

Permutations and Combinations

Question1

The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is _____.

JEE Main 2025 (Online) 23rd January Evening Shift

Answer: 17280

Solution:

A : number of ways that all boys sit together = $5! \times 5!$

B : number of ways if no 2 boys

sit together = $4! \times 5!$

$A \cap B = \phi$

Required no. of ways = $5! \times 5! + 4! \times 5! = 17280$

Question2

The number of 3 -digit numbers, that are divisible by 2 and 3 , but not divisible by 4 and 9 , is _____.

JEE Main 2025 (Online) 24th January Morning Shift

Answer: 125

Solution:

No. of 3 digits = $999 - 99 = 900$

No. of 3 digit numbers divisible by 2 & 3 i.e. by 6

$$\frac{900}{6} = 150$$

No. of 3 digit numbers divisible by 4 & 9 i.e. by 36

$$\frac{900}{36} = 25$$

\therefore No of 3 digit numbers divisible by 2&3 but not by 4&9

$$150 - 25 = 125$$

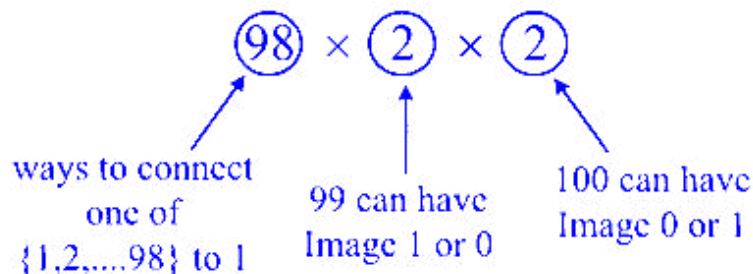
Question3

Number of functions $f : \{1, 2, \dots, 100\} \rightarrow \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98 , is equal to _____.

JEE Main 2025 (Online) 24th January Evening Shift

Answer: 392

Solution:



392 Ans.

Question4

The number of natural numbers, between 212 and 999, such that the sum of their digits is 15, is _____.

JEE Main 2025 (Online) 28th January Evening Shift

Answer: 64

Solution:

x	y	z
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Let $x = 2 \Rightarrow y + z = 13$

$(4, 9), (5, 8), (6, 7), (7, 6), (8, 5), (9, 4), \rightarrow 6$

Let $x = 3 \rightarrow y + z = 12$

$(3, 9), (4, 8), \dots, (9, 3) \rightarrow 7$

Let $x = 4 \rightarrow y + z = 11$

$(2, 9), (3, 8), \dots, (9, 1) \rightarrow 9$

Let $x = 5 \rightarrow y + z = 10$

$(1, 9), (2, 8), \dots, (9, 1) \rightarrow 10$

Let $x = 6 \rightarrow y + z = 9$

$(0, 9), (1, 8), \dots, (9, 0) \rightarrow 9$

Let $x = 7 \rightarrow y + z = 8$

$(0, 9), (1, 7), \dots, (8, 0) \rightarrow 9$

Let $x = 8 \rightarrow y + z = 7$

$(0, 7), (1, 6), \dots, (7, 0) \rightarrow 8$

Let $x = 9 \rightarrow y + z = 6$

$(0, 6), (1, 5), \dots, (6, 0) \rightarrow 7$

Total = $6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 = 64$

Question5

The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is _____.

JEE Main 2025 (Online) 29th January Morning Shift

Answer: 1405

Solution:

(i) Single letter is used, then no. of words = 5

(ii) Two distinct letters are used, then no. of words

$${}^5C_2 \times \left(\frac{6!}{2!4!} \times 2 + \frac{6!}{3!3!} \right) = 10(30 + 20) = 500$$

(iii) Three distinct letters are used, then no. of words

$${}^5C_3 \times \frac{6!}{2!2!2!} = 900$$

Total no. of words = 1405

Question6

If the number of seven-digit numbers, such that the sum of their digits is even, is $m \cdot n \cdot 10^n$; $m, n \in \{1, 2, 3, \dots, 9\}$, then $m + n$ is equal to _____

JEE Main 2025 (Online) 3rd April Morning Shift

Answer: 14

Solution:

When numbers are uniformly distributed, half of them have even digit sums and half have odd digit sums.

Number of 7-digit numbers with even digit sum =

$$\frac{1}{2} \cdot 9 \cdot 10^6 = 4.5 \cdot 10^6$$

Note that $9 \cdot 5 \cdot 10^5 = 4.5 \cdot 10^6$

$$m + n = 9 + 5 = 14$$

Question 7

All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by W_n . Let the probability $P(W_n)$ of choosing the word W_n satisfy $P(W_n) = 2P(W_{n-1})$, $n > 1$.

If $P(\text{CDBEA}) = \frac{2^\alpha}{2^\beta - 1}$, $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to : _____

JEE Main 2025 (Online) 3rd April Morning Shift

Answer: 183

Solution:

Firstly, by this rule, we note:

$$P(W_1) = p$$

$$P(W_2) = 2p$$

$$P(W_3) = 4p$$

...

$$P(W_n) = 2^{n-1}p$$

To find the initial probability p , consider the total probability must sum to 1 across all 120 possible words (since $5! = 120$):

$$\sum_{n=1}^{120} P(W_n) = 1$$

This is a geometric series sum where:

$$p(1 + 2 + 2^2 + \dots + 2^{119}) = 1$$

Since the series sum $1 + 2 + 2^2 + \dots + 2^{119}$ is equal to $2^{120} - 1$, we have:

$$p(2^{120} - 1) = 1 \Rightarrow p = \frac{1}{2^{120} - 1}$$

Thus, the probability for the n -th word is:

$$P(W_n) = \frac{2^{n-1}}{2^{120} - 1} \quad (\text{i})$$

Next, determine the position of "CDBEA". Starting from the first letter:

Words beginning with 'A': $4! = 24$

Words beginning with 'B': $4! = 24$

Words beginning with 'C':

CA*: $3! = 6$

CB*: $3! = 6$

CDA :** $2! = 2$

CDBA*: $1! = 1$

Summing these, the position of "CDBEA" is the 64th word.

Substitute into equation (i):

$$P(\text{CDBEA}) = P(W_{64}) = \frac{2^{63}}{2^{120}-1}$$

Given $P(\text{CDBEA}) = \frac{2^\alpha}{2^\beta-1}$, we find:

$$\alpha = 63$$

$$\beta = 120$$

Thus, the sum $\alpha + \beta = 63 + 120 = 183$.

Question8

Let m and n, ($m < n$), be two 2-digit numbers. Then the total numbers of pairs (m, n), such that $\gcd(m, n) = 6$, is _____ .

JEE Main 2025 (Online) 4th April Evening Shift

Answer: 64

Solution:

$$m = 6a, n = 6b$$

$$\text{So } \gcd(m, n) = 6 \Rightarrow \gcd(a, b) = 1$$

$$m = 6a \geq 10 \Rightarrow a \geq \left\lceil \frac{10}{6} \right\rceil = 2$$

$$m = 6a \leq 99 \Rightarrow a \leq \left\lfloor \frac{99}{6} \right\rfloor = 16$$

So $a, b \in \{2, 3, \dots, 16\}$, and we count how many coprime pairs (a, b) with $a < b, \gcd(a, b) = 1$

$$a = 2 \Rightarrow b = 3, 5, 7, 9, 11, 13, 15 \Rightarrow 7$$

$$a = 3 \Rightarrow b = 4, 5, 7, 8, 10, 11, 13, 14, 16 \Rightarrow 9$$

$$a = 4 \Rightarrow b = 5, 7, 9, 11, 13, 15 \Rightarrow 6$$

$$a = 5 \Rightarrow b = 6, 7, 8, 9, 11, 12, 13, 14, 16 \Rightarrow 9$$

$$a = 6 \Rightarrow b = 7, 11, 13 \Rightarrow 3$$

$$a = 7 \Rightarrow b = 8, 9, 10, 11, 12, 13, 15, 16 \Rightarrow 8$$

$$a = 8 \Rightarrow b = 9, 11, 13, 15 \Rightarrow 4$$

$$a = 9 \Rightarrow b = 10, 11, 13, 14, 16 \Rightarrow 5$$

$$a = 10 \Rightarrow b = 11, 13 \Rightarrow 2$$

$$a = 11 \Rightarrow b = 12, 13, 14, 15, 16 \Rightarrow 5$$

$$a = 12 \Rightarrow b = 13, 17 \times \rightarrow \text{only } 13 \text{ is valid} \Rightarrow 1$$

$$a = 13 \Rightarrow b = 14, 15, 16 \Rightarrow 3$$

$$a = 14 \Rightarrow b = 15, \Rightarrow 1$$

$$a = 15 \Rightarrow b = 16 \Rightarrow 1$$

$$\begin{aligned} \text{Total} &= 7 + 9 + 6 + 9 + 3 + 8 + 4 + 5 + 2 + 5 + 1 \\ &+ 3 + 1 + 1 = 64 \end{aligned}$$

Question9

From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is :

JEE Main 2025 (Online) 22nd January Morning Shift

Options:

A. 6084

B. 5148

C. 14950

D. 4356

Answer: B

Solution:

First, note that we are choosing **5 distinct letters** (in strictly increasing alphabetical order) such that the **middle (third) letter is 'M'**. Symbolically, if we denote the chosen letters as:

$$L_1 < L_2 < L_3 < L_4 < L_5,$$

we want $L_3 = M$. The English alphabet has 26 letters, and M is the 13th.

Step 1: Letters before M

The letters before M are $\{A, B, C, \dots, L\}$.

There are **12** letters here (A through L).

We need to pick **2** of these 12 letters to occupy L_1 and L_2 .

The number of ways to choose 2 letters out of 12 is $^{12}C_2$.

Step 2: Letters after M

The letters after M are $\{N, O, P, \dots, Z\}$.

There are **13** letters here (N through Z).

We need to pick **2** of these 13 letters to occupy L_4 and L_5 .

The number of ways to choose 2 letters out of 13 is $^{13}C_2$.

Step 3: Multiply the choices

Since these choices are independent (picking the two letters before M and two letters after M), the total number of ways is:

$$^{12}C_2 \times ^{13}C_2$$

Calculate each combination:

$$^{12}C_2 = \frac{12 \times 11}{2} = 66, \quad ^{13}C_2 = \frac{13 \times 12}{2} = 78.$$

So,

$$^{12}C_2 \times ^{13}C_2 = 66 \times 78 = 5148.$$

Answer: 5148 (Option B)

Question10

In a group of 3 girls and 4 boys, there are two boys B_1 and B_2 . The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is :

JEE Main 2025 (Online) 22nd January Evening Shift

Options:

A. 120

B. 96

C. 72

D. 144

Answer: D

Solution:

Let's break the problem down step by step:

There are two blocks because all girls must stand together and all boys must stand together. The two blocks can be arranged in:

2 ways (i.e., girls first then boys, or boys first then girls).

The girls can be arranged among themselves in:

$3! = 6$ ways.

For the boys (4 in total), they must be arranged such that the specific boys B_1 and B_2 are not adjacent.

First, calculate the total number of arrangements of 4 boys:

$4! = 24$.

Next, count the arrangements where B_1 and B_2 are adjacent. Think of B_1 and B_2 as a single unit. This unit can be arranged in:

$2! = 2$ ways (since B_1 and B_2 can swap positions).

Now, with this new unit, we have 3 units in total (the B_1B_2 unit and the other 2 boys), which can be arranged in:

$3! = 6$ ways.

So, the number of arrangements where B_1 and B_2 are adjacent is:

$2! \times 3! = 2 \times 6 = 12$.

Therefore, the number of valid arrangements for the boys where B_1 and B_2 are not adjacent is:

$24 - 12 = 12$.

Finally, multiply all the factors together:

Total ways = $2 \times 6 \times 12 = 144$.

Thus, the number of ways in which the girls and boys can stand in the queue under the given conditions is 144.

This corresponds to Option D.

Question11

The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is :

JEE Main 2025 (Online) 23rd January Morning Shift

Options:

A. 34000

B. 37000

C. 35000

D. 36000

Answer: D

Solution:

DAUGHTER

Total words = 8 !

Total words in which vowels are together = $6! \times 3!$ words in which all vowels are not together

$$= 8! - 6! \times 3!$$

$$= 6![56 - 6]$$

$$= 720 \times 50$$

$$= 36000$$

Question12

Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be

invited for a picnic if 5 of them must be from group A and the remaining 3 from group B , is equal to :

JEE Main 2025 (Online) 24th January Evening Shift

Options:

A. 8925

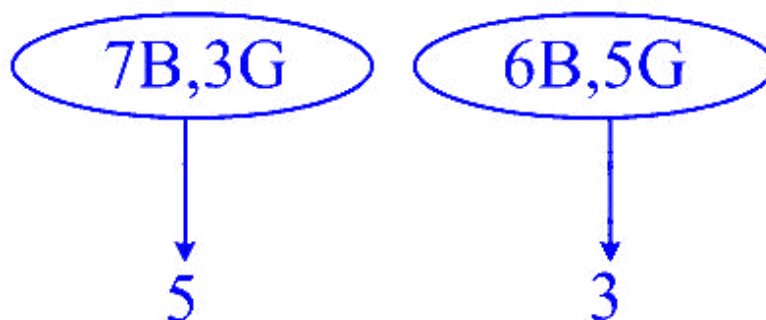
B. 9100

C. 8575

D. 8750

Answer: A

Solution:



C-I (3G&2 B)&(1G&2 B)

C-II (2G&3 B)&(2G&1 B)

C-III (1G&4 B)&(3G&0 B)

Total = C-I + C-II + C-III

$$= {}^7C_2 \cdot {}^3C_3 \cdot {}^6C_2 \cdot {}^5C_1 + {}^7C_3 \cdot {}^3C_2 \cdot {}^6C_1 \cdot {}^5C_2 + {}^7C_4 \cdot {}^3C_1 \cdot {}^6C_0 \cdot {}^5C_3$$

$$= 8925$$

Question13

The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0 , 1, 2, 3, 4, 5, 6, 7, such that the sum of their first and last digits should not be more than 8 , is

JEE Main 2025 (Online) 28th January Morning Shift

Options:

A. 5720

B. 5719

C. 4608

D. 4607

Answer: D

Solution:

Case I 5___0

Case II 5___1

5 2

5 3

6 0

6 1

6 2

7 0

Case IX 7___1

$9 \times (8 \times 8 \times 8) = 4608$ but 50000 is not included, so total numbers $4608 - 1 = 4607$

Question14

Let ${}^nC_{r-1} = 28$, ${}^nC_r = 56$ and ${}^nC_{r+1} = 70$. Let $A(4 \cos t, 4 \sin t)$, $B(2 \sin t, -2 \cos t)$ and $C(3r - n, r^2 - n - 1)$ be the vertices of a triangle ABC , where t is a parameter. If $(3x - 1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC , then α equals

Options:

A. 18

B. 8

C. 20

D. 6

Answer: C

Solution:

$${}^nC_{r-1} = 28, {}^nC_r = 56$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{28}{56}$$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{1}{2}$$

$$\frac{r}{(n-r+1)} = \frac{1}{2}$$

$$3r = n + 1 \quad \dots (i)$$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{56}{70}$$

$$\frac{(r+1)}{(n-r)} = \frac{56}{70} \Rightarrow 9r = 4n - 5 \quad \dots (ii)$$

By (i) & (ii)

$$(r = 3), (n = 8)$$

$$A(4 \cos t, 4 \sin t) \quad B(2 \sin t, -2 \cos t) \quad C(3r - n, r^2 - n - 1)$$

$$A(4 \cos t, 4 \sin t) \quad B(2 \sin t, -2 \cos t) \quad C(1, 0)$$

$$(3x - 1)^2 + (3y)^2 = (4 \cos t + 2 \sin t)^2 + (4 \sin t - \cos t)^2$$

$$(3x - 1)^2 + (3y)^2 = 20 \quad \therefore \text{option (1)}$$

Question15

Let P be the set of seven digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then the number of elements in the set P is :

Options:

A.

164

B.

158

C.

161

D.

173

Answer: C

Solution:

(i) number of numbers created using

$$1111133 = \frac{7!}{5!2!} \Rightarrow 21$$

(ii) number of numbers created using

$$1111223 = \frac{7!}{4!2!} \Rightarrow 105$$

(iii) number of numbers created using

$$1112222 = \frac{7!}{4!3!} \Rightarrow 35$$

$$\text{Total} = 161$$

Question16

If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440th position in this arrangement is :

JEE Main 2025 (Online) 29th January Evening Shift

Options:

A.

PRNAKU

B.

PRKAUN

C.

PRKANU

D.

PRNAUK

Answer: B

Solution:

A, K, N, P, R, U

$$\boxed{A} \dots\dots\dots = \underline{5} = 120$$

$$\boxed{K} \dots\dots\dots = \underline{5} = 120$$

$$\boxed{N} \dots\dots\dots = \underline{5} = 120$$

$$\boxed{P} \boxed{A} \dots\dots\dots = \underline{4} = 24$$

$$\boxed{P} \boxed{K} \dots\dots\dots = \underline{4} = 24$$

$$\boxed{P} \boxed{N} \dots\dots\dots = \underline{4} = 24$$

$$\boxed{P} \boxed{R} \boxed{A} \dots\dots\dots = \underline{3} = 6$$

$$\boxed{P} \boxed{R} \boxed{K} \boxed{A} \boxed{N} \boxed{U} \quad \quad = 1$$

$$\boxed{P} \boxed{R} \boxed{K} \boxed{A} \boxed{U} \boxed{N} \quad \quad = 1$$

Total = 440

$\Rightarrow 440^{\text{th}}$ word

Question17

The number of sequences of ten terms, whose terms are either 0 or 1 or 2 , that contain exactly five 1 s and exactly three 2 s , is equal to :

JEE Main 2025 (Online) 2nd April Morning Shift

Options:

A. 360

B. 2520

C. 1820

D. 45

Answer: B

Solution:

To find the number of sequences of ten terms, each being either 0, 1, or 2, containing exactly five 1s and exactly three 2s, follow these steps:

Determine the Remaining Terms: Since there are 5 ones and 3 twos, you will need 2 zeros to fill the sequence (because $5 + 3 + 2 = 10$).

Calculate the Number of Arrangements: You have a total of 10 positions to fill with these numbers (5 ones, 3 twos, 2 zeros). The formula for calculating permutations of a multiset is:

$$\frac{10!}{5! \times 3! \times 2!}$$

Where:

10! is the factorial of the total number of terms.

5! is the factorial for the number of 1s.

3! is the factorial for the number of 2s.

2! is the factorial for the number of 0s.

