Linear Inequalities

Question1

If the equation $\mathrm{a(b-c)x^2+b(c-a)x+c(a-b)=0}$ has equal roots, where $\mathrm{a+c=15}$ and $\mathrm{b=\frac{36}{5}}$, then a^2+c^2 is equal to __________

JEE Main 2025 (Online) 23rd January Morning Shift

Answer: 117

Solution:

To solve the given problem, we start with the quadratic equation:

$a(b-c)x^2+b(c-a)x+c(a-b)=0$

Given that the roots are equal (let's assume both roots are 1), we know the sum of the roots, $\alpha+\beta$, is twice the value of one root, which leads us to:

$\alpha+\beta=2$

Using the formula for the sum of roots for a quadratic equation, $\alpha+\beta=-\frac{b(c-a)}{a(b-c)}$, we set this equal to 2:

$-\frac{b(c-a)}{a(b-c)}=2$

Solving for this:

$-bc+ab=2ab-2ac2ac=ab+bc2ac=b(a+c)$

Given that $a+c=15$ and $b=\frac{36}{5}$, substitute these into the equation:

$2ac=15b2ac=15\times\frac{36}{5}=108ac=54$

Now, using the equation $a+c=15$ and $ac=54$, find a^2+c^2:

$a^2+c^2=(a+c)^2-2ac=15^2-2\times54a^2+c^2=225-108=117$

Therefore, a^2+c^2 is equal to 117.

Question2

If the set of all $a \in \mathbf{R} - \{1\}$, for which the roots of the equation $(1 - a)x^2 + 2(a - 3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to ________.

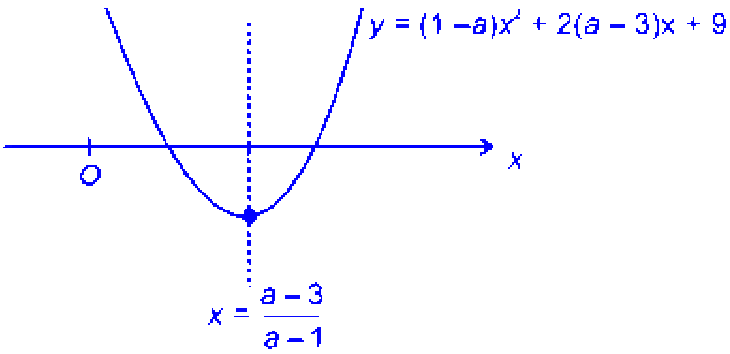
JEE Main 2025 (Online) 2nd April Evening Shift

Answer: 7

Solution:

$f(x) = (1 - a)x^2 + 2(a - 3)x + 9, f(0) = 9 > 0$
$D \geq 0 \Rightarrow 4(a - 3)^2 \geq 4(1 - a) \cdot 9$

$\Rightarrow a \in (-\infty, -3] \cup [0, \infty)$ ..... (i)



$x_1 + x_2 = \dfrac{-2(a - 3)}{1 - a}, x_1 x_2 = \dfrac{9}{1 - a}$
$x_1 + x_2 > 0 \Rightarrow \dfrac{a - 3}{a - 1} > 0 \Rightarrow a \in (-\infty, 1) \cup (3, \infty) \ldots$ (ii)

$x_1 x_2 > 0 \Rightarrow 1 - a > 0 \Rightarrow a \in (-\infty, 1) \ldots$ (iii)

$\Rightarrow$ Interaction of (i), (ii) and (iii)
$a \in (-\infty, -3] \cup [0, 1)$
$\Rightarrow \quad \alpha = 3, \beta = 0, \gamma = 1 \Rightarrow 2\alpha + \beta + \gamma = 7$

Question3

Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0, \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then $16(M + m)$ equals :

JEE Main 2025 (Online) 22nd January Evening Shift

Options:

A. 27

B. 17

C. 25

D. 24

Answer: C

Solution:

To find the sum of the fourth powers of the roots α_θ and β_θ of the quadratic equation $2x^2 + (\cos\theta)x - 1 = 0$, we start analyzing the expression $\alpha_\theta^4 + \beta_\theta^4$.

