Sequences and Series

Question1

The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88 , then $q - 2p$ is equal to ________ .

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Answer: 474

Solution:

$S_{11} = \frac{11}{2}(2a + 10d) = 88$
$a + 5d = 8$
$a = 8 - 5 \times \frac{3}{2} = \frac{1}{2}$
Roots are
$\mathrm{T}_{10} = \mathrm{a} + 9\mathrm{~d} = \frac{1}{2} + 9 \times \frac{3}{2} = 14$
$\mathrm{T}_{11} = \mathrm{a} + 10\mathrm{~d} = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$
$\frac{\mathrm{p}}{3} = \mathrm{T}_{10} + \mathrm{T}_{11} = 14 + \frac{31}{2} = \frac{59}{2}$
$\mathrm{p} = \frac{177}{2}$
$\frac{\mathrm{q}}{3} = \mathrm{T}_{10} \times \mathrm{T}_{11} = 7 \times 31 = 217$
$\mathrm{q} = 651$
$\mathrm{q} - 2\mathrm{p}$
$= 651 - 177$
$= 474$

Question2

The interior angles of a polygon with *n* sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then *n* is equal to _______.

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Answer: 20

Solution:

$$\frac{n}{2}(2a + (n-1)6) = (n-2) \cdot 180^\circ$$
$$an + 3n^2 - 3n = (n-2) \cdot 180^\circ \quad .... (1)$$

Now according to question

$$a + (n-1)6^\circ = 219^\circ$$
$$\Rightarrow a = 225^\circ - 6n^\circ \quad .... (2)$$

Putting value of a from equation (2) in (1)

We get

$$\left(225n - 6n^2\right) + 3n^2 - 3n = 180n - 360$$
$$\Rightarrow 2n^2 - 42n - 360 = 0$$
$$\Rightarrow n2 - 14n - 120 = 0$$
$$n = 20, -6 \text{ (rejected)}$$

Question3

Let $a_1, a_2, \ldots, a_{2024}$ be an Arithmetic Progression such that $a_1 + \left(a_5 + a_{10} + a_{15} + \ldots + a_{2020}\right) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + \ldots + a_{2024}$ is equal to __________.

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Answer: 11132

Solution:

$$a_1 + a_5 + a_{10} + \ldots\ldots + a_{2020} + a_{2024} = 2233$$

In an A.P. the sum of terms equidistant from ends is equal.

$$a_1 + a_{2024} = a_5 + a_{2020} = a_{10} + a_{2015} \ldots\ldots$$
$$\Rightarrow 203 \text{ pairs}$$
$$\Rightarrow 203\left(a_1 + a_{2024}\right) = 2233$$

Hence,

$$S_{2024} = \frac{2024}{2}(a_1 + a_{2024})$$
$$= 1012 \times 11$$
$$= 11132$$

Question4

If the sum of the first 10 terms of the series $\frac{4\cdot1}{1+4\cdot1^4}+\frac{4\cdot2}{1+4\cdot2^4}+\frac{4\cdot3}{1+4\cdot3^4}+\ldots\ldots$ is $\frac{\mathrm{m}}{\mathrm{n}}$, where $\gcd(\mathrm{m},\mathrm{n})=1$, then $\mathrm{m}+\mathrm{n}$ is equal to ________________

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Answer: 441

Solution:

$$\frac{4.1}{1+4.1^4}+\frac{4.2}{1+4.2^4}+\frac{4.3}{1+4.3^4}+\ldots.$$
$$T_r=\frac{4r}{1+4r^4}=\frac{4r}{4r^4+4r^2+1-4r^2}$$
$$=\frac{4r}{\left(2r^2+1\right)^2-(2r)^2}$$
$$T_r=\frac{4r}{\left(2r^2-2r+1\right)\left(2r^2+2r+1\right)}$$
$$T_r=\frac{\left(2r^2+2r+1\right)-\left(2r^2-2r+1\right)}{\left(2r^2-2r+1\right)\left(2r^2+2r+1\right)}$$
$$T_r=\left(\frac{1}{r^2+(r-1)^2}-\frac{1}{r^2+(r+1)^2}\right)$$
$$\sum_{r=1}^{10}T_r=\left(\frac{1}{0^2+1^2}-\frac{1}{1^2+2^2}+\frac{1}{1^2+2^2}-\frac{1}{2^2+3^2}+\ldots.\right.$$
$$\left.\frac{1}{9^2+10^2}-\frac{1}{10^2+11^2}\right)$$
$$=1-\frac{1}{221}$$
$$=\frac{220}{221}$$
$$\therefore\quad m+n=220+221$$
$$=441$$

Question5

Let $a_1,a_2,a_3,\ldots$ be a G.P. of increasing positive terms. If $a_1a_5=28$ and $a_2+a_4=29$, then a_6 is equal to:

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Options:

A. 812

B. 784

C. 628

D. 526

Answer: B

Solution:

First, let us denote the first term of the G.P. by $a_1 = A$ and the common ratio (which is > 1, since the G.P. is increasing) by r. Then the terms are:

$$a_1 = A, \quad a_2 = Ar, \quad a_3 = Ar^2, \quad a_4 = Ar^3, \quad a_5 = Ar^4, \quad a_6 = Ar^5, \ \ldots$$

We are given:

$a_1 \cdot a_5 = 28$, i.e.

$$A \cdot (Ar^4) \ = \ A^2 r^4 \ = \ 28. \quad (1)$$

$a_2 + a_4 = 29$, i.e.

$$Ar \ + \ Ar^3 \ = \ A(r + r^3) \ = \ 29. \quad (2)$$

We want to find $a_6 = Ar^5$.

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1. Solve for r

From (1):

$$A^2 r^4 = 28 \quad \implies \quad A^2 = \frac{28}{r^4}.$$

From (2):

$$A\left(r + r^3\right) = 29 \quad \implies \quad A = \frac{29}{r + r^3}.$$

Plug A from (2) into (1). After some algebra (or by a systematic approach), one finds that

$$r^2 = 28 \quad \implies \quad r = \sqrt{28} \ = \ 2\sqrt{7}$$

(since $r > 1$, we take the positive root).

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2. Solve for A

From (2), using $r = 2\sqrt{7}$:

$$r + r^3 = 2\sqrt{7} + (2\sqrt{7})^3 = 2\sqrt{7} + 56\sqrt{7} = 58\sqrt{7}.$$

Hence,

$$A = \frac{29}{58\sqrt{7}} = \frac{1}{2\sqrt{7}}.$$

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3. Find a_6

$$a_6 = A\,r^5 = \frac{1}{2\sqrt{7}} \ \times \ (2\sqrt{7})^5.$$

Compute $(2\sqrt{7})^5$. First, $(2\sqrt{7})^2 = 28$; thus

$$(2\sqrt{7})^5 = (2\sqrt{7})^4 \cdot (2\sqrt{7}) = 784 \times (2\sqrt{7}) = 1568\sqrt{7}.$$

Therefore,

$a_6 = \frac{1}{2\sqrt{7}} \cdot 1568\sqrt{7} = \frac{1568}{2}$ (since $\sqrt{7}$ cancels) $= 784$.

Answer: $a_6 = 784$**.**

Hence, the correct option is **Option B**.

Question6

Suppose that the number of terms in an A.P. is $2k, k \in N$. If the sum of all odd terms of the A.P. is 40 , the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27 , then k is equal to:

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Options:

A. 8

B. 6

C. 4

D. 5

Answer: D

Solution:

$$a_1, a_2, a_3, \ldots\ldots, a_{2k} \quad \rightarrow \mathrm{A.P.}$$
$$\sum_{r=1}^{k} a_{2r-1} = 40, \sum_{r=1}^{k} a_{2r} = 55, a_{2k} - a_1 = 27$$
$$\frac{k}{2}[2a_1 + (k-1)2\,d] = 40, \frac{k}{2}[2a_2 + (k-1)2\,d] = 55,$$
$$d = \frac{27}{2k-1}$$
$$a_1 = \frac{40}{k} - (k-1)d = \frac{55}{k} - kd$$
$$d = \frac{15}{k} \Rightarrow \frac{27}{2k-1} = \frac{15}{k} \Rightarrow 9k = 10k - 5$$
$$\therefore k = 5$$

Question7

If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

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Options:

A. -120

B. -1200

C. -1080

D. -1020

Answer: C

Solution:

The first term, $a = 3$

Common difference, d

The formula for the sum of the first n terms of an A.P. is:

$S_n = \frac{n}{2}[2a + (n-1)d]$

Given:

$S_4 = \frac{1}{5}(S_8 - S_4)$

This implies:

$5S_4 = S_8 - S_4 \quad \Rightarrow \quad 6S_4 = S_8$

Substituting the sum formulas:

$6 \cdot \frac{4}{2}[2 \times 3 + (4-1)d] = \frac{8}{2}[2 \times 3 + (8-1)d]$

Simplifying:

$6 \times 2[6 + 3d] = 4[6 + 7d]$

$12(6 + 3d) = 4(6 + 7d)$

$72 + 36d = 24 + 28d$

$36d - 28d = 24 - 72$

$8d = -48$

$d = -6$

Now, to find S_{20}:

$S_{20} = \frac{20}{2}[2 \times 3 + (20-1)(-6)]$

$S_{20} = 10[6 + 19 \times (-6)]$

$S_{20} = 10[6 - 114]$

$S_{20} = 10 \times (-108)$

$S_{20} = -1080$

Thus, the sum of the first 20 terms is -1080.

Question8

Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term -p and common difference p is $\sqrt{2026\ \mathrm{S}_{2025}}$, then the absolute difference betwen 20^{th} and 15^{th} terms of the A.P. is

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Options:

A. 20

B. 45

C. 90

D. 25

Answer: D

Solution:

To find the value of S_{2025}, calculate the partial sum

$S_n = \sum_{n=1}^{2025} \frac{1}{n(n+1)} = \sum_{n=1}^{2025} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

This series is telescopic, meaning most terms cancel out with each other:

$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{2025} - \frac{1}{2026}\right)$

Simplifying the expression, we find:

$= \frac{1}{1} - \frac{1}{2026} = 1 - \frac{1}{2026} = \frac{2025}{2026}$

Now, evaluate $\sqrt{2026 \cdot S_{2025}}$:

$\sqrt{2026 \cdot \frac{2025}{2026}} = \sqrt{2025} = 45$

For the arithmetic progression with the first term $-p$ and common difference p, the sum of the first six terms is given by:

$\frac{6}{2}[-2p + (6-1)p] = 3(5p) = 15p$

Given that:

$15p = 45$

We find:

$p = 3$

To determine the absolute difference between the 20^{th} and 15^{th} terms of the A.P., compute:

$|A_{20} - A_{15}| = |(-p + 19p) - (-p + 14p)|$

Substituting the value of p:

$= |18p - 13p| = |5p| = 5 \times 5 = 25$

Thus, the absolute difference is $\boxed{25}$.

Question9

If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \frac{1}{7^3}(5+3\alpha) + \ldots\ldots\ldots\ldots\infty$, then the value of α is :

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Options:

A. $\frac{1}{7}$

B. 1

C. $\frac{6}{7}$

D. 6

Answer: D

Solution:

$$\text{Let } S = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \ldots$$
$$\frac{1}{7}S = \frac{1}{7}(5) + \frac{1}{7^2}(5+\alpha) + \ldots\infty$$
$$\frac{6}{7}(S) = 5 + \frac{1}{7}\alpha\left(\frac{1}{1-\frac{1}{7}}\right)$$
$$6 = 5 + \frac{\alpha}{6} \Rightarrow \alpha = 6$$

Question10

In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to :

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Options:

A. 525

B. 505

C. 510

D. 515

Answer: D

Solution:

Let a & d are first term and common diff of an AP.

$$S_{40}=\frac{40}{2}[2a+39\,d]=1030 \quad \text{..... (1)}$$
$$S_{12}=\frac{12}{2}[2a+11\,d]=57 \quad \text{..... (2)}$$

by (1) & (2)

$$a=-\frac{7}{2} \quad d=\frac{3}{2}$$
$$\therefore\ S_{30}-S_{10}=\frac{30}{2}[2a+29\,d]-\frac{10}{2}[2a+9\,d]$$
$$=20a-390\,d$$
$$=515$$

Question11

Let T_r be the r^{th} term of an A.P. If for some m, $T_m=\frac{1}{25}$, $T_{25}=\frac{1}{20}$, and $20\sum\limits_{r=1}^{25} T_r=13$, then $5\,m\sum\limits_{r=m}^{2\,m} T_r$ is equal to

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Options:

A. 98

B. 126

C. 112

D. 142

Answer: B

Solution:

To solve this problem, we start by analyzing the terms of an arithmetic progression (A.P.) where:

$T_m=\frac{1}{25}$

$T_{25}=\frac{1}{20}$

$20\sum\limits_{r=1}^{25} T_r=13$

The formula for the r^{th} term of an A.P. is:

$T_r = a + (r-1)d$

Given:

$T_m = a + (m-1)d = \frac{1}{25} \quad \text{(Equation 1)}$

$T_{25} = a + 24d = \frac{1}{20}$

The sum of the first 25 terms (S_{25}) is given by:

$S_{25} = \frac{25}{2} \times (2a + 24d)$

Substituting into the equation for the sum:

$20 \times \frac{25}{2} \times (2a + 24d) = 13$

This simplifies to:

$a = \frac{1}{500}$

Substituting $a = \frac{1}{500}$ into $T_{25} = a + 24d = \frac{1}{20}$, we find:

$\frac{1}{500} + 24d = \frac{1}{20}$

$24d = \frac{1}{20} - \frac{1}{500}$

$d = \frac{1}{500}$

Using Equation 1 again:

$\frac{1}{500} + \frac{m-1}{500} = \frac{1}{25}$

$\frac{m}{500} = \frac{1}{25}$

$m = 20$

Now to find $5m \sum_{r=m}^{2m} T_r$:

$5m = 5 \times 20 = 100$

$\sum_{r=20}^{40} T_r = \frac{21}{2} \times (T_{20} + T_{40})$

Since $m = 20$, the summation covers terms from T_{20} to T_{40}, and:

$100 \times \sum_{r=20}^{40} T_r = 126$

Therefore, $5m \sum_{r=m}^{2m} T_r = 126$.

Question12

Let $\langle a_{\mathrm{n}} \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and $2a_{\mathrm{n}+2} = 5a_{\mathrm{n}+1} - 3a_{\mathrm{n}}, \mathrm{n} = 0, 1, 2, 3, \ldots$. Then $\sum_{k=1}^{100} a_k$ is equal to

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Options:

A. $3a_{100} + 100$

B. $3a_{100} - 100$

C. $3a_{99} - 100$

D. $3a_{99} + 100$

Answer: B

Solution:

$$a_0 = 0, a_1 = \frac{1}{2}$$
$$2a_{n+2} = 5a_{n+1} - 3a_n$$
$$2x^2 - 5x + 3 = 0 \Rightarrow x = 1, 3/2$$
$$\therefore a_n = A(1)^n + B\left(\frac{3}{2}\right)^n$$
$$\left.\begin{array}{ll} \mathrm{n} = 0 & 0 = \mathrm{A} + \mathrm{B} \\ \mathrm{n} = 1 & \frac{1}{2} = \mathrm{A} + \frac{3}{2}\mathrm{B} \end{array}\right] \begin{array}{l} \mathrm{A} = -1 \\ \mathrm{B} = 1 \end{array}$$
$$\Rightarrow a_n = -1 + \left(\frac{3}{2}\right)^n$$
$$\sum_{k=1}^{100} a_k = \sum_{k=1}^{100}(-1) + \left(\frac{3}{2}\right)^k$$
$$= -100 + \frac{\left(\frac{3}{2}\right)\left(\left(\frac{3}{2}\right)^{100} - 1\right)}{\frac{3}{2} - 1}$$
$$= -100 + 3\left(\left(\frac{3}{2}\right)^{100} - 1\right)$$
$$= 3 \cdot (\mathrm{a}_{100}) - 100$$

Question13

For positive integers n, if $4a_n = \left(n^2 + 5n + 6\right)$ and $S_n = \sum_{k=1}^{n}\left(\frac{1}{a_k}\right)$, then the value of $507S_{2025}$ is :

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Options:

A.

540

B.

675

C.

1350

D.

135

Answer: B

Solution:

$$a_n = \frac{n^2+5n+6}{4}$$
$$S_n = S_n = \sum_{k=1}^{n} \frac{1}{a_k} = \sum_{1}^{n} \frac{4}{k^2+5k+6}$$
$$= 4\sum_{k=1}^{n} \frac{1}{(k+2)(k+3)}$$
$$= 4\sum_{k=1}^{n} \frac{1}{k+2} - \frac{1}{k+3}$$
$$= 4\left(\frac{1}{3}-\frac{1}{4}\right) + 4\left(\frac{1}{4}-\frac{1}{5}\right) + \ldots\ldots..$$

$4\left(\frac{1}{n+2} - \frac{1}{n+3}\right)$
$= 4\left(\frac{1}{3} - \frac{1}{n+3}\right)$
$= \frac{4n}{3(n+3)}$
$^{507}\ S_{2025} = \frac{(507)(4)(2025)}{3(2028)}$
$= 675$

Question14

Consider an A. P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

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Options:

A.

108

B.

90

C.

122

D.

84

Answer: B

Solution:

$S_3 = 3a + 3\,d = 54$
$\Rightarrow a + d = 18$
$S_{20} = 10(2a + 19\,d)$
$\Rightarrow 10(36 + 17\,d)$
$\Rightarrow 1600 < 10(36 + 17\,d) < 1800$
$\Rightarrow 160 < 36 + 17\,d < 180$
$\Rightarrow 124 < 17\,d < 144$
$\Rightarrow 7\frac{5}{17} < d < 8\frac{8}{17}$
Common difference will be natural number
$\Rightarrow d = 8 \Rightarrow a = 10$
$\Rightarrow a_{11} = 10 + 10 \times 8 = 90$

Question15

Let $a_1, a_2, a_3, \ldots$ be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5} a_1, a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is :

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Options:

A. 18

B. 17

C. 11

D. 10

Answer: C

Solution:

Given:

$\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5} a_1$

This means:

$a_1 + a_3 + \cdots + a_{23} = -\frac{72}{5} a_1$

Express each term in A.P.:

The general term of the A.P. is given by $a_k = a + (k-1)d$. Thus, for the odd indices:

$a_1 + (a+2d) + (a+4d) + \cdots + (a+22d)$

Simplify the equation:

$12a + 2d(1+2+\cdots+11) = -\frac{72}{5}a$

Use the formula for sum of an arithmetic series (first 11 terms):

$1+2+\cdots+11 = \frac{11\times12}{2} = 66$

So, the equation becomes:

$12a + 2d(66) = -\frac{72}{5}a$

$12a + 132d = -\frac{72}{5}a$

Solve for the relationship between a and d:

$132d = -\frac{72}{5}a - 12a$

$132d = -\frac{132}{5}a$

So,

$a = -5d \quad \text{(i)}$

Second condition:

$\sum_{k=1}^{n} a_k = 0 \quad \Rightarrow \quad S_n = 0$

$\frac{n}{2}[2a + (n-1)d] = 0$

$2a = -(n-1)d \quad \text{(ii)}$

Combine equation (i) and (ii):

Substitute $a = -5d$ into equation (ii):

$2(-5d) = -(n-1)d$

$-10d = -(n-1)d$

Therefore:

$n - 1 = 10$

$n = 11$

Thus, $n = 11$.

Question16

The number of terms of an A.P. is even; the sum of all the odd terms is 24 , the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

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Options:

A. 6

B. 4

C. 8

D. 10

Answer: B

Solution:

Let the number of terms be $2n$

$$\begin{array}{c} T_1 + T_3 + T_5 \ldots T_{2n-1} = 24 \\ T_2 + T_4 + T_6 \ldots T_{2n} = 30 \\ \hline T_2 - T_1) + (T_4 - T_3) + \ldots (T_{2n} - T_{2n-1}) = 6 \end{array}$$

$nd = 6$

$(a + (2n+1)d) - a = \frac{21}{2}$

$\Rightarrow 2nd - d = \frac{21}{2}$

$\Rightarrow 12 - \frac{21}{2} = d$

$\Rightarrow d = \frac{3}{2}$

$\therefore n = 4$

$\therefore$ Total terms $= 8$

Question17

The sum $1 + 3 + 11 + 25 + 45 + 71 + \ldots$ upto 20 terms, is equal to

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Options:

A. 7240

B. 8124

C. 7130

D. 6982

Answer: A

Solution:

$T_r = 3r^2 - 7r + 5$ using second order difference

$$\sum_{r=1}^{20}\left(3r^2 - 7r + 5\right) = 3\Sigma r^2 - 7\Sigma r + 5\Sigma(1)$$

$$= \frac{3(n)(n+1)(2n+1)}{6} - \frac{7n(n+1)}{2} - 5n, n = 20$$

$$= 7240$$

Question18

Let $a_1, a_2, a_3, \ldots$. be a G.P. of increasing positive numbers. If $a_3 a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24\left(a_1 + a_2 + a_3\right)$ is equal to

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Options:

A. 128

B. 129

C. 131

D. 130

Answer: B

Solution:

We start with the given sequence $a_1, a_2, a_3, \ldots$ of an increasing geometric progression (G.P.). Two key conditions are provided:

$a_3 \cdot a_5 = 729$

$a_2 + a_4 = \frac{111}{4}$

Calculating using given conditions:

Using the sequence terms:

$a_3 = a \cdot r^2, \quad a_5 = a \cdot r^4,$

Then:

$a_3 \cdot a_5 = (a \cdot r^2)(a \cdot r^4) = a^2 \cdot r^6 = 729 = 27^2$

It follows:

$a \cdot r^3 = 27 \quad \Rightarrow \quad a_4 = a \cdot r^3 = 27 \quad \text{(i)}$

Using the second condition:

$a_2 + a_4 = \frac{111}{4}$

Substituting $a_4 = 27$:

$a_2 = \frac{111}{4} - 27$

$a_2 = a \cdot r = \frac{3}{4} \quad \text{(ii)}$

Solving for r and a:

From equation (i) and (ii), we derive r^2:

$r^2 = \frac{4 \cdot 27}{3} = 4 \times 9 = 36$

Since the G.P. is increasing, $r = 6$.

Solve for a:

Substituting $r = 6$ into $a \cdot r = \frac{3}{4}$:

$a \cdot 6 = \frac{3}{4} \quad \Rightarrow \quad a = \frac{3}{24} = \frac{1}{8}$

Calculating the requested sum:

The task is to find:

$24(a_1 + a_2 + a_3) = 24(a + ar + ar^2)$

Substitute the values:

$= 24 \times \frac{1}{8}(1 + 6 + 6^2) = 3 \times (1 + 6 + 36) = 3 \times 43 = 129$

Therefore, the value is 129.

Question19

The sum $1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \ldots$ upto ∞ terms, is equal to

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Options:

A. $3e$

B. $2e$

C. $4e$

D. $6e$

Answer: B

Solution:

The series given is:

$1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \ldots$

In general, the r-th term of the series, denoted as T_r, takes the form:

$T_r = \frac{r^2}{r!}$

To simplify T_r, we write it in terms of factorials:

$T_r = \frac{r}{(r-1)!} = \frac{(r-1)+1}{(r-1)!} = \frac{1}{(r-2)!} + \frac{1}{(r-1)!}$

Thus, the series can be expressed as:

$\sum_{r=1}^{\infty} T_r = \sum_{r=1}^{\infty} \left(\frac{1}{(r-2)!} + \frac{1}{(r-1)!} \right)$

This breaks into two parts:

$= \sum_{r=1}^{\infty} \frac{1}{(r-2)!} + \sum_{r=1}^{\infty} \frac{1}{(r-1)!}$

Each part is a well-known series. Specifically:

$\sum_{r=1}^{\infty} \frac{1}{(r-2)!} = e$, starting from the term where $r-2=0$, which aligns with the expansion of e.

$\sum_{r=1}^{\infty} \frac{1}{(r-1)!} = e$, for the terms starting where $r-1=0$.

Consequently, the sum of the entire series is:

$e+e=2e$

Question20

$1+3+5^2+7+9^2+\ldots$ upto 40 terms is equal to

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Options:

A. 40870

B. 41880

C. 43890

D. 33980

Answer: B

Solution:

$$\begin{aligned}
&1+3+5^2+7+9^2+\ldots \text{ upto 40 terms} \\
&\left(1^2+5^2+9^2+\ldots\right)+(3+7+11+\ldots) \\
&=\left(\sum_{k=1}^{20}(4k-3)^2\right)+\frac{20}{2}[6+(20-1)4] \\
&=16\sum_{k=1}^{20}k^2-24\sum_{k=1}^{20}k+9\times 20+10[82] \\
&=16\left(\frac{20\times 21\times 41}{6}\right)-24\left(\frac{20\times 21}{2}\right)+1000 \\
&=45920-5040+1000 \\
&=41880
\end{aligned}$$

Question21

Let $A = \{1, 6, 11, 16, \ldots\}$ and $B = \{9, 16, 23, 30, \ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

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Options:

A. 3814

B. 4003

C. 4027

D. 3761

Answer: D

Solution:

1^{st} A.P. : $1, 6, 11 \ldots \quad \Rightarrow T_n = S_n - 4$
2^{nd} A.P.: $9, 16, 23 \ldots \quad \Rightarrow T_m = 2 + 7m$

Let's find when they are equal for the first time:

$$\begin{aligned} & 5n - 4 = 2 + 7m \\ & \Rightarrow 5n - 7m = 6 \\ & \Rightarrow n = 4, m = 2 \end{aligned}$$

$\Rightarrow$ 16 is the first term, common difference will be

$\text{LCM}\left(d_1, d_2\right) = \text{LCM}(5, 7) = 35$

$\Rightarrow$ Common terms will be 16, 51, 86 . . .

The last term of 1^{st} A.P.

$= T_{2025} = 5 \times 2025 - 4 = 10121$

$$\begin{aligned} & \Rightarrow \text{ Common term must be less than that} \\ & \Rightarrow 35n - 19 \\ & \Rightarrow 35n - 19 \leq 10121 \Rightarrow 35n \leq 10140 \\ & \quad \Rightarrow n \leq 289.7 \\ & \quad \Rightarrow n = 289 \\ & \Rightarrow \text{in } n(A \cup B) = n(A) + n(B) - n(A \cap B) \\ & \quad = 2025 + 2025 - 289 \\ & \quad = 3761 \end{aligned}$$

Question22

Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common

differences of $\mathrm{AP}'\mathrm{s}$ in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $\mathrm{p} - \mathrm{q}$ is equal to

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Options:

A. 540

B. 450

C. 600

D. 630

Answer: A

Solution:

Let the terms in A be $a_1 - d, a_1, a_1 + d$

and in B be $a_2 - D, a_2, a_2 + D$

Now $3a_1 = 36$

$\Rightarrow \quad a_1 = 12$

and $3a_2 = 36$

$\Rightarrow \quad a_2 = 12$

Now $(12 - d)(12)(12 + d) = p$

and $(12 - D)(12)(12 + D) = q$

Also $\frac{p+q}{p-q} = \frac{19}{5}$

$\Rightarrow 12q = 7p$
$\Rightarrow 12(12 - D)(12)(12 + D) = 7(12 - d)(12)(12 + \mathrm{d})$
$\Rightarrow 12(9 - d)(12)(15 - d) = 7(12 - d)(12)(12 + \mathrm{d})$
$\Rightarrow 12\left(135 - d^2 - 6d\right) = 7\left(144 - d^2\right)$
$\Rightarrow d = 6, D = 9$
$p = 6 \times 12 \times 18 = 1296$
$q = 756$
$p - q = 540$

Question23

If the sum of the first 20 terms of the series $\frac{4 \cdot 1}{4+3 \cdot 1^2+1^4} + \frac{4 \cdot 2}{4+3 \cdot 2^2+2^4} + \frac{4 \cdot 3}{4+3 \cdot 3^2+3^4} + \frac{4 \cdot 4}{4+3 \cdot 4^2+4^4} + \ldots$ is $\frac{\mathrm{m}}{\mathrm{n}}$, where m and n are coprime, then $\mathrm{m} + \mathrm{n}$ is equal to :

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Options:

A. 423

B. 421

C. 422

D. 420

Answer: B

Solution:

$$S_n = \sum_{r=1}^{n} \frac{4r}{4+3r^2+r^4}$$
$$= 2\sum_{r=1}^{n} \frac{2r}{\left(r^2+2\right)^2 - r^2} = 2\sum_{r=1}^{n} \frac{\left(r^2+2+r\right)-\left(r^2+2-r\right)}{\left(r^2+2+r\right)\left(r^2+2-r\right)}$$
$$= 2\sum_{r=1}^{n}\left(\frac{1}{r^2+2-r} - \frac{1}{r^2+2+r}\right)$$
$$S_{20} = 2\left[\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{8}\right)+\ldots\right]$$
$$= 2\left(\frac{1}{2} - \frac{1}{20^2+2+20}\right)$$
$$= 2\left(\frac{1}{2}-\frac{1}{422}\right)$$
$$= 2\left(\frac{422-2}{422\times 2}\right) = \frac{420}{422} = \frac{210}{211} = \frac{m}{n}$$
$$m+n = 421$$

Question24

Let x_1, x_2, x_3, x_4 be in a geometric progression. If $2, 7, 9, 5$ are subtracted respectively from x_1, x_2, x_3, x_4, then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}\left(x_1 x_2 x_3 x_4\right)$ is:

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Options:

A. 18

B. 216

C. 36

D. 72

Answer: B

Solution:

Given the sequence x_1, x_2, x_3, x_4 in geometric progression:

$x_1 = a$

$x_2 = ar$

$x_3 = ar^2$

$x_4 = ar^3$

When you subtract 2, 7, 9, and 5 from x_1, x_2, x_3, x_4 respectively, the sequence becomes an arithmetic progression. Thus, the new sequence is:

$a - 2$

$ar - 7$

$ar^2 - 9$

$ar^3 - 5$

For these to form an arithmetic progression, the common differences must be equal, so:

$(ar - 7) - (a - 2) = (ar^2 - 9) - (ar - 7)$

Simplifying gives:

$a(r - 1) - 5 = ar(r - 1) - 2$

$a(r - 1)(r - 1) = -3 \quad (i)$

$(ar - 7) - (a - 2) = (ar^3 - 5) - (ar^2 - 9)$

Simplifying gives:

$a(r - 1) - 5 = ar^2(r - 1) + 4$

$a(r - 1)(r^2 - 1) = -9 \quad (ii)$

Using the ratio of equations (ii) and (i):

$\frac{a(r-1)(r^2-1)}{a(r-1)(r-1)} = \frac{-9}{-3}$

$r + 1 = 3 \implies r = 2$

Plugging back into equation (i):

$a(1)(1) = -3 \implies a = -3$

So, the sequence x_1, x_2, x_3, x_4 is:

$x_1 = -3$

$x_2 = -6$

$x_3 = -12$

$x_4 = -24$

The expression for $\frac{1}{24}(x_1 \cdot x_2 \cdot x_3 \cdot x_4)$ is:

$\frac{1}{24}((-3) \cdot (-6) \cdot (-12) \cdot (-24)) = 216$

Question25

If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is :

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Options:

A.

757

B.

755

C.

750

D.

760

Answer: A

Solution:

To solve this problem, we have to work with two equations derived from the geometric progression (G.P.):

$\mathrm{ar}+\mathrm{ar}^3+\mathrm{ar}^5=21$

$\mathrm{ar}^7+\mathrm{ar}^9+\mathrm{ar}^{11}=15309$

From these equations, we can extract the terms:

Equation (1): $\mathrm{ar}(1+\mathrm{r}^2+\mathrm{r}^4)=21$

Equation (2): $\mathrm{ar}^7(1+\mathrm{r}^2+\mathrm{r}^4)=15309$

Now, divide equation (2) by equation (1):

$\frac{\mathrm{ar}^7}{\mathrm{ar}}=\frac{15309}{21}$

From this division, we get:

$\mathrm{r}^6=729$

Which implies:

$\mathrm{r}=3 \quad$ (since both terms and ratios are positive)

Using this value of r, calculate the sum of the first nine terms of the G.P. using the formula for the sum of a G.P.:

$S_n=\frac{\mathrm{a}(\mathrm{r}^9-1)}{\mathrm{r}-1}$

Substitute known values:

$\Rightarrow \frac{a(3^9-1)}{3-1} = \frac{a\cdot(19683-1)}{2}$

Given that $\frac{7}{91} \times (19683-1)/2 = \frac{7\times19682}{91\times2}$:

$= \frac{7\times19682}{91\times2} = \frac{9841}{13} = 757$

Therefore, the sum of the first nine terms of the G.P. is **757**.

Question26

Let a_n be the n^{th} term of an A.P. If $S_n = a_1 + a_2 + a_3 + \ldots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to :

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Options:

A.

65

B.

56

C.

70

D.

64

Answer: D

Solution:

Sum of n terms of an AP (S_n): $S_n = \frac{n}{2}[2a + (n-1)d]$ where 'a' is the first term and 'd' is the common difference.

nth term of an AP (a_n): $a_n = a + (n-1)d$

Given Information:

$S_n = 700$ (The sum of the first n terms is 700)

$a_6 = 7$ (The 6th term is 7)

$S_7 = 7$ (The sum of the first 7 terms is 7)

Steps to Solve:

Use S_7 to find a relationship between 'a' and 'd':

$S_7 = \frac{7}{2}[2a + (7-1)d] = 7$

$\frac{7}{2}[2a + 6d] = 7$

$2a + 6d = 2$

$a + 3d = 1$ **(Equation 1)**

Use a_6 to find another relationship between 'a' and 'd':

$a_6 = a + (6-1)d = 7$

$a + 5d = 7$ **(Equation 2)**

Solve the system of equations (Equation 1 and Equation 2) to find 'a' and 'd':

Subtract Equation 1 from Equation 2:

$(a + 5d) - (a + 3d) = 7 - 1$

$2d = 6$

$d = 3$

Substitute d = 3 into Equation 1:

$a + 3(3) = 1$

$a + 9 = 1$

$a = -8$

Find 'n' using $S_n = 700$:

$S_n = \frac{n}{2}[2a + (n-1)d] = 700$

$\frac{n}{2}[2(-8) + (n-1)(3)] = 700$

$n[-16 + 3n - 3] = 1400$

$n[3n - 19] = 1400$

$3n^2 - 19n - 1400 = 0$

Solve the quadratic equation for 'n':

We can use the quadratic formula: $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$n = \frac{19 \pm \sqrt{(-19)^2 - 4(3)(-1400)}}{2(3)}$

$n = \frac{19 \pm \sqrt{361 + 16800}}{6}$

$n = \frac{19 \pm \sqrt{17161}}{6}$

$n = \frac{19 \pm 131}{6}$

We have two possible values for n:

$n = \frac{19 + 131}{6} = \frac{150}{6} = 25$

$n = \frac{19 - 131}{6} = \frac{-112}{6} = -\frac{56}{3}$ (Since 'n' must be a positive integer, we discard this solution)

So, $n = 25$

Find a_n (which is a_{25}):

$a_{25} = a + (25-1)d$

$a_{25} = -8 + (24)(3)$

$a_{25} = -8 + 72$

$a_{25} = 64$

Therefore, $a_n = 64$. The correct answer is Option D.

Question27

If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\infty=\frac{\pi^4}{90}$,

$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\infty=\alpha$,

$\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\ldots\infty=\beta$,

then $\frac{\alpha}{\beta}$ is equal to :

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Options:

A.

23

B.

14

C.

18

D.

15

Answer: D

Solution:

$$\text{If } \frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\infty=\frac{\pi^4}{90} \quad \text{....... (i)}$$
$$\beta=\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\ldots,$$
$$=\frac{1}{16}\left[\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\right],$$

$=\frac{1}{16}\times\frac{\pi^4}{90}$ using (ii) .... (ii)

$$\alpha = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots \infty$$
$$\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \ldots\ldots\right)$$
$$-\left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \ldots\ldots\right)$$
$$\alpha = \frac{\pi^4}{90} - \frac{1}{16} \times \frac{\pi^4}{90} \quad \text{[using (i) and (ii)]}$$
$$\alpha = \frac{16-1}{16 \times 90} \times \pi^4 = \frac{15}{16 \times 90}\pi^4 = \frac{\pi^4}{96}$$
$$\therefore \frac{\alpha}{\beta} = \frac{\frac{\pi^4}{96}}{\frac{\pi^4}{16 \times 90}} = \frac{16 \times 90}{96} = 15$$

Question28

The number of common terms in the progressions 4, 9, 14, 19,......, up to 25^{th} term and 3, 6, 9, 12, up to 37^{th} term is :

[27-Jan-2024 Shift 1]

Options:

A.

9

B.

5

C.

7

D.

8

Answer: C

Solution:

4, 9, 14, 19, ... , up to 25 th term

$T_{25} = 4 + (25 - 1)5 = 4 + 120 = 124$

3, 6, 9, 12, ... , up to 37 th term

$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$

Common difference of I st series $d_1 = 5$

Common difference of II nd series $d_2 = 3$

First common term $= 9$, and

their common difference $= 15(\text{LCM}^{2.}$ of d_1 and $d_2)$ then common terms are

9, 24, 39, 54, 69, 84, 99

Question29

If

$$8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \ldots \infty,$$

then the value of p is_____

[27-Jan-2024 Shift 1]

Answer: 9

Solution:

$$8 = \frac{3}{1 - \frac{1}{4}} + \frac{p \cdot \frac{1}{4}}{\left(1 - \frac{1}{4}\right)^2}$$

(sum of infinite terms of A.G.P $= \frac{a}{1 - r} + \frac{dr}{(1 - r)^2}$)

$\Rightarrow \frac{4p}{9} = 4 \Rightarrow p = 9$

Question30

The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is :-

[27-Jan-2024 Shift 2]

Options:

A.

-118

B.

-110

C.

-115

D.

-100

Answer: C

Solution:

$20,\ 19\frac{1}{4},\ 18\frac{1}{2},\ 17\frac{3}{4},\ \ldots\ldots,\ -129\frac{1}{4}$

This is A.P. with common difference

$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$

$-129\frac{1}{4},\ \ldots\ldots\ldots\ldots..19\frac{1}{4},\ 20$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

Required term =

$-129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right)$

$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$

Question31

If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

[29-Jan-2024 Shift 1]

Options:

A.

7

B.

4

C.

5

D.