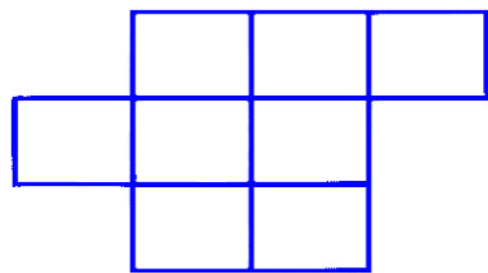
Result: Simplifying the calculation gives:

$$\frac{10!}{5! \times 3! \times 2!} = 2520$$

Thus, there are 2520 possible sequences with the given conditions.

Question18

The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :



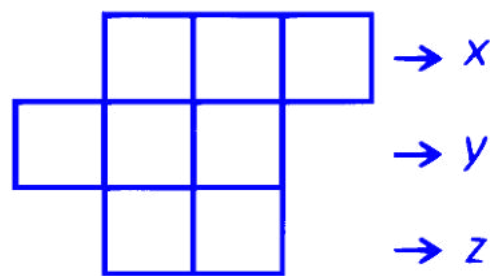
JEE Main 2025 (Online) 2nd April Evening Shift

Options:

- A. 5880
- B. 840
- C. 960
- D. 5760

Answer: D

Solution:



Let x, y, z be the number of box which are filled

$$\Rightarrow 1 \leq x \leq 3, 1 \leq y \leq 3, 1 \leq z \leq 2$$

x	y	z	Number of ways
3	1	1	${}^3C_3 \cdot {}^3C_1 \cdot {}^2C_1 = 6$

x	y	z	Number of ways
2	2	1	${}^3C_2 \cdot {}^3C_2 \cdot {}^2C_1 = 18$
1	3	1	${}^3C_1 \cdot {}^3C_3 \cdot {}^2C_1 = 6$
2	1	2	${}^3C_2 \cdot {}^3C_1 \cdot {}^2C_2 = 9$
1	2	2	${}^3C_1 \cdot {}^3C_2 \cdot {}^2C_2 = 9$

Total ways = (48) to fill boxes

Now to arrange a, b, c, d and e

Number of ways will be $48 \cdot 5! = 5760$

Question19

Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O . In the first quadrant, P_1, P_2, \dots, P_{12} are 12 points on line L_1 and Q_1, Q_2, \dots, Q_9 are 9 points on line L_2 . Then the total number of triangles, that can be formed having vertices at three of the 22 points O, $P_1, P_2, \dots, P_{12}, Q_1, Q_2, \dots, Q_9$, is:

JEE Main 2025 (Online) 3rd April Evening Shift

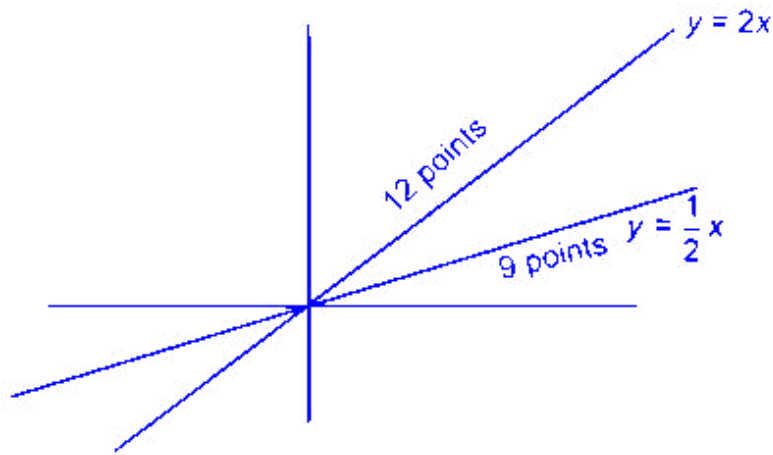
Options:

- A. 1026
- B. 1188
- C. 1134
- D. 1080

Answer: C

Solution:

Total triangles



(2 points as $y = \frac{x}{2}$, 1 point on $y = \frac{x}{2}$)

+2 (points and $y = \frac{x}{2}$, 1 point on $y = 2x$)

+(1 point on $y = 2x$, 1) point on $y = \frac{x}{2}$ and origin)

$$= {}^9C_2, {}^{12}C_1 + {}^9C_1 {}^{12}C_2 + {}^9C_1 \cdot {}^{12}C_1 \cdot {}^1C_1$$

$$= 1134$$

Question20

From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include atleast 4 batsmen and atleast 4 bowlers. One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is

JEE Main 2025 (Online) 7th April Morning Shift

Options:

A. 145

B. 165

C. 155

D. 135

Answer: C

Solution:

1 Captain, 1 vice-captain are already present

⇒ We need to select 8 players such that atleast 3 batsman and bowler must be there

Batsman	Bowler	Number of ways
3	5	${}^6C_3 \cdot {}^5C_5 = 20$
4	4	${}^6C_4 \cdot {}^5C_4 = 75$
5	3	${}^6C_5 \cdot {}^5C_3 = 60$

Total = 155 ways

Question21

There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is

JEE Main 2025 (Online) 8th April Evening Shift

Options:

A.

230

B.

210

C.

200

D.

220

Answer: B

Solution:

To count the number of distinct triangles:

Start with all possible triples of points.

From 12 points, the number of ways to choose any 3 is

$$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220.$$

Subtract the “invalid” triples that are collinear.

The only collinear sets of three points arise from the single line containing the 5 collinear points.

Number of ways to pick 3 points from those 5 is

$$\binom{5}{3} = 10.$$

Valid triangles = total triples – collinear triples.

$$220 - 10 = 210.$$

Hence, the total number of triangles that can be formed is **210**.

Correct option: B

Question22

Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then :

[27-Jan-2024 Shift 2]

Options:

A.

$$\alpha \in \mathbb{N} \text{ and } \beta \notin \mathbb{N}$$

B.

$$\alpha \notin \mathbb{N} \text{ and } \beta \in \mathbb{N}$$

C.

$$\alpha \in \mathbb{N} \text{ and } \beta \in \mathbb{N}$$

D.

$$\alpha \notin \mathbb{N} \text{ and } \beta \notin \mathbb{N}$$

Answer: C

Solution:

$$\alpha = \frac{(4!)!}{(4!)^{3!}}, \beta = \frac{(5!)!}{(5!)^{4!}}$$

$$\alpha = \frac{(24)!}{(4!)^6}, \beta = \frac{(120)!}{(5!)^{24}}$$

Let 24 distinct objects are divided into 6 groups of 4 objects in each group.

$$\text{No. of ways of formation of group} = \frac{24!}{(4!)^6 \cdot 6!} \in \mathbb{N}$$

Similarly,

Let 120 distinct objects are divided into 24 groups of 5 objects in each group.

No. of ways of formation of groups

$$= \frac{(120)!}{(5!)^{24} \cdot 24!} \in \mathbb{N}$$

Question23

All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

[29-Jan-2024 Shift 1]

Answer: 553

Solution:

Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

GTWENTY = 1
Total = 553

Question24

Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

[29-Jan-2024 Shift 2]

Options:

A.

18

B.

16

C.

12

D.

15

Answer: D

Solution:

3 Shelf empty : $(8, 0, 0, 0) \rightarrow 1$ way

2 shelf empty: $\left. \begin{array}{l} (7, 1, 0, 0) \\ (6, 2, 0, 0) \\ (5, 3, 0, 0) \\ (4, 4, 0, 0) \end{array} \right\} \rightarrow 4 \text{ ways}$

1 shelf empty : $\left. \begin{array}{l} (6, 1, 1, 0) \quad (3, 3, 2, 0) \\ (5, 2, 1, 0) \quad (4, 2, 2, 0) \\ (4, 3, 1, 0) \end{array} \right\} \rightarrow 5 \text{ ways}$

0 Shelf empty : $\left. \begin{array}{l} (1, 2, 3, 2) \quad (5, 1, 1, 1) \\ (2, 2, 2, 2) \\ (3, 3, 1, 1) \\ (4, 2, 1, 1) \end{array} \right\} \rightarrow 5 \text{ ways}$

Total = 15 ways

Question25

In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections: A,B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _____

[30-Jan-2024 Shift 2]

Answer: 11376

Solution:

If 4 questions from each section are selected

Remaining 3 questions can be selected either in (1, 1, 1) or (3, 0, 0) or (2, 1, 0)

$$\begin{aligned}\therefore \text{Total ways} &= {}^8C_5 \cdot {}^6C_5 \cdot {}^6C_5 + {}^8C_6 \cdot {}^6C_5 \cdot {}^6C_4 \times 2 + {}^8C_5 \cdot {}^6C_6 \cdot {}^6C_4 \times 2 + {}^8C_4 \cdot {}^6C_6 \cdot {}^6C_5 \times 2 + {}^8C_7 \cdot {}^6C_4 \cdot {}^6C_4 \\ &= 56 \cdot 6 \cdot 6 + 28 \cdot 6 \cdot 15 \cdot 2 + 56 \cdot 15 \cdot 2 + 70 \cdot 6 \cdot 2 + 8 \cdot 15 \cdot 15 \\ &= 2016 + 5040 + 1680 + 840 + 1800 = 11376\end{aligned}$$

Question 26

The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____

[31-Jan-2024 Shift 1]

Answer: 3734

Solution:

We have I, T, T, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$$= {}^8C_1 \times \frac{4!}{3!} = 32$$

Number of words with selection (a, a, b, b)

$$= \frac{4!}{2!2!} = 6$$

Number of words with selection (a, a, b, c)

$$= {}^2C_1 \times {}^8C_2 \times \frac{4!}{2!} = 672$$

Number of words with selection (a, b, c, d)

$$= {}^9C_4 \times 4! = 3024$$

$$\therefore \text{total} = 3024 + 672 + 6 + 32$$

$$= 3734$$

Question27

The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

[31-Jan-2024 Shift 2]

Options:

A.

406

B.

130

C.

142

D.

136

Answer: D

Solution:

After giving 2 apples to each child 15 apples left now 15 apples can be distributed in $^{15+3-1}C_2 = ^{17}C_2$ ways

$$= \frac{17 \times 16}{2} = 136$$

Question28

If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

[1-Feb-2024 Shift 1]

Options:

A.

47

B.

53

C.

51

D.

43

Answer: C

Solution:

Total ways to partition 5 into 4 parts are :

$$5, 0, 0, 0 \Rightarrow 1 \text{ way}$$

$$4, 1, 0, 0 \Rightarrow \frac{5!}{4!} = 5 \text{ ways}$$

$$3, 2, 0, 0, \Rightarrow \frac{5!}{3!2!} = 10 \text{ ways}$$

$$2, 2, 0, 1 \Rightarrow \frac{5!}{2!2!2!} = 15 \text{ ways}$$

$$2, 1, 1, 1 \Rightarrow \frac{5!}{2!(1!)^3 3!} = 10 \text{ ways}$$

$$3, 1, 1, 0 \Rightarrow \frac{5!}{3!2!} = 10 \text{ ways}$$

$$\text{Total} \Rightarrow 1 + 5 + 10 + 15 + 10 + 10 = 51 \text{ ways}$$

Question29

The lines L_1, L_2, \dots, L_{20} are distinct. For $n = 1, 2, 3, \dots, 10$ all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \dots, L_{20}\}$ is equal to :

[1-Feb-2024 Shift 2]

Answer: 101

Solution:

$L_1, L_3, L_5, \dots, L_{19}$ are Parallel

$L_2, L_4, L_6, \dots, L_{20}$ are Concurrent

$$\text{Total points of intersection} = {}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1 = 101$$

Question30

The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is__

[24-Jan-2023 Shift 1]

Answer: 60

Solution:

Solution:

Even digits occupy at even places

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = \frac{24 \times 120}{4 \times 12} = 60$$

Question31

A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

[24-Jan-2023 Shift 1]

Answer: 546

Solution:

Solution:

For at most two language courses

$$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$$

Question32

The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

[24-Jan-2023 Shift 2]

Options:

A. 120

B. 168

C. 220

D. 48

Answer: B

Solution:

Solution:

Four digit numbers greater than 7000 = $2 \times 4 \times 3 \times 2 = 48$

Five digit number = $5! = 120$

Total number greater than 7000 = $120 + 48 = 168$

Question33

Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of nonempty subsets of S that have the sum of all elements a multiple of 3, is _____.

[25-Jan-2023 Shift 1]

Answer: 43

Solution:

Solution:

Elements of the type $3k = 3$

Elements of the type $3k + 1 = 1, 7, 9$

Elements of the type $3k + 2 = 2, 5, 11$

Subsets containing one element $S_1 = 1$

Subsets containing two elements

$$S_2 = {}^3C_1 \times {}^3C_1 = 9$$

Subsets containing three elements

$$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$$

Subsets containing four elements

$$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$$

Subsets containing five elements

$$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$$

Subsets containing six elements $S_6 = 1$

Subsets containing seven elements $S_7 = 1$

$$\Rightarrow \text{sum} = 43$$

Question34

The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 without repetition, is

[25-Jan-2023 Shift 2]

Options:

A. 6

B. 12

C. 120

D. 72

Answer: D

Solution:

Solution:

Numbers between 5000&10000

Using digits 1, 3, 5, 7, 9

Total Numbers = $3 \times 4 \times 3 \times 2 = 72$

Question35

Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 orange, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is _____
[25-Jan-2023 Shift 2]

Answer: 6860

Solution:

Solution:

7 Red apple(RA), 5 white apple(WA), 8 oranges (O)

5 fruits to be selected (Note:- fruits taken different)

Possible selections :- (2O, 1 RA, 2 WA) or (2O ,

2RA, 1WA) or (3O, 1RA, 1WA)

$$\Rightarrow {}^8C_2 {}^7C_1 {}^5C_2 + {}^8C_2 {}^7C_2 {}^5C_1 + {}^8C_3 {}^7C_1 {}^5C_1$$

$$\Rightarrow 1960 + 2940 + 1960$$

$$\Rightarrow 6860$$

Question36

If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is _____.
[29-Jan-2023 Shift 1]

Answer: 32

Solution:

Solution:

1	2				
---	---	--	--	--	--

$$= {}^7C_4 = 35$$

1	3				
---	---	--	--	--	--

$$= {}^6C_4 = 15$$

1	4				
---	---	--	--	--	--

$$= {}^5C_4 = 5$$

1	5				
---	---	--	--	--	--

$$= {}^4C_4 = 1$$

2	3				
---	---	--	--	--	--

$$= {}^6C_4 = 15$$

71 words

245678 \rightarrow 72th word

$$2 + 4 + 5 + 6 + 7 + 8 = 32$$

$$= {}^6C_4 = 15$$

71 words

$$245678 \rightarrow 72^{\text{th}} \text{ word}$$

$$2 + 4 + 5 + 6 + 7 + 8 = 32$$

Question37

Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _____.

[29-Jan-2023 Shift 1]

Answer: 1436

Solution:

Solution:

No of 5 digit numbers starting with digit 1

$$= 5 \times 5 \times 5 \times 5 = 625$$

No of 5 digit numbers starting with digit 2

$$= 5 \times 5 \times 5 \times 5 = 625$$

No of 5 digit numbers starting with 31

$$= 5 \times 5 \times 5 = 125$$

No of 5 digit numbers starting with 32

$$= 5 \times 5 \times 5 = 125$$

No of 5 digit numbers starting with 33

$$= 5 \times 5 \times 5 = 125$$

No of 5 digit numbers starting with 351

$$= 5 \times 5 = 25$$

No of 5 digit numbers starting with 352

$$= 5 \times 5 = 25$$

No of 5 digit numbers starting with 3531 = 5

No of 5 digit numbers starting with 3532 = 5

Before 35337 will be 4 numbers,

So rank of 35337 will be 1690

So, in descending order serial number will be

$$3125 - 1690 + 1 = 1436$$

Question38

The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48 , is
[29-Jan-2023 Shift 2]

Options:

A. 472

B. 432

C. 507

D. 400

Answer: B

Solution:

Solution:

Total 3 digit number = 900

Divisible by 3 = 300 (Using $\frac{900}{3} = 300$)

Divisible by 4 = 225 (Using $\frac{900}{4} = 225$)

Divisible by 3 & 4 = 108, ...

(Using $\frac{900}{12} = 75$)

Number divisible by either 3 or 4

= $300 + 225 - 75 = 450$

We have to remove divisible by 48 ,

144, 192,, 18 terms

Required number of numbers = $450 - 18 = 432$

Question39

The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is:
[29-Jan-2023 Shift 2]

Options:

A. 89

B. 84

C. 86

D. 79

Answer: A

Solution:

Solution:

Lets arrange the letters of OUGHT in alphabetical order.

G, H, O, T, U

Words starting with

G----- $\rightarrow 4!$

H----- $\rightarrow 4!$

O----- $\rightarrow 4!$

TG---- $\rightarrow 3!$

TH--- $\rightarrow 3!$

TO G-- $\rightarrow 2!$

T O H -- $\rightarrow 2!$

T O U G H $\rightarrow 1!$

— Total = 89

Question40

The total number of 4-digit numbers whose greatest common divisor with 54 is 2 , is _____.

[29-Jan-2023 Shift 2]

Answer: 3000

Solution:

Solution:

N should be divisible by 2 but not by 3

$N = (\text{Numbers divisible by 2}) - (\text{Numbers divisible by 6})$

$$N = \frac{9000}{2} - \frac{9000}{6} = 4500 - 1500 = 3000$$

Question41

Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5 , and are divisible by 15 , is equal to _____
[30-Jan-2023 Shift 1]

Answer: 21

Solution:

Solution:

For number to be divisible by 15 , last digit should be 5 and sum of digits must be divisible by 3 .
Possible combinations are

1	2	1	5
---	---	---	---

Numbers = 3

2	2	3	5
---	---	---	---

Numbers = 3

3	3	1	5
---	---	---	---

1	1	5	5
---	---	---	---

Numbers = 3

2	3	5	5
---	---	---	---

Numbers = 3

3	3	5	5
---	---	---	---

Total Numbers = 21

Question42

The number of ways of selecting two numbers a and b ,
 $a \in \{2, 4, 6, \dots, 100\}$ and $b \in \{1, 3, 5, \dots, 99\}$ such that 2 is the
remainder when $a + b$ is divided by 23 is
[30-Jan-2023 Shift 2]

Options:

A. 186

B. 54

C. 108

D. 268

Answer: C

Solution:

Solution:

$a \in \{2, 4, 6, 8, 10, \dots, 100\}$

$b \in \{1, 3, 5, 7, 9, \dots, 99\}$

Now, $a + b \in \{25, 71, 117, 163\}$

(i) $a + b = 25$, no. of ordered pairs (a, b) is 12

(ii) $a + b = 71$, no. of ordered pairs (a, b) is 35

(iii) $a + b = 117$, no. of ordered pairs (a, b) is 42

(iv) $a + b = 163$, no. of ordered pairs (a, b) is 19 \therefore total = 108 pairs

Question43

The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____.