The equation can be rewritten with its roots using:

$\alpha + \beta = -\frac{\cos\theta}{2}, \quad \alpha\beta = -\frac{1}{2}$

We need to calculate $\alpha^2 + \beta^2$ and $\alpha^2\beta^2$:

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(-\frac{\cos\theta}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = \frac{\cos^2\theta}{4} + 1$

$\alpha^2\beta^2 = (\alpha\beta)^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

Substitute these into:

$\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = \left(\frac{\cos^2\theta}{4} + 1\right)^2 - \frac{1}{2}$

Maximize and minimize $\left(\frac{\cos^2\theta}{4} + 1\right)^2$:

Zero of $\cos\theta$ leads to:

$\left(\frac{0^2}{4} + 1\right)^2 = 1$

Max value $\cos^2\theta = 1$:

$\left(\frac{1}{4} + 1\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

Substitute back:

Max: $\frac{25}{16} - \frac{1}{2} = \frac{25}{16} - \frac{8}{16} = \frac{17}{16}$

Min: $1 - \frac{1}{2} = \frac{1}{2}$

Finally, compute $16(M+m)$:

$16\left(\frac{17}{16}+\frac{1}{2}\right)=16\left(\frac{17}{16}+\frac{8}{16}\right)=16\times\frac{25}{16}=25$

Question4

The product of all the rational roots of the equation $\left(x^2-9x+11\right)^2-(x-4)(x-5)=3$, is equal to

JEE Main 2025 (Online) 24th January Morning Shift

Options:

A. 7

B. 21

C. 28

D. 14

Answer: D

Solution:

To solve the given equation, start by rewriting the expression:

$(x^2-9x+11)^2-(x-4)(x-5)=3$

First, simplify the second part of the expression:

$(x-4)(x-5)=x^2-9x+20$

Now the equation becomes:

$(x^2-9x+11)^2-(x^2-9x+20)=3$

Introduce a substitution for simplification:

Let $t=x^2-9x$

Thus, the equation transforms to:

$t^2+22t+121-t-20-3=0$

Simplify further:

$t^2+21t+98=0$

Factor the quadratic:

$(t+14)(t+7)=0$

This gives:

$t = -7 \quad \text{or} \quad t = -14$

Address each case where $t = x^2 - 9x$:

$x^2 - 9x = -7$

$x^2 - 9x + 7 = 0$

Solving this quadratic equation, we find the roots:

$x = \frac{9 \pm \sqrt{81 - 4 \times 7}}{2} = \frac{9 \pm \sqrt{53}}{2}$

$x^2 - 9x = -14$

$x^2 - 9x + 14 = 0$

Solving this quadratic equation:

$x = \frac{9 \pm \sqrt{81 - 4 \times 14}}{2} = \frac{9 \pm \sqrt{25}}{2}$

$x = \frac{9 \pm 5}{2} = 7 \quad \text{or} \quad x = 2$

The rational roots from the second equation are 7 and 2. Thus, the product of all the rational roots is:

$7 \times 2 = 14$

Question5

The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x - 3|, |x + 2|\}$ is :

JEE Main 2025 (Online) 24th January Evening Shift

Options:

A. 2

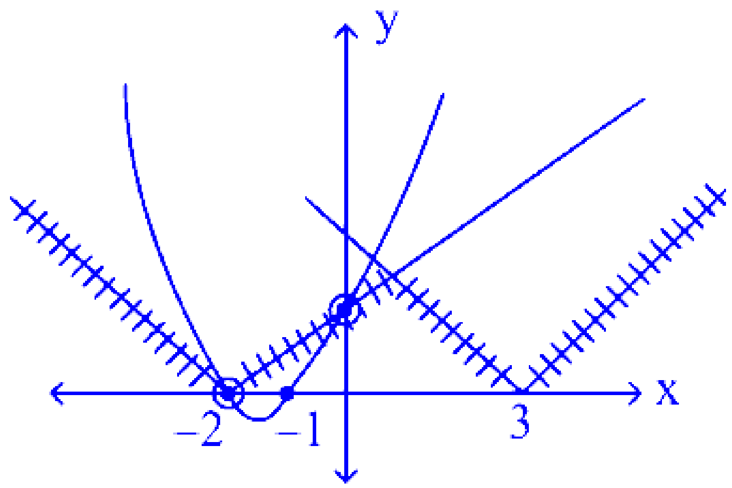
B. 3

C. 1

D. 0

Answer: A

Solution:



Only 2 solutions.

Question6

The sum, of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$, is

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. $6(2 - \sqrt{2})$

B. $3(3 - \sqrt{2})$

C. $3(2 - \sqrt{2})$

D. $6(3 - \sqrt{2})$

Answer: A

Solution:

To find the sum of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$:

Case I: $x \geq \frac{3}{2}$

For $x \geq \frac{3}{2}$, the expression $|2x - 3|$ becomes $2x - 3$. Thus, the equation becomes:

$x^2 + 2x - 3 - 4 = 0$

Simplifying gives:

$x^2 + 2x - 7 = 0$

Solving this quadratic equation, we find:

$x = 2\sqrt{2} - 1$

Case II: $x < \frac{3}{2}$

For $x < \frac{3}{2}$, the expression $|2x - 3|$ becomes $-(2x - 3) = -2x + 3$. The equation therefore becomes:

$x^2 + 3 - 2x - 4 = 0$

Simplifying gives:

$x^2 - 2x - 1 = 0$

Solving this quadratic equation, we obtain:

$x = 1 - \sqrt{2}$

Sum of the Squares of the Roots

The sum of the squares of the roots is:

$(2\sqrt{2} - 1)^2 + (1 - \sqrt{2})^2$

Calculating each term:

$(2\sqrt{2} - 1)^2 = (2\sqrt{2})^2 - 2 \cdot 2\sqrt{2} \cdot 1 + 1^2 = 8 - 4\sqrt{2} + 1 = 9 - 4\sqrt{2}$

$(1 - \sqrt{2})^2 = 1^2 - 2 \cdot 1 \cdot \sqrt{2} + (\sqrt{2})^2 = 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$

Adding these results:

$(9 - 4\sqrt{2}) + (3 - 2\sqrt{2}) = 12 - 6\sqrt{2}$

This simplifies to $6(2 - \sqrt{2})$.

Thus, the sum of the squares of the roots is $6(2 - \sqrt{2})$.

Question7

Let $f : \mathbf{R} - \{0\} \rightarrow (-\infty, 1)$ be a polynomial of degree 2 , satisfying $f(x) f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. If $f(\mathrm{~K}) = -2\mathrm{~K}$, then the sum of squares of all possible values of K is :

JEE Main 2025 (Online) 28th January Evening Shift

Options:

A.

9

B.

1

C.

6

D.

7

Answer: C

Solution:

as $f(x)$ is a polynomial of degree two let it be

$f(x)=ax^2+bx+c \quad (a\neq 0)$

on satisfying given conditions we get

$C=1\&a=\pm 1$

hence $f(x)=1\pm x^2$

also range $\in(-\infty,1]$ hence

$f(x)=1-x^2$
now $f(k)=-2k$

$1-\mathrm{k}^2=-2\mathrm{k}\rightarrow \mathrm{k}^2-2\mathrm{k}-1=0$

let roots of this equation be $\alpha\&\beta$ then $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=4-2(-1)=6$

Question8

The number of solutions of the equation

$$\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0 \text{ is :}$$

JEE Main 2025 (Online) 29th January Morning Shift

Options:

A.

3

B.

2

C.

