{5}$f or$\theta\in(0,\pi)$, then the value of the integral $\int_0^\theta\sin x\,d\ x$ is equal to
[2021, 22 July Shift-II]

Options:

A. 1

B. 2

C. -1

D. 0

Answer: A

Solution:

Solution:
Let $z = (1-\cos\theta + 2i\sin\theta)^{-1}$

$$\Rightarrow z = \frac{1}{1-\cos\theta+2i\sin\theta}$$

$$= \frac{1}{1-\cos\theta+2i\sin\theta} \times \frac{1-\cos\theta-2i\sin\theta}{1-\cos\theta-2i\sin\theta}$$

$$= \frac{(1-\cos\theta)-2i\sin\theta}{(1-\cos\theta)^2-(2i\sin\theta)^2}$$

$$= \frac{2\sin^2\frac{\theta}{2}-4i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{4\sin^4\frac{\theta}{2}+16\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}\right)}{4\sin^2\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$$

$$= \frac{\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$$

Now, $\mathrm{Re}(z) = \dfrac{\sin\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$

$$= \frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)}$$

Given, $\mathrm{Re}(z) = \frac{1}{5}$

$$\Rightarrow \frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$$

$$\Rightarrow 1+3\cos^2\frac{\theta}{2} = \frac{5}{2} \Rightarrow \cos^2\frac{\theta}{2} = \frac{1}{2}$$

$$\Rightarrow \cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$$

$$\therefore \theta = 2n\pi \pm \frac{\pi}{2}$$

Given, range is $\theta \in (0, \pi)$.

$$\therefore \theta = \frac{\pi}{2}$$

Now, $\int_0^{\theta} \sin x\, dx = \int_0^{\frac{\pi}{2}} \sin x\, dx$

$$\int_0^{\frac{\pi}{2}} \sin x\, dx = -\cos x\Big]_0^{\pi/2}$$

$$= -\left(\cos\frac{\pi}{2}-\cos 0\right)$$

$$= -(0-1) = 1$$

Question177

If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3z\omega}\right)$ is (Here, arg(z) denotes the principal argument of complex number z) [2021, 20 July Shift-1]

Options:

A. $\frac{\pi}{4}$

B. $-\frac{3\pi}{4}$

C. $-\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

Answer: B

Solution:

Solution:

$|zW| = 1, \arg(z) - \arg(w) = \frac{3\pi}{2}$

Let $z = re^{i\theta}$

$w = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)} \Rightarrow \bar{z} = re^{-i\theta}$

$w\bar{z} = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)} \cdot re^{-i\theta}$

$\Rightarrow w\bar{z} = e^{i\left(\theta - \frac{3\pi}{2} - \theta\right)} = e^{-i\frac{3\pi}{2}}$

$\Rightarrow w\bar{z} = \cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)$

$\Rightarrow w\bar{z} = 0 + i$

$\Rightarrow w\bar{Z} = i$

$\left(\frac{1 - 2w\bar{z}}{1 + 3w\bar{z}}\right) = \left(\frac{1 - 2i}{1 + 3i} \times \frac{1 - 3i}{1 - 3i}\right)$

$= \frac{1 - 2i - 3i + 6i^2}{10} = \frac{-5 - 5i}{10}$

$\therefore \arg = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$

Question178

Let Z_1 and Z_2 be two complex numbers such that $\arg(Z_1 - Z_2) = \frac{\pi}{4}$ and Z_1, Z_2 satisfy the equation $|Z - 3| = \mathrm{Re}(Z)$. Then, the imaginary part of $Z_1 + Z_2$ is equal to
[2021, 27 Aug. Shift-11]

Answer: 6

Solution:

Solution:

Let $Z_1 = a_1 + ib_1, Z_2 = a_2 + ib_2$

$Z_1 - Z_2 = (a_1 - a_2) + i(b_1 - b_2)$

$\arg(Z_1 - Z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{b_1 - b_2}{a_1 - a_2}\right) = \frac{\pi}{4}$

11 $\Rightarrow b_1 - b_2 = a_1 - a_2$

Also, $|Z_1 - 3| = \mathrm{Re}(Z_1)$

$\Rightarrow (a_1 - 3)^2 + b_1^2 = a_1^2$

and $|Z_2 - 3| = \mathrm{Re}(Z_2)$

$\Rightarrow (a_2 - 3)^2 + b_2^2 = a_2^2$

$\Rightarrow (a_1 - 3)^2 - (a_2 - 3)^2 + b_1^2 - b_2^2$

$= a_1^2 - a_2^2$

$\Rightarrow (a_1 - a_2)(a_1 + a_2 - 6) + (b_1 - b_2)(b_1 + b_2)$

$= (a_1 - a_2)(a_1 + a_2)$

$\Rightarrow a_1 + a_2 - 6 + b_1 + b_2 = a_1 + a_2$

$\Rightarrow b_1 + b_2 = 6$

$\Rightarrow \; 1m(Z_1 + Z_2) = 6$
[using Eq. (i).]

Question179

The sum of the roots of the equation $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$ is [2021, 31 Aug. Shift-II]

Options:

A. $\log_2 14$

B. $\log_2 11$

C. $\log_2 12$

D. $\log_2 13$

Answer: B

Solution:

Solution:
$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$
$\Rightarrow$
$x + 1 - 2\log_2(3 + 2^x) + \log_2\left(\frac{10 \cdot 2^x - 1}{2^x}\right) = 0$
$\Rightarrow x + 1 - 2\log_2(3 + 2^x) + \log_2(10 \cdot 2^x - 1)$
$\Rightarrow 1 + \log_2\left(\frac{10 \cdot 2^x - 1}{(3 + 2^x)^2}\right) = 0$
$\Rightarrow \; \frac{10 \cdot 2^x - 1}{9 + (2^x)^2 + 6 \cdot 2^x} = \frac{1}{2}$
$\Rightarrow \; (2^x)^2 - 14 \cdot 2^x + 11 = 0$
Let $2^x = y$
$\Rightarrow \; y^2 - 14y + 11 = 0$
Let $2^x = y$
$\Rightarrow \; y^2 - 14y + 11 = 0$
$y = \frac{14 \pm \sqrt{152}}{2} = 7 \pm \frac{\sqrt{152}}{2}$
$y_1 = 7 + \frac{\sqrt{152}}{2}$,
$y_2 = 7 - \frac{\sqrt{152}}{2}$
$\Rightarrow \; 2^{x_1} = 7 + \frac{\sqrt{152}}{2}$,
$2^{x_2} = 7 - \frac{\sqrt{152}}{2}$
$\Rightarrow x_1 = \log_2\left(7 + \frac{\sqrt{152}}{2}\right)$
$x_2 = \log_2\left(7 - \frac{\sqrt{152}}{2}\right)$
$\therefore$ Sum of roots $= x_1 + x_2$
$= \log_2\left(49 - \frac{152}{4}\right) = \log_2 11$

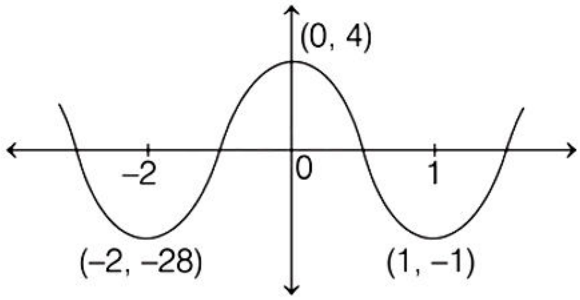
Question180

The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is [2021, 27 Aug. Shift-I]

Answer: 4

Solution:

Solution:
Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4 = 0$
Differentiating w.r.t. x_1
$f'(x) = 12x^3 + 12x^2 - 24x = 0$
$\Rightarrow 12x(x^2 + x - 2) = 0$
$\Rightarrow x(x + 2)(x - 1) = 0$
Critical point $x = 0, 1, -2$



Graph of $y = f(x)$
Number of real roots $= 4$

Question181

The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)$ $(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is
[2021, 27 Aug. Shift-II]

Options:

A. $\left(1, \frac{5}{2}\right]$

B. $[2, 3)$

C. $\left[-\frac{1}{2}, 1\right)$

D. $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

Answer: A

Solution:

Solution:
Given,
$(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)$
$(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$
Let $y = 3x^2 + 4x + 2$
Then, given equation becomes
$(y + 1)^2 - (k + 1)y(y + 1) + ky^2 = 0$
$\Rightarrow y^2 + 2y + 1 - ky^2 - ky - y^2 - y + ky^2 = 0$
$\Rightarrow\ y + 1 - ky = 0$
$\Rightarrow\ y(1 - k) = -1$
$\Rightarrow\ y = \dfrac{1}{k - 1}$
$\Rightarrow 3x^2 + 4x + 2 - \dfrac{1}{k - 1} = 0$
For real roots, $D \geq 0$
$\Rightarrow\ 16 - 4 \cdot 3 \cdot \left(2 - \dfrac{1}{k - 1}\right) \geq 0$
$\Rightarrow\ -8 + \dfrac{12}{k - 1} \geq 0 \Rightarrow \dfrac{3}{k - 1} \geq 2$

$\Rightarrow \dfrac{3-2k+2}{k-1} \geq 0 \Rightarrow \dfrac{2k-5}{k-1} \leq 0$

$\Rightarrow k \in \left(1, \dfrac{5}{2}\right]$ $[\because k \neq 1]$

Question182

The sum of all integral values of k (k ≠ 0) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is [2021,26 Aug. Shift-I]

Answer: 66

Solution:

Solution:

$\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k} \Rightarrow x \in R - \{1, 2\}$

$k(2x-4-x+1) = 2(x^2-3x+2)$

$k(x-3) = 2(x^2-3x+2)$

$2x^2-(6+k)x+3k+4 = 0$

For no real roots $b^2-4ac < 0$

$\therefore (k+6)^2-8\cdot(3k+4) < 0$

$\Rightarrow \quad k^2-12k-4 < 0$

$\Rightarrow (k-6)^2-32 < 0$

$\Rightarrow \quad (k-6)^2 < 32$

$\Rightarrow \quad -4\sqrt{2} < k-6 < 4\sqrt{2}$

$\Rightarrow 6-4\sqrt{2} < k < 6+4\sqrt{2}$

Integral $k \in \{1, 2, 3, 4, \ldots..11\}$

Sum $= 66$

Question183

If $(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation [2021, 26 Aug. Shift-III]

Options:

A. $x^2-(\sqrt{3}-1)x-\sqrt{3} = 0$

B. $x^2+(\sqrt{3}+1)x+\sqrt{3} = 0$

C. $x^2+(\sqrt{3}-1)x-\sqrt{3} = 0$

D. $x^2-(\sqrt{3}+1)x+\sqrt{3} = 0$

Answer: A

Solution:

Solution:

$(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$

$2^{100}e^{i100\frac{\pi}{6}} = 2^{99}(p+iq)$

$\Rightarrow 2e^{i\frac{2\pi}{3}} = p+iq$

$\Rightarrow 2\left[\cos\left(\pi-\frac{\pi}{3}\right)+i\sin\left(\pi-\frac{\pi}{3}\right)\right] = p+iq$

$\Rightarrow \quad (-1+i\sqrt{3}) = p+iq$

$\Rightarrow p = -1$ and $q = \sqrt{3}$

Equation whose roots are -1 and $\sqrt{3}i$ is

$\Rightarrow\ (x+1)(x-\sqrt{3})=0$
$x^2-(\sqrt{3}-1)x-\sqrt{3}=0$

Question184

Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2-x+2\lambda=0$ and α and γ are the roots of equation $3x^2-10x+27\lambda=0$, then $\frac{\beta\gamma}{\lambda}$ is equal to
[2021,26 Aug. Shift-II]

Answer: 18

Solution:

Solution:
We have, α is common root of the equations $x^2-x+2\lambda=0$ and $3x^2-10x+27\lambda=0$.
Now, common root of these equations is $(3\alpha^2-10\alpha+27\lambda)-(3\alpha^2-3\alpha+6\lambda)=0 \Rightarrow\ -7\alpha+21\lambda=0$
$\Rightarrow\ \alpha=3\lambda$
Again, α is root of $x^2-x+2\lambda=0$
$\therefore\ \alpha^2-\alpha+2\lambda=0$
$\Rightarrow\ (3\lambda)^2-3\lambda+2\lambda=0$
$\Rightarrow\ 9\lambda^2-\lambda=0$
$\Rightarrow\ \lambda(9\lambda-1)=0$
$\Rightarrow\ \lambda=0,\ \frac{1}{9}$
$\Rightarrow\ \lambda=\frac{1}{9}\ [\because \lambda \neq 0]$
$\therefore\ \alpha=3\lambda=3\times\frac{1}{9}=\frac{1}{3}$
Again, α and β are roots of the equation
$x^2-x+2\lambda=0$

11 $\therefore\ \alpha+\beta=\frac{-(-1)}{1}=1$
$\Rightarrow \beta=1-\alpha=1-\frac{1}{3}=\frac{2}{3}$
And α and γ are the roots of the equation $3x^2-10x+27\lambda=0$
$\therefore\ \alpha+\gamma=\frac{-(-10)}{3}=\frac{10}{3}$
$\Rightarrow\ \gamma=\frac{10}{3}-\alpha=\frac{10}{3}-\frac{1}{3}=\frac{9}{3}=3$
$\therefore\ \frac{\beta\gamma}{\lambda}=\frac{\left(\frac{2}{3}\right)\times(3)}{\left(\frac{1}{9}\right)}=18$

Question185

If $S=\left\{z \in C : \frac{z-i}{z+2i} \in R\right\}$, then
[2021, 27 Aug. Shift-1]

Options:

A. S contains exactly two elements.

B. S contains only one element.

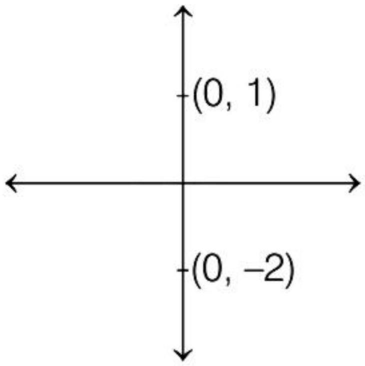
C. S is a circle in the complex plane.

D. S is a straight line in the complex plane.

Answer: D

Solution:

Solution:
Given, $\frac{z-i}{z+2i} \in R$



$\Rightarrow \arg\left(\frac{z-i}{z+2i}\right)=0$ or π
$\Rightarrow i, -2i, z$ are collinear.
$\Rightarrow S$ is a straight line in the complex plane.

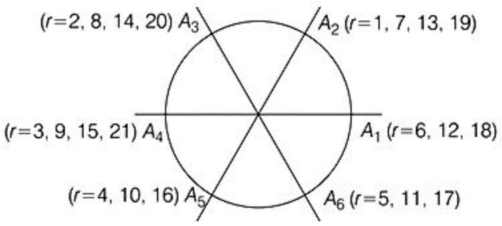
Question186

Let $z=\frac{1-i\sqrt{3}}{2}$ and $i=\sqrt{-1}$. Then the value of $21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$ is
[2021, 26 Aug. Shift-1]

Answer: 13

Solution:

Solution:
$z=\frac{1}{2}-\frac{\sqrt{3}}{2}i=e^{\frac{-i\pi}{3}}$
Again, $z^r+\frac{1}{z^r}=z^r+\bar{z}^r=2\mathrm{Re}(z^r)$
$[\because |z^r|=1]\quad =2\cos\left(\frac{r\pi}{3}\right)$
$21+\sum_{r=1}^{21}\left(z^r+\frac{1}{z^r}\right)^3=21+\sum_{r=1}^{21}8\cos^3\left(\frac{\pi r}{3}\right)$



Now, all the diametric ends will cancel out each other. Only a single value at A_L will remain which is -1.
So, 21+ 8(-1) $=13$

Question187

The equation $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle with
[2021, 26 Aug. Shift-I]

Options:

A. centre at $(0, -1)$ and radius $\sqrt{2}$

B. centre at $(0, 1)$ and radius $\sqrt{2}$

C. centre at $(0, 0)$ and radius $\sqrt{2}$

D. centre at $(0, 1)$ and radius 2

Answer: B

Solution:

Solution:
We have, $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4} \Rightarrow \arg(z-1) - \arg(z+1) = \frac{\pi}{4}$
Let $z = x + iy$
$\arg[(x-1)+iy] - \arg[(x+1)+iy] = \frac{\pi}{4}$
$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{\pi}{4}$
$\Rightarrow \left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \frac{y}{x-1} \cdot \frac{y}{x+1}}\right) = \tan\frac{\pi}{4}$
$\Rightarrow \frac{y(x+1) - y(x-1)}{(x^2-1) + y^2} = 1$
$\Rightarrow 2y = x^2 + y^2 - 1$
$\Rightarrow x^2 + y^2 - 2y - 1 = 0$
$\Rightarrow x^2 + (y-1)^2 = 2$
$\Rightarrow x^2 + (y-1)^2 = (\sqrt{2})^2$
Which is a circle with Centre $(0, 1)$ and Radius $= \sqrt{2}$ units

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Question188

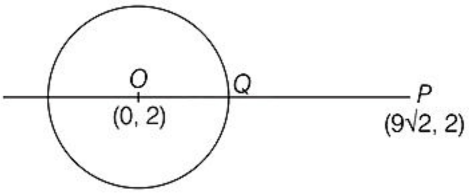
A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $|z - 9\sqrt{2} - 2i|^2$ equal to
[2021, 31 Aug. Shift-I]

Answer: 98

Solution:

Solution:
$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$
If
$z = x + iy$
$\arg[(x-2)+iy] - \arg[(x+2)+iy] = \frac{\pi}{4}$
$\Rightarrow \tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$
$\Rightarrow \frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \frac{y}{x-2} \cdot \frac{y}{x+2}} = \tan\left(\frac{\pi}{4}\right)$

$1 \Rightarrow \frac{xy + 2y - xy + 2y}{x^2 + y^2 - 4} = 1$
$\Rightarrow 4y = x^2 + y^2 - 4$
$\Rightarrow x^2 + y^2 - 4y - 4 = 0$
z is a circle.
Centre $= (0, 2)$, Radius $= (2\sqrt{2})$
$|z - 9\sqrt{2} - 2i|^2$ is the distance of $(9\sqrt{2}, 2)$ from any point on circle. Distance will be minimum when $(9\sqrt{2}, 2)$ will lie on the line joining the centre.