6

Answer: D

Solution:

$$a + ar + ar^2 + ar^3 + \ldots + a^{63}$$

$$= 7(a + ar^2 + ar^4 \ldots . + ar^{62})$$

$$\Rightarrow \frac{a(1-r^{64})}{1-r} = \frac{7a(1-r^{64})}{1-r^2}$$

$$r = 6$$

Question32

In an A.P., the sixth terms a6 = 2. If the $a_1a_4a_5$ is the greatest, then the common difference of the A.P., is equal to

[29-Jan-2024 Shift 1]

Options:

A.

3/2

B.

8/5

C.

2/3

D.

5/8

Answer: B

Solution:

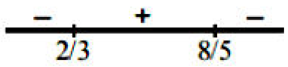
$$a_6 = 2 \Rightarrow a + 5d = 2$$

$$a_1 a_4 a_5 = a(a+3d)(a+4d)$$

$$= (2-5d)(2-2d)(2-d)$$

$$f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3$$

$$f'(d) = -2(5d-8)(3d-2)$$



$$d = \frac{8}{5}$$

Question33

If $\log_e a$, $\log_e b$, $\log_e c$ are in an A.P. and $\log_e a - \log_e 2b$, $\log_e 2b - \log_e 3c$, $\log_e 3c - \log_e a$ are also in an A.P, then a:b:c is equal to

[29-Jan-2024 Shift 2]

Options:

A.

9 : 6 : 4

B.

16 : 4 : 1

C.

25 : 10 : 4

D.

6 : 3 : 2

Answer: A

Solution:

$\log_e a, \log_e b, \log_e c$ are in A.P.

$\therefore b^2 = ac$ ......(i)

Also

$\log_e\left(\frac{a}{2b}\right), \log_e\left(\frac{2b}{3c}\right), \log_e\left(\frac{3c}{a}\right)$ are in A.P.

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$$

$$\frac{b}{c} = \frac{3}{2}$$

Putting in eq. (i) $b^2 = a \times \frac{2b}{3}$

$$\frac{a}{b} = \frac{3}{2}$$

$$a : b : c = 9 : 6 : 4$$

Question34

If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = 1/8$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots + an$, then $S_{20} - S_{18}$ is equal to

[29-Jan-2024 Shift 2]

Options:

A.

2^{15}

B.

-2^{18}

C.

2^{18}

D.

-2^{15}

Answer: D

Solution:

Let r' th term of the GP be ar^{n-1}. Given,

$$2a_r = a_{r+1} + a_{r+2}$$

$$2ar^{n-1} = ar^n + ar^{n+1}$$

$$\frac{2}{r} = 1 + r$$

$$r^2 + r - 2 = 0$$

Hence, we get, $r = -2$(as $r \neq 1$)

So, $S_{20} - S_{18} =$ (Sum upto 20 terms) - (Sum upto 18 terms $) = T_{19} + T_{20}$

$$T_{19} + T_{20} = ar^{18}(1 + r)$$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

Question35

Let Sa denote the sum of first n terms an arithmetic progression. If S_{20} = 790 and S_{10} = 145, then $S_{15} - S_5$ is :

[30-Jan-2024 Shift 1]

Options:

A.

395

B.

390

C.

405

D.

Answer: A

Solution:

$$S_{20} = \frac{20}{2}[2a + 19d] = 790$$

$2a + 19d = 79$ ....(1)

$$S_{10} = \frac{10}{2}[2a + 9d] = 145$$

$2a + 9d = 29$ ....(2)

From (1) and (2) $a = -8, d = 5$

$$S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$$

$$= \frac{15}{2}[-16 + 70] - \frac{5}{2}[-16 + 20]$$

$$= 405 - 10$$

$$= 395$$

Question36

Let α = $1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 +$....... upto 10 terms and $\beta = \sum_{n=1}^{10} n^4.$ **If 4α − β = 55k + 40, then k is equal to**

[30-Jan-2024 Shift 1]

Answer: 353

Solution:

$\alpha = 1^2 + 4^2 + 8^2 \ldots.$

$t_n = a^2 + bn + c$

$1 = a + b + c$

$4 = 4a + 2b + c$

$8 = 9a + 3b + c$

On solving we get, a = $\frac{1}{2}$, b = $\frac{3}{2}$, c = -1

$\alpha = \sum_{n=1}^{10} \left(\frac{n^2}{2} + \frac{3n}{2} - 1 \right)^2$

$4\alpha = \sum_{n=1}^{10} (n^2 + 3n - 2)^2, \beta = \sum_{n=1}^{10} n^4$

$4\alpha - \beta = \sum_{n=1}^{10} (6n^3 + 5n^2 - 12n + 4) \ = 55(353) + 40$

Question37

Let a and b be be two distinct positive real numbers. Let 11th term of a GP, whose first term is a and third term is b, is equal to pth term of another GP, whose first term is a and fifth term is b. Then p is equal to

[30-Jan-2024 Shift 2]

Options:

A.

20

B.

25

C.

21

D.

24

Answer: C

Solution:

1^{st} GP $\Rightarrow t_1 = a, t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$

$t_{11} = ar^{10} = a(r^2)^5 = a \cdot \left(\frac{b}{a}\right)^5$

$2^{nd}G.P. \Rightarrow T_1 = a, T_5 = ar^4 = b$

$\Rightarrow r^4 = \left(\frac{b}{a}\right) \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$

$T_p = ar^{p-1} = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$t_{11} = T_p \Rightarrow a\left(\frac{b}{a}\right)^5 = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$\Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$

Question38

Let S_n be the sum to n-terms of an arithmetic progression 3,7,11,.......

If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals____

[30-Jan-2024 Shift 2]

Answer: 9

Solution:

$S_n = 3 + 7 + 11 + \ldots\ldots n$ terms

$= \frac{n}{2}(6 + (n-1)4) = 3n + 2n^2 - 2n$

$= 2n^2 + n$

$\sum_{k=1}^{n} S_k = 2\sum_{k=1}^{n} K^2 + \sum_{k=1}^{n} K$

$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= n(n+1)\left[\frac{2n+1}{3} + \frac{1}{2}\right]$

$= \frac{n(n+1)(4n+5)}{6}$

$\Rightarrow 40 < \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 42$

$40 < 4n + 5 < 42$

$35 < 4n < 37$

$n = 9$

Question39

The sum of the series $\frac{1}{1-3\cdot 1^2+1^4} + \frac{2}{1-3\cdot 2^2+2^4} + \frac{3}{1-3\cdot 3^2+3^4} + \ldots$ up to 10 terms is

[31-Jan-2024 Shift 1]

Options:

A.

45/109

B.

$-\frac{45}{109}$

C.

55/109

D.

$-\frac{55}{109}$

Answer: D

Solution:

General term of the sequence,

$$T_r = \frac{r}{1-3r^2+r^4}$$

$$T_r = \frac{r}{r^4-2r^2+1-r^2}$$

$$T_r = \frac{r}{(r^2-1)^2-r^2}$$

$$T_r = \frac{r}{(r^2-r-1)(r^2+r-1)}$$

$$T_r = \frac{\frac{1}{2}[(r^2+r-1)-(r^2-r-1)]}{(r^2-r-1)(r^2+r-1)}$$

$$= \frac{1}{2}\left[\frac{1}{r^2-r-1} - \frac{1}{r^2+r-1}\right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2}\left[\frac{1}{-1} - \frac{1}{109}\right] = \frac{-55}{109}$$

Question40

Let 2^{nd}, 8^{th} and 44^{th} , terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of G.P. If the first term of A.P. is 1 then the sum of first 20 terms is equal to-

[31-Jan-2024 Shift 2]

Options:

A.

980

B.

960

C.

990

D.

970

Answer: D

Solution:

$1+d,\quad 1+7d, 1+43d$ are in GP

$(1+7d)^2=(1+d)(1+43d)$

$1+49d^2+14d=1+44d+43d^2$

$6d^2-30d=0$

$d=5$

$S_{20}=\frac{20}{2}[2\times 1+(20-1)\times 5]$

$=10[2+95]$

$=970$

Question41

Let 3, a, b, c be in A.P. and 3, a−1, b+1, c+9 be in G.P. Then, the arithmetic mean of a, b and c is :

[1-Feb-2024 Shift 1]

Options:

A.

-4

B.

-1

C.

13

D.

11

Answer: D

Solution:

$3, a, b, c \rightarrow$ A.P $\Rightarrow 3, 3+d, 3+2d, 3+3d$

$3, a-1, b+1, c+9 \rightarrow$ G.P $\Rightarrow 3, 2+d, 4+2d, 12+3d$

$a = 3 + d \quad (2+d)^2 = 3(4+2d)$

$b = 3 + 2d \quad d = 4, -2$

$c = 3 + 3d$

If $d = 4$ G.P $\Rightarrow 3, 6, 12, 24$

$a = 7$

$b = 11$

$c = 15$

$\frac{a+b+c}{3} = 11$

Question42

Let 3, 7, 11, 15,....,403 and 2, 5, 8, 11, . ., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to____

[1-Feb-2024 Shift 1]

Answer: 6699

Solution:

$3, 7, 11, 15, \ldots, 403$

$2, 5, 8, 11, \ldots, 404$

$LCM(4, 3) = 12$

$11, 23, 35, \ldots$ let (403)

$403 = 11 + (n-1) \times 12$

$\frac{392}{12} = n - 1$

$33.66 = n$

$n = 33$

Sum $\frac{33}{2}(22 + 32 \times 12)$

$= 6699$

Question43

Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

[1-Feb-2024 Shift 2]

Options:

A.

800

B.

890

C.

790

D.

690

Answer: C

Solution:

$S_{10} = 390$

$\frac{10}{2}[2a + (10-1)d] = 390$

$\Rightarrow 2a + 9d = 78$

$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a = 3d$

From (1) & (2) $a = 3$ & $d = 8$

$S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8)$

$= \frac{15 \times 118 - 5 \times 38}{2} = 790$

Question44

If three successive terms of a G.P. with common ratio $r(r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r, then

3[r] + [−r] is equal to :

[1-Feb-2024 Shift 2]

Answer: 1

Solution:

a, ar, $ar^2 \rightarrow$ G.P.

Sum of any two sides > third side

$a+ar>ar^2,\ a+ar^2>ar,\ ar+ar^2>a$

$r^2-r-1<0$

$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ ....(1)

$r^2-r+1>0$

always true

$r^2+r-1>0$

$r \in \left(-\infty, -\frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$

Taking intersection of (1),(2)

$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

As $r>1$

$r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$[r]=1[-r]=-2$

$3[r]+[-r]=1$

Question45

For three positive integers p, q, r, $x^{pq^2}=y^{qr}=z^{p^2r}$ and $r=pq+1$ such that $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r-p-q$ is equal to
[24-Jan-2023 Shift 1]

Options:

A. 2

B. 6

C. 12

D. −6

Answer: A

Solution:

Solution:

$pq^2 = \log_x \lambda$

$qr = \log_y \lambda$

$p^2 r = \log_z \lambda$

$\log_y x = \frac{qr}{pq^2} = \frac{r}{pq} \ldots . (1)$

$\log_x z = \frac{pq^2}{p^2 r} = \frac{q^2}{pr} \ldots . . . . (2)$

$\log_z y = \frac{p^2 r}{qr} = \frac{p^2}{q} \ldots . . (3)$

$3, \frac{3r}{pq}, \frac{3p^2}{q}, \frac{7q^2}{pr}$ in A.P

$\frac{3r}{pq} - 3 = \frac{1}{2}$

$r = \frac{7}{6} pq \ldots . . . (4)$

$r = pq + 1$

$pq = 6 \ldots . . . (5)$

$r = 7 \ldots . . (6)$

$\frac{3p^2}{q} = 4$

After solving $p = 2$ and $q = 3$

Question46

The 4 th term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is
[24-Jan-2023 Shift 1]

Answer: 12

Solution:

Solution:

$T_4 = 500$ where $a =$ first term,

$r =$ common ratio $= \frac{1}{m}, m \in N$

$ar^3 = 500$

$\frac{a}{m^3} = 500$

$S_n - S_{n-1} = ar^{n-1}$
$S_6 > S_5 + 1$ and $S_7 - S_6 < \frac{1}{2}$
$S_6 - S_5 > 1$ $\frac{a}{m^6} < \frac{1}{2}$
$ar^5 > 1$ $m^3 > 10^3$
$\frac{500}{m^2} > 1$ $m > 10$
$m^2 < 500$ ......(1)
From (1) and (2)
$m = 11, 12, 13 \ldots\ldots\ldots, 22$

So number of possible values of m is 12

Question47

If $\frac{1^3 + 2^3 + 3^3 + \ldots\ldots \text{ upto } n \text{ terms}}{1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \ldots\ldots \text{ upto } n \text{ terms}} = \frac{9}{5}$, then the value of n is [24-Jan-2023 Shift 2]

Answer: 5

Solution:

Solution:

$1^3 + 2^3 + 3^3 \ldots.. + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \ldots + n^n + \text{terms} =$

$\sum_{r=1}^{n} r(2r+1) = \sum_{r=1}^{n} (2r^2 + r)$

$= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{6}(2(2n+1)+3)$

$= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3}$

$= \frac{n^2(n+1)^2}{4}$

$\Rightarrow \frac{\frac{5n(n+1)}{2} \times \frac{(4n+5)}{3}}{2} = \frac{9}{5}$

$\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3}$

$\Rightarrow 15n(n+1) = 18(4n+5)$

$\Rightarrow 15n^2 + 15n = 72n + 90$

$\Rightarrow 15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow (n-5)(5n+6) = 0$

$\Rightarrow n = \frac{-6}{5}$ or 5

$\Rightarrow n = 5.$

Question48

Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as A, A + 1, A + 2, respectively. Let a, b, c be the 7^{th}, 9^{th}, 17^{th} terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix}$ **+ 70 = 0**

If a = 29, then the sum of first 20 terms of an AP whose first term is c − a − b and common difference is $\frac{d}{12}$, is equal to ________.
[25-Jan-2023 Shift 1]

Answer: 495

Solution:

Solution:

$$\begin{vmatrix} A+6d & 7 & 1 \\ 2(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix} + 70 = 0$$
$\Rightarrow A = -7$ and $d = 6$
$\therefore c - a - b = 20$
$S_{20} = 495$

Question49

For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then, 16a + 12b is equal to ________.
[25-Jan-2023 Shift 2]

Answer: 3

Solution:

Solution:

$a, b, \frac{1}{18} \rightarrow GP$

$\frac{a}{18} = b^2$

$\frac{1}{a}, 10, \frac{1}{b} \rightarrow AP$

$\frac{1}{a}+\frac{1}{b}=20$

$\Rightarrow a+b=20ab$, from eq. (i) ; we get

$\Rightarrow 18b^2+b=360b^3$

$\Rightarrow 360b^2-18b-1=0 \quad \{\because b\neq 0\}$

$\Rightarrow b=\frac{18\pm\sqrt{324+1440}}{720}$

$\Rightarrow b=\frac{18+\sqrt{1764}}{720} \quad \{\because b>0\}$

$\Rightarrow b=\frac{1}{12}$

$\Rightarrow =18\times\frac{1}{144}=\frac{1}{8}$

Now, $16a+12b=16\times\frac{1}{8}+12\times\frac{1}{12}=3$

Question50

Let a_1, a_2, a_3, ... be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24 , then $a_1a_9+a_2a_4a_9+a_5+a_7$ is equal to ______.
[29-Jan-2023 Shift 1]

Answer: 60

Solution:

Solution:

$a_4\cdot a_6=9\Rightarrow(a_5)^2=9\Rightarrow a_5=3$

$\&a_5+a_7=24\Rightarrow a_5+a_5r^2=24\Rightarrow(1+r^2)=8\Rightarrow r=\sqrt{7}$

$\Rightarrow a=\frac{3}{49}$

$\Rightarrow a_1a_9+a_2a_4a_9+a_5+a_7=9+27+3+21=60$

Question51

Let $\{a_k\}$ and $\{b_k\}$, $k\in N$, be two G.P.s with common ratio r_1 and r_2 respectively such that $a_1=b_1=4$ and $r_1<r_2$. Let $c_k=a_k+b_k$, $k\in N$. If $c_2=5$ and $c_3=\frac{13}{4}$ then $\sum_{k=1}^{\infty}c_k-(12a_6+8b_4)$ is equal to__
[29-Jan-2023 Shift 2]

Answer: 9

Solution:

Solution:
Given that
$c_k = a_k + b_k$ and $a_1 = b_1 = 4$
also $a_2 = 4r_1\ a_3 = 4r_1^2$
$b_2 = 4r_2\ b_3 = 4r_2^2$
Now $c_2 = a_2 + b_2 = 5$ and $c_3 = a_3 + b_3 = \frac{13}{4}$
$\Rightarrow r_1 + r_2 = \frac{5}{4}$ and $r_1^2 + r_2^2 = \frac{13}{16}$
Hence $r_1 r_2 = \frac{3}{8}$ which gives $r_1 = \frac{1}{2}$ & $r_2 = \frac{3}{4}$
$\sum_{k=1^{\infty}c}^{k} - (12a_6 + 8b_4)$
$= \frac{4}{1-r_1} + \frac{4}{1-r_2} - \left(\frac{48}{32} + \frac{27}{2}\right)$
$= 24 - 15 = 9$

Question52

Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall n \ge 2$. If $S = \sum_{n=1^{10}} \frac{b_n}{2^n}$ and $T = \sum_{n=1^{8}} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to ______.

[29-Jan-2023 Shift 2]

Answer: 461

Solution:

Solution:

As, $S = \frac{b_1}{2} + \frac{b_2}{2^2} + \ldots\ldots + \frac{b_9}{2^9} + \frac{b_{10}}{2^{10}}$

$\Rightarrow \frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \ldots\ldots + \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}}$

subtracting

$\Rightarrow \frac{S}{2} = \frac{b_1}{2} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} \ldots\ldots + \frac{a_9}{2^{10}}\right) - \frac{b_{10}}{2^{11}}$

$\Rightarrow S = b_1 - \frac{b_{10}}{2^{10}} + \left(\frac{a_1}{2} + \frac{a_2}{2^2} \ldots\ldots\ldots + \frac{a_9}{2^9}\right)$

$\Rightarrow \frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} \ldots\ldots\ldots + \frac{a_9}{2^{10}}\right)$

subtracting

$\Rightarrow \frac{s}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2} - \frac{a_9}{2^{10}}\right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{8}{2^9}\right)$

$$\Rightarrow \frac{S}{2} = \frac{a_1+b_1}{2} - \frac{(b_{10}+2a_9)}{2^{11}} + \frac{T}{4}$$
$$\Rightarrow 2S = 2(a_1+b_1) - \frac{(b_{10}+2a_9)}{2^9} + T$$
$$\Rightarrow 2^7(2S-T) = 2^8(a_1+b_1) - \frac{(b_{10}+2a_9)}{4}$$
Also, $b_n - b_{n-1} = a_{n-1}$
$\therefore b_{10} - b_1 = a_1 + a_2 + \ldots + a_9$
$= 1+2+4+7+11+16+22+29+37$
$\Rightarrow b_{10} = 130($ As $b_1 = 1)$
$\therefore 2^7(2S-T) = 2^8(1+1) - (130 + 2 \times 37)$
$2^9 - \frac{204}{4} = 461$

Question53

If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + \ldots\ldots + a_{25}$ is equal to : [30-Jan-2023 Shift 1]

Options:

A. $\frac{51}{144}$

B. $\frac{49}{138}$

C. $\frac{50}{141}$

D. $\frac{52}{147}$

Answer: C

Solution:

Solution:
Option (3)
If $a_n = \frac{-2}{4n^2-16n+15}$ then $a_1 + a_2 + \ldots\ldots . a_{25}$
$$\Rightarrow \sum_{n=1}^{25} a_n = \sum \frac{-2}{4n^2-16n+15}$$
$$= \sum \frac{-2}{4n^2-6n-10n+15}$$
$$= \sum \frac{-2}{2n(2n-3)-5(2n-3)}$$
$$= \sum \frac{-2}{(2n-3)(2n-5)}$$
$$= \sum \frac{1}{2n-3} - \frac{1}{2n-5}$$
$$= \frac{1}{47} - \frac{1}{(-3)}$$
$$= \frac{50}{141}$$

Question54

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n}$ Then $a^2 - b + c$ is equal to _______.
[30-Jan-2023 Shift 1]

Answer: 26

Solution:

Solution:

$$\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)}$$

$$= \sum_{n=0}^{\infty} \frac{1}{(n-3)!} + \sum_{n=0}^{\infty} \frac{3}{(n-2)!}$$

$$+ \sum_{n=0}^{\infty} \frac{1}{(n-1)!} + \sum_{n=0}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$= e + 3e + e + \frac{1}{2}\left(e - \frac{1}{e}\right) - \frac{1}{2}\left(e + \frac{1}{e}\right)$$

$$= 5e - \frac{1}{e}$$

$\therefore a^2 - b + c = 26$

Question55

Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., and $\log_a b$, $\log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to
[30-Jan-2023 Shift 2]

Options:

A. 343

B. 216

C. $\frac{343}{8}$

D. $\frac{125}{8}$

Answer: B

Solution:

Solution:

As a^3, b^3, c^3 be in A.P. $\rightarrow a^3 + c^3 = 2b^3$ . . . (1)
$\log_a^{\ b}, \log_c^{\ a}, \log_b^{\ c}$ are in G.P.
$$\therefore \frac{\log b}{\log a} \cdot \frac{\log c}{\log b} = \left(\frac{\log a}{\log c}\right)^2$$
$\therefore (\log a)^3 = (\log c)^3 \Rightarrow a = c$ . . . (2)
From (1) and (2)
$a = b = c$
$$T_1 = \frac{a + 4b + c}{3} = 2a;\ d = \frac{a - 8b + c}{10} = \frac{-6a}{10} = \frac{-3}{5}a$$
$$\therefore S_{20} = \frac{20}{2}\left[4a + 19\left(-\frac{3}{5}a\right)\right]$$
$$= 10\left[\frac{20a - 57a}{5}\right]$$
$= -74a$
$\therefore -74a = -444 \Rightarrow a = 6$
$\therefore abc = 6^3 = 216$

Question56

The parabolas : $ax^2 + 2bx + cy = 0$ and $d\,x^2 + 2ex + f\,y = 0$ intersect on the line $y = 1$. If a, b, c, d , e, f are positive real numbers and a, b, c are in G.P., then [30-Jan-2023 Shift 2]

Options:

A. d, e, f are in A.P.

B. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

C. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

D. d, e, f are in G.P.

Answer: C

Solution:

Solution:
$ax^2 + 2bx + c = 0$
$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 (\because b^2 = ac)$
$\Rightarrow (x\sqrt{a} + \sqrt{c})^2 = 0$
$x^2 - \frac{\sqrt{c}}{\sqrt{a}}$ ......
Now, $dx^2 + 2ex + f = 0$
$$\Rightarrow d\left(\frac{c}{a}\right) + 2e\left[-\frac{\sqrt{c}}{\sqrt{a}}\right] + f = 0$$
$$\Rightarrow \frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$$
$$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2e\sqrt{\frac{1}{ac}}$$
$$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}[\text{ as } b = \sqrt{ae}]$$
$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Question57

The 8^{th} common term of the series
$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots.$
$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots$
is ________.
[30-Jan-2023 Shift 2]

Answer: 151

Solution:

Solution:
$T_8 = 11 + (8 - 1) \times 20$
$= 11 + 140 = 151$

Question58

Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$. Then
$S = \left\{x \in R : \tan^{-1}(\sqrt{f(x)} + \sin^{-1}(\sqrt{f(x) + 1})) = \frac{\pi}{2}\right\}$:
[31-Jan-2023 Shift 1]

Options:

A. contains exactly two elements

B. contains exactly one element

C. is an infinite set

D. is an empty set

Answer: A

Solution:

Solution:
$\left(x + \frac{1}{2}\right)^2 = \left(y + \frac{1}{4}\right)$
$y = (x^2 + x)$
$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \pi/2$

$0 \le x^2 + x + 1 \le 1$
$x^2 + x \le 0 \ldots$ (1)
Also $x^2 + x \ge 0 \ldots$ (2)
$\therefore x^2 + x = 0 \Rightarrow x = 0, -1$
S contains 2 element.

Question59

Let $a_1, a_2, \ldots\ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$$12\left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \ldots \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}}\right)$$

is equal to _______.
[31-Jan-2023 Shift 1]

Answer: 8

Solution:

Solution:
$2a_7 = a_5$ (given)
$2(a_1 + 6d) = a_1 + 4d$
$a_1 + 8d = 0 \ldots$ (1)
$a_1 + 10d = 18 \ldots$ (2)
By (1) and (2) we get $a_1 = -72, d = 9$
$a_{18} = a_1 + 17d = -72 + 153 = 81$
$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \ldots\ldots \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d}\right)$$

$$12\left(\frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d}\right) = \frac{12(9-3)}{9} = \frac{12 \times 6}{6} = 8$$

Question60

Let $a_1, a_2, a_3, \ldots.$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :
[31-Jan-2023 Shift 2]

Options:

A. 24

B. $\frac{33}{4}$

C. $\frac{381}{4}$

D. 9

Answer: A

Solution:

Solution:
$a+6d = 3 \ldots (1)$
$Z = a(a+3d)$
$= (3-6d)(3-3d)$
$= 18d^2 - 27d + 9$
Differentiating with respect to d
$\Rightarrow 36d - 27 = 0$
$\Rightarrow d = \frac{3}{4}$, from (1) $a = \frac{-3}{2}$, ($Z =$ minimum)
Now, $S_n = \frac{n}{2}\left(-3+(n-1)\frac{3}{4}\right) = 0$
$\Rightarrow n = 5$
Now,
$n! - 4a_{n(n+2)} = 120 - 4(a_{35})$
$= 120 - 4(a+(35-1)d)$
$= 120 - 4\left(\frac{-3}{2} + 34 \cdot \left(\frac{3}{4}\right)\right)$
$= 120 - 4\left(\frac{-6+102}{4}\right)$
$= 120 - 96 = 24$

Question61

The sum $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \ldots\ldots + 15.29^2$ is ________. [31-Jan-2023 Shift 2]

Answer: 6952

Solution:

Solution:
Separating odd placed and even placed terms we get
$S = (1.1^2 + 3.5^2 + \ldots .15 \cdot (29)^2) - (2.3^2 + 4.7^{2.}$
$+ \ldots + 14.(27)^2$
$S = \sum_{n=1}^{8}(2n-1)(4n-3)^2 - \sum_{n=1}^{7}(2n)(4n-1)^2$
Applying summation formula we get
$= 29856 - 22904 = 6952$

Question62

The sum to 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots$ is:-
[1-Feb-2023 Shift 1]

Options:

A. $\frac{59}{111}$

B. $\frac{55}{111}$

C. $\frac{56}{111}$

D. $\frac{58}{111}$

Answer: B

Solution:

Solution:

$T_r = \frac{(r^2+r+1)-(r^2-r+1)}{2(r^4+r^2+1)}$

$\Rightarrow T_r = \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right]$

$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$

$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$

$T_3 = \frac{1}{2}\left[\frac{1}{7} - \frac{1}{13}\right]$

$T_{10} = \frac{1}{2}\left[\frac{1}{91} - \frac{1}{111}\right]$

$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{55}{111}$

Question63

Let $a_1 = 8, a_2, a_3, \ldots a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is ______.
[1-Feb-2023 Shift 1]

Answer: 754

Solution:

Solution:

$$a_1 + a_2 + a_3 + a_4 = 50$$
$$\Rightarrow 32 + 6d = 50$$
$$\Rightarrow d = 3$$
$$\text{and, } a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$$
$$\Rightarrow 32 + (4n - 10) \cdot 3 = 170$$
$$\Rightarrow n = 14$$
$$a_7 = 26,\ a_8 = 29$$
$$\Rightarrow a_7 \cdot a_8 = 754$$

Question64

The number of 3 -digit numbers, that are divisible by either 2 or 3 but not divisible by 7 is _______.
[1-Feb-2023 Shift 1]

Answer: 514

Solution:

Solution:
Divisible by $2 \rightarrow 450$
Divisible by $3 \rightarrow 300$
Divisible by $7 \rightarrow 128$
Divisible by $2\&7 \rightarrow 64$
Divisible by $3\&7 \rightarrow 43$
Divisible by $2\&3 \rightarrow 150$
Divisible by $2, 3\&7 \rightarrow 21$
$\therefore$ Total numbers $= 450 + 300 - 150 - 64 - 43 + 21 = 514$

Question65

Which of the following statements is a tautology ?
[1-Feb-2023 Shift 2]

Options:

A. $p \rightarrow (p \Lambda (p \rightarrow q))$

B. $(p \Lambda q) \rightarrow (\sim(p) \rightarrow q))$

C. $(p \Lambda (p \rightarrow q)) \rightarrow \sim q$

D. $p V (p \Lambda q)$

Answer: B

Solution:

Solution:
(i)$p \to (p \wedge (p \to q))$
$(\sim p) \vee (p \wedge (\sim p \vee q))$
$(\sim p) \vee (f \vee (p \wedge q))$
$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$
$= \sim p \vee q$
(ii) $(p \wedge q) \to (\sim p \to q)$
$\sim (p \wedge q) \vee (p \vee q) = t$
$\{a, b, d\} \vee \{a, b, c\} = V$
Tautology
(iii) $(p \wedge (p \to q)) \to \sim q$
$\sim (p \wedge (\sim p \vee q)) \vee \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$
Not tantology
(iv) $p \vee (p \wedge q) = p$
Not tautology.

Question66

The sum of the common terms of the following three arithmetic progressions.
3, 7, 11, 15, …………, 399
2, 5, 8, 11, ………….., 359 and
2, 7, 12, 17, ……, 197, is equal to ________.
[1-Feb-2023 Shift 2]

Answer: 321

Solution:

Solution:
3, 7, 11, 15, …………, 399 $d_1 = 4$
2, 5, 8, 11, ………….., 359 $d_2 = 3$
2, 7, 12, 17, ……, 197 $d_3 = 5$
$\mathrm{LCM}(d_1, d_2, d_3) = 60$
Common terms are 47, 107, 167
Sum = 321

Question67

The sum of the first 20 terms of the series 5 + 11 + 19 + 29 + 41 + …… is :
[6-Apr-2023 shift 1]

Options:

A. 3450

B. 3420

C. 3520

D. 3250

Answer: C

Solution:

Solution:

$$s_n = 5+11+19+29+41+.....+T_n$$
$$S_n = \quad 5+11+19+29+...+T_{n-1}+T_n$$
$$0 = 5+\left\{\underbrace{6+8+10+12+....}_{(n-1)\text{terms}}\right\}-T_n$$

$T_n = 5+\frac{(n-1)}{2}[2\cdot 6+(n-2)\cdot 2]$

$T_n = 5+(n-1)(n+4) = 5+n^2+3n-4 = n^2+3n+1$

Now $S_{20} = \sum_{n=1}^{20} T_n = \sum_{n=1}^{20} n^2+3n+1$

$S_{20} = \frac{20.21.41}{6}+\frac{3.20.21}{2}+20$

$S_{20} = 2870+630+20$

$S_{20} = 3520$

Question68

Let $a_1, a_2, a_3, ..., a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then :

$\lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+.......+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ **is**

[6-Apr-2023 shift 1]

Options:

A. $\frac{1}{\sqrt{d}}$

B. 1

C. $\sqrt{d}$

D. 0

Answer: B

Solution:

Solution:

$$\lim_{n \to \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_1} - \sqrt{a_2}}{a_1 - a_2} + \frac{\sqrt{a_2} - \sqrt{a_3}}{a_2 - a_3} + \ldots\ldots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{a_{n-1} - a_n} \right)$$

$$= \lim_{n \to \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_1} - \sqrt{a_2} + \sqrt{a_2} + \sqrt{a_3} + \ldots\ldots + \sqrt{a_{n-1}} - \sqrt{a_n}}{-d} \right)$$

$$= \lim_{n \to \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_n} - \sqrt{a_1}}{d} \right)$$

$$= \lim_{n \to \infty} \frac{1}{\sqrt{n}} \left(\frac{\sqrt{a_1 + (n-1)d} - \sqrt{a_1}}{\sqrt{d}} \right)$$

$$= \lim_{n \to \infty} \frac{1}{\sqrt{d}} \left(\sqrt{\frac{a_1}{n} + d - \frac{d}{n}} - \frac{\sqrt{a_1}}{n} \right)$$

$$= 1$$

Question69

If gcd(m, n) = 1 and $1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$ then $m^2 - n^2$ is equal to : [6-Apr-2023 shift 2]

Options:

A. 180

B. 220

C. 200

D. 240

Answer: D

Solution:

Solution:

$$(1-2)(1+2) + (3-4)(3+4) + \ldots\ldots + (2021-2022)(2021+2022) + (2023)^2 = (1012)m^2n$$

$$\Rightarrow (-1)[1+2+3+4+\ldots+2022] + (2023)^2 = (1012)m^2n$$

$$\Rightarrow (-1)\frac{(2022)(2023)}{2} + (2023)^2 = (1012)m^2n$$

$$\Rightarrow (2023)[2023 - 1011] = (1012)m^2n$$

$$\Rightarrow (2023)(1012) = (1012)m^2n$$

$$\Rightarrow m^2n = 2023$$

$$\Rightarrow m^2n = (17)^2 \times 7$$

$$m = 17, n = 7$$

$$m^2 - n^2 = (17)^2 - 7^2 = 289 - 49 = 240$$

Ans. Option 4

Question70

If $(20)^{19}+2(21)(20)^{18}+3(21)^{2}(20)^{17}+\ldots+20(21)^{19}=k(20)^{19}$, then k is equal to ________ :
[6-Apr-2023 shift 2]

Answer: 400

Solution:

Solution:

$S=(20)^{19}+2(21)(20)^{18}+\ldots\ldots+20(21)^{19}$

$\frac{21}{20}S=21(20)^{18}+2(21)^{9}(20)^{17}+\ldots\ldots.+(21)^{20}$

Subtract

$\left(1-\frac{21}{20}\right)S=(20)^{19}+(21)(20)^{18}+(21)^{2}(20)^{17}+\ldots\ldots+(21)^{19}-(21)^{20}$

$$\left(\frac{-1}{20}\right)S=(20)^{19}\left[\frac{1-\left(\frac{21}{20}\right)^{20}}{1-\frac{21}{20}}\right]-(21)^{20}$$

$\left(\frac{-1}{20}\right)S=(21)^{20}-(20)^{20}-(21)^{20}$

$S=(20)^{21}=K(20)^{19}$ (given)

$K=(20)^{2}=400$

Question71

Let S_K $\frac{1+2+\ldots+K}{K}$ and $\sum_{j=1}^{n}S_j^{\,2}=\frac{n}{A}(Bn^2+Cn+D)$, where A, B, C, D $\in$ N and A has least value. Then
[8-Apr-2023 shift 1]

Options:

A. $A+B$ is divisible by D

B. $A+B=5(D-C)$

C. $A+C+D$ is not divisible by B

D. $A+B+D$ is divisible by 5

Answer: A

Solution:

Solution:

$S_k=\frac{k+1}{2}$

$S_k^{\,2}=\frac{k^2+1+2k}{4}$

$$\therefore \sum_{j=1}^{n} S_j^{\,2} = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n + n(n+1)\right]$$
$$= \frac{n}{4}\left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1\right]$$
$$= \frac{n}{4}\left[\frac{2n^2+3n+1}{6} + n + 2\right]$$
$$= \frac{n}{4}\left[\frac{2n^2+9n+13}{6}\right] = \frac{n}{24}[2n^2+9n+13]$$
A = 24, B = 2, C = 9, D = 13

Question72

Let a_n be the n^{th} term of the series $5+8+14+23+35+50+\ldots$. and $Sn = \sum_{k=1}^{n} a_K$. Then $S_{30} - a_{40}$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 11260

B. 11280

C. 11290

D. 11310

Answer: C

Solution:

Solution:
$$S_n = 5+8+14+23+35+50+\ldots+a_n$$
$$S_n = 5+8+14+23+35+\ldots+a_n$$

$$O = 5+3+6+9+12+15+\ldots-a_n$$
$$a_n = 5+(3+6+9+\ldots(n-1)\ \text{terms}\ )$$
$$a_n = \frac{3n^2-3n+10}{2}$$
$$a_{40} = \frac{3(40)^2-3(40)+10}{2} = 2345$$
$$S_{30} = \frac{3\sum_{n=1}^{30} n^2 - 3\sum_{n=1}^{30} n + 10\sum_{n=1}^{30} 1}{2}$$
$$= \frac{\frac{3\times30\times31\times61}{6} - \frac{3\times30\times31}{2} + 10\times30}{2}$$
$$S_{30} = 13635$$
$$S_{30} - a_{40} = 13635 - 2345$$
$$= 11290(\ \text{Option}\ (3))$$

Question73

Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x+y+z)^2$ is equal to _______.
[8-Apr-2023 shift 2]

Answer: 150

Solution:

Solution:

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$
$$2y^2 = xz$$
$$\frac{2}{y} = \frac{x+z}{xz} = \frac{x+z}{2y^2}$$
$$x + z = 4y$$
$$xy + yz + zx = \frac{3}{\sqrt{2}}xyz$$
$$y(x+z) + zx = \frac{3}{\sqrt{2}}xz \cdot y$$
$$4y^2 + 2y^2 = \frac{3}{\sqrt{2}}y \cdot 2y^2$$
$$6y^2 = 3\sqrt{2}y^3$$
$$y = \sqrt{2}$$
$$x + y + z = 5y = 5\sqrt{2}$$
$$3(x+y+z)^2 = 3 \times 50 = 150$$

Question74

Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033 , then the sum of these terms is equal to :
[10-Apr-2023 shift 1]

Options:

A. 210

B. 220

C. 231

D. 241

Answer: C

Solution:

Solution:
Let a, ar, ar^2 be three terms of GP
Given : $a^2+(ar)^2+(ar^2)^2=33033$
$a^2(1+r^2+r^4)=11^2.3.7.13$
$\Rightarrow a=11$ and $1+r^2+r^4=3.7.13$
$\Rightarrow r^2(1+r^2)=273-1$
$\Rightarrow r^2(r^2+1)=272=16\times17$
$\Rightarrow r^2=16$
$\therefore r=4\ [\because r>0]$
Sum of three terms $=a+ar+ar^2=a(1+r+r^2)$
$=11(1+4+16)$
$=11\times21=231$