[30-Jan-2023 Shift 2]

Answer: 240

Solution:

Solution:

Digits are 1, 2, 2, 2, 3, 3, 5

If unit digit 5 , then total numbers = $\frac{6!}{3!2!}$

If unit digit 3 , then total numbers = $\frac{6!}{3!}$

If unit digit 1 , then total numbers = $\frac{6!}{3!2!}$

\therefore total numbers = $60 + 60 + 120 = 240$

Question44

Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to _____.

[31-Jan-2023 Shift 1]

Answer: 710

Solution:

Solution:

1000 – 2799

Divisible by 3

$1002 + (n - 1)3 = 2799$

$n = 600$

Divisible by 11

$$1 - 2799 \rightarrow \left[\frac{2799}{11} \right] = [254] = 254$$

$$1 - 999 = \left[\frac{999}{11} \right] = 90$$

$$1000 - 2799 = 254 - 90 = 164$$

Divisible by 33

$$1 - 2799 \rightarrow \left[\frac{2799}{33} \right] = 84$$

$$1 - 999 \rightarrow \left[\frac{999}{33} \right] = 30$$

$$1000 - 2799 \rightarrow 54$$

$$\therefore n(3) + n(11) - n(33)$$

$$600 + 164 - 54 = 710$$

Question45

Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is _____.
[31-Jan-2023 Shift 1]

Answer: 2997

Solution:

Solution:

$$42920 = 1$$

$$42922 = 1$$

$$42923 = 1$$

$$= 2997$$

Question46

If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to:
[31-Jan-2023 Shift 2]

Answer: 45

Solution:

Solution:

$$\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \frac{11}{21}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$$

$$\Rightarrow \frac{2n+1}{(n+1)(n+2)} = \frac{11}{42}$$

$$\Rightarrow n = 5$$

$$\Rightarrow n^2 + n + 15 = 25 + 5 + 15 = 45$$

Question47

The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is _____.

[1-Feb-2023 Shift 1]

Answer: 50400

Solution:

Solution:

Vowels : A,A,A,I,I,O

Consonants : S,S,S,S,N,N,T

∴ Total number of ways in which vowels come together

$$= \frac{{}^8P_4}{{}^2P_2} \times \frac{{}^6P_3}{{}^2P_2} = 50400$$

Question48

Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1$, $y \geq 3$, $z \geq 4$, is equal to _____.

[1-Feb-2023 Shift 2]

Answer: 105

Solution:

Solution:

$${}^{15}C_2 = \frac{15 \times 14}{2} = 105$$

Question49

The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6 , is _____.
[1-Feb-2023 Shift 2]

Answer: 81

Solution:

Solution:

Taking single digit $\rightarrow 444444 \quad \frac{6!}{6!} = 1$

Taking two digit \rightarrow

(4, 5) 444555 (4, 9) 444999

$$\frac{5!}{3!2!} = 10 \quad \frac{5!}{3!2!} = 10$$

Taking three digit

$$4, 5, 9, 4, 4, 4 \Rightarrow \frac{5!}{3!} = 20$$

$$4, 5, 9, 5, 5, 5 \Rightarrow \frac{5!}{4!} = 5$$

$$4, 5, 9, 9, 9, 9 \Rightarrow \frac{5!}{4!} = 5$$

$$4, 5, 9, 4, 5, 9 \Rightarrow \frac{5!}{2!2!} = 30$$

$$\text{Total} = 81$$

Question50

The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is _____.
[6-Apr-2023 shift 1]

Answer: 171

Solution:

Solution:

20 distinct oranges distributed among 3 children so
that each child gets at least one orange

$$= 3^{20} - {}^3C_1 2^{20} + {}^3C_2 1^{20}$$

Question51

**All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is :
[6-Apr-2023 shift 2]**

Options:

A. 580

B. 578

C. 576

D. 582

Answer: D

Solution:

Solution:

B———— = 5! = 120

C———— = 5! = 120

I———— = 5! = 120

L———— = 5! = 120

PB———— = 4! = 24

PC———— = 4! = 24

PI———— = 4! = 24

PL———— = 4! = 24

PUBC———— = 2! = 2

PUBI———— = 2! = 2

PUBLIC _____ = 1
PUBLIC _____ - $\frac{= 1}{582}$
Rank = 582
Ans. Option 4

Question52

The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is _____ :
[6-Apr-2023 shift 2]

Answer: 432

Solution:

Solution:
Case I 2 vowels different, 2 consonant different
 $({}^3C_2)({}^4C_2)(4!)$
 $= (3)(6)(24)$
 $= 432$

Question53

The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is.
[8-Apr-2023 shift 1]

Options:

- A. 16800
- B. 14800
- C. 18000
- D. 33600

Answer: A

Solution:

Solution:

IEEEE,

NNN, DD, P, C

$$\frac{8!}{3!2!} \times \frac{6!}{4!} = 16800$$

Question54

The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is
[8-Apr-2023 shift 1]

Options:

A. $7(720)^2$

B. 720

C. $7(360)^2$

D. $126(5!)^2$

Answer: D

Solution:

Solution:

$$6! \times {}^7C_5 \times 5!$$

$$\Rightarrow 720 \times 21 \times 120$$

$$\Rightarrow 2 \times 360 \times 7 \times 3 \times 120$$

$$\Rightarrow 126 \times (5!)^2$$

Question55

If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is $(6!)k$, is equal to
[8-Apr-2023 shift 2]

Options:

A. 1890

B. 945

C. 2835

D. 5670

Answer: D

Solution:

Solution:

$M_2A_2T_2$ HEICS

= total words - when C&S are together

$$\frac{11!}{2!2!2!} - \frac{10!}{2!2!2!} \times 2$$

$$\frac{10!}{2!2!2!} \times 9$$

$$= \frac{9 \times 10 \times 9 \times 8 \times 7}{8} \times 6$$

$$= 5670 \times 6$$

$$k = 5670 \quad (\text{Option 4})$$

Question56

The number of permutations of the digits 1, 2, 3, ..., 7 without repetition, which neither contain the string 153 nor the string 2467, is

[10-Apr-2023 shift 1]

Answer: 4898

Solution:

Solution:

Numbers are 1, 2, 3, 4, 5, 6, 7

Numbers having string (154) = (154), 2, 3, 6, 7 = 5 !

Numbers having string (2467) = (2467), 1, 3, 5 = 4 !

Number having string (154) and (2467)

$$= (154), (2467) = 2!$$

$$\text{Now } n(154 \cup 2467) = 5! + 4! - 2!$$

$$= 120 + 24 - 2 = 142$$

$$\text{Again total numbers} = 7! = 5040$$

$$\text{Now required numbers} = n(\text{neither } 154 \text{ nor } 2467)$$

$$= 5040 - 142$$

$$= 4898$$

Question57

Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840, then the total numbers of persons, who participated in the tournament, is _____.

[10-Apr-2023 shift 1]

Answer: 16

Solution:

Solution:

Let number of couples = n

$$\therefore {}^nC_2 \times {}^{n-2}C_2 \times 2 = 840$$

$$\Rightarrow n(n-1)(n-2)(n-3) = 840 \times 2$$

$$= 21 \times 40 \times 2$$

$$= 7 \times 3 \times 8 \times 5 \times 2$$

$$n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$$

$$\therefore n = 8$$

Hence, number of persons = 16.

Question58

Eight persons are to be transported from city A to city B in three cars different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is

[10-Apr-2023 shift 2]

Options:

A. 1120

B. 560

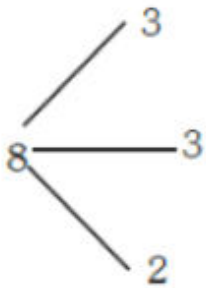
C. 3360

D. 1680

Answer: D

Solution:

Solution:



$$\begin{aligned}\text{Ways} &= \frac{8!}{3!3!2!2!} \times 3! \\ &= \frac{8!}{4} \\ &= 56 \times 30 \\ &= 1680\end{aligned}$$

Question59

The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _____.

[10-Apr-2023 shift 2]

Answer: 26664

Solution:

Solution:

2, 1, 2, 3

$$-- \frac{1}{2!} = 3$$

$$-- \times 2 \quad 3! = 6$$

$$--x3 \frac{3!}{2!} = 3$$

$$\text{Sum of digits of unit place} = 3 \times 1 + 6 \times 2 + 3 \times 3 = 24$$

Required sum

$$= 24 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1$$

$$= 24 \times 1111$$

$$= 26664$$

Question60

The number of triplets (x, y, z), where x, y, z are distinct non negative integers satisfying $x + y + z = 15$, is :
[11-Apr-2023 shift 1]

Options:

A. 136

B. 114

C. 80

D. 92

Answer: B

Solution:

Solution:

$$x + y + z = 15$$

$$\text{Total no. solution} = {}^{15+3-1}C_3 = 136... (1)$$

$$\text{Let } x = y \neq z$$

$$2x + z = 15 \Rightarrow z = 15 - 2x$$

$$\Rightarrow x \in \{0, 1, 2, \dots, 7\} - \{5\}$$

$\therefore 7$ solutions

\therefore there are 21 solutions in which exactly

Two of x, y, z are equal ... (2)

There is one solution in which $x = y = z...$ (3)

$$\text{Required answer} = 136 - 21 - 1 = 114$$

Question61

In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the

students sits on the allotted seat, is _____.
[11-Apr-2023 shift 1]

Answer: 44

Solution:

Solution:

Derangement of 5 students

$$\begin{aligned}D_5 &= 5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right) \\&= 120 \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} \right) \\&= 60 - 20 + 5 - 1 \\&= 40 + 4 \\&= 44\end{aligned}$$

Question62

If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is
[11-Apr-2023 shift 2]

Options:

- A. 102
- B. 103
- C. 101
- D. 104

Answer: A

Solution:

Solution:

5 2 1 3 4

T H A M S

4 1 0 0 0

4! 3! 2! 1! 0!

$$\Rightarrow 4 \times 4! + 3! \times 1 + 0 + 0 + 0$$

$$\Rightarrow 96 + 6 = 102$$

$$\text{Rank THAMS} = 102 + 1 = 103$$

Question63

The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to
[12-Apr-2023 shift 1]

Options:

A. 132

B. 120

C. 72

D. 96

Answer: B

Solution:

Solution:

5 x x x 0

7 x x x 0

5 x x x 5

9 x x x 0

9 x x x 5

$$\text{So Required numbers} = 5 \times {}^4P_3 = 120$$

Question64

Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

[12-Apr-2023 shift 1]

Answer: 1260

Solution:

Solution:

abc or cba

abc

cba

$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

Question65

The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is _____.

[13-Apr-2023 shift 1]

Answer: 413

Solution:

Solution:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12, x_i \in \{1, 2, 3, 4\}$$

$$\text{No. of solutions} = {}^{5+7-1}C_{7-1} - \frac{7!}{6!} - \frac{7!}{5!} = 413$$

Question66

All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is
[13-Apr-2023 shift 2]

Options:

A. 328

B. 327

C. 324

D. 326

Answer: B

Solution:

Solution:

$$\begin{array}{|c|c|c|c|c|c|} \hline A & & & & & \\ \hline \end{array} \overset{5!}{=} 120$$

$$\begin{array}{|c|c|c|c|c|c|} \hline D & & & & & \\ \hline \end{array} \overset{5!}{=} 120$$

$$\begin{array}{|c|c|c|c|c|c|} \hline M & A & & & & \\ \hline \end{array} \overset{4!}{=} 24$$

$$\begin{array}{|c|c|c|c|c|c|} \hline M & D & & & & \\ \hline \end{array} \overset{4!}{=} 24$$

$$\begin{array}{|c|c|c|c|c|c|} \hline M & N & & & & \\ \hline \end{array} \overset{4!}{=} 24$$

$$\begin{array}{|c|c|c|c|c|c|} \hline & & & & 3! & \\ \hline \text{M} & \text{O} & \text{A} & & & \\ \hline \end{array} = 6$$

$$\begin{array}{|c|c|c|c|c|c|} \hline & & & & 3! & \\ \hline \text{M} & \text{O} & \text{D} & & & \\ \hline \end{array} = 6$$

$$\begin{array}{|c|c|c|c|c|c|} \hline & & & & 2! & \\ \hline \text{M} & \text{O} & \text{N} & \text{A} & & \\ \hline \end{array} = 2$$

$$\begin{array}{|c|c|c|c|c|c|} \hline \text{M} & \text{O} & \text{N} & \text{D} & \text{A} & \text{Y} \\ \hline \end{array} = 1$$

$$\begin{aligned} \text{Rank} &= 120 + 120 + 24 + 24 + 24 + 6 + 6 + 2 + 1 \\ &= 327 \end{aligned}$$

Question67

Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is _____.
[13-Apr-2023 shift 2]

Answer: 16

Solution:

Solution:

a	b	2
---	---	---

$(a, b) = (1, 3), (3, 1), (2, 2), (2, 5), (5, 2), (3, 4), (4, 3), (5, 5)$
= 8 numbers

a	b	4
---	---	---

$(a, b) = (1, 1), (1, 4), (4, 1), (2, 3), (3, 2), (4, 4), (3, 5), (5, 3)$ = 8 numbers
total $8 + 8 = 16$

Question68

The total number of three-digit numbers, divisible by 3 , which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is

[15-Apr-2023 shift 1]

Options:

A. 21

B. 18

C. 20

D. 22

Answer: D

Solution:

Solution:

(1, 1, 1)(3, 3, 3)(5, 5, 5)(8, 8, 8)

(5, 5, 8)(8, 8, 5)(1, 3, 5)(1, 3, 8)

$$\text{Total number} = 1 + 1 + 1 + 1 + \frac{3!}{2!} + \frac{3!}{2!} + 3! + 3! = 22$$

Question69

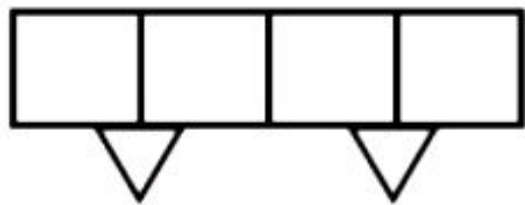
A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is

[15-Apr-2023 shift 1]

Answer: 72

Solution:

Solution:



Sum of first two digits

Sum of last two digits = α

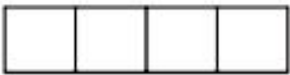
Case-I : $\alpha = 7$

$2 \times 12 = 24$ ways.

<div><div>70</div><div>07</div></div>	<div><div></div><div>16</div></div>
	<div><div>25</div></div>
	<div><div>34</div></div>
	<div><div>43</div></div>
	<div><div>52</div></div>
	<div><div>61</div></div>

Case – II : $\alpha = 8$

<div><div></div><div>17</div></div>	<div><div></div><div>26</div></div>
<div><div></div><div>71</div></div>	<div><div></div><div>62</div></div>
	<div><div></div><div>35</div></div>
	<div><div></div><div>53</div></div>



2×8 ways

= 16 ways

Case-III : $\alpha = 9$

<div>□□</div>	<div>□□</div>
27	36
72	63
	45
	54



2×8 ways

= 16 ways

Case IV : $\alpha = 10$

<div>□□</div>	<div>□□</div>
37	46
73	64

2×4 ways

8 ways

Case V : $\alpha = 11$

<div>□□</div>	<div>□□</div>
47	56
74	65

2×4 ways

8 ways

Ans. $24 + 16 + 16 + 8 + 8 = 72$

Question70

The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is
[25-Jul-2022-Shift-1]

Answer: 1492

Solution:

Solution:

M	A	N	K	I	N	D
---	---	---	---	---	---	---

$$\left(\frac{4 \times 6!}{2!} \right) + (5! \times 0) + \left(\frac{4! \times 3}{2!} \right) + (3! \times 2) + (2! \times 1) + (1! \times 1) + (0! \times 0) + 1 = 1492$$

Question71

The number of 5-digit natural numbers, such that the product of their digits is 36 , is _____.
[26-Jul-2022-Shift-1]

Answer: 180

Solution:

Solution:

Factors of 36 = $2^2 \cdot 3^2 \cdot 1$

Five-digit combinations can be

(1, 2, 2, 3, 3)(1, 4, 3, 3, 1), (1, 9, 2, 2, 1)

(1, 4, 9, 11)(1, 2, 3, 6, 1)(1, 6, 6, 1, 1)

i.e., total numbers

$$\frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{3!} + \frac{5!}{2!} + \frac{5!}{3!2!}$$
$$= (30 \times 3) + 20 + 60 + 10 = 180$$

Question72

Numbers are to be formed between 1000 and 3000 , which are divisible by 4 , using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is _____.
[26-Jul-2022-Shift-2]

Answer: 30

Solution:

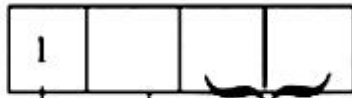
Solution:

Here 1st digit is 1 or 2 only

Case-I

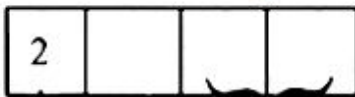
If first digit is 1

Then last two digits can be 24, 32, 36, 52, 56, 64



$$1 \times 3 \times 6 = 18 \text{ ways}$$

Case - II If first digit is 2 then last two digit can be 16, 36, 56, 64



$$1 \times 3 \times 4 = 12 \text{ ways}$$

Total ways = $12 + 18 = 30$ ways

Question 73

Let S be the sample space of all five digit numbers. If p is the probability that a randomly selected number from S , is a multiple of 7 but not divisible by 5, then $9p$ is equal to
[27-Jul-2022-Shift-1]

Options:

- A. 1.0146
- B. 1.2085
- C. 1.0285
- D. 1.1521

Answer: C

Solution:

Solution:

Among the 5 digit numbers,

First number divisible by 7 is 10003 and last is 99995 .

\Rightarrow Number of numbers divisible by 7 .

$$= \frac{99995 - 10003}{7} + 1$$

$$= 12857$$

First number divisible by 35 is 10010 and last is 99995.