PQ $= OP - OQ$
$= 9\sqrt{2} - 2\sqrt{2} = 7\sqrt{2}$
$PO^2 = (7\sqrt{2})^2 = 98$

Question189

If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z-(3+3i)|$ is [2021, 31 Aug. Shift-II]

Options:

A. $2\sqrt{2}-1$

B. $6\sqrt{2}$

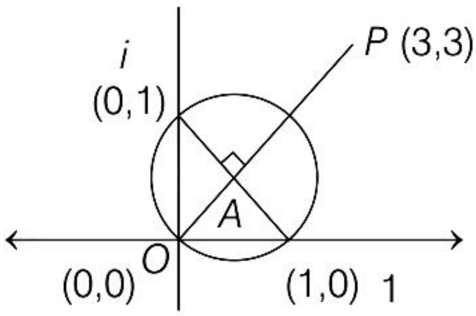
C. $3\sqrt{2}$

D. $2\sqrt{2}$

Answer: D

Solution:

Solution:
Let $z = x + iy$
c $\frac{z-i}{z-1} = \frac{x+i(y-1)}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy}$
$= \frac{x(x-1)+y(y-1)}{(x-1)^2+y^2} + i\left[\frac{(x-1)(y-1)-xy}{(x-1)^2+y^2}\right]$
As $\frac{z-i}{z-1}$ is purely imaginary,
$x^2+y^2-x-y = 0$
$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2 = 0$
This is a circle with centre $\left(\frac{1}{2}, \frac{1}{2}\right)$, radius $= \frac{1}{2}$ which passes through origin as shown in the figure.



Minimum $|z-(3+3i)| = OP - OA$
$\sqrt{(3-0)^2+(3-0)^2} - \sqrt{\left(\frac{1}{2}-0\right)^2+\left(\frac{1}{2}-0\right)^2}$
$= 3\sqrt{2} - \sqrt{2}$
$= 2\sqrt{2}$

Question190

The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}$, $i = \sqrt{-1}$, is a positive integer, is [2021, 26 Aug. Shift-11]

Answer: 6

Solution:

Solution:
We have,

$$\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(1-i)^n(1-i)^{-2}}$$

$$= \left(\frac{2i}{1-i}\right)^n (1-i)^2$$

$$= \left[\frac{2i(1+i)}{(1-i)(1+i)}\right]^n (1+i^2-2i)$$

$$= \left(\frac{2i-2}{2}\right)^n (1-1-2i)$$

$$= (i-1)^n(-2i)$$

If $n=1$, $(i-1)(-2i) = -2i^2+2i = 2+2i$
If $n=2$, $-2i(i-1)^2 = -2i(-2i) = -4$
If $n=4$, $-2i(i-1)^4 = -2i(-2i)(-2i) = 8i$
If $n=6$, $-2i(i-1)^6 = -2i(-2i)(-2i)(-2i) = 16$
So, least value of n for which given complex is positive is 6 .

Question191

The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2+ax+b=0$, α^2-2 is also a root of this equation, is [2021, 01 Sep. Shift-II]

Options:

A. 6

B. 2

C. 4

D. 8

Answer: A

Solution:

Solution:
Given equation $x^2+ax+b=0$
It has two roots (not necessarily real α and β)
$\Rightarrow$ Either $\alpha=\beta$ or $\alpha\neq\beta$
1. If $\alpha=\beta \Rightarrow \alpha=\alpha^2-2 \Rightarrow \alpha=-1, 2$
When $\alpha=-1$, then $(a, b)=(2, 1)$
When $\alpha=2$, then $(a, b)=(-4, 4)$
II. If $\alpha\neq\beta$, then
(a) $\alpha=\alpha^2-2$ and $\beta=\beta^2-2$
Here, $(\alpha, \beta)=(2, -1)$ or $(-1, 2)$
Hence $(a, b)=(-\alpha-\beta, \alpha\beta)=(-1, -2)$
(b) $\alpha=\beta^2-2$ and $\beta=\alpha^2-2$
Then $\alpha-\beta=\beta^2-\alpha^2=(\beta-\alpha)(\beta+\alpha)$
$\because\ \alpha\neq\beta$
$\Rightarrow\ \alpha+\beta=\beta^2+\alpha^2-4$
or $\alpha+\beta=(\alpha+\beta)^2-2\alpha\beta-4$
$\Rightarrow\ -1=1-2\alpha\beta-4 \Rightarrow \alpha\beta=-1$
$\Rightarrow\ (a, b)=(-\alpha-\beta, \alpha\beta)=(1, -1)$
(c) $\alpha=\alpha^2-2=\beta^2-2$ and $\alpha\neq\beta \Rightarrow \alpha=-\beta$
Thus, $\alpha=2, \beta=-2$
or $\alpha=-1, \beta=1$
$\therefore\ (a, b)=(0, -4)$ and $(0, -1)$
(d) $\beta=\alpha^2-2=\beta^2-2$ and $\alpha\neq\beta$(as in (c))
$\Rightarrow$ We get 6 pairs of (a, b)

They are $(2, 1), (-4, 4), (-1, -2), (1, -1)$,
$(0, -4)$, and $(0, -1)$.

Question192

If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta\in[0,2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:
[Jan. 7, 2020 (II)]

Options:

A. $\pi-\tan^{-1}\left(\frac{4}{3}\right)$

B. $\pi-\tan^{-1}\left(\frac{3}{4}\right)$

C. $-\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: A

Solution:

Solution:
Let $z = \frac{3+i\sin\theta}{4-i\cos\theta}$, after rationalising
$z = \frac{(3+i\sin\theta)}{(4-i\cos\theta)} \times \frac{(4+i\cos\theta)}{(4+i\cos\theta)}$
As z is purely real
$\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\frac{3}{4}$
$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$
$= \pi + \tan^{-1}\left(-\frac{4}{3}\right) = \pi - \tan^{-1}\left(\frac{4}{3}\right)$

Question193

Let z be a complex number such that $\left|\frac{Z-i}{Z+2i}\right| = 1$ and $|Z| = \frac{5}{2}$. Then the value of $|Z+3i|$ is:
[Jan. 9, 2020 (I)]

Options:

A. $\sqrt{10}$

B. $\frac{7}{2}$

C. $\frac{15}{4}$

D. $2\sqrt{3}$

Answer: B

Solution:

Solution:
Let $z = x + iy$
Then, $\left|\frac{z-i}{z+2i}\right| = 1 \Rightarrow x^2 + (y-1)^2$
$= x^2 + (y+2)^2 \Rightarrow -2y + 1 = 4y + 4$

$\Rightarrow\ 6y = -3 \Rightarrow y = -\frac{1}{2}$

$\because\ |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$

$\Rightarrow\ x^2 = \frac{24}{4} = 6$

$\therefore\ z = x + iy\ \ \Rightarrow z = \pm\sqrt{6} - \frac{i}{2}$

$|z + 3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$

$\Rightarrow\ |z + 3i| = \frac{7}{2}$

Question194

If z be a complex number satisfying |Re(z)|+ | I m(Z) | = 4, then |Z | cannot be: [Jan. 9, 2020 (II)]

Options:

A. $\sqrt{\frac{17}{2}}$

B. $\sqrt{10}$

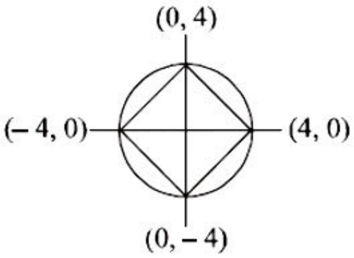
C. $\sqrt{7}$

D. $\sqrt{8}$

Answer: C

Solution:

Solution:
$z = x + iy\ |x| + |y| = 4$
$|z| = \sqrt{x^2 + y^2}$
Minimum value of
$|z| = 2\sqrt{2}$
Maximum value of
$|z| = 4\ |z| \in [\sqrt{8}, \sqrt{16}]$
So, $|z|$ can't be $\sqrt{7}$.



Question195

Let $\alpha = \frac{-1 + i\sqrt{3}}{2}$. If $a = (1 + \alpha)\sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation: [Jan. 8, 2020 (II)]

Options:

A. $x^2 + 101x + 100 = 0$

B. $x^2 - 102x + 101 = 0$

C. $x^2 - 101x + 100 = 0$

D. $x^2 + 102x + 101 = 0$

Answer: B

Solution:

Solution:
Let $\alpha = \omega$, $b = 1 + \omega^3 + \omega^6 + \ldots . = 101$
$a = (1 + \omega)(1 + \omega^2 + \omega^4 + \ldots . . \omega^{198} + \omega^{200})$
$= (1 + \omega) \frac{(1 - (\omega^2)^{101})}{1 - \omega^2} = \frac{(\omega + 1)(\omega^{202} - 1)}{(\omega^2 - 1)}$
$\Rightarrow \ a = \frac{(1 + \omega)(1 - \omega)}{1 - \omega^2} = 1$
Required equation $= x^2 - (101 + 1)x + (101) \times 1 = 0$
$\Rightarrow \ x^2 - 102x + 101 = 0$

Question196

If $\operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a : [Jan. 7, 2020 (I)]

Options:

A. circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.

B. straight line whose slope is $-\frac{2}{3}$.

C. straight line whose slope is $\frac{3}{2}$.

D. circle whose diameter is $\frac{\sqrt{5}}{2}$.

Answer: D

Solution:

Solution:
$\because z = x + iy$
$\left(\frac{z-1}{2z+i}\right) = \frac{(x-1) + iy}{2(x + iy) + i}$
$= \frac{(x-1) + iy}{2x + (2y+1)i} \times \frac{2x - (2y+1)i}{2x - (2y+1)i}$
$\operatorname{Re}\left(\frac{z+1}{2z+i}\right) = \frac{2x(x-1) + y(2y+1)}{(2x)^2 + (2y+1)^2} = 1$
$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{3}{4}\right)^2 = \left(\frac{\sqrt{5}}{4}\right)^2$

Question197

The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is: [Jan. 9, 2020 (I)]

Options:

A. 1

B. 3

C. 2

D. 4

Answer: A

Solution:

Solution:
Let $e^x = t \in (0, \infty)$
Given equation
$t^4 + t^3 - 4t^2 + t + 1 = 0$
$\Rightarrow t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$
$\Rightarrow \left(t^2 + \frac{1}{t^2}\right) + \left(t + \frac{1}{t}\right) - 4 = 0$
Let $t + \frac{1}{t} = y$
$(y^2 - 2) + y - 4 = 0 \quad \Rightarrow \quad y^2 + y - 6 = 0$
$y^2 + y - 6 = 0 \quad \Rightarrow \quad y = -3, 2$
$\Rightarrow \quad y = 2 \quad \Rightarrow t + \frac{1}{t} = 2$
$\Rightarrow \quad e^x + e^{-x} = 2$
$x = 0$, is the only solution of the equation
Hence, there only one solution of the given equation.

Question198

The least positive value of ' a ' for which the equation, $2x^2 + (a-10)x + \frac{33}{2} = 2a$ has real roots is ______

[Jan. 8, 2020 (I)]

Answer: 8

Solution:

Solution:
Since, $2x^2 + (a-10)x + \frac{33}{2} = 2a$ has real roots,
$\therefore \ D \geq 0$
$\Rightarrow \ (a-10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0$
$\Rightarrow \ (a-10)^2 - 4(33 - 4a) \geq 0$
$\Rightarrow \ a^2 - 4a - 32 \geq 0$
$\Rightarrow \ (a-8)(a+4) \geq 0$
$\Rightarrow \ a \leq -4 \cup a \geq 8$
$\Rightarrow \ a \in (-\infty, -4] \cup [8, \infty)$

Question199

If the equation, $x^2 + bx + 45 = 0 (b \in R)$ has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then: [Jan. 8, 2020 (I)]

Options:

A. $b^2 - b = 30$

B. $b^2 + b = 72$

C. $b^2 - b = 42$

D. $b^2 + b = 12$

Answer: A

Solution:

Solution:

Let $z = \alpha \pm i\beta$ be the complex roots of the equation
So, sum of roots $= 2\alpha = -b$ and
Product of roots $= \alpha^2 + \beta^2 = 45$
$(\alpha + 1)^2 + \beta^2 = 40$
Given, $|z + 1| = 2\sqrt{10}$
$\Rightarrow\ (\alpha + 1)^2 - \alpha^2 = -5\ [\because \beta^2 = 45 - \alpha^2]$
$\Rightarrow\ 2\alpha + 1 = -5\ \Rightarrow\ 2\alpha = -6$
Hence, $b = 6$ and $b^2 - b = 30$

Question200

Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true?
[Jan. 7, 2020 (II)]

Options:

A. $p_3 = p_5 - p_4$

B. $P_5 = 11$

C. $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

D. $p_5 = p_2 \cdot p_3$

Answer: D

Solution:

Solution:
$\alpha^5 = 5\alpha + 3$
$\beta^5 = 5\beta + 3$
$p_5 = 5(\alpha + \beta) + 6 = 5(1) + 6$
$\left[\because \text{ from } x^2 - x - 1 = 0, \alpha + \beta = \frac{-b}{a} = 1\right]$
$p_5 = 11$ and $p_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$
$p_2 = 3$ and $p_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1$
$= 2(1) + 2 = 4$
$p_2 \times p_3 = 12$ and $p_5 = 11 \Rightarrow p_5 \neq p_2 \times p_3$

Question201

Let α and $\overline{\beta}$ be two real roots of the equation $(k + 1)\tan^2 x - \sqrt{2} . \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is:
[Jan. 7, 2020 (I)]

Options:

A. $10\sqrt{2}$

B. 10

C. 5

D. $5\sqrt{2}$

Answer: B

Solution:

Solution:

$(k+1)\tan^2 x - \sqrt{2}\lambda \tan x + (k-1) = 0$
$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$ [Sum of roots]
$\tan\alpha \cdot \tan\beta = \frac{k-1}{k+1}$ [Product of roots]
$\therefore \tan(\alpha+\beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$
$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$
$\lambda = 10$

Question202

Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : [Jan. 9, 2020 (II)]

Options:

A. 25

B. 26

C. 28

D. 24

Answer: A

Solution:

Solution:
$ax^2 - 2bx + 5 = 0$
If α and α are roots of equations, then sum of roots
$2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$
and product of roots $= \alpha^2 = \frac{5}{a} \Rightarrow \frac{b^2}{a^2} = \frac{5}{a}$
$\Rightarrow b^2 = 5a \ (a \neq 0)$ . . .(i)
For $x^2 - 2bx - 10 = 0$
$\alpha + \beta = 2b$ . . .(ii)
and $\alpha\beta = -10$ . . .(iii)
$\alpha = \frac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$
$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$
By eqn. (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$
$\Rightarrow 20a^2 = 5a$
$\Rightarrow a = \frac{1}{4}$ and $b^2 = \frac{5}{4}$
$\alpha^2 = 20$ and $\beta^2 = 5$
Now, $\alpha^2 + \beta^2 = 5 + 20 = 25$

Question203

If the four complex numbers z, $\bar{z}$, $\bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then |z| is equal to : [Sep. 05, 2020 (I)]

Options:

A. $4\sqrt{2}$

B. 4

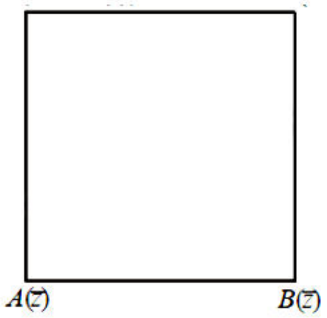
C. $2\sqrt{2}$

D. 2

Answer: C

Solution:

Solution:
$D(z-2\mathrm{Re}(z))$ $C(\bar{z}-2\mathrm{Re}(\bar{z}))$



Let $z=x+iy$
$\because$ Length of side of square $=4$ units
Then, $|z-\bar{z}|=4 \Rightarrow |2iy|=4 \Rightarrow |y|=2$
Also, $|z-(z-2\mathrm{Re}(z))|=4$
$\Rightarrow |2\mathrm{Re}(z)|=4 \Rightarrow |2x|=4 \Rightarrow |x|=2$
$\therefore |z|=\sqrt{x^2+y^2}=\sqrt{4+4}=2\sqrt{2}$

Question204

The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is:
[Sep. 05, 2020 (II)]

Options:

A. -2^{15}

B. $2^{15}i$

C. $-2^{15}i$

D. 6^5

Answer: C

Solution:

Solution:
$\because -1+\sqrt{3}i=2\cdot e^{\frac{2\pi}{3}i}$ and $1-i=\sqrt{2}\cdot e^{-\frac{i\pi}{4}}$
$\therefore \left(\frac{-1+\sqrt{3}i}{1-i}\right)^{30}=\left(\sqrt{2}e^{\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)i}\right)^{30}$
$=2^{15}\cdot e^{-\frac{\pi}{2}i}=-2^{15}\cdot i.$

Question205

If $\left(\frac{1+i}{1-i}\right)^{m/2}=\left(\frac{1+i}{i-1}\right)^{n/3}=1$, (n,m $\in$ N), then the greatest common divisor of the least values of m and n is________.
[Sep. 03, 2020 (I)]

Answer: 4

Solution:

Solution:
Given that $\left(\frac{1+i}{1-i}\right)^{m/2}=\left(\frac{1+i}{i-1}\right)^{n/3}=1$
$\Rightarrow\left(\frac{(1+i)^2}{2}\right)^{m/2}=\left(\frac{(1+i)^2}{-2}\right)^{n/3}=1$
$\Rightarrow i^{m/2}=(-i)^{n/3}=1$
m(least) = 8, n(least) = 12
GCD(8, 12) = 4

Question206

If Z_1, Z_2 are complex numbers such that $Re(z_1)=|Z_1-1|, Re(Z_2)=|Z_2-1|$ and $arg(Z_1-Z_2)=\frac{\pi}{6}$, then $Im(Z_1+Z_2)$ is equal to:
[Sep. 03, 2020 (II)]

Options:

A. $\frac{2}{\sqrt{3}}$

B. $2\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

Solution:
Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$
$\because |z_1-1|=Re(z_1)$
$\Rightarrow(x_1-1)^2+y_1^2=x_1^2$
$\Rightarrow y_1^2-2x_1+1|=0$ ....(i)
$|z_2-1|=Re(z_2)\Rightarrow(x_2-1)^2+y_2^2=x_2^2$
$\Rightarrow y_2^2-2x_2+1=0$ ....(ii)
From eqn. (i) - (ii)
$y_1^2-y_2^2-2(x_1-x_2)=0$
$\Rightarrow y_1+y_2=2\left(\frac{x_1-x_2}{y_1-y_2}\right)$ ....(iii)
$\because arg(z_1-z_2)=\frac{\pi}{6}$
$\Rightarrow\tan^{-1}\left(\frac{y_1-y_2}{x_1-x_2}\right)=\frac{\pi}{6}$
$\Rightarrow\frac{y_1-y_2}{x_1-x_2}=\frac{1}{\sqrt{3}}$
$\Rightarrow\frac{2}{y_1+y_2}=\frac{1}{\sqrt{3}}\left[\text{From, }\frac{y_1-y_2}{x_1-x_2}=\frac{2}{y_1+y_2}\right]$
$\therefore y_1+y_2=2\sqrt{3}\Rightarrow Im(z_1+z_2)=2\sqrt{3}$