Question75

If $f(x)=\frac{(\tan 1^{\circ})x+\log_e(123)}{x\log_e(1234)-(\tan 1^{\circ})}$, x > 0, then the least value of $f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)$ is: [10-Apr-2023 shift 1]

Options:

A. 2

B. 4

C. 8

D. 0

Answer: B

Solution:

Solution:
$f(x)=\frac{(\tan 1)x+\log 123}{x\log 1234-\tan 1}$
Let $A=\tan 1, B=\log 123, C=\log 1234$
$f(x)=\frac{Ax+B}{xC-A}$
$f(f(x))=\frac{A\left(\frac{Ax+B}{xC-A}\right)+B}{C\left(\frac{Ax+B}{CX-A}\right)-A}$
$=\frac{A^2x+AB+xBC-AB}{ACx+BC-ACx+A^2}$
$=\frac{x(A^2+BC)}{(A^2+BC.}=x$
$f(f(x))=x$
$f\left(f\left(\frac{4}{x}\right)\right)=\frac{4}{x}$
$f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)$
$AM\geq GM$
$x+\frac{4}{x}\geq4$

Question76

The sum of all those terms, of the arithmetic progression 3, 8, 13, …., 373, which are not divisible by 3 , is equal to _______.
[10-Apr-2023 shift 1]

Answer: 9525

Solution:

Solution:
A.P: $3, 8, 13 \ldots \ldots 373$
$T_n = a + (n-1)d$
$373 = 3 + (n-1)5$
$\Rightarrow n = \frac{370}{5}$
$\Rightarrow n = 75$
$\text{Now Sum} = \frac{n}{2}[a + 1]$
$= \frac{75}{2}[3 + 373] = 14100$
Now numbers divisible by 3 are,
$3, 18, 33 \ldots \ldots 363$
$363 = 3 + (k-1)15$
$\Rightarrow k - 1 = \frac{360}{15} = 24 \Rightarrow k = 25$
$\text{Now, sum} = \frac{25}{2}(3 + 363) = 4575 \text{ s}$
$\therefore \text{ req. sum} = 14100 - 4575$
$= 9525$

Question77

If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots.$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to
[10-Apr-2023 shift 2]

Options:

A. 220

B. 227

C. 226

D. 223

Answer: D

Solution:

Solution:
$S_n = 4 + 11 + 21 + 34 + 50 + \ldots + n$ terms
Difference are in A.P.
Let $T_n = an^2 + bn + c$
$T_1 = a + b + c = 4$
$T_2 = 4a + 2b + c = 11$
$T_3 = 9a + 3b + c = 21$
By solving these 3 equations
$a = \frac{3}{2}, b = \frac{5}{2}, c = 0$
So $T_n = \frac{3}{2}n^2 + \frac{5}{2}n$
$S_n = \Sigma T_n$
$= \frac{3}{2}\sum n^2 + \frac{5}{2}\sum n$
$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2}\frac{(n)(n+1)}{2}$
$= \frac{n(n+1)}{4}[2n+1+5]$
$S_n = \frac{n(n+1)}{4}(2n+6) = \frac{n(n+1)(n+3)}{2}$
$\frac{1}{60}\left(\frac{29 \times 30 \times 32}{2} - \frac{9 \times 10 \times 12}{2}\right) = 223$

Question78

Suppose $a_1, a_2, 2, a_3, a_4$ be in an arithemetico-geometric progression. If the common ratio of the corresponding geometric progression in 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to _______. [10-Apr-2023 shift 2]

Answer: 16

Solution:

Solution:
$\frac{(a-2d)}{4}, \frac{(a-d)}{2}, a, 2(a+d), 4(a+2d)$
$a = 2$
$\left(\frac{1}{4} + \frac{1}{2} + 1 + 6\right) \times 2 + (-1 + 2 + 8)d = \frac{49}{2}$
$2\left(\frac{3}{4} + 7\right) + 9d = \frac{49}{2}$
$9d = \frac{49}{2} - \frac{62}{4} = \frac{98 - 62}{4} = 9$
$d = 1$
$\Rightarrow a_4 = 4(a + 2d)$
$= 16$

Question79

Let $x_1, x_2, \ldots.., x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200 . If $y_i = i(x_i - i)$, $1 \le i \le 100$, then the mean of $y_1, y_2, \ldots . y_{100}$ is :
[11-Apr-2023 shift 1]

Options:

A. 10051.50

B. 10100

C. 10101.50

D. 10049.50

Answer: D

Solution:

Solution:

$$\text{Mean} = 200$$
$$\Rightarrow \frac{\frac{100}{2}(2 \times 2 + 99d)}{100} = 200$$
$$\Rightarrow 4 + 99d = 400$$
$$\Rightarrow d = 4$$
$$y_i = i(xi - 1)$$
$$= i(2 + (i-1)4 - i) = 3i^2 - 2i$$
$$\text{Mean} = \frac{\Sigma y_i}{100}$$
$$= \frac{1}{100}\sum_{i=1}^{100} 3i^2 - 2i$$
$$= \frac{1}{100}\left\{\frac{3 \times 100 \times 101 \times 201}{6} - \frac{2 \times 100 \times 101}{2}\right\}$$
$$= 101\left\{\frac{201}{2} - 1\right\} = 101 \times 99.5$$
$$= 10049.50$$

Question80

Let $S = S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots. + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to _______.
[11-Apr-2023 shift 1]

Answer: 2175

Solution:

Solution:

$$S = 109 + \frac{108}{5} + \frac{107}{5^2} \ldots\ldots + \frac{1}{5^{108}}$$

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} \ldots\ldots \cdot \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 - \frac{1}{5} - \frac{1}{5^2} \ldots\ldots - \frac{1}{5^{108}} - \frac{1}{5^{109}}$$

$$= 109 - \left(\frac{1}{5} \frac{\left(1 - \frac{1}{5^{109}}\right))}{\left(1 - \frac{1}{5}\right)} \right)$$

$$= 109 - \frac{1}{4}\left(1 - \frac{1}{5^{109}}\right)$$

$$= 109 - \frac{1}{4} + \frac{1}{4} \times \frac{1}{5^{109}}$$

$$S = \frac{5}{4}\left(109 - \frac{1}{4} + \frac{1}{4.5^{109}}\right)$$

$$16S = 20 \times 109 - 5 + \frac{1}{5^{108}}$$

$$16S - (25)^{-54} = 2180 - 5 = 2175$$

Question81

Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is [11-Apr-2023 shift 2]

Options:

A. 55

B. 108

C. 90

D. 110

Answer: C

Solution:

Solution:
Given $a + b + c + d = 11$
$(a, b, c, d > 0\}$
$(a^5b^3c^2d)$ max. = ?
Let assume Numbers -
$\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2},$
We know A.M. $\geq$ G.M.

$$\frac{\frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} + d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 \cdot 3^3 \cdot 2^2 \cdot 1} \right)^{\frac{1}{11}}$$

$$\frac{11}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 \cdot 3^3 \cdot 2^2 \cdot 1} \right)^{\frac{1}{11}}$$

$a^5 \cdot b^3 \cdot c^2 \cdot d \le 5^5 \cdot 3^3 \cdot 2^2$
$\max(a^5 b^3 c^2 d) = 5^5 \cdot 3^3 \cdot 2^2 = 337500$
$= 90 \times 3750 = \beta \times 3750$
$\beta = 90$
Option (C) 90 correct

Question82

For $k \in N$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots$ is 10 , then the value of k is ______
[11-Apr-2023 shift 2]

Answer: 2

Solution:

Solution:

$$10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots \text{ upto } \infty$$
$$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots\ldots \text{ upto } \infty$$
$$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \ldots\ldots \text{upto } \infty$$

$$S = 9\left(1 - \frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} \ldots\ldots \text{ upto } \infty$$
$$\frac{S}{k} = \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \ldots\ldots \text{upto } \infty$$

$$\left(1 - \frac{1}{k}\right)S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + \ldots\ldots \infty$$
$$9\left(1 - \frac{1}{k}\right)^2 = \frac{4}{k} + \frac{\frac{1}{k^3}}{\left(1 - \frac{1}{k}\right)}$$
$$9(k-1)^3 = 4k(k-1) + 1$$
$$k = 2$$

Question83

Let <a_n> be a sequence such that $a_1 + a_2 + .. + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}$. If $28 \sum_{k=1}^{10} \frac{1}{a_k} = p_1 p_2 p_3 \ldots p_m$, where $p_1, p_2 \ldots\ldots p_m$ are the first m prime numbers, then m is equal to
[12-Apr-2023 shift 1]

Options:

A. 8

B. 5

C. 6

D. 7

Answer: C

Solution:

Solution:

$$a_n = S_n - S_{n-1} = \frac{n^2+3n}{(n+1)(1+2)} - \frac{(n-1)(n+2)}{n(n+1)}$$
$$\Rightarrow a_n = \frac{4}{n(n+1)(1+2)}$$
$$\Rightarrow 28\sum_{k-1}^{10}\frac{1}{a_k} = 28\sum_{k-1}^{10}\frac{k(k+1)(k+2)}{4}$$
$$= \frac{7}{4}\sum_{k-1}^{10}(k(k+1)(k+2)(k+3)-(k-1)k(k+1)(k+2))$$
$$= \frac{7}{4}\cdot 10\cdot 11\cdot 12\cdot 13 = 2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13$$

So $m = 6$

Question84

Let s_1, s_2, s_3, ..., s_{10} respectively be the sum to 12 terms of 10 A.P. s_m whose first terms are 1, 2, 3, ..., 10 and the common differences are 1, 3, 5, ..., 19 respectively. Then $\sum\limits_{i=1}^{10} s_i$ is equal to
[13-Apr-2023 shift 1]

Options:

A. 7260

B. 7380

C. 7220

D. 7360

Answer: A

Solution:

Solution:

$$S_k = 6(2k+(11)(2k-1))$$
$$S_k = 6(2k+22k-11)$$
$$S_k = 144k-66$$
$$\sum_1^{10} S_k = 144\sum_{k=1}^{10} k - 66\times 10$$

$= 144 \times \frac{10 \times 11}{2} - 660$
$= 7920 - 660$
$= 7260$

Question85

The sum to 20 terms of the series $2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - \ldots\ldots..$ is equal to _______.
[13-Apr-2023 shift 1]

Answer: 1310

Solution:

Solution:
$(2^2 - 3^2 + 4^2 - 5^2 + 20 \text{ terms }) + (2^2 + 4^2 + \ldots. + 10 \text{ terms })$
$-(2 + 3 + 4 + 5 + \ldots. + 11) + 4[1 + 2^2 + \ldots..10^2]$
$-\left[\frac{21 \times 22}{2} - 1\right] + 4 \times \frac{10 \times 11 \times 21}{6}$
$= 1 - 231 + 14 \times 11 \times 10$
$= 1540 + 1 - 231$
$= 1310$

Question86

Let a_1, a_2, a_3, ... be a G. P. of increasing positive numbers. Let the sum of its 6^{th} and 8^{th} terms be 2 and the product of its 3^{rd} and 5^{th} terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 2

B. 3

C. $3\sqrt{3}$

D. $2\sqrt{2}$

Answer: B

Solution:

Solution:

$a_3 \cdot a_5 = \frac{1}{9}$
$\Rightarrow ar^2 \cdot ar^4 = \frac{1}{9}$
$\Rightarrow (ar^3)^2 = \frac{1}{9}$
$\Rightarrow ar^3 = \frac{1}{3} \ldots$ (i)
$a_6 + a_8 = 2$
$\Rightarrow ar^5 + ar^7 = 2$
$\Rightarrow ar^3(r^2 + r^4) = 2$
$\Rightarrow \frac{1}{3}r^2(1 + r^2) = 2$
$\Rightarrow r^2(1 + r^2) = 2 \times 3$
$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$
$a = \frac{1}{3} \times \frac{1}{r^3}$
$= \frac{1}{3} \times \frac{1}{2\sqrt{2}} = \frac{1}{6\sqrt{2}}$
$6(a_2 + a_4)(a_4 + a_6)$
$\Rightarrow 6(ar + ar^3)(ar^3 + ar^5)$
$\Rightarrow 6\left(\frac{ar^3}{r^2} + \frac{1}{3}\right)\left(\frac{1}{3} + \frac{1}{3}r^2\right) = 3$

Question87

Let [α] denote the greatest integer ≤α. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots + [\sqrt{120}]$ is equal to _______.
[13-Apr-2023 shift 2]

Answer: 825

Solution:

Solution:
$S = [\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots + [\sqrt{120}]$
$[\sqrt{1}] \rightarrow [\sqrt{3}] = 1 \times 3$
$[\sqrt{4}] \rightarrow [\sqrt{8}] = 2 \times 5$
$[\sqrt{9}] \rightarrow [\sqrt{15}] = 3 \times 7$
$\vdots$
$[\sqrt{100}] \rightarrow [\sqrt{120}] = 10 \times 21$
$S = 1 \times 3 + 2 \times 5 + 3 \times 7 + \ldots + 10 \times 21$
$= \sum_{r=1}^{10} r(2r + 1)$
$= 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$
$= \frac{2 \times 10 \times 11 \times 21}{6} + \frac{10 \times 11}{2}$
$= 770 + 55$
$= 825$

Question88

Let $f(x) = \sum_{k=1}^{10} kx^k$, $x \in R$ If $2f'(2) - f'(2) = 119(2)^n + 1$ then n is equal to _______ [13-Apr-2023 shift 2]

Answer: 10

Solution:

Solution:

$$f(x) = \sum_{k=1}^{10} kx\,x^k$$

$$\Rightarrow f(x) = x + 2x^2 + 3x^3 + \cdots + 9x^9 + 10x^{10} - (i)$$

$$xf(x) = x^2 + 2x^3 + \ldots + 9x^{10} + 10x^{11} \ldots \text{ (ii)}$$

"(i) − (ii)'

$$f(x)(1-x) = x + x^2 + x^3 + \cdots + x^{10} - 10x^{11}$$

$$f(x)(1-x) = \frac{x(1-x^{10})}{1-x} - 10x^{11}$$

$$f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(2) = 2 + g(2)^{11}$$

$$(1-x)^2 f(x) = x(1-x^{10}) - 10x^{11}(1-x)$$

diff. w.r.t. x

$$(1-x)^2 f'(2) + f(2)2(1-x)(-1)$$

$$= x(-10x^9) + (1-x^{10}) - 10x^{11}(-1) - (1-x)(110)x^{10}$$

put $x = 2$

$$f'(2) + f(2)(2) = -10(2)^{10} + 1 - 2^{10} + 10(2)^{11} - 110(2)^{10} + 110(2)^{11}$$

$$= (-121)2^{10} + (120)2^{11} + 1$$

$$= 2^{10}(240 - 121) + 1$$

$$= 119(2)^{10} + 1$$

$$n = 10$$

Question89

Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$ is equal to [15-Apr-2023 shift 1]

Options:

A. $2(A_1 + A_2)G_1G_3$

B. $(A_1 + A_2)^2 G_1G_3$

C. $2(A_1 + A_2)G_1^2G_3^2$

D. $(A_1 + A_2)G_1{}^2G_3{}^2$

Answer: B

Solution:

Solution:
a, A_1, A_2, b are in A.P.
$d = \frac{b-a}{3}$; $A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$
$A_2 = \frac{a+2b}{3}$
$A_1 + A_2 = a + b$
a, G_1, G_2, G_3, b are in G.P.
$r = \left(\frac{b}{a}\right)^{\frac{1}{4}}$
$G_1 = (a^3b)^{\frac{1}{4}}$
$G_2 = (a^2b^2)^{\frac{1}{4}}$
$G_3 = (ab^3)^{\frac{1}{4}}$
$G_1{}^4 + {}_2{}^4 + G_3{}^4 + G_1{}^2G_3{}^2 =$
$a^3b + a^2b^2 + ab^3 + (a^3b)^{\frac{1}{2}} \cdot (ab^3)^{\frac{1}{2}}$
$= a^3b + a^2b^2 + ab^3 + a^2 \cdot b^2$
$= ab(a^2 + 2ab + b^2)$
$= ab(a+b)^2$
$= G_1 \cdot G_3 \cdot (A_1 + A_2)^2$

Question90

If the sum of the series $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2\cdot 3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{2^2\cdot 3} + \frac{1}{2\cdot 3^2} - \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{2^2\cdot 3} + \frac{1}{2^2\cdot 3^2} - \frac{1}{2\cdot 3^2} + \frac{1}{3^4}\right) + \ldots\ldots.$ **is** $\frac{\alpha}{\beta}$**, where** α **and** β **are co-prime, then** $\alpha + 3\beta$ **is equal to ________**
[15-Apr-2023 shift 1]

Answer: 7

Solution:

Solution:

$$P\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2\cdot 3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{2^2\cdot 3}+\frac{1}{2\cdot 3^2}-\frac{1}{3^3}\right)+\ldots P\left(\frac{1}{2}+\frac{1}{3}\right)=\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{3^4}\right)+\ldots$$

$$\frac{5P}{6}=\frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1+\frac{1}{3}}$$

$$\frac{5P}{6}=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$$

$$\therefore P=\frac{1}{2}=\frac{\alpha}{\beta}\quad \therefore \alpha=1, \beta=2$$

$$\alpha+3\beta=7$$

Question91

If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference
1 , and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to :
[24-Jun-2022-Shift-1]

Options:

A. 48

B. 96

C. 92

D. 104

Answer: B

Solution:

Solution:

$$\sum_{i=1}^{n} a_i = 192$$

$$\Rightarrow a_1+a_2+a_3+\ldots\ldots+a_n=192$$

$$\Rightarrow \frac{n}{2}[a_1+a_n]=192$$

$$\Rightarrow a_1+a_n=\frac{384}{n}\ldots\ldots\ (1)$$

$$\text{Now, } \sum_{i=1}^{\frac{n}{2}} a_{2i}=120$$

$$\Rightarrow a_2+a_4+a_6+\ldots\ldots+a_n=120$$

Here total $\frac{n}{2}$ terms present.

$\frac{n}{}$

$\therefore \frac{\overline{2}}{2}[a_2 + a_n] = 120$

$\Rightarrow \frac{n}{4}[a_1 + 1 + a_n] = 120$

$\Rightarrow a_1 + a_n + 1 = \frac{480}{n}$.....

Subtracting (1) from (2), we get

$1 = \frac{480}{n} - \frac{384}{n}$

$\Rightarrow 1 = \frac{96}{n}$

$\Rightarrow$n $= 96$

Question92

If $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \ldots + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$, then the remainder when K is divided by 6 is : [25-Jun-2022-Shift-1]

Options:

A. 1

B. 2

C. 3

D. 5

Answer: D

Solution:

Solution:

$$\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \ldots + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$$

$$\Rightarrow \frac{1}{2 \cdot 3^{10}} \left[\frac{\left(\frac{3}{2}\right)^{10} - 1}{\frac{3}{2} - 1} \right] = \frac{K}{2^{10} \cdot 3^{10}}$$

$$= \frac{3^{10} - 2^{10}}{2^{10} \cdot 3^{10}} = \frac{K}{2^{10} \cdot 3^{10}} \Rightarrow K = 3^{10} - 2^{10}$$

Now $K = (1+2)^{10} - 2^{10}$

$= {}^{10}C_0 + {}^{10}C_1 2 + {}^{10}C_2 2^3 + \ldots + {}^{10}C_{10} 2^{10} - 2^{10}$

$= {}^{10}C_0 + {}^{10}C_1 2 + 6\lambda + {}^{10}C_9 \cdot 2^9$

$= 1 + 20 + 5120 + 6\lambda$

$= 5136 + 6\lambda + 5$

$= 6\mu + 5$

$\lambda, \mu \in N$

$\therefore$ remainder $= 5$

Question93

The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, .$ is equal to
[25-Jun-2022-Shift-1]

Answer: 98

Solution:

Solution:

$S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \ldots$

$= \sum_{r=1}^{100} \left(\frac{3^r - 2^r}{3^r} \right)$

$= 100 - \frac{2}{3} \frac{\left(1 - \left(\frac{2}{3}\right)^{100}\right)}{1/3}$

$= 98 + 2\left(\frac{2}{3}\right)^{100}$

$\therefore [S] = 98$

Question94

The sum $1+2\cdot3+3\cdot3^2+\ldots\ldots\ldots+10.3^9$ is equal to
[25-Jun-2022-Shift-2]

Options:

A. $\frac{2\cdot3^{12}+10}{4}$

B. $\frac{19\cdot3^{10}+1}{4}$

C. $5\cdot3^{10}-2$

D. $\frac{9\cdot3^{10}+1}{2}$

Answer: B

Solution:

Solution:
Let $S=1.3^0+2.3^1+3.3^2+\ldots\ldots+10.3^9$
$3S=1.3^1+2.3^2+\ldots\ldots\ldots+10.3^{10}$
$-2S=(1.3^0+1.3^1+1.3^2+\ldots\ldots+1.3^9)-10.3^{10}$
$\Rightarrow S=\frac{1}{2}\left[10.3^{10}-\frac{3^{10}-1}{-3-1}\right]$
$\Rightarrow S=\frac{19.3^{10}+1}{4}$

Question95

Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$ and $B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$. Then $A+B$ is equal to
[26-Jun-2022-Shift-1]

Answer: 1100

Solution:

Solution:

$$A = \sum_{i=1}^{10}\sum_{j=1}^{10} \min\{i, j\}$$

$$B = \sum_{i=1}^{10}\sum_{j=1}^{10} \max\{i, j\}$$

$$A = \sum_{j=1}^{10} \min(i, 1) + \min(j, 2) + \ldots \min(i, 10)$$

$$= (\underbrace{1 + 1 + 1 + 1 + \ldots + 1}_{19 \text{ times}}) + (\underbrace{2 + 2 + 2\ldots + 2}_{17 \text{ times}}) + (\underbrace{3 + 3 + 3\ldots + 3}_{15 \text{ times}}) + \ldots (1)1 \text{ times}$$

$$B = \sum_{j=1}^{10} \max(i, 1) + \max(j, 2) + \ldots \max(i, 10)$$

$$= (\underbrace{10 + 10 + \ldots + 10}_{19 \text{ times}}) + (\underbrace{9 + 9 + \ldots + 9}_{17 \text{ times}}) + \ldots + 11 \text{ times}$$

$$A + B = 20(1 + 2 + 3 + \ldots + 10)$$

$$= 20 \times \frac{10 \times 11}{2} = 10 \times 110 = 1100$$

Question96

If $A = \sum_{n=1}^{\infty} \frac{1}{(3 + (-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3 + (-1)^n)^n}$, then $\frac{A}{B}$ is equal to : [26-Jun-2022-Shift-2]

Options:

A. $\frac{11}{9}$

B. 1

C. $-\frac{11}{9}$

D. $-\frac{11}{3}$

Answer: C

Solution:

Solution:

$$A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n} \text{ and } B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$$

$$A = \frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \dots\dots$$

$$B = \frac{-1}{2} + \frac{1}{4^2} - \frac{1}{2^3} + \frac{1}{4^4} + \dots\dots$$

$$A = \frac{\frac{1}{2}}{1-\frac{1}{4}} + \frac{\frac{1}{16}}{1-\frac{1}{16}}, B = \frac{-\frac{1}{2}}{1-\frac{1}{4}} + \frac{\frac{1}{16}}{1-\frac{1}{16}}$$

$$A = \frac{11}{15}, B = \frac{-9}{15}$$

$$\therefore \frac{A}{B} = \frac{-11}{9}$$

Question97

If $a_1(>0), a_2, a_3, a_4, a_5$ are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to____
[26-Jun-2022-Shift-2]

Answer: 40

Solution:

Solution:

Let G.P. be $a_1 = a, a_2 = ar, a_3 = ar^2,$

$\because 3a_2 + a_3 = 2a_4$

$\Rightarrow 3ar + ar^2 = 2ar^3$

$\Rightarrow 2ar^2 - r - 3 = 0$

$\therefore r = -1$ or $\frac{3}{2}$

$\because a_1 = a > 0$ then $r \neq -1$

Now, $a_2 + a_4 = 2a_3 + 1$

$ar + ar^3 = 2ar^2 + 1$

$a\left(\frac{3}{2} + \frac{27}{8} - \frac{9}{2}\right) = 1$

$\therefore a = \frac{8}{3}$

$\therefore a_2 + a_4 + 2a_5 = a(r + r^3 + 2r^4)$

$= \frac{8}{3}\left(\frac{3}{2} + \frac{27}{8} + \frac{81}{8}\right) = 40$

Question98

$x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1$, $abc \neq 0$, then :
[27-Jun-2022-Shift-1]

Options:

A. x, y, z are in A.P.

B. x, y, z are in G.P.

C. $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

D. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Answer: C

Solution:

Solution:
$x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}; y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}; z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$
Now,
$a, b, c \rightarrow AP$
$1-a, 1-b, 1-c \rightarrow AP$

$\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c} \rightarrow HP$

$x, y, z \rightarrow HP$

$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \rightarrow AP$

Question99

If the sum of the first ten terms of the series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+\ldots$ is $\frac{m}{n}$, where m and n are co-prime numbers, then $m+n$ is equal to

[27-Jun-2022-Shift-1]

Answer: 276

Solution:

Solution:

$T_r=\frac{r}{(2r^2)^2+1}$

$=\frac{r}{(2r^2+1)^2-(2r)^2}$

$=\frac{1}{4}\frac{4r}{(2r^2+2r+1)(2r^2-2r+1)}$

$S_{10}=\frac{1}{4}\sum_{r=1}^{10}\left(\frac{1}{(2r^2-2r+1)}-\frac{1}{(2r^2+2r+1)}\right)$

$=\frac{1}{4}\left[1-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+\ldots+\frac{1}{181}-\frac{1}{221}\right]$

$\Rightarrow S_{10}=\frac{1}{4}\cdot\frac{220}{221}=\frac{55}{221}=\frac{m}{n}$

$\therefore m+n=276$

Question100

Let $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+\ldots..$ Then 4S is equal to

[27-Jun-2022-Shift-2]

Options:

A. $\left(\frac{7}{3}\right)^2$

B. $\frac{7^3}{3^2}$

C. $\left(\frac{7}{3}\right)^3$

D. $\frac{7^2}{3^3}$

Answer: C

Solution:

Solution:

$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots\ldots$ (i)

$\frac{1}{7}S = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots\ldots\ldots$ (ii)

(i) - (ii)

$\frac{6}{7}S = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \ldots\ldots\ldots\ldots$ (iii)

$\frac{6}{7^2}S = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \ldots\ldots\ldots\ldots$ (iv)

(iii) - (iv)

$\left(\frac{6}{7}\right)^2 S = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \ldots\ldots.$

$= 2\left[\frac{1}{1 - \frac{1}{7}}\right] = 2\left(\frac{7}{6}\right)$

$\therefore 4S = 8\left(\frac{7}{6}\right)^3 = \left(\frac{7}{3}\right)^3$

Question101

If $a_1, a_2, a_3\ldots\ldots$ and $b_1, b_2, b_3\ldots\ldots$ are A.P., and $a_1 = 2$, $a_{10} = 3$, $a_1b_1 = 1 = a_{10}b_{10}$, then a_4b_4 is equal to - [27-Jun-2022-Shift-2]

Options:

A. $\frac{35}{27}$

B. 1

C. $\frac{27}{28}$

D. $\frac{28}{27}$

Answer: D

Solution:

Solution:

$a_1, a_2, a_3\ldots$ are in A.P. (Let common difference is d_1)

$b_1, b_2, b_3\ldots$ are in A.P. (Let common difference is d_2)

and $a_1 = 2, a_{10} = 3, a_1b_1 = 1 = a_{10}b_{10}$

$\because a_1b_1 = 1$

$\therefore b_1 = \frac{1}{2}$
$a_{10}b_{10} = 1$
$\therefore b_{10} = \frac{1}{3}$
Now, $a_{10} = a_1 + 9d_1 \Rightarrow d_1 = \frac{1}{9}$
$b_{10} = b_1 + 9d_2 \Rightarrow d_2 = \frac{1}{9}\left[\frac{1}{3} - \frac{1}{2}\right] = -\frac{1}{54}$
Now, $a_4 = 2 + \frac{3}{9} = \frac{7}{3}$
$b_4 = \frac{1}{2} - \frac{3}{54} = \frac{4}{9}$

$\therefore a_4b_4 = \frac{28}{27}$

Question102

Let A_1, A_2, A_3, ... be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1256}$ and $A_2 + A_4 = \frac{7}{36}$, then the value of $A_6 + A_8 + A_{10}$ is equal to
[28-Jun-2022-Shift-1]

Options:

A. 33

B. 37

C. 43

D. 47

Answer: C

Solution:

Solution:
$A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$
$(A_4)^4 = \frac{1}{1296}$
$A_4 = \frac{1}{6}$
$A_2 + A_4 = \frac{7}{36}$
$A_2 = \frac{1}{36}$
$A_6 = 1$
$A_8 = 6$
$A_{10} = 36$
$A_6 + A_8 + A_{10} = 43$

Question103

If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is :
[28-Jun-2022-Shift-2]

Options:

A. 21

B. 22

C. 23

D. 24

Answer: C

Solution:

Solution:
$a, A_1, A_2 \ldots\ldots\ldots . A_n, 100$
Let d be the common difference of above A.P. then
$\frac{a+d}{100-d} = \frac{1}{7}$
$\Rightarrow 7a + 8d = 100 \ldots\ldots (i)$
and $a + n = 33$
and $100 = a + (n+1)d$
$\Rightarrow 100 = a + (34-a)\frac{(100-7a)}{8}$
$\Rightarrow 800 = 8a + 7a^2 - 338a + 3400$
$\Rightarrow 7a^2 - 330a + 2600 = 0$
$\Rightarrow a = 10, \frac{260}{7}, \text{ but } a \neq \frac{260}{7}$
$\therefore n = 23$

Question104

Let for n = 1, 2, ……, 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of

$$\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$$ **is equal to____**
[28-Jun-2022-Shift-2]

Answer: 41651

Solution:

Solution:

$$S_n = \frac{n^2}{1-\frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{n+2} = (n^2+1) - \frac{2}{n+2}$$

Now $\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$

$$= \frac{1}{26} + \sum_{n=1}^{50}\left\{(n^2-n) + 2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\}$$

$$= \frac{1}{26} + \frac{50\times51\times101}{6} - \frac{50\times51}{2} + 2\left(\frac{1}{2} - \frac{1}{52}\right)$$

$$= 1 + 25\times17(101-3)$$

$$= 41651$$

Question105

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to:

[29-Jun-2022-Shift-1]

Options:

A. $\frac{6}{343}$

B. $\frac{7}{216}$

C. $\frac{8}{343}$

D. $\frac{49}{216}$

Answer: B

Solution:

Solution:

$a_{n+2} = 2a_{n+1} - a_n + 1 \& a_0 = a_1 = 0$

$a_2 = 2a_1 - a_0 + 1 = 1$

$a_3 = 2a_2 - a_1 + 1 = 3$

$a_4 = 2a_3 - a_2 + 1 = 6$

$a_5 = 2a_4 - a_3 + 1 = 10$

$$\sum_{n=2}^{\infty} \frac{a_n}{7^n} = \frac{a_2}{7^2} + \frac{a_3}{7^3} + \frac{a_4}{7^4} + \ldots$$

$$s = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \ldots$$

$$\frac{1}{7}s = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \ldots$$

$$\frac{6s}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \ldots$$

$$\frac{6s}{49} = \frac{1}{7^3} + \frac{2}{7^4} + \ldots$$

$$\frac{36s}{49} = \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \ldots$$

$$\frac{36s}{49}=\frac{\frac{1}{7^2}}{1-\frac{1}{7}}$$
$$\frac{36s}{49}=\frac{7}{49\times 6}$$
$$s=\frac{7}{216}$$

Question106

The sum of the infinite series $1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+\frac{51}{6^5}+\frac{70}{6^6}+\ldots\ldots$ is equal to: [29-Jun-2022-Shift-2]

Options:

A. $\frac{425}{216}$

B. $\frac{429}{216}$

C. $\frac{288}{125}$

D. $\frac{280}{125}$

Answer: C

Solution:

Solution:

Question107

Let 3, 6, 9, 12, upto 78 terms and 5, 9, 13, 17, upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to_____ [29-Jun-2022-Shift-2]

Answer: 2223

Solution:

Solution:
1st AP :
3, 6, 9, 12, ... upto 78 terms
$t_{78}=3+(78-1)3$
$=3+77\times 3$

$= 234$
2nd AP:
$5, 9, 13, 17, \ldots\ldots$ upto 59 terms
$t_{59} = 5 + (59 - 1)4$
$= 5 + 58 \times 4$
$= 237$
Common term's AP :
First term $= 9$
Common difference of first $AP = 3$
And common difference of second $AP = 4$
$\therefore$ Common difference of common terms
$AP = LCM(3, 4) = 12$
$\therefore$ New AP $= 9, 21, 33, \ldots\ldots$
$t_n = 9 + (n - 1)12 \le 234$
$\Rightarrow n \le \frac{237}{12}$
$\Rightarrow n = 19$
$\therefore S_{19} = \frac{19}{2}[2.9 + (19 - 1)12]$
$= 19(9 + 108)$
$= 2223$

Question108

Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \ge$ sl ant2. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to [25-Jul-2022-Shift-1]

Answer: 27560

Solution:

Solution:
Given,
$a_n = a_{n-1} + 2$
$\Rightarrow a_n - a_{n-1} = 2$
$\therefore$ In this series between any two consecutives terms difference is 2 . So this is an A.P. with common difference 2.
Also given $a_1 = 1$
$\therefore$ Series is $= 1, 3, 5, 7 \ldots\ldots$
$\therefore a_n = 1 + (n - 1)2 = 2n - 1$
Also $b_n = a_n + b_{n-1}$
When $n = 2$ then
$b_2 - b_1 = a_2 = 3$
$. \Rightarrow b_2 - 1 = 3$ [Given $b_1 = 1$]
$\Rightarrow b_2 = 4$
When $n = 3$ then
$b_3 - b_2 = a_3$
$\Rightarrow b_3 - 4 = 5$
$\Rightarrow b_3 = 9$
$\therefore$ Series is $= 1, 4, 9 \ldots\ldots$
$= 1^2, 2^2, 3^2 \ldots\ldots n^2$

$\therefore b_n = n^2$

Now, $\sum_{n=1}^{15}(a_n \cdot b_n)$

$= \sum_{n=1}^{15}[(2n-1)n^2]$

$= \sum_{n=1}^{15} 2n^3 - \sum_{n=1}^{15} n^2$

$= 2(1^3 + 2^3 + \ldots 15^3) - (1^2 + 2^2 + \ldots 15^2)$

$= 2 \times \left(\frac{15 \times 16}{2}\right)^2 - \left(\frac{15(16) \times 31}{6}\right)$

$= 27560$

Question109

The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to
[25-Jul-2022-Shift-2]

Options:

A. $\frac{7}{87}$

B. $\frac{7}{29}$

C. $\frac{14}{87}$

D. $\frac{21}{29}$

Answer: B

Solution:

Solution:

$\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)} = \frac{3}{4}\sum_{n=1}^{21} \frac{1}{4n-1} - \frac{1}{4n+3}$

$= \frac{3}{4}\left[\left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{11}\right) + \ldots + \left(\frac{1}{83} - \frac{1}{87}\right)\right]$

$= \frac{3}{4}\left[\frac{1}{3} - \frac{1}{87}\right] = \frac{3}{4}\frac{84}{3.87} = \frac{7}{29}$

Question110

Consider two G.Ps. $2, 2^2, 2^3, \ldots\ldots$ and $4, 4^2, 4^3, \ldots\ldots$ of 60 and n terms respectively. If the geometric mean of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :
[26-Jul-2022-Shift-1]

Options:

A. 560

B. 1540

C. 1330

D. 2600

Answer: C

Solution:

Solution:
Given G.P's $2, 2^2, 2^3, \ldots .60$ terms
$4, 4^2, \ldots . n$ terms
Now, G.M $= 2^{\frac{225}{8}}$
$(2.2^2 \ldots 4.4^2 \ldots)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$
$\left(2^{\frac{n^2+n+1830}{6+n}}\right) = 2^{\frac{225}{8}}$
$\Rightarrow \frac{n^2+n+1830}{60+n} = \frac{225}{8}$
$\Rightarrow 8n^2 - 217n + 1140 = 0$
$n = \frac{57}{8}, 20, \quad \text{so} \quad n = 20$
$\therefore \sum_{k=1}^{20} k(20-k) = 20 \times \frac{20 \times 21}{2} - \frac{20 \times 21 \times 41}{6}$
$= \frac{20 \times 21}{2}\left[20 - \frac{41}{3}\right] = 1330$

Question111

The series of positive multiples of 3 is divided into sets :
{3}, {6, 9, 12}, {15, 18, 21, 24, 27}, ... Then the sum of the elements in the 11 th set is equal to __________.
[26-Jul-2022-Shift-1]

Answer: 6993

Solution:

Solution:
$\underset{1-\text{term}}{\{3 \times 1\}}, \underset{3 \text{ terms}}{\{3 \times 2, 3 \times 3, 3 \times 4\}}, \underset{5-\text{terms}}{\{3 \times 5, 3 \times 6, 3 \times 7, 3 \times 8, 3 \times 9\}}, \ldots$

$\therefore 11^{\text{th}}$ set will have $1 + (10)2 = 21$ term
Also upto 10^{th} set total $3 \times k$ type terms will be $1 + 3 + 5 + \ldots\ldots + 19 = 100-$ term
$\therefore$ Set $11 = \{3 \times 101, 3 \times 102, \ldots\ldots . 3 \times 121\}$
$\therefore$ Sum of elements $= 3 \times (101 + 102 + \ldots + 121)$
$= \frac{3 \times 222 \times 21}{2} = 6993$

Question112

If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then $m+n$ is equal to ________.
[26-Jul-2022-Shift-2]

Answer: 166

Solution:

Solution:
$\sum_{k=1}^{10} \frac{k}{k^4+k^2+1}$
$= \frac{1}{2}\left[\sum_{k=1}^{10}\left(\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right)\right.$.
$= \frac{1}{2}\left[1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{13} + \ldots + \frac{1}{91} - \frac{1}{111}\right]$
$= \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{110}{2.111} = \frac{55}{111} = \frac{m}{n}$
$\therefore m+n = 55+111 = 166$

Question113

Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is ________.
[26-Jul-2022-Shift-2]

Answer: 53

Solution:

Solution:
$d_1 = \frac{199-100}{2} \notin I$
$d_2 = \frac{199-100}{3} = 33$
$d_3 = \frac{199-100}{4} \notin I$
$d_n = \frac{199-100}{i+1} \in I$
$d_i = 33+11, 9$
Sum of CD's $= 33+11+9$
$= 53$

Question114

Suppose a_1, a_2, ..., a_n, .. be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and , 110 < a_{15} < 120, then the sum of the first ten terms of the progression is equal to
[27-Jul-2022-Shift-1]

Options:

A. 290

B. 380

C. 460

D. 510

Answer: B

Solution:

Solution:
$\because a_1, a_2, \dots a_n$ be an A.P of natural numbers and

$$\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}[2a_1 + 4d]}{\frac{9}{2}[2a_1 + 8d]} = \frac{5}{17}$$

$\Rightarrow 34a_1 + 68d = 18a_1 + 72d$
$\Rightarrow 16a_1 = 4d$
$\therefore d = 4a_1$
And $110 < a_{15} < 120$
$\therefore 110 < a_1 + 14d < 120 \Rightarrow 110 < 57a_1 < 120$
$\therefore a_1 = 2(\because a_i \in N)$
$d = 8$
$\therefore S_{10} = 5[4 + 9 \times 8] = 380$

Question115

Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \le 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _______.
[27-Jul-2022-Shift-1]

Answer: 10620

Solution:

Solution:
$\because |f(n)| \leq 800$
$\Rightarrow -800 \leq 2n^2 - n - 1 \leq 800$
$\Rightarrow 2n^2 - n - 801 \leq 0$
$\therefore n \in \left[\frac{-\sqrt{6409}+1}{4}, \frac{\sqrt{6409}+1}{4}\right]$ and $n \in z$
$\therefore n = -19, -18, -17, \ldots\ldots\ldots, 19, 20.$
$\therefore \sum(2x^2 - x - 1) = 2\sum x^2 - \sum x - \sum 1.$
$= 2.2 \cdot (1^2 + 2^2 + \ldots + 19^2) + 2.20^2 - 20 - 40$
$= 10620$

Question116

Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5 . Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is 10ar, n^{th} term is a_n and the common difference is $10ar^2$, is equal to :
[27-Jul-2022-Shift-2]

Options:

A. $21a_{11}$

B. $22a_{11}$

C. $15a_{16}$

D. $14a_{16}$

Answer: A

Solution:

Solution:
Let first term of G.P. be a and common ratio is r Then, $\frac{a}{1-r} = 5 \ldots\ldots$ (i)
$a\frac{(r^5-1)}{(r-1)} = \frac{98}{25} \Rightarrow 1 - r^5 = \frac{98}{125}$
$\therefore r^5 = \frac{27}{125}, r = \left(\frac{3}{5}\right)^{\frac{3}{5}}$
$\therefore$ Then, $S_{21} = \frac{21}{2}[2 \times 10ar + 20 \times 10ar^2]$
$= 21[10ar + 10.10ar^2]$
$= 21a_{11}$

Question117

$\frac{2^3-1^3}{1\times 7}+\frac{4^3-3^3+2^3-1^3}{2\times 11}+\frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times 15}+\cdots+\frac{30^3-29^3+28^3-27^3+\ldots+2^3-1^3}{15\times 63}$ **is equal to ________.**

[27-Jul-2022-Shift-2]

Answer: 120

Solution:

Solution:

$$T_n=\frac{\sum_{k=1}^{n}[(2k)^3-(2k-1)^3]}{n(4n+3)}$$

$$=\frac{\sum_{k=1}^{n}4k^2+(2k-1)^2+2k(2k-1)}{n(4n+3)}$$

$$=\frac{\sum_{k=1}^{n}(12k^2-6k+1)}{n(4n+3)}$$

$$=\frac{2n(2n^2+3n+1)-3n^2-3n+n}{n(4n+3)}$$

$$=\frac{n^2(4n+3)}{n(4n+3)}=n$$

$$\therefore T_n=n$$

$$S_n=\sum_{n=1}^{15}T_n=\frac{15\times 16}{2}=120$$

Question118

Consider the sequence $a_1, a_2, a_3, \ldots$ such that a_1-1, a_2-2 and $a_{n+2}=\frac{2}{a_{n+1}}+a_{-n}$ for $n-1, 2, 3, \ldots$ If $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdot\left(\frac{a_3+\frac{1}{a_4}}{a_5}\right)\ldots\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right)=2^{\alpha}({}^{61}C_{31})$, then α is equal to :

[28-Jul-2022-Shift-1]

Options:

A. −30

B. −31

C. −60

D. −61

Answer: C

Solution:

Solution:

$a_{n+2} = \dfrac{2}{a_{n+1}} + a_n$

$\Rightarrow a_n a_{n+1} + 1 = a_{n+1} a_{n+2} - 1$

$\Rightarrow a_{n+2} a_{n+1} - a_n \cdot a_{n+1} = 2$

For

$n = 1\ a_3 a_2 - a_1 a_2 = 2$

$n = 2\ a_4 a_3 - a_3 a_2 = 2$

$n = 3\ a_5 a_4 - a_4 a_3 = 2$

.