⇒ Number of numbers divisible by 35

$$= \frac{99995 - 10010}{35} + 1$$

$$= 2572$$

Hence number of number divisible by 7 but not by 5

$$= 12857 - 2572$$

$$= 10285$$

$$9P. = \frac{10285}{90000} \times 9$$

$$= 1.0285$$

Question74

Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from {A, B, C, D, E } or a number from {1, 2, 3, 4, 5} with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from {1, 2, 3, 4, 5} is $\alpha \times 5^6$, then α is equal to _____. [28-Jul-2022-Shift-1]

Answer: 7073

Solution:

Solution:

If password is 6 character long, then

Total number of ways having atleast one number = $10^6 - 5^6$

Similarly, if 7 character long = $10^7 - 5^7$

and if 8-character long = $10^8 - 5^8$

Number of password = $(10^6 + 10^7 + 10^8) - (5^6 + 5^7 + 5^8)$

$$= 5^6(2^6 + 5 \cdot 2^7 + 25 \cdot 2^8 - 1 - 5 - 25)$$

$$= 5^6(64 + 640 + 6400 - 31)$$

$$= 7073 \times 5^6$$

$$\therefore \alpha = 7073$$

Question75

A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then $b + 3g$ is equal to

 .
[28-Jul-2022-Shift-2]

Answer: 17

Solution:

Solution:

$${}^bC_3 \cdot {}^gC_2 = 168$$

$$\Rightarrow \frac{b(b-1)(b-2)}{6} \cdot \frac{g(g-1)}{2} = 168$$

$$\Rightarrow b(b-1)(b-2) \cdot g(g-1) = 2^5 \cdot 3^2 \cdot 7$$

$$\Rightarrow b(b-1)(b-2) \cdot g(g-1) = 6 \cdot 7 \cdot 8 \cdot 3 \cdot 2$$

$$\therefore b = 8 \text{ and } g = 3$$

$$\therefore b + 3g = 17$$

Question76

Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, \dots, 1000\}$. If

$A = \{a_1 + a_2 + \dots + a_k : k \in \mathbb{N}, a_1, a_2, a_3, \dots, a_k \in S\}$, then the sum of all the elements in the set $T - A$ is equal to _____.

[29-Jul-2022-Shift-1]

Answer: 11

Solution:

Solution:

Here $S = \{4, 6, 9\}$

And $T = \{9, 10, 11, \dots, 1000\}$

We have to find all numbers in the form of $4x + 6y + 9z$, where $x, y, z \in \{0, 1, 2, \dots\}$.

If a and b are coprime number then the least number from which all the number more than or equal to it can be express as $ax + by$ where $x, y \in \{0, 1, 2, \dots\}$ is $(a-1) \cdot (b-1)$.

Then for $6y + 9z = 3(2y + 3z)$

All the number from $(2-1) \cdot (3-1) = 2$ and above can be express as $2x + 3z$ (say t).

$$\begin{aligned}\text{Now } 4x + 6y + 9z &= 4x + 3(t+2) \\ &= 4x + 3t + 6\end{aligned}$$

again by same rule $4x + 3t$, all the number from $(4-1)(3-1) = 6$ and above can be express from $4x + 3t$

Then $4x + 6y + 9z$ express all the numbers from 12 and above.

again 9 and 10 can be express in form $4x + 6y + 9z$.

Then set $A = \{9, 10, 12, 13, \dots, 1000\}$.

Then $T - A = \{11\}$

Only one element 11 is there.

Sum of elements of $T - A = 11$

Question77

The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is _____.

[29-Jul-2022-Shift-2]

Answer: 6

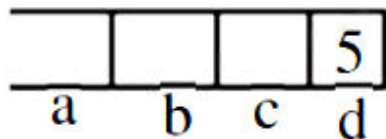
Solution:

Solution:

4 digit numbers

For divisibility by 55, no. should be div. by 5 and 11 both

Also, for divisibility by 11



$$a + c = b + 5$$

$$\text{for } b = 1 \quad a = 2, \quad c = 4$$

$$a = 4, \quad c = 2$$

$$\text{for } b = 2 \quad a = 3, \quad c = 4$$

$$a = 4, \quad c = 3$$

$$\text{for } b = 3 \quad a = 6, \quad c = 2$$

$$a = 2, \quad c = 6$$

\therefore 6 possible four digit no.s are div. by 55

(II) 5 digit number is not possible



(Not possible)

Question78

In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is ____
[24-Jun-2022-Shift-1]

Answer: 40

Solution:

Solution:

Let student marks x correct answers and y incorrect. So

$$3x - 2y = 5 \text{ and } x + y \leq 5 \text{ where } x, y \in W$$

Only possible solution is $(x, y) = (3, 2)$

Students can mark correct answers by only one choice but for an incorrect answer, there are two choices. So
total number of ways of scoring 5 marks $= {}^5C_3(1)^3 \cdot (2)^2 = 40$

Question79

The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is ____
[24-Jun-2022-Shift-2]

Answer: 576

Solution:

Solution:

Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 → Difference of sum at even place & odd place is divisible by 11 .

Let number of the form abcdefg

$$\therefore (a + c + e + g) - (b + d + f) = 11x$$

$$a + b + c + d + e + f = 31$$

$$\therefore \text{either } a + c + e + g = 21 \text{ or } 10$$

$$\therefore b + d + f = 10 \text{ or } 21$$

Case-1

$$a + c + e + g = 21$$

$$b + d + f = 10$$

$$(b, d, f) \in \{(1, 2, 7)(2, 3, 5)(1, 4, 5)\}$$

$$(a, c, e, g) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$$

Case-2

$$a + c + e + g = 10$$

$$b + d + f = 21$$

$$(a, b, e, g) \in \{1, 2, 3, 4\}$$

$$(b, d, f) \in \{(5, 7, 9)\}$$

$$\therefore \text{Total number in case 2} = 3! \times 4! = 144$$

$$\therefore \text{Total numbers} = 144 + 432 = 576$$

Question80

The sum of all the elements of the set $\{\alpha \in \{1, 2, \dots, 100\} : \text{HCF}(\alpha, 24) = 1\}$ is ____
[24-Jun-2022-Shift-2]

Answer: 1633

Solution:

Solution:

The numbers upto 24 which gives g.c.d. with 24 equals to 1 are 1, 5, 7, 11, 13, 17, 19 and 23.

Sum of these numbers = 96

There are four such blocks and a number 97 is there upto 100 .

\therefore Complete sum

$$= 96 + (24 \times 8 + 96) + (48 \times 8 + 96) + (72 \times 8 + 96) + 97$$

$$= 1633$$

Question81

The number of 3-digit odd numbers, whose sum of digits is a multiple of 7 , is _____
[25-Jun-2022-Shift-1]

Answer: 63

Solution:

Solution:

For odd numbers, unit place shall be 1, 3, 5, 7 or 9 .

$xyxy$, $xyxy$, $xyxy$, $xyxz$, $xyx9$ are the type of numbers.

If $xy1$ then

$$x+y = 6, 13, 20 \dots$$

$$\text{Total number} = 6 + 6 + 0 + \dots = 12$$

If $x y 3$ then

$$x+y = 4, 11, 18, \dots$$

$$\text{Total number} = 4 + 8 + 1 + 0 \dots = 13$$

Similarly for $x y 5$, we have

$$x+y = 2, 9, 16, \dots$$

$$\text{Total number} = 2 + 9 + 3 = 14$$

for $x y 7$ we have

$$x+y = 0, 7, 14, \dots$$

$$\text{Total number} = 0 + 7 + 5 = 12 \text{ ways}$$

And for $x y 9$ we have

$$x+y = 5, 12, 19 \dots$$

$$\text{Total number} = 5 + 7 + 0 \dots = 12 \text{ ways}$$

\therefore Total odd numbers whose sum of digits is a multiple of 7 is 63 .

Question82

The total number of three-digit numbers, with one digit repeated exactly two times, is _____
[25-Jun-2022-Shift-2]

Answer: 243

Solution:

Solution:

C – 1 : All digits are non-zero

$${}^9C_2 \cdot 2 \cdot \frac{3!}{2} = 216$$

C – 2: One digit is 0

$$0, 0, x \Rightarrow {}^9C_1 \cdot 1 = 9$$

$$0, x, x \Rightarrow {}^9C_1 \cdot 2 = 18$$

$$\text{Total} = 216 + 27 = 243$$

Question83

There are ten boys B_1, B_2, \dots, B_{10} and five girls G_1, G_2, \dots, G_5 in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is _____
[26-Jun-2022-Shift-1]

Answer: 1120

Solution:

Solution:

Number of ways when B_1 and B_2 are not together

= Total number of ways of selecting 3 boys – B_1 and B_2 are together

$$= {}^{10}C_3 - {}^8C_1$$

$$= \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} - 8$$

$$= 112$$

Number of ways to select 3 girls = ${}^5C_3 = 10$

\therefore Total number of ways = $112 \times 10 = 1120$

Question84

The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _____
[26-Jun-2022-Shift-2]

Answer: 150

Solution:

Solution:

$$\because x \in [100, 999], x \in N$$

$$\text{Then } \frac{x}{2} \in [50, 499], \frac{x}{2} \in N$$

Number whose G.C.D. with 18 is 1 in this range have the required condition. There are 6 such number from 18×3 to 18×4 . Similarly from 18×4 to 18×5, 26×18 to 27×18

$$\therefore \text{Total numbers} = 24 \times 6 + 6 = 150$$

The extra numbers are 53, 487, 491, 493, 497 and 499.

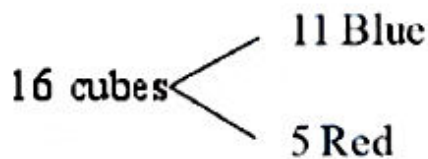
Question85

The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is
[27-Jun-2022-Shift-1]

Answer: 56

Solution:

Solution:



$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$$

$$x_1, x_6 \geq 0, \quad x_2, x_3, x_4, x_5 \geq 2$$

$$x_2 = t_1 + 2$$

$$x_3 = t_2 + 2$$

$$x_4 = t_3 + 2$$

$$x_5 = t_4 + 2$$

$$x_1, t_1, t_2, t_3, t_4, x_6 \geq 0$$

$$\text{No. of solutions} = {}^{6+3-1}C_3 = {}^8C_3 = 56$$

Question86

The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is
[28-Jun-2022-Shift-1]

Options:

A. 36

B. 48

C. 60

D. 72

Answer: D

Solution:

Solution:

To make a no. divisible by 3 we can use the digits 1, 2, 5, 6, 7 or 1, 2, 3, 5, 7.

Using 1, 2, 5, 6, 7, number of even numbers is

$$= 4 \times 3 \times 2 \times 1 \times 2 = 48$$

Using 1, 2, 3, 5, 7, number of even numbers is

$$= 4 \times 3 \times 2 \times 1 \times 1 = 24$$

Required answer is 72.

Question87

Let $A = \{1, a_1, a_2, \dots, a_{18}, 77\}$ be a set of integers with

$1 < a_1 < a_2 < \dots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$

contain exactly 39 elements. Then, the value of $a_1 + a_2 + \dots + a_{18}$ is equal to _____

[28-Jun-2022-Shift-1]

Answer: 702

Solution:

Solution:

$a_1, a_2, a_3, \dots, a_{18}, 77$

are in AP i.e. 1, 5, 9, 13, ..., 77.

Hence $a_1 + a_2 + a_3 + \dots + a_{18} = 5 + 9 + 13 + \dots$ 18 terms = 702

Question88

The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives at least 4 and at most 7 candies, C_3 receives at least 2 and at most 6 candies, is equal to:

[28-Jun-2022-Shift-2]

Options:

A. 205

B. 615

C. 510

D. 430

Answer: D

Solution:

Solution:

By multinomial theorem, no. of ways to distribute 30 identical candies among four children C_1, C_2 and C_3, C_4

$$\begin{aligned} &= \text{Coefficient of } x^{30} \text{ in } (x^4 + x^5 + \dots + x^7)(x^2 + x^3 + \dots + x^6)(1 + x + x^2 + \dots)^2 \\ &= \text{Coefficient of } x^{24} \text{ in } \frac{(1-x^4)}{1-x} \frac{(1-x^5)}{1-x} \frac{(1-x^{31})^2}{(1-x)^2} \\ &= \text{Coefficient of } x^{24} \text{ in } (1-x^4-x^5+x^9)(1-x)^{-4} \\ &= {}^{27}C_{24} - {}^{23}C_{20} - {}^{22}C_{19} + {}^{18}C_{15} = 430 \end{aligned}$$

Question89

Let $b_1 b_2 b_3 b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, \dots, 100\}$ for $1 \leq i \leq 4$ and $b_i \neq b_j$ for $i \neq j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1 b_2 b_3 b_4$ is equal to ____
[29-Jun-2022-Shift-1]

Answer: 18915

Solution:

Solution:

$b_i \in \{1, 2, 3, \dots, 100\}$

Let A = set when $b_1 b_2 b_3$ are consecutive

$$n(A) = \frac{97 + 97 + \dots + 97}{98 \text{ times}} = 97 \times 98$$

Similarly when $b_2 b_3 b_4$ are consecutive

$$N(A) = 97 \times 98$$

$$n(A \cap B) = \frac{97 + 97 + \dots + 97}{98 \text{ times}} = 97 \times 98$$

Similarly when $b_2 b_3 b_4$ are consecutive

$$n(B) = 97 \times 98$$

$$n(A \cap B) = 97$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{Number of permutation} = 18915$$

Question90

The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to _____
[29-Jun-2022-Shift-2]

Answer: 1086

Solution:

Solution:

Let the number is abcd, where a,b,c are divisible by d.

No. of such numbers

$$d = 1, 9 \times 10 \times 10 = 900$$

$$d = 2, 4 \times 5 \times 5 = 100$$

$$d = 3, 3 \times 4 \times 4 = 48$$

$$d = 4, 2 \times 3 \times 3 = 18$$

$$d = 5, 1 \times 2 \times 2 = 4$$

$$d = 6, 7, 8, 9 \quad 4 \times 4 = 16$$

$$\text{Total} = 1086$$

Question91

The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is _____
[2021, 26 Feb. Shift-II]

Answer: 1000

Solution:

Solution:

Let x be four digit number, then $\gcd(x, 18) = 3$

This implies x is divisible by 3 but not divisible by 9 .

The 4-digit numbers which is an odd multiple of 3 are 1005, 1011, 1017, 9999

These are 1499 in counting i.e. total number of 4-digit numbers which is odd multiple of 3 are 1499.

Now, The 4-digit numbers which is an odd multiple of 9 are, 1017, 1035, ...999 These, are total 499.

Then, required 4-digit numbers

$$= 1499 - 499 = 1000$$

Question92

**The total number of two digit numbers n' , such that $3^n + 7^n$ is a multiple of 10 , is
[2021, 25 Feb. Shift-II]**

Answer: 45

Solution:

Solution:

We may write, $7^n = (10 - 3)^n$ or $7^n = 10K + (-3)^n$ (using expansion)

$$\therefore 7^n + 3^n = 10K + (-3)^n + 3^n$$

$$= \begin{cases} 10k & n = \text{odd} \\ 10k + 2 \cdot 3^n & n = \text{even} \end{cases}$$

Let $n = \text{even} = 2t, t \in \mathbb{N}$

$$\text{Then, } 3^n = 3^{2t} = 9^t = (10 - 1)^t$$

$$= 10p + (-1)^t$$

$$= 10p \pm 1$$

If $n = \text{even}$, then $7^n + 3^n$ will never be multiple of 10.

This implies $n = 0$ dd

$$n = 11, 13, 15, \dots 99 \quad (\text{since, } n \text{ is two digit})$$

$$\Rightarrow 10 < n < 100$$

Total possible 'n' are 45.

Question93

The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is
[2021, 25 Feb. Shift-I]

Answer: 32

Solution:

Solution:

Given, digits = {1, 2, 3, 4, 5}

Numbers divisible by 3 (sum of digits divisible by 3).

--	--	--

Case I When sum is 12 \rightarrow 3, 4, 5 $\rightarrow 3! = 6$

Case II When sum is 9 \rightarrow 2, 3, 4 $\rightarrow 3! = 6$

Case III When sum is 9 \rightarrow 1, 3, 5 $\rightarrow 3! = 6$

Case IV When sum is 6 \rightarrow 1, 2, 3 $\rightarrow 3! = 6$

So, total numbers divisible by 3 = $6 \times 4 = 24$

Numbers divisible by 5 (ending with 5)

So, total numbers divisible by 5 = 12

Numbers divisible by 15, are 145, 415, 345, 435

i.e. total 4 numbers are divisible by both 3 and 5.

		5
--	--	---

4×3 \uparrow
1

$= 4 \times 3 = 12$

i.e. divisible by 15.

Hence, the required numbers which are divisible by 3 or 5 = $24 + 12 - 4 = 32$

Question94

A natural number has prime factorisation given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$.

Then, the number of odd divisors of n, including 1, is
[2021, 26 Feb. Shift-II]

Options:

A. 11

B. 6

C. 6x

D. 12

Answer: D

Solution:

Solution:

Given, $n = 2^x 3^y 5^z$

and $y + z = 5$

$$\frac{1}{y} + \frac{1}{z} = \frac{5}{6} \text{ or } \frac{y+z}{yz} = \frac{5}{6}$$

This implies,

$$y + z = 5 \text{ and } yz = 6$$

$$\text{Put } y = \frac{6}{z} \text{ in } y + z = 5$$

$$\Rightarrow \frac{6}{z} + z = 5 \text{ or } z^2 - 5z + 6 = 0$$

$$\Rightarrow z^2 - 3z - 2z + 6 = 0$$

$$(z - 3)(z - 2) = 0$$

$$\Rightarrow z = 3 \text{ or } 2$$

Using $y = \frac{6}{z}$, we get $y = 2$ or 3

For calculating the odd divisor x must be 0 i.e. $x = 10$

$$\therefore n = 2^0 3^3 5^2 \text{ or } n = 2^0 3^2 5^3$$

Total number of odd divisor of n is equal to

$$= (3 + 1)(2 + 1)$$

$$= (4)(3) = 12$$

Question95

The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is
[2021, 26 Feb. Shift-I]

Options:

A. 42

B. 82

C. 77

D. 35

Answer: C

Solution:

Solution:

To form a seven digit number with sum of digits 10 , all the digits can't be 1, 2 or 3 . Hence, seven digit number must have the following cases,

Case 1. Using 1, 1, 1, 1, 1, 2, 3

Possible seven digit numbers will be

$$= \frac{7!}{5!} = 7 \times 6 = 42$$

Case 2. Using 2, 2, 2, 1, 1, 1, 1

Possible numbers will be

$$= \frac{7!}{3!4!} = \frac{7 \times 6 \times 5}{3 \times 2} = 35$$

No more cases will be formed

Hence, total number of seven digit numbers possible

$$= 42 + 35 = 77$$

Question 96

The total number of positive integral solutions (x, y, z), such that $xyz = 24$ is

[2021, 25 Feb. Shift-1]

Options:

A. 36

B. 24

C. 45

D. 30

Answer: D

Solution:

Solution:

Given, $xyz = 24$

$$\Rightarrow xyz = 2^3 \cdot 3^1$$

$$\text{Let } x = 2^{a_1} \cdot 3^{b_1},$$

$$y = 2^{a_2} \cdot 3^{b_2},$$

$$z = 2^{a_3} \cdot 3^{b_3}$$

where, $a_1, a_2, a_3 \in \{0, 1, 2, 3\}$

$$b_1, b_2, b_3 \in \{0, 1\}$$

Case I $a_1 + a_2 + a_3 = 3$

\therefore Non-negative solution

$$= {}^{3+3-1}C_{3-1} = {}^5C_2 = 10$$

Case II $b_1 + b_2 + b_3 = 1$

\therefore Non-negative solution

$$= {}^{1+3-1}C_{3-1} = {}^3C_2 = 3$$

\therefore Total solutions $= 10 \times 3 = 30$

Question97

The students S_1, S_2, \dots, S_{10} are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then, the total number of possibilities of forming such groups is

[2021, 24 Feb. Shift-II]

Answer: 31650

Solution:

Solution:

Given, total students $= 10$

number of groups $= 3$ (i.e. A, B and C)

Each group has atleast one student but group C has atmost 3 students.