Question207

Let $z=x+iy$ be a non-zero complex number such that $z^2=i|z|^2$, where $i=\sqrt{-1}$, then z lies on the:
[Sep. 06, 2020 (II)]

Options:

A. line, $y = -x$

B. imaginary axis

C. line, $y = x$

D. real axis

Answer: C

Solution:

Solution:
Let $z = x + iy$
$\because z^2 = i|z|^2$
$\therefore x^2 - y^2 + 2ixy = i(x^2 + y^2)$
$\Rightarrow x^2 - y^2 = 0$ and $2xy = x^2 + y^2$
$\Rightarrow (x - y)(x + y) = 0$ and $(x - y)^2 = 0$
$\Rightarrow x = y$

Question208

If a and b are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1 + i\sqrt{3}}{2}$, then $a + b$ is equal to : [Sep. 04, 2020 (II)]

Options:

A. 9

B. 24

C. 33

D. 57

Answer: A

Solution:

Solution:
Given that, $\alpha = \dfrac{-1 + \sqrt{3}i}{2} = \omega$
$\therefore (2 + \omega)^4 = a + b\omega \Rightarrow (4 + \omega^2 + 4\omega)^2 = a + b\omega$
$\Rightarrow (\omega^2 + 4(1 + \omega))^2 = a + b\omega$
$\Rightarrow (\omega^2 - 4\omega^2)^2 = a + b\omega$
$[\because 1 + \omega = -\omega^2]$
$\Rightarrow (-3\omega^2)^2 = a + b\omega \Rightarrow 9\omega^4 = a + b\omega$
$\Rightarrow 9\omega = a + b\omega \ (\because \omega^3 = 1)$
On comparing ,$a = 0, b = 9$
$\Rightarrow a + b = 0 + 9 = 9$

Question209

The value of $\left(\dfrac{1 + \sin\frac{2\pi}{9} + i\cos\frac{2\pi}{9}}{1 + \sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}} \right)^3$ is :

[Sep. 02, 2020 (I)]

Options:

A. $\frac{1}{2}(1 - i\sqrt{3})$

B. $\frac{1}{2}(\sqrt{3}-i)$

C. $-\frac{1}{2}(\sqrt{3}-i)$

D. $-\frac{1}{2}(1-i\sqrt{3})$

Answer: C

Solution:

Solution:

$$\left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$=\left(\frac{2\cos^2\frac{5\pi}{36}+i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}\right)^3$$

$$=\left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3=\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)^6$$

$$=\cos\left(6\times\frac{5\pi}{36}\right)+i\sin\left(6\times\frac{5\pi}{36}\right)=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$=-\frac{\sqrt{3}}{2}+i\frac{1}{2}=-\frac{1}{2}(\sqrt{3}-i)$$

Question210

The imaginary part of $(3+2\sqrt{-54})^{1/2}-(3-2\sqrt{-54})^{1/2}$ can be: [Sep. 02, 2020 (II)]

Options:

A. $-\sqrt{6}$

B. $-2\sqrt{6}$

C. 6

D. $\sqrt{6}$

Answer: B

Solution:

Solution:
$3+2\sqrt{-54}=3+6\sqrt{6}i$
Let $\sqrt{3+6\sqrt{6}i}=a+ib$
$\Rightarrow a^2-b^2=3$ and $ab=3\sqrt{6}$
$\Rightarrow a^2+b^2=\sqrt{(a^2-b^2)^2+4a^2b^2}=15$
So, $a=\pm3$ and $b=\pm\sqrt{6}$
$\sqrt{3+6\sqrt{6}i}=\pm(3+\sqrt{6}i)$
Similarly, $\sqrt{3-6\sqrt{6}i}=\pm(3-\sqrt{6}i)$
$\operatorname{Im}\left(\sqrt{3+6\sqrt{6}i}-\sqrt{3-6\sqrt{6}i}\right)=\pm2\sqrt{6}$

Question211

If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is:

[Sep. 06, 2020 (I)]

Options:

A. 2

B. 3

C. 1

D. 4

Answer: A

Solution:

Solution:
$\because \alpha + \beta = 64, \alpha\beta = 256$
$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(2^8)^{5/8}} = \frac{64}{32} = 2$

Question212

If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to:
[Sep. 06, 2020 (II)]

Options:

A. $2\alpha(\alpha + 1)$

B. $-2\alpha(\alpha + 1)$

C. $2\alpha(\alpha - 1)$

D. $2\alpha^2$

Answer: B

Solution:

Solution:
Let α and β be the roots of the given quadratic equation,
$2x^2 + 2x - 1 = 0$
Then, $\alpha + \beta = -\frac{1}{2} \Rightarrow -1 = 2\alpha + 2\beta$
and $4\alpha^2 + 2\alpha - 1 = 0$ [$\because \alpha$ is root of eq. (i)]
$\Rightarrow 4\alpha^2 + 2\alpha + 2\alpha + 2\beta = 0 \Rightarrow \beta = -2\alpha(\alpha + 1)$

Question213

The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is:
[Sep. 05, 2020 (I)]

Options:

A. $\frac{5}{9}$

B. $\frac{25}{81}$

C. $\frac{5}{27}$

D. $\frac{25}{9}$

Answer: B

Solution:

Solution:
Let $|x| = y$ then
$9y^2 - 18y + 5 = 0$
$\Rightarrow 9y^2 - 15y - 3y + 5 = 0$
$\Rightarrow (3y-1)(3y-5) = 0$
$\Rightarrow y = \frac{1}{3}$ or $\frac{5}{3} \Rightarrow |x| = \frac{1}{3}$ or $\frac{5}{3}$
Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$
Product $= \frac{25}{81}$

Question214

If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$ the the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to : [Sep. 05, 2020 (II)]

Options:

A. $\frac{27}{32}$

B. $\frac{1}{24}$

C. $\frac{3}{8}$

D. $\frac{27}{16}$

Answer: D

Solution:

Solution:
Let α and β be the roots of the quadratic equation $7x^2 - 3x - 2 = 0$
$\therefore \alpha + \beta = \frac{3}{7}, \alpha\beta = \frac{-2}{7}$
Now, $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$
$= \frac{\alpha - \alpha\beta(\alpha+\beta) + \beta}{1-(\alpha^2+\beta^2)+(\alpha\beta)^2}$
$= \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1-(\alpha+\beta)^2 + 2\alpha\beta + (\alpha\beta)^2}$
$= \frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} + 2\times\frac{-2}{7} + \frac{4}{49}} = \frac{27}{16}$

Question215

Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by $\mathrm{Re}(u) + \mathrm{I\,m}(u) = 1$ intersects the y -axis at the points P and Q where PQ = 5, then the value of k is : [Sep. 04, 2020 (I)]

Options:

A. $3/2$

B. 1/2

C. 4

D. 2

Answer: D

Solution:

Solution:
$u=\frac{2(x+iy)+i}{(x+iy)-ki}=\frac{2x+i(2y+1)}{x+i(y-k)}$
Real part of $u=\operatorname{Re}(u)=\frac{2x^2+(y-K)(2y+1)}{x^2+(y-K)^2}$
Imaginary part of u
$=\operatorname{I}m(u)=\frac{-2x(y-K)+x(2y+1)}{x^2+(y-K)^2}$
$\because \operatorname{Re}(u)+\operatorname{I}m(u)=1$
$\Rightarrow 2x^2+2y^2-2Ky+y-K-2xy+2Kx+2xy+x$
$=x^2+y^2+K^2-2Ky$
Since, the curve intersect at y -axis
$\therefore x=0$
$\Rightarrow y^2+y-K(K+1)=0$
Let y_1 and y_2 are roots of equations if $x=0$
$\because y_1+y_2=-1$
$y_1y_2=-(K^2+K)$
$\therefore (y_1-y_2)^2=(1+4K^2+4K)$
Given $PQ=5\Rightarrow |y_1-y_2|=5$
$\Rightarrow 4K^2+4K-24=0\Rightarrow K=2$ or -3
as $K>0, \therefore K=2$

Question216

Let $\lambda\neq 0$ be in R. If α and β are roots of the equation, $x^2-x+2\lambda=0$ and α and γ are the roots of the equation, $3x^2-10x+27\lambda=0$, then $\frac{\beta\gamma}{\lambda}$ is equal to
[Sep. 04, 2020 (II)]

Options:

A. 27

B. 18

C. 9

D. 36

Answer: B

Solution:

Solution:
Since α is common root of $x^2-x+2\lambda=0$ and $3x^2-10x+27\lambda=0$
$\therefore 3\alpha^2-10\alpha+27\lambda=0$ ....(i)
$3\alpha^2-3\alpha+6\lambda=0$ ....(ii)
$\therefore$ On subtract, we get $\alpha=3\lambda$
Now, $\alpha\beta=2\lambda\Rightarrow 3\lambda\cdot\beta=2\lambda\Rightarrow\beta=\frac{2}{3}$
$\Rightarrow\alpha+\beta=1\Rightarrow 3\lambda+\frac{2}{3}=1\Rightarrow\lambda=\frac{1}{9}$ and
$\alpha\gamma=9\lambda\Rightarrow 3\lambda\cdot\gamma=9\lambda\Rightarrow\gamma=3$
$\therefore \frac{\beta\gamma}{\lambda}=18$

Question217

If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$ then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to
[Sep. 03, 2020 (I)]

Options:

A. $\frac{9}{4}(9 + q^2)$

B. $\frac{9}{4}(9 - q^2)$

C. $\frac{9}{4}(9 + p^2)$

D. $\frac{9}{4}(9 - p^2)$

Answer: D

Solution:

Solution:
$\alpha \cdot \beta = 2$ and $\alpha + \beta = -p$ also $\frac{1}{\alpha} + \frac{1}{\beta} = -q$
$\Rightarrow p = 2q$
Now $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$
$= \left[\alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right]\left[\alpha\beta + \frac{1}{\alpha\beta} + 2\right]$
$= \frac{9}{2}\left[\frac{5}{2} - \frac{\alpha^2 + \beta^2}{2}\right] = \frac{9}{4}[5 - (p^2 - 4)]$
$= \frac{9}{4}(9 - p^2)$ $[\because \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta]$

Question218

The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0,1) is :
[Sep. 03, 2020 (II)]

Options:

A. (0,2)

B. (2,4]

C. (1,3]

D. (-3,-1)

Answer: C

Solution:

Solution:
The given quadratic equation is
$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$
$\because$ One root is in the interval (0,1)
$\therefore f(0)f(1) \le 0$
$\Rightarrow 2(\lambda^2 + 1 - 4\lambda + 2) \le 0$
$\Rightarrow 2(\lambda^2 - 4\lambda + 3) \le 0$
$(\lambda - 1)(\lambda - 3) \le 0 \Rightarrow \lambda \in [1, 3]$
But at $\lambda = 1$, both roots are 1 so $\lambda \ne 1$
$\therefore \lambda \in (1, 3]$

Question219

Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3, ..., then : [Sep. 02, 2020 (I)]

Options:

A. $6S_6 + 5S_5 = 2S_4$

B. $6S_6 + 5S_5 + 2S_4 = 0$

C. $5S_6 + 6S_5 = 2S_4$

D. $5S_6 + 6S_5 + 2S_4 = 0$

Answer: C

Solution:

Solution:
Since, α and β are the roots of the equation
$5x^2 + 6x - 2 = 0$
Then, $5\alpha^2 + 6\alpha - 2 = 0, 5\beta^2 + 6\beta - 2 = 0$
$5\alpha^2 + 6\alpha = 2$
$5S_6 + 6S_5 = 5(\alpha^6 + \beta^6) + 6(\alpha^5 + \beta^5)$
$= (5\alpha^4 + 6\alpha^5) + (5\beta^6 + 6\beta^5)$
$= \alpha^4(5\alpha^2 + 6\alpha) + \beta^4(5\beta^2 + 6\beta)$
$= 2(\alpha^4 + \beta^4) = 2S_4$

Question220

If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is: [Jan. 12, 2019 (I)]

Options:

A. $2 - \sqrt{3}$

B. $4 - 3\sqrt{2}$

C. $-2 + \sqrt{2}$

D. $4 - 2\sqrt{3}$

Answer: B

Solution:

Solution:
Let roots of the quadratic equation are α, β.
Given, $\lambda = \frac{\alpha}{\beta}$ and $\lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$
$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = 1 \ldots \text{(i)}$
The quadratic equation is, $3m^2x^2 + m(m-4)x + 2 = 0$
$\therefore \alpha + \beta = \frac{m(4-m)}{3m^2} = \frac{4-m}{3m}$ and $\alpha\beta = \frac{2}{3m^2}$
Put these values in eq (1)
$$\frac{\left(\frac{4-m}{3m}\right)^2}{\frac{2}{3m^2}} = 3$$

$\Rightarrow (m-4)^2 = 18 \Rightarrow m = 4 \pm \sqrt{18}$
Therefore, least value is
$4-\sqrt{18} = 4-3\sqrt{2}$

Question221

If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is :
[Jan. 11, 2019 (I)]

Options:

A. -81

B. 100

C. 144

D. -300

Answer: D

Solution:

Solution:
Let α and β be the roots of the equation,
$81x^2 + kx + 256 = 0$
Given $(\alpha)^{\frac{1}{3}} = \beta \Rightarrow \alpha = \beta^3$
$\because$ Product of the roots $= \frac{256}{81}$
$\therefore (\alpha)(\beta) = \frac{256}{81}$
$\Rightarrow \beta^4 = \left(\frac{4}{3}\right)^4 \Rightarrow \beta = \frac{4}{3} \Rightarrow \alpha = \frac{64}{27}$
Sum of the roots $= -\frac{k}{81}$
$\therefore \alpha + \beta = -\frac{k}{81} \Rightarrow \frac{4}{3} + \frac{64}{27} = -\frac{k}{81}$
$\Rightarrow k = -300$

Question222

Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$ $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3) . Then the number of elements in S is:
[Jan. 10, 2019 (I)]

Options:

A. 18

B. 12

C. 10

D. 11

Answer: D

Solution:

Solution:
Consider the quadratic equation
$(c-5)x^2 - 2cx + (c-4) = 0$
Now, $f(0), f(3) > 0$ and $f(0) . f(2) < 0$
$\Rightarrow (c-4)(4c-49) > 0$ and $(c-4)(c-24) < 0$

$\Rightarrow c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty\right)$ and $c \in (4, 24)$

$\Rightarrow c \in \left(\frac{49}{4}, 24\right)$

Integral values in the interval $\left(\frac{49}{4}, 24\right)$ are $13, 14, \ldots, 23$

$\therefore\ S = \{13, 14, \ldots, 23\}$

Question223

The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is:
[Jan. 10, 2019 (II)]

Options:

A. $\frac{15}{8}$

B. 1

C. $\frac{4}{9}$

D. 2

Answer: D

Solution:

Solution:
The given quadratic equation is
$x^2 + (3 - \lambda)x + 2 = \lambda$
Sum of roots $= \alpha + \beta = \lambda - 3$
Product of roots $= \alpha\beta = 2 - \lambda$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (\lambda - 3)^2 - 2(2 - \lambda)$
$= \lambda^2 - 4\lambda + 5 = (\lambda - 2)^2 + 1$
For least $(\alpha^2 + \beta^2)\lambda = 2$

Question224

Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:
[Jan. 9, 2019 (I)]

Options:

A. -256

B. 512

C. -512

D. 256

Answer: A

Solution:

Solution:
Consider the equation
$x^2 + 2x + 2 = 0$
$x = \dfrac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i$
Let $\alpha = -1 + i, \beta = -1 - i$
$\alpha^{15} + \beta^{15} = (-1 + i)^{15} + (-1 - i)^{15}$

$= \left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15} + \left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$
$= (\sqrt{2})^{15}\left[e^{\frac{i45\pi}{4}} + e^{\frac{-i45\pi}{4}}\right]$
$= (\sqrt{2})^{15} \cdot 2\cos\frac{45\pi}{4} = (\sqrt{2})^{15} \cdot 2\cos\frac{3\pi}{4}$
$= \frac{-2}{\sqrt{2}}(\sqrt{2})^{15}$
$= -2(\sqrt{2})^{14} = -256$

Question225

The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is:
[Jan. 09, 2019 (II)]

Options:

A. 3

B. 2

C. 4

D. 5

Answer: A

Solution:

Solution:
The roots of $6x^2 - 11x + \alpha = 0$ are rational numbers.
$\therefore$ Discriminant D must be perfect square number.
$D = (-11)^2 - 4 \cdot 6 \cdot \alpha$
$= 121 - 24\alpha$ must be a perfect square
Hence, possible values for α are
$\alpha = 3, 4, 5$
$\therefore$3 positive integral values are possible.

Question226

If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval:
[Jan. 09, 2019 (II)]

Options:

A. (-5,-4)

B. (4,5)

C. (5,6)

D. (3,4)

Answer: B

Solution:

Solution:
Given quadratic equation is: $x^2 - mx + 4 = 0$
Both the roots are real and distinct.
So, discriminant $B^2 - 4AC > 0$
$\therefore\ m^2 - 4 \cdot 1 \cdot 4 > 0$
$\therefore\ (m-4)(m+4) > 0$
$\therefore\ m \in (-\infty, -4) \cup (4, \infty)$ ....(i)
Since, both roots lies in [1,5]

$\therefore\ -\frac{-m}{2} \in (1, 5)$
$\Rightarrow\ m \in (2, 10)$
And $1 \cdot (1 - m + 4) > 0 \Rightarrow m < 5$
$\therefore\ m \in (-\infty, 5)\ \ldots$ (iii)
And $1 \cdot (25 - 5m + 4) > 0 \Rightarrow m < \frac{29}{5}$
$\therefore\ m \in \left(-\infty\ \frac{29}{5}\right)\ \ldots(iv)$
From (i), (ii), (iii) and (iv), $m \in (4, 5)$

Question227

Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to: [Jan. 09, 2019 (II)]

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. 0

Answer: A

Solution:

Solution:
$\because z_0$ is a root of quadratic equation
$x^2 + x + 1 = 0$
$\therefore\ z_0 = \omega$ or $\omega^2 \Rightarrow z_0^3 = 1$
$\therefore\ z = 3 + 6iz_0^{81} - 3iz_0^{93}$
$= 3 + 6i((z_0)^3)^{27} - 3i((z_0)^3)^{31}$
$= 3 + 6i - 3i$
$= 3 + 3i$
$\therefore\ \arg(z) \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$

Question228

If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to: [Jan. 10, 2019 (I)]

Options:

A. $\frac{3}{4}$

B. $\frac{5}{4}$

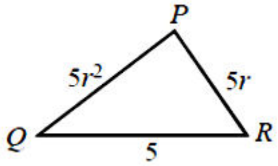
C. $\frac{7}{4}$

D. $\frac{3}{2}$

Answer: C

Solution:

Solution:



ΔPQR is possible if

$5+5r>5r^2$

$\Rightarrow\ 1+r>r^2$

$\Rightarrow\ r^2-r-1<0$

$\Rightarrow\left(r-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\left(r-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)<0$

$\Rightarrow r\in\left(\frac{-\sqrt{5}+1}{2},\frac{\sqrt{5}+1}{2}\right)$

$\because\ \frac{7}{4}\notin\left(\frac{-\sqrt{5}+1}{2},\frac{\sqrt{5}+1}{2}\right)\ \therefore r\neq\frac{7}{4}$

Question229

If $\frac{z-\alpha}{z+\alpha}(\alpha\in R)$ is a purely imaginary number and $|z|=2$, then a value of α is:
[Jan. 12, 2019 (I)]

Options:

A. 2

B. 1

C. $\frac{1}{2}$

D. $\sqrt{2}$

Answer: A

Solution:

Solution:

Let $t=\frac{z-\alpha}{z+\alpha}$

$\because$ t is purely imaginary number.