.

.

$n = n\quad \dfrac{a_{n+2} a_{n+1} - a_n a_{n+1} = 2}{a_{n+2} a_{n+1} = 2n + a_1 a_2}$

Now,

$\dfrac{(a_1 a_2 + 1)}{a_2 a_3} \cdot \dfrac{(a_2 a_3 + 1)}{a_3 a_4} \cdot \dfrac{(a_3 a_4 + 1)}{a_4 a_5} \ldots\ldots \cdot \dfrac{(a_{30} a_{31} + 1)}{a_{31} a_{32}}$

$= \dfrac{3}{4} \cdot \dfrac{5}{6} \cdot \dfrac{7}{8} \ldots\ldots \cdot \dfrac{61}{62}$

$= 2^{-60} ({}^{61}C_{31})$

Question119

For p, q $\in R_s$ consider the real valued function $f(x) = (x - p)^2 - q$, $x \in R$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $f(a_i) \mid = 500$ for all i = 1, 2, 3, 4, then the absolute difference between the roots of $f(x) = 0$ is _______.
[28-Jul-2022-Shift-1]

Answer: 50

Solution:

Solution:

$\because a_1, a_2, a_3, a_4$

$\therefore a_2 = p - 3d$, $a_2 = p - d$, $a_3 = p + d$ and $a_4 = p + 3d$

Where $d > 0$

$\because |f(a_i)| = 500$

$\Rightarrow |9d^2 - q| = 500$

and $|d^2 - q| = 500$

either $9d^2 - q = d^2 - q$

$\Rightarrow d = 0$ not acceptable

$\therefore 9d^2 - q = q - d^2$

$\therefore 5d^2 - q = 0$

Roots of $f(x) = 0$ are $p + \sqrt{q}$ and $p - \sqrt{q}$

$\therefore$ absolute difference between roots $= |2\sqrt{q}| = 50$

Question120

Let $x_1, x_2, x_3, \ldots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\bar{x}$ is the mean of new data, then the greatest integer less than or equal to $\bar{x}$ is ________.
[28-Jul-2022-Shift-1]

Answer: 142

Solution:

Solution:
$x_1, x_2, x_3, \ldots, x_{20}$ are in G.P.

$x_1 = 3, r = \frac{1}{2}$

$\bar{x} = \frac{\sum x_i^2 - 2x_i i + i^2}{20}$

$= \frac{1}{20}\left[12\left(1 - \frac{1}{2^{40}}\right) - 6\left(4 - \frac{11}{2^{18}}\right) + 70 \times 41\right]$

$$\left\{\begin{aligned} S &= 1 + 2\cdot\frac{1}{2} + 3\cdot\frac{1}{2^2} + \ldots \\ \frac{S}{2} &= \frac{1}{2} + \frac{2}{2^2} + \ldots \end{aligned}\right.$$

$\cdot\ \frac{S}{2} = 2\left(1 - \frac{1}{2^{20}}\right) - \frac{20}{2^{20}} = 4 - \frac{11}{2^{18}}\}$

$\therefore [\bar{x}] = \left[\frac{2858}{20} - \left(\frac{12}{240} - \frac{66}{2^{18}}\right)\cdot\frac{1}{20}\right]$

$= 142$

Question121

$\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \ldots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then m . n is equal to_______.
[28-Jul-2022-Shift-2]

Answer: 12

Solution:

Solution:

$$\frac{1}{3^{12}}+5\left(\frac{2^0}{3^{12}}+\frac{2^1}{3^{11}}+\frac{2^2}{3^{10}}+\ldots..+\frac{2^{11}}{3}\right)=2^n\cdot m$$
$$\Rightarrow \frac{1}{3^{12}}+5\left(\frac{1}{3^{12}}\frac{((6)^2-1)}{(6-1)}\right)=2^n\cdot m$$
$$\Rightarrow \frac{1}{3^{12}}+\frac{5}{5}\left(\frac{1}{3^{12}}\cdot 2^{12}\cdot 3^{12}-\frac{1}{3^{12}}\right)=2^n\cdot m$$
$$\Rightarrow \frac{1}{3^{12}}+2^{12}-\frac{1}{3^{12}}=2^n\cdot m$$
$\Rightarrow 2^n\cdot m=2^{12}$
$\Rightarrow m=1$ and $n=12$
$m\cdot n=12$

Question122

If $\frac{1}{(20-a)(40-a)}+\frac{1}{(40-a)(60-a)}+\ldots+\frac{1}{(180-a)(200-a)}=\frac{1}{256}$, then the maximum value of a is :
[29-Jul-2022-Shift-1]

Options:

A. 198

B. 202

C. 212

D. 218

Answer: C

Solution:

Solution:
$$\frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{40-a}+\frac{1}{40-a}-\frac{1}{60-a}+\ldots+\frac{1}{180-a}-\frac{1}{200-a}\right)=\frac{1}{256}$$
$$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{200-a}\right)=\frac{1}{256}$$
$$\Rightarrow \frac{1}{20}\left(\frac{180}{(20-a)(200-a)}\right)=\frac{1}{256}$$
$\Rightarrow (20-a)(200-a)=9.256$
OR $a^2-220a+1696=0$
$\Rightarrow a=212, 8$

Question123

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty}\frac{a_r}{2^r}=4$, then $4a_2$ is equal to ________.
[29-Jul-2022-Shift-1]

Answer: 16

Solution:

Solution:
Given

$$S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \ldots\ldots\infty$$

$$\frac{1}{2}S = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots\infty$$

$$\frac{S}{2} = \frac{a_1}{2} + \frac{(a_2 + a_1)}{2^2} + \frac{(a_3 + a_2)}{2^3} + \ldots\ldots\infty$$

$$\Rightarrow \frac{S}{2} = \frac{a_1}{2} + \frac{d}{2}$$

$$\Rightarrow a_1 + d = a_2 = 4 \Rightarrow 4a_2 = 16$$

Question124

If $\frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \ldots + \frac{1}{100 \times 101 \times 102} = \frac{k}{101}$, then 34k is equal to ________.
[29-Jul-2022-Shift-1]

Answer: 286

Solution:

Solution:

$$S = \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \ldots + \frac{1}{100 \times 101 \times 102}$$

$$= \frac{1}{(3-1) \cdot 1}\left[\frac{1}{2 \times 3} - \frac{1}{101 \times 102}\right]$$

$$= \frac{1}{2}\left(\frac{1}{6} - \frac{1}{101 \times 102}\right)$$

$$= \frac{143}{102 \times 101} = \frac{k}{101}$$

$$\therefore 34k = 286$$

Question125

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 - a_1 - 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1$, $\forall n \geq 0$. Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to
[29-Jul-2022-Shift-2]

Options:

A. 483

B. 528

C. 575

D. 624

Answer: B

Solution:

Solution:
$a_0 = 0, a_1 = 0$
$a_{n+2} = 3a_{n+1} - 2a_{n+1} : n \geq 0$
$a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1$
$n = 0 \quad a_2 - a_1 = 2(a_1 - a_0) + 1$
$n = 1 \quad a_3 - a_2 = 2(a_2 - a_1) + 1$
$n = 2 \quad a_4 - a_3 = 2(a_3 - a_2) + 1$
$n = n \quad a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1$
$(a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n+1) = 0$
$a_{n+2} = 2a_{n+1} + (n+1)$
$n \to n-2$
$a_n - 2a_{n-1} = n-1$
Now $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$
$= (a_{25} - 2a_{24})(a_{23} - 2a_{22}) = (24)(22) = 528$

Question126

$\sum_{r=1}^{20}(r^2+1)(r!)$ is equal to
[29-Jul-2022-Shift-2]

Options:

A. $22! - 21\,!$

B. $22! - 2(21!)$

C. $21! - 2(20!)$

D. $21! - 20\,!$

Answer: B

Solution:

Solution:
Given,
$\sum_{r=1}^{20}(r^2+1)(r!)$
Let, $f(r) = (r^2+1)(r!)$
$= (r^2)(r!) + r!$
$= r(rr!) + r!$
$= r[(r+1-1)r!] + r!$
$= r[(r+1)r! - r!] + r!$
$= r[(r+1)! - (r!)] + r!$
$= r(r+1)! - r(r!) + r! = (r+2-2)(r+1)! - r(r!) + r!$

$= (r+2)(r+1)! - 2(r+1)! - [(r+1-1)(r!)] + r!$
$= (r+2)! - 2(r+1)! - (r+1)! + r! + r!$
$= (r+2)! - 3(r+1)! + 2r!$
$= [(r+2)! - (r+1)!] - 2[(r+1)! - r!]$
$\therefore \sum_{r=1}^{20} f(r)$
$= \sum_{r=1}^{20} [(r+2)! - (r+1)!] - 2\sum_{r=1}^{20} [(r+1)! - r!]$
$= [(22! + 21! + 20! + \ldots. + 4! + 3!) - (21! + 20! + 19! + \ldots. + 3! + 2!] - 2[(21! + 20! + \ldots + 3! + 2!) - (20! + 19! + \ldots\ldots 1!)]$
$= [(22!) - (2!)] - 2[(21)! - (1!)]$
$= 22! - 2! - 2\cdot(21)! + 2.1!$
$= 22! - 2\cdot(21)!$

Question127

If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \ldots \infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$,

then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is

[24-Feb-2021 Shift 1]

Options:

A. $2\sqrt{3}$

B. $\frac{3}{2}$

C. $\sqrt{3}$

D. $\frac{1}{2}$

Answer: D

Solution:

Solution:
$e^{(\cos^2 x + \cos^4 x + \ldots \infty)\ell n2} = 2^{\cos^2 x + \cos^4 x + \ldots \infty} = 2^{\cot^2 x}$
Now $t^2 - 9t + 9 = 0 \Rightarrow t = 1, 8$
$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$
$\Rightarrow \frac{2\sin x}{\sin x + \sqrt{3}\cos x} = \frac{2}{1 + \sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$

Question128

Let $A = \{x : x$ is 3-digit number $]$ $B = \{x : x = 9k + 2, k \in I\}$ and
$C = \{x : x = 9k + \ell, k \in I, \ell \in I, O < \ell < 9\}$ for some $\ell(0 < \ell < 9)$ If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then ℓ is equal to
[24-Feb-2021 Shift 1]

Answer: 5

Solution:

Solution:
B and C will contain three digit numbers of the form $9k+2$ and $9k+\ell$ respectively. We need to find sum of all elements in the set $B\cup C$ effectively.
Now, $S(B\cup C) = S(B)+S(C)-S(B\cap C)$ where $S(k)$ denotes sum of elements of set k .
Also $B = \{101, 110, \ldots\ldots 992\}$
$\therefore S(B) = \frac{100}{2}(101+992) = 54650$

Case-I : If $\ell = 2$
then $B\cap C = B$
$\therefore S(B\cup C) = S(B)$
which is not possible as given sum is
274 × 400=109600

Case-II : If $\ell \neq 2$
then $B\cap C = \varphi$
$\therefore S(B\cup C) = S(B)+S(C) = 400\times 274$

$\Rightarrow 54650+\sum_{k=11}^{110} 9k+\ell = 109600$

$\Rightarrow 9\sum_{k=11}^{110} k+\sum_{k=11}^{110} \ell = 54950$
$\Rightarrow 9\left(\frac{100}{2}(11+110)\right)+\ell(100) = 54950$
$\Rightarrow 54450+100\ell = 54950$
$\Rightarrow \ell = 5$

Question129

The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1 and the third term is α_1 then 2α is
[2021, 24 Feb. Shift-II]

Answer: 9

Solution:

Solution:
Let four numbers in GP be a, ar, ar^2, ar^3 .
According to the question,
$a+ar+ar^2+ar^3 = \frac{65}{12} \quad \cdots\cdots (i)$
and $\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3} = \frac{65}{18}$

$\Rightarrow \frac{1}{a}\left(\frac{1+r+r^2+r^3}{r^3} \right) = \frac{65}{18} \quad \cdots\cdots (ii)$

Dividing Eq. (i) by (ii), we get

$$\frac{a(1+r+r^2+r^3)}{\frac{1}{a}\frac{(1+r+r^2+r^3)}{r^3}} = \frac{65/12}{65/18}$$

$\Rightarrow a^2r^3 = \frac{18}{12}$

$\Rightarrow a^2r^3 = \frac{3}{2}$

Also, product of first three terms $= 1$

$a \times ar \times ar^2 = 1$

$\Rightarrow \quad a^3r^3 = 1$

$\Rightarrow \ a^3 \times \frac{3}{2a^2} = 1 \ \left[\because r^3 = \frac{3/2}{a^2} \right]$

$\Rightarrow \ a = \frac{2}{3}$

and $r^3 = \frac{3/2}{(2/3)^2} = \left(\frac{3}{2} \right)^3 \Rightarrow \ r = \frac{3}{2}$

According to the question, third term

$\alpha = ar^2 = \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} = \frac{3}{2} \therefore \ 2\alpha = 2 \times \frac{3}{2} = 3$ third term

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Question130

The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to [2021, 26 Feb. Shift-II]

Options:

A. $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

B. $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

C. $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

D. $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

Answer: B

Solution:

Solution:

Let $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!} = S$

$= \sum_{n=1}^{\infty} \frac{4n^2+24n+40}{4(2n+1)!}$

$= \sum_{n=1}^{\infty} \frac{(2n+1)^2+(2n+1)\cdot 10+29}{4(2n+1)!}$

$$= \frac{1}{4}\left[\sum_{n=1}^{\infty}\frac{(2n+1)^2}{(2n+1)(2n)!}+\sum_{n=1}^{\infty}\frac{(2n+1)\cdot 10}{(2n+1)(2n)!}+\sum_{n=1}^{\infty}\frac{29}{(2n+1)!}\right]$$

$$= \frac{1}{4}\left[\sum_{n=1}^{\infty}\frac{(2n+1)}{(2n)!}+\sum_{n=1}^{\infty}\frac{10}{(2n)!}+\sum_{n=1}^{\infty}\frac{29}{(2n+1)!}\right] \quad \cdots\cdots (1)$$

Now,

$$= \sum_{n=1}^{\infty}\frac{(2n+1)}{(2n)!} = \sum_{n=1}^{\infty}\frac{2n}{(2n)!}+\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$= \sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

Now,

$$= \sum_{n=1}^{\infty}\frac{1}{(2n-1)!} = \frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots = \frac{e-\frac{1}{e}}{2} \quad \cdots\cdots (ii)$$

and $\sum_{n=1}^{\infty}\frac{1}{(2n)!} = \frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots = \frac{e+\frac{1}{e}-2}{2} \quad \cdots\cdots (iii)$

and $\sum_{n=1}^{\infty}\frac{1}{(2n+1)!} = \frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+\ldots = \frac{e-\frac{1}{e}-2}{2} \quad \cdots\cdots (iv)$

Using Eqs. (ii), (iii), (iv) in (i),

$$S = \frac{1}{4}\left[\frac{e-\frac{1}{e}}{2}+11\&\left(\frac{e+\frac{1}{e}-2}{2}\right)\right.$$

$$\left.+29\&\left(\frac{e-\frac{1}{e}-2}{2}\right)\right]$$

$$= \frac{1}{4}\left[\frac{e}{2}-\frac{1}{2e}+\frac{11e}{2}+\frac{11}{2e}+\frac{29e}{2}-\frac{29}{2e}-4\right]$$

$$= \frac{41e}{8}-\frac{19}{8e}-10$$

Question131

The sum of the infinite series $1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots$ is equal to [2021, 26 Feb. Shift-1]

Options:

A. $\frac{13}{4}$

B. $\frac{9}{4}$

C. $\frac{15}{4}$

D. $\frac{11}{4}$

Answer: A

Solution:

Solution:

Given, $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots$

Let, $S_1 = \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots$

Multiply $1/3$ in series Eq. (i),

$\frac{S_1}{3} = \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \ldots$

Subtract Eq. (ii) from Eq. (i), we get

$S_1 - \frac{S_1}{3} = \frac{2}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \ldots$

$\Rightarrow \frac{2S_1}{3} = \frac{2}{3} + \left[\frac{5}{3^2} + \frac{5}{3^3} + \ldots\right]$

$= \frac{2}{3} + \left[\frac{5/3^2}{1-1/3}\right]$ $\left[\because \frac{5}{3^2} + \frac{5}{3^3} + \ldots\right.$ is a geometric series with $r = 1/3$, sum upto infinity of this series is $\frac{a}{1-r}$, where $a =$ first term]

$= \frac{2}{3} + \left[\frac{5}{6}\right] = \frac{9}{6} = \frac{3}{2}$

$\Rightarrow S_1 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$

$\therefore S = 1 + S_1$

$= 1 + \frac{9}{4} = \frac{13}{4}$

Question132

If the arithmetic mean and geometric mean of the pth and qth terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then $p + q$ is equal to [2021, 26 Feb. Shift-II]

Answer: 10

Solution:

Solution:

If AM and GM satisfy the equation $4x^2 - 9x + 5 = 0$, then AM and GM are nothing but roots of this quadratic equation,

$4x^2 - 9x + 5 = 0$

$\Rightarrow 4x^2 - 4x - 5x + 5 = 0$

$\Rightarrow 4x(x-1) - 5(x-1) = 0$

$\Rightarrow (x-1)(4x-5) = 0$

$\Rightarrow x = 1, \frac{5}{4}$

Then, $AM = \frac{5}{4}$ and $GM = 1$ $[\because AM \geq GM]$

Again, the given series is

$-16, 8, -4, 2 \ldots\ldots$

which is a geometric progression series with common ratio $\frac{-1}{2}$, then

pth term $= -16\left(\frac{-1}{2}\right)^{p-1} = t_p$

qth term $= -16\left(\frac{-1}{2}\right)^{q-1} = t_q$

Arithmetic mean $= \frac{5}{4}$

$\Rightarrow \frac{t_p + t_q}{2} = \frac{5}{4}$

Geometric mean $= 1$

$\Rightarrow \sqrt{t_p t_q} = 1$

$\because \sqrt{t_p t_q} = 1$

$\Rightarrow (-16)\left(\frac{-1}{2}\right)^{p-1}(-16)\left(\frac{-1}{2}\right)^{q-1} = 1$

$\Rightarrow (-16)^2\left(\frac{-1}{2}\right)^{p+q-2} = 1$

$\Rightarrow (-2^4)^2\left(\frac{-1}{2}\right)^{p+q-2} = 1$

$\Rightarrow (-2)^8 \frac{(+1)^{p+q-2}}{(-2)^{p+q-2}} = 1$

$\Rightarrow (-2)^8(+1)^{p+q-2} = (-2)^{p+q-2}$

$\Rightarrow (-2)^8 = (-2)^{p+q-2}$

$\Rightarrow p+q-2 = 8$

$\Rightarrow p+q = 10$

Question133

In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25 . Then, the sum of 4th, 6th and 8th terms is equal to [2021,26 Feb. Shift-1]

Options:

A. 30

B. 26

C. 35

D. 32

Answer: C

Solution:

Solution:

Let the first term of geometric series be' a' and common ratio be 'r'.

Then, n th term of given series is given as

$T_n = ar^{n-1}$

Now, given that sum of second and sixth term is 25/2.

i.e. $T_2 + T_6 = 25/2$

$\Rightarrow ar + ar^5 = 25/2$

$\Rightarrow ar(1+r^4) = 25/2 \quad \cdots\cdots (i)$

Also, given that product of third and fifth term is 25.

l rl i.e. $(T_3)(T_5) = 25$

$\Rightarrow (ar^2)(ar^4) = 25$

$\Rightarrow a^2r^6 = 25 \quad \cdots\cdots (ii)$

Squaring Eq. (i), we get$a^2r^2(1+r^4)^2 = \left(\frac{25}{2}\right)^2 \quad \cdots\cdots (iii)$

Divide Eq. (ii) by (iii),

$\Rightarrow \quad \frac{a^2r^2(1+r^4)^2}{a^2r^6} = \frac{(25)^2}{4(25)}$

$\Rightarrow \quad \frac{(1+r^4)^2}{r^4} = \frac{25}{4}$

$\Rightarrow \quad 4(1+r^4)^2 = 25r^4$

$\Rightarrow \quad 4(1+r^8+2r^4) = 25r^4$

$\Rightarrow \quad 4r^8 - 17r^4 + 4 = 0$

$\Rightarrow \quad 4r^8 - 16r^4 - r^4 + 4 = 0$

$\Rightarrow \quad 4r^4(r^4-4) - 1(r^4+(-4)) = 0$

$\Rightarrow \quad (r^4-4)(4r^4-1) = 0$

Gives, $r^4 = 40rr^4 = 1/4$

We have to find sum of 4 th, 6th and 8th term, i.e.

$T_4 + T_6 + T_8 \quad = ar^3 + ar^5 + ar^7$

$= ar(r^2 + r^4 + r^6)$

$= ar^3(1 + r^2 + r^4) \quad \cdots\cdots (iv)$

Using Eq. (ii),

$(ar^3)^2 = 25$

$\Rightarrow ar^3 = 5$

Also, we take $r^4 = 4$ because given series is increasing and $r^2 = 2$.

$\therefore \; T_4 + T_6 + T_8 \quad = 5(1+2+4)$

$= 5(7) = 35$

Question134

The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to [2021, 25 Feb. Shift-II]

Options:

A. $a+1$

B. $a + \frac{1}{a}$

C. $2\sqrt{a}$

D. $2a$

Answer: C

Solution:

Solution:

We already know, Arithmetic mean $\geq$ Geometric mean,

Let us take AM and GM of two terms a^{a^*} and a^{1-a^x},

$\Rightarrow \quad AM = \frac{a^{a^x} + a^{1-a^x}}{2}$

and $GM = \sqrt{a^{a^x} \cdot a^{1-a^x}}$

$\because AM \geq GM \Rightarrow \frac{a^{a^x} + a^{1-a^x}}{2} \geq \sqrt{a^{a^x} \cdot a^{1-a^x}}$

$\Rightarrow a^{a^x} + a^{1-a^x} \geq 2\sqrt{a^1}$

$\therefore$ Minimum value of $f(x) = a^{a^x} + a^{1-a^x}$ is $2\sqrt{a}$.

Question135

If $0 < \theta, \varphi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta_1$ $y = \sum_{n=0}^{\infty} \sin^{2n}\varphi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\varphi$, then [2021, 25 Feb. Shift-1]

Options:

A. $xy - z = (x+y)z$

B. $xy + yz + zx = z$

C. $xyz = 4$

D. $xy + z = (x+y)z$

Answer: D

Solution:

Solution:

Given, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$

$y = \sum_{n=0}^{\infty} \sin^{2n}\varphi$

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\varphi$

$\Rightarrow \ x = 1 + \cos^2\theta + \cos^4\theta + \ldots\infty$

$\therefore \ x = \dfrac{1}{1-\cos^2\theta} = \operatorname{cosec}^2\theta$

$\Rightarrow \ y = 1 + \sin^2\varphi + \sin^4\varphi + \ldots\infty$

$\therefore \ x = \dfrac{1}{1-\cos^2\theta} = \operatorname{cosec}^2\theta \quad \cdots\cdots (i)$

$\Rightarrow \ y = 1 + \sin^2\varphi + \sin^4\varphi + \ldots\infty$

$\therefore \ y = \dfrac{1}{1-\sin^2\varphi} = \sec^2\varphi \quad \cdots\cdots (ii)$

$\Rightarrow \ z = 1 + \cos^2\theta \cdot \sin^2\varphi + \cos^4\theta\sin^4\varphi + \ldots\infty$

$\therefore \ z = \dfrac{1}{1-\cos^2\theta\sin^2\varphi} \ \ldots$ (iii)

From Eqs. (i), (ii) and (iii), we get

$$z = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} \left[\begin{array}{l} \because \cos^2\theta = 1 - \frac{1}{x} \\ \because \sin^2\varphi = 1 - \frac{1}{y} \end{array}\right]$$

$$z = \frac{xy}{xy-(x-1)(y-1)}$$

$$z = \frac{xy}{xy - xy + x + y - 1}$$

$\Rightarrow \; xz + yz - z = xy$
$\Rightarrow \; xy + z = (x + y)z$

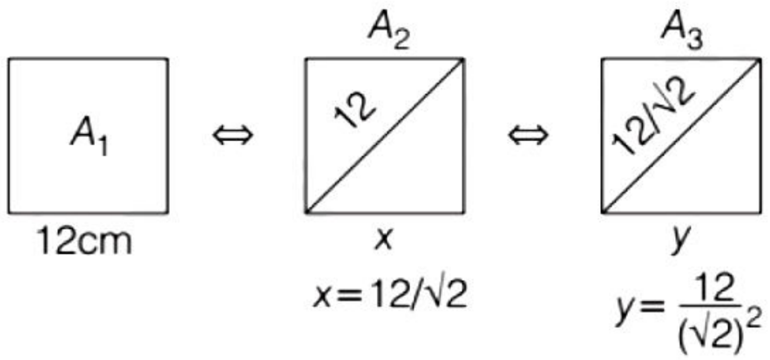
Question136

Let A_1, A_2, A_3, ... be squares, such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12cm, then the smallest value of n for which area of A_n is less than one, is ....
[2021, 25 Feb. Shift-I]

Answer: 9

Solution:

Solution:
According to the question, length of side of A_1 square is $12 \text{ mathrm} \sim \text{cm}$.



$\because$ Side lengths are in GP.
$\therefore \; T_n = \frac{12}{(\sqrt{2})^{n-1}}$
(Side of nth square i.e. A_n)
$\therefore \; \text{Area} = (\text{ Side })^2 = \left(\frac{12}{(\sqrt{2})^{n-1}}\right)^2 = \frac{144}{2^{n-1}}$
According to the question, the area of A_n square <1
$\Rightarrow \; 2^{n-1} > 144$
Here, the smallest possible value of is $= 9$.

Question137

Consider an arithmetic series and a geometric series having four initial terms from the set { 11, 8, 21, 16, 26, 32, 4 }. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to.........
[2021, 16 March Shift-1]

Answer: 3

Solution:

Solution:
Given, set $\{11, 8, 21, 16, 26, 32, 4\}$
By observation, we can say that
$AP = \{11, 16, 21, 26, \ldots\}$
$GP = \{4, 8, 16, 32, \ldots\}$
$5m + 6 = 4.2^{n-1}$
$5m + 6 = 2^{n+1}$
So, $(2^{n+1} - 6)$ should be a multiple of 5 . The unit digit of 2^k is $2, 4, 6, 8$. So, when 6 is subtracted from 2^{n+1}, the possible unit digits will be $6, 8, 0, 2$. Only 0 is divisible by 5. Hence, 2^{n+1} unit digit has to be 6 .
$2^{n+1} = 2^4, 2^8, 2^{12}, 2^{16} \ldots$
As, 2^{16} will not be a 4 digit number, so, common terms $= \{16, 256, 4096\}$
$\therefore$ Number of common terms $= 3$

Question138

Let $\frac{1}{16}$, a and b be in G. P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in (a) P., where a, b > 0. Then, 72(a + b) is equal to ....
[2021, 16 March Shift-II]

Answer: 14

Solution:

Solution:
Given, $GP = \frac{1}{16}, a, b$
$\Rightarrow \quad a^2 = \frac{b}{16}$
and given, $AP = 1/a, 1/b, 6$
$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + 6$
$\Rightarrow \quad \frac{2}{16a^2} = \frac{1}{a} + 6$
$\Rightarrow \quad \frac{1}{8a^2} = \frac{1 + 6a}{a}$
$\Rightarrow \quad 1 = 8a(1 + 6a)$

$\Rightarrow \quad 48a^2 + 8a - 1 = 0$
$\Rightarrow \ (4a+1)(12a-1) = 0$
$\Rightarrow \quad a = -1/4 \ \text{or} \ 1/12$
As per the question, $a > 0$
$\therefore \ a = 1/12$
$b = 16a^2 = 16 \cdot \frac{1}{144} = \frac{1}{9}$
$\therefore \ 72(a+b) = 72\left(\frac{1}{12} + \frac{1}{9}\right)$
$= 6 + 8$
$= 14$

Question139

If α, β are natural numbers, such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$, then the slope of the line passing through (α, β) and origin is
[2021,18 March Shift-1]

Options:

A. 540

B. 550

C. 530

D. 510

Answer: B

Solution:

Solution:
Given,
$100^{\alpha} - 199 \,.\, \beta = \ (100)(100) + (99)(101)$
$+(98)(102) + \ldots + (1)(199)$
$\Rightarrow 100^{\alpha} - 199\beta \ = \sum_{x=0}^{99} (100-x)(100+x)$
$= \sum_{x=0}^{99} (100^2 - x^2)$
$= \sum_{x=0}^{99} (100)^2 - \sum_{x=0}^{99} (x)^2$
$= (100)^3 - \frac{99 \times 100 \times 199}{6}$
$\Rightarrow (100)^{\alpha} - (199)\ \beta = (100)^3 - (199)(1650)$
On comparing, we get $\alpha = 3, \beta = 1650$
Then, the slope of the line passing through (α, β) and origin is
$= \frac{\beta - 0}{\alpha - 0} = \frac{\beta}{\alpha} = \frac{1650}{3} = 550$

Question140

$\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\ldots+\frac{1}{(201)^2-1}$

is equal to

[2021, 18 March Shift-1]

Options:

A. $\frac{101}{404}$

B. $\frac{25}{101}$

C. $\frac{101}{408}$

D. $\frac{99}{400}$

Answer: B

Solution:

Solution:

$$\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\ldots+\frac{1}{(201)^2-1}$$
$$=\sum_{r=1}^{100}\frac{1}{(2r+1)^2-1}$$
$$=\sum_{r=1}^{100}\frac{1}{4r^2+4r+1-1}$$
$$=\sum_{r=1}^{100}\frac{1}{2r(2r+2)}=\sum_{r=1}^{100}\frac{1}{4(r)(r+1)}$$
$$=\frac{1}{4}\sum_{r=1}^{100}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$
$$=\frac{1}{4}\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{100}-\frac{1}{101}\right)\right]$$
$$=\frac{1}{4}\left[1-\frac{1}{101}\right]=\frac{1}{4}\times\frac{100}{101}$$
$$=\frac{25}{101}$$

Question141

Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If (S_2-S_1) is 1000 , then the sum of the first 6n terms of the arithmetic progression is equal to

[2021, 18 March Shift-11]

Options:

A. 1000

B. 7000

C. 5000

D. 3000

Answer: D

Solution:

Solution:
Given, $S_1 = S_{2n}$ and $S_2 = S_{4n}$
and $S_2 - S_1 = 1000 \Rightarrow S_{4n} - S_{2n} = 1000$
$\Rightarrow \frac{4n}{2}[2a + (4n - 1)d] - \frac{2n}{2}[2a + (2n - 1)d] = 1000$
$\Rightarrow 2n[2a + (4n - 1)d] - n[2a + (2n - 1)d] = 1000$
$\Rightarrow 2an + n(8n - 2 - 2n + 1)d = 1000$
$\Rightarrow 2an + n(6n - 1)d = 1000$
$\Rightarrow n[2a + (6n - 1)d] = 1000$
$\Rightarrow S_{6n} = \frac{6n}{2}[2a + (6n - 1)d]$
$= \frac{6}{2} \times (1000)$
$= 6 \times 500 = 3000$

Question142

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to
[2021, 27 July Shift-1]

Answer: 3

Solution:

Solution:
$\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right) \rightarrow AP$
$\Rightarrow 2\log_3(2^x - 5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$
$\Rightarrow \log_3(2^x - 5)^2 = \log_3\left[2 \cdot \left(2^x - \frac{7}{2}\right)\right]$
$\Rightarrow (2^x - 5)^2 = 2 \cdot 2^x - 7$
$\Rightarrow (2^x)^2 + 25 - 10 \cdot 2^x - 2 \cdot 2^x + 7 = 0$
$\Rightarrow (2^x)^2 - 12 \cdot 2^x + 32 = 0$
$\Rightarrow (2^x - 4)(2^x - 8) = 0$
$\Rightarrow 2^x = 4$ or $8 \Rightarrow x = 2$ or 3
If $x = 2$, then $\log_3(2^x - 5) = \log_3(2^2 - 5)$
Here, argument is negative, so, $x \neq 2$.
Hence, $x = 3$

Question143

If $\tan\left(\frac{\pi}{9}\right)$, x, $\tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right)$, y, $\tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x-2y|$ is equal to
[2021, 27 July Shift-II]

Options:

A. 4

B. 3

C. 0

D. 1

Answer: C

Solution:

Solution:

If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in AP.

So, $x = \frac{1}{2}\left[\tan\frac{\pi}{9} + \tan\left(\frac{7\pi}{18}\right)\right]$

($\because$ if a, b, c are in AP, so, $b = \frac{a+c}{2}$)

And $\tan\left(\frac{\pi}{9}\right), y_1 \tan\left(\frac{5\pi}{18}\right)$ are in AP.