\therefore There are 3 cases depending on number of students in group C.

Case I C has 1 student, then $\left. \begin{matrix} A \\ B \end{matrix} \right] \leftarrow 9 \text{ students}$

$$\therefore \text{Number of ways} = {}^{10}C_1 \times [2^9 - 2]$$

Case II C has 2 students, then $\left. \begin{matrix} A \\ B \end{matrix} \right] \leftarrow 8 \text{ Students.}$

$$\therefore \text{Number of ways} = {}^{10}C_2 \times [2^8 - 2]$$

Case III C has 3 students, then $\left. \begin{matrix} A \\ B \end{matrix} \right\} \leftarrow 7 \text{ Students.}$

$$\therefore \text{Number of ways} = {}^{10}C_3 \times [2^7 - 2]$$

\therefore Required number of possibilities

$$= {}^{10}C_1(2^9 - 2) + {}^{10}C_2(2^8 - 2) + {}^{10}C_3(2^7 - 2)$$

$$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3]$$

$$= 20 - 90 - 240$$

$$= 128[40 + 90 + 120] - 350$$

$$= (128 \times 250) - 350 = 31650$$

Question98

A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

[24-Feb-2021 Shift 1]

Options:

A. 1625

B. 575

C. 560

D. 1050

Answer: A

Solution:

Solution:

Indians	Foreigners	Number of Ways
2	4	${}^6C_2 \times {}^8C_4 = 1050$
3	6	${}^6C_3 \times {}^8C_6 = 560$
4	8	${}^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

Question99

**The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is
[2021, 18 March Shift-I]**

Options:

- A. 26664
- B. 122664
- C. 122234
- D. 22264

Answer: A

Solution:

Solution:

Given, digits are = 1, 2, 2, 3

\therefore Total distinct numbers = $4!/2! = 12$

1 at unit place \Rightarrow Number of such

numbers = $\frac{3!}{2!} = 3$

2 at unit place \Rightarrow Number of such numbers = $3! = 6$

3 at unit place \Rightarrow Number of such numbers = $\frac{3!}{2!} = 3$

\therefore Sum of digits at unit place is

$3 \times 1 + 6 \times 2 + 3 \times 3 = 24$

Hence, sum of all 4 digit such numbers

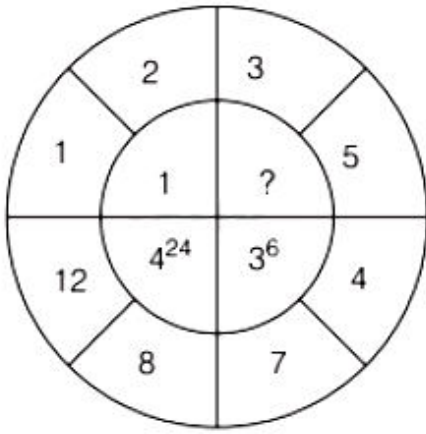
$= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

$= 1111 \times 24$

$= 26664$

Question100

The missing value in the following figure is



[2021, 18 March Shift-I]

Answer: 4

Solution:

Solution:

As, we observe the pattern Inside number

Inside number = (difference)^{(difference)!}

= (Greater number - Smaller number)^{(Greater number - Smaller number)!}

i.e. $1 = (2 - 1)^{(2-1)!}$, $4^{24} = (12 - 8)^{(12-8)!}$,

$3^6 = (7 - 4)^{(7-4)!}$

$\therefore ? = (5 - 3)^{(5-3)!}$

$\therefore \text{Required number} = 2^{2!} = 2^{2 \times 1} = 4$

Question101

If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to

[2021, 18 March Shift-II]

Answer: 160

Solution:

Solution:

$$\begin{aligned} & \sum_{r=1}^{10} r![(r+1)(r+2)(r+3) - 9(r+1) + 8] \\ &= \sum_{r=1}^{10} [\{(r+3)! - (r+1)!\} - 8\{(r+1)! - r!\}] \\ &= (13! + 12! - 2! - 3!) - 8(11! - 1) \\ &= (12 \cdot 13 + 12 - 8) \cdot 11! - 8 + 8 \\ &= (160)(11!) \\ \therefore \alpha &= 160 \end{aligned}$$

Question102

The number of times the digit 3 will be written when listing the integers from 1 to 1000 is
[2021, 18 March Shift-1]

Answer: 300

Solution:

Solution:

Let the number be xyz , $0 \leq x, y, z \leq 9$

Case I '3' appears only one time $\Rightarrow {}^3C_1 \times 9 \times 9 = 243$

Case II '3' appears two times $\Rightarrow {}^3C_2 \times 2 \times 9 = 54$

Case III '3' appears three times

$$\Rightarrow {}^3C_3 \times 3 = 3$$

$$\therefore \text{Total} = 243 + 54 + 3 = 300$$

Question103

Team A consists of 7 boys and n girls and Team B has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams, when a boy plays against a boy and a girl plays against a girl, then n is equal to
[2021, 17 March Shift-I]

Options:

A. 5

B. 2

C. 4

D. 6

Answer: C

Solution:

Solution:

	Boys	Girls
Team A	7	n
Team B	4	6

Number of matches between Team A and Team B when a boy play against a boy $({}^7C_1 \times {}^4C_1) = 28$

Similarly, number of matches between Team A and Team B when a girl play against a girl

$$({}^nC_1 \times {}^6C_1) = 6n$$

According to question,

$$28 + 6n = 52$$

$$6n = 24$$

$$n = 4$$

Question104

If the sides AB, BC and CA of a triangle $\triangle ABC$ have 3,5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to [2021, 17 March Shift-II]

Options:

A. 364

B. 240

C. 333

D. 360

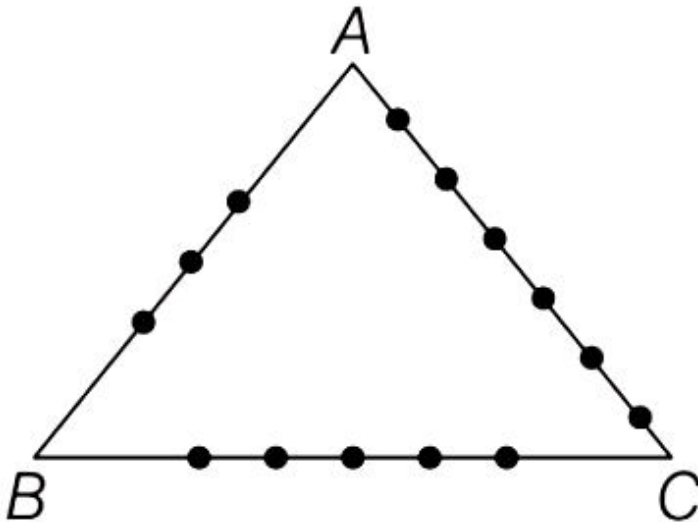
Answer: C

Solution:

Solution:

Method (I) (Proper Method)

Whenever we construct a triangle, we must require three non-collinear points.



∴ Total number of triangles using the points 3, 5 and 6 which are on the sides AB, BC and CA

= Either taking (one point from AB, BC and CA)

or (one point from AB and two points from BC)

or (one point from BC and two points from AB)

or (one point from AB and two points from AC)

or (one point from AC and two points from AB)

or (one point from BC and two points from AC)

or (one point from BC and two points from AC)

or (one point from AC and two points from BC)

⇒ Total number of triangles

$$r = ({}^3C_1 \times {}^5C_1 \times {}^6C_1) + ({}^3C_1 \times {}^5C_2)$$

$$+ ({}^3C_1 \times {}^6C_2) + ({}^6C_1 \times {}^3C_2) + ({}^5C_2)$$

$$+ ({}^6C_2)$$

$$= 90 + 30 + 15 + 45 + 18 + 75 + 60$$

$$= 333 \quad \left[\text{using } {}^nC_r = \frac{n!}{r!(n-r)!} \right]$$

$$\text{and } n! = 1 \times 2 \times 3 \times \dots \times n]$$

Method (II) (Direct Method)

Total number of points = 3 + 5 + 6 = 14

Then, when we construct a triangle, we must select 3 points out of 14 but these points never be collinear.

$$\therefore \text{Total number of triangles formed} = {}^{14}C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3 = 333$$

Question105

Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA, respectively. Let α be the number

of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then, $(\beta - \alpha)$ is equal to
[2021, 16 March Shift-II]

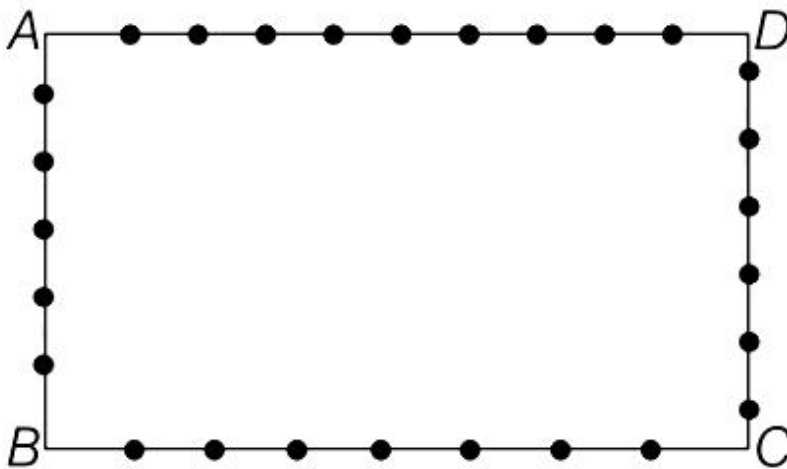
Options:

- A. 795
- B. 1173
- C. 1890
- D. 717

Answer: D

Solution:

Solution:



Number of triangles that can be formed from the points on 3 of the sides.

$${}^5C_1 {}^7C_1 {}^6C_1 + {}^5C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^9C_1 + {}^6C_1 {}^7C_1 {}^9C_1$$

$$= 210 + 315 + 270 + 378$$

$$\Rightarrow \alpha = 1173$$

Number of quadrilaterals that can be formed by taking one point from each of the four vertex

$${}^5C_1 {}^7C_1 {}^6C_1 {}^9C_1 = 5 \times 6 \times 7 \times 9 = 1890$$

$$\Rightarrow \beta = 1890$$

$$\therefore \beta - \alpha = 1890 - 1173$$

$$= 717$$

Question 106

Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is
[2021, 20 July Shift-1]

Options:

A. $1/66$

B. $1/11$

C. $1/9$

D. $2/11$

Answer: B

Solution:

Solution:

$$\begin{array}{ccccccc} \text{E} & \text{X} & \text{A} & \text{M} & & \text{I} & \text{N} & \text{A} & \text{T} & \text{I} & \text{O} & \text{N} \\ \underbrace{}_3 & & & & & \underbrace{}_7 & & & & & & \\ & & & & & & & & & & & \end{array}$$

Let x = When M is at fourth place = $\frac{10!}{2!2!2!}$

Let y = Total number of words = $\frac{11!}{2!2!2!}$

Probability = $\frac{x}{y} = \frac{1}{11}$

Question 107

There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include atleast 4 bowlers, 5 batsman and 1 wicketkeeper, is
[2021, 20 July Shift-I]

Answer: 777

Solution:

Solution:

Total number of players = 15

Bowlers = 6 , Batsman = 7 , Wicket keepers = 2

Bowlers	Batsman	Wicket Keepers	Total
4 + 1	5	1	${}^6C_5 \times {}^7C_5 \times {}^2C_1 = 252$
4	5 + 1	1	${}^6C_4 \times {}^7C_6 \times {}^2C_1 = 210$
4	5	1 + 1	${}^6C_4 \times {}^7C_5 \times {}^2C_2 = 315$

Total = 252 + 210 + 315 = 777

Question108

If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10000 is equal to

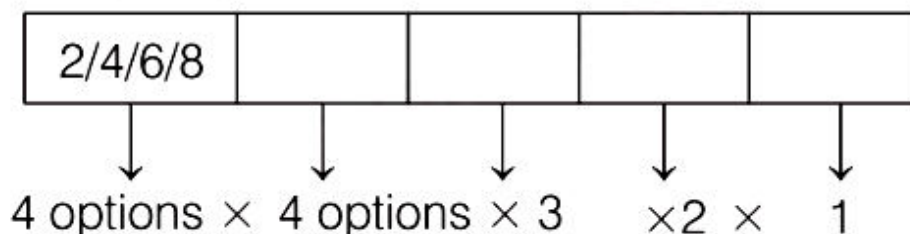
[2021, 22 July Shift-II]

Answer: 96

Solution:

Solution:

0, 2, 4, 6, 8



∴ Total = 4 × 4 × 3 × 2 × 1 = 96

Question109

If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to
[2021, 25 July Shift-II]

Options:

- A. 1
- B. 4
- C. 2
- D. 3

Answer: C

Solution:

Solution:

Given, ${}^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$$\Rightarrow \frac{n!}{(n-r)(n-r-1)!} = \frac{n!}{(n-r-1)!}$$

$$\Rightarrow n-r = 1 \quad \dots(i)$$

and

$$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{{}^nC_r = {}^nC_{r-1}}{(r-1)!(n-r+1)!}$$

$$\Rightarrow \frac{1}{r(n-r)!} = \frac{1}{(n-r+1)(n-r)!}$$

$$\Rightarrow n-r+1 = r$$

From Eq. (i),

$$1+1 = r \Rightarrow r = 2$$

Question110

There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is $100k$, then k is equal to

[2021, 25 July Shift-I]

Answer: 238

Solution:

Solution:

10(5)	11(6)	12(8)
2+1	2	2+3
2	2+1	2+3
2	2	2+4
	⌋	⌋
	5	5

$$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5 = 8400$$

$$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5 = 11200$$

$$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6 = 4200$$

$$\text{Total} = 8400 + 11200 + 4200 = 23800$$

According to the question, $100K = 23800$

$$K = 238$$

Question111

Let $n \in \mathbb{N}$ and $[x]$ denote the greatest integer less than or equal to x .

If the sum of $(n+1)$ terms ${}^nC_0, 3 \cdot {}^nC_1, 5 \cdot {}^nC_2, 7 \cdot {}^nC_3, \dots$ is equal to

$2^{100} \cdot 101$, then $2 \left\lfloor \frac{n-1}{2} \right\rfloor$ is equal to

[2021, 25 July Shift-II]

Answer: 98

Solution:

Solution:

We have,

$$1^n C_0 + 3^n C_1 + 5^n C_2 + \dots + (2n+1)^n C_n$$

$$T_r = (2r+1)^n C_r$$

$$\text{Now, } \text{sum}(S) = \sum T_r$$

$$S = \sum (2r+1)^n C_r$$

$$= 2 \sum r^n C_r + \sum^n C_r$$

$$= 2(n2^{n-1}) + 2^n = n \cdot 2^n + 2^n$$

$$\therefore S = 2^n(n+1)$$

$$\text{Given that, } S = 2^{100} \cdot 101$$

$$\Rightarrow 2^n(n+1) = 2^{100} \cdot 101$$

$$\Rightarrow n = 100$$

$$\text{Now, } 2 \left[\frac{n-1}{2} \right] = 2 \left[\frac{100-1}{2} \right] = 2 \left[\frac{99}{2} \right]$$

$$= 2[49.5] = 2 \times 49 = 98$$

($\because [x]$ is greatest integer function)

Question112

Let n be a non-negative integer. Then the number of divisors of the form " $4n+1$ " of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to
[2021, 27 July shift-II]

Answer: 924

Solution:

Solution:

$$\text{Let } N = (10)^{10} \cdot (11)^{11} \cdot (13)^{13}$$

$$N = 2^{10} \cdot 5^{10} \cdot 11^{11} \cdot 13^{13}$$

Now, power of 2 must be zero. Power of 5 can be anything. Power of 13 can be anything. But power of 11 should be even. So, required number of divisor is $= 1 \times 11 \times 14 \times 6 = 924$

Question113

The sum of all three-digit numbers less than or equal to 500, that are formed without using the digit 1 and they all are multiple of 11, is
[2021, 26 Aug. Shift-II]

Answer: 7744

Solution:

Solution:

Multiples of 11 such that they are of 3 -digit and less than 500.

121, 132, ..., 495

$$cn = \frac{495 - 121}{11} + 1 = 35$$

$$S = \frac{35}{2}(121 + 495) = 10780$$

Again, multiplies of 11 which are 3-digits, less than 500 and having 1 at hundred's place are

121, 132, ..., 198

$$n_1 = \left(\frac{198 - 121}{11} \right) + 1 = 8$$

$$S_1 = \frac{8}{2}(121 + 198) = 1276$$

The multiply of 11 which are of 3 -digits, less than 500 and having 1 at ten's place are 319, 418

$$\therefore S_2 = 319 + 418 = 737$$

The multiple of 11 which are 3-digits, less than 500 and having 1 at unit place are 231, 341, 451

$$\therefore S_3 = 231 + 341 + 451 = 1023$$

$$\begin{aligned} \therefore \text{Required sum} &= S - S_1 - S_2 - S_3 \\ &= 7744 \end{aligned}$$

Question114

**The number of three-digit even numbers, formed by the digits 0,1 , 3, 4, 6, 7, if the repetition of digits is not allowed, is
[2021, 26 Aug. Shift-I]**

Answer: 52

Solution:

Solution:

Case I When 0 is at unit place

$$\frac{5}{(5)} \times \frac{4}{(4)} \times \frac{0}{(1)} = 20$$

Case II When 4 or 6 are at unit place

$$\frac{1}{(4)} \times \frac{\frac{6}{4}}{(4)} \times \frac{6}{(2)} = 32$$

[0 cannot be come at hundredth place]

∴ Total number of required

$$= 20 + 32 = 52$$

Question115

A number is called a palindrome if it reads the same backward as well as forward For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55 , is
[2021, 27 Aug. Shift-I]

Answer: 100

Solution:

Solution:

Form of six digit palindrome number $xyzzyx$

This will be divisible by 55 Hence, $x = 5$ and $5yzzzy5$ will be divisible by 11 .