$\therefore\ t+\bar{t}=0$

$\Rightarrow\ \frac{z-\alpha}{z+\alpha}+\frac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$

$\Rightarrow\ (z-\alpha)(\bar{z}+\alpha)+(\bar{z}-\alpha)(z+\alpha)=0$

$\Rightarrow\ z\bar{z}-\alpha^2+z\bar{z}-\alpha^2=0$

$\Rightarrow\ z\bar{z}-\alpha^2=0$

$\Rightarrow\ |z|^2-\alpha^2=0$

$\Rightarrow\ \alpha^2=4$

$\Rightarrow\ \alpha=\pm 2$

Question230

Let z_1 and z_2 be two complex numbers satisfying $|z_1|=9$ and $|z_2|-|3|-|4i||=|4$. Then the minimum value of $|z_1-z_2|$ is :
[Jan. 12, 2019 (II)]

Options:

A. 0

B. $\sqrt{2}$

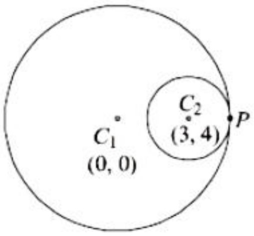
C. 1

D. 2

Answer: A

Solution:

Solution:
$|z_1| = 9, |z_2 - 3 - 4i| = 4$
z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$
z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$
So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



Question231

Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$) Then $|z|$ is equal to : [Jan. 11, 2019 (II)]

Options:

A. $\frac{\sqrt{34}}{3}$

B. $\frac{5}{3}$

C. $\frac{\sqrt{41}}{4}$

D. $\frac{5}{4}$

Answer: B

Solution:

Solution:
Since, $|z| + z = 3 + i$
Let $z = a + ib$, then
$|z| + z = 3 + i \Rightarrow \sqrt{a^2 + b^2} + a + ib = 3 + i$
Compare real and imaginary coefficients on both sides
$b = 1, \sqrt{a^2 + b^2} + a = 3$
$\sqrt{a^2 + 1} = 3 - a$
$a^2 + 1 = a^2 + 9 - 6a$
$6a = 8 \Rightarrow a = \frac{4}{3}$
Then
$|z| = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}$

Question232

Let z_1 and z_2 be any two non-zero complex numbers such that $3z_1| = 4z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then [Jan. 10 2019 (II)]

Options:

A. $Re(z) = 0$

B. $|z| = \sqrt{\frac{5}{2}}$

C. $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$

D. $I\,m(z) = 0$

E. None of Above

Answer: E

Solution:

Solution:
(none)
Let $z_1 = r_1e^{i\theta}$ and $z_2 = r_2e^{i\varphi}$
$3\,|\,z_1\,| \; = 4\,|\,z_2\,| \Rightarrow 3r_1 = 4r_2$
$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = \frac{3r_1}{2r_2}e^{i(\theta-\varphi)} + \frac{2}{3}\frac{r_2}{r_1}e^{i(\varphi-\theta)}$
$= \frac{3}{2}\times\frac{4}{3}(\cos(\theta-\varphi) + i\sin(\theta-\varphi)) +$
$\frac{2}{3}\times\frac{3}{4}[\cos(\theta-\varphi) - i\sin(\theta-\varphi)]$
$z = \left(2 + \frac{1}{2}\right)\cos(\theta-\varphi) + i\left(2 - \frac{1}{2}\right)\sin(\theta-\varphi)$
$\therefore \; |\,z\,| = \sqrt{\frac{25}{4}\cos^2(\theta-\varphi) + \frac{9}{4}\sin^2(\theta-\varphi)}$
$= \sqrt{\frac{16}{4}\cos^2(\theta-\varphi) + \frac{9}{4}} \Rightarrow \frac{3}{2} \leq |z| \leq \frac{5}{2}$

Question233

Let A = $\left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta}\right.$ is purely imaginary }.
Then the sum of the elements in A is:
[Jan. 9 2019 (I)]

Options:

A. $\frac{5\pi}{6}$

B. π

C. $\frac{3\pi}{4}$

D. $\frac{2\pi}{3}$

Answer: D

Solution:

Solution:
Suppose $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$
Since, z is purely imaginary, then $z + \bar{z} = 0$
$\Rightarrow \frac{3+2i\sin\theta}{1-2i\sin\theta} + \frac{3-2i\sin\theta}{1+2i\sin\theta} = 0$
$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta) + (3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta}$
$= 0$
$\Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$
$\Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$
Now, the sum of elements in $A = -\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$

Question234

Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}(i=\sqrt{-1})$, where x and y are real numbers then $y-x$ equals:
[Jan. 11, 2019 (I)]

Options:

A. 91

B. -85

C. 85

D. -91

Answer: A

Solution:

Solution:
$-(6+i)^3=x+iy$
$\Rightarrow\ -[216+i^3+18i(6+i)]=x+iy$
$\Rightarrow-[216-i+108i-18]=x+iy$
$\Rightarrow\ -216+i-108i+18=x+iy$
$\Rightarrow\ -198-107i=x+iy$
$\Rightarrow\ x=-198,\ y=-107$
$\Rightarrow\ y-x=-107+198=91$

Question235

Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I (z) respectively denote the real and imaginary parts of z, then:
[Jan. 10, 2019 (II)]

Options:

A. $I(z)=0$

B. $R(z)>0$ and $I(z)>0$

C. $R(z)<0$ and $I(z)>0$

D. $R(z)=-(c)$

Answer: A

Solution:

Solution:
$z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$
$=\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^5+\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^5$
$=\left(e^{i\frac{\pi}{6}}\right)^5+\left(e^{-i\frac{\pi}{6}}\right)^5=2\cos\frac{\pi}{6}=\sqrt{3}$
$\Rightarrow\ I(z)=0,\ Re(z)=\sqrt{3}$

Question236

Let z ∈C with I m(Z) = 10 and it satisfies $\frac{2z-n}{2z+n}=2i-1$ for some natural number n.Then: [April 12, 2019 (II)]

Options:

A. $n=20$ and $\mathrm{Re}(z)=-10$

B. $n=40$ and $\mathrm{Re}(z)=10$

C. $n=40$ and $\mathrm{Re}(z)=-10$

D. $n=20$ and $\mathrm{Re}(z)=10$

Answer: C

Solution:

Solution:
Let $\mathrm{Re}(z)=x$ i.e., $z=x+10i$
$2z-n=(2i-1)(2z+n)$
$(2x-n)+20i=(2i-1)((2x+n)+20i)$
On comparing real and imaginary parts,
$-(2x+n)-40=2x-n$ and $20=4x+2n-20$
$\Rightarrow 4x=-40$ and $40=-40+2n$
$\Rightarrow x=-10$ and $n=40$
Hence, $\mathrm{Re}(z)=-10$

Question237

The equation $|Z-i|=|Z-1|, i=\sqrt{-1}$, represents: [April 12, 2019 (I)]

Options:

A. a circle of radius $\frac{1}{2}$

B. the line through the origin with slope 1.

C. a circle of radius 1.

D. the line through the origin with slope -1.

Answer: B

Solution:

Solution:
Given equation is, $|z-1|=|z-i|$
$\Rightarrow\ (x-1)^2+y^2=x^2+(y-1)^2$ [Here ,$z=x+iy$]
$\Rightarrow\ 1-2x=1-2y \Rightarrow x-y=0$
Hence, locus is straight line with slope 1.

Question238

if a > 0 and $Z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to:

[April 10, 2019 (I)]

Options:

A. $-\frac{1}{2}-\frac{3}{5}i$

B. $-\frac{3}{5}-\frac{1}{5}i$

C. $\frac{1}{5}-\frac{3}{5}i$

D. $-\frac{1}{5}+\frac{3}{5}i$

Answer: A

Solution:

Solution:
$z = \frac{(1+i)^2}{a-i} \times \frac{a+i}{a+i}$
$z = \frac{(1-1+2i)(a+i)}{a^2+1} = \frac{2ai-2}{a^2+1}$
$|z| = \sqrt{\left(\frac{-2}{a^2+1}\right)^2 + \left(\frac{2a}{a^2+1}\right)^2} = \sqrt{\frac{4+4a^2}{(a^2+1)^2}}$
$\Rightarrow |z| = \sqrt{\frac{4(1+a^2)}{(1+a^2)^2}} = \frac{2}{\sqrt{1+a^2}}$ ....(i)
Since, it is given that $|z| = \sqrt{\frac{2}{5}}$
Then, from equation (i),
$\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{1+a^2}}$
Now, square on both side; we get
$\Rightarrow 1 + a^2 = 10 \Rightarrow a = \pm 3$
Since, it is given that $a > 0 \Rightarrow a = 3$ Then, $z = \frac{(1+i)^2}{a-i} = \frac{1+i^2+2i}{3-i} = \frac{2i}{3-i}$
$= \frac{2i(3+i)}{10} = \frac{-1+3i}{5}$
Hence, $\bar{z} = \frac{-1}{5} - \frac{3}{5}i$

Question239

If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then: [April 10, 2019 (II)]

Options:

A. $\bar{z}\omega = i$

B. $z\bar{\omega} = \frac{-1+i}{\sqrt{2}}$

C. $\bar{z}\omega = -i$

D. $z\bar{\omega} = \frac{1-i}{\sqrt{2}}$

Answer: C

Solution:

Solution:
Given $|z\omega| = 1 \ldots$ (i)
and $\arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$
$\therefore \frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0 \quad \left[\because \mathrm{Re}\left(\frac{z}{\omega}\right) = 0\right]$
$\Rightarrow \bar{z}\omega = -z\bar{\omega}$
from equation (i), $z\bar{z}\omega\bar{\omega} = 1$ [using $z\bar{z} = |z|^2$]
$(\bar{z}\omega)^2 = -1 \Rightarrow \bar{z}\omega = \pm i$
from equation (ii), $-\arg(\bar{z}) - \arg\omega = \frac{\pi}{2} - \arg(\bar{z}\omega) = \frac{-\pi}{2}$
Hence, $\bar{z}w = -i$

Question240

Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then :
[April 09, 2019 (II)]

Options:

A. $5\text{Re}(\omega) > 4$

B. $4\text{I m}(\omega) > 5$

C. $5\text{Re}(\omega) > 1$

D. $5\text{I m}(\omega) < 1$

Answer: C

Solution:

Solution:
$\omega = \frac{5+3z}{5-5z} \Rightarrow 5\omega - 5\omega z = 5 + 3z$
$\Rightarrow 5\omega - 5 = z(3+5\omega) \Rightarrow z = \frac{5(\omega-1)}{3+5\omega}$
$\because |z| < 1, \therefore 5|\omega - 1| < |3 + 5\omega|$
$\Rightarrow 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) < 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega}$
$(\because |z|^2 = z\bar{z})$
$\Rightarrow 16 < 40\omega + 40\bar{\omega} \Rightarrow \omega + \bar{\omega} > \frac{2}{5} \Rightarrow 2\text{Re}(\omega) > \frac{2}{5}$
$\Rightarrow \text{Re}(\omega) > \frac{1}{5}$

Question241

If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}(i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to:
[April 08, 2019 (II)]

Options:

A. 0

B. 1

C. $(-1+2i)^9$

D. -1

Answer: D

Solution:

Solution:
$\left(\frac{\sqrt{3}}{2} + \frac{i}{2} = -i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -i\omega\right.$
where ω is imaginary cube root of unity.
Now, $(1 + iz + z^5 + iz^8)^9$
$= (1 + \omega - i\omega^2 + i\omega^2)^9 = (1+\omega)^9$
$= (-\omega^2)^9 = -\omega^{18} = -1 \quad (\because 1 + \omega + \omega^2 = 0)$

Question242

If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 00, \theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})(\alpha-\beta)^{24}}$ is equal to :
[April 10, 2019 (I)]

Options:

A. $\frac{2^{12}}{(\sin\theta-4)^{12}}$

B. $\frac{2^{12}}{(\sin\theta+8)^{12}}$

C. $\frac{2^{12}}{(\sin\theta-8)^{6}}$

D. $\frac{2^{6}}{(\sin\theta+8)^{12}}$

Answer: B

Solution:

Solution:
Given equation is,
$x^2+x\sin\theta-2\sin\theta=0$
$\alpha+\beta=-\sin\theta$ and $\alpha\beta=-2\sin\theta$
$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$
$\therefore\ |\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\sin^2\theta+8\sin\theta}$
$\therefore\ \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$

Question243

The number of real roots of the equation $5+|2^x-1|=2^x(2^x-2)$ is: [April 10, 2019 (II)]

Options:

A. 3

B. 2

C. 4

D. 1

Answer: D

Solution:

Solution:
Let $2^x-1=t$
$5+|t|=(t+1)(t-1)\Rightarrow|t|=t^2-6$
When $t>0,t^2-t-6=0\Rightarrow t=3$ or -2
$t=-2$(rejected)
When $t<0,t^2+t-6=0\Rightarrow t=-3$ or 2 (both rejected)
$\therefore 2^x-1=3\Rightarrow 2^x=4\Rightarrow x=2$

Question244

Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2+px+q=0$, then: [April 10, 2019 (II)]

Options:

A. $p^2-4q+12=0$

B. $q^2-4p-16=0$

C. $q^2 + 4p + 14 = 0$

D. $p^2 - 4q - 12 = 0$

Answer: D

Solution:

Solution:
Since $2-\sqrt{3}$ is a root of the quadratic equation
$x^2 + px + q = 0$
$\therefore 2+\sqrt{3}$ is the other root
$\Rightarrow x^2 + px + q = [x-(2-\sqrt{3})][x-(2+\sqrt{3})]$
$= x^2 - (2+\sqrt{3})x - (2-\sqrt{3})x + (2^2 - (\sqrt{3})^2)$
$= x^2 - 4x + 1$
Now, by comparing $p = -4$, $q = 1$
$\Rightarrow p^2 - 4q - 12 = 16 - 4 - 12 = 0$

Question245

If m is chosen in the quadratic equation $(m^2+1)x^2 - 3x + (m^2+1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is: [April 09, 2019 (II)]

Options:

A. $10\sqrt{5}$

B. $8\sqrt{3}$

C. $8\sqrt{5}$

D. $4\sqrt{3}$

Answer: C

Solution:

Solution:
Sum of roots $= \dfrac{3}{m^2+1}$
$\because$ sum of roots is greatest. $\therefore m = 0$
Hence equation becomes $x^2 - 3x + 1 = 0$
Now, $\alpha + \beta = 3$, $\alpha\beta = 1 \Rightarrow |-\alpha - \beta| = \sqrt{5}$
$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)| = \sqrt{5}(9-1) = 8\sqrt{5}$

Question246

The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, $(x > 0)$ is equal to: [April 8, 2019 (I)]

Options:

A. 9

B. 12

C. 4

D. 10

Answer: D

Solution:

Solution:
Let $\sqrt{x} = a$
$\therefore$ given equation will become:
$|a-2| + a(a-4) + 2 = 0$
$\Rightarrow \ |a-2| + a^2 - 4a + 4 - 2 = 0$
$\Rightarrow |a-2| + (a-2)^2 - 2 = 0$
Let $|a-2| = y$(Clearly $y \geq 0$)
$\Rightarrow \ y + y^2 - 2 = 0$
$\Rightarrow \ y = 1$ or -2 (rejected) $\Rightarrow \ |a-2| = 1 \Rightarrow a = 1, 3$
When $\sqrt{x} = 1 \Rightarrow x = 1$
When $\sqrt{x} = 3 \Rightarrow x = 9$
Hence, the required sum of solutions of the equation
$= 10$

Question247

If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is: [April 8, 2019 (I)]

Options:

A. 2

B. 5

C. 4

D. 3

Answer: C

Solution:

Solution:
The given quadratic equation is $x^2 - 2x + 2 = 0$
Then, the roots of the this equation are $\frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$
Now, $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$
or $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = -i$ So , $\frac{\alpha}{\beta} = \pm i$
Now, $\left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$
$\Rightarrow$ n must be a multiple of 4
Hence, the required least value of $n = 4$

Question248

The set of all α ∈ R, for which $w = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all z ∈ C satisfying |z| = 1 and Rez ≠ 1, is [Online April 15, 2018]

Options:

A. $\{0\}$

B. an empty set

C. $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$

D. equal to R

Answer: A

Solution:

Solution:
$\because |z| = 1 \& Re z \neq 1$
Suppose $z = x + iy \Rightarrow x^2 + y^2 = 1$...... (i)
Now, $w = \frac{1 + (1 - 8\alpha)z}{1 - z}$
$\Rightarrow w = \frac{1 + (1 - 8\alpha)(x + iy)}{1 - (x + iy)}$
$\Rightarrow w = \frac{1 + (1 - 8\alpha)(x + iy))((1 - x) + iy)}{1 - (x + iy))((1 - x) + iy)}$
$\Rightarrow w = \frac{[(1 + x(1 - 8\alpha)(1 - x) - (1 - 8\alpha)y^2]}{(1 - x)^2 + y^2}$
$+ i \frac{[(1 + x(1 - 8\alpha))y - (1 - 8\alpha)y(1 - x)]}{(1 - x)^2 + y^2}$
As, w is purely imaginary. So,
$Re w = \frac{[(1 + x(1 - 8\alpha))(1 - x) - (1 - 8\alpha)y^2]}{(1 - x)^2 + y^2} = 0$
$\Rightarrow (1 - x) + x(1 - 8\alpha)(1 - x) = (1 - 8\alpha)y^2$
$\Rightarrow (1 - x) + x(1 - 8\alpha) - x^2(1 - 8\alpha) = (1 - 8x)y^2$
$\Rightarrow (1 - x) + x(1 - 8\alpha) = 1 - 8\alpha$[From (i), $x^2 + y^2 = 1$]
$\Rightarrow 1 - 8\alpha = 1 \Rightarrow \alpha = 0$
$\therefore \alpha \in \{0\}$

Question249

The least positive integer n for which $\left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)^n = 1$, is

[Online April 16, 2018]

Options:

A. 2

B. 6

C. 5

D. 3

Answer: D

Solution:

Solution:
Let $1 = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)$
$\therefore 1 = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 + i\sqrt{3}}{1 + i\sqrt{3}} \right)$
$= \left(\frac{-2 + i2\sqrt{3}}{4} \right) = \left(\frac{1 - i\sqrt{3}}{-2} \right)$
Also, $1 = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 - i\sqrt{3}}{1 - i\sqrt{3}} \right)$
$= \left(\frac{4}{-2 - i2\sqrt{3}} \right) = \left(\frac{-2}{1 + i\sqrt{3}} \right)$
Now, $\left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)^3 = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)$
$= \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{-2}{1 + i\sqrt{3}} \right) \times \left(\frac{1 - i\sqrt{3}}{-2} \right) = 1$
$\therefore$ least positive integer n is 3 .