1

D.

4

Answer: D

Solution:

Consider $\frac{1}{\sqrt{\mathrm{x}}}=\alpha \quad \mathrm{x}>0$

$$\left(9\alpha^2-9\alpha+2\right)\left(2\alpha^2-7\alpha+3\right)=0$$
$$(3\alpha-2)(3\alpha-1)(\alpha-3)(2\alpha-1)=0$$
$$\alpha=\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 3$$
$$x=9,4, \frac{9}{4}, \frac{1}{9}$$

So, no. of solutions $=4$

Question9

If the set of all $a \in \mathbf{R}$, for which the equation $2x^2+(a-5)x+15=3a$ has no real root, is the interval (α, β), and $X=|x \in Z ; \alpha<x<\beta|$, then $\sum\limits_{x \in X} x^2$ is equal to:

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Options:

A.

2139

B.

2119

C.

2109

D.

2129

Answer: A

Solution:

$$\begin{aligned}
&(a-5)^2-8(15-3a)<0\\
&a^2+14a+25-120<0\\
&a^2+14a-95<0\\
&(a+19)(a-5)<0\\
&a\in(-19,5)\\
&\therefore -19<x<5\\
&\therefore \sum_{x\in X} x^2=\left(1^2+2^2+\ldots.+4^2\right)+\left(1^2+2^2+\ldots+18^2\right)\\
&=\frac{4\times5\times9}{6}+\frac{18\times19\times37}{6}\\
&=30+2109\\
&=2139
\end{aligned}$$

Question10

Let $\mathrm{P_n}=\alpha^{\mathrm{n}}+\beta^{\mathrm{n}}, \mathrm{n}\in\mathrm{N}$. If $\mathrm{P}_{10}=123, \mathrm{P}_9=76, \mathrm{P}_8=47$ and $\mathrm{P}_1=1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. $x^2+x-1=0$

B. $x^2-x+1=0$

C. $x^2+x+1=0$

D. $x^2-x-1=0$

Answer: A

Solution:

Given:

$P_{10} = 123$

$P_9 = 76$

$P_8 = 47$

$P_1 = 1$

We know that:

$P_n = \alpha^n + \beta^n$

According to Newton's identities, we have the relation:

$P_{10} = P_9 + P_8$

This implies:

$P_{10} - P_9 - P_8 = 0$

From $P_1 = 1$, it follows that:

$\alpha + \beta = 1$

$\alpha\beta = 1$

Now, we are tasked with finding the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. For such a quadratic equation:

Using the relationship between roots and coefficients, the equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is:

$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$

Substitute the known sum and product of α and β:

$x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$

Given that $\alpha + \beta = 1$ and $\alpha\beta = 1$, we can simplify:

$x^2 - \left(\frac{1}{1}\right)x + \frac{1}{1} = 0$

This simplifies to:

$x^2 + x - 1 = 0$

Thus, the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is:

$x^2 + x - 1 = 0$

Question11

Let α and β be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q_n = \gamma^n + \hat{o}^n$, then $\frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25}-Q_{23}}{Q_{24}}$ is equal to

JEE Main 2025 (Online) 3rd April Morning Shift

Options:

A. 4

B. 3

C. 5

D. 7

Answer: C

Solution:

$$x^2 + 3x - 1 = 0 \begin{matrix} \nearrow \gamma \\ \searrow \delta \end{matrix} \Rightarrow x^2 - 1 = -3x$$

$$\Rightarrow \quad P^n = \gamma^n + \delta^n$$
$$P_{25} - P_{23} = \left(\gamma^{25} - \gamma^{23}\right) + \left(\delta^{25} - \delta^{23}\right)$$
$$= \gamma^{23}\left(\gamma^2 - 1\right) + \delta^{23}\left(\delta^2 - 1\right)$$
$$= \gamma^{23}(-3\gamma) + \delta^{23}(-3\delta) = -3\left[\gamma^{24} + \delta^{24}\right]$$