$\Rightarrow (5 + z + y) - (y + z + 5)$ is divisible by 11 which is true for all values of y and $z \Rightarrow y$ and z can be chosen in 10×10 ways Number of such number = 100

Question116

Let $S = \{1, 2, 3, 4, 5, 6, 9\}$. Then, the number of elements in the set $T = \{A \subset \text{eq}S : A \neq \varnothing \text{ and the sum of all the elements of } A \text{ is not a multiple of } 3 \}$ is
[2021, 27 Aug. Shift-II]

Answer: 80

Solution:

Solution:

$$S = \{1, 2, 3, 4, 5, 6, 9\}$$

3 n Type numbers 3, 6, 9

$3n - 1$ Type numbers 2, 5

$3n - 2$ Type numbers 1, 4

Let N_p = Number of Subset of S

containing p element which are not divisible by 3 .

For $P = 1$

$${}^2C_1 + {}^2C_1 = 4$$

For $P = 2$

$${}^3C_1 {}^2C_1 + {}^3C_1 {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$$

For $P = 3$

$${}^3C_1 ({}^2C_2 + {}^2C_2) + {}^3C_2 ({}^2C_1 + {}^2C_1) + {}^2C_2 {}^2C_1 \\ + {}^2C_1 {}^2C_2 = 22$$

For $P = 4$

$${}^3C_1 [{}^2C_2 {}^2C_1 + {}^2C_1 {}^2C_2] + {}^3C_2 ({}^2C_2 + {}^2C_2) \\ + {}^3C_3 ({}^2C_1 + {}^2C_1) = 22$$

For $P = 5$

$${}^3C_2 ({}^2C_2 {}^2C_1 + {}^2C_1 {}^2C_2) + {}^3C_3 ({}^2C_2 + {}^2C_2) = 14$$

For $P = 6$

$${}^3C_3 ({}^2C_2 {}^2C_1 + {}^2C_1 {}^2C_2) = 4$$

Total Subsets

$$= 4 + 14 + 22 + 22 + 14 + 4 = 80$$

Question117

**The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is
[2021, 31 Aug. Shift-1]**

Answer: 576

Solution:

Solution:

VOWELS (2 Vowel +4 consonant)

All consonants must not be together

Total possibility of formation of 6 letter word = 6 !

The number of arrangement when all the consonant comes together = $3! \times 4!$

Number of arrangement when all the consonants never come together
= Total - All consonant together = $6! - 3!4! = 576$

Question118

The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r$
= $(k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is:
[Jan. 7, 2020 (II)]

Options:

- A. 3
- B. 2
- C. 6
- D. 4

Answer: D

Solution:

Solution:

$$\frac{36}{r+1} \times {}^{35}C_r (k^2 - 3) = {}^{35}C_r \cdot 6$$

$$\Rightarrow k^2 - 3 = \frac{r+1}{6}$$

$$\Rightarrow k^2 = 3 + \frac{r+1}{6}$$

r can be 5, 35 for $k \in$

$r = 5, k = \pm 2$

$r = 35, k = \pm 3$

Hence, number of ordered pairs = 4

Question119

An urn contains 5 red marbles, 4 black marbles and 3 white marbles.
Then the number of ways in which 4 marbles can be drawn so that at
the most three of them are red is _____.
[NA Jan. 8, 2020 (I)]

Answer: 490

Solution:

Solution:

0 Red, 1 Red, 2 Red, 3 Red

Number of ways of selecting atmost three red balls

$$= {}^7C_4 + {}^5C_1 \cdot {}^7C_3 + {}^5C_2 \cdot {}^7C_2 + {}^5C_3 \cdot {}^7C_1$$

$$= 35 + 175 + 210 + 70 = 490$$

Question 120

If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then:

[Jan. 8, 2020 (I)]

Options:

A. $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$

B. $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

C. $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

D. $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$

Answer: C

Solution:

Solution:

We know nC_r is greatest at middle term.

$$\text{So, } a = ({}^{19}C_p)_{\max} = {}^{19}C_{10} = {}^{19}C_9$$

$$b = \binom{20}{q}_{\max} = {}^{20}C_{10}$$

$$c = \binom{21}{6}_{\max} = {}^{21}C_{10} = {}^{21}C_{11}$$

$$\text{Now, } \frac{a}{19}C_9 = \frac{b}{\frac{20}{10} \cdot {}^{19}C_9} = \frac{c}{\frac{21}{11} \cdot \frac{20}{10} {}^{19}C_9}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{2} = \frac{c}{42/11} \quad \therefore \quad \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

Question121

The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is

_____.
[NA Jan. 8, 2020 (II)]

Answer: 2454

Solution:

Solution:

EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I : If all are different, then

$${}^8P_4 = \frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$$

Case II : If two are same and two are different, then ${}^3C_1 \cdot {}^7C_2 \cdot \frac{4!}{2!} = 3 \cdot 21 \cdot 12 = 756$

Case III : If two are same and other two are same, then

$${}^3C_2 \cdot \frac{4!}{2!2!} = 3 \cdot 6 = 18$$

$$\therefore \text{Total cases} = 1680 + 756 + 18 = 2454$$

Question122

$\sum_r {}^{25}C_r$ and $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + \dots + (101) \cdot C_{25} = 2^{25} \cdot k$, then k is equal to

[NA Jan. 9, 2020 (II)]

Answer: 51

Solution:

Solution:

$$\begin{aligned}
\sum_{r=0}^{25} (4r+1)^{25} C_r &= 4 \sum_{r=0}^{25} r \cdot {}^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r \\
&= 4 \sum_{r=1}^{25} r \times \frac{25}{r} {}^{24}C_{r-1} + 2^{25} = 100 \sum_{r=1}^{25} {}^{24}C_{r-1} + 2^{25} \\
&= 100 \cdot 2^{24} + 2^{25} = 2^{25}(50+1) = 51 \cdot 2^{25}
\end{aligned}$$

Hence, by comparison $k = 51$

Question123

If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is $336k$, then k is equal to:

[Jan. 9, 2020 (I)]

Options:

A. 4

B. 6

C. 7

D. 8

Answer: D

Solution:

Solution:

Number of five digit numbers with 2 at 10^{th} place
 $= 8 \times 8 \times 7 \times 6 = 2688$

\therefore It is given that, number of five digit number with 2 at

10^{th} place $= 336k$

$\therefore 336k = 2688 \Rightarrow k = 8$

Question124

Total number of 6 -digit numbers in which only and all the five digits 1,3,5,7 and 9 appear, is:

[Jan. 7, 2020 (I)]

Options:

A. $\frac{1}{2}(6!)$

B. $6!$

C. 5^6

D. $\frac{5}{2}(6!)$

Answer: D

Solution:

Solution:

Five digits numbers be 1,3,5,7,9

For selection of one digit, we have 5C_1

choice. And six digits can be arrange in $\frac{6!}{2!}$ ways.

Hence, total such numbers = $\frac{5 \cdot 6!}{2!} = \frac{5}{2} \cdot 6!$

Question125

Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

[Sep. 06, 2020 (I)]

Options:

A. $2!3!4!$

B. $(3!)^3 \cdot (4!)$

C. $(3!)^2 \cdot (4!)$

D. $3!(4!)^3$

Answer: B

Solution:

Solution:

Number of arrangement

$= (3! \times 3! \times 4!) \times 3! = (3!)^3 4!$

Question126

The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots \text{ up to } 51^{\text{th}} \text{ term}) + (1! - 2! + 3! - \dots \text{ up to } 51^{\text{th}} \text{ term})$ is equal to :
[Sep. 03, 2020 (I)]

Options:

- A. $1 - 51(51)!$
- B. $1 + (51)!$
- C. $1 + (52)!$
- D. 1

Answer: C

Solution:

Solution:

We know, $(r+1) \cdot {}^rP_{r-1} = (r+1) \cdot \frac{r!}{1!} = (r+1)!$

So, $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + \dots \text{ 51 terms}) +$

$(1! - 2! + 3! - \dots \text{ upto 51 terms})$

$= [2! - 3! + 4! - \dots + 52!] + [1! - 2! + 3! - \dots + 51!]$

$= 52! + 1! = 52! + 1$

Question127

If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is _____.

[NA Sep. 02, 2020 (I)]

Answer: 309

Solution:

Solution:

M - 3

O - 4

T - 6

H - 2

E - 1

R - 5

$$\begin{aligned}\Rightarrow & 2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1 \\ & = 240 + 48 + 18 + 2 + 1 = 309\end{aligned}$$

Question128

The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____.

[NA Sep. 06, 2020 (II)]

Answer: 120

Solution:

Solution:

For vowels not together

Number of ways to arrange L, T, T, R = $\frac{4!}{2!}$

Then put both E in 5 gaps formed in 5C_2 ways.

$$\therefore \text{No. of ways} = \frac{4!}{2!} \cdot {}^5C_2 = 120$$

Question129

The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _____.

[NA Sep. 05, 2020 (I)]

Answer: 240

Solution:

Solution:

$S \rightarrow 2, L \rightarrow 2, A, B, Y, U$

$$\therefore \text{Required number of ways} = {}^2C_1 \times {}^5C_2 \times \frac{4!}{2!} = 240$$

Question130

There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:

[Sep. 05, 2020 (II)]

Options:

A. 3000

B. 1500

C. 2255

D. 2250

Answer: D

Solution:

Solution:

Since, each section has 5 questions.

\therefore Total number of selection of 5 questions

$$= 3 \times {}^5C_1 \times {}^5C_1 \times {}^5C_3 + 3 \times {}^5C_1 \times {}^5C_2 \times {}^5C_2$$

$$= 3 \times 5 \times 5 \times 10 + 3 \times 5 \times 10 \times 10$$

$$= 750 + 1500 = 2250$$

Question131

A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four

of the answers are correct, is
[NA Sep. 04, 2020 (II)]

Answer: 135

Solution:

Solution:

Select any 4 correct questions in 6C_4 ways.

Number of ways of answering wrong question = 3

\therefore Required number of ways = ${}^6C_4(1)^4 \times 3^2 = 135$

Question132

The total number of 3 -digit numbers, whose sum of digits is 10, is
[NA Sep. 03, 2020 (II)]

Answer: 54

Solution:

Solution:

Let xyz be the three digit number

$x + y + z = 10, x \leq 1, y \geq 0, z \geq 0$

$x - 1 = t \Rightarrow x = 1 + t \quad x - 1 \geq 0, t \geq 0$

$t + y + z = 10 - 1 = 9 \quad 0 \leq t, z, z \leq 9$

\therefore Total number of non-negative integral solution = ${}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11 \cdot 10}{2} = 55$

But for $t = 9, x = 10$, so required number of integers

$= 55 - 1 = 54$

Question133

Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected

by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is:
[Sep. 02, 2020 (II)]

Options:

A. 201

B. 200

C. 101

D. 199

Answer: A

Solution:

Solution:

Number of two consecutive stations (Blue lines) = n

Number of two non-consecutive stations (Red lines) = ${}^nC_2 - n$

Now, according to the question, ${}^nC_2 - n = 99n$

$$\Rightarrow \frac{n(n-1)}{2} - 100n = 0 \Rightarrow n(n-1-200) = 0$$

$$\Rightarrow n-1-200 = 0 \Rightarrow n = 201$$

Question134

Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

[Jan. 9, 2019 (I)]

Options:

A. 500

B. 200

C. 300

D. 350

Answer: C

Solution:

Solution:

Since, the number of ways to select 2 girls is 5C_2 .

Now, 3 boys can be selected in 3 ways.

(a) Selection of A and selection of any 2 other boys (except B) in 5C_2 ways

(b) Selection of B and selection of any 2 twoother boys (except A) in 5C_2 ways

(c) Selection of 3 boys (except A and B) in 5C_3 ways

Hence, required number of different teams

$$= {}^5C_2({}^5C_2 + {}^5C_2 + {}^5C_3) = 300$$

Question135

**The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to:
[Jan. 09, 2019 (II)]**

Options:

A. 374

B. 372

C. 375

D. 250

Answer: A

Solution:

Solution:

Number of numbers with one digit = 4 = 4

Number of numbers with two digits = $4 \times 5 = 20$

Number of numbers with three digits = $4 \times 5 \times 5$

= 100 Number of numbers with four digits = $2 \times 5 \times 5 \times 5$

= 250

\therefore Total number of numbers = $4 + 20 + 100 + 250$

= 374

Question136

Let S be the set of all triangles in the xy -plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is:

[Jan. 09, 2019 (II)]

Options:

- A. 9
- B. 18
- C. 36
- D. 32

Answer: C

Solution:

Solution:

One of the possible $\triangle OAB$ is $A(a, 0)$ and $B(0, b)$.

$$\text{Area of } \triangle OAB = \frac{1}{2} |ab|$$

$$\therefore |ab| = 100$$

$$|a| |b| = 100$$

But $100 = 1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20$ or 10×10

\therefore For 1×100 , $a = 1$ or -1 and $b = 100$ or -100

\therefore Total possible pairs are 8

Total possible pairs for $1 \times 100, 2 \times 50, 4 \times 25$ or 5×20 are 4×8

And for 10×10 total possible pairs are 4

\therefore Total number of possible triangles with integral coordinates are $4 \times 8 + 4 = 36$

Question137

$$\text{If } \sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right)^3 = \frac{k}{21}, \text{ then } k \text{ equals:}$$

[Jan. 10, 2019 (I)]

Options:

- A. 400

B. 50

C. 200

D. 100

Answer: D

Solution:

Solution:

Consider the expression,

$$\begin{aligned}\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} &= \frac{{}^{20}C_{i-1}}{{}^{21}C_1} \\&= \frac{20!}{(i-1)!(21-i)!} \times \frac{i!(21-i)!}{21!} = \frac{i}{21} \\ \therefore \sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right)^3 &= \sum_{i=1}^{20} \left(\frac{i}{21} \right)^3 = \frac{(1)}{(21)^3} \sum_{i=1}^{20} i^3 \\&= \frac{1}{(21)^3} \times \left(\frac{20 \times 21}{2} \right)^2 = \frac{100}{21} \\ \therefore \sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right)^3 &= \frac{k}{21} \\ \therefore k &= 100\end{aligned}$$

Question138

Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i , the label of the ball drawn from the i^{th} box, ($i = 1, 2, 3$). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

[Jan. 12, 2019 (I)]

Options:

A. 120

B. 82

C. 240

D. 164

Answer: A

Solution:

Solution:

Collecting different labels of balls drawn = $10 \times 9 \times 8$

\therefore arrangement is not required.

\therefore the number of ways in which the balls can be chosen is,

$$\frac{10 \times 9 \times 8}{3!} = 120$$

Question139

There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is

[Jan. 12, 2019 (II)]

Options:

A. 12

B. 11

C. 9

D. 7

Answer: A

Solution:

Solution:

$${}^mC_2 \times 2 = {}^mC_1 \cdot {}^2C_1 \times 2 + 84$$

$$m(m-1) = 4m + 84$$

$$m^2 - 5m - 84 = 0$$

$$m^2 - 12m - 7m - 84 = 0$$

$$m(m-12) + 7(m-12) = 0$$

$$m = 12, m = -7$$

$$\therefore m > 0$$

$$m = 12$$

Question140

The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is:

[April 08, 2019 (II)]

Options:

A. 288

B. 360

C. 306

D. 310

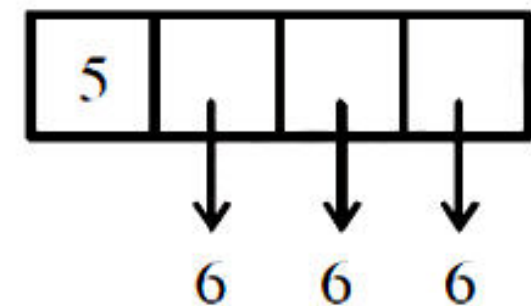
Answer: D

Solution:

Solution:

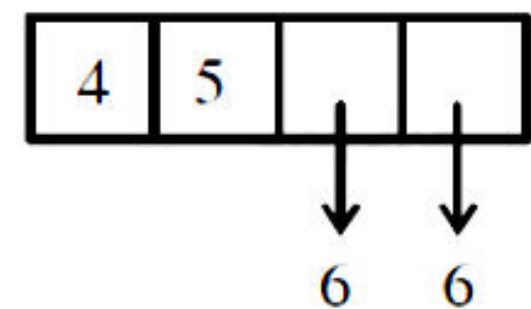
0, 1, 2, 3, 4, 5

Number of four-digit number starting with 5 is,



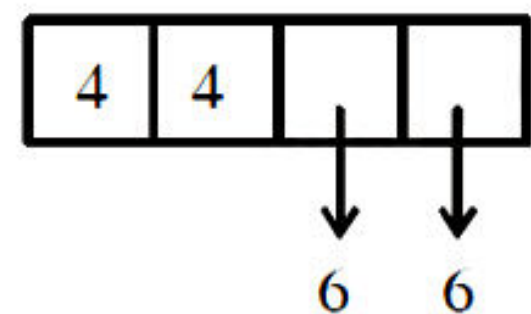
$$= 6 \times 6 \times 6 = 216$$

Number of four-digit numbers starting with 45 is,



$$= 6 \times 6 = 36$$

Number of four-digit numbers starting with 44 is,



$$= 6 \times 6 = 36$$

Number of four-digit numbers starting with 43 and greater than 4321 is,



$(5/4/3)$ 3 6

$$= 3 \times 6 = 18$$

Number of four-digit numbers starting with 432 and greater than 4321 is,



4

$$= 4$$

Hence, required numbers = $216 + 36 + 36 + 18 + 4 = 310$.

Question 141

A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then:

[April 9, 2019 (I)]

Options:

A. $m + n = 68$

B. $m = n = 78$

C. $n = m - 8$

D. $m = n = 68$

Answer: B

Solution:

Solution:

Since, m = number of ways the committee is formed with at least 6 males

$$= {}^8C_6 \cdot {}^5C_5 + {}^8C_7 \cdot {}^5C_4 + {}^8C_8 \cdot {}^5C_3 = 78$$

and n = number of ways the committee is formed with at least 3 females

$$= {}^5C_3 \cdot {}^8C_8 + {}^5C_4 \cdot {}^8C_7 + {}^5C_5 \cdot {}^8C_6 = 78$$

Hence, $m = n = 78$

Question142

All possible numbers are formed using the digits 1,1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is:

[April 8, 2019 (I)]

Options:

A. 180

B. 175

C. 160

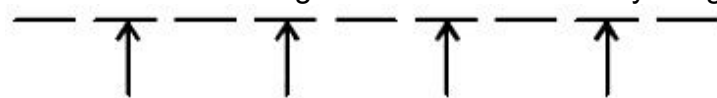
D. 162

Answer: A

Solution:

Solution:

∴ There are total 9 digits and out of which only 3 digits are odd.