Question250

Let p, q and r be real numbers ($p \neq q, r \neq 0$), such that the roots of the equation $\frac{1}{x + p} + \frac{1}{x + q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to.

[Online April 16, 2018]

Options:

A. $p^2+q^2+r^2$

B. p^2+q^2

C. $2(p^2+q^2)$

D. $\frac{p^2+q^2}{2}$

Answer: B

Solution:

Solution:
$\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$
$\frac{x+p+x+q}{(x+p)(x+q)}=\frac{1}{r}$
$(2x+p+q)r=x^2+px+qx+pq$
$x^2+(p+q-2r)x+pq-pr-qr=0$
Let α and β be the roots.
$\therefore \alpha+\beta=-(p+q-2r)$ ....(i)
$\& \alpha\beta=pq-pr-qr$ ....(ii)
$\because \alpha=-\beta$ (given)
$\therefore$ in eq. (1), we get
$\Rightarrow -(p+q-2r)=0$ ....(iii)
Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$=(-(p+q-2r))^2-2(pq-pr-qr)\ldots$ (from (i) and (ii))
$=p^2+q^2+4r^2+2pq-4pr-4qr-2pq+2pr+2qr$
$=p^2+q^2+4r^2-2pr-2qr$
$=p^2+q^2+2r(2r-p-q)$ ... (from (iii))
$=p^2+q^2+0$
$=p^2+q^2$

Question251

If an angle A of a ΔABC satisfies $5\cos A+3=0$, then the roots of the quadratic equation, $9x^2+27x+20=0$ are.
[Online April 16, 2018]

Options:

A. $\sin A$, $\sec A$

B. $\sec A$, $\tan A$

C. $\tan A$, $\cos A$

D. $\sec A$, $\cot A$

Answer: B

Solution:

Solution:
Here, $9x^2+27x+20=0$
$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$\Rightarrow x=\frac{-27\pm\sqrt{27^2-4\times 9\times 20}}{2\times 9}$
$\Rightarrow x=-\frac{4}{3},-\frac{5}{3}$
Given, $\cos A=-\frac{3}{5}$

$\therefore \sec A = \frac{1}{\cos A} = -\frac{5}{3}$
Here, A is an obtuse angle.
$\therefore \tan A = -\sqrt{\sec^2 A - 1} = -\frac{4}{3}$
Hence, roots of the equation are $\sec A$ and $\tan A$

Question252

If tan A and tan B are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3\sin^2(A+B) - 10\sin(A+B)\cdot\cos(A+B) - 25\cos^2(A+B)$ is
[Online April 15, 2018]

Options:

A. 25

B. -25

C. -10

D. 10

Answer: B

Solution:

Solution:
As $\tan A$ and $\tan B$ are the roots of $3x^2 - 10x - 25 = 0$,
So, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{10}{3}}{1 + \frac{25}{3}} = \frac{10/3}{28/3} = \frac{5}{14}$
Now, $\cos 2(A+B) = -1 + 2\cos^2(A+B)$
$= \frac{1 - \tan^2(A+B)}{1 + \tan^2(A+B)} \Rightarrow \cos^2(A+B) = \frac{196}{221}$
$\therefore 3\sin^2(A+B) - 10\sin(A+B)\cos(A+B) - 25\cos^2(A+B)$
$= \cos^2(A+B)[3\tan^2(A+B) - 10\tan(A+B) - 25]$
$= \frac{75 - 700 - 4900}{196} \times \frac{196}{221} = -\frac{5525}{196} \times \frac{196}{221} = -25$

Question253

If f (x) is a quadratic expression such that f + f = 0 and -1 is a root of f (x) = 0, then the other root of f (x) = 0 is
[Online April 15, 2018]

Options:

A. $-\frac{5}{8}$

B. $-\frac{8}{5}$

C. $\frac{5}{8}$

D. $\frac{8}{5}$

Answer: D

Solution:

Solution:
If a and -1 are the roots of the polynomial, then we get
$f(x) = x^2 + (1-a)x - a$

$\therefore\ f(1) = 2 - 2a$
and $f(2) = 6 - 3a$
As, $f(1) + f(2) = 0 \Rightarrow 2 - 2a + 6 - 3a = 0 \Rightarrow a = \frac{8}{5}$
Therefore, the other root is $\frac{8}{5}$

Question254

If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to : [2018]

Options:

A. 0

B. 1

C. 2

D. -1

Answer: B

Solution:

Solution:
α, β are roots of $x^2 - x + 1 = 0$
$\therefore\ \alpha = -\omega$ and $\beta = -\omega^2$
where ω is cube root of unity
$\therefore\ \alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega)^{107}$
$= -[\omega^2 + \omega] = -[-1] = 1$

Question255

If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is [Online April 15, 2018]

Options:

A. 20

B. $2\sqrt{5}$

C. $2\sqrt{7}$

D. $4\sqrt{2}$

Answer: B

Solution:

Solution:
Let, the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ are α and β
Also roots of the given equation are
$\frac{\lambda - 2 \pm \sqrt{4 - 4\lambda + \lambda^2 - 40 + 4\lambda}}{2} = \frac{\lambda - 2 \pm \sqrt{\lambda^2 - 36}}{2}$
The magnitude of the difference of the roots is $|\sqrt{\lambda^2 - 36}$
So, $\alpha^3 + \beta^3 = \frac{(\lambda - 2)^3}{4} + \frac{3(\lambda - 2)(\lambda^2 - 36)}{4}$
$= \frac{(\lambda - 2)(4\lambda^2 - 4\lambda - 104)}{4} = (\lambda - 2)(\lambda^2 - \lambda - 26) = f(\lambda)$
As $f(\lambda)$ attains its minimum value at $\lambda = 4$
Therefore, the magintude of the difference of the roots is
$|i\sqrt{20}| = 2\sqrt{5}$

Question256

If $|z-3+2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is [Online April 15, 2018]

Options:

A. $\sqrt{13}$

B. $2\sqrt{13}$

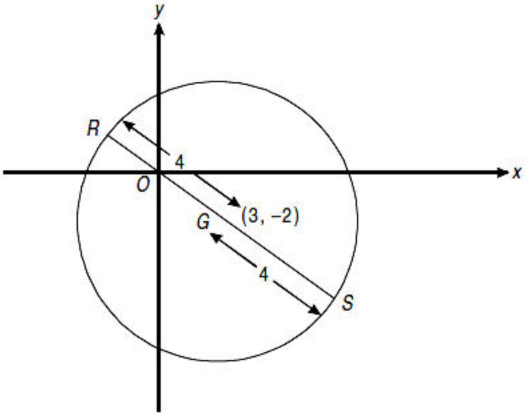
C. 8

D. $4+\sqrt{13}$

Answer: B

Solution:

Solution:
$|z-(3-2i)| \leq 4$ represents a circle whose centre is (3,-2) and radius $=4$
$|z| = |z-0|$ represents the distance of point z ' from origin (0,0)



Suppose RS is the normal of the circle passing through origin ' O ' and G is its center (3,-2) .
Here, OR is the least distance and OS is the greatest distance
$OR = RG - OG$ and $OS = OG + GS$
As, $RG = GS = 4$
$OG = \sqrt{3^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$
From (i), $OR = 4-\sqrt{13}$ and $OS = 4+\sqrt{13}$
So, required difference $=(4+\sqrt{13})-(4-\sqrt{13})$
$=\sqrt{13}+\sqrt{13}=2\sqrt{13}$

Question257

If, for a positive integer n, the quadratic equation, $x(x+1)+(x+1)(x+2)+.....+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to: [2017]

Options:

A. 11

B. 12

C. 9

D. 10

Answer: A

Solution:

Solution:

We have, $\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$\sum_{r=1}^{n}(x^2+xr+(r-1)x+r^2-r)=10n$

$\Rightarrow \sum_{r=1}^{n}(x^2+(2r-1)x+r(r-1))=10n$

$\Rightarrow nx^2+\{1+3+5+....+(2n-1)\}x$
$+\{1.2+2.3+....+(n-1)n\}=10n$

$\Rightarrow nx^2+n^2x+\frac{(n-1)n(n+1)}{3}=10n$

$\Rightarrow x^2+nx+\frac{n^2-31}{3}=0$

Let α and $\alpha+1$ be its two solutions
($\because$ it has two consequtive integral solutions)

$\Rightarrow \alpha+(\alpha+1)=-n$

$\Rightarrow \alpha=\frac{-n-1}{2}$ ....(i)

Also $\alpha(\alpha+1)=\frac{n^2-31}{3}$ ....(ii)

Putting value of (i) in (ii), we get

$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right)=\frac{n^2-31}{3}$

$\Rightarrow n^2=121 \Rightarrow n=11$

Question258

The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)}=1$ is:
[Online April 9, 2017]

Options:

A. 16

B. 14

C. -4

D. -5

Answer: C

Solution:

Solution:
$(x-1)(x^2+5x-50)=0$
$\Rightarrow (x-1)(x+10)(x-5)=0$
$\Rightarrow x=1, 5, -10$
Sum $=-4$

Question259

Let p(x) be a quadratic polynomial such that p(0) = 1. If p(x) leaves remainder 4 when divided by $x-1$ and it leaves remainder 6 when divided by $x+1$; then
[Online April 8, 2017]

Options:

A. $p(b)=11$

B. $p(b)=19$

C. $p(-2)=19$

D. $p(-2)=11$

Answer: C

Solution:

Solution:
Let $p(x) = ax^2 + bx + c$
$\because p(0) = 1 \Rightarrow c = 1$
Also, $p(1) = 4$ & $p(-1) = 6$
$\Rightarrow a + b + 1 = 4$ & $a - b + 1 = 6$
$\Rightarrow a + b = 3$ & $a - b = 5$
$\Rightarrow a = 4$ & $b = -1$
$p(x) = 4x^2 - x + 1$
$p(b) = 16 - 2 + 1 = 15$
$p(-2) = 16 + 2 + 1 = 19$

Question260

A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary, is:
[2016]

Options:

A. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

B. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: B

Solution:

Solution:
Rationalizing the given expression $\frac{(2 + 3i\sin\theta)(1 + 2i\sin\theta)}{1 + 4\sin^2\theta}$
For the given expression to be purely imaginary, real part of the above expression should be equal to zero.
$\Rightarrow \frac{2 - 6\sin^2\theta}{1 + 4\sin^2\theta} = 0 \quad \Rightarrow \sin^2\theta = \frac{1}{3}$
$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Question261

The point represented by $2 + i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by :
[Online April 9, 2016]

Options:

A. $1 + i$

B. $2 + 2i$

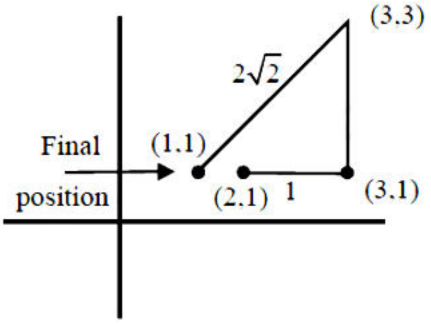
C. $-2 - 2i$

D. $-1 - i$

Answer: A

Solution:

Solution:



So new position is at the point 1 + i

Question262

The sum of all real values of x satisfying the equation $(x^2-5x+5)^{x^2+4x-60}=1$ is: [2016]

Options:

A. 6

B. 5

C. 3

D. -4

Answer: C

Solution:

Solution:
$(x^2-5x+5)^{x^2+4x-60}=1$
Case I
$x^2-5x+5=1$ and $x^2+4x-60$ can be any real number $\Rightarrow x=1,4$
Case II
$x^2-5x+5=-1$ and $x^2+4x-60$ has to be an even number
$\Rightarrow x=2,3$
where 3 is rejected because for $x=3$,
$x^2+4x-60$ is odd
Case III
x^2-5x+5 can be any real number and
$x^2+4x-60=0$
$\Rightarrow x=-10,6$
$\Rightarrow$ Sum of all values of x
$=-10+6+2+1+4=3$

Question263

If x is a solution of the equation, $\sqrt{2x+1}-\sqrt{2x-1}=1$, $\left(x\geq\frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to: [Online April 10, 2016]

Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $2\sqrt{2}$

D. 2

Answer: A

Solution:

Solution:
$\sqrt{2x+1} - \sqrt{2x-1} = 1$
$\Rightarrow\ 2x + 1 + 2x - 1 - 2\sqrt{4x^2 - 1} = 1$
$\Rightarrow\ 4x - 1 = 2\sqrt{4x^2 - 1}$
$\Rightarrow\ 16x^2 - 8x + 1 = 16x^2 - 4$
$\Rightarrow\ 8x = 5$
$\Rightarrow\ x = \frac{5}{8}$ which satisfies equation (i)
So, $\sqrt{4x^2 - 1} = \frac{3}{4}$

Question264

If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to :
[Online April 9, 2016]

Options:

A. 2

B. 3

C. $\sqrt{3}$

D. $\sqrt{2}$

Answer: C

Solution:

Solution:
$x^2 + bx - 1 = 0$ common root
$x^2 + x + b = 0$
$x = \frac{b+1}{b-1}$
Put $x = \frac{b+1}{b-1}$ in equation
$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$
Put $x = \frac{b+1}{b-1}$ in equation
$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$
$(b+1)^2 + (b+1)(b-1) + b(b-1)^2 = 0$
$b^2 + 1 + 2b + b^2 - 1 + b(b^2 - 2b + 1) = 0$
$2b^2 + 2b + b^3 - 2b^2 + b = 0$
$b^3 + 3b = 0$
$b(b^2 + 3) = 0$
$b^2 = -3$
$b = \pm\sqrt{3}i$
$|b| = \sqrt{3}$

Question265

If z is a non-real complex number, then the minimum value of $\frac{1 mz^5}{(1 mz)^5}$ is :
[Online April 11, 2015]

Options:

A. -1

B. -4

C. -2

D. -5

Answer: B

Solution:

Solution:
Let $z = re^{i\theta}$
Consider $\frac{Imz^5}{(Imz)^5} = \frac{r^5(\sin 5\theta)}{r^5(\sin\theta)^5}$
$(\because e^{i\theta} = \cos\theta + i\sin\theta)$
$= \frac{\sin 5\theta}{\sin^5\theta} = \frac{16\sin^5\theta - 20\sin^3\theta + 5\sin\theta}{\sin^5\theta}$
$= \frac{16\sin^5\theta}{\sin^5\theta} - \frac{20\sin^3\theta}{\sin^5\theta} + \frac{5\sin\theta}{\sin^5\theta}$
$= 5\operatorname{cosec}^4\theta - 20\operatorname{cosec}^2\theta + 16$
minimum value of $\frac{Imz^5}{(Imz)^5}$ is -4

Question266

A complex number z is said to be unimodular if |z| = 1.
Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a:
[2015]

Options:

A. circle of radius 2.

B. circle of radius $\sqrt{2}$.

C. straight line parallel to x -axis

D. straight line parallel to y-axis.

Answer: A

Solution:

Solution:
$\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$
$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$
$\Rightarrow (z_1 - 2z_2)(\overline{z_1 - 2z_2}) = (2 - z_1\bar{z}_2)(\overline{2 - z_1\bar{z}_2})$
$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$
$\Rightarrow (z_1\bar{z}_1) - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4z_2\bar{z}_2$
$= 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 + z_1\bar{z}_1 z_2\bar{z}_2$
$\Rightarrow |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$
$\Rightarrow |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$
$(|z_1|^2 - 4)(1 - |z_2|^2) = 0$
$\because |z_2| \neq 1$
$\therefore |z_1|^2 = 4$
$\Rightarrow |z_1| = 2$
$\Rightarrow$ Point z_1 lies on circle of radius 2

Question267

Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to:
[2015]

Options:

A. 3

B. -3

C. 6

D. -6

Answer: A

Solution:

Solution:
$\alpha, \beta = \frac{6 \pm \sqrt{36+8}}{2} = 3 \pm \sqrt{11}$
$\alpha = 3 + \sqrt{11}, \beta = 3 - \sqrt{11}$
$\therefore a_n = (3+\sqrt{11})^n - (3-\sqrt{11})^n \; \frac{a_{10} - 2a_8}{2a_9}$
$= \frac{(3+\sqrt{11})^{10} - (3-\sqrt{11})^{10} - 2(3+\sqrt{11})^8 + 2(3-\sqrt{11})^8}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]}$
$= \frac{(3+\sqrt{11})^8[(3+\sqrt{11})^2 - 2] + (3-\sqrt{11})^8[2 - (3-\sqrt{11})^2]}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]}$
$= \frac{(3+\sqrt{11})^8(9+11+6\sqrt{11}-2) + (3-\sqrt{11})^8(2-9-11+6\sqrt{11})}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]}$
$= \frac{6(3+\sqrt{11})^9 - 6(3-\sqrt{11})^9}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]} = \frac{6}{2} = 3$

Question268

If the two roots of the equation, $(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is
[Online April 11, 2015]
:

Options:

A. $\left(0, \frac{1}{2}\right)$

B. $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

C. $\left(-\frac{1}{2}, 0\right)$

D. $(-\infty, -2) \cup (2, \infty)$

Answer: B

Solution:

Solution:
$(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$
$\Rightarrow (a-1)(x^2 + x + 1)(x^2 - x + 1) + (a+1)(x^2 + x + 1)^2 = 0$
$\Rightarrow (x^2 + x + 1)[(a-1)(x^2 - x + 1) + (a+1)(x^2 + x + 1)] = 0$
$\Rightarrow (x^2 + x + 1)(ax^2 + x + a) = 0$
For roots to be distinct and real, $a \neq 0$ and $1 - 4a^2 > 0$

$\Rightarrow a \neq 0$ and $a^2 < \frac{1}{4}$

$\Rightarrow a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

Question269

If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation:
[Online April 10, 2015]

Options:

A. exists and is equal to $-\frac{1}{2}$.

B. exists and is equal to $\frac{1}{2}$

C. exists and is equal to 1 .

D. does not exist.

Answer: B

Solution:

Solution:
$\alpha = 2 + 3i;\ \beta = 2 - 3i,\ \gamma = ?$

$\alpha\beta\gamma = \frac{13}{2}\left[\text{ since product of roots } = \frac{d}{a}\right]$