Now, $x - 2y = \frac{1}{2}\left[\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right] - \left(\tan\frac{\pi}{9} + \tan\frac{5\pi}{18}\right)$

$$\Rightarrow |x-2y| = \left|\frac{\cot\frac{\pi}{9} - \tan\frac{\pi}{9}}{2} - \tan\frac{5\pi}{18}\right| \quad \left\{\begin{array}{c} \because \tan\frac{5\pi}{18} = \cot\frac{2\pi}{9} \\ \text{and } \tan\frac{7\pi}{18} = \cot\frac{\pi}{9} \end{array}\right\}$$

$= \left|\cot\frac{2\pi}{9} - \cot\frac{2\pi}{9}\right| = 0$

$\left[\because \cot 2A = \frac{2\cot^2 A^{-1}}{2\cot A}\right]$

Question144

Let S_n be the sum of the first n terms of an arithmetic progression, If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is
[2021, 25 July Shift-1]

Options:

A. 6

B. 4

C. 2

D. 8

Answer: A

Solution:

Solution:
Let $S_n = An^2 + Bn = n(An + B) S_{3n} = 3S_{2n}$
$\Rightarrow 3n[A(3n) + B] = 3 \cdot 2n \cdot [A(2n) + B]$
$\Rightarrow\ 3An + B = 4An + 2B$
$\Rightarrow\ An + B = 0$
$\therefore\ \frac{S_{4n}}{S_{2n}} = \frac{4n[A(4n) + B]}{2n[A(2n) + B]}$
$= 2\left(\frac{4An - An}{2An - An}\right)$
$= 2 \times 3 = 6$

Question145

Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to [2021,22 July Shift-II]

Options:

A. 1862

B. 1842

C. 1852

D. 1872

Answer: A

Solution:

Solution:
$S_n = An^2 + Bn$
$S_{10} = 100A + 10B = 530$
$S_5 = 25A + 5B = 140$ Solving both equations, we get $B = 3$ and
$A = 5$
$\therefore\ S_n = 5n^2 + 3n$
$\therefore S_{20} - S_6 = 5(20^2 - 6^2) + 3(20 - 6)$
$= 5 \cdot 26.14 + 3 \cdot 14$
$= 14(130 + 3) = 14 \times 133 = 1862$

Question146

The sum of all the elements in the set $\{ n \in \{1, 2, \ldots 100\} :$ HCF of n and 2040 is 1$\}$ is equal to
[2021, 22 July Shift-II]

Answer: 1251

Solution:

Solution:
$n \in \{1, 2, 3, \ldots\ldots 100\}$
$2040 = 2^3 \times 3 \times 5 \times 17$
If HCF of n and 2040 is 1, n should not be a multiple of $2, 3, 5, 17$.
$n \in \{ 1, 7, 11, 13, 19, 23, 29, 31, 37, 41, 43, 47 , 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 97 \}$
$\sum_n = | 1251 |$

Question147

If sum of the first 21 terms of the series
$\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \ldots\ldots$ **where $x > 0$**
is 504 , then x is equal to
[2021, 20 July Shift-II]

Options:

A. 243

B. 9

C. 7

D. 81

Answer: D

Solution:

Solution:
Let $S = \log_{9^{1/2}} x + \log_{9^{1/3}} x + \ldots$
Using property, $\log_{a^b} x = \frac{1}{b} \log_a x$
$S \ = 2\log_9 x + 3\log_9 x + \ldots + 22\log_9 x$
$= \log_9 x(2 + 3 + 4 + \ldots + 22)$
$= \log_9 x\left[\frac{21}{2}(4 + 20) \right] = \log_9 x(21 \times 12)$
$\therefore S = 252\log_9 x$
Given, $S = 504$, then

$252\log_9 x = 504$
$\Rightarrow \log_9 x = 2$
$\Rightarrow x = (9)^2 = 81$

Question148

For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$

where $\alpha > 0$. Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$

[2021, 20 July Shift-II]

Answer: 9

Solution:

Solution:
Given,

$$\frac{1}{\alpha(\alpha+1)\ldots+(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}, \alpha > 0$$

$$\Rightarrow \frac{1}{\alpha(\alpha+1)\ldots(\alpha+20)} = \frac{A_0}{\alpha} + \frac{A_1}{\alpha+1}$$

$$+\ldots+ \frac{A_{20}}{\alpha+20}$$

$$A_{14} = \frac{1}{(-14)(-13)\ldots(-1)(1)\ldots(6)} = \frac{1}{14!6!}$$

$$A_{15} = \frac{1}{(-15)(-14)\ldots(-1)(1)\ldots(5)} = \frac{-1}{15!5!}$$

$$A_{13} = \frac{1}{(-13)\ldots(-1)(1)\ldots(7)} = \frac{-1}{13!7!}$$

$$\frac{A_{14}}{A_{13}} = \frac{-13!7!}{14!6!} = \frac{-7}{14} = \frac{-1}{2}$$

$$\Rightarrow \frac{A_{15}}{A_{13}} = \frac{13!7!}{15!5!} = \frac{42}{15\times 14} = \frac{1}{5}$$

$$\therefore 100\left(\frac{A_{14}}{A_{13}} + \frac{A_{15}}{A_{13}}\right)^2 = 100\left(-\frac{1}{2} + \frac{1}{5}\right)^2$$

$$= 100\left(\frac{-3}{10}\right)^2 = 9$$

Question149

If the value of

$$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\ldots \text{ upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots \text{ upto } \infty\right)}$$

is l , then l^2 is equal to ....
[2021, 25 July Shift-I]

Answer: 3

Solution:

Solution:
Let $1=\alpha^\beta$

$\alpha=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\ldots\infty \quad \cdots\cdots(\text{i})$

$\frac{\alpha}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\ldots. \quad \cdots\cdots(\text{ii})$

Subtracting Eq. (ii) from Eq. (i),

$\frac{2\alpha}{3}=1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\ldots..$

$=\frac{4}{3}\left(\frac{1}{1-\frac{1}{3}}\right)=2$

$\therefore\ \alpha=3$

$\beta=\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\ldots.\right)$

$=\log_{0.25}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)=\log_{\frac{1}{4}}\frac{1}{2}=\frac{1}{2}$

$\therefore\ L=3^{1/2}$

$\Rightarrow\ L^2=3$

Question150

Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1=1$, $a_2=1$ and $a_{n+2}=2a_{n+1}+a_n$ for all $n\geq 1$. Then the value of $47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to ....
[2021, 20 July Shift-II]

Answer: 7

Solution:

Solution:

Let $\sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = x$ i.e. $\sum_{n=1}^{\infty} \frac{a_n}{8^n} = x$

Given, $a_{n+2} = 2a_{n+1} + a_n$

Divide the whole by 8^n ,

$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$

$\Rightarrow \ 8^2 \cdot \frac{a_{n+2}}{8^{n+2}} = 8 \cdot 2 \frac{a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n} \Rightarrow$

$64\left(\frac{a_{n+2}}{8^{n+2}}\right) = 16\left(\frac{a_{n+1}}{8^{n+1}}\right) + \frac{a_n}{8^n}$

Now, take the summation,

$64 \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = 16 \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty} \frac{a_n}{8^n} \quad \cdots\cdots (i)$

$\because \ \sum_{n=1}^{\infty} \frac{a_n}{8^n} = x$

i.e. $\frac{a_1}{8} + \frac{a_2}{8^2} + \frac{a_3}{8^3} + \frac{a_4}{8^4} + \ldots = x$

$\Rightarrow \ \frac{a_3}{8^3} + \frac{a_4}{8^4} + \ldots = x - \frac{a_1}{8} - \frac{a_2}{8^2}$

$\Rightarrow \ \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = x - \frac{a_1}{8} - \frac{a_2}{8^2} \quad \cdots\cdots (ii)$

Again, $\frac{a_2}{8^2} + \frac{a_3}{8^3} + \ldots = x - \frac{a_1}{8}$

$\Rightarrow \ \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} = x - \frac{a_1}{8} \quad \cdots\cdots (iii)$

From Eqs. (i), (ii) and (iii),

$64\left(x - \frac{a_1}{8} - \frac{a_2}{64}\right) = 16\left(x - \frac{a_1}{8}\right) + x$

Use $a_1 = 1 = a_2$

$64\left(x - \frac{1}{8} - \frac{1}{64}\right) = 16\left(x - \frac{1}{8}\right) + x$

$\Rightarrow 64x - 9 = 2(8x - 1) + x$

$\Rightarrow 64x - 16x - x = 9 - 2 \Rightarrow 47x = 7$

$\therefore 47 \sum \frac{a_n}{2^{3n}} = 7$

Question151

Let a_1, a_2, a_3...... be an (a)P.

If $\frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots.. + a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to

[2021,31 Auq. Shift-II]

Options:

A. $\frac{19}{21}$

B. $\frac{100}{121}$

C. $\frac{21}{19}$

D. $\frac{121}{100}$

Answer: C

Solution:

Solution:

$$\frac{a_1+a_2+\ldots+a_{10}}{a_1+a_2+\ldots+a_p}=\frac{100}{p^2}$$

$$\Rightarrow \frac{S_{10}}{S_p}=\frac{100}{p^2}\Rightarrow S_p=\frac{S_{10}\cdot p^2}{100}$$

$$\frac{a_{11}}{a_{10}}=\frac{S_{11}-S_{10}}{S_{10}-S_9}=\frac{S_{10}\cdot\frac{121}{100}-S_{10}}{S_{10}-S_{10}\cdot\frac{81}{100}}$$

$$=\frac{\frac{121}{100}-1}{1-\frac{81}{100}}=\frac{21}{19}$$

Question152

The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is [2021, 31 Aug. Shift-II]

Answer: 5143

Solution:

Solution:
Total 4-digit number
$=9\times10\times10\times10=9000$
4 -digit number divisible by 7
$1001, 1008, \ldots, 9996$
Number of 4-digit number divisible by 7
$=\frac{9996-1001}{7}+1=1286$
4 -digit number divisible by 3
$1002, 1005, \ldots, 9999$
Number of 4-digit number divisible by 3
$=\frac{9999-1002}{3}+1=3000$
4 digit number divisible by 21
$1008, 1031, \ldots, 9996$
Number of 4 - digit number divisible by 21
$=\frac{9996-1008}{21}+1=429$

$\therefore$ Number of 4-digit numbers neither divisible by 7 nor 3
$= 9000 - 1286 - 3000 + 429 = 5143$

Question153

Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d . If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to
[2021, 31 Aug. Shift-I]

Options:

A. $7 - 7\sqrt{3}$

B. $7 + \sqrt{3}$

C. $7 - \sqrt{3}$

D. $7 + 3\sqrt{3}$

Answer: B

Solution:

Solution:
Let three numbers be $\frac{a}{r}$, a, ar.
According to the question, $\frac{a}{r}, 2a, ar \rightarrow AP$
$4a = ar + \frac{a}{r} \Rightarrow r + \frac{1}{r} = 4$
$\Rightarrow r^2 - 4r + 1 = 0 \Rightarrow r = 2 \pm \sqrt{3}$
T_4 of GP $= 3r^2$
$3r^2 = ar^2$
$a = 3$
$r = 2 + \sqrt{3}$
$d = 2a - \frac{a}{r} = 3\sqrt{3}$
$r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3}$
$= 7 + 4\sqrt{3} - 3\sqrt{3} = 7 + \sqrt{3}$

Question154

If $0 < x < 1$, then
$\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots\ldots$, is equal to
[2021, 27 Aug. Shift-1]

Options:

A. $x\left(\frac{1+x}{1-x}\right)+\log_e(1-x)$

B. $x\left(\frac{1-x}{1+x}\right)+\log_e(1-x)$

C. $\frac{1-x}{1+x}+\log_e(1-x)$

D. $\frac{1+x}{1-x}+\log_e(1-x)$

Answer: A

Solution:

Solution:
We have,
$\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+\ldots$
$=\left(2-\frac{1}{2}\right)x^2+\left(2-\frac{1}{3}\right)x^3+\left(2-\frac{1}{4}\right)x^4+\ldots$
$=2(x^2+x^3+x^4+\ldots)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\right)$
$=2\cdot\frac{x^2}{1-x}-[-\log_e(1-x)-x]$
[using sum of infinite $GP=\frac{a}{1-r}$ and logarithmic series]

$=\frac{2x^2}{1-x}+x+\log_e(1-x)$
$=\frac{2x^2+x-x^2}{1-x}+\log_e(1-x)$
$=\frac{x^2+x}{1-x}+\log_e(1-x)$
$=x\left(\frac{1+x}{1-x}\right)+\log_e(1-x)$

Question155

If $x, y \in R, x>0$
$y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}+\ldots$
upto ∞ terms and
$\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y}=\frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to
[2021, 27 Aug. Shift-1]

Options:

A. $(10^6, 6)$

B. $(10^4, 6)$

C. $(10^2, 3)$

D. $(10^6, 9)$

Answer: D

Solution:

Solution:

Given, $\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y} = \frac{4}{\log_{10}x}$

$\Rightarrow \frac{2(1+2+3+\ldots+y)}{3(1+2+3+\ldots+y)} = \frac{4}{\log_{10}x}$

$\Rightarrow \frac{2}{3} = \frac{4}{\log_{10}x} \Rightarrow \log_{10}x = 6$

$\Rightarrow x = 10^6$

Now, $y = \log_{10}x + \log_{10}x^{\frac{1}{3}} + \log_{10}x^{\frac{1}{9}} + \ldots$ upto ∞ terms.

$= \log_{10}\left(x \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{9}} \ldots \infty \text{ terms}\right)$

$= \log_{10}x^{1+\frac{1}{3}+\frac{1}{9}+\ldots\infty \text{ terms}}$

$= \log_{10}x^{\frac{1}{1-\frac{1}{3}}} = \log_{10}x^{3/2}$

$= \log_{10}(10^6)^{\frac{3}{2}}$ $[\because x = 10^6]$

$\Rightarrow y = 6 \times \frac{3}{2} = 9$

$\therefore x = 10^6, y = 9$

$(x, y) = (10^6, 9)$

Question156

If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^{4+\cdots}$ then the value of e^{1+y} at $x = \frac{1}{2}$ is [2021, 27 Aug. Shift-II]

Options:

A. $\frac{1}{2}e^2$

B. $2e$

C. $\frac{1}{2}\sqrt{e}$

D. $2e^2$

Answer: A

Solution:

Solution:

$y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ldots$

$\Rightarrow y=\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+\left(1-\frac{1}{4}\right)x^4+$

$=(x^2+x^3+x^4+\ldots)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\right)$

$=\frac{x^2}{1-x}+x-\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}\ldots\right)$

$\therefore\ y=\frac{x}{1-x}+\ln(1-x)$

Put $x=\frac{1}{2}$, we get

$y=1-\ln 2$

Then, $e^{1+y}=e^{1+1-\ln 2}=e^{2-\ln 2}$

$=e^2\cdot e^{\ln 2^{-1}}$

$=\frac{1}{2}e^2$

Question157

If the sum of an infinite GP a, ar, ar^2, ar^3, ... is 15 and the sum of the squares of its each term is 150 , then the sum of ar^2, ar^4, ar^6, ... is [2021, 26 Aug. Shift-1]

Options:

A. $5/2$

B. $1/2$

C. $25/2$

D. $9/2$

Answer: B

Solution:

Solution:

We have, sum of infinite GP a, ar, ar^2, ... is

$S_\infty=\frac{a}{1-r}=15\quad\cdots\cdots(i)$

and sum of infinite GP a^2, a^2r^2, a^2r^4, ... is

$cS_\infty{}'=\frac{a^2}{1-r^2}=150$

$\Rightarrow\left(\frac{a}{1-r}\right)\left(\frac{a}{1+r}\right)=150\quad\cdots\cdots(ii)$

Divide Eq. (ii) by Eq. (i)

$\frac{a}{1+r}=10\quad\cdots\cdots(iii)$

Divide Eq. (iii) by Eq. (i)

$\frac{1-r}{1+r}=\frac{10}{15}=\frac{2}{3}$

$\Rightarrow 3-3r=2+2r$

$\Rightarrow 1=5r\Rightarrow r=\frac{1}{5}$

Now, putting $r = \frac{1}{5}$ in Eq. (iii), we get

$$\frac{a}{1+\frac{1}{5}} = 10$$

$\Rightarrow \frac{5a}{6} = 10 \Rightarrow a = 12$

Now, sum of $ar^2, ar^4, ar^6, \ldots, \infty$

$$S_\infty'' = \frac{ar^2}{1-r^2} = \frac{12 \cdot \left(\frac{1}{25}\right)}{\frac{24}{25}} = \frac{1}{2}$$

Question158

Let $a_1, a_2 \ldots\ldots, a_{10}$ be an AP with common difference -3 and $b_1, b_2, \ldots\ldots, b_{10}$ be a GP with common ratio 2.
Let $c_k = a_k + b_k$, $k = 1, 2, \ldots\ldots, 10$. If $c_2 = 12$ and
$c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to
[2021, 26 Aug. Shift-II]

Answer: 2021

Solution:

Solution:
$a_1, a_2, a_3, \ldots, a_{10}$ arein APcommon
difference $= -3$
$b_1, b_2, b_3, \ldots, b_{10}$ arein GPcommon ratio $= 2$
Since,c $= a_k + b_k$, $k = 1, 2, 3 \ldots\ldots, 10$
$\therefore\ c_2 = a_2 + b_2 = 12$
$c_3 = a_3 + b_3 = 13$
Now, $c_3 - c_2 = 1$
$\Rightarrow\ (a_3 - a_2) + (b_3 - b_2) \neq 1$
$\Rightarrow\ -3 + (2b_2 - b_2) \neq 1$
$\Rightarrow\ b_2 = 4$
$\therefore\ a_2 = 8$
So, AP is $11, 8, 5, \ldots.$
and GP is $2, 4, 8, \ldots.$
Now, $\sum_{k=1}^{10} c_k = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$

$$= \left(\frac{10}{2}\right)[22 + 9(-3)] + 2\left(\frac{2^{10}-1}{2-1}\right)$$
$$= 5(22-27) + 2(1023)$$
$$= 2046 - 25 = 2021$$

Question159

The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$ is
[2021, 31 Aug. Shift-1]

Options:

A. 1

B. 120/121

C. 99/100

D. 143 ⁄ 144

Answer: B

Solution:

Solution:

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$$
$$= \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + \ldots.$$
$$= \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \ldots. \left(\frac{1}{10^2} - \frac{1}{11^2}\right)$$
$$= \frac{1}{1^2} - \frac{1}{11^2} = 1 - \frac{1}{121} = \frac{120}{121}$$

Question160

If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$, then 160 Sis equal to
[2021, 31 Aug. Shift-II]

Answer: 305

Solution:

Solution:

$$S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots + \infty \quad \cdots\cdots \text{(i)}$$
$$\frac{S}{5} = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \frac{19}{5^5} + \ldots \infty \quad \cdots\cdots \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i),

$$c\ \frac{4S}{5} = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \ldots \infty$$
$$\frac{4S}{5} - \frac{7}{5} = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \ldots \infty = K \quad \cdots\cdots \text{(iii)}$$
$$\frac{K}{5} = \frac{2}{5^3} + \frac{4}{5^4} + \frac{6}{5^5} + \frac{8}{5^6} + \ldots \infty \quad \ldots \text{(iv)}$$

Subtracting Eq. (iv) from Eq (iii),

$$\frac{4K}{5}=\frac{2}{5^2}+\frac{2}{5^3}+\frac{2}{5^4}+\frac{2}{5^5}+\ldots\infty$$

$$\frac{4K}{5}=\frac{2}{25}\left(\frac{1}{1-1/5}\right)=\frac{1}{10}\Rightarrow K=\frac{1}{8}$$

From Eq. (iii),

$$\frac{4S}{5}-\frac{7}{5}=\frac{1}{8}\Rightarrow S=\frac{61}{32}$$

Now, $106S=160\times\frac{61}{32}=305$

Question161

Let $a_1, a_2, \ldots\ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}}=\frac{4}{9}$. If the sum of this AP is 189 , then $a_6 a_{16}$ is equal to

[2021, 01 Sep. Shift-II]

Options:

A. 57

B. 72

C. 48

D. 36

Answer: B

Solution:

Solution:

Let d be the common difference of an AP

$a_1, a_2, \ldots, a_{21}$ and $\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}}=\frac{4}{9}$

$\Rightarrow \sum_{n=1}^{20}\frac{1}{a_n(a_n+d)}=\frac{4}{9}$

$\Rightarrow \frac{1}{d}\sum_{n=1}^{20}\left(\frac{1}{a_n}-\frac{1}{a_n+d}\right)=\frac{4}{9}$

$\Rightarrow \frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_3}-\frac{1}{a_4}+\ldots+\frac{1}{a_{20}}-\frac{1}{a_{21}}\right]=\frac{4}{9}$

$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{21}}\right)=\frac{4}{9}$

$\Rightarrow \frac{1}{d}\left(\frac{a_{21}-a_1}{a_1 a_{21}}\right)=\frac{4}{9}\Rightarrow a_1 a_{21}=45$

$\Rightarrow a_1(a_1+20d)=45 \quad \cdots\cdots(i)$

$\Rightarrow \frac{21}{2}(2a_1+20d)=189$

Also sum of first 21 terms $=189$

$\Rightarrow \frac{21}{2}(2a_1+20d)=189$

$\Rightarrow a_1+10d=9 \quad \cdots\cdots(ii)$

By Eqs. (i) and (ii), we get $a_1=3, d=3/5$

or $a_1 = 15, d = -\frac{3}{5}$

So, $a_6 a_{16} = (a_1 + 5d)(a_1 + 15d) = 72$

Question162

Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3.$
$(n-3) + \ldots + (n-1) \cdot 1, n \geq 4$.

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to

[2021, 01 Sep. Shift-II]

Options:

A. $\frac{e-1}{3}$

B. $\frac{e-2}{6}$

C. $\frac{e}{3}$

D. $\frac{e}{6}$

Answer: A

Solution:

Solution:

$S_n = 1 \cdot (n-1) + 2(n-2) + 3(n-3) + \ldots + (n-1) \cdot 1, n \geq 4$

$= \sum_{r=1}^{n-1} r(n-r) = \frac{n(n^2-1)}{6}$

$= \frac{n(n-1)(n+1)}{6}$

$\frac{2S_n}{n!} = \frac{(n+1)}{3(n-2)!}$

$\Rightarrow \sum_{n=4}^{\infty}\left(\frac{2s_n}{n!} - \frac{1}{(n-2)!}\right)$

$= \sum_{n=4}^{\infty} \frac{n-2}{3(n-2)!} = \frac{1}{3}\sum_{n=4}^{\infty} \frac{1}{(n-3)!}$

$= \frac{1}{3}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\right) = \frac{e-1}{3}$

Question163

The number of terms common to the two A.P.'s 3, 7, 11, ... 407 and 2, 9, 16, ..., 709 is ______.

[NA Jan. 9, 2020 (II)]

Answer: 14

Solution:

Solution:
First common term of both the series is 23 and common difference is $7 \times 4 = 28$
$\because$ Last term $\leq 407 \Rightarrow 23 + (n-1) \times 28 \leq 407$
$\Rightarrow (n-1) \times 28 \leq 384$
$\Rightarrow \ n \leq \frac{384}{28} + 1$
$\Rightarrow n \leq 14.71$
Hence, $n = 14$

Question164

If the 10 th term of an A.P. is $\frac{1}{20}$ and its 20 th term is $\frac{1}{10}$, then the sum of its first 200 terms is:
[Jan. 8, 2020 (II)]

Options:

A. 50

B. 50 $\frac{1}{4}$

C. 100

D. 100 $\frac{1}{2}$

Answer: D

Solution:

Solution:
$T_{10} = \frac{1}{20} = a + 9d$ . . . (i)
$T_{20} = \frac{1}{10} = a + 19d$ . . . (ii)
Solving equations (i) and (ii), we get
$a = \frac{1}{200}, d = \frac{1}{200}$
$\Rightarrow S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right] = \frac{201}{2} = 100\frac{1}{2}$

Question165

Let $f : R \rightarrow R$ be such that for all $x \in R$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^{x} + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is:
[Jan. 8, 2020 (I)]

Options:

A. 2

B. 3

C. 0

D. 4

Answer: B

Solution:

Solution:
If $2^{1-x} + 2^{1+x}$, $f(x)$, $3^{x} + 3^{-x}$ are in A.P., then
$$f(x) = \left(\frac{2^{1+x} + 2^{1-x} + 3^{x} + 3^{-x}}{2} \right)$$
$$2f(x) = 2\left(2^{x} + \frac{1}{2^{x}}\right) + \left(3^{x} + \frac{1}{3^{x}}\right)$$
Using $AM \geq GM$
$f(x) \geq 3$

Question166

Five numbers are in A.P., whose sum is 25 and product is 2520 . If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is: [Jan. 7, 2020 (I)]

Options:

A. 27

B. 7

C. $\frac{21}{2}$

D. 16

Answer: D

Solution:

Solution:
Let 5 terms of A.P. be
$a-2d, a-d, a, a+d, a+2d$
$\text{Sum} = 25 \Rightarrow 5a = 25 \Rightarrow a = 5$
Product $= 2520$
$(5-2d)(5-d)5(5+d)(5+2d) = 2520$
$\Rightarrow (25-4d^{2})(25-d^{2}) = 504$
$\Rightarrow 625 - 100d^{2} - 25d^{2} + 4d^{4} = 504$
$\Rightarrow 4d^{4} - 125d^{2} + 625 - 504 = 0$
$\Rightarrow 4d^{4} - 125d^{2} + 121 = 0$
$\Rightarrow 4d^{4} - 121d^{2} - 4d^{2} + 121 = 0$

$\Rightarrow (d^2 - 1)(4d^2 - 121) = 0$
$\Rightarrow d = \pm 1, d = \pm \frac{11}{2}$
$d = \pm 1$ and $d = -\frac{11}{2}$, does not give $\frac{-1}{2}$ as a term
$\therefore\ d = \frac{11}{2}$
$\therefore$ Largest term $= 5 + 2d = 5 + 11 = 16$

Question167

The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \ldots$ to ∞ is equal to: [Jan. 9, 2020 (I)]

Options:

A. $2^{\frac{1}{2}}$

B. $2^{\frac{1}{4}}$

C. 1

D. 2

Answer: A

Solution:

Solution:
$2^{\frac{1}{4} + \frac{2}{16} + \frac{3}{48} + \ldots \infty}$
$= 2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \infty} = \sqrt{2}$

Question168

Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to : [Jan. 9, 2020 (II)]

Options:

A. 300

B. 225

C. 175

D. 150

Answer: D

Solution:

Solution:
Let G.P. be $a, ar, ar^2 \ldots\ldots$
$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots.. + a_{201} = 200$
$\Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200$
$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots.. + a_{200} = 100$
$\Rightarrow \frac{ar(r^{200}-1)}{r^2-1} = 100$
From equations (i) and (ii), $r = 2$ and
$a_2 + a_3 + \ldots.. + a_{200} + a_{201} = 300$
$\Rightarrow r(a_1 + \ldots.. + a_{200}) = 300$
$\Rightarrow \sum_{n=1}^{200} a_n = \frac{300}{r} = 150$

Question169

If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then : [Jan. 9, 2020 (II)]

Options:

A. $x(1+y) = 1$

B. $y(1-x) = 1$

C. $y(1+x) = 1$

D. $x(1-y) = 1$

Answer: B

Solution:

Solution:
$y = 1 + \cos^2\theta + \cos^4\theta + \ldots.$
$\Rightarrow \ y = \frac{1}{1-\cos^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$
$x = 1 - \tan^2\theta + \tan^4\theta + \ldots.$
$x = \frac{1}{1-(-\tan^2\theta)} = \frac{1}{\sec^2\theta} \Rightarrow x = \cos^2\theta$
$y = \frac{1}{\sin^2\theta} \Rightarrow y = \frac{1}{1-x}$
$\therefore \ y(1-x) = 1$

Question170

The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \ldots + 49^2 + 49 + 1$, is:
[Jan. 7, 2020 (I)]

Options:

A. 32

B. 63

C. 60

D. 65

Answer: B

Solution:

Solution:

$\frac{(49)^{126} - 1}{48} = \frac{((49)^{63} + 1)(49^{63} - 1)}{48}\left[\because S_n = \frac{a(r^n - 1)}{r - 1}\right]$

$\therefore\ K = 63$

Question171

Let $a_1, a_2, a_3, \ldots$ be a G. P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to:
[Jan. 7, 2020 (II)]

Options:

A. -513

B. -171

C. 171

D. $\frac{511}{3}$

Answer: B

Solution:

Solution:

Since, $a_1 + a_2 = 4 \Rightarrow a_1 + a_1 r = 4 \ldots$ (i)

$a_3 + a_4 = 16 \quad \Rightarrow a_1 r^2 + a_1 r^3 = 16 \ldots$ (ii)

From eqn. (i), $a_1 = \frac{4}{1 + r}$ and substituting the value of a_1, in eqn (ii),

$$\left(\frac{4}{1+r}\right)^{r^2}+\left(\frac{4}{1+r}\right)^{r^3}=16$$
$$\Rightarrow 4r^2(1+r)=16(1+r)$$
$$\Rightarrow r^2=4 \quad \therefore r=\pm 2$$
$$r=2,\ a_1(1+2)=4 \Rightarrow a_1=\frac{4}{3}$$
$$r=-2,\ a_1(1-2)=4 \Rightarrow a_1=-4$$
$$\sum_{i=1}^{a} a_i=\frac{a_1(r^q-1)}{r-1}=\frac{(-4)((-2)^9-1)}{-2-1}$$
$$=\frac{4}{3}(-513)=4\lambda \Rightarrow \lambda=-171$$

Question172

The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+\ldots+x^{10}$ is: [Jan. 7, 2020 (II)]

Options:

A. 210

B. 330

C. 120

D. 420

Answer: B

Solution:

Solution:

The given series is in G.P. then $S_n=\frac{a(1-r^n)}{1-r}$

$$\frac{(1+x)^{10}\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$$

$$\Rightarrow \frac{(1+x)^{10}[(1+x)^{11}-x^{11}]}{(1+x)^{11}\times\frac{1}{(1+x)}}=(1+x)^{11}-x^{11}$$

$\therefore$ Coefficient of x^7 is ${}^{11}C_7={}^{11}C_{11-7}={}^{11}C_4=330$

Question173

The sum, $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to __________. [Jan. 8, 2020 (II)]

Answer: 504

Solution:

Solution:

$$\left[\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}\right] \frac{1}{4}\left[\sum_{n=1}^{7}(2n^3+3n^2+n)\right]$$

$$= \frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2+3\left(\frac{7.8.15}{6}\right)+\frac{7.8}{2}\right]$$

$$\Rightarrow \frac{1}{4}[2\times 49\times 16+28\times 15+28]$$

$$= \frac{1}{4}[1568+420+28]=504$$

Question174

The sum $\sum_{k=1}^{20}(1+2+3+\ldots+k)$ is
[Jan. 8, 2020 (I)]

Answer: 1540

Solution:

Solution:
Given series can be written as

$$\sum_{k=1}^{20}\frac{k(k+1)}{2}=\frac{1}{2}\sum_{k=1}^{20}(k^2+k)$$

$$= \frac{1}{2}\left[\frac{20(21)(41)}{6}+\frac{20(21)}{2}\right]$$

$$= \frac{1}{2}\left[\frac{420\times 41}{6}+\frac{20\times 21}{2}\right]=\frac{1}{2}[2870+210]=1540$$

Question175

If the sum of the first 40 terms of the series, $3+4+8+9+13+14+18+19+\ldots$ is (102)m, then m is equal to:
[Jan. 7, 2020 (II)]

Options:

A. 20

B. 25

C. 5

D. 10

Answer: A

Solution:

Solution:
$S = 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 \ldots\ldots 40$ terms
$S = 7 + 17 + 27 + 37 + 47 + \ldots\ldots 20$ terms
$S_{40} = \frac{20}{2}[2 \times 7 + (19)10] = 10[14 + 190]$
$= 10[2040] = (102)(20)$
$\Rightarrow\ m = 20$

Question176

If the sum of first 11 terms of an A.P., $a_1, a_2, a_3, \ldots$ is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, \ldots, a_{23}$ is ka_1, where k is equal to : [Sep. 02, 2020 (II)]

Options:

A. $-\frac{121}{10}$

B. $\frac{121}{10}$

C. $\frac{72}{5}$

D. $-\frac{72}{5}$

Answer: D

Solution:

Solution:
Let common difference be d
$\because S_{11} = 0 \quad \therefore \frac{11}{2}\{2a_1 + 10 \cdot d\} = 0$
$\Rightarrow a_1 + 5d = 0 \Rightarrow d = -\frac{a_1}{5} \ldots$ (i)
Now, $S = a_1 + a_3 + a_5 + \ldots + a_{23}$
$= a_1 + (a_1 + 2d) + (a_1 + 4d) + \ldots. + (a_1 + 22d)$
$= 12a_1 + 2d\ \frac{11 \times 12}{2}$
$= 12\left[a_1 + 11 \cdot \left(-\frac{a_1}{5}\right)\right]$ (From (i))
$= 12 \times \left(-\frac{6}{5}\right)a_1 = -\frac{72}{5}a_1$

Question177

If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:
[Sep. 03, 2020 (I)]

Options:

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{7}$

Answer: A

Solution:

Solution:
Given $a = 3$ and $S_{25} = S_{40} - S_{25}$
$\Rightarrow 2S_{25} = S_{40}$
$\Rightarrow 2\times\frac{25}{2}[6+24d] = \frac{40}{2}[6+39d]$
$\Rightarrow 25[6+24d] = 20[6+39d]$
$\Rightarrow 5(2+8d) = 4(2+13d)$
$\Rightarrow 10+40d = 8+52d$
$\Rightarrow d = \frac{1}{6}$

Question178

In the sum of the series $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+\ldots$ upto n^{th} term is 488 and then n^{th} term is negative, then :
[Sep. 03, 2020 (II)]

Options:

A. $n = 60$

B. n^{th} term is -4

C. $n = 41$

D. n^{th} term is $-4\frac{2}{5}$

Answer: B

Solution:

Solution:

$S_n = 20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots.$

$\because S_n = 488$

$488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(-\frac{2}{5}\right)\right]$ $488 = \frac{n}{2}(101-n) \Rightarrow n^2 - 101n + 2440 = 0$

$\Rightarrow n = 61$ or 40

For $n = 40 \Rightarrow T_n > 0$

For $n = 61 \Rightarrow T_n < 0$

n^{th} term $= T_{61} = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$

Question179

Let $a_1, a_2, \ldots., a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \ldots. + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :

[Sep. 04, 2020 (II)]

Options:

A. (2490,249)

B. (2480,249)

C. (2480,248)

D. (2490,248)

Answer: D

Solution:

Solution:

Given that $a_1 = 1$ and $a_n = 300$ and $d \in Z$

$\therefore 300 = 1 + (n-1)d$

$\Rightarrow d = \frac{299}{(n-1)} = \frac{23 \times 13}{(n-1)}$

$\because d$ is an integer

$\therefore n - 1 = 13$ or 23

$\Rightarrow n = 14$ or 24 $(\because 15 \leq n \leq 50)$

$\Rightarrow n = 24$ and $d = 13$

$a_{20} = 1 + 19 \times 13 = 248$

$s_{20} = \frac{20}{2}(2 + 19 \times 13) = 2490$

Question180

If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4 - 2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P is:

[Sep. 05, 2020 (I)]

Options:

A. 66

B. 81

C. 65

D. 78

Answer: A

Solution:

Solution:
Given that $3^{2\sin 2\alpha - 1}$, 14, $3^{4 - 2\sin 2\alpha}$ are in A.P.
So, $3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha} = 28$
$\Rightarrow \frac{3^{2\sin 2\alpha}}{3} + \frac{81}{3^{2\sin 2\alpha}} = 28$
Let $3^{2\sin 2\alpha} = x$
$\Rightarrow \frac{x}{3} + \frac{81}{x} = 28$
$\Rightarrow x^2 - 84x + 243 = 0 \Rightarrow x = 81, x = 3$
When $x = 81 \Rightarrow \sin 2\alpha = 2$ (Not possible)
When $x = 3 \Rightarrow \alpha = \frac{\pi}{12}$
$\therefore a = 3^0 = 1, d = 14 - 1 = 13$
$a_6 = a + 5d = 1 + 65 = 66$

Question181

If the sum of the first 20 terms of the series $\log_{(7^{1/2})}x + \log_{(7^{1/3})}x + \log_{(7^{1/4})}x + \ldots$ is 460, then x is equal to :
[Sep. 05, 2020 (II)]

Options:

A. 7^2

B. $7^{\frac{1}{2}}$

C. e^2

D. $7^{\frac{46}{21}}$

Answer: A

Solution:

Solution:

$S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + \ldots 20$ terms
$\because S = 460$
$\Rightarrow \log_7(x^2 \cdot x^3 \cdot x^4 \cdot \ldots\ldots x^{21}) = 460$
$\Rightarrow \log_7 x^{(2+3+4\ldots\ldots 21)} = 460$
$\Rightarrow (2+3+4+\ldots\ldots+21)\log_7 x = 460$
$\Rightarrow \frac{20}{2}(2+21)\log_7 x = 460$
$\Rightarrow \log_7 x = \frac{460}{230} = 2 \Rightarrow x = 7^2 = 49$

Question182

If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, $x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is :
[Sep. 06, 2020 (I)]

Options:

A. $\frac{2}{3}$

B. $\frac{1}{9}$

C. $\frac{1}{3}$

D. $\frac{4}{9}$

Answer: D

Solution:

Solution:
Let $f(1) = k$, then $f(2) = f(1+1) = k^2$
$f(3) = f(2+1) = k^3$
$\sum_{x=1}^{\infty} f(x) = 2 \Rightarrow k + k^2 + k^3 + \ldots \infty = 2$
$\Rightarrow \frac{k}{1-k} = 2 \Rightarrow k = \frac{2}{3}$
Now, $\frac{f(4)}{f(2)} = \frac{k^4}{k^2} = k^2 = \frac{4}{9}$.

Question183

Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then :
[Sep. 06, 2020 (I)]

Options:

A. a, c, p are in A.P.

B. a, c, p are in G.P.

C. a, b, c, d are in G.P.

D. a, b, c, d are in A.P.

Answer: C

Solution:

Solution:
Rearrange given equation, we get
$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2)$
$+(c^2p^2 - 2cd\,p + d^2) = 0$
$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$
$\therefore ap - b = bp - c = cp - d = 0$
$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ $\therefore$ a, b, c, d are in G.P.