$$\Rightarrow \frac{P_{25} - P_{23}}{P_{24}} = (-3)$$

Similarly,

$$x^2 + \sqrt{3}x - 16 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \quad Q_n = \alpha^n + \beta^n$$

$$\Rightarrow Q_{25} + \sqrt{3}Q_{24} = \left(\alpha^{25} + \sqrt{3}\alpha^{24}\right) + \left(\beta^{25} + \sqrt{3}\beta^{24}\right)$$
$$= \alpha^{23}\left(\alpha^2 + \sqrt{3}\alpha\right) + \beta^{23}\left(\beta^2 + \sqrt{3}\beta\right)$$
$$= \alpha^{23}(16) + 16\beta^{23}$$
$$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2 \cdot Q_{23}} = \frac{16\left(\alpha^{23} + \beta^{23}\right)}{2\left(\alpha^{23} + \beta^{23}\right)} = 8$$
$$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2Q_{23}} + \frac{\left(P_{25} - P_{23}\right)}{P_{24}} = 8 + (-3) = 5$$

Question12

Let the equation $x(x+2)(12-k) = 2$ have equal roots. Then the distance of the point $\left(k, \frac{k}{2}\right)$ from the line $3x + 4y + 5 = 0$ is

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. 15

B. 12

C. $5\sqrt{3}$

D. $15\sqrt{5}$

Answer: A

Solution:

Given the equation $x(x+2)(12-k)=2$, we want it to have equal roots. To achieve this, we need to manipulate it into a quadratic form in terms of x:

$x^2+2x-\frac{2}{12-k}=0$

For this quadratic equation to have equal (repeated) roots, the discriminant D must be zero. The discriminant D for the equation $ax^2+bx+c=0$ is given by:

$D=b^2-4ac$

Substituting $a=1$, $b=2$, and $c=-\frac{2}{12-k}$ into the discriminant formula, we get:

$4-4\left(-\frac{2}{12-k}\right)=0$

Simplifying the expression:

$1+\frac{2}{12-k}=0$

Solving for k:

$\frac{2}{12-k}=-1 \quad \Rightarrow \quad 2=-(12-k) \quad \Rightarrow \quad 2=-12+k \quad \Rightarrow \quad k=14$

Now, consider the point $(k, \frac{k}{2})$, which becomes $(14,7)$. We need to find its distance from the line $3x+4y+5=0$. The formula for the distance d from a point (x_1,y_1) to a line $Ax+By+C=0$ is:

$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Substituting $(x_1,y_1)=(14,7)$ and the line coefficients $A=3$, $B=4$, $C=5$:

$d=\frac{|3(14)+4(7)+5|}{\sqrt{3^2+4^2}}$

Calculating the numerator:

$3\times 14+4\times 7+5=42+28+5=75$

And the denominator:

$\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

Therefore, the distance is:

$d = \frac{75}{5} = 15$

Thus, the distance is 15.

Question13

Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to

JEE Main 2025 (Online) 4th April Morning Shift

Options:

A. 6

B. 5

C. 8

D. 7

Answer: A

Solution:

$x^2 + 4x - n = 0$ has integer roots

$$\Rightarrow x = \frac{-4 \pm \sqrt{16 + 4n}}{2} = -2 \pm \sqrt{4 + n}$$

For x to be integer $4 + n$ must be perfect squares

$n \in [20, 100]$
$n + 4 \in [24, 104] = S$

$\{25, 36, \ldots 10^2\} \in S \Rightarrow 5^2, 6^2, \ldots 10^2 \Rightarrow 6$ values of n

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Question14

Let the set of all values of $p \in \mathbb{R}$, for which both the roots of the equation $x^2 - (p + 2)x + (2p + 9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 5

B. 0

C. 20

D. 9

Answer: A

Solution:

To find the set of all values of $p \in \mathbb{R}$ for which both roots of the equation $x^2 - (p+2)x + (2p+9) = 0$ are negative real numbers, follow these steps:

Discriminant Condition:

The equation's discriminant D must be non-negative for real roots:

$(p+2)^2 - 4(2p+9) \geq 0$

Simplifying this:

$p^2 + 4p + 4 - 8p - 36 \geq 0 \quad \Rightarrow \quad p^2 - 4p - 32 \geq 0$

This can be factored as:

$(p-8)(p+4) \geq 0$

Meaning $p \in (-\infty, -4] \cup [8, \infty)$. …… (1)

Sum of Roots Condition:

The sum of the roots (which is $p+2$) must be negative:

$p + 2 < 0 \quad \Rightarrow \quad p < -2$ …… (2)

Product of Roots Condition:

The product of the roots $(2p+9)$ must be positive:

$2p + 9 > 0 \quad \Rightarrow \quad p > -\frac{9}{2}$ …… (3)

Determine the Valid Interval:

Combine the results from conditions (1), (2), and (3). From conditions (1) and (2), we find $p < -2$:

Intersection of $(-\infty, -4]$ and $(-\frac{9}{2}, -2)$ gives:

$p \in \left(-\frac{9}{2}, -4\right]$

Calculate $\beta - 2\alpha$:

With $\alpha = -\frac{9}{2}$ and $\beta = -4$, compute:

$\beta - 2\alpha = -4 - 2\left(-\frac{9}{2}\right) = -4 + 9 = 5$

Therefore, the difference $\beta - 2\alpha$ is 5.

Question15

The number of real roots of the equation $x|x-2| + 3|x-3| + 1 = 0$ is :

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

4

B.

3

C.

2

D.

1

Answer: D

Solution:

(I) $x < 2$

$-x^2 + 2x - 3x + 9 + 1 = 0$

$\Rightarrow x^2 + x - 10 = 0$

$$\Rightarrow x = \underset{\times}{\frac{-1+\sqrt{41}}{2}}, \underset{\checkmark}{\frac{-1-\sqrt{41}}{2}}$$

(II) $2 \leq x < 3$
$\Rightarrow \mathrm{x}^2 - 2\mathrm{x} - 3\mathrm{x} + 9 + 1 = 0$
$\Rightarrow \mathrm{x}^2 - 5\mathrm{x} + 10 = 0$
$\mathrm{D} < 0$
(III) $\mathrm{x} \geq 3$
$\mathrm{x}^2 - 2\mathrm{x} + 3\mathrm{x} - 9 + 2 = 0$
$\Rightarrow \mathrm{x}^2 + \mathrm{x} - 8 = 0$

$\mathrm{x} = \frac{-1+\sqrt{32}}{2}, \frac{-1-\sqrt{32}}{2}$

$\times \qquad \times$

1 real roots

Question16

The sum of the squares of the roots of $|x-2|^2 + |x-2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x-3| - 5 = 0$, is

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

24

B.

26

C.

36

D.

30

Answer: C

Solution:

$|x-2|^2+2|x-2|-|x-2|-2=0$
$\Rightarrow(|x-2|+2)(|x-2|-1)=0$
$\Rightarrow|x-2|=1$
$\Rightarrow x=2\pm1=3,1$
$\Rightarrow$ sum of square of roots $=9+1=10$
$x^2-2|x-3|-5=0$
Case-I $x-3\geq0$
$\Rightarrow x^2-2x+1=0$
$\Rightarrow(x-1)^2=0$
$\Rightarrow x=1$
But $x\geq3$
$\Rightarrow x\in\phi$
Case-II $x-3<0$
$x^2+2x-11=0, D>0\Rightarrow$ Real & distinct roots
$f(x)=x^2+2x-11$
$f(3)>0, \frac{-p}{2a}=-1<3$
$\Rightarrow$ both roots $<3,$ both roots acceptable
Sum of square of roots $=(\alpha+\beta)^2-2\alpha\beta$
$=4+22=26$
$\Rightarrow$ Final sum $=10+26=36$