∴ Number of ways to arrange odd digits first $= {}^4C_3 \cdot \frac{3!}{2!}$

Hence, total number of 9 digit numbers

$$= \left({}^4C_3 \cdot \frac{3!}{2!} \right) \cdot \frac{6!}{2!4!} = 180$$

Question143

The number of 6 digit numbers that can be formed using the digits 0,1,2,5,7 and 9 which are divisible by 11 and no digit is repeated, is:
[April 10, 2019 (I)]

Options:

- A. 72
- B. 60
- C. 48
- D. 36

Answer: B

Solution:

Solution:

Given digit 0, 1, 2, 5, 7, 9

a_1	a_2	a_3	a_4	a_5	a_6
-------	-------	-------	-------	-------	-------

$$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11K$$

Therefore, (1,2,9) (0,5,7)

Number of ways to arranging them

$$= 3! \times 3! + 3! \times 2 \times 2 = 6 \times 6 + 6 \times 4 = 6 \times 10 = 60$$

Question144

Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is:

[April 10, 2019 (II)]

Options:

- A. 170
- B. 180
- C. 210
- D. 190

Answer: A

Solution:

Solution:

$$\text{Total number of beams} = {}^{20}C_2 - 20 = 190 - 20 = 170$$

Question145

The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct is:
[April 12, 2019 (I)]

Options:

A. $2^{20} - 1$

B. 2^{21}

C. 2^{20}

D. $2^{20} + 1$

Answer: C

Solution:

Solution:

Number of ways of selecting 10 objects

= (10I, 0D) or (9I, 1D) or (8I, 1D) or ... (0I, 10D)

Here, D signifies distinct object and I indicates identical object

$$= 1 + {}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{10} = \frac{2^{21}}{2} = 2^{20}$$

Question146

A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :
[April 12, 2019 (II)]

Options:

A. 28

B. 27

C. 25

D. 24

Answer: C

Solution:

Solution:

Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons

$$= {}^{n+5}C_3 - {}^nC_3 - {}^5C_3 = 1750$$

$$\Rightarrow \frac{(n+5)!}{3!(n+2)!} - \frac{n!}{3!(n-3)!} - \frac{5!}{3!2!} = 1750$$

$$\Rightarrow \frac{(n+5)(n+4)(n+3)}{6} - \frac{n(n-1)(n-2)}{6} = 1760$$

$$\Rightarrow n^2 + 3n - 700 = 0 \Rightarrow n = 25 \text{ [} n = -28 \text{ rejected]}$$

Question147

n– digit numbers are formed using only three digits 2,5 and 7 . The smallest value of n for which 900 such distinct numbers can be formed, is

[Online April 15, 2018]

Options:

A. 6

B. 8

C. 9

D. 7

Answer: D

Solution:

Solution:

Required n digit numbers is 3^n as each place can be filled by 2,5,7

So smallest value of n such that $3^n > 900$. Therefore $n = 7$.

Question148

The number of four letter words that can be formed using the letters of the word BARRACK is
[Online April 15, 2018]

Options:

A. 144

B. 120

C. 264

D. 270

Answer: D

Solution:

Solution:

If all four letters are different then the number of words

$${}^5C_4 \times 4! = 120$$

If two letters are R and other two different letters are chosen from B, A, C, K then the number of words

$$= {}^4C_2 \times \frac{4!}{2!} = 72$$

If two letters are A and other two different letters are chosen from B, R, C, K then the number of words

$$= {}^4C_2 \times \frac{4!}{2!} = 72$$

If word is formed using two R 's and two A 's then the number of words $= \frac{4!}{2!2!} = 6$

Therefore, the number of four-letter words that can be formed $= 120 + 72 + 72 + 6 = 270$

Question149

The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4, (repetition of digits is not allowed) and are multiple of 3 is?
[Online April 16, 2018]

Options:

A. 30

B. 48

C. 24

D. 36

Answer: A

Solution:

Solution:

The thousands place can only be filled with 2,3 or 4, since the number is greater than 2000 .

For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3 .

It the sum of digits of the number is divisible by 3, then the number itself is divisible by 3

Case 1: If we take 2 at thousands place.

The remaining digits can be filled as:

0,1 and 3 as $2 + 1 + 0 + 3 = 6$ is divisible by 3 0,3 and 4 as $2 + 3 + 0 + 4 = 9$ is divisible by 3

In both the above combinations the remaining three digits can be arranged in $3!$ ways.

\therefore Total number of numbers in this case $= 2 \times 3! = 12$.

Case 2: If we take 3 at thousands place. The remaining digits can be filled as:

0,1 and 2 as $3 + 1 + 0 + 2 = 6$ is divisible by 3 .

0,2 and 4 as $3 + 2 + 0 + 4 = 9$ is divisible by 3 .

In both the above combinations, the remaining three digits can be arranged in $3!$ ways. Total number of numbers in this case $= 2 \times 3! = 12$

Case 3 : If we take 4 at thousands place.

The remaining digits can be filled as:

0,2 and 3 as $4 + 2 + 0 + 3 = 9$ is divisible by 3 .

In the above combination, the remaining three digits can be arranged in $3!$ ways.

\therefore Total number of numbers in this case $= 3! = 6$

\therefore Total number of numbers between 2000 and 5000 divisible by 3 are $12 + 12 + 6 = 30$

Question150

From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is:

[2018]

Options:

A. less than 500

B. at least 500 but less than 750

C. at least 750 but less than 1000

D. at least 1000

Answer: D

Solution:

Solution:

$$\begin{aligned}\therefore \text{Required number of ways} &= {}^6C_4 \times {}^3C_1 \times 4! \\ &= 15 \times 3 \times 24 = 1080\end{aligned}$$

Question151

If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is:

[Online April 8, 2017]

Options:

A. 44th

B. 45th

C. 46th

D. 47th

Answer: C

Solution:

Solution:

E, E, N, Q, U

(i) E = $4! = 24$

(ii) N..... = $\frac{4!}{2} = 12$

(iii) QE = $3! = 6$

(iv) QN..... = $\frac{3!}{2!} = 3$

(v) QUEEN = 1

\therefore Required rank

$$= (24) + (12) + (6) + (3) + (1) = 46\text{th}$$

Question152

The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is:
[Online April 9, 2017]

Options:

A. $5 \times 6!$

B. $6 \times 6!$

C. $7!$

D. $5 \times 7!$

Answer: A

Solution:

Solution:

4 boys and 2 girls in circle

$$\Rightarrow 5! \times \frac{6!}{4!2!} \times 2!$$

$$\Rightarrow 5 \times 6!$$

Question153

A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :
[2017]

Options:

A. 484

B. 485

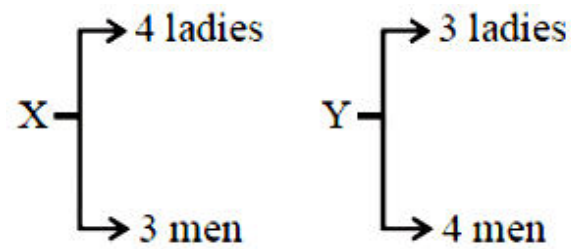
C. 468

D. 469

Answer: B

Solution:

Solution:



Possible cases for X are

- (1) 3 ladies, 0 man
- (2) 2 ladies, 1 man
- (3) 1 lady, 2 men
- (4) 0 ladies, 3 men

Possible cases for Y are

- (1) 0 ladies, 3 men
- (2) 1 lady, 2 men
- (3) 2 ladies, 1 man
- (4) 3 ladies, 0 man

$$\begin{aligned}\text{No. of ways} &= {}^4C_3 \cdot {}^4C_3 + ({}^4C_2 \cdot {}^3C_1)^2 + ({}^4C_1 \cdot {}^3C_2)^2 + ({}^3C_3)^2 \\ &= 16 + 324 + 144 + 1 = 485\end{aligned}$$

Question154

If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is:
[2016]

Options:

A. 52nd

B. 58th

C. 46th

D. 59th

Answer: B

Solution:

Solution:

ALLMS

No. of words starting with

$$A : A_ _ _ _ \frac{4!}{2!} = 12$$

$$L : L_ _ _ _ 4! = 24$$

$$M : M_ _ _ _ \frac{4!}{2!} = 12$$

$$S : SA_ _ _ \frac{3!}{2!} = 3$$

$$: SL_ _ _ 3! = 6$$

SMALL \rightarrow 58th word

Question155

If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E , then the total number of all such words is :

[Online April 9, 2016]

Options:

A. 110

B. 59

C. $\frac{11!}{(2!)^3}$

D. 56

Answer: B

Solution:

Solution:

M, E E E, D, I, T, RR, AA, NN

R--E

Two empty places can be filled with identical letters [EE, AA, NN] \Rightarrow 3 ways

Two empty places, can be filled with distinct letters [M, E, D, I, T, R, A, N] $\Rightarrow {}^8P_2$

$$\therefore \text{Number of words } 3 + {}^8P_2 = 59$$

Question156

The value of $\sum_{r=1}^{15} r^2 \left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}} \right)$ is equal to
[Online April 9, 2016]

Options:

A. 1240

B. 560

C. 1085

D. 680

Answer: D

Solution:

Solution:

$$\begin{aligned} & \sum_{r=1}^{15} r^2 \left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}} \right) \\ &= \frac{16-r}{r} \\ &= \sum_{r=1}^{15} r^2 \left(\frac{16-r}{r} \right) = \sum_{r=1}^{15} r(16-r) \\ &= 16 \sum_{r=1}^{15} r - \sum_{r=1}^{15} r^2 \\ &= \frac{16 \times 15 \times 16}{2} - \frac{15 \times 31 \times 16}{6} \\ &= 8 \times 15 \times 16 - 5 \times 8 \times 31 = 1920 - 1240 = 680 \end{aligned}$$

Question157

If $\frac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11$, then n satisfies the equation :
[Online April 10, 2016]

Options:

A. $n^2 + n - 110 = 0$

B. $n^2 + 2n - 80 = 0$

C. $n^2 + 3n - 108 = 0$

$$D. n^2 + 5n - 84 = 0$$

Answer: C

Solution:

Solution:

$$\frac{n+2}{n-2} = 11$$

$$\Rightarrow \frac{(n+2)(n+1)n(n-1)(n-2)(n-3)}{\frac{6.5.4.3.2.1}{(n-2)(n-3)}} = 11$$

$$\Rightarrow (n+2)(n+1)n(n-1) = 11.10.9.4$$

$$\Rightarrow n = 9$$

$$n^2 + 3n - 108 = (9)^2 + 3(9) - 108$$

$$= 81 + 27 - 108$$

$$= 108 - 108 = 0$$

Question 158

The sum $\sum_{r=1}^{10} (r^2 + 1) \times (r!)$ is equal to
[Online April 10, 2016]

Options:

A. $11 \times (11!)$

B. $10 \times (11!)$

C. $(11!)$

D. $101 \times (10!)$

Answer: B

Solution:

Solution:

$$\sum_{r=1}^{10} (r^2 + 1) \mid r$$

$$\mid r+r$$

$$T_1 = (r^2 + 1 + r - r) \mid r = (r^2 + r) \mid r - (r - r) \mid r$$

$$T_1 = r \mid r+r - (r-1) \mid r$$

$$T_1 = 1 \mid \underline{\quad} \mid 2 - 0$$

$$T_2 = 2 \mid \underline{\quad} \mid 3 - 1 \mid \underline{\quad} \mid 2$$

$$T_3 = 3 \mid \underline{\quad} \mid 4 - 2 \mid \underline{\quad} \mid 3$$

$$T_{10} = 10 \mid \underline{\quad} \mid 11 - 9 \mid \underline{\quad} \mid 10$$

$$\sum_{r=1}^{10} (r^2 + 1) \mid \underline{\quad} \mid r = 10 \mid \underline{\quad} \mid 11$$

Question159

The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0,0),(0,41) and (41,0) is :
[2015]

Options:

A. 820

B. 780

C. 901

D. 861

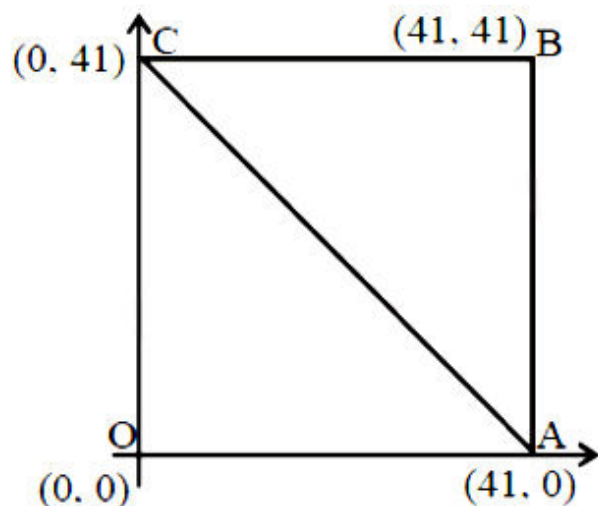
Answer: B

Solution:

Solution:

Total number of integral points inside the square OABC = $40 \times 40 = 1600$

No. of integral points on AC



= No. of integral points on OB

= 40 [namely (1, 1), (2, 2) ... (40, 40)]

\therefore No. of integral points inside the ΔOAC
 $= \frac{1600 - 40}{2} = 780$

Question160

**The number of integers greater than 6,000 that can be formed, using the digits 3,5,6,7 and 8 , without repetition, is:
[2015]**

Options:

- A. 120
- B. 72
- C. 216
- D. 192

Answer: D

Solution:

Solution:

Four digits number can be arranged in $3 \times 4!$ ways.

Five digits number can be arranged in $5!$ ways.

Number of integers $= 3 \times 4! + 5! = 192$

Question161

**The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is:
[Online April 10, 2015]**

Options:

- A. 1120
- B. 1880
- C. 1960
- D. 1240

Answer: D

Solution:

Solution:

Number of ways of selecting a man and a woman for a team from 15 men and 15 women

$$= 15 \times 15 = (15)^2$$

Number of ways of selecting a man and a woman for next team out of the remaining 14 men and 14 women.

$$= 14 \times 14 = (14)^2$$

Similarly for other teams

Hence required number of ways

$$= (15)^2 + (14)^2 + \dots + (1)^2 = \frac{15 \times 16 \times 31}{6} = 1240$$

Question162

Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$ each having at least three elements is :

[2015]

Options:

A. 275

B. 510

C. 219

D. 256

Answer: C

Solution:

Solution:

Given

$$n(A) = 4, n(B) = 2, n(A \times B) = 8$$

Required number of subsets

$$= {}^8C_3 + {}^8C_4 + \dots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$$

$$= 256 - 1 - 8 - 28 = 219$$

Question163

If in a regular polygon the number of diagonals is 54, then the number of sides of this polygon is
[Online April 11, 2015]

Options:

A. 12

B. 6

C. 10

D. 9

Answer: A

Solution:

Solution:

Number of diagonal = 54

$$\Rightarrow \frac{n(n-3)}{2} = 54$$

$$\Rightarrow n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$$

$$\Rightarrow n(n-12) + 9(n-12) = 0$$

$$\Rightarrow n = 12, -9 \Rightarrow n = 12 (\because n \neq -9)$$

Question164

The sum of the digits in the unit's place of all the 4 -digit numbers formed by using the numbers 3,4,5 and 6, without repetition, is:
[Online April 9, 2014]

Options:

A. 432

B. 108

C. 36

D. 18

Answer: B

Solution:

Solution:

With 3 at unit place,

total possible four digit number (without repetition) will be $3! = 6$

With 4 at unit place,

total possible four digit numbers will be $3! = 6$

With 5 at unit place, total possible four digit numbers will be $3! = 6$ With 6 at unit place, total possible four digit numbers will be $3! = 6$

Sum of unit digits of all possible numbers

$$= 6 \times 3 + 6 \times 4 + 6 \times 5 + 6 \times 6$$

$$= 6[3 + 4 + 5 + 6]$$

$$= 6[18] = 108$$

Question165

An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is:

[Online April 11, 2014]

Options:

A. 72(7!)

B. 18(7!)

C. 40(7!)

D. 36(7!)

Answer: D

Solution:

Solution:

We know that any number is divisible by 9 if sum of the digits of the number is divisible by 9.

Now sum of the digits from 0 to 9

$$= 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

Hence to form 8 digits numbers which are divisible by 9, a pair of digits either 0 and 9, 1 and 8, 2 and 7, 3 and 6 or 4 and 5 are not used.

Digits which are not used to form 8 digits number divisible by 9	Number of 8 digits numbers which are divisible by 9
0 and 9	$8 \times 7!$
1 and 8	$7 \times 7!$
2 and 7	$7 \times 7!$
3 and 6	$7 \times 7!$
4 and 5	$7 \times 7!$

Hence total number of 8 digits numbers which are divisible by 9

$$= 8 \times (7!) + 7 \times (7!) + 7 \times (7!) + 7 \times (7!) + 7 \times (7!)$$

$$= 36 \times (7!)$$

Question 166

8-digit numbers are formed using the digits 1,1,2,2,2,3,4 4. The number of such numbers in which the odd digits do not occupy odd places, is:

[Online April 12, 2014]

Options:

A. 160

B. 120

C. 60

D. 48

Answer: B

Solution:

Solution:

In 8 digits numbers, 4 places are odd places.

Also, in the given 8 digits, there are three odd digits 1, 1 and 3

$$\text{No. of ways three odd digits arranged at four even places} = \frac{{}^4P_3}{2!} = \frac{4!}{2!}$$

$$\text{No. of ways the remaining five digits 2,2,2,4 and 4 arranged at remaining five places} = \frac{5!}{3!2!}$$

Hence, required number of 8 digits number

$$= \frac{4!}{2!} \times \frac{5!}{3!2!} = 120$$

Question167

Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval:

[Online April 19, 2014]

Options:

- A. [8,9]
- B. [10,12)
- C. (11,13]
- D. (14,17)

Answer: B

Solution:

Solution:

Let no. of men = n

No. of women = 2

Total participants = $n + 2$

No. of games that M_1 plays with all other men = $2(n - 1)$

These games are played by all men

M_2, M_3, \dots, M_n

So, total no. of games among men = $n \cdot 2(n - 1)$.

However, we must divide it by '2', since each game is counted twice (for both players).

So, total no. of games among all men

= $n(n - 1)$ (i)

Now, no. of games M_1 plays with W_1 and $W_2 = 4$

(2 games with each)

Total no. of games that M_1, M_2, \dots, M_n play with W_1 and $W_2 = 4n$ (ii)

..... (ii)

Given : $n(n - 1) - 4n = 66 \Rightarrow n = 11, -6$

As the number of men can't be negative.