Question184

The common difference of the A.P. $b_1, b_2, \ldots, b_m$ is 2 more than the common difference of A.P. $a_1, a_2, \ldots, a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to:
[Sep. 06, 2020 (II)]

Options:

A. 81

B. -127

C. -81

D. 127

Answer: C

Solution:

Solution:
Let common difference of series
$a_1, a_2, a_3, \ldots\ldots, a_n$ be d
$\because a_{40} = a_1 + 39d = -159$ . . . (i)
and $a_{100} = a_1 + 99d = -399$ . . . (ii)
From equations (i) and (ii),
$d = -4$ and $a_1 = -3$
Since, the common difference of $b_1, b_2, \ldots\ldots, b_n$ is 2 more than common difference of $a_1, a_2, \ldots\ldots, a_n$
$\therefore$ Common difference of $b_1, b_2, b_3, \ldots..$ is (-2)
$\because b_{100} = a_{70}$

$\Rightarrow b_1 + 99(-2) = (-3) + 69(-4)$
$\Rightarrow b_1 = 198 - 279 \Rightarrow b_1 = -81$

Question185

Suppose that a function $f : R \to R$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(a) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to _______.
[NA Sep. 06, 2020 (II)]

Answer: 5

Solution:

Solution:
$\because f(x+y) = f(x) \cdot f(y) \quad \forall x \in R$ and $f(1) = 3$
$\Rightarrow f(x) = 3^x \Rightarrow f(i) = 3^i$
$\Rightarrow \sum_{i=1}^{n} f(i) = 363$
$\Rightarrow 3 + 3^2 + 3^3 + \ldots + 3^n = 363$
$\Rightarrow \frac{3(3^n - 1)}{3 - 1} = 363 \quad \left[\because S_n = \frac{a(r^n - 1)}{(r - 1)}\right]$
$\Rightarrow 3^n - 1 = \frac{363 \times 2}{3} = 242$
$\Rightarrow 3^n = 243 = 3^5 \Rightarrow n = 5$

Question186

If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \ldots + 2 \times 3^9 + 3^{10} = S - 2^{11}$ then S is equal to:
[Sep. 05, 2020 (I)]

Options:

A. $3^{11} - 2^{12}$

B. 3^{11}

C. $\frac{3^{11}}{2} + 2^{10}$

D. $2 \cdot 3^{11}$

Answer: B

Solution:

Solution:

Given sequence are in G.P. and common ratio 3

$$\therefore \frac{2^{10}\left(\left(\frac{3}{2}\right)^{11}-1\right)}{\left(\frac{3}{2}-1\right)} = S - 2^{11}$$

$$\Rightarrow 2^{10} \frac{\left(\frac{3^{11}-2^{11}}{2^{11}}\right)}{\frac{1}{2}} = S - 2^{11}$$

$$\Rightarrow 3^{11} - 2^{11} = S - 2^{11} \Rightarrow S = 3^{11}$$

Question187

If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is:
[Sep. 05, 2020 (II)]

Options:

A. $\frac{1}{26}(3^{49}-1)$

B. $\frac{1}{26}(3^{50}-1)$

C. $\frac{2}{13}(3^{50}-1)$

D. $\frac{1}{13}(3^{50}-1)$

Answer: B

Solution:

Solution:
Let the first term be ' a ' and common ratio be ' r '.
$\because ar(1+r+r^2) = 3$ . . . (i)
and $ar^5(1+r+r^2) = 243$ . . . (ii)
From (i) and (ii),
$r^4 = 81 \Rightarrow r = 3$ and $a = \frac{1}{13}$

$$\therefore S_{50} = \frac{a(r^{50}-1)}{r-1} = \frac{3^{50}-1}{26} \quad \left[\because S_n = \frac{a(r^n-1)}{(r-1)}\right]$$

Question188

Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q+p) : (2q-p)$ is :
[Sep. 04, 2020 (I)]

Options:

A. 3: 1

B. 9: 7

C. 5: 3

D. 33: 31

Answer: B

Solution:

Solution:
Let $\alpha, \beta, \gamma, \delta$ be in G.P., then $\alpha\delta = \beta\gamma$

$\Rightarrow \frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha-\beta}{\alpha+\beta}\right| = \left|\frac{\gamma-\delta}{\gamma+\delta}\right|$

$\Rightarrow \frac{\sqrt{9-4p}}{3} = \frac{\sqrt{36-4q}}{6}$

$\Rightarrow 36 - 16p = 36 - 4q \Rightarrow q = 4p$

$\therefore \frac{2q+p}{2q-p} = \frac{8p+p}{8p-p} = \frac{9p}{7p} = \frac{9}{7}$

Question189

The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots \text{ to } \infty\right)}$ is equal to ________.

[NA Sep. 03, 2020 (I)]

Answer: 4

Solution:

Solution:

$(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots\infty\right)}$

$0.16^{\log_{2.5}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)} \left[\because S_\infty = \frac{a}{1-r}\right]$

$= 0.16^{\log_{2.5}\left(\frac{1}{2}\right)}$

$= (2.5)^{-2\log_{2.5}\left(\frac{1}{2}\right)} = \left(\frac{1}{2}\right)^{-2} = 4$

Question190

The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :
[Sep. 02, 2020 (I)]

Options:

A. $(-\infty, -9] \cup [3, \infty)$

B. $[-3, \infty)$

C. $(-\infty, -3] \cup [9, \infty)$

D. $(-\infty, 9]$

Answer: C

Solution:

Solution:
Let terms of G.P. be $\frac{a}{r}, a, ar$
$\therefore a\left(\frac{1}{r}+1+r\right) = S \ldots$ (i)
and $a^3 = 27$
$\Rightarrow a = 3 \ldots$ (ii)
Put $a = 3$ in eqn. (1), we get
$S = 3 + 3\left(r + \frac{1}{r}\right)$
If $f(x) = x + \frac{1}{x}$, then $f(x) \in (-\infty, -2] \cup [2, \infty)$
$\Rightarrow 3f(x) \in (-\infty, -6] \cup [6, \infty)$
$\Rightarrow 3 + 3f(x) \in (-\infty, -3] \cup [9, \infty)$
Then, it concludes that
$S \in (-\infty, -3] \cup [9, \infty)$

Question191

If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \ldots$ is :
[Sep. 02, 2020 (I)]

Options:

A. $\frac{x+y-xy}{(1+x)(1+y)}$

B. $\frac{x+y+xy}{(1+x)(1+y)}$

C. $\frac{x+y-xy}{(1-x)(1-y)}$

D. $\frac{x+y+xy}{(1-x)(1-y)}$

Answer: C

Solution:

Solution:

$S=(x+y)+(x^2+y^2+xy)+(x^3+x^2y+xy^2+y^3)+\ldots\infty$

$=\frac{1}{x-y}[(x^2-y^2)+(x^3-y^3)+(x^4-y^4+\ldots\infty]$

$=\frac{1}{x-y}\left[\frac{x^2}{1-x}-\frac{y^2}{1-y}\right]=\frac{(x-y)(x+y-xy)}{(x-y)(1-x)(1-y)}$

$\left[\because S_\infty=\frac{a}{1-r}\right]$

$=\frac{x+y-xy}{(1-x)(1-y)}$

Question192

Let S be the sum of the first 9 terms of the series:
$\{x+ka\}+\{x^2+(k+2)a\}+\{x^3+(k+4)a\}+\{x^4+(k+6)a\}+\ldots$ where $a\neq 0$ and $x\neq 1$. If $S=\frac{x^{10}-x+45a(x-1)}{x-1}$, then k is equal to:
[Sep. 02, 2020 (II)]

Options:

A. -5

B. 1

C. -3

D. 3

Answer: C

Solution:

Solution:

$S=(x+x^2+x^3+\ldots 9\text{ terms })$

$+a[k+(k+2)++(k+4)+\ldots 9\text{ terms }]\Rightarrow S=\frac{x(x^9-1)}{x-1}+\frac{9}{2}[2ak+8\times(2a)]$

$\Rightarrow S=\frac{x^{10}-x}{x-1}+\frac{9a(k+8)}{1}=\frac{x^{10}-x+45a(x-1)}{x-1}\Big(\text{ Given })$

$\Rightarrow\frac{x^{10}-x+9a(k+8)(x-1)}{x-1}=\frac{x^{10}-x+45a(x-1)}{x-1}$

$\Rightarrow 9a(k+8)=45a\Rightarrow k+8=5\Rightarrow k=-3$

Question193

If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to

______

[Sep. 03, 2020 (II)]

Answer: 39

Solution:

Solution:
Let m arithmetic mean be $A_1, A_2 \ldots A_m$ and G_1, G_2, G_3 be geometric mean.
The A.P. formed by arithmetic mean is,
$3, A_1, A_2, A_3, \ldots\ldots A_m, 243$
$\therefore d = \frac{243-3}{m+1} = \frac{240}{m+1}$
The G.P. formed by geometric mean
$3, G_1, G_2, G_3, 243$
$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$
$\because A_4 = G_2$
$\Rightarrow 3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$
$\Rightarrow 3 + \frac{960}{m+1} = 27 \Rightarrow m+1 = 40 \Rightarrow m = 39.$

Question194

If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \ldots\ldots + (1 - 20^2)\ 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to :
[Sep. 04, 2020 (I)]

Options:

A. (10,97)

B. (11,103)

C. (10,103)

D. (11,97)

Answer: B

Solution:

Solution:
The given series is
$1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \ldots (1 - 20^2 \cdot 19)$
$S = 1 + \sum_{r=1}^{10} [1 - (2r)^2(2r-1)]$
$= 1 + \sum_{r=1}^{10} (1 - 8r^3 + 4r^2) = 1 + 10 - \sum_{r=1}^{10} (8r^3 - 4r^2)$
$= 11 - 8\left(\frac{10 \times 11}{2}\right)^2 + 4 \times \left(\frac{10 \times 11 \times 21}{6}\right)$
$= 11 - 2 \times (110)^2 + 4 \times 55 \times 7$
$= 11 - 220(110 - 7)$

$= 11 - 220 \times 103 = \alpha - 220\beta$
$\Rightarrow \alpha = 11, \beta = 103$
$\therefore (\alpha, \beta) = (11, 103)$

Question195

Let $f : R \to R$ be a function which satisfies $f(x+y) = f(x) + f(y)$, $\forall x, y \in R$. If $f(a) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k)$, $n \in N$, then the value of n, for which $g(n) = 20$, is : [Sep. 02, 2020 (II)]

Options:

A. 5

B. 20

C. 4

D. 9

Answer: A

Solution:

Solution:
Given: $f(x+y) = f(x) + f(y)$, $\forall x, y \in R$, $f(1) = 2$
$\Rightarrow f(2) = f(1) + f(1) = 2 + 2 = 4$
$f(3) = f(1) + f(2) = 2 + 4 = 6$
$f(n-1) = 2(n-1)$
Now, $g(n) = \sum_{k=1}^{n-1} f(k)$
$= f(1) + f(2) + f(3) + \ldots . f(n-1)$
$= 2 + 4 + 6 + \ldots .. + 2(n-1)$
$= 2[1 + 2 + 3 + \ldots \ldots + (n-1)]$
$= 2 \times \frac{(n-1)(n)}{2} = n^2 - n$
$\because g(n) = 20$ (given)
So, $n^2 - n = 20$
$\Rightarrow n^2 - n - 20 = 0$
$\Rightarrow (n-5)(n+4) = 0$
$\Rightarrow n = 5$ or $n = -4$ (not possible)

Question196

Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to: [Jan. 09, 2019 (II)]

Options:

A. 2

B. $\frac{1}{2}$

C. $\frac{7}{13}$

D. 4

Answer: D

Solution:

Solution:
Let first term and common difference be A and D respectively.
$\therefore a = A + 6D$, $b = A + 10D$ and $c = A + 12D$
Since, a, b, c are in G.P.
Hence, relation between a, b and c is,
$\therefore b^2 = a \cdot c.$
$\therefore (A + 10D)^2 = (A + 6D)(A + 12D)$
$\therefore 14D + A = 0$
$\therefore A = -14D$
$\therefore a = -8D$, $b = -4D$ and $c = -2D$
$\therefore \frac{a}{c} = \frac{-8D}{-2D} = 4$

Question197

If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then x cannot be:
[Jan. 09, 2019 (I)]

Options:

A. -2

B. -3

C. 4

D. 2

Answer: D

Solution:

Solution:
$\because a, b, c,$ are in G.P.
$\Rightarrow b^2 = ac$
Since, $a + b + c = xb$
$\Rightarrow a + c = (x - 1)b$
Take square on both sides, we get
$a^2 + c^2 + 2ac = (x - 1)^2 b^2$
$\Rightarrow a^2 + c^2 = (x - 1)^2 ac - 2ac [\because b^2 = ac]$
$\Rightarrow a^2 + c^2 = ac[(x - 1)^2 - 2]$
$\Rightarrow a^2 + c^2 = ac[x^2 - 2x - 1]$
$\because a^2 + c^2$ are positive and $b^2 = ac$ which is also positive.

Then, x^2-2x-1 would be positive but for $x=2$, x^2-2x-1 is negative.
Hence, x cannot be taken as 2 .

Question198

The sum of the following series $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+\ldots+5^2)}{11}+\ldots$ up to 15 terms, is:

[Jan. 09, 2019 (II)]

Options:

A. 7520

B. 7510

C. 7830

D. 7820

Answer: D

Solution:

Solution:

$S=1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+3^2+4^2+5^2)}{11}+\ldots$

$S=\frac{3\cdot(1)^2}{3}+\frac{6\cdot(1^2+2^2)}{5}+\frac{9\cdot(1^2+2^2+3^2)}{7}+\frac{12\cdot(1^2+2^2+3^2+4^2)}{9}+\ldots$

Now, n^{th} term of the series,

$t_n=\frac{3n\cdot(1^2+2^2+\ldots+n^2)}{(2n+1)}$

$\Rightarrow t_n=\frac{3n\cdot n(n+1)(2n+1)}{6(2n+1)}=\frac{n^3+n^2}{2}$

$\therefore\ S_n=\Sigma t_n=\frac{1}{2}\left\{\left(\frac{n(n+1)}{2}\right)^2+\frac{n(n+1)(2n+1)}{6}\right\}$

$=\frac{n(n+1)}{4}\left(\frac{n(n+1)}{2}+\frac{2n+1}{3}\right)$

Hence, sum of the series upto 15 terms is,

$S_{15}=\frac{15\times16}{4}\left\{\frac{15\cdot16}{2}+\frac{31}{3}\right\}$

$=60\times120+60\times\frac{31}{3}$

$=7200+620=7820$

Question199

The sum of all values of $\theta\in\left(0,\ \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$ is:

[Jan. 10, 2019 (I)]

Options:

A. π

B. $\frac{5\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{8}$

Answer: C

Solution:

Solution:

$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow \cos^2 2\theta(1 - \cos^2 2\theta) = \frac{1}{4} \ldots$ (i)

$\because$ G.M. $\leq$ A.M.

$\therefore (\cos^2 2\theta)(1 - \cos^2 2\theta) \leq \left(\frac{\cos^2 2\theta + (1 - \cos^2 2\theta)}{2} \right)^2$

$= \frac{1}{4} \ldots$ (ii)

So, from equation (i) and (ii), we get.

G.M. = A.M.

It is possible only if

$\cos^2 2\theta = 1 - \cos^2 2\theta$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} \Rightarrow \cos 2\theta = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8}$ $\therefore$ Sum $= \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$

Question200

Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0 (0 < \theta < 45^\circ)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty} \left(\alpha^n + \frac{(-1)^n}{\beta^n} \right)$ is equal to :

[Jan. 11, 2019 (II)]

Options:

A. $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

B. $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

C. $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

D. $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

Answer: C

Solution:

Solution:
$x^2 \sin\theta - x(\sin\theta \cdot \cos\theta + 1) + \cos\theta = 0.$
$x^2 \sin\theta - x\sin\theta \cdot \cos\theta - x + \cos\theta = 0$
$x\sin\theta(x - \cos\theta) - 1(x - \cos\theta) = 0$
$(x - \cos\theta)(x\sin\theta - 1) = 0$
$\therefore x = \cos\theta, \operatorname{cosec}\theta, \theta \in (0, 45^\circ)$
$\therefore \ \alpha = \cos\theta, \beta = \operatorname{cosec}\theta$

$$\sum_{n=0}^{\infty} \alpha^n = 1 + \cos\theta + \cos^2\theta + \ldots\infty = \frac{1}{1-\cos\theta}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\beta^n} = 1 - \frac{1}{\operatorname{cosec}\theta} + \frac{1}{\operatorname{cosec}^2\theta} - \frac{1}{\operatorname{cosec}^3\theta} + \ldots\infty$$

$$= 1 - \sin\theta + \sin^2\theta - \sin^3\theta + \ldots\infty$$

$$= \frac{1}{1+\sin\theta}$$

$$\therefore \sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right) = \sum_{n=0}^{\infty}\alpha^n + \sum_{n=0}^{\infty}\frac{(-1)^n}{\beta^n}$$

$$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$$

Question201

Let $a_1, a_2, \ldots, a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals: [Jan. 11, 2019 (I)]

Options:

A. 5^4

B. $4(5^2)$

C. 5^3

D. $2(5^2)$

Answer: A

Solution:

Solution:
Let $a_1 = a, a_2 = ar, a_3 = ar^2 \ldots a_{10} = ar^9$
where $r =$ common ratio of given G.P.
Given, $\frac{a_3}{a_1} = 25$
$\Rightarrow \frac{ar^2}{a} = 25$
$\Rightarrow r = \pm 5$
Now, $\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = (\pm 5)^4 = 5^4$

Question202

The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :
[Jan. 11, 2019 (I)]

Options:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{2}{9}$

D. $\frac{4}{9}$

Answer: B

Solution:

Solution:
Let the terms of infinite series are $a, ar, ar^2, ar^3, \ldots$
So, $\frac{a}{1-r} = 3$
Since, sum of cubes of its terms is $\frac{27}{19}$ that is sum of a^3,
$a^3r^3, \ldots \infty$ is $\frac{27}{19}$
So, $\frac{a^3}{1-r^3} = \frac{27}{19}$
$\Rightarrow \frac{a}{1-r} \times \frac{a^2}{(1+r^2+r)} = \frac{27}{19}$
$\Rightarrow \frac{9(1+r^2-2r) \times 3}{1+r^2+r} = \frac{27}{19}$
$\Rightarrow 6r^2 - 13r + 6 = 0$
$\Rightarrow (3r-2)(2r-3) = 0$
$\Rightarrow r = \frac{2}{3}$, or $\frac{3}{2}$
As $|r| < 1$
So, $r = \frac{2}{3}$

Question203

Let $S_n = 1 + q + q^2 + \ldots. + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \ldots. + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$, then α is equal to :
[Jan. 11, 2019 (II)]

Options:

A. 2^{99}

B. 202

C. 200

D. 2^{100}

Answer: D

Solution:

Solution:

$$S_n = \left(\frac{1-q^{n+1}}{1-q}\right), T_n = \frac{1-\left(\frac{q+1}{2}\right)^{n+1}}{1-\left(\frac{q+1}{2}\right)}$$

$$\Rightarrow T_{100} = \frac{1-\left(\frac{q+1}{2}\right)^{101}}{1-\left(\frac{q+1}{2}\right)}$$

$$Sn = \frac{1}{1-q} - \frac{q^{n+1}}{1-q}, T_{100} = \frac{2^{101}-(q+1)^{101}}{2^{100}(1-q)}$$

Now, ${}^{101}C_1 + {}^{101}C_2S_1 + {}^{101}C_3S_2 + \ldots + {}^{101}C_{101}S_{100}$

$$= \left(\frac{1}{1-q}\right)({}^{101}C_2 + \ldots + {}^{101}C_{101})$$

$$- \frac{1}{1-q}({}^{101}C_2q^2 + {}^{101}C_3q^3 + \ldots + {}^{101}C_{101}q^{101}) + 101$$

$$= \frac{1}{1-q}(2^{101} - 1 - 101) - \left(\frac{1}{1-q}\right)((1+q)^{101} - 1$$

$$- {}^{101}C_1q) + 101$$

$$= \frac{1}{1-q}[2^{101} - 102 - (1+q)^{101} + 1 + 101q] + 101$$

$$= \frac{1}{1-q}[2^{101} - 101 + 101q - (1+q)^{101}] + 101$$

$$= \left(\frac{1}{1-q}\right)[2^{101} - (1+q)^{101}] = 2^{100}T_{100}$$

Hence, by comparison $\alpha = 2^{100}$

Question204

The product of three consecutive terms of a G.P. is 512 . If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is :
[Jan. 12, 2019 (I)]

Options:

A. 36

B. 32

C. 24

D. 28

Answer: D

Solution:

Solution:

Let three terms of a G.P. be $\frac{a}{r}$, a, ar

$\frac{a}{r} \cdot a \cdot ar = 512$

$a^3 = 512 \Rightarrow a = 8$

4 is added to each of the first and the second of three terms then three terms are, $\frac{8}{r} + 4,\ 8 + 4,\ 8r$.

$\because \frac{8}{r} + 4,\ 12,\ 8r$ form an A.P.

$\therefore\ 2 \times 12 = \frac{8}{r} + 8r + 4$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (2r - 1)(r - 2) = 0$

$\Rightarrow r = \frac{1}{2}$ or 2

Therefore, sum of three terms $= \frac{8}{2} + 8 + 16 = 28$

Question205

Let $S_k = \frac{1 + 2 + 3 + ... + k}{k}$ If $S_1{}^2 + S_2{}^2 + ... + S_{10}{}^2 = \frac{5}{12}A$. Then A is equal to [Jan. 12, 2019 (I)]

Options:

A. 283

B. 301

C. 303

D. 156

Answer: C

Solution:

Solution:

$\because 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2}$

$\therefore S_k = \frac{k(k+1)}{2k} = \frac{k+1}{2}$

$\Rightarrow \frac{5}{12}A = \frac{1}{4}[2^2 + 3^3 + \ldots + 11^2]$

$= \frac{1}{4}[1^2 + 2^2 + \ldots + 11^2 - 1]$

$= \frac{1}{4}\left[\frac{11(11+1)(2 \times 11 + 1)}{6} - 1\right]$

$\frac{1}{4}\left[\frac{11 \times 12 \times 23}{6} - 1\right]$

$= \frac{1}{4}[505]$

$A = \frac{505}{4} \times \frac{12}{5} = 303$

Question206

If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots\ldots$ is equal to 225k then k is equal to : [Jan. 12, 2019 (II)]

Options:

A. 108

B. 27

C. 54

D. 9

Answer: B

Solution:

Solution:

$S = \left(\frac{3}{4}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{9}{4}\right)^3 + (3)^3 + \ldots$

$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + \ldots$

Let the general term of S be

$T_r = \left(\frac{3r}{4}\right)^3$, then

$255K = \sum_{r=1}^{15} T_r = \left(\frac{3}{4}\right)^3 \sum_{r=1}^{15} r^3$

$255K = \frac{27}{64} \times \left(\frac{15 \times 16}{2}\right)^2$

$\Rightarrow K = 27$

Question207

I f $^nC_4$, $^nC_5$ and $^nC_6$ are in A.P., then n can be [Jan. 12, 2019 (II)]

Options:

A. 9

B. 14

C. 11

D. 12

Answer: B

Solution:

Solution:
Since ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.

$$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$$

$$2 = \frac{5}{n-4} + \frac{n-5}{5}$$

$$\Rightarrow\ 12(n-4) = 30 + n^2 - 9n + 20$$

$$\Rightarrow\ n^2 - 21n + 98 = 0$$

$$(n-7)(n-14) = 0$$

$$(n-7)(n-14) = 0$$

$$n = 7,\ n = 14$$

Question208

If 1 9th term of a non-zero A.P. is zero, then its (49th term): (29th term) is : [Jan. 11, 2019 (II)]

Options:

A. 4: 1

B. 1: 3

C. 3: 1

D. 2: 1

Answer: C

Solution:

Solution:
Let first term and common difference of AP be a and d respectively, then

$$t_n = a + (n-1)d$$

$$\therefore\ t_{19} = a + 18d = 0\ \therefore\ a = -18d$$

$$\therefore\ \frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d}$$

$$= \frac{-18d + 48d}{-18d + 28d} = \frac{30d}{10d} = 3$$

$$t_{49} : t_{29} = 3 : 1$$

Question209

The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is:
[Jan. 10, 2019 (I)]

Options:

A. 1256

B. 1465

C. 1365

D. 1356

Answer: D

Solution:

Solution:
Two digit positive numbers which when divided by 7 yield 2 as remainder are 12 terms i.e, $16, 23, 30, \ldots, 93$
Two digit positive numbers which when divided by 7 yield
5 as remainder are 13 terms i.e ,$12, 19, 26, \ldots, 96$
By using AP sum of $16, 23, \ldots, 93$, we get
$S_1 = 16 + 23 + 30 + \ldots + 93 = 654$
By using AP sum of $12, 19, 26, \ldots, 96$, we get
$S_1 = 12 + 19 + 26 + \ldots + 96 = 702$
$\therefore$ required $\text{Sum} = S_1 + S_2 = 654 + 702 = 1356$

Question210

Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$ If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to:
[Jan. 09, 2019 (I)]

Options:

A. 52

B. 57

C. 47

D. 42

Answer: A

Solution:

Solution:
$S = \sum_{i=1}^{30} a_i = \frac{30}{2}[2a_1 + 29d]$
$T = \sum_{i=1}^{15} a_{(2i-1)} = \frac{15}{2}[2a_1 + 28d]$

Since, $S-2T=75$
$\Rightarrow 30a_1+435d-30a_1-420d=75$
$\Rightarrow d=5$
Also, $a_5=27 \Rightarrow a_1+4d=27 \Rightarrow a_1=7$
Hence, $a_{10}=a_1+9d=7+9\times 5=52$

Question211

If three distinct numbers a, b, c are in G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root, then which one of the following statements is correct?
[April 08, 2019 (II)]

Options:

A. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

B. d, e, f are in A.P.

C. d, e, f are in G.P.

D. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

Answer: A

Solution:

Solution:
Since a, b, c are in G.P.
$\therefore b^2=acc$
Given equation is, $ax^2+2bx+c=0$
$\Rightarrow ax^2+2\sqrt{ac}x+c=0 \Rightarrow (\sqrt{a}x+\sqrt{c})^2=0$
$\Rightarrow x=-\sqrt{\frac{c}{a}}$
Also, given that $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root.
$\Rightarrow x=-\sqrt{\frac{c}{a}}$ must satisfy $dx^2+2ex+f=0$
$\Rightarrow d\cdot\frac{c}{a}+2e\left(-\sqrt{\frac{c}{a}}\right)+f=0$
$\frac{d}{a}-\frac{2e}{\sqrt{ac}}+\frac{f}{c}=0 \Rightarrow \frac{d}{a}-\frac{2e}{b}+\frac{f}{c}=0$
$\Rightarrow \frac{2e}{b}=\frac{d}{a}+\frac{f}{c} \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Question212

For $x \in R$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series $\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\cdot s+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is

[April 12, 2019 (I)]

Options:

A. -153

B. -133

C. -131

D. -135

Answer: B

Solution:

Solution:

$\because [x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]\ldots\left[x+\frac{n-1}{n}\right]=[nx]$

and $[x]+[-x]=-1(x\notin z)$

$\therefore\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-100\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]$

$=-100-\left\{\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{100}\right]+\ldots\left[\frac{1}{3}+\frac{99}{100}\right]\right\}$

$=-100-\left[\frac{100}{3}\right]=-133$

Question213

The sum $\frac{3\times 1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}\Bigg)+\ldots\ldots$ upto 10^{th} term, is :

[April 10, 2019 (I)]

Options:

A. 680

B. 600

C. 660

D. 620

Answer: C

Solution:

Solution:

r^{th} term of the series,

$T_r=\frac{(2r+1)(1^3+2^3+3^3+\ldots+r^3)}{1^2+2^2+3^2+\ldots+r^2}$

$T_r=(2r+1)\left(\frac{r(r+1)}{2}\right)^2\times\frac{6}{r(r+1)(2r+1)}=\frac{3r(r+1)}{2}$

$\therefore$ sum of 10 terms is $= S = \sum_{r=1}^{10} T_r = \frac{3}{2}\sum_{r=1}^{10}(r^2 + r)$

$= \frac{3}{2}\left\{\frac{10 \times (10+1)(2 \times 10+1)}{6} + \frac{10 \times 11}{2}\right\}$

$= \frac{3}{2} \times 5 \times 11 \times 8 = 660$

Question214

The sum $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + ... + \frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15} - \frac{1}{2}(1+2+3+...+15$ is equal to : [April 10, 2019 (II)]

Options:

A. 620

B. 1240

C. 1860

D. 660

Answer: A

Solution:

Solution:

Let, $S = 1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots 15$ terms

$T_n = \frac{1^3+2^3+\ldots n^3}{1+2+\ldots n} = \frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)}{2}} = \frac{n(n+1)}{2}$

Now, $S = \frac{1}{2}\left(\sum_{n=1}^{15} n^2 + \sum_{n=1}^{15} n\right) = \frac{1}{2}\left(\frac{15(16)(31)}{6} + \frac{15(16)}{2}\right)$

$= 680$

$\therefore$ required sum is, $680 - \frac{1}{2}\frac{15(16)}{2} = 680 - 60 = 620$

Question215

The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + ...$ upto 11^{th} term is: [April 09, 2019 (II)]

Options:

A. 915

B. 946

C. 945

D. 916

Answer: B

Solution:

Solution:
$1 + 2.3 + 3.5 + 4.7 + \ldots\ldots.$ Let, $S = (2.3 + 3.5 + 4.7 + \ldots\ldots)$
Now, $S_{10} = \sum_{n=1}^{10} (n+1)(2n+1) = \sum_{n=1}^{10} (2n^2 + 3n + 1)$
$= \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$
Put $n = 10$
$= \frac{2.10.11.21}{6} + \frac{3.10.11}{2} + 10 = 945$
Hence required sum of the series $= 1 + 945 = 946$

Question216

Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is:
[April 09, 2019 (II)]

Options:

A. 157

B. 262

C. 225

D. 190

Answer: D

Solution:

Solution:
Number of balls used in equilateral triangle
$= \frac{n(n+1)}{2}$
$\because$ side of equilateral triangle has n -balls
$\therefore$ no. of balls in each side of square is $= (n-2)$
According to the question, $\frac{n(n+1)}{2} + 99 = (n-2)^2$
$\Rightarrow n^2 + n + 198 = 2n^2 - 8n + 8$
$\Rightarrow n^2 - 9n - 190 = 0 \Rightarrow (n-19)(n+10) = 0$
$\Rightarrow n = 19$
Number of balls used to form triangle
$= \frac{n(n+1)}{2} = \frac{19 \times 20}{2} = 190$

Question217

The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to :
[April 08, 2019 (II)]

Options:

A. $2 - \frac{3}{2^{17}}$

B. $1 - \frac{11}{2^{20}}$

C. $2 - \frac{11}{2^{19}}$

D. $2 - \frac{21}{2^{20}}$

Answer: C

Solution:

Solution:
Let, $S = \sum_{k=1}^{20} k \cdot \frac{1}{2^k}$
$S = \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \cdot \frac{1}{2^3} + \ldots. + 20 \cdot \frac{1}{2^{20}} \ldots$ (i)
$\frac{1}{2}S = \frac{1}{2^2} + 2 \cdot \frac{1}{2^3} + \ldots. + 19 \frac{1}{2^{20}} + 20 \frac{1}{2^{21}} \ldots$ (ii)
On subtracting equations (ii) by (i),
$\frac{S}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots. + \frac{1}{2^{20}} \right) - 20 \frac{1}{2^{21}}$
$= \frac{\frac{1}{2}\left(1 - \frac{1}{2^{20}}\right)}{1 - \frac{1}{2}} - 20 \cdot \frac{1}{2^{21}} = 1 - \frac{1}{2^{20}} - 10 \cdot \frac{1}{2^{20}}$
$\frac{S}{2} = 1 - 11 \cdot \frac{1}{2^{20}} \Rightarrow S = 2 - 11 \cdot \frac{1}{2^{19}} = 2 - \frac{11}{2^{19}}$

Question218

Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :
[April 12, 2019 (I)]

Options:

A. -260

B. -410

C. -320

D. -380

Answer: C

Solution:

Solution:
Given, $S_4 = 16$ and $S_6 = -48$
$\Rightarrow \ 2(2a + 3d) = 16 \Rightarrow 2a + 3d = 8 \ldots$ (i)
And $3[2a + 5d] = -48 \Rightarrow 2a + 5d = -16$
$\Rightarrow 2d = -24$ [using equation(i)]
$\Rightarrow d = -12$ and $a = 22$
$\therefore \ S_{10} \frac{10}{2} = (44 + 9(-12)) = -320$

Question219

If $a_1, a_2, a_3, \ldots\ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is : [April 12, 2019 (II)]

Options:

A. 200

B. 280

C. 120

D. 150

Answer: A

Solution:

Solution:
Let the common difference of the A.P. is 'd'.
Given, $a_1 + a_7 + a_{16} = 40$
$\Rightarrow \ a_1 + a_1 + 6d + a_1 + 15d = 40$
$\Rightarrow \ 3a_1 + 21d = 40$
$\Rightarrow \ a_1 + 7d = \frac{40}{3}$
Now, sum of first 15 terms of this A.P. is,
$S_{15} = \frac{15}{2}[2a_1 + 14d] = 15(a_1 + 7d)$
$= 15\left(\frac{40}{3}\right) = 200$
[Using(i)]

Question220

If $a_1, a_2, a_3, \ldots . a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots . + a_{16} = 114$ then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :
[April 10, 2019 (I)]

Options:

A. 98

B. 76

C. 38

D. 64

Answer: B

Solution:

Solution:
$a_1 + a_4 + a_7 + \ldots + a_{16} = 114$
$\Rightarrow 3(a_1 + a_{16}) = 114$
$\Rightarrow a_1 + a_{16} = 38$
Now, $a_1 + a_6 + a_{11} + a_{16} = 2(a_1 + a_{16}) = 2 \times 38 = 76$

Question221

Let the sum of the first n terms of a non-constant A.P. $a_1, a_2, a_3, \ldots\ldots\ldots\ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to:
[April 09, 2019 (I)]

Options:

A. $(50, 50 + 46A)$

B. $(50, 50 + 45A)$

C. $(A, 50 + 45A)$

D. $(A, 50 + 46A)$

Answer: D

Solution:

Solution:
$\because S_n = \left(50 - \frac{7A}{2}\right)n + n^2 \times \frac{A}{2} \Rightarrow a_1 = 50 - 3S$
$\therefore\ d = a_2 - a_1 = S_{n_2} - S_{n_1} - S_{n_1} \Rightarrow d = \frac{A}{2} \times 2 = A$
Now, $a_{50} = a_1 + 49 \times d$

$= (50-3A)+49A = 50+46A$
So, $(d, a_{50}) = (A, 50+46A)$

Question222

Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function f satisfies $f(x+y) = f(x)f(y)$ for all natural numbers x, y and $f(a) = 2$ Then the natural number 'a ' is: [April 09, 2019 (I)]

Options:

A. 2

B. 16

C. 4

D. 3

Answer: D

Solution:

Solution:
$\because f(x+y) = f(x)\cdot f(y)$
$\Rightarrow \text{Let} f(x) = t^x$
$\because f(1) = 2$
$\therefore t' = 2$
$\Rightarrow f(x) = 2^x$
Since, $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$
Then, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10}-1)$
$\Rightarrow 2^a \sum_{k=1}^{10} 2^k = 16(2^{10}-1)$
$\Rightarrow 2^a \times \frac{((2^{10})-1)\times 2}{(2-1)} = 16\times(2^{10}-1)2.2^a = 16$
$\Rightarrow a = 3$

Question223

If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11 th term is: [April 09, 2019 (II)]

Options:

A. -35

B. 25

C. -36

D. -25

Answer: D

Solution:

Solution:
Let three terms of A.P. are $a-d, a, a+d$
Sum of terms is, $a-d+a+a+d=33 \Rightarrow a=11$
Product of terms is, $(a-d)a(a+d)=11(121-d^2)=1155$
$\Rightarrow 121-d^2=105 \Rightarrow d=\pm 4$ if $d=4$
$T_{11}=T_1+10d=7+10(4)=47$
if $d=-4$
$T_{11}=T_1+10d=15+10(-4)=-25$

Question224

The sum of all natural numbers ' n ' such that 100 < n < 200 and H.C.F. (91, n) > 1 is :
[April 08, 2019 (I)]

Options:

A. 3203

B. 3303

C. 3221

D. 3121

Answer: D

Solution:

Solution:
$\because 91=13\times 7$
Then, the required numbers are either divisible by 7 or 13 .
$\therefore$ Sum of such numbers = Sum of no. divisible by $7+$
sum of the no. divisible by $13-$ Sum of the numbers divisible by 91
$=(105+112+\ldots+196)+(104+117+\ldots+195)-182$
$=2107+1196-182=3121$

Question225

If α, β and γ are three consecutive terms of a nonconstant G.P. such that the equations $\alpha x^2+2\beta x+\gamma=0$ and $x^2+x-1=0$ have a common root, then $\alpha(\beta+\gamma)$ is equal to :
[April 12, 2019 (II)]

Options:

A. 0

B. $\alpha\beta$

C. $\alpha\gamma$

D. $\beta\gamma$

Answer: D

Solution:

Solution:
$\because \alpha, \beta, \gamma$ are three consecutive terms of a non- constant G.P.
$\therefore\ \beta^2 = \alpha\gamma$
So roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$ are
$\dfrac{-2\beta \pm 2\sqrt{\beta^2 - \alpha\gamma}}{2\alpha} = \dfrac{\beta}{\alpha}$
$\because\ \alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root.
$\therefore$ this root satisfy the equation $x^2 + x - 1 = 0$
$\beta^2 - \alpha\beta - \alpha^2 = 0$
$\Rightarrow\ \alpha\gamma - \alpha\beta - \alpha^2 = 0 \Rightarrow \alpha + \beta = \gamma$
Now, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$
$= \alpha\beta + \beta^2 = (\alpha + \beta)\beta = \beta\gamma$

Question226

Let a, b and c be in G.P. with common ratio r, where a ≠ 0 and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4 th term of this A.P. is: [April 10, 2019 (II)]

Options:

A. $\frac{2}{3}a$

B. 5a

C. $\frac{7}{3}a$

D. a

Answer: D

Solution:

Solution:
$\because a, b, c$ are in G.P. $\Rightarrow b = ar, c = ar^2$
$\because 3a, 7b, 15c$ are in A.P.
$\Rightarrow 3a$, 7 ar, 15 ar ^{2} are in A.P.
$\therefore 14ar = 3a + 15ar^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow r = \frac{1}{3}$ or $\frac{3}{5}$

$\because r < \frac{1}{2} \quad \therefore r = \frac{3}{5}$ rejected

Fourth term $= 15ar^2 + 7ar - 3a$

$= a(15r^2 + 7r - 3) = a\left(\frac{15}{9} + \frac{7}{3} - 3\right) = a$

Question227

Let $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots, (x_i \neq 0$ for $i = 1, 2, \ldots, n)$ be in A.P.
such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$,

then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to.