Question17

Let S be the set of all real roots of the equation, $3^x(3^x-1)+2=3^x-1\,|+3^x-2\,|$. Then S: [Jan. 8, 2020 (II)]

Options:

A. contains exactly two elements.

B. is a singleton.

C. is an empty set.

D. contains at least four elements.

Answer: B

Solution:

Solution:
Let $3^x=y$
$\therefore\ y(y-1)+2=|y-1|+|y-2|$
Case 1: when $y>2$
$y^2-y+2=y-1+y-2$
$y^2-3y+5=0$

$\because$ $D < 0$[$\therefore$ Equation not satisfy.]
Case 2 : when $1 \le y \le 2$
$y^2 - y^2 + 2 = y - 1 - y + 2$
$y^2 - y + 1 = 0$
$\because$ $D < 0$[$\therefore$ Equation not satisfy.]
Case 3: when $y \le 1$
$y^2 - y + 2 = -y + 1 - y + 2$
$y^2 + y - 1 = 0$
$\therefore y = \dfrac{-1 + \sqrt{5}}{2}$
$= \dfrac{-1 - \sqrt{5}}{2}$ [$\therefore$ Equation not Satisfy]
$\because$ Only one $-1 + \dfrac{\sqrt{5}}{2}$ satisfy equation

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Question18

If $A = \{x \in R: |x| < 2\}$ and $B = \{x \in R: |x - 2| \geq 3\}$; then : [Jan. 9, 2020 (II)]

Options:

A. $A \cap B = (-2, -1)$

B. $B - A = R - (-2, 5)$

C. $A \cup B = R - (2, 5)$

D. $A - B = [-1, 2)$

Answer: B

Solution:

Solution:
$A = \{x : x \in (-2, 2)\}$
$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$
$A \cap B = \{x : x \in (-2, -1]\}$
$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$
$A - B = \{x : x \in (-1, 2)\}$
$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

Question19

Consider the two sets :
$A = \{m \in R$: both the roots of $x^2 - (m + 1)x + m + 4 = 0$ are real $\}$ and

B = [−3, 5) Which of the following is not true?
[Sep. 03, 2020 (I)]

Options:

A. $A - B = (-\infty, -3) \cup (5, \infty)$

B. $A \cap B = \{-3\}$

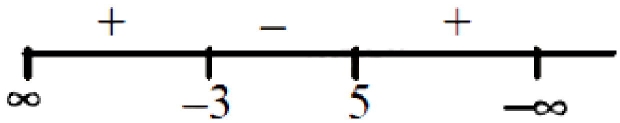
C. $B - A = (-3, 5)$

D. $A \cup B = R$

Answer: A

Solution:

Solution:
$A = \{m \in R : x^2 - (m+1)x + m + 4 = 0$ has real roots
$D \geq 0$
$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$
$\Rightarrow m^2 - 2m - 15 \geq 0$



$A = \{(-\infty, -3] \cup [5, \infty)\}$
$B = [-3, 5) \Rightarrow A - B = (-\infty, -3) \cup [5, \infty)$

Question20

The region represented by $\{z = x + iy \in C: |z| - Re(z) \leq 1\}$ is also given by the inequality:
[Sep. 06, 2020 (I)]

Options:

A. $y^2 \geq 2(x+1)$

B. $y^2 \leq 2\left(x + \frac{1}{2}\right)$

C. $y^2 \leq x + \frac{1}{2}$

D. $y^2 \geq x + 1$

Answer: B

Solution:

Solution:
$\because |z| - \text{Re}(z) \le 1 (\because z = x + iy)$
$\Rightarrow \sqrt{x^2+y^2} - x \le 1 \Rightarrow \sqrt{x^2+y^2} \le 1 + x$
$\Rightarrow x^2 + y^2 \le 1 + x^2 + 2x$
$\Rightarrow y^2 \le 1 + 2x \Rightarrow y^2 \le 2\left(x + \frac{1}{2}\right)$