So, $n = 11$

Question168

A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is :
[Online April 9, 2013]

Options:

- A. 40
- B. 41
- C. 16
- D. 32

Answer: B

Solution:

Solution:

L	O	Y	\Rightarrow	L	O	Y
2	2	4		1	1	2
≥ 1	≥ 1	$2 \leq$		1	2	1
				2	1	1
				2	2	0

Required number of ways

$$\begin{aligned}
 &= {}^2C_1 \times {}^2C_1 \times {}^2C_2 + {}^2C_1 \times {}^2C_2 \times {}^4C_1 + {}^2C_2 \times {}^2C_1 \times {}^4C_1 + {}^2C_2 \times {}^2C_2 \times {}^4C_0 \\
 &= 2 \times 2 \times \frac{4 \times 3}{2} + 2 \times 1 \times 4 + 1 \times 2 \times 4 + 1 \times 1 \times 1 \\
 &= 24 + 8 + 8 + 1 = 41
 \end{aligned}$$

Question169

The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is:
[Online April 22, 2013]

Options:

- A. ${}^{30}C_7$

B. ${}^{21}C_8$

C. ${}^{21}C_7$

D. ${}^{30}C_8$

Answer: C

Solution:

Solution:

30 marks to be allotted to 8 questions. Each question has to be given ≥ 2 marks

Let questions be a, b, c, d, e, f, g, h

and $a + b + c + d + e + f + g + h = 30$

Let $a = a_1 + 2$ so, $a_1 \geq 0$

$b = a_2 + 2$ so, $a_2 \geq 0, \dots, a_8 \geq 0$

So, $a_1 + a_2 + \dots + a_8 + 2 + 2 + \dots + 2 = 30$

$\Rightarrow a_1 + a_2 + \dots + a_8 = 30 - 16 = 14$

So, this is a problem of distributing 14 articles in 8 groups.

Number of ways $= {}^{14+8-1}C_{8-1} = {}^{21}C_7$

Question 170

**On the sides AB, BC, CA of a $\triangle ABC$, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are :
[Online April 23, 2013]**

Options:

A. 210

B. 205

C. 215

D. 220

Answer: B

Solution:

Solution:

Required number of triangles
 $= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 205$

Question171

5 - digit numbers are to be formed using 2,3,5,7,9 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, then p : q is :

[Online April 25, 2013]

Options:

A. 6: 5

B. 3: 2

C. 4: 3

D. 5: 3

Answer: D

Solution:

Solution:

p: 0 0 0 0 0 place
5 4 3 2 1 ways

Total no. of ways = $5! = 120$

Since all numbers are $>20,000$

\therefore all numbers 2,3,5,7,9 can come at first place.

q: 0 0 0 0 0 place
3 4 3 2 1 ways

Total no. of ways = $3 \times 4! = 72$

(\because 2 and 9 can not be put at first place)

So, $p : q = 120 : 72 = 5 : 3$

Question172

Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

[2013]

Options:

A. 256

B. 220

C. 219

D. 211

Answer: C

Solution:

Solution:

Given

$$n(A) = 2, n(B) = 4, n(A \times B) = 8$$

Required number of subsets

$$\begin{aligned} &= {}^8C_3 + {}^8C_4 + \dots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 \\ &= 256 - 1 - 8 - 28 = 219 \end{aligned}$$

Question173

Let T_n be the number of all possible triangles formed by joining vertices of an n -sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is :
[2013]

Options:

A. 7

B. 5

C. 10

D. 8

Answer: B

Solution:

Solution:

We know,

$$T_n = {}^nC_3, T_{n+1} = {}^{n+1}C_3$$

$$\text{ATQ, } T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$$

$$\Rightarrow {}^nC_2 = 10$$

$$\Rightarrow n = 5$$

Question174

If the number of 5 -element subsets of the set $A = \{a_1, a_2, \dots, a_{20}\}$ of 20 distinct elements is k times the number of 5 -element subsets containing a_4 , then k is

[Online May 7, 2012]

Options:

A. 5

B. $\frac{20}{7}$

C. 4

D. $\frac{10}{3}$

Answer: C

Solution:

Solution:

Set $A = \{a_1, a_2, \dots, a_{20}\}$ has 20 distinct elements.

We have to select 5 -element subset.

\therefore Number of 5 -element subsets $= {}^{20}C_5$

According to question

$${}^{20}C_5 = ({}^{19}C_4) \cdot k$$

$$\Rightarrow \frac{20!}{5!15!} = k \cdot \left(\frac{19!}{4!15!} \right)$$

$$\Rightarrow \frac{20}{5} = k \Rightarrow k = 4$$

Question175

Statement 1: If A and B be two sets having p and q elements respectively, where $q > p$. Then the total number of functions from set

A to set B is q^p

Statement 2 : The total number of selections of p different objects out of q objects is qC_p

[Online May 12, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

C. Statement 1 is false, Statement 2 is true

D. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.

Answer: D

Solution:

Solution:

Statement -1: $n(A) = p$, $n(B) = q$, $q > p$

Total number of functions from $A \rightarrow B = q^p$

It is a true statement.

Statement -2: The total number of selections of p different objects out of q objects is qC_p

It is also a true statement and it is a correct explanation for statement - 1 also.

Question176

The number of arrangements that can be formed from the letters a, b, c, d, e, f taken 3 at a time without repetition and each arrangement containing at least one vowel, is

[Online May 19, 2012]

Options:

A. 96

B. 128

C. 24

D. 72

Answer: A

Solution:

Solution:

There are 2 vowels and 4 consonants in the letters a, b, c, d, e, f

If we select one vowel, then number of arrangements

$$= {}^2C_1 \times {}^4C_2 \times 3! = 2 \times \frac{4 \times 3}{2} \times 3 \times 2 = 72$$

If we select two vowels, then number of arrangements

$$= {}^2C_2 \times {}^4C_1 \times 3! = 1 \times 4 \times 6 = 24$$

Hence, total number of arrangements

$$= 72 + 24 = 96$$

Question177

If $n = {}^mC_2$, then the value of nC_2 is given by
[Online May 19, 2012]

Options:

A. $3({}^{m+1}C_4)$

B. ${}^{m-1}C_4$

C. ${}^{m+1}C_4$

D. $2({}^{m+2}C_4)$

Answer: A

Solution:

Solution:

$$n = {}^mC_2 = \frac{m(m-1)}{2}$$

Since m and $(m-1)$ are two consecutive natural numbers, therefore their product is an even natural number. So

$\frac{m(m-1)}{2}$ is also a natural number.

$$\text{Now } \frac{m(m-1)}{2} = \frac{m^2 - m}{2}$$

$$\therefore \frac{m(m-1)}{2}C_2 = \frac{\left(\frac{m^2 - m}{2}\right)\left(\frac{m^2 - m}{2} - 1\right)}{2}$$

$$\begin{aligned}
&= \frac{m(m-1)(m^2-m-2)}{8} \\
&= \frac{m(m-1)[m^2-2m+m-2]}{8} \\
&= \frac{m(m-1)[m(m-2)+1(m-2)]}{8} \\
&= \frac{m(m-1)(m-2)(m+1)}{8} \\
&= \frac{3 \times (m+1)m(m-1)(m-2)}{4 \times 3 \times 2 \times 1} = 3({}^{m+1}C_4)
\end{aligned}$$

Question178

If seven women and seven men are to be seated around a circular table such that there is a man on either side of every woman, then the number of seating arrangements is
[Online May 26, 2012]

Options:

A. $6!7!$

B. $(6!)^2$

C. $(7!)^2$

D. $7!$

Answer: A

Solution:

Solution:

7 women can be arranged around a circular table in $6!$ ways.

Among these 7 men can sit in $7!$ ways.

Hence, number of seating arrangement = $7! \times 6!$

Question179

Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is:
[2012]

Options:

A. 880

B. 629

C. 630

D. 879

Answer: D

Solution:

Solution:

Given that number of white balls = 10

Number of green balls = 9

and Number of black balls = 7

∴ Required probability

$$= (10 + 1)(9 + 1)(7 + 1) - 1$$

$$= 11 \cdot 10 \cdot 8 - 1 = 879$$

[∴ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind is

$$(p + 1)(q + 1)(r + 1) - 1]$$

Question180

**There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. Then :
[2011RS]**

Options:

A. $N \leq 100$

B. $100 < N \leq 140$

C. $140 < N \leq 190$

D. $N > 190$

Answer: A

Solution:

Solution:

Number of required triangles

$$\begin{aligned} &= {}^{10}C_3 - {}^6C_3 \\ &= \frac{10 \times 9 \times 8}{6} - \frac{6 \times 5 \times 4}{6} = 120 - 20 = 100 \end{aligned}$$

Question181

Statement-1: The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is 9C_3

Statement-2: The number of ways of choosing any 3 places from 9 different places is 9C_3

[2011]

Options:

- A. Statement- 1 is true, Statement- 2 is true; Statement- 2 is not a correct explanation for Statement- 1.
- B. Statement- 1 is true, Statement- 2 is false.
- C. Statement- 1 is false, Statement- 2 is true.
- D. Statement- 1 is true, Statement- 2 is true; Statement- 2 is a correct explanation for Statement- 1.

Answer: A

Solution:

Solution:

The number of ways of distributing 10 identical balls in 4 distinct boxes

$$= {}^{10-1}C_{4-1} = {}^9C_3$$

Question182

There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

[2010]

Options:

- A. 36
- B. 66
- C. 108
- D. 3

Answer: C

Solution:

Solution:

Two balls are taken from each urn Total number of ways

$$\begin{aligned} &= {}^3C_2 \times {}^9C_2 \\ &= 3 \times \frac{9 \times 8}{2} = 3 \times 36 = 108 \end{aligned}$$

Question183

From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is:

[2009]

Options:

- A. at least 500 but less than 750
- B. at least 750 but less than 1000
- C. at least 1000
- D. less than 500

Answer: C

Solution:

Solution:

4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in ${}^6C_4 \times {}^3C_1$ ways

Then 4 novels with one dictionary in the middle can be arranged in $4!$ ways.

∴ Total ways of arrangement
 $= {}^6C_4 \times {}^3C_1 \times 4! = 1080$

Question184

**How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
 [2008]**

Options:

A. $8 \cdot {}^6C_4 \cdot {}^7C_4$

B. $6 \cdot 7 \cdot {}^8C_4$

C. $6 \cdot 8 \cdot {}^7C_4$

D. $7 \cdot {}^6C_4 \cdot {}^8C_4$

Answer: D

Solution:

Solution:

First let us arrange M, I, I, I, I, P, P

Which can be done in $\frac{7!}{4!2!}$ ways

*M * I * I * I * I * P * P*

Now 4S can be kept at any of the * places in 8C_4 ways so that no two S are adjacent.

Total required ways

$$= \frac{7!}{4!2!} {}^8C_4 = \frac{7!}{4!2!} {}^8C_4 = 7 \times {}^6C_4 \times {}^8C_4$$

Question185

The set $S = \{1, 2, 3, \dots, 12\}$ is to be partitioned into three sets A, B, C of equal size.

**Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \emptyset$. The number of ways to partition S is
 [2007]**

Options:

A. $\frac{12!}{(4!)^3}$

B. $\frac{12!}{(4!)^4}$

C. $\frac{12!}{3!(4!)^3}$

D. $\frac{12!}{3!(4!)^4}$

Answer: A

Solution:

Solution:

Set $S = \{1, 2, 3, \dots, 12\}$

$A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \varnothing$

\therefore Each sets contain 4 elements.

\therefore The number of ways to partition

$$= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4$$

$$= \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!} = \frac{12!}{(4!)^3}$$

Question186

At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is
[2006]

Options:

A. 5040

B. 6210

C. 385

D. 1110

Answer: C

Solution:

Solution:

The number of ways can vote

$$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 \\ = 10 + 45 + 120 + 210 = 385$$

Question187

If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number [2005]

Options:

A. 601

B. 600

C. 603

D. 602

Answer: A

Solution:

Solution:

Alphabetical order is

A, C, H, I, N, S

No. of words starting with A = $5! = 120$

No. of words starting with C = $5! = 120$

No. of words starting with H = $5! = 120$

No. of words starting with I = $5! = 120$

No. of words starting with N = $5! = 120$

SACHIN -1

∴ Sachin appears at serial no. 601

Question188

The value of ${}^{50}C_4 + \sum_{r=1}^6 {}^{56-r}C_3$ is [2005]

Options:

A. ${}^{55}C_4$

B. ${}^{55}C_3$

C. ${}^{56}C_3$

D. ${}^{56}C_4$

Answer: D

Solution:

Solution:

$$\begin{aligned} & {}^{50}C_4 + \sum_{r=1}^6 {}^{56-r}C_3 \\ &= {}^{50}C_4 + \left[\begin{array}{l} {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 \\ \quad + {}^{51}C_3 + {}^{50}C_3 \end{array} \right] \end{aligned}$$

We know

$$\begin{aligned} & [{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r] \\ &= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 \\ &= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 \end{aligned}$$

Proceeding in the same way, we get

$$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$$

Question189

How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order [2004]

Options:

A. 480

B. 240

C. 360

D. 120

Answer: C

Solution:

Solution:

Total number of arrangements of letters in the word GARDEN = $6! = 720$ there are two vowels A and E, in half of the arrangements A precedes E and other half A follows E. So, numbers of word with vowels in alphabetical order in

$$\frac{1}{2} \times 720 = 360$$

Question190

The range of the function $f(x) = {}^{7-x}P_{x-3}$ is
[2004]

Options:

- A. $\{1, 2, 3, 4, 5\}$
- B. $\{1, 2, 3, 4, 5, 6\}$
- C. $\{1, 2, 3, 4, \}$
- D. $\{1, 2, 3, \}$

Answer: D

Solution:

Solution:

${}^{7-x}P_{x-3}$ is defined if

$$7-x \geq 0, x-3 \geq 0 \text{ and } 7-x \geq x-3$$

$$\Rightarrow 3 \leq x \leq 5 \text{ and } x \in I$$

$$\therefore x = 3, 4, 5$$

$$\therefore f(3) = {}^{7-3}P_{3-3} = {}^4P_0 = 1$$

$$\therefore f(4) = {}^{7-4}P_{4-3} = {}^3P_1 = 3$$

$$\therefore f(5) = {}^{7-5}P_{5-3} = {}^2P_2 = 2$$

$$\text{Hence range} = \{1, 2, 3\}$$

Question191

The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is [2004]

Options:

A. 8C_3

B. 21

C. 3^8

D. 5

Answer: B

Solution:

Solution:

We know that the number of ways of distributing n identical items among r persons, when each one of them receives at least one item is ${}^{n-1}C_{r-1}$

\therefore The required number of ways

$$= {}^{8-1}C_{3-1} = {}^7C_2 = \frac{7!}{2!5!} = \frac{7 \times 6}{2 \times 1} = 21$$

Question192

The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by [2003]

Options:

A. $6! \times 5!$

B. 6×5

C. 30

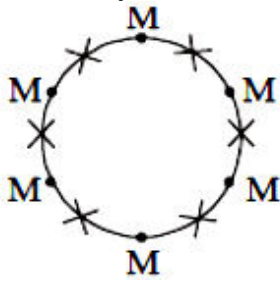
D. 5×4

Answer: A

Solution:

Solution:

No. of ways in which 6 men can be arranged at a round table $= (6 - 1)! = 5!$



Now women can be arranged in 6P_5

$= 6!$ ways

Total Number of ways $= 6! \times 5!$

Question193

A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is [2003]

Options:

A. 346

B. 140

C. 196

D. 280

Answer: C

Solution:

Solution:

According to given question two cases are possible.

(i) Selecting 4 out of first five question and 6 out of remaining question

$$= {}^5C_4 \times {}^8C_6 = 140 \text{ ways}$$

(ii) Selecting 5 out of first five question and 5 out of remaining

$$8 \text{ questions} = {}^5C_5 \times {}^8C_5 = 56 \text{ ways}$$

Therefore, total number of choices

$$= 140 + 56 = 196$$

Question194

If nC_r denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals
[2003]

Options:

A. ${}^{n+1}C_{r+1}$

B. ${}^{n+2}C_r$

C. ${}^{n+2}C_{r+1}$

D. ${}^{n+1}C_r$

Answer: C

Solution:

Solution:

$$\begin{aligned} & {}^nC_{r+1} + {}^nC_{r-1} + 2{}^nC_r \\ &= {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1} \\ & [\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r] \\ &= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1} \end{aligned}$$

Question195

The sum of integers from 1 to 100 that are divisible by 2 or 5 is
[2002]

Options:

A. 3000

B. 3050

C. 3600

D. 3250

Answer: B

Solution:

Solution:

Required sum

$$\begin{aligned} &= (2 + 4 + 6 + \dots + 100) + (5 + 10 + 15 + \dots + 100) \\ &\quad - (10 + 20 + \dots + 100) \\ &= 2(1 + 2 + 3 \dots + 50) + 5(1 + 2 + 3 + \dots + 50) \\ &= 2550 + 1050 - 530 = 3050. \end{aligned}$$

Question196

Number greater than 1000 but less than 4000 is formed using the digits 0,1,2,3,4 (repetition allowed). Their number is [2002]

Options:

A. 125

B. 105

C. 374

D. 625

Answer: C

Solution:

Solution:

Total number of numbers

$$= 3 \times 5 \times 5 \times 5 - 1 = 374$$

Question197

Total number of four digit odd numbers that can be formed using 0,1,2,3,5,7 (using repetition allowed) are [2002]

Options:

A. 216

B. 375

C. 400

D. 720

Answer: D

Solution:

Solution:

Total number of numbers formed using 0,1,2,3,5,7
 $= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720$

Question198

Five digit number divisible by 3 is formed using 0,1,2,3,4 6 and 7 without repetition. Total number of such numbers are [2002]

Options:

A. 312

B. 3125

C. 120

D. 216

Answer: D

Solution:

Solution:

We know that a number is divisible by 3 only when the sum of the digits is divisible by 3 . The given digits are 0,1,2,3,4,5

Here the possible number of combinations of 5 digits out of 6 are ${}^5C_4 = 5$, which are as follows -

$1 + 2 + 3 + 4 + 5 = 15 = 3 \times 5$ (divisible by 3)

$0 + 2 + 3 + 4 + 5 = 14$ (not divisible by 3)

$0 + 1 + 3 + 4 + 5 = 13$ (not divisible by 3)

$0 + 1 + 2 + 4 + 5 = 12 = 3 \times 4$ (divisible by 3)

$0 + 1 + 2 + 3 + 5 = 11$ (not divisible by 3)

$0 + 1 + 2 + 3 + 4 = 10$ (not divisible by 3)

Thus the number should contain the digits 1,2,3,4,5 or the digits 0,1,2,4,5

Taking 1, 2, 3, 4, 5, the 5 digit numbers are $= 5! = 120$

Taking 0, 1, 2, 4, 5, the 5 digit numbers are $= 5! - 4! = 96$

\therefore Total number of numbers $= 120 + 96 = 216$