[Online April 16, 2018]

Options:

A. 3

B. $\frac{13}{8}$

C. $\frac{13}{4}$

D. $\frac{1}{8}$

Answer: C

Solution:

Solution:

$\because \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots, \frac{1}{x_n}$ are in A.P

$x_1 = 4$ and $x_{21} = 20$

Let $'d'$ be the common difference of this A.P

its 21^{st} term $= \frac{1}{x_{21}} = \frac{1}{x_1} + [(21-1) \times d]$

$\Rightarrow d = \frac{1}{20} \times \left(\frac{1}{20} - \frac{1}{4}\right) \Rightarrow d = -\frac{1}{100}$

Also $x_n > 50$ (given).

$\therefore \frac{1}{x_n} = \frac{1}{x_1} + [(n-1) \times d]$

$\Rightarrow x_n = \frac{x_1}{1 + (n-1) \times d \times x_1}$

$\therefore \frac{x_1}{1 + (n-1) \times d \times x_1} > 50$

$\Rightarrow \frac{4}{1 + (n-1) \times \left(-\frac{1}{100}\right) \times 4} > 50$

$\Rightarrow 1 + (n-1) \times \left(-\frac{1}{100}\right) \times 4 < \frac{4}{50}$

$\Rightarrow -\frac{1}{100}(n-1) < -\frac{23}{100}$
$\Rightarrow n-1 > 23 \Rightarrow n > 24$
Therefore, $n = 25$.
$\Rightarrow \sum_{i=1}^{25} \frac{1}{x_i} = \frac{25}{2}\left[\left(2\times\frac{1}{4}\right) + (25-1)\times\left(-\frac{1}{100}\right)\right] = \frac{13}{4}$

Question228

If $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h^2}, \ldots \cdot \frac{1}{h_n}$ are two A.P's such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 \cdot h_{10}$ equals.

[Online April 15, 2018]

Options:

A. 2560

B. 2650

C. 3200

D. 1600

Answer: A

Solution:

Solution:
Suppose d_1 is the common difference of the A.P. $x_1, x_2, \ldots . x_n$ then
$\because x_8 - x_3 = 5d_1 = 12 \Rightarrow d_1 = \frac{12}{5} = 2.4$
$\Rightarrow x_5 = x_3 + 2d_1 = 8 + 2\times\frac{12}{5} = 12.8$
Suppose d_2 is the common difference of the A.P
$\frac{1}{h_1}, \frac{1}{h_2}, \ldots \cdot \frac{1}{h_n}$ then
$5d_2 = \frac{1}{20} - \frac{1}{8} = \frac{-3}{40} \Rightarrow d_2 = \frac{-3}{200}$
$\because \frac{1}{h_{10}} = \frac{1}{h_7} + 3d_2 = \frac{1}{200} \Rightarrow h_{10} = 200$
$\Rightarrow x_5 \cdot h_{10} = 12.8 \times 200 = 2560$

Question229

If b is the first term of an infinite G. P whose sum is five, then b lies in the interval.

[Online April 15, 2018]

Options:

A. $(-\infty, -10)$

B. $(10, \infty)$

C. (0,10)

D. (-10,0)

Answer: C

Solution:

Solution:
First term $= \mathrm{b}$ and common ratio $= \mathrm{r}$
For infinite series, $\mathrm{Sum} = \dfrac{\mathrm{b}}{1-\mathrm{r}} = 5$
$\Rightarrow \mathrm{b} = 5(1-\mathrm{r})$
So, interval of $\mathrm{b} = (0,\ 10)$ as, $-1 < \mathrm{r} < 1$ for infinite G.P.

Question230

Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \geq p$ is
[Online April 15, 2018]

Options:

A. 5

B. 7

C. 11

D. 9

Answer: B

Solution:

Solution:
$A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$
Which is a G.P. with $a = \frac{3}{4}, r = \frac{-3}{4}$ and number of terms $= n$

$$\therefore A_n = \frac{\frac{3}{4} \times \left(1 - \left(\frac{-3}{4}\right)^n\right)}{1 - \left(\frac{-3}{4}\right)} = \frac{\frac{3}{4} \times \left(1 - \left(\frac{-3}{4}\right)^n\right)}{\frac{7}{4}}$$

$\Rightarrow A_n = \frac{3}{7}\left[1 - \left(\frac{-3}{4}\right)^n\right] \ldots$ (i)
As, $B_n = 1 - A_n$
For least odd natural number p, such that $B_n > A_n$
$\Rightarrow 1 - A_n > A_n \quad \Rightarrow 1 > 2 \times A_n \Rightarrow A_n < \frac{1}{2}$
From eqn. (i), we get
$\frac{3}{7} \times \left[1 - \left(\frac{-3}{4}\right)^n\right] < \frac{1}{2} \Rightarrow 1 - \left(\frac{-3}{4}\right)^n < \frac{7}{6}$

$\Rightarrow 1-\frac{7}{6}<\left(\frac{-3}{4}\right)^n \Rightarrow \frac{-1}{6}<\left(\frac{-3}{4}\right)^n$

As n is odd, then $\left(\frac{-3}{4}\right)^n=-\frac{3^n}{4}$

So $\frac{-1}{6}<-\left(\frac{3}{4}\right)^n \Rightarrow \frac{1}{6}>\left(\frac{3}{4}\right)^n$

$\log\left(\frac{1}{6}\right)=n\log\left(\frac{3}{4}\right) \Rightarrow 6.228<n$

Hence, n should be 7 .

Question231

If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a<b<c$ and $a+b+c=\frac{3}{4}$, then the value of a is [Online April 15, 2018]

Options:

A. $\frac{1}{4}-\frac{1}{3\sqrt{2}}$

B. $\frac{1}{4}-\frac{1}{4\sqrt{2}}$

C. $\frac{1}{4}-\frac{1}{\sqrt{2}}$

D. $\frac{1}{4}-\frac{1}{2\sqrt{2}}$

Answer: D

Solution:

Solution:
$\because a, b, c$ are in A.P. then $a+c=2b$
also it is given that,
$a+b+c=\frac{3}{4}\ldots$ (i)

$\Rightarrow 2b+b=\frac{3}{4} \Rightarrow b=\frac{1}{4}\ldots$ (ii)

Again it is given that, a^2, b^2, c^2 are in G.P. then
$(b^2)^2=a^2c^2 \Rightarrow ac=\pm\frac{1}{16}\ldots$ (iii)
From (i), (ii) and (iii), we get;
$a\pm\frac{1}{16a}=\frac{1}{2} \quad \Rightarrow 16a^2-8a\pm 1=0$
Case I: $16a^2-8a+1=0$

$\Rightarrow a=\frac{1}{4}\left(\right.$ not possible as $\left.a<b\right)$

Case II: $16a^2-8a-1=0 \Rightarrow a=\frac{8\pm\sqrt{128}}{32}$

$\Rightarrow a=\frac{1}{4}\pm\frac{1}{2\sqrt{2}}$

$\therefore a=\frac{1}{4}-\frac{1}{2\sqrt{2}} \quad (\because a<b)$

Question232

The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$ is? [Online April 16, 2018]

Options:

A. $38 + \frac{1}{2^{20}}$

B. $39 + \frac{1}{2^{19}}$

C. $39 + \frac{1}{2^{20}}$

D. $38 + \frac{1}{2^{19}}$

Answer: D

Solution:

Solution:

The general term of the given series $= \frac{2 \times 2^r - 1}{2^r}$, where $r \geq 0$

$\therefore$ req. $\text{sum} = 1 + \sum_{r=1}^{19} \frac{2 \times 2^r - 1}{2^r}$

Now, $\sum_{r=1}^{19} \left(\frac{2 \times 2^r - 1}{2^r} \right) = \sum_{r=1}^{19} \left(2 - \frac{1}{2^r} \right)$

$= 2(19) - \frac{\frac{1}{2}\left(1 - \left(\frac{1}{2}\right)^{19}\right)}{1 - \frac{1}{2}} = 38 + \frac{\left(\frac{1}{2}\right)^{19} - 1}{1}$

$= 38 + \left(\frac{1}{2}\right)^{19} - 1 = 37 + \left(\frac{1}{2}\right)^{19}$

$\therefore$ req. $\text{sum} = 1 + 37 + \left(\frac{1}{2}\right)^{19} = 38 + \left(\frac{1}{2}\right)^{19}$

Question233

Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \ldots$ If $B - 2A = 100\lambda$, then λ is equal to [2018]

Options:

A. 248

B. 464

C. 496

D. 232

Answer: A

Solution:

Solution:
Here, $B-2A$
$= \sum_{n=1}^{40} a_n - 2\sum_{n=1}^{20} a_n = \sum_{n=21}^{40} a_n - 2\sum_{n=1}^{20} a_n$
$B-2A = (21^2 + 2.22^2 + 23^2 + 2.24^2 + \ldots.. + 40^2)$
$-(1^2 + 2.2^2 + 3^2 + 2.4^2 \ldots.. + 20^2)$
$= 20[22 + 2.24 + 26 + 2.28 + \ldots.. + 60]$
$= 20[\underbrace{}(22 + 24 + 26\ldots 60)_{20 \text{ terms}} + \underbrace{}(24 + 28 + \ldots. + 60)_{10 \text{ terms}}]$
$20\left[\frac{20}{2}(22+60) + \frac{10}{2}(24+60)\right]$
$= 10[20.82 + 10.84]$
$= 100[164 + 84] = 100.248$

Question234

Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^{\ 2} + a_2^{\ 2} + \ldots + a_{17}^{\ 2} = 140m$, then m is equal to [2018]

Options:

A. 68

B. 34

C. 33

D. 66

Answer: B

Solution:

Solution:
$\because \sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$
$\Rightarrow a_1 + 24d = 32$
$N\text{ow}, a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$
From eq. (i) & (ii) we get; $d = 1$ and $a_1 = 8$
Also, $\sum_{r=1}^{17} a_r^{\ 2} = \sum_{r=1}^{17} [8 + (r-1)1]^2 = 140m$
$\Rightarrow \sum_{r=1}^{17} (r+7)^2 = 140m$
$\Rightarrow \sum_{r=1}^{17} (r^2 + 14r + 49) = 140m$
$\Rightarrow \left(\frac{17 \times 18 \times 35}{6}\right) + 14\left(\frac{17 \times 18}{2}\right) + (49 \times 17) = 140$
$\Rightarrow m = 34$

Question235

For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then [2017]

Options:

A. a, b and c are in G.P.

B. b, c and a are in G.P.

C. b, c and a are in A.P.

D. a, b and c are in A.P.

Answer: C

Solution:

Solution:
\We have
$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$
$\Rightarrow 225a^2 + 9b^2 + 25c^2 - 75ac = 45ab + 15bc$
$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 75ac - 45ab - 15bc = 0$
$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$
It is possible when $15a - 3b = 0, 3b - 5c = 0$ and $5c - 15a = 0$
$\Rightarrow 15a = 3b \Rightarrow b = 5a$
$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$
$\Rightarrow a + b = \frac{c}{3} + \frac{5c}{3} = \frac{6c}{3}$
$\Rightarrow a + b = 2c$
$\Rightarrow b, c, a$ are in $A.P.$

Question236

If three positive numbers a, b and c are in A.P. such that abc = 8, then the minimum possible value of b is : [Online April 9, 2017]

Options:

A. 2

B. $4^{\frac{1}{3}}$

C. $4^{\frac{2}{3}}$

D. 4

Answer: A

Solution:

Solution:
By Arithmetic Mean:
$a + c = 2b$
Consider $a = b = c = 2$
$\Rightarrow abc = 8$
$\Rightarrow a + b = 2b$
$\therefore$ minimum possible value of $b = 2$

Question237

If the arithmetic mean of two numbers a and b, $a > b > 0$, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to: [Online April 8, 2017]

Options:

A. $\frac{\sqrt{6}}{2}$

B. $\frac{3\sqrt{2}}{4}$

C. $\frac{7\sqrt{3}}{12}$

D. $\frac{5\sqrt{6}}{12}$

Answer: D

Solution:

Solution:
A.T.Q.
A.M. = 5 G.M.
$\frac{a+b}{2} = 5\sqrt{ab}$
$\frac{a+b}{\sqrt{ab}} = 10$
$\therefore \quad \frac{a}{b} = \frac{10+\sqrt{96}}{10-\sqrt{96}} = \frac{10+4\sqrt{6}}{10-4\sqrt{6}}$
Use Componendo and Dividendo
$\frac{a+b}{a-b} = \frac{20}{8\sqrt{6}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

Question238

Let a, b, c $\in$ R . I f f (x) = $ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x+y) = f(x) + f(y) + xy$, $\forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to :
[2017]

Options:

A. 255

B. 330

C. 165

D. 190

Answer: B

Solution:

Solution:
$f(x) = ax^2 + bx + c$
$f(1) = a + b + c = 3 \Rightarrow f(1) = 3$
Now $f(x+y) = f(x) + f(y) + xy$ ...
Put $x = y = 1$ in eqn (i)
$f(2) = f(1) + f(1) + 1 = 2f(1) + 1$
$f(2) = 7$
$\Rightarrow f(3) = 12$
$S_n = 3 + 7 + 12 + ... + t_n$
$S_n = \ 3 + 7 + 12 + ... + t_{n-1} + t_n$
$0 = 3 + 4 + 5.........$ to n term $-t_n$
$t_n = 3 + 4 + 5 + ...$ upto n terms
$t_n = \frac{(n^2 + 5n)}{2}$
$S_n = \sum t_n = \sum \frac{(n^2 + 5n)}{2}$
$S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right]$
$= \frac{n(n+1)(n+8)}{6}$
$S_{10} = \frac{10 \times 11 \times 18}{6} = 330$

Question239

Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + ... + \frac{1+2+......+n}{1^3+2^3+......+n^3}$, If $100S_n = n$, then n is equal to :
[Online April 9, 2017]

Options:

A. 199

B. 99

C. 200

D. 19

Answer: A

Solution:

Solution:

$$T_n = \frac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2}$$

$$\Rightarrow T_n = \frac{2}{n(n+1)}$$

$$\Rightarrow S_n = \sum T_n = 2\sum_{n=1}^{n}\left(\frac{1}{n} - \frac{1}{n+1}\right) = 2\left\{1 - \frac{1}{n+1}\right\}$$

$$\Rightarrow S_n = \frac{2n}{n+1}$$

$$\because 100S_n = n$$

$$\Rightarrow 100 \times \frac{2n}{n+1} = n$$

$$\Rightarrow n + 1 = 200$$

$$\Rightarrow n = 199$$

Question240

If the sum of the first n terms of the series $\sqrt{3} + \sqrt{75} + \sqrt{243} + \sqrt{507} + \ldots\ldots$ is $435\sqrt{3}$, then n equals:
[Online April 8, 2017]

Options:

A. 18

B. 15

C. 13

D. 29

Answer: B

Solution:

Solution:

$$\because \sqrt{3}[1 + \sqrt{25} + \sqrt{81} + \sqrt{69} + \ldots\ldots] = 435\sqrt{3}$$

$$\Rightarrow \sqrt{3}[1 + 5 + 9 + 13 + \ldots.. + T_n] = 435\sqrt{3}$$

$$\Rightarrow \sqrt{3} \times \frac{n}{2}[2 + (n-1)_4] = 435\sqrt{3}$$

$$\Rightarrow 2n + 4n^2 - 4n = 870$$

$$\Rightarrow 4n^2 - 2n - 870 = 0$$

$$\Rightarrow 2n^2 - n - 435 = 0$$

$$n = \frac{1 \pm \sqrt{1 + 4 \times 2 \times 435}}{4} = \frac{1 \pm 59}{4}$$

$$\therefore n = \frac{1 + 59}{4} = 15; \text{ or } n = \frac{1 - 59}{4} = 14.5$$

Question241

Let $a_1, a_2, a_3, \ldots, a_n$, be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to : [Online April 10, 2016]

Options:

A. 306

B. 204

C. 153

D. 612

Answer: A

Solution:

Solution:

$a_3 + a_7 + a_{11} + a_{15} = 72$

$(a_3 + a_{15}) + (a_7 + a_{11}) = 72$

$a_3 + a_{15} + a_7 + a_{11} = 2(a_1 + a_{17})$

$a_1 + a_{17} = 36$

$S_{17} = \frac{17}{2}[a_1 + a_{17}] = 17 \times 18 = 306$

Question242

If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : [2016]

Options:

A. 1

B. $\frac{7}{4}$

C. $\frac{8}{5}$

D. $\frac{4}{3}$

Answer: D

Solution:

Solution:
Let the GP be a, ar and ar ^{2} then $a = A + d$; $ar = A + 4d$;
$ar^2 = A + 8d$
$\Rightarrow \frac{ar^2 - ar}{ar - a} = \frac{(A + 8d) - (A + 4d)}{(A + 4d) - (A + d)}$
$r = \frac{4}{3}$

Question243

Let z = 1+ ai be a complex number, a > 0, such that z^3 is areal number. Then the sum $1 + z + z^2 + ... + z^{11}$ is equal to:
(Online April 10, 2016)

Options:

A. $1365\sqrt{3}i$

B. $-1365\sqrt{3}i$

C. $-1250\sqrt{3}i$

D. $1250\sqrt{3}\ i$

Answer: B

Solution:

Solution:
$z = 1 + ai$
$z^2 = 1 - a^2 + 2ai$
$z^2 \cdot z = \{(1 - a^2) + 2ai\}\ \ \{1 + ai\}$
$= (1 - a^2) + 2ai + (1 - a^2)\ \ ai - 2a^2$
$\because z^3$ is real $\Rightarrow 2a + (1 - a^2)a = 0$
$a(3 - a^2) = 0 \Rightarrow a = \sqrt{3}(a > 0)$
$1 + z + z^2 ....z^{11} = \frac{z^{12} - 1}{z - 1} = \frac{(1 + \sqrt{3}i)^{12} - 1}{1 + \sqrt{3}i - 1}$
$= \frac{(1 + \sqrt{3}i)^{12} - 1}{\sqrt{3}i}$
$(1 + \sqrt{3}i)^{12} = 2^{12}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{12}$
$= 2^{12}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{12} = 2^{12}(\cos 4\pi + i\sin 4\pi) = 2^{12}$
$\Rightarrow \frac{2^{12} - 1}{\sqrt{3}i} = \frac{4095}{\sqrt{3}i} = -\frac{4095}{3}\sqrt{3}i = -1365\sqrt{3}i$

Question244

If A > 0, B > 0 and $A + B = \frac{\pi}{6}$, then the minimum value of $\tan A + \tan B$ is :
[Online April 10, 2016]

Options:

A. $\sqrt{3}-\sqrt{2}$

B. $4-2\sqrt{3}$

C. $\frac{2}{\sqrt{3}}$

D. $2-\sqrt{3}$

Answer: B

Solution:

Solution:
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{1 - \tan A \tan B}$ where $y = \tan A + \tan B$
$\Rightarrow \tan A \tan B = 1 - \sqrt{3}y$
Also $AM \geq GM$
$\Rightarrow \frac{\tan A + \tan B}{2} \geq \sqrt{\tan A \tan B}$
$\Rightarrow \ y \geq 2\sqrt{1-\sqrt{3}y}$
$\Rightarrow \ y^2 \geq 4 - 4\sqrt{3}y$
$\Rightarrow y^2 + 4\sqrt{3}y - 4 \geq 0$
$\Rightarrow \ y \leq -2\sqrt{3} - 4$ or $y \geq -2\sqrt{3} + 4$
($y \leq -2\sqrt{3} - 4$ is not possible as $\tan A \tan B > 0$)

Question245

Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to : [Online April 9, 2016]

Options:

A. 342

B. 216

C. 258

D. 270

Answer: B

Solution:

Solution:
$x + y + z = 12$
$AM \geq GM$

$$\frac{3\left(\frac{x}{3}\right)+4\left(\frac{y}{4}\right)+5\left(\frac{z}{5}\right)}{12} \geq 12\sqrt{\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^4\left(\frac{z}{5}\right)^5}$$

$$\frac{x^3y^4z^5}{3^34^45^5} \leq 1$$

$x^3y^4z^5 \leq 3^3 \cdot 4^4 \cdot 5^5$

$x^3y^4z^5 \leq (0.1)(600)^3$

But, given $x^3y^4z^5 = (0.1)(600)^3$

∴ all the number are equal

$\therefore \ \frac{x}{3} = \frac{y}{4} = \frac{z}{5}(=k)$

$x = 3k; y = 4k; z = 5k$

$x + y + z = 12$

$3k + 4k + 5k = 12$

$k = 1 \therefore \ x = 3; y = 4; z = 5$

$\therefore x^3 + y^3 + z^3 = 216$

Question246

If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...$, is $\frac{16}{5}$m then m is equal to : [2016]

Options:

A. 100

B. 99

C. 102

D. 101

Answer: D

Solution:

Solution:

$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 ... + \left(\frac{44}{5}\right)^2$

$S = \frac{16}{25}(2^2 + 3^2 + 4^2 + ... + 11^2)$

$= \frac{16}{25}\left(\frac{11(11+1)(22+1)}{6} - 1\right) \quad = \frac{16}{25} \times 505 = \frac{16}{5} \times 101$

$\Rightarrow \frac{16}{5}m = \frac{16}{5} \times 101$

$\Rightarrow m = 101$

Question247

For $x \in R, x \neq -1$, if $(1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \dots + x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to :
[Online April 9, 2016]

Options:

A. $\frac{2017!}{17!2000!}$

B. $\frac{2016!}{17!1999!}$

C. $\frac{2016!}{16!}$

D. $\frac{2017!}{2000!}$

Answer: A

Solution:

Solution:

$S = (1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014}$
$+ \dots + x^{2015}(1+x) + x^{2016} \dots$ (i)

$\left(\frac{x}{1+x}\right)S = x(1+x)^{2015} + x^2(1+x)^{2014} + \dots +$
$x^{2016} + \frac{x^{2017}}{1+x} \dots$ (ii)

Subtracting (i) from (ii)

$\frac{S}{1+x} = (1+x)^{2016} - \frac{x^{2017}}{1+x}$

$\therefore S = (1+x)^{2017} - x^{2017}$

$a_{17} =$ coefficient of $x^{17} = {}^{2017}C_{17} = \frac{2017!}{17!2000!}$

Question248

If m is the A.M. of two distinct real numbers l and $n (l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.
[2015]

Options:

A. $4l\,mn^2$

B. $4l^2m^2n^2$

C. $4l^2mn$

D. $4l\,m^2n$

Answer: D

Solution:

Solution:
$m = \frac{1+n}{2}$ and common ratio of G.P.
$= r = \left(\frac{n}{1}\right)^{\frac{1}{4}}$
$\therefore \; G_1 = 1^{3/4} n^{1/4}, G_2 = 1^{1/2} n^{1/2}, G_3 = 1^{1/4} n^{3/4}$
$G_1{}^4 + 2G_2{}^4 + G_3{}^4 = 1^3 n + 21^2 n^2 + 1 n^3$
$= \ln(1+n)^2 = \ln \times (2m)^2 = 4l\, m^2 n$

Question249

The sum of the 3 rd and the 4 th terms of a G.P. is 60 and the product of its first three terms is 1000 . If the first term of this G.P. is positive, then its 7 th term is : [Online April 11, 2015]

Options:

A. 7290

B. 640

C. 2430

D. 320

Answer: D

Solution:

Solution:
Let a, ar and ar ^{2} be the first three terms of G.P According to the question
$a(ar)(ar^2) = 1000 \Rightarrow (ar)^3 = 1000 \Rightarrow ar = 10$
and $ar^2 + ar^3 = 60 \Rightarrow ar(r + r^2) = 60$
$\Rightarrow r^2 + r - 6 = 0$
$\Rightarrow r = 2, -3$
$a = 5, a = -\frac{10}{3}$ (reject)
Hence, $T_7 = ar^6 = 5(2)^6 = 5 \times 64 = 320$

Question250

The sum of first 9 terms of the series.

$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$

[2015]

Options:

A. 142

B. 192

C. 71

D. 96

Answer: D

Solution:

Solution:

n^{th} term of series $= \dfrac{\left[\dfrac{n(n+1)}{2}\right]^2}{n^2} = \dfrac{1}{4}(n+1)^2$

Sum of n term $= \Sigma \dfrac{1}{4}(n+1)^2 = \dfrac{1}{4}[\Sigma n^2 + 2\Sigma n + n]$

$= \dfrac{1}{4}\left[\dfrac{n(n+1)(2n+1)}{6} + \dfrac{2n(n+1)}{2} + n\right]$

Sum of 9 terms

$= \dfrac{1}{4}\left[\dfrac{9 \times 10 \times 19}{6} + \dfrac{18 \times 10}{2} + 9\right] = \dfrac{384}{4} = 96$

Question251

If $\sum\limits_{n=1}^{5} \dfrac{1}{n(n+1)(n+2)(n+3)} = \dfrac{k}{3}$, then k is equal to [Online April 11, 2015]

Options:

A. $\frac{1}{6}$

B. $\frac{17}{105}$

C. $\frac{55}{336}$

D. $\frac{19}{112}$

Answer: C

Solution:

Solution:
General term of given expression can be written as

$T_r = \dfrac{1}{3}\left[\dfrac{1}{n(n+1)(n+2)} - \dfrac{1}{(n+1)(n+2)(n+3)}\right]$

on taking summation both the side, we get

$\sum\limits_{r=1}^{5} T_r = \dfrac{1}{3}\left[\dfrac{1}{6} - \dfrac{1}{6.7.8}\right] = \dfrac{k}{3}$

$\Rightarrow \frac{1}{3} \times \frac{1}{6}\left(1 - \frac{1}{56}\right) = \frac{k}{3} \Rightarrow \frac{1}{3} \times \frac{1}{6} \times \frac{55}{56} = \frac{k}{3}$

$\Rightarrow k = \frac{55}{336}$

Question252

The value of $\sum_{r=16}^{30} (r+2)(r-3)$ is equal to :
[Online April 10, 2015]

Options:

A. 7770

B. 7785

C. 7775

D. 7780

Answer: D

Solution:

Solution:
$\sum_{r=1}^{20} (r^2 - r - 6) = 7780$

Question253

Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$ If p, q, r are in A . P and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is:
[2014]

Options:

A. $\frac{\sqrt{34}}{9}$

B. $\frac{2\sqrt{13}}{9}$

C. $\frac{\sqrt{61}}{9}$

D. $\frac{2\sqrt{17}}{9}$

Answer: B

Solution:

Solution:
Let p, q, r are in AP
$\Rightarrow 2q = p + r \ldots$ (i)
Given $\frac{1}{\alpha} + \frac{1}{\beta} = 4$
$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = 4$
We have $\alpha + \beta = -q/p$ and $\alpha\beta = \frac{r}{p}$
$-xq$
$\Rightarrow \frac{p}{\frac{r}{p}} = 4 \Rightarrow q = -4r \ldots$ (ii)
From (i), we have
$2(-4r) = p + r$
$p = -9r \ldots$(iii)
Now, $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}} = \frac{\sqrt{q^2 - 4pr}}{|p|}$
From (ii) and (iii)
$= \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$

Question254

The sum of the first 20 terms common between the series 3 +7 + 11 + 15 + ... and 1 + 6 + 11 + 16 + ..., is
[Online April 11, 2014]

Options:

A. 4000

B. 4020

C. 4200

D. 4220

Answer: B

Solution:

Solution:
Given $n = 20; S_{20} = ?$
Series (1) $\rightarrow 3, 7, x11, 15, 19, 23, 27, x31, 35, 39, 43, 47\ 51, 55, 59\ldots$
Series (2) $\rightarrow 1, 6, x11, 16, 21, 26, x31, 36, 41, 46, x51, 56$ 61,66,71
The common terms between both the series are $11, 31, 51, 71\ldots$
Above series forms an Arithmetic progression (A.P).
Therefore, first term $(a) = 11$ and
common difference $(d) = 20$
Now, $S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{20} = \frac{20}{2}[2 \times 11 + (20-1)20]$
$S_{20} = 10[22 + 19 \times 20]$

$S_{20} = 10 \times 402 = 4020$
$\therefore S_{20} = 4020$

Question255

Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4 th term is:
[Online April 9, 2014]

Options:

A. 8

B. 16

C. 20

D. 24

Answer: C

Solution:

Solution:
Let a be the first term and d be the common difference of given A.P.
Second term, $a + d = 12$ . . . (i)
Sum of first nine terms,
$S_9 = \frac{9}{2}(2a + 8d) = 9(a + 4d)$
Given that S_9 is more than 200 and less than 220
$\Rightarrow 200 < S_9 < 220$
$\Rightarrow 200 < 9(a + 4d) < 220$
$\Rightarrow 200 < 9(a + d + 3d) < 220$
Putting value of $(a + d)$ from equation (i)
$200 < 9(12 + 3d) < 220$
$\Rightarrow 200 < 108 + 27d < 220$
$\Rightarrow 200 - 108 < 108 + 27d - 108 < 220 - 108$
$\Rightarrow 92 < 27d < 112$
Possible value of d is 4
$27 \times 4 = 108$
Thus, $92 < 108 < 112$
Putting value of d in equation (i)
$a + d = 12$
$a = 12 - 4 = 8$
4^{th} term $= a + 3d = 8 + 3 \times 4 = 20$

Question256

Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is:
[2014]

Options:

A. $2-\sqrt{3}$

B. $2+\sqrt{3}$

C. $\sqrt{2}+\sqrt{3}$

D. $3+\sqrt{2}$

Answer: B

Solution:

Solution:
Let a, ar, ar^2 are in G.P.
According to the question
a, $2ar, ar^2$ are in A.P.
$\Rightarrow 2 \times 2ar = a + ar^2$
$\Rightarrow 4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$
$r = \dfrac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$
Since $r > 1$
$\therefore\ r = 2-\sqrt{3}$ is rejected
Hence, $r = 2+\sqrt{3}$

Question257

The least positive integer n such that $1 - \frac{2}{3} - \frac{2}{3^2} - .... - \frac{2}{3^{n-1}} < \frac{1}{100}$, is: [Online April 12, 2014]

Options:

A. 4

B. 5

C. 6

D. 7

Answer: B

Solution:

Solution:
$1 - \dfrac{2}{3} - \dfrac{2}{3^2} \cdots \dfrac{2}{3^{n-1}} < \dfrac{1}{100}$

$\Rightarrow 1 - \dfrac{2}{3}\left[\dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \ldots \dfrac{1}{3^{n-1}}\right] < \dfrac{1}{100}$

$\Rightarrow \dfrac{1 - 2\left[\frac{1}{3}\left(\frac{1}{3^n} - 1\right)\right]}{\frac{1}{3} - 1} < \dfrac{1}{100}$

$\Rightarrow 1 - 2\left[\dfrac{3^n - 1}{2.3^n}\right] < \dfrac{1}{100}$

$\Rightarrow 1 - \left[\dfrac{3^n - 1}{3^n}\right] < \dfrac{1}{100}$

$\Rightarrow 1 - 1 + \dfrac{1}{3^n} < \dfrac{1}{100} \Rightarrow 100 < 3^n$

Thus, least value of n is 5

Question258

In a geometric progression, if the ratio of the sum of first 5 terms to the sum of their reciprocals is 49, and the sum of the first and the third term is 35 . Then the first term of this geometric progression is: [Online April 11, 2014]

Options:

A. 7

B. 21

C. 28

D. 42

Answer: C

Solution:

Solution:
According to Question

$\Rightarrow \dfrac{S_5}{S_5'} = 49$ \{here, $S_5 =$ Sum of first 5 terms and $S_5 =$ Sum of their reciprocals)

$\Rightarrow \dfrac{\dfrac{a(r^5 - 1)}{(r - 1)}}{\dfrac{a^{-1}(r^{-5} - 1)}{(r^{-1} - 1)}} = 49$

$\Rightarrow \dfrac{a(r^5 - 1) \times (r^{-1} - 1)}{a^{-1}(r^{-5} - 1) \times (r - 1)} = 49$

or $\dfrac{a^2(1 - r^5) \times (1 - r) \times r^5}{(1 - r^5) \times (1 - r) \times r} = 49$

$\Rightarrow a^2r^4 = 49 \Rightarrow a^2r^4 = 7^2$

$\Rightarrow ar^2 = 7$ . . . (i)

Also, given, $S_1 + S_3 = 35$

$a + ar^2 = 35$ . . . (ii)

Now substituting the value of eq. (i) in eq. (ii)

$a + 7 = 35$

$a = 28$

Question259

The coefficient of x^{50} in the binomial expansion of $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\ldots+x^{1000}$ is:
[Online April 11, 2014]

Options:

A. $\frac{(1000)!}{(50)!(950)!}$

B. $\frac{(1000)!}{(49)!(951)!}$

C. $\frac{(1001)!}{(51)!(950)!}$

D. $\frac{(1001)!}{(50)!(951)!}$

Answer: D

Solution:

Solution:
Let given expansion be
$S=(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\ldots+\ldots+x^{1000}$
Put $1+x=t$
$S=t^{1000}+xt^{999}+x^2(t)^{998}+\ldots+x^{1000}$
This is a G.P with common ratio $\frac{x}{t}$

$$S=\frac{t^{1000}\left[1-\left(\frac{x}{t}\right)^{1001}\right]}{1-\frac{x}{t}}$$

$$=\frac{(1+x)^{1000}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]}{1-\frac{x}{1+x}}$$

$$=\frac{(1+x)^{1001}[(1+x)^{1001}-x^{1001}]}{(1+x)^{1001}}$$

$=[(1+x)^{1001}-x^{1001}]$
Now coeff of x^{50} in above expansion is equal to coeff of x^{50} in $(1+x)^{1001}$ which is ${}^{1001}C_{50}$

$$=\frac{(1001)!}{50!(951)!}$$

Question260

Let G be the geometric mean of two positive numbers a and b, and M be the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M}$: G is 4 : 5, then a : b can be:
[Online April 12, 2014]

Options:

A. 1: 4

B. 1: 2

C. 2: 3

D. 3: 4

Answer: A

Solution:

Solution:

$G = \sqrt{ab}$

$M = \dfrac{\frac{1}{a} + \frac{1}{b}}{2}$

$M = \dfrac{a+b}{2ab}$

Given that $\dfrac{1}{M} : G = 4 : 5$

$\dfrac{2ab}{(a+b)\sqrt{ab}} = \dfrac{4}{5}$

$\Rightarrow \dfrac{a+b}{2\sqrt{ab}} = \dfrac{5}{4}$

$\Rightarrow \dfrac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \dfrac{5+4}{5-4}$

{Using Componendo & Dividendo}

$\Rightarrow \dfrac{(\sqrt{a})^2 + (\sqrt{b})^2 + 2\sqrt{ab}}{(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab}} = \dfrac{9}{1}$

$\Rightarrow \left(\dfrac{\sqrt{b} + \sqrt{a}}{\sqrt{b} - \sqrt{a}}\right)^2 = \dfrac{9}{1} \Rightarrow \dfrac{\sqrt{b} + \sqrt{a}}{\sqrt{b} - \sqrt{a}} = \dfrac{3}{1}$

$\Rightarrow \dfrac{\sqrt{b} + \sqrt{a} + \sqrt{b} - \sqrt{a}}{\sqrt{b} + \sqrt{a} - \sqrt{b} + \sqrt{a}} = \dfrac{3+1}{3-1}$

{Using Componendo & Dividendo}

$\sqrt{\dfrac{b}{a}} = \dfrac{4}{2} = 2$

$\dfrac{b}{a} = \dfrac{4}{1}$

$\dfrac{a}{b} = \dfrac{1}{4} \Rightarrow a : b = 1 : 4$

Question261

If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + \dots + 10(11)^9 = k(10)^9$, then k is equal to: [2014]

Options:

A. 100

B. 110

C. $\frac{121}{10}$

D. $\frac{441}{100}$

Answer: A

Solution:

Solution:
Given that $10^9 + 2 \cdot (11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$
Let $x = 10^9 + 2 \cdot (11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9$ . . . (i)
Multiplied by $\frac{11}{10}$ on both the sides
$\frac{11}{10}x = 11.10^8 + 2 . (11)^2 \cdot (10)^7 + \ldots + 9(11)^9 + 11^{10}$ . . . (ii)
Subtract (ii) from (i), we get
$x\left(1 - \frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2 \times (10)^7 + \ldots + 11^9 - 11^{10}$

$$\Rightarrow \ - \frac{x}{10} = 10^9 \left[\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1} \right] - 11^{10}$$

$\Rightarrow \ - \frac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$
$\Rightarrow x = 10^{11} = k \cdot 10^9$ Given
$\Rightarrow k = 100$

Question262

The number of terms in an A.P. is even; the sum of the odd terms in it is 24 and that the even terms is 30 . If the last term exceeds the first term by 10 $\frac{1}{2}$, then the number of terms in the A.P. is:
[Online April 19, 2014

Options:

A. 4

B. 8

C. 12

D. 16

Answer: B

Solution:

Solution:
Let a, d and $2n$ be the first term, common difference and total number of terms of an A.P. respectively i.e. a $+(a + d) + (a + 2d) + \ldots + (a + (2n - 1)d)$
No. of even terms $= n$, No. of odd terms $= n$
Sum of odd terms :
$S_o = \frac{n}{2}[2a + (n - 1)(2d)] = 24$
$\Rightarrow n[a + (n - 1)d] = 24$ . . . (i)
Sum of even terms:
$S_e = \frac{n}{2}[2(a + d) + (n - 1)2d] = 30$
$\Rightarrow n[a + d + (n - 1)d] = 30$ . . . (ii)
Subtracting equation (i) from (ii), we get
$nd = 6$ . . . (iii)

Also, given that last term exceeds the first term by $\frac{21}{2}$

$a+(2n-1)d = a+\frac{21}{2}$

$2nd - d = \frac{21}{2}$

$\Rightarrow 2\times 6-\frac{21}{2}=d \quad (\because nd = 6)$

$d = \frac{3}{2}$

Putting value of d in equation (3) $n=\frac{6\times 2}{3}=4$

Total no. of terms $=2n=2\times 4=8$

Question263

If the sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots\ldots+$ up to 20 terms is equal to $\frac{k}{21}$, then k is equal to:
[Online April 9, 2014]

Options:

A. 120

B. 180

C. 240

D. 60

Answer: A

Solution:

Solution:
n^{th} term of given series is
$$\frac{2n+1}{\frac{n(n+1)(2n+1)}{6}}=\frac{6}{n(n+1)}$$
Let n^{th} term, $a_n=6\left[\frac{1}{n}-\frac{1}{n+1}\right]$
Sum of 20 terms, $S_{20}=a_1+a_2+a_3+\ldots.+a_{20}$

$S_{20}=6\left(\frac{1}{1}-\frac{1}{2}\right)+6\left(\frac{1}{2}-\frac{1}{3}\right)+6\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots$

$+6\left(\frac{1}{18}-\frac{1}{19}\right)+6\left(\frac{1}{19}-\frac{1}{20}\right)+6\left(\frac{1}{20}-\frac{1}{21}\right)$

$S_{20}=\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots\right.$

$\left.+\left(\frac{1}{18}-\frac{1}{19}\right)+\left(\frac{1}{19}-\frac{1}{20}\right)+\left(\frac{1}{20}-\frac{1}{21}\right)\right]$

$S_{20}=6\left(1-\frac{1}{21}\right)=\frac{120}{21}$ . . . (i)

Given that $S_{20}=\frac{k}{21}$ . . . (ii)

On comparing (i) and (ii), we get
$k=120$

Question264

Let $a_1, a_2, a_3, \ldots$ be an A.P, such that $\frac{a_1+a_2+\ldots+a_p}{a_1+a_2+a_3+\ldots+a_q} = \frac{p^3}{q^3}$; $p \neq q$. Then $\frac{a_6}{a_{21}}$ is equal to: [Online April 9, 2013]

Options:

A. $\frac{41}{11}$

B. $\frac{31}{121}$

C. $\frac{11}{41}$

D. $\frac{121}{1861}$

Answer: B

Solution:

Solution:

$\frac{a_1+a_2+a_3+\ldots\ldots+a_p}{a_1+a_2+a_3+\ldots\ldots+a_q} = \frac{p^3}{q^3}$

$\Rightarrow \frac{a_1+a_2}{a_1} = \frac{8}{1} \Rightarrow a_1+(a_1+d) = 8a_1$

$\Rightarrow d = 6a_1$

Now $\frac{a_6}{a_{21}} = \frac{a_1+5d}{a_1+20d}$

$= \frac{a_1+5\times 6a_1}{a_1+20\times 6a_1} = \frac{1+30}{1+120} = \frac{31}{121}$

Question265

The sum of the series: $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots\ldots$ upto 10 terms, is [Online April 9, 2013]

Options:

A. $\frac{18}{11}$

B. $\frac{22}{13}$

C. $\frac{20}{11}$

D. $\frac{16}{9}$

Answer: C

Solution:

Solution:

$T_r = \dfrac{1}{1+2+3+\ldots+r} = \dfrac{2}{r(r+1)}$ $S_{10} = 2\sum_{r=1}^{10}\dfrac{1}{r(r+1)} = 2\sum_{r=1}^{10}\left[\dfrac{r+1}{r(r+1)} - \dfrac{r}{r(r+1)}\right]$

$= 2\sum_{r=1}^{10}\left(\dfrac{1}{r} - \dfrac{1}{r+1}\right)$

$= 2\left[\left(\dfrac{1}{1}-\dfrac{1}{2}\right)+\left(\dfrac{1}{2}-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{4}\right)+\ldots+\left(\dfrac{1}{10}-\dfrac{1}{11}\right)\right]$

$= 2\left[1-\dfrac{1}{11}\right] = 2\times\dfrac{10}{11} = \dfrac{20}{11}$

Question266

If $a_1, a_2, a_3, \ldots, a_n, \ldots.$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P., is :
[Online April 23, 2013]

Options:

A. 10m

B. 12m

C. 13m

D. 15m

Answer: C

Solution:

Solution:
If d be the common difference, then
$m = a_4 - a_7 + a_{10} = a_4 - a_7 + a_7 + 3d = a_7$
$S_{13} = \dfrac{13}{2}[a_1 + a_{13}] = \dfrac{13}{2}[a_1 + a_7 + 6d]$
$= \dfrac{13}{2}[2a_7] = 13a_7 = 13m$

Question267

Given sum of the first n terms of an A.P. is $2n + 3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is:
[Online April 22, 2013]

Options:

A. $n+4n^2$

B. $6n^2-n$

C. n^2+4n

D. $3n+2n^2$

Answer: B

Solution:

Solution:
Given $S_n=2n+3n^2$
Now, first term $=2+3=5$
second term $=2(2)+3(4)=16$
third term $=2(3)+3(9)=33$
Now, sum given in option (b) only has the same first term and difference between 2nd and 1 st term is double also.