Question21

The number of integral values of m for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is : [Jan. 12, 2019 (II)]

Options:

A. 3

B. 8

C. 7

D. 6

Answer: C

Solution:

Solution:
Let the given quadratic expression
$(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, is positive for all $x \in R$
then
$1 + 2m > 0$ . . .(i)
$D < 0$
$\Rightarrow \ 4(1+3m)^2 - 4(1+2m)4(1+m) < 0$
$\Rightarrow \ 1 + 9m^2 + 6m - 4[1 + 2m^2 + 3m] < 0$
$\Rightarrow \ m^2 - 6m - 3 < 0$
$\Rightarrow \ m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$
From (i)
$\therefore \ m > -\frac{1}{2}$
$m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$
Then, integral values of $m = \{0, 1, 2, 3, 4, 5, 6\}$
Hence, number of integral values of $m = 7$

Question22

The number of integral values of m for which the equation $(1+m^2)x^2-2(1+3m)x+(1+8m)=0$ has no real root is : [April 08, 2019 (II)]

Options:

A. 1

B. 2

C. infinitely many

D. 3

Answer: C

Solution:

Solution:
Given equation is
$(1+m^2)x^2-2(1+3m)x+(1+8m)=0$
$\because$ equation has no real solution
$\therefore D<0$
$\Rightarrow 4(1+3m)^2<4(1+m^2)(1+8m)$
$\Rightarrow 1+9m^2+6m<1+8m+m^2+8m^3$
$\Rightarrow 8m^3-8m^2+2m>0$
$\Rightarrow 2m(4m^2-4m+1)>0 \Rightarrow 2m(2m-1)^2>0$
$\Rightarrow m>0$ and $m \neq \frac{1}{2}$
$\left[\because \frac{1}{2}\right.$ is not an integer]
$\Rightarrow$ number of integral values of m are infinitely many.

Question23

All the pairs (x,y) that satisfy the inequality $2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot\frac{1}{4^{\sin^2 y}}\leq 1$ also satisfy the equation: [April 10, 2019 (I)]

Options:

A. $2|\sin x|=3\sin y$

B. $2\sin x=\sin y$

C. $\sin x = 2 \sin y$

D. $\sin x = |\sin y|$

Answer: D

Solution:

Solution:
Given inequality is,
$2\sqrt{\sin^2 x - 2 \sin x + 5} \leq 2^{2\sin^2 y}$
$\Rightarrow \sqrt{\sin^2 x - 2 \sin x + 5} \leq 2\sin^2 y$
$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \leq 2\sin^2 y$
It is true if $\sin x = 1$ and $|\sin y| = 1$
Therefore, $\sin x = |\sin y|$

Question24

If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1, x \in R$, then the equation $f(x) = 0$ has : [Online April 9, 2014]

Options:

A. no solution

B. one solution

C. two solutions

D. more than two solutions

Answer: B

Solution:

Solution:
$f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$
Put $f(x) = 0$
$\Rightarrow \ 0 = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$
$\Rightarrow \ \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x = 1$
$\Rightarrow \ 3^x + 4^x = 5^x \ldots$(i)
For $x = 1$
$3^1 + 4^1 > 5^1$
For $x = 3$

$3^3 + 4^3 = 91 < 5^3$
Only for $x = 2$, equation (i) Satisfy
So, only one solution $(x = 2)$

Question25

If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is
[2002]

Options:

A. less than 1

B. equal to 1

C. greater than 1

D. anyreal no.

Answer: A

Solution:

Solution:
$\because (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$
$\Rightarrow 2(a^2 + b^2 + c^2 - ab - bc - ca) > 0$
$[\because a^2 + b^2 + c^2 = 1]$
$\Rightarrow 2 > 2(ab + bc + ca) \Rightarrow ab + bc + ca < 1$