$\Rightarrow (4 + 9)\gamma = \frac{13}{2} \Rightarrow \gamma = \frac{1}{2}$

Question270

If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$:
[2014]

Options:

A. is strictly greater than $\frac{5}{2}$

B. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

C. is equal to $\frac{5}{2}$

D. lie in the interval (1,2)

Answer: D

Solution:

Solution:
We know minimum value of $|Z_1 + Z_2|$ is

$|Z_1| - |Z_2|$. Thus minimum value of $\left|Z + \frac{1}{2}\right|$ is $|Z| - \frac{1}{2}|$

$\leq \left|Z + \frac{1}{2}\right| \leq \left|Z\right| + \frac{1}{2}$

Since, $|Z| \geq 2$ therefore

$2 - \frac{1}{2} < \left|Z + \frac{1}{2}\right| < 2 + \frac{1}{2}$

$\Rightarrow \frac{3}{2} < \left|Z + \frac{1}{2}\right| < \frac{5}{2}$

Question271

For all complex numbers z of the form $1 + i\alpha$, $\alpha \in R$, if $z^2 = x + iy$, then [Online April 19, 2014]

Options:

A. $y^2 - 4x + 2 = 0$

B. $y^2 + 4x - 4 = 0$

C. $y^2 - 4x - 4 = 0$

D. $y^2 + 4x + 2 = 0$

Answer: B

Solution:

Solution:
Let $z = 1 + i\alpha, \alpha \in R$
$z^2 = (1 + i\alpha)(1 + i\alpha)$
$x + iy = (1 + 2i\alpha - \alpha^2)$
On comparing real and imaginary parts, we get
$x = 1 - \alpha^2, y = 2\alpha$
Now, consider option (b), which is
$y^2 + 4x - 4 = 0$
LHS :$y^2 + 4x - 4 = (2\alpha)^2 + 4(1 - \alpha^2) - 4$
$= 4\alpha^2 + 4 - 4\alpha^2 - 4$
$= 0 = R.H.S$
Hence, $y^2 + 4x - 4 = 0$

Question272

Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z + \frac{1}{z}$ is: [Online April 12, 2014]

Options:

A. zero

B. anynon-zero real number other than 1 .

C. any non-zero real number.

D. a purely imaginary number.

Answer: C

Solution:

Solution:
Let $z = x + iy$
$\frac{z-i}{z+i}$ is purely imaginary means its real part is zero.
$\frac{x+iy-i}{x+iy+i} = \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$
$= \frac{x^2 - 2ix(y+1) + xi(y-1) + y^2 - 1}{x^2 + (y+1)^2}$
$= \frac{x^2+y^2-1}{x^2+(y+1)^2} - \frac{2xi}{x^2+(y+1)^2}$
for pure imaginary, we have
$\frac{x^2+y^2-1}{x^2+(y+1)^2} = 0$

$\Rightarrow x^2 + y^2 = 1$
$\Rightarrow (x+iy)(x-iy) = 1$
$\Rightarrow x + iy = \dfrac{1}{x - iy} = z$
and $\dfrac{1}{z} = x - iy$
$z + \dfrac{1}{z} = (x+iy) + (x-iy) = 2x$
$\left(z + \dfrac{1}{z}\right)$ is any non-zero real number

Question273

If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then $\arg\left(\dfrac{z_1}{z_4}\right) + \arg\left(\dfrac{z_2}{z_3}\right)$ equals:
[Online April 11, 2014]

Options:

A. 0

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. π

Answer: A

Solution:

Solution:
Consider $\arg\left(\dfrac{z_1}{z_4}\right) + \arg\left(\dfrac{z_2}{z_3}\right)$
$= \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$
$= (\arg(z_1) + \arg(z_2)) - (\arg(z_3) + \arg(z_4))$
given $\left(\begin{matrix} z_2 = \bar{z}_1 \\ z_4 = \bar{z}_3 \end{matrix}\right)$
$= (\arg(z_1) + \arg(\bar{z}_1)) - (\arg(z_3) + \arg(\bar{z}_3))$
$\left\{\begin{matrix} \text{also } (\arg(\bar{z}_1) = -\arg(z_1)) \\ \arg(\bar{z}_3) = -\arg(z_3) \end{matrix}\right\}$
$= (\arg(z_1) - \arg(z_1)) - (\arg(z_3) - \arg(z_3))$
$= 0 - 0 = 0$

Question274

Let $w(I\,mw \neq 0)$ be a complex number. Then the set of all complex number z satisfying the equation $w - \overline{w}z = k(1-z)$, for some real number k, is
[Online April 9, 2014]

Options:

A. $\{z : |z| = 1\}$

B. $\{z : z = \bar{z}\}$

C. $\{z : z \neq 1\}$

D. $\{z : |z| = 1, z \neq 1\}$

Answer: D

Solution:

Solution:
Consider the equation
$w - \bar{w}z = k(1 - z), k \in R$
Clearly $z \neq 1$ and $\frac{w - \bar{w}z}{1 - z}$ is purely real
$\therefore \overline{\left(\frac{w - \bar{w}z}{1 - z}\right)} = \frac{w - \bar{w}z}{1 - z}$
$\Rightarrow \frac{\bar{w} - w\bar{z}}{1 - \bar{z}} = \frac{w - \bar{w}z}{1 - z}$
$\Rightarrow \bar{w} - \bar{w}z - w\bar{z} + wz\bar{z} = w - w\bar{z} - \bar{w}z + \bar{w}z\bar{z}$
$\Rightarrow \bar{w} + w|z|^2 = w + \bar{w}|z|^2$
$\Rightarrow (w - \bar{w})(|z|^2) = w - \bar{w}$
$\Rightarrow |z|^2 = 1 \ (\because Im\, w \neq 0)$
$\Rightarrow |z| = 1$ and $z \neq 1$
$\therefore$ The required set is $\{z : |z| = 1, z \neq 1\}$

Question275

**If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval:
[2014]**

Options:

A. (-2,-1)

B. $(-\infty, -2) \cup (2, \infty)$

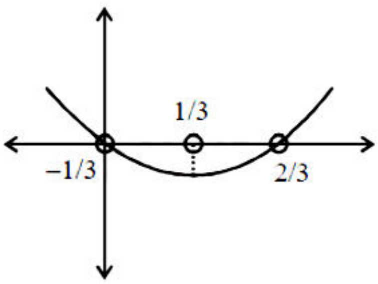
C. $(-1,0) \cup (0, 1)$

D. (1,2)

Answer: C

Solution:

Solution:
Consider $-3(x - [x])^2 + 2[x - [x]) + a^2 = 0$
$\Rightarrow \ 3\{x\}^2 - 2\{x\} - a^2 = 0 \ \ (\because x - [x] = \{x\})$
$\Rightarrow \ 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2, a \neq 0$
$\Rightarrow \ a^2 = 3\{x\}\left(\{x\} - \frac{2}{3}\right)$



Now, $\{x\} \in (0, 1)$ and $\frac{-2}{3} \leq a^2 < 1$ (by graph)
Since, x is not an integer
$\therefore \ a \in (-1, 1) - \{0\}$
$\Rightarrow a \in (-1, 0) \cup (0, 1)$

Question276

**The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has;
[Online April 19, 2014]**

Options:

A. no solution

B. exactly one solution

C. exactly two solution

D. exactly four solution

Answer: A

Solution:

Solution:
Consider $\sqrt{3x^2+x+5}=x-3$
Squaring both the sides, we get
$3x^2+x+5=(x-3)^2$
$\Rightarrow 3x^2+x+5=x^2+9-6x$
$\Rightarrow 2x^2+7x-4=0$
$\Rightarrow 2x^2+8x-x-4=0$
$\Rightarrow 2x(x+4)-1(x+4)=0$
$\Rightarrow x=\frac{1}{2}$ or $x=-4$
For $x=\frac{1}{2}$ and $x=-4$
L.H.S. $\neq$ R.H.S. of equation, $\sqrt{3x^2+x+5}=x-3$
Also, for every $x \in R$, LH S $\neq$ RH S of the given equation.
$\therefore$ Given equation has no solution.

Question277

The sum of the roots of the equation, $x^2+|2x-3|-4=0$, is: [Online April 12, 2014]

Options:

A. 2

B. -2

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: C

Solution:

Solution:
$x^2+|2x-3|-4=0$

$$|2x-3|=\begin{cases}(2x-3) & \text{if} \quad x>\frac{3}{2}\\ -(2x-3) & \text{if} \quad x<\frac{3}{2}.\end{cases}$$

for $x>\frac{3}{2}$, $x^2+2x-3-4=0$
$x^2+2x-7=0$
$x=\frac{-2\pm\sqrt{4+28}}{2}=\frac{-2\pm4\sqrt{2}}{2}=-1\pm2\sqrt{2}$
Here $x=2\sqrt{2}-1 \quad \left\{2\sqrt{2}-1<\frac{3}{2}\right\}$
for $x<\frac{3}{2}$
$x^2-2x+3-4=0$
$\Rightarrow x^2-2x-1=0$
$\Rightarrow \quad x=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$
Here $x=1-\sqrt{2} \quad \left\{(1-\sqrt{2})<\frac{3}{2}\right\}$
Sum of roots $:(2\sqrt{2}-1)+(1-\sqrt{2})=\sqrt{2}$

Question278

If α and β are roots of the equation, $x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to:
[Online April 11, 2014]

Options:

A. $248\sqrt{2}$

B. $280\sqrt{2}$

C. $-32\sqrt{2}$

D. $-280\sqrt{2}$

Answer: D

Solution:

Solution:
$x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$
or, $x^2 - 4\sqrt{2}kx + 2k^4 - 1 = 0$
$\alpha + \beta = 4\sqrt{2}k$ and $\alpha \cdot \beta = 2k^4 - 1$
Squaring both sides, we get
$(\alpha+\beta)^2 = (4\sqrt{2}k)^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 32k^2$
$66 + 2\alpha\beta = 32k^2$
$66 + 2(2k^4 - 1) = 32k^2$
$66 + 4k^4 - 2 = 32k^2 \Rightarrow 4k^4 - 32k^2 + 64 = 0$
or, $k^4 - 8k^2 + 16 = 0 \Rightarrow (k^2)^2 - 8k^2 + 16 = 0$
$\Rightarrow (k^2 - 4)(k^2 - 4) = 0 \Rightarrow k^2 = 4, k^2 = 4$
$\Rightarrow k = \pm 2$
Now, $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)$
$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}k)[66 - (2k^4 - 1)]$
Putting $k = -2$, ($k = +2$ cannot be taken because it does not satisfy the above equation)
$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}(-2))[66 - 2(-2)^4 - 1]$
$\alpha^3 + \beta^3 = (-8\sqrt{2})(66 - 32 + 1) = (-8\sqrt{2})$
$\therefore \alpha^3 + \beta^3 = -280\sqrt{2}$

Question279

If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0 (a \neq 0, a, b, \in R)$, then the equation, $x(x + b^3) + (a^3 - 3abx) = 0$ as roots :
[Online April 9, 2014]

Options:

A. $\alpha^{3/2}$ and $\beta^{3/2}$

B. $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$

C. $\sqrt{\alpha\beta}$ and $\alpha\beta$

D. $\alpha^{-\frac{3}{2}}$ and β^{-3} ?

Answer: A

Solution:

Solution:

Let $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ be the roots of $ax^2 + bx + 1 = 0$
$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \left(\frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} \right) = -\frac{b}{a}$
$\frac{1}{\sqrt{\alpha}\sqrt{\beta}} = \frac{1}{a} \Rightarrow a = \sqrt{\alpha\beta}$
$b = -(\sqrt{\alpha} + \sqrt{\beta})$
$x(x + b^3) + (a^3 - 3abx) = 0$
$\Rightarrow \ x^2 + (b^3 - 3ab)x + a^3 = 0$
Putting values of a and b, we get
$x^2 + [(-\sqrt{\alpha} + \sqrt{\beta})^3 + 3(\sqrt{\alpha\beta})(\sqrt{\alpha} + \sqrt{\beta})] + (\alpha\beta)^{3/2} = 0$
$\Rightarrow x^2 - [\alpha^{3/2} + \beta^{3/2} + 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta}) - 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta})]x + (\alpha\beta)^{3/2} = 0$
$\Rightarrow \ x^2 - (\alpha^{3/2} + \beta^{3/2})x + \alpha^{3/2}\beta^{3/2} = 0$
Roots of this equation are $\alpha^{3/2}, \beta^{3/2}$

Question280

If non-zero real numbers b and c are such that min f (x) > max g(x), where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2 (x \in R)$ then $\left| \frac{c}{b} \right|$ lies in the interval:
[Online April 19, 2014]

Options:

A. $\left(0, \frac{1}{2}\right)$

B. $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

C. $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$

D. $(\sqrt{2}, \infty)$

Answer: D

Solution:

Solution:
We have
$f(x) = x^2 + 2bx + 2c^2$
and $g(x) = -x^2 - 2cx + b^2, (x \in R)$
$\Rightarrow f(x) = (x + b)^2 + 2c^2 - b^2$
and $g(x) = -(x + c)^2 + b^2 + c^2$
Now, $f_{min} = 2c^2 - b^2$ and $g_{max} = b^2 + c^2$
Given :$\min f(x) > \max g(x)$
$\Rightarrow 2c^2 - b^2 > b^2 + c^2$
$\Rightarrow c^2 > 2b^2$
$\Rightarrow |c| > |b|\sqrt{2}$
$\Rightarrow \frac{|c|}{|b|} > \sqrt{2} \Rightarrow \left| \frac{c}{b} \right| > \sqrt{2}$
$\Rightarrow \left| \frac{c}{b} \right| \in (\sqrt{2}, \infty)$

Question281

If equations $ax^2 + bx + c = 0 (a, b, c \in R, a \neq 0)$ and $2x^2 + 3x + 4 = 0$ have a common root, then a : b : c equals:
[Online April 9, 2014]

Options:

A. 1: 2: 3

B. 2: 3: 4

C. 4: 3: 2

D. 3: 2: 1

Answer: B

Solution:

Solution:
Let α, β be the common roots of both the equations.
For first equation $ax^2 + bx + c = 0$
we have
$\alpha + \beta = \frac{-b}{a}$ . . .(i)
$\alpha \cdot \beta = \frac{c}{a}$ . . .(ii)
For second equation $2x^2 + 3x + 4 = 0$
we have
$\alpha + \beta = \frac{-3}{2}$ . . .(iii)
$\alpha \cdot \beta = \frac{2}{1}$ . . . (iv)
Now, from (i) & (iii) & from (ii) & (iv)
$\frac{-b}{a} = \frac{-3}{2}$ $\frac{c}{a} = \frac{2}{1}$
$\frac{b}{a} = \frac{3/2}{1}$
Therefore on comparing we get $a = 1, b = \frac{3}{2}$ & $c = 2$
putting these values in first equation, we get
$x^2 + \frac{3}{2}x + 2 = 0$ or $2x^2 + 3x + 4 = 0$
from this, we get $a = 2, b = 3; c = 4$
or $a : b : c = 2 : 3 : 4$

Question282

If z is a complex number of unit modulus and argument θ, then arg$\left(\frac{1+z}{1+\bar{z}}\right)$ equals: [2013]

Options:

A. $-\theta$

B. $\frac{\pi}{2} - \theta$

C. θ

D. $\pi - \theta$

Answer: C

Solution:

Solution:
Given $|z| = 1, \arg z = \theta$
$\Rightarrow \bar{z} = \frac{1}{z}$
$\therefore \arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{1+z}{1+\frac{1}{z}}\right) = \arg(z) = \theta$

Question283

Let z satisfy $|z| = 1$ and $z = 1 - \bar{z}$.
Statement 1 : z is a real number.

Statement 2: Principal argument of z is $\frac{\pi}{3}$
[Online April 25, 2013]

Options:

A. Statement 1 is true Statement 2 is true; Statement 2 is a correct explanation for Statement 1 .

B. Statement 1 is false; Statement 2 is true

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

Answer: B

Solution:

Solution:
Let $z = x + iy, \bar{z} = x - iy$
Now, $z = 1 - \bar{z}$
$\Rightarrow x + iy = 1 - (x - iy)$
$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$
Now, $|z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$
$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$
Now, $\tan\theta = \frac{y}{x}$ (θ is the argument)
$= \frac{\sqrt{3}}{2} \div \frac{1}{2}$ (+ ve since only principal argument)
$= \sqrt{3}$
$\Rightarrow \theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$
Hence, z is not a real number
So, statement- 1 is false and 2 is true.

Question284

Let $a = Im\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number.

The set $A = \{a : |z| = 1 \text{ and } z \neq \pm 1\}$ is equal to:
[Online April 23, 2013]

Options:

A. (-1,1)

B. [-1,1]

C. [0,1)

D. (-1,0]

Answer: A

Solution:

Solution:
Let $z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$
Now, $\frac{1+z^2}{2iz} = \frac{1 + x^2 - y^2 + 2ixy}{2i(x+iy)} = \frac{(x^2 - y^2 + 1) + 2ixy}{2ix - 2y}$
$= \frac{(x^2 - y^2 + 1) + 2ixy}{-2y + 2ix} \times \frac{-2y - 2ix}{-2y - 2ix}$
$= \frac{y(x^2 + y^2 - 1) + x(x^2 + y^2 + 1)i}{2(x^2 + y^2)}$
$a = \frac{x(x^2 + y^2 + 1)}{2(x^2 + y^2)}$
Since, $|z| = 1 \Rightarrow \sqrt{x^2 + y^2} = 1 \Rightarrow x^2 + y^2 = 1$

$\therefore a = \dfrac{x(1+1)}{2 \times 1} = x$
Also $z \neq 1 \Rightarrow x + iy \neq 1$
$\therefore A = (-1, 1)$

Question285

If $Z_1 \neq 0$ and Z_2 be two complex numbers such that $\frac{Z_2}{Z_1}$ is a purely imaginary number, then $\left|\frac{2Z_1 + 3Z_2}{2Z_1 - 3Z_2}\right|$ is equal to:
[Online April 9, 2013]

Options:

A. 2

B. 5

C. 3

D. 1

Answer: D

Solution:

Solution:
Let $z_1 = 1 + i$ and $z_2 = 1 - i$
$\dfrac{z_2}{z_1} = \dfrac{1-i}{1+i} = \dfrac{(1-i)(1-i)}{(1+i)(1-i)} = -i$
$\dfrac{2z_1 + 3z_2}{2z_1 - 3z_2} = \dfrac{2 + 3\left(\frac{z_2}{z_1}\right)}{2 - 3\left(\frac{z_2}{z_1}\right)} = \dfrac{2-3i}{2+3i}$
$\left|\dfrac{2z_1 + 3z_2}{2z_1 - 3z_2}\right| = \left|\dfrac{2-3i}{2+3i}\right| = \left|\dfrac{2-3i}{2+3i}\right| \quad \left[\because \left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}\right]$
$= \dfrac{\sqrt{4+9}}{\sqrt{4+9}} = 1$

Question286

If p and q are non-zero real numbers and $\alpha^3 + \beta^3 = -p$, $\alpha\beta = q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is:
[Online April 25, 2013]

Options:

A. $px^2 - qx + p^2 = 0$

B. $qx^2 + px + q^2 = 0$

C. $px^2 + qx + p^2 = 0$

D. $qx^2 - px + q^2 = 0$

Answer: B

Solution:

Solution:
Given $\alpha^3 + \beta^3 = -p$ and $\alpha\beta = q$
Let $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$ be the root of required quadratic equation.

So, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{-p}{q}$
and $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = q$
Hence, required quadratic equation is
$x^2 - \left(\frac{-p}{q}\right)x + q = 0$
$\Rightarrow x^2 + \frac{p}{q}x + q = 0 \Rightarrow qx^2 + px + q^2 = 0$

Question287

If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$ such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set :
[Online April 22, 2013]

Options:

A. $\{2, -5\}$

B. $\{-3, 2\}$

C. $\{-2, 5\}$

D. $\{3, -5\}$

Answer: C

Solution:

Solution:
Given quadratic eqn. is
$x^2 + px + \frac{3p}{4} = 0$
So, $\alpha + \beta = -p, \alpha\beta = \frac{3p}{4}$
Now, given $|\alpha - \beta| = \sqrt{10}$
$\Rightarrow \alpha - \beta = \pm\sqrt{10}$
$\Rightarrow (\alpha - \beta)^2 = 10 \Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 10$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 10$
$\Rightarrow p^2 - 4 \times \frac{3p}{4} = 10 \Rightarrow p^2 - 3p - 10 = 0$
$\Rightarrow p = -2, 5 \Rightarrow p \in \{-2, 5\}$

Question288

If a complex number z statisfies the equation $z + \sqrt{2} \mid z + 1 \mid + i = 0$, then |z| is equal to :
[Online April 22, 2013]

Options:

A. 2

B. $\sqrt{3}$

C. $\sqrt{5}$

D. 1

Answer: C

Solution:

Solution:
Given equation is
$z + \sqrt{2} \mid z + 1 \mid + i = 0$
put $z = x + iy$ in the given equation.
$(x + iy) + \sqrt{2} \mid x + iy + 1 \mid + i = 0$

$\Rightarrow x + iy + \sqrt{2}\left[\sqrt{(x+1)^2 + y^2}\right] + i = 0$
Now, equating real and imaginary part, we get
$x + \sqrt{2}\sqrt{(x+1)^2 + y^2} = 0$ and
$y + 1 = 0 \Rightarrow y = -1$
$\Rightarrow x + \sqrt{2}\sqrt{(x+1)^2 + (-1)^2} = 0 \quad (\because y = -1)$
$\Rightarrow \sqrt{2}\sqrt{(x+1)^2 + 1} = -x$
$\Rightarrow 2[(x+1)^2 + 1] = x^2$
$\Rightarrow x^2 + 4x + 4 = 0$
$\Rightarrow x = -2$
Thus, $z = -2 + i(-1) \Rightarrow |z| = \sqrt{5}$

Question289

If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$ a, b, c $\in$ R, have a common root, then a : b : c is [2013]

Options:

A. 1: 2: 3

B. 3: 2: 1

C. 1: 3: 2

D. 3: 1: 2

Answer: A

Solution:

Solution:
Given equations are
$x^2 + 2x + 3 = 0$ ... (i)
$ax^2 + bx + c = 0$ ... (ii)
Roots of equation (i) are imaginary roots in order pair.
According to the question (ii) will also have both roots same as (i).
Thus $\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda$(say)
$\Rightarrow \quad a = \lambda, b = 2\lambda, c = 3\lambda$
Hence, required ratio is 1: 2: 3

Question290

The least integral value α of x such that $\frac{x-5}{x^2 + 5x - 14} > 0$ satisfies: [Online April 23, 2013]

Options:

A. $\alpha^2 + 3\alpha - 4 = 0$

B. $\alpha^2 - 5\alpha + 4 = 0$

C. $\alpha^2 - 7\alpha + 6 = 0$

D. $\alpha^2 + 5\alpha - 6 = 0$

Answer: A

Solution:

Solution:
$\frac{x-5}{x^2 + 5x - 14} > 0 \Rightarrow x^2 + 5x - 14 < x - 5$
$\Rightarrow x^2 + 4x - 9 < 0$

$\Rightarrow \alpha = -5, -4, -3, -2, -1, 0, 1$
$\alpha = -5$ does not satisfy any of the options
$\alpha = -4$ satisfy the option (a) $\alpha^2 + 3\alpha - 4 = 0$

Question291

The values of ' a ' for which one root of the equation $x^2 - (a+1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2 , are given by :
[Online April 9, 2013]

Options:

A. $3 < a < 10$

B. $a \geq 10$

C. $-2 < a < 3$

D. $a \leq -2$

Answer: C

Solution:

Solution:
$x^2 - (a+1)x + a^2 + a - 8 = 0$
Since roots are different, therefore $D > 0$
$\Rightarrow (a+1)^2 - 4(a^2 + a - 8) > 0$
$\Rightarrow (a-3)(3a+1) < 0$
There are two cases arises
Case I. $a - 3 > 0$ and $3a + 1 < 0$
$\Rightarrow a > 3$ and $a < -\frac{11}{3}$
Hence, no solution in this case
Case II :$a - 3 < 0$ and $3a + 11 > 0$
$\Rightarrow a < 3$ and $a > -\frac{11}{3}$
$\therefore -\frac{11}{3} < a < 3 \Rightarrow -2 < a < 3$

Question292

$|z_1 + z_2|^2 + |z_1 - z_2|^2$ is equal to
[Online May 26, 2012]

Options:

A. $2(|z_1| + |z_2|)$

B. $2(|z_1|^2 + |z_2|^2)$

C. $|z_1||z_2|$

D. $|z_1|^2 + |z_2|^2$

Answer: B

Solution:

Solution:
$|z_1 + z_2|^2 + |z_1 - z_2|^2$
$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2| + |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$
$= 2|z_1|^2 + 2|z_2|^2 = 2[|z_1|^2 + |z_2|^2]$

Question293

Let Z and W be complex numbers such that $|Z| = |W|$, and $\arg Z$ denotes the principal argument of Z.
Statement 1: If $\arg Z + \arg W = \pi$, then $Z = -\overline{W}$
Statement 2: $|Z| = |W|$, implies $\arg Z - \arg \overline{W} = \pi$
[Online May 19, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1 .

C. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1 .

D. Statement 1 is false, Statement 2 is true.

Answer: A

Solution:

Solution:
Let $|Z| = |W| = r \Rightarrow Z = re^{i\theta}, W = re^{i\varphi}$
where $\theta + \varphi = \pi$
$\therefore \overline{W} = re^{-i\varphi}$
Now, $Z = re^{i(\pi - \varphi)} = re^{i\pi} \times e^{-i\varphi} = -re^{-i\varphi}$
$= -\overline{W}$
Thus, statement- 1 is true but statement- 2 is false.

Question294

Let Z_1 and Z_2 be any two complex number. Statement 1: $|Z_1 - Z_2| \geq |Z_1| - |Z_2|$
Statement 2: $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$
[Online May 7, 2012]

Options:

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is false, Statement 2 is true.

Answer: B

Solution:

Solution:
Statement -1 and 2 both are true. It is fundamental property. But Statement -2 is not correct explanation for Statement -1.

Question295

If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies:
[2012]

Options:

A. either on the real axis or on a circle passing through the origin.

B. on a circle with centre at the origin

C. either on the real axis or on a circle not passing through the origin.

D. on the imaginary axis.

Answer: A

Solution:

Solution:

$\frac{z^2}{z-1} = \frac{\overline{z}^2}{\overline{z}-1}\left[\because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}\right]$

$\Rightarrow z\overline{z}z - z^2 = z \cdot \overline{z} \cdot \overline{z} - \overline{z}^2$

$\Rightarrow |z|^2 \cdot z - z^2 = |z|^2 \cdot \overline{z} - \overline{z}^2$

$\Rightarrow |z|^2 (z-\overline{z}) - (z-\overline{z})(z+\overline{z}) = 0$

$\Rightarrow (z-\overline{z})(|z|^2 - (z+\overline{z})) = 0$

Either $z - \overline{z} = 0$ or $|z|^2 - (z+\overline{z}) = 0$

Either $z = \overline{z} \Rightarrow$ real axis

or $|z|^2 = z + \overline{z} \Rightarrow z\overline{z} - z - \overline{z} = 0$

represents a circle passing through origin.

Question296

Let p, q, r ∈ R and r > p > 0. If the quadratic equation $px^2 + qx + r = 0$ has two complex roots α and β, then |α|+ |β| is
[Online May 19, 2012]

Options:

A. equal tol

B. less than 2 but not equal to 1

C. greater than 2

D. equal to 2

Answer: C

Solution:

Solution:

Given quadratic equation is $px^2 + qx + r = 0$

$D = q^2 - 4pr$

Since α and β are two complex root

$\therefore \beta = \overline{\alpha} \Rightarrow |\beta| = |\overline{\alpha}| \Rightarrow |\beta| = |\alpha| \quad (\because |\overline{\alpha}| = |\alpha|)$

Consider

$|\alpha| + |\beta| = |\alpha| + |\alpha| \quad (\because |\beta| = |\alpha|)$

$= 2|\alpha| > 2.1 = 2 \ (\because |\alpha| > 1)$

Hence, $|\alpha| + |\beta|$ is greater than 2

Question297

If the sum of the square of the roots of the equation $x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$ is least, then α is equal to
[Online May 12, 2012]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

Solution:

Solution:
Given equation is
$x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$
Let x_1 and x_2 be two roots of quadratic equation
$\therefore x_1 + x_2 = \sin\alpha - 2$ and $x_1 x_2 = -(1 + \sin\alpha)$
$(x_1 + x_2)^2 = (\sin\alpha - 2)^2 = \sin^2\alpha + 4 - 4\sin\alpha$
$\Rightarrow\ x_1^{\ 2} + x_2^{\ 2} = \sin^2\alpha + 4 - 4\sin\alpha - 2x_1x_2$
$= \sin^2\alpha + 4 - 4\sin\alpha + 2(1 + \sin\alpha)$
$= \sin^2\alpha - 2\sin\alpha + 6$
Now, By putting
$\alpha = \frac{\pi}{6}, \alpha = \frac{\pi}{4}, \alpha = \frac{\pi}{3}$ and $\alpha = \frac{\pi}{2}$ in (i) one by one
We get least value of $x_1^{\ 2} + x_2^{\ 2}$ at $\frac{\pi}{2}$
Hence, $\alpha = \frac{\pi}{2}$

Question298

The value of k for which the equation $(k - 2)x^2 + 8x + k + 4 = 0$ has both roots real, distinct and negative is
[Online May 7, 2012]

Options:

A. 6

B. 3

C. 4

D. 1

Answer: B

Solution:

Solution:
$(k - 2)x^2 + 8x + k + 4 = 0$
If real roots then,
$8^2 - 4(k - 2)(k + 4) > 0$
$\Rightarrow\ k^2 + 2k - 8 < 16$
$\Rightarrow k^2 + 6k - 4k - 24 < 0$
$\Rightarrow (k + 6)(k - 4) < 0$
$\Rightarrow\ -6 < k < 4$
If both roots are negative
then $\alpha\beta$ is $+$ ve
$\Rightarrow\ \frac{k + 4}{k - 2} > 0 \Rightarrow k > -4$
Also, $\frac{k - 2}{k + 4} > 0 \Rightarrow k > 2$
Roots are real so $-6 < k < 4$
So, 6 and 4 are not correct.
Since, $k > 2$, so 1 is also not correct value of k.
$\therefore k = 3$

Question299

If $\omega(\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals
[2011]

Options:

A. (1,1)

B. (1,0)

C. (-1,1)

D. (0,1)

Answer: A

Solution:

Solution:
$(1+\omega)^7 = A + B\omega$
$(-\omega^2)^7 = A + B\omega (\because \omega^{14} = \omega^{12} \cdot \omega^2 = \omega^2)$
$-\omega^2 = A + B\omega$
$1 + \omega = A + B\omega$
$\Rightarrow A = 1, B = 1$

Question300

Let for $a \neq a_1 \neq 0$ $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(b)$ is : [2011 RS]

Options:

A. 3

B. 9

C. 6

D. 18

Answer: D

Solution:

Solution:
$p(x) = 0$
$\Rightarrow f(x) = g(x)$
$\Rightarrow ax^2 + bx + c = a_1x^2 + b_1x + c_1$
$\Rightarrow (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0$
It has only one solution, $x = -1$
$\Rightarrow b - b_1 = a - a_1 + c - c_1 \ \ldots (i)$
Sum of roots $\frac{-(b - b_1)}{(a - a_1)} = -1 - 1$
$\Rightarrow \frac{b - b_1}{2(a - a_1)} = 1$ ....(ii)
$\Rightarrow b - b_1 = 2(a - a_1)$
Now $p(-2) = 2$
$\Rightarrow f(-2) - g(-2) = 2$
$\Rightarrow 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$
$\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2 \ldots (iii)$
From equations, (i), (ii) and (iii)
$a - a_1 = c - c_1 = \frac{1}{2}(b - b_1) = 2$
Now, $p(2) = f(2) - g(2)$
$= 4(a - a_1) + 2(b - b_1) + (c - c_1)$
$= 8 + 8 + 2 = 18$

Question301

Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3) . Rahul made a mistake in writing down coefficient of x to get roots (3,2) . The correct roots of equation are :
[2011 RS]

Options:

A. 6,1

B. 4,3

C. -6,-1

D. -4,-3

Answer: A

Solution:

Solution:
Let the correct equation be
$ax^2 + bx + c = 0$
Now, Sachin's equation
$ax^2 + bx + c' = 0$
Given that, roots found by Sachin's are 4 and 3
$\Rightarrow -\frac{b}{a} = 7$ ....(i)
Rahul's equation, $ax^2 + b'x + c = 0$
Given that roots found by Rahul's are 3 and 2
$\Rightarrow \frac{c}{a} = 6$ ....(ii)
From (i) and (ii), roots of the correct equation
$x^2 - 7x + 6 = 0$ are 6 and 1

Question302

Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\text{Re} z = 1$ then it is necessary that :
[2011]

Options:

A. $\beta \in (-1, 0)$

B. $|\beta| = 1$

C. $\beta \in (1, \infty)$

D. $\beta \in (0, 1)$

Answer: C

Solution:

Solution:
Since both the roots of given quadratic equation lie in the line $\text{Re} z = 1$ i.e., $x = 1$, hence real part of both the roots are 1
Let both roots be $1 + i\alpha$ and $1 - i\alpha$
Product of the roots, $1 + \alpha^2 = \beta$
$\because \alpha^2 + 1 \geq 1$
$\therefore \beta \geq 1 \Rightarrow \because \beta \in (1, \infty)$

Question303

The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals
[2010]

Options:

A. 1

B. 2

C. ∞

D. 0

Answer: A

Solution:

Solution:
Let $z = x + iy$
$|z - 1| = |z + 1| \Rightarrow (x - 1)^2 + y^2 = (x + 1)^2 + y^2$
$\Rightarrow x = 0 \Rightarrow \text{Re } z = 0$
$|z - 1| = |z - i| \Rightarrow (x - 1)^2 + y^2 = x^2 + (y - 1)^2$
$\Rightarrow x = y$
$|z + 1| = |z - i| \Rightarrow (x + 1)^2 + y^2 = x^2 + (y - 1)^2$
$\Rightarrow x = -y$
Only (0,0) will satisfy all conditions.
$\Rightarrow$ Number of complex number $z = 1$

Question304

If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2000} =$ [2010]

Options:

A. -1

B. 1

C. 2

D. -2

Answer: B

Solution:

Solution:
$x^2 - x + 1 = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1 - 4}}{2}$
$x = \dfrac{1 \pm \sqrt{3}i}{2}$
$\alpha = \dfrac{1}{2} + i\dfrac{\sqrt{3}}{2} = -\omega^2$
$\beta = \dfrac{1}{2} - \dfrac{i\sqrt{3}}{2} = -\omega$
$\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$
$= -\omega^2 - \omega = 1$

Question305

If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is : [2009]

Options:

A. less than 4ab

B. greater than $-4ab$

C. less than $-4ab$

D. greater than $4ab$

Answer: B

Solution:

Solution:
Given that roots of the equation
$bx^2 + cx + a = 0$ are imaginary
$\therefore\ c^2 - 4ab < 0$
Let $y = 3b^2x^2 + 6bcx + 2c^2$
$\Rightarrow 3b^2x^2 + 6bcx + 2c^2 - y = 0$
As x is real, $D \geq 0$
$\Rightarrow 36b^2c^2 - 12b^2(2c^2 - y) \geq 0$
$\Rightarrow 12b^2(3c^2 - 2c^2 + y) \geq 0 [\because b^2 \geq 0]$
$\Rightarrow\ c^2 + y \geq 0\ \Rightarrow\ y \geq -c^2$
But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$
$\therefore$ we get $y \geq -c^2 > -4ab$
$\Rightarrow\ y > -4ab$

Question306

If $z - \frac{4}{z}\Big| = 2$, then the maximum value of $|z|$ is equal to :
[2009]

Options:

A. $\sqrt{5} + 1$

B. 2

C. $2 + \sqrt{2}$

D. $\sqrt{3} + 1$

Answer: A

Solution:

Solution:
Given that $\left|z - \frac{4}{z}\right| = 2$
$|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right| \leq \left|z - \frac{4}{z}\right| + \frac{4}{|z|}$
$\Rightarrow |z| \leq 2 + \frac{4}{|z|}$
$\Rightarrow |z|^2 - 2|z| - 4 \leq 0$
$\Rightarrow \left(|z| - \frac{2+\sqrt{20}}{2}\right)\left(|z| - \frac{2-\sqrt{20}}{2}\right) \leq 0$
$\Rightarrow (|z| - (1+\sqrt{5}))(|z| - (1-\sqrt{5})) \leq 0$
$\frac{+}{-\infty}\ -\ \frac{+}{\infty}$
$(1-\sqrt{5})(1+\sqrt{5})$
$\Rightarrow\ (-\sqrt{5}+1) \leq |z| \leq (\sqrt{5}+1)$
$\Rightarrow\ |z|\max = \sqrt{5} + 1$

Question307

The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

[2009]

Options:

A. 1

B. 4

C. 3

D. 2

Answer: D

Solution:

Solution:
Let the roots of equation $x^2 - 6x + a = 0$ be α and 4 β and that of the equation
$x^2 - cx + 6 = 0$ be α and 3β.Then
$\alpha + 4\beta = 6$ ... (i) $4\alpha\beta = a$... (ii)
and $\alpha + 3\beta = c$... (iii) $3\alpha\beta = 6$... (iv)
$\Rightarrow$ $a = 8$(from (ii) and (iv))
$\therefore$ The equation becomes $x^2 - 6x + 8 = 0$
$\Rightarrow$ $(x - 2)(x - 4) = 0$
$\Rightarrow$ roots are 2 and 4
$\Rightarrow \alpha = 2, \beta = 1$ $\therefore$ Common root is 2

Question308

The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is [2008]

Options:

A. $\frac{-1}{i-1}$

B. $\frac{1}{i+1}$

C. $\frac{-1}{i+1}$

D. $\frac{1}{i-1}$

Answer: C

Solution:

Solution:
$\overline{\left(\frac{1}{i-1}\right)} = \frac{1}{\overline{(i-1)}} = \frac{1}{-i-1} = \frac{-1}{i+1}$

Question309

If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is [2007]

Options:

A. 6

B. 0

C. 4

D. 10

Answer: A

Solution:

Solution:
$|z+1| = |z+4-3| \le |z+4| + |-3| \le |3| + |-3|$
$\Rightarrow |z+1| \le 6 \Rightarrow |z+1|_{max} = 6$

Question310

If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is
[2007]

Options:

A. $(3, \infty)$

B. $(-\infty, -3)$

C. (-3,3)

D. $(-3, \infty)$

Answer: C

Solution:

Solution:
Let α and β are roots of the equation
$x^2 + ax + 1 = 0$
$\alpha + \beta = -a$ and $\alpha\beta = 1$
Given that $|\alpha - \beta| < \sqrt{5}$
$\Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} < \sqrt{5}$
$(\because (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta)$
$\Rightarrow \sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 - 4 < 5$
$\Rightarrow a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$
$\Rightarrow a \in (-3, 3)$

Question311

All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 lie in the interval
[2006]

Options:

A. $-2 < m < 0$

B. $m > 3$

C. $-1 < m < 3$

D. $1 < m < 4$

Answer: C

Solution:

Solution:
Given equation is $x^2 - 2mx + m^2 - 1 = 0$
$\Rightarrow (x-m)^2 - 1 = 0$
$\Rightarrow (x-m+1)(x-m-1) = 0$
$\Rightarrow x = m-1, m+1$

$m-1>-2$ and $m+1<4$
$\Rightarrow m>-1$ and $m<3 \Rightarrow -1<m<3$

Question312

If the roots of the quadratic equation $x^2+px+q=0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively, then the value of $2+q-p$ is
[2006]

Options:

A. 2

B. 3

C. 0

D. 1

Answer: B

Solution:

Solution:
Given that $x^2+px+q=0$
Sum of roots $=\tan 30^\circ+\tan 15^\circ=-p$
Product of roots $=\tan 30^\circ \cdot \tan 15^\circ=q$
$\tan 45^\circ=\dfrac{\tan 30^\circ+\tan 15^\circ}{1-\tan 30^\circ \cdot \tan 15^\circ} \Rightarrow \dfrac{-p}{1-q}=1$
$\Rightarrow -p=1-q \Rightarrow q-p=1$
$\therefore 2+q-p=3$

Question313

If $z^2+z+1=0$, where z is complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots\ldots\ldots+\left(z^6+\frac{1}{z^6}\right)^2$ is
[2006]

Options:

A. 18

B. 54

C. 6

D. 12

Answer: D

Solution:

Solution:
$z^2+z+1=0 \Rightarrow z=\omega$ or ω^2
So, $z+\dfrac{1}{z}=\omega+\omega^2=-1$
$\left[\because \dfrac{1}{z}=\omega^2\right.$ and $\left.1+\omega+\omega^2=0\right]$
$z^2+\dfrac{1}{z^2}=\omega^2+\omega=-1$
$[\because \omega^3=1]$
$z^3+\dfrac{1}{z^3}=\omega^3+\omega^3=2$

$z^4 + \frac{1}{z^4} = -1, z^5 + \frac{1}{z^5} = -1$
and $z^6 + \frac{1}{z^6} = 2$
$\therefore$ The given sum $= 1 + 1 + 4 + 1 + 1 + 4 = 12$

Question314

If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is [2006]

Options:

A. $\frac{1}{4}$

B. 41

C. 1

D. $\frac{17}{7}$

Answer: B

Solution:

Solution:
$y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$
$3x^2(y - 1) + 9x(y - 1) + 7y - 17 = 0$
$D \geq 0 \ \because$ x is real
$81(y - 1)^2 - 4 \times 3(y - 1)(7y - 17) \geq 0$
$\Rightarrow (y - 1)(y - 41) \leq 0 \Rightarrow 1 \leq y \leq 41$
$\therefore$ Max value of y is 41

Question315

If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on [2005]

Options:

A. an ellipse

B. a circle

C. a straight line

D. a parabola

Answer: C

Solution:

Solution:
Given that $w = \frac{z}{z - \frac{1}{3}i}$
$\Rightarrow |w| = \frac{|z|}{\left|z - \frac{1}{3}i\right|} = 1 \quad \left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$
$\Rightarrow |z| = \left|z - \frac{1}{3}i\right|$

⇒ distance of z from origin and point $\left(0, \frac{1}{3}\right)$ is same hence z lies on bisector of the line joining points (0,0) and $(0, 1/3)$
Hence z lies on a straight line.

Question316

If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg $z_1 - \arg z_2$ is equal to
[2005]

Options:

A. $\frac{\pi}{2}$

B. $-\pi$

C. 0

D. $\frac{-\pi}{2}$

Answer: C

Solution:

Solution:
$|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence arg $z_1 - \arg z_2 = 0$.

Question317

If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are
[2005]

Options:

A. $-1, -1+2\omega, -1-2\omega^2$

B. -1,-1,-1

C. $-1, 1-2\omega, 1-2\omega^2$

D. $-1, 1+2\omega, 1+2\omega^2$

Answer: C

Solution:

Solution:
$\because (x-1)^3 + 8 = 0 \Rightarrow (x-1) = (-2)(1)^{1/3}$
$\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$
or $x = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$

Question318

In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $-\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0, a \neq 0$ then
[2005]

Options:

A. $a = b + c$

B. $c = a + b$

C. $b = c$

D. $b = a + c$

Answer: B

Solution:

Solution:
$\tan\left(\frac{P}{2}\right), \tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$
$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$
$\tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right) = \frac{c}{a}$
$$\frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$$
$\left[\because P + Q = \frac{\pi}{2}\right]$
$$\Rightarrow \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = 1 \Rightarrow -\frac{b}{a} = \frac{a - c}{a}$$
$\Rightarrow -b = a - c \Rightarrow c = a + b$

Question319

If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals [2005]

Options:

A. -2

B. 3

C. 2

D. 1

Answer: D

Solution:

Solution:
Let $\alpha, \alpha + 1$ be roots
Then $\alpha + \alpha + 1 = b =$ sum of roots
$\alpha(\alpha + 1) = c =$ product of roots
$\therefore\ b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1) = 1$

Question320

If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval [2005]

Options:

A. (5,6]

B. $(6, \infty)$

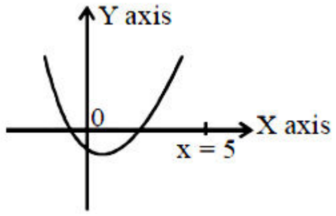
C. $(-\infty, 4)$

D. [4,5]

Answer: C

Solution:

Solution:
Given that both roots of quadratic equation are less than 5 then (i)



Discriminant ≥ 0
$4k^2 - 4(k^2 + k - 5) \geq 0$
$4k^2 - 4k^2 - 4k + 20 \geq 0$
$4k \leq 20 \Rightarrow k \leq 5$
(ii) $p(5) > 0$
$\Rightarrow f(5) > 0;\ 25 - 10k + k^2 + k - 5 > 0$
$\Rightarrow k^2 - 9k + 20 > 0$
$\Rightarrow k(k-4) - 5(k-4) > 0$
$\Rightarrow (k-5)(k-4) > 0$

(iii) $\dfrac{\text{Sum of roots}}{2} < 5$
$\Rightarrow -\dfrac{b}{2a} = \dfrac{2k}{2} < 5$
$\Rightarrow k < 5$
The intersection of (i), (ii) & (iii) gives
$k \in (-\infty, 4)$

Question321

The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is
[2005]

Options:

A. 1

B. 0

C. 3

D. 2

Answer: A

Solution:

Solution:
Given equation is $x^2 - (a-2)x - a - 1 = 0$
$\Rightarrow \alpha + \beta = a - 2;\ \alpha\beta = -(a+1)$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$\quad = a^2 - 2a + 6 = (a-1)^2 + 5$
For min. value of $\alpha^2 + \beta^2$, $a - 1 = 0$
$\Rightarrow \quad a = 1$

Question322

If $z = x - iy$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) / (p^2 + q^2)$ is equal to
[2004]

Options:

A. -2

B. -1

C. 2

D. 1

Answer: A

Solution:

Solution:
Given that $z^{\frac{1}{3}} = p + iq$
$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$
$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$
Comparing both side, we get
$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$ .... (i)
and $y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$......... (ii)
Adding (i) and (ii), we get
$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \quad \therefore \left(\frac{x}{p} + \frac{y}{q}\right) / (p^2 + q^2) = -2$

Question323

Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then arg z equals
[2004]

Options:

A. $\frac{5\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:
Given that arg $zw = \pi$
$\Rightarrow \arg z + \arg w = \pi$
$\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$
Replace i by $-i$, we get
$\therefore z = iw \Rightarrow \arg z = \frac{\pi}{2} + \arg w$
$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z$ (from (i))
$\arg z = \frac{3\pi}{4}$

Question324

If $|z^2-1| = |z|^2+1$, then z lies on
[2004]

Options:

A. an ellipse

B. the imaginary axis

C. a circle

D. the real axis

Answer: B

Solution:

Solution:
Given that $|z^2-1| = |z|^2+1 \Rightarrow |z^2-1|^2 = (z\bar{z}+1)^2$
$[\because |z|^2 = z\bar{z}]$
$\Rightarrow (z^2-1)(\bar{z}^2-1) = (z\bar{z}+1)^2 (\because \overline{z_1-z_2} = \bar{z}_1 - \bar{z}_2)$
$\Rightarrow z^2\bar{z}^2 - z^2 - \bar{z}^2 + 1 = z^2\bar{z}^2 + 2z\bar{z} + 1$
$\Rightarrow z^2 + 2z\bar{z} + \bar{z}^2 = 0$
$\Rightarrow (z+\bar{z})^2 = 0 \Rightarrow z = -\bar{z}$
$\Rightarrow z$ is purely imaginary

Question325

If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots, then the value of $^\circ q'$ is
[2004]

Options:

A. 4

B. 12

C. 3

D. $\frac{49}{4}$

Answer: D

Solution:

Solution:
Given that 4 is a root of $x^2+px+12=0$
$\Rightarrow 16+4p+12=0 \Rightarrow p=-7$
Now, the equation $x^2+px+q=0$ has equal roots.
$\therefore D=0$
$\Rightarrow p^2-4q=0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$

Question326

If $(1-p)$ is a root of quadratic equation $x^2+px+(1-p)=0$ then its root are
[2004]

Options:

A. -1,2

B. -1,1

C. 0,-1

D. 0,1

Answer: C

Solution:

Solution:
Let the second root be α.
Then $\alpha + (1-p) = -p \Rightarrow \alpha = -1$
Also $\alpha \cdot (1-p) = 1-p$
$\Rightarrow (\alpha - 1)(1-p) = 0 \Rightarrow p = 1 [\because \alpha = -1]$
$\therefore$ Roots are $\alpha = -1$ and $1 - p = 0$

Question327

If $\left(\frac{1+i}{1-i}\right)^{x} = 1$ then
[2003]

Options:

A. $x = 2n + 1$, where n is any positive integer

B. $x = 4n$, where n is any positive integer

C. $x = 2n$, where n is any positive integer

D. $x = 4n + 1$, where n is any positive integer.

Answer: B

Solution:

Solution:
Given that
$\left(\frac{1+i}{1-i}\right)^{x} = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^{x} = 1$
$\left(\frac{1+i^2+2i}{1+1}\right)^{x} = 1 \Rightarrow (i)^{x} = 1; \quad \therefore x = 4n; \quad n \in I^{+}$

Question328

If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $\operatorname{Arg}(z) - \operatorname{Arg}(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to
[2003]

Options:

A. -1

B. 1

C. $-i$

D. i

Answer: A

Solution:

Solution:
$|\bar{z}\omega| = |\bar{z}||\omega| = |z||\omega| = |z\omega| = 1 [\because |\bar{z}| = |z|]$
$\operatorname{Arg}(\bar{z}\omega) = \arg(\bar{z}) + \arg(\omega)$

$= -\arg(z) + \arg \omega = -\frac{\pi}{2}$
$[\because \arg(\bar{z}) = -\arg(z)]$
$\therefore z\omega = -1$

Question329

The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is [2003]

Options:

A. 3

B. 2

C. 4

D. 1

Answer: C

Solution:

Solution:
Given that $x^2 - 3|x| + 2 = 0 \Rightarrow |x|^2 - 3|x| + 2 = 0$
$\Rightarrow (|x| - 2)(|x| - 1) = 0$
$\Rightarrow |x| = 1, 2 \Rightarrow x = \pm 1, \pm 2$
$\therefore$ No. of solution $= 4$

Question330

The value of ' a ' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is [2003]

Options:

A. $-\frac{1}{3}$

B. $\frac{2}{3}$

C. $-\frac{2}{3}$

D. $\frac{1}{3}$

Answer: B

Solution:

Solution:
Let one roots of given equation be α
$\therefore$ Second roots be 2α then
$\alpha + 2\alpha = 3\alpha = \frac{1 - 3a}{a^2 - 5a + 3}$
$\Rightarrow \alpha = \frac{1 - 3a}{3(a^2 - 5a + 3)}$ ....(i)
and $\alpha.2\alpha = 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$
$\therefore 2\left[\frac{1}{9}\frac{(1 - 3a)^2}{(a^2 - 5a + 3)^2}\right] = \frac{2}{a^2 - 5a + 3}$
[from (i)]
$\frac{(1 - 3a)^2}{(a^2 - 5a + 3)} = 9$

$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$
$\Rightarrow 39a = 26 \Rightarrow a = \frac{2}{3}$

Question331

Let Z_1 and Z_2 be two roots of the equation $Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then [2003]

Options:

A. $a^2 = 4b$

B. $a^2 = b$

C. $a^2 = 2b$

D. $a^2 = 3b$

Answer: D

Solution:

Solution:
Given that $Z^2 + aZ + b = 0$;
$Z_1 + Z_2 = -a$ & $Z_1 Z_2 = b$
$0, Z_1, Z_2$ form an equilateral triangle $\therefore 0^2 + Z_1{}^2 + Z_2{}^2 = 0 . Z_1 + Z_1 \cdot Z_2 + Z_2 \cdot 0$
(for an equilateral triangle,
$Z_1{}^2 + Z_2{}^2 + Z_3{}^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$)
$\Rightarrow Z_1{}^2 + Z_2{}^2 = Z_1 Z_2$
$\Rightarrow (Z_1 + Z_2)^2 = 3Z_1 Z_2$
$\therefore a^2 = 3b$

Question332

If $|z - 4| < |z - 2|$, its solution is given by [2002]

Options:

A. $Re(z) > 0$

B. $Re(z) < 0$

C. $Re(z) > 3$

D. $Re(z) > 2$

Answer: C

Solution:

Solution:
Given that $|z - 4| < |z - 2$
Let $z = x + iy$
$\Rightarrow |(x - 4) + iy)| < |(x - 2) + iy|$
$\Rightarrow (x - 4)^2 + y^2 < (x - 2)^2 + y^2$
$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$
$\Rightarrow x > 3 \Rightarrow Re(z) > 3$

Question333

z and w are two non zero complex numbers such that $|z| = |w|$ and $\text{Arg}z + \text{Arg}w = \pi$ then z equals [2002]

Options:

A. $\overline{\omega}$

B. $-\overline{\omega}$

C. ω

D. $-\omega$

Answer: B

Solution:

Solution:
Let $|z| = |\omega| = r$
$\therefore z = re^{i\theta}, \omega = re^{i\varphi}$ where $\theta + \varphi = \pi$
$\therefore z = re^{i(\pi - \varphi)} = re^{i\pi} \cdot e^{-i\varphi = -re^{-i\varphi}} = -\overline{\omega}$
$[\because e^{i\pi} = -1$ and $\overline{\omega} = re^{-i\varphi}]$

Question334

The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be [2002]

Options:

A. an ellipse

B. a hyperbola

C. a circle

D. none of these

Answer: B

Solution:

Solution:
Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a$ ... (i)
$|z_0 - z_2| = r + b$ ....(ii)
Subtract (ii) from (i)
we get $|z_0 - z_1| - |z_0 - z_2| = a - b$.
$\therefore$ Locus of centre z_0 is $|z - z_1| - |z - z_2|$
$= a - b$, which represents a hyperbola.

Question335

If p and q are the roots of the equation $x^2 + px + q = 0$, then [2002]

Options:

A. $p = 1, q = -2$

B. $p = 0, q = 1$

C. $p = -2, q = 0$

D. $p = -2, q = 1$

Answer: A

Solution:

Solution:
$p + q = -p \Rightarrow q = 2p$
and $pq = q \Rightarrow q(p-1) = 0$
$\Rightarrow q = 0$ or $p = 1$
If $q = 0$, then $p = 0$
or $p = 1$, then $q = -2$.

Question336

Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ [2002]

Options:

A. is always positive

B. is always negative

C. does not exist

D. none of these

Answer: A

Solution:

Solution:
Product of real roots $= \frac{c}{a} = \frac{9}{t^2} > 0, \forall t \in R$
$\therefore$ Product of real roots is always positive.

Question337

Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then [2002]

Options:

A. $a + b + 4 = 0$

B. $a + b - 4 = 0$

C. $a - b - 4 = 0$

D. $a - b + 4 = 0$

Answer: A

Solution:

Solution:
Let α and β are roots of the equation $x^2 + ax + b = 0$ and γ and δ be the roots of the equation $x^2 + bx + a = 0$ respectively
$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a$
Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$
$\Rightarrow a^2 - 4b = b^2 - 4a$
$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$
$\Rightarrow a + b + 4 = 0 \quad (\because a \neq b)$

Question338

If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is [2002]

Options:

A. $3x^2 - 19x + 3 = 0$

B. $3x^2 + 19x - 3 = 0$

C. $3x^2 - 19x - 3 = 0$

D. $x^2 - 5x + 3 = 0.$

Answer: A

Solution:

Solution:
Given that $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3;$
$\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$
or $x^2 - 5x + 3 = 0$
$\therefore \alpha + \beta = 5$ and $\alpha\beta = 3$
Thus, the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is
$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$
$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$
or $3x^2 - 19x + 3 = 0$