Question268

Given a sequence of 4 numbers, first three of which are in G.P. and the last three are in A.P. with common difference six. If first and last terms of this sequence are equal, then the last term is : [Online April 25, 2013]

Options:

A. 16

B. 8

C. 4

D. 2

Answer: B

Solution:

Solution:
Let a, b, c, d be four numbers of the sequence.
Now, according to the question $b^2=ac$ and $c-b=6$ and a
$-c=6$
Also, given $a=d$
$\therefore b^2=ac \Rightarrow b^2=a\left[\frac{a+b}{2}\right]$ $(\because 2c=a+b)$
$\Rightarrow a^2-2b^2+ab=0$
Now, $c-b=6$ and $a-c=6$
gives $a-b=12$
$\Rightarrow b=a-12$
$\therefore a^2-2b^2+ab=0$
$\Rightarrow a^2-2(a-12)^2+a(a-12)=0$
$\Rightarrow a^2-2a^2-288+48a+a^2-12a=0$

$\Rightarrow 36a = 288 \Rightarrow a = 8$
Hence, last term is $d = a = 8$.

Question269

The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ... is [2013]

Options:

A. $\frac{7}{81}(179 - 10^{-20})$

B. $\frac{7}{9}(99 - 10^{-20})$

C. $\frac{7}{81}(179 + 10^{-20})$

D. $\frac{7}{9}(99 + 10^{-20})$

Answer: C

Solution:

Solution:
Let $S = \frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + ...+$ up to 20 terms
$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + ...+\right.$ up to 20 terms]
Multiply and divide by 9
$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...\right.$ up to 20 terms]

$$= \frac{7}{9}\left[\begin{array}{c}\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) \\ +\ldots\ldots \text{ up to 20 terms}\end{array}\right]$$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^{20}\right)}{1 - \frac{1}{10}}\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}[179 + (10)^{-20}]$$

Question270

The value of $1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots\ldots\ldots + 25^2$ is [Online April 25, 2013]

Options:

A. 2925

B. 1469

C. 1728

D. 1456

Answer: A

Solution:

Solution:
Consider $1^2+3^2+5^2+\ldots\ldots+25^2$
n^{th} term $T_n=(2n-1)^2, n=1,\ldots\ldots 13$

$$\text{Now, } S_n=\sum_{n=1}^{13} T_n=\sum_{n=1}^{13}(2n-1)^2$$

$$=\sum_{n=1}^{13} 4n^2+\sum_{n=1}^{13} 1-\sum_{n=1}^{13} 4n=4\sum n^2+13-4\sum n$$

$$=4\left[\frac{n(n+1)(2n+1)}{6}\right]+13-4\frac{n(n+1)}{2}$$

Put $n=13$, we get
$S_n=26\times14\times9+13-26\times14$
$=3276+13-364=2925$

Question271

The sum of the series :
$(b)^2+2(d)^2+3(6)^2+\ldots$ upto 10 terms is :
[Online April 23, 2013]

Options:

A. 11300

B. 11200

C. 12100

D. 12300

Answer: C

Solution:

Solution:
$2^2+2(4)^2+3(6)^2+\ldots\ldots$ upto 10 terms
$=2^2[1^3+2^3+3^3+\ldots\ldots$ upto 10 terms]

$$=4\cdot\left(\frac{10\times11}{2}\right)^2=12100$$

Question272

The sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots$ upto 11 -terms is:
[Online April 22, 2013]

Options:

A. $\frac{7}{2}$

B. $\frac{11}{4}$

C. $\frac{11}{2}$

D. $\frac{60}{11}$

Answer: C

Solution:

Solution:
Given sum is
$\frac{3}{12}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots$
n th term $=T_n$
$=\frac{2n+1}{\frac{n(n+1)(2n+1)}{6}}=\frac{6}{n(n+1)}$
or $T_n=6\left[\frac{1}{n}-\frac{1}{n+1}\right]$
$\therefore\ S_n=\sum T_n=6\sum\frac{1}{n}-6\sum\frac{1}{n+1}=\frac{6n}{n}-\frac{6}{n+1}$
$=6-\frac{6}{n+1}=\frac{6n}{n+1}$
So, sum upto 11 terms means
$S_{11}=\frac{6\times 11}{11+1}=\frac{66}{12}=\frac{33}{6}=\frac{11}{2}$

Question273

The sum of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots+2(2m)^2$ is
[Online May 7, 2012]

Options:

A. $m(2m+1)^2$

B. $m^2(m+2)$

C. $m^2(2m+1)$

D. $m(m+2)^2$

Answer: A

Solution:

Solution:
The sum of the given series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2$
$+2.6^2 + ... + 2(2m)^2$ is $\frac{2m(2m+1)^2}{2} = m(2m+1)^2$

Question274

The difference between the fourth term and the first term of a Geometrical Progresssion is 52 . If the sum of its first three terms is 26, then the sum of the first six terms of the progression is
[Online May 7, 2012]

Options:

A. 63

B. 189

C. 728

D. 364

Answer: C

Solution:

Solution:
Let $a, ar, ar^2, ar^3, ar^4, ar^5$ be six terms of a G.P. where ' a ' is first term and r is common ratio.
According to given conditions, we have
$ar^3 - a = 5 \Rightarrow a(r^3 - 1) = 52$ . . . (i)
and $a + ar + ar^2 = 26$
$\Rightarrow a(1 + r + r^2) = 26$ . . . (ii)
To find: $a(1 + r + r^2 + r^3 + r^4 + r^5)$
Consider
$a[1 + r + r^2 + r^3 + r^4 + r^5]$
$= a[1 + r + r^2 + r^3(1 + r + r^2)]$
$= a[1 + r + r^2][1 + r^3]$ . . . (iii)
Divide (i) by (ii), we get
$\frac{r^3 - 1}{1 + r + r^2} = 2$
we know $r^3 - 1 = (r - 1)(1 + r + r^2)$
$\therefore r - 1 = 2 \Rightarrow r = 3$ and $a = 2$
$\therefore a(1 + r + r^2 + r^3 + r^4 + r^5) = a(1 + r + r^2)(1 + r^3)$
$= 2(1 + 3 + 9)(1 + 27) = 26 \times 28 = 728$

Question275

If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$ then

[Online May 12, 2012]

Options:

A. a, b, c, d are in A.P.

B. $ab = cd$

C. $ac = bd$

D. a, b, c, d are in G.P.

Answer: D

Solution:

Solution:
The given relation can be written as
$(a^2p^2 - 2abp + b^2) + (b^2p^2 + c^2 - 2bpc) + (c^2p^2 + d^2 - 2pcd) \le 0$
or $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$ . . . (i)
Since a, b, c, d and p are all real, the inequality (i) is possible only when each of factor is zero.
i.e., $ap - b = 0$, $bp - c = 0$ and $cp - d = 0$
or $p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$
or a, b, c, d are in G.P.

Question276

The sum of the series $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + .$ upto 15 terms is [Online May 12, 2012]

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Solution:
Given series is $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots$
n^{th} term $= \frac{1}{\sqrt{n}+\sqrt{n+1}}$
$\therefore 15^{th}$ term $= \frac{1}{\sqrt{15}+\sqrt{16}}$
Thus, given series upto 15 terms is
$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots + \frac{1}{\sqrt{15}+\sqrt{16}}$

This can be re-written as
$\frac{1-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+\frac{\sqrt{3}-\sqrt{4}}{-1}+.....+\frac{\sqrt{15}-\sqrt{16}}{-1}$
(Byrationalization)
$=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}+.....-\sqrt{14}+\sqrt{15}$
$=-1+\sqrt{16}=-1+4=3$
Hence, the required sum $=3$

Question277

Suppose θ and φ(≠0) are such that sec(θ + φ), sec θ and sec(θ − φ) are in A.P. If $\cos\theta = k\cos\left(\frac{\varphi}{2}\right)$ for some k, then k is equal to

[Online May 19, 2012]

Options:

A. $\pm\sqrt{2}$

B. ± 1

C. $\pm\frac{1}{\sqrt{2}}$

D. ± 2

Answer: A

Solution:

Solution:
Since, $\sec(\theta-\varphi)$, $\sec\theta$ and $\sec(\theta+\varphi)$ are in A.P., $\therefore 2\sec\theta=\sec(\theta-\varphi)+\sec(\theta+\varphi)$
$\Rightarrow \frac{2}{\cos\theta}=\frac{\cos(\theta+\varphi)+\cos(\theta-\varphi)}{\cos(\theta-\varphi)\cos(\theta+\varphi)}$
$\Rightarrow 2(\cos^2\theta-\sin^2\varphi)=\cos\theta[2\cos\theta\cos\varphi]$
$\Rightarrow \cos^2\theta(1-\cos\varphi)=\sin^2\varphi=1-\cos^2\varphi$
$\Rightarrow \cos^2\theta=1+\cos\varphi=2\cos^2\frac{\varphi}{2}$
$\therefore \cos\theta=\sqrt{2}\cos\frac{\varphi}{2}$
But given $\cos\theta=k\cos\frac{\varphi}{2}$
$\therefore k=\sqrt{2}$

Question278

The sum of the series $1+\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+...$ upto n terms is

[Online May 19, 2012]

Options:

A. $\frac{7}{6}n+\frac{1}{6}-\frac{2}{3.2^{n-1}}$

B. $\frac{5}{3}n - \frac{7}{6} + \frac{1}{2.3^{n-1}}$

C. $n + \frac{1}{2} - \frac{1}{2.3^{n}}$

D. $n - \frac{1}{3} - \frac{1}{3.2^{n-1}}$

Answer: C

Solution:

Solution:

Given series is $1 + \frac{4}{3} + \frac{10}{9} + \frac{28}{27} + \ldots.n$ terms

$= 1 + \left(1 + \frac{1}{3}\right) + \left(1 + \frac{1}{9}\right) + \left(1 + \frac{1}{27}\right) + \ldots.n$ terms

$= (1 + 1 + 1 + \ldots. + n$ terms $)$

$+ \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots.n \text{ terms}\right)$

$$= n + \frac{\frac{1}{3}\left(1 - \frac{1}{3^{n}}\right)}{1 - \frac{1}{3}} = n + \frac{1}{3} \times \frac{3}{2}[1 - 3^{-n}]$$

$$= n + \frac{1}{2}[1 - 3^{-n}] = n + \frac{1}{2} - \frac{1}{2.3^{n}}$$

Question279

If the A.M. between p^{th} and q^{th} terms of an A.P. is equal to the A.M. between r^{th} and s^{th} terms of the same A.P. then $p + q$ is equal to [Online May 26, 2012]

Options:

A. $r + s - 1$

B. $r + s - 2$

C. $r + s + 1$

D. $r + s$

Answer: D

Solution:

Solution:

Given : $\frac{a_p + a_q}{2} = \frac{a_r + a_S}{2}$

$\Rightarrow a + (p - 1)d + a + (q - 1)d$

$= a + (r - 1)d + a + (s - 1)d$

$\Rightarrow 2a + (p + q)d - 2d = 2a + (r + s)d - 2d$

$\Rightarrow (p + q)d = (r + s)d \Rightarrow p + q = r + s$

Question280

If 100 times the 100 th term of an AP with non zero common difference equals the 50 times its 50 th term, then the 150 th term of this AP is:
[2012]

Options:

A. -150

B. 150 times its 50 th term

C. 150

D. Zero

Answer: D

Solution:

Solution:
Let ' $'a'$ is the first term and 'd ' is the common difference of an A . P.
Now, According to the question $100a_{100} = 50a_{50}$
$100(a + 99d) = 50(a + 49d)$
$\Rightarrow 2a + 198d = a + 49d \Rightarrow a + 149d = 0$
Hence, $T_{150} = a + 149d = 0$

Question281

Statement-1: The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400)$ is 8000
Statement-2: $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$, for any natural number n.
[2012]

Options:

A. Statement- 1 is false, Statement- 2 is true.

B. Statement- 1 is true, statement- 2 is true; statement- 2 is a correct explanation for Statement- 1 .

C. Statement- 1 is true, statement- 2 is true; statement- 2 is not a correct explanation for Statement- 1 .

D. Statement- 1 is true, statement- 2 is false.

Answer: B

Solution:

Solution:

n th term of the given series

$= T_n = (n-1)^2 + (n-1)n + n^2$

$= \frac{((n-1)^3 - n^3)}{(n-1) - n} = n^3 - (n-1)^3$

$\Rightarrow S_n = \sum_{k=1}^{n} [k^3 - (k-1)^3] \Rightarrow 8000 = n^3$

$\Rightarrow n = 20$ which is a natural number.

Hence, both the given statements are true.

and statement 2 is correct explanation for statement 1.

Question282

Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is
[2011]

Options:

A. $\alpha - \beta$

B. $\frac{\alpha - \beta}{100}$

C. $\beta - \alpha$

D. $\frac{\alpha - \beta}{200}$

Answer: B

Solution:

Solution:

Let A.P. be $a, a+d, a+2d, \ldots\ldots\ldots$

$a_2 + a_4 + \ldots\ldots\ldots + a_{200} = \alpha$

$\Rightarrow \frac{100}{2}[2(a+d) + (100-1)2d] = \alpha \ldots (i)$

and $a_1 + a_3 + a_5 + \ldots\ldots\ldots + a_{199} = \beta$

$\Rightarrow \frac{100}{2}[2a + (100-1)2d] = \beta \ldots$ (ii)

Subtracting (ii) from (i), we get

$d = \frac{\alpha - \beta}{100}$

Question283

If the sum of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + \ldots 2.6^2 + \ldots$ upto n terms, when n is even, is $\frac{n(n+1)^2}{2}$, then the sum of the series, when n is odd, is
[Online May 26, 2012]

Options:

A. $n^2(n+1)$

B. $\frac{n^2(n-1)}{2}$

C. $\frac{n^2(n+1)}{2}$

D. $n^2(n-1)$

Answer: C

Solution:

Solution:
If n is odd, the required sum is
$1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + 2(n-1)^2 + n^2$
$= \frac{(n-1)(n-1+1)^2}{2} + n^2$ ($\because n-1$ is even)
$= \left(\frac{n-1}{2} + 1\right)n^2 = \frac{n^2(n+1)}{2}$

Question284

A man saves ₹200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹40 more than the saving of immediately previous month. His total saving from the start of service will be ₹11040 after
[2011]

Options:

A. 19 months

B. 20 months

C. 21 months

D. 18 months

Answer: C

Solution:

Solution:
Let number of months $= n$
$\therefore 200 \times 3 + (240 + 280 + 320 + \ldots + (n-3)^{th}$ term $) = 11040$
$\Rightarrow \frac{n-3}{2}[2 \times 240 + (n-4) \times 40] = 11040 - 600$
$\Rightarrow (n-3)[240 + 20n - 80] = 10440$
$\Rightarrow (n-3)(20n + 160) = 10440$
$\Rightarrow (n-3)(n+8) = 522$
$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n+26)(n-21)=0$
$\therefore\ n = 21$

Question285

A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference-2, then the time taken by him to count all notes is [2010]

Options:

A. 34 minutes

B. 125 minutes

C. 135 minutes

D. 24 minutes

Answer: A

Solution:

Solution:
Till 10^{th} minute number of counted notes $= 1500$
Remaining notes $= 4500 - 1500 = 3000$
$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$
$n^2 - 149n + 3000 = 0$
$\Rightarrow n = 125, 24$
But $n = 125$ is not possible
$\therefore$ Total time $= 24 + 10 = 34$ minutes.

Question286

The sum to infinite term of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \cdot s$ is [2009]

Options:

A. 3

B. 4

C. 6

D. 2

Answer: A

Solution:

Solution:

Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots\ldots\infty$ . . . (i)

Multiplying both sides by $\frac{1}{3}$, we get

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots\ldots\ldots\infty$ . . . (ii)

Subtracting eqn. (ii) from eqn. (i), we get

$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots\ldots\ldots\infty$

$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots\ldots\ldots\infty$

$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1 - \frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3$

Question287

The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48 . If the terms of the geometric progression are alternately positive and negative, then the first term is [2008]

Options:

A. -4

B. -12

C. 12

D. 4

Answer: B

Solution:

Solution:

AT Q

$a + ar = 12$

$ar^2 + ar^3 = 48$

$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$

($\because$ terms are alternately + ve and $-$ve)

$\Rightarrow a = -12$

Question288

In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals

[2007]

Options:

A. $\sqrt{5}$

B. $\frac{1}{2}(\sqrt{5}-1)$

C. $\frac{1}{2}(1-\sqrt{5})$

D. $\frac{1}{2}\sqrt{5}$

Answer: B

Solution:

Solution:
Let the series $a, ar, ar^2, ...$ are in geometric progression.
Given that, $a = ar + ar^2$
$\Rightarrow 1 = r + r^2 \quad \Rightarrow r^2 + r - 1 = 0$
$\Rightarrow r = \dfrac{-1 \pm \sqrt{1-4\times -1}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$
$\Rightarrow r = \dfrac{\sqrt{5}-1}{2}$[$\because$ terms of G.P. are positive $\therefore r$ should be positive]

Question289

The sum of series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ......$ upto infinity is [2007]

Options:

A. $e^{-\frac{1}{2}}$

B. $e^{+\frac{1}{2}}$

C. e^{-2}

D. e^{-1}

Answer: D

Solution:

Solution:
We know that $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + ...\infty$
$\text{Put} x = -1$

$\therefore\ e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\ldots \infty$

$\therefore\ e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \ldots\ldots\ldots \infty$

Question290

Let a_1, a_2, a_3………… be terms on A.P. If $\frac{a_1 + a_2 + \ldots\ldots\ldots a_p}{a_1 + a_2 + \ldots\ldots\ldots + a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals [2006]

Options:

A. $\frac{41}{11}$

B. $\frac{7}{2}$

C. $\frac{2}{7}$

D. $\frac{11}{41}$

Answer: D

Solution:

Solution:
Given that

$$\frac{S_p}{S_q} = \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

Put $p = 11$ and $q = 41$

$$\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$$

Question291

The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is [2006]

Options:

A. i

B. 1

C. -1

D. $-i$

Answer: D

Solution:

Solution:
$\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)$
$=i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11}-i\sin\frac{2k\pi}{11}\right)\ [\because e^{i\theta}=\cos\theta+i\sin\theta]$
$=i\sum_{k=1}^{10}e^{-\frac{2k\pi}{11}i}=i\left\{\sum_{k=0}^{10}e^{-\frac{2k\pi}{11}i}-1\right\}$
$=i\left[1+e^{-\frac{2\pi}{11}i}+e^{-\frac{4\pi}{11}i}+\ldots.11\text{ terms}\right]-i$
$=i\left[\frac{1-\left(e^{-\frac{2\pi}{11}}\right)^{11}}{1-e^{-\frac{2\pi}{11}i}}\right]-i=i\left[\frac{1-e^{-2\pi i}}{1-e^{-\frac{2\pi}{11}i}}\right]-i$
$=i\times 0-i\ [\because e^{-2\pi i}=1]$
$=-i$

Question292

If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3\ldots\ldots$ then a_n is
[2006]

Options:

A. $\frac{b^n-a^n}{b-a}$

B. $\frac{a^n-b^n}{b-a}$

C. $\frac{a^{n+1}-b^{n+1}}{b-a}$

D. $\frac{b^{n+1}-a^{n+1}}{b-a}$

Answer: D

Solution:

Solution:
$(1-ax)^{-1}(1-bx)^{-1}$
$=(1+ax+a^2x^2+\ldots)(1+bx+b^2x^2+\ldots)$
$\therefore$ Coefficient of x^n
$x^n=b^n+ab^{n-1}+a^2b^{n-2}+\ldots\ldots+a^{n-1}b+a^n$
{ which is a G.P. with $r=\frac{a}{b}$

Its sum is $= \left. \dfrac{b^n\left[1-\left(\frac{a}{b}\right)^{n+1}\right]}{1-\frac{a}{b}} \right\}$

$= \dfrac{b^{n+1}-a^{n+1}}{b-a} \quad \therefore a_n = \dfrac{b^{n+1}-a^{n+1}}{b-a}$

Question293

If $a_1, a_2, \ldots\ldots\ldots, a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + \ldots\ldots\ldots + a_{n-1}a_n$ is equal to [2006]

Options:

A. $n(a_1 - a_n)$

B. $(n-1)(a_1 - a_n)$

C. na_1a_n

D. $(n-1)a_1a_n$

Answer: D

Solution:

Solution:

$\because a_1, a_2, a_3 \ldots\ldots a_n$ are in H.P.

$\therefore \dfrac{1}{a_1}, \dfrac{1}{a_2}, \dfrac{1}{a_3} \ldots \dfrac{1}{a_n}$ are in A.P.

$\therefore \dfrac{1}{a_2} - \dfrac{1}{a_1} = \dfrac{1}{a_3} - \dfrac{1}{a_2} = \ldots\ldots\ldots = \dfrac{1}{a_n} - \dfrac{1}{a_{n-1}} = d$ (say)

Then $a_1a_2 = \dfrac{a_1 - a_2}{d}, a_2a_3 = \dfrac{a_2 - a_3}{d},$

$\ldots\ldots\ldots\ldots a_{n-1}a_n = \dfrac{a_{n-1} - a_n}{d}$

Adding all equations, we get

$\therefore a_1a_2 + a_2a_3 + \ldots\ldots\ldots + a_{n-1}a_n$

$= \dfrac{a_1 - a_2}{d} + \dfrac{a_2 - a_3}{d} + \ldots. + \dfrac{a_{n-1} - a_n}{d}$

$= \dfrac{1}{d}[a_1 - a_2 + a_2 - a_3 + \ldots. + a_{n-1} - a_n] = \dfrac{a_1 - a_n}{d}$

Also, $\dfrac{1}{a_n} = \dfrac{1}{a_1} + (n-1)d$

$\Rightarrow \dfrac{a_1 - a_n}{a_1a_n} = (n-1)d$

$\Rightarrow \dfrac{a_1 - a_n}{d} = (n-1)a_1a_n$

Which is the required result.

Question294

If the coefficients of rth, (r + 1) th , and (r + 2) th terms in the the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation
[2005]

Options:

A. $m^2 - m(4r - 1) + 4r^2 - 2 = 0$

B. $m^2 - m(4r + 1) + 4r^2 + 2 = 0$

C. $m^2 - m(4r + 1) + 4r^2 - 2 = 0$

D. $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

Answer: C

Solution:

Solution:
Coeficient of r^{th} , $(r+1)^{th}$ and $(r+2)^{th}$ terms is ${}^mC_{r-1}$, ${}^mC_r$ and ${}^mC_{r+1}$ resp.
Given that ${}^mC_{r-1}, {}^mC_r, {}^mC_{r+1}$ are in A.P.
$2{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$
$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = \frac{r}{m-r+1} + \frac{m-r}{r+1}$
$\Rightarrow m^2 - m(4r + 1) + 4r^2 - 2 = 0$

Question295

If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P and $|a| < 1, |b| < 1, |c| < 1$ then x, y, z are in
[2005]

Options:

A. G.P.

B. A.P.

C. Arithmetic - Geometric Progression

D. H.P.

Answer: D

Solution:

Solution:
$x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$
$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$

$z=\sum_{n=0}^{\infty} c^n=\frac{1}{1-c} \Rightarrow c=1-\frac{1}{z}$
a, b, c are in A.P. $\Rightarrow \ 2b=a+c$
$2\left(1-\frac{1}{y}\right)=1-\frac{1}{x}+1-\frac{1}{z}$
$\frac{2}{y}=\frac{1}{x}+\frac{1}{z} \Rightarrow x, y, z$ are in H.P.

Question296

The sum of the series $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+$..................... ad inf. is [2005]

Options:

A. $\frac{e-1}{\sqrt{e}}$

B. $\frac{e+1}{\sqrt{e}}$

C. $\frac{e-1}{2\sqrt{e}}$

D. $\frac{e+1}{2\sqrt{e}}$

Answer: D

Solution:

Solution:
We know that
$\frac{e^x+e^{-x}}{2}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}$............
Putting $x=\frac{1}{2}$, we get

$$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+\ldots\ldots\infty=\frac{e^{\frac{1}{2}}+e^{\frac{-1}{2}}}{2}$$
$$=\frac{\sqrt{e}+\frac{1}{\sqrt{e}}}{2}=\frac{e+1}{2\sqrt{e}}$$

Question297

Let T_r be the rth term of an A.P. whose first term is a and common difference is d . If for some positive integers m, n, $m \neq n$, $T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals [2004]

Options:

A. $\frac{1}{m}+\frac{1}{n}$

B. 1

C. $\frac{1}{mn}$

D. 0

Answer: D

Solution:

Solution:
$T_m = a+(m-1)d = \frac{1}{n} \ldots$ (i)
$T_n = a+(n-1)d = \frac{1}{m} \ldots$ (ii)
Subtracting (ii) from (i), we get
$(m-n)d = \frac{1}{n}-\frac{1}{m} \Rightarrow d = \frac{1}{mn}$
From (i) $a = \frac{1}{mn} \Rightarrow a-d = 0$

Question298

Let two numbers have arithmetic mean 9 and geometric mean 4 . Then these numbers are the roots of the quadratic equation [2004]

Options:

A. $x^2-18x-16=0$

B. $x^2-18x+16=0$

C. $x^2+18x-16=0$

D. $x^2+18x+16=0$

Answer: B

Solution:

Solution:
Let two numbers be a and b then $\frac{a+b}{2}=9$
$\Rightarrow a+b=18$ and $\sqrt{ab}=4 \Rightarrow ab=16$
$\therefore$ Equation with roots a and b is
$x^2-(a+b)x+ab=0 \Rightarrow x^2-18x+16=0$

Question299

The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+.....$ is
[2004]

Options:

A. $\frac{(e^2-2)}{e}$

B. $\frac{(e-1)^2}{2e}$

C. $\frac{(e^2-1)}{2e}$

D. $\frac{(e^2-1)}{2}$

Answer: B

Solution:

Solution:
We know that
$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}....\infty$
$\therefore e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+......$
and $e^{-1} = 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+.......$
$\therefore e+e^{-1} = 2\left[1+\frac{1}{2!}+\frac{1}{4!}+...\right]$
$\therefore \frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+...... = \frac{e+e^{-1}}{2}-1$
$= \frac{e^2+1-2e}{2e} = \frac{(e-1)^2}{2e}$

Question300

The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum
[2004]

Options:

A. $\left[\frac{n(n+1)}{2}\right]^2$

B. $\frac{n^2(n+1)}{2}$

C. $\frac{n(n+1)^2}{4}$

D. $\frac{3n(n+1)}{2}$

Answer: B

Solution:

Solution:
If n is odd, the required sum is
$1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + 2 \cdot (n-1)^2 + n^2$
$= \dfrac{(n-1)(n-1+1)^2}{2} + n^2$
[$\because (n-1)$ is even
$\therefore$ using given formula for the sum of $(n-1)$ terms.
$= \left(\dfrac{n-1}{2} + 1\right) n^2 = \dfrac{n^2(n+1)}{2}$

Question301

If $S_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{^nC_r}$, then $\frac{t_n}{S_n}$ is equal to [2004]

Options:

A. $\dfrac{2n-1}{2}$

B. $\frac{1}{2}n - 1$

C. $n - 1$

D. $\frac{1}{2}n$

Answer: D

Solution:

Solution:
$S_n = \dfrac{1}{^nC_0} + \dfrac{1}{^nC_1} + \dfrac{1}{^nC_2} + ... + \dfrac{1}{^nC_n}$
$t_n = \dfrac{0}{^nC_0} + \dfrac{1}{^nC_1} + \dfrac{2}{^nC_2} + ... + \dfrac{n}{^nC_n}$
$t_n = \dfrac{n}{^nC_n} + \dfrac{n-1}{^nC_{n-1}} + \dfrac{n-2}{^nC_{n-2}} + ... + \dfrac{0}{^nC_0}$
Adding (i) and (ii), we get,
$2t_n = (n)\left[\dfrac{1}{^nC_0} + \dfrac{1}{^nC_1} + ... \dfrac{1}{^nC_n}\right] = nS_n$
$\because {^nC_r} = {^nC_{n-r}}$
$\therefore \dfrac{t_n}{S_n} = \dfrac{n}{2}$

Question302

If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in
[2003]

Options:

A. Arithmetic - Geometric Progression

B. Arithmetic Progression

C. Geometric Progression

D. Harmonic Progression.

Answer: D

Solution:

Solution:
$ax^2 + bx + c = 0, \alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$
AT Q, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$
$\alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} \Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$
On simplification $2a^2c = ab^2 + bc^2$
$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$ [Divide both side by abc]
$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P.
$\therefore \frac{a}{c}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

Question303

The sum of the series $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4}$............ up to ∞ is equal to
[2003]

Options:

A. $\log_e\left(\frac{4}{e}\right)$

B. $2\log_e 2$

C. $\log_e 2 - 1$

D. $\log_e 2$

Answer: A

Solution:

Solution:

Let $S = \frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} \ldots\ldots\ldots\ldots.\infty$

$T_n = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$\therefore\ S = \left(\frac{1}{1} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{4} - \frac{1}{5}\right) \ldots.$

$= 1 - 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} \ldots\ldots\ldots\ldots.\infty\right]$

$\left[\because \log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\infty\right]$

$= 1 - 2[-\log(1+1) + 1] = 2\log 2 - 1 = \log\left(\frac{4}{e}\right)$

Question304

If 1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P. then x equals [2002]

Options:

A. $\log_3 4$

B. $1 - \log_3 4$

C. $1 - \log_4 3$

D. $\log_4 3$

Answer: B

Solution:

Solution:

1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P.

$\because$a, b, c are in A . P then $b = a + c$

$\Rightarrow 2\log_9(3^{1-x} + 2) = 1 + \log_3(4.3^x - 1)$

$\because\ \log_{b^q} a^p = \frac{p}{q}\log_b a$

$\Rightarrow \log_3(3^{1-x} + 2) = \log_3 3 + \log_3(4.3^x - 1)$

$\Rightarrow \log_3(3^{1-x} + 2) = \log_3[3(4 \cdot 3^x - 1)]$

$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$

$\Rightarrow 3.3^{-x} + 2 = 12.3^x - 3$ Put$3^x = t$

$\Rightarrow \frac{3}{t} + 2 = 12t - 3 \Rightarrow 12t^2 - 5t - 3 = 0$

Hence $t = -\frac{1}{3}, \frac{3}{4}$

$\Rightarrow 3^x = \frac{3}{4}\left(\right.$ as $3^x \neq -\text{ve}\left.\right)$

$\Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x = \log_3 3 - \log_3 4$
$\Rightarrow x = 1 - \log_3 4$

Question305

Sum of infinite number of terms of GP is 20 and sum of their square is 100 . The common ratio of GP is
[2002]

Options:

A. 5

B. $\frac{3}{5}$

C. $\frac{8}{5}$

D. $\frac{1}{5}$

Answer: B

Solution:

Solution:
Let $a =$ first term of G.P. and $r =$ common ratio of G.P.
Then G.P. is a, ar, ar^2
Given $S_\infty = 20 \Rightarrow \frac{a}{1-r} = 20$
$\Rightarrow a = 20(1-r) \ldots$ (i)
Also $a^2 + a^2r^2 + a^2r^4 + \ldots$ to $\infty = 100$
$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow \frac{[20(1-r)]^2}{1-r^2} = 100$ [from (i)]
$\Rightarrow \frac{400(1-r)^2}{(1-r)(1+r)} = 100 \Rightarrow 4(1-r) = 1+r$
$\Rightarrow 1 + r = 4 - 4r \Rightarrow 5r = 3 \Rightarrow r = 3/5$

Question306

Fifth term of a GP is 2, then the product of its 9 terms is
[2002]

Options:

A. 256

B. 512

C. 1024

D. none of these

Answer: B

Solution:

Solution:

$\because a_4 = 2 \Rightarrow ar^4 = 2$

Now, $a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$

$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

Question307

$1^3 - 2^3 + 3^3 - 4^3 + \ldots + 9^3 =$ [2002]

Options:

A. 425

B. -425

C. 475

D. -475

Answer: A

Solution:

Solution:

$1^3 - 2^3 + 3^3 - 4^3 + \ldots\ldots + 9^3$

$= 1^3 + 2^3 + 3^3 + \ldots\ldots + 9^3 - 2(2^3 + 4^3 + 6^3 + 8^3)$

$\left[\because \Sigma n^3 = \left(\frac{n(n+1)}{2}\right)^2\right]$

$= \left[\frac{9 \times 10}{2}\right]^2 - 2.2^3[1^3 + 2^3 + 3^3 + 4^3]$

$= (45)^2 - 16 \cdot \left[\frac{4 \times 5}{2}\right]^2 = 2025 - 1600 = 425$

Question308

The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \ldots \infty$ is [2002]

Options:

A. 1

B. 2

C. $\frac{3}{2}$

D. 4

Answer: B

Solution:

Solution:
Let $P = 2^{1/4} \cdot 2^{2/8} \cdot 2^{3/16} \ldots\ldots\ldots \infty$
$= 2^{1/4 + 2/8 + 3/16 + \ldots\ldots\ldots \infty}$
Now, let $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \ldots\ldots\ldots \infty \ldots\ldots (i)$
$\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \ldots\ldots\ldots \infty$ . . . ..(ii)
Subtracting (ii) from (i)
$\Rightarrow \frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\ldots\ldots \infty$
or $\frac{1}{2}S = \frac{a}{1-r} = \frac{1/4}{1-1/2} = \frac{1}{2} \Rightarrow S = 1$
$\therefore\ P = 2^S = 2$
