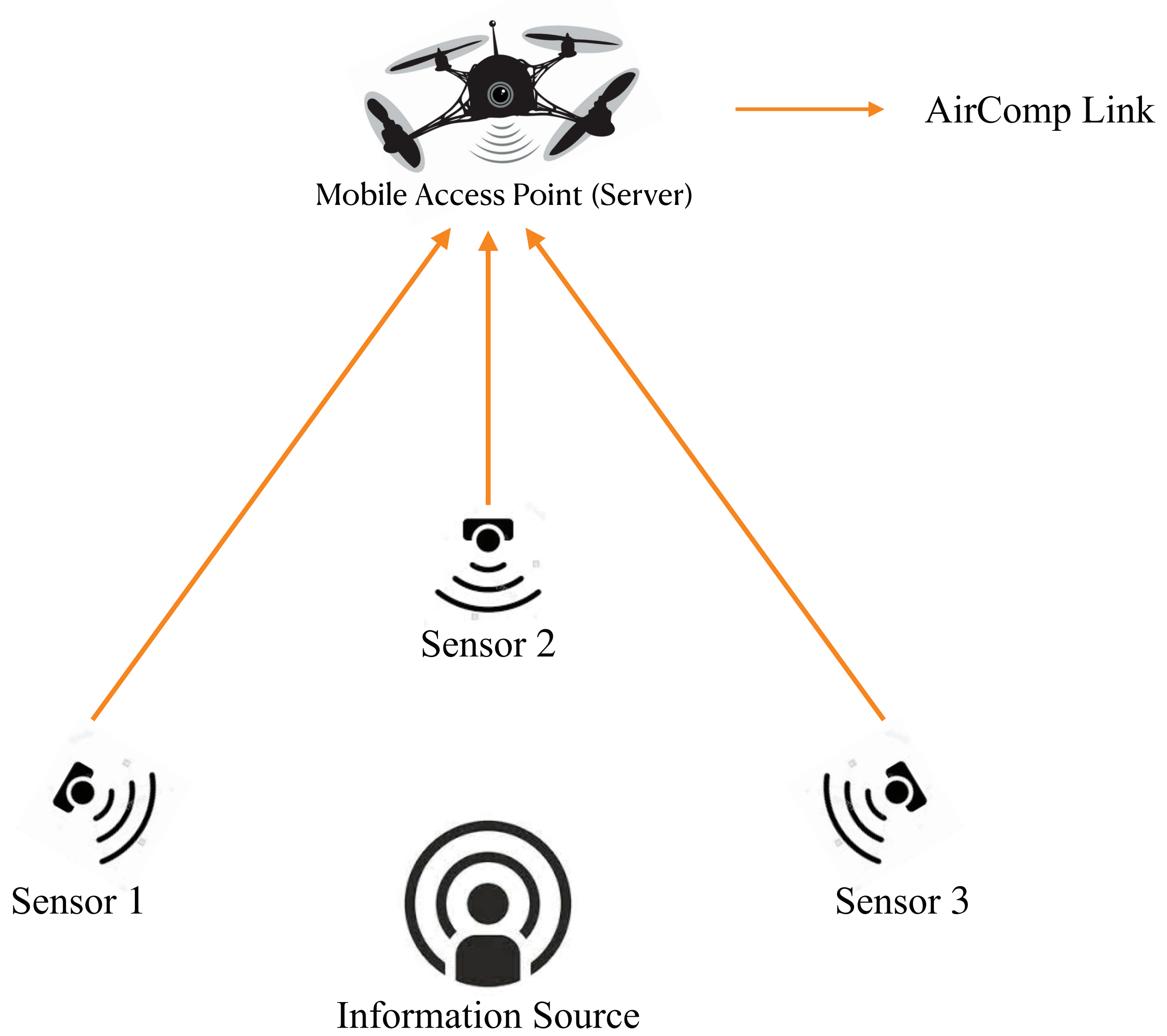
Background

- A Mobile Server
  - Real-time inference applications
  - Collect features from sensors
- Multiple Sensors
  - Sensing the same source
  - Each observation is a corrupted version of ground-true data [1],
- AirComp
  - Aggregate the features to average the corruptions.
- Metric: Discriminant Gain



[1] J.-J. Xiao, S. Cui, Z.-Q. Luo and A. J. Goldsmith, "Power scheduling of universal decentralized estimation in sensor networks", IEEE Trans. Signal Processing, vol. 54, no. 2, pp. 413-422, Feb. 2006.

System Model

- Network Model
  - One mobile server (access point) with N receive antennas
  - K sensors with single transmit antennas
- Sensing Data (Features)
  - The sensed data (feature) vector, denoted as $\mathbf{x}$, has M dimensions.
  - For the k-th sensor, its observed feature vector is $\mathbf{x}_k$, given as

$$\mathbf{x}_k = \mathbf{x} + \mathbf{d}_k,$$

    - $\mathbf{x}$ is the ground-true data (feature).
    - $\mathbf{d}_k$ is the observation distortion with the following distribution,

$$\mathbf{d}_k \sim \mathcal{N}\left(\mathbf{0}, \mathbf{D}_k\right).$$

      - $D_k = \mathrm{diag}\{\delta_{k,1}^2, \delta_{k,2}^2, \ldots, \delta_{k,M}^2\}$ is the diagonal covariance matrix.

Discriminant Gain Model

- The ground-true feature $\mathbf{x}$ has the following distribution

$$f(\mathbf{x}) = \frac{1}{L} \sum_{\ell=1}^{L} \mathcal{N}\left(\boldsymbol{\mu}_\ell, \boldsymbol{\Sigma}\right),$$

  - L is the total number of classes.
  - $\boldsymbol{\mu}_\ell = \left[\mu_{\ell,1}, \mu_{\ell,2}, \ldots, \mu_{\ell,M}\right]^T$, is the mean vector of the $\ell-$th class.
  - $\boldsymbol{\Sigma} = \mathrm{diag}\left\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2\right\}$, is the covariance matrix.
    - $\boldsymbol{\Sigma}$ is a diagonal matrix, as PCA is used to pre-process the features and different feature elements are independent.
    - All classes are assumed to have the same variance.

Discriminant Gain Model

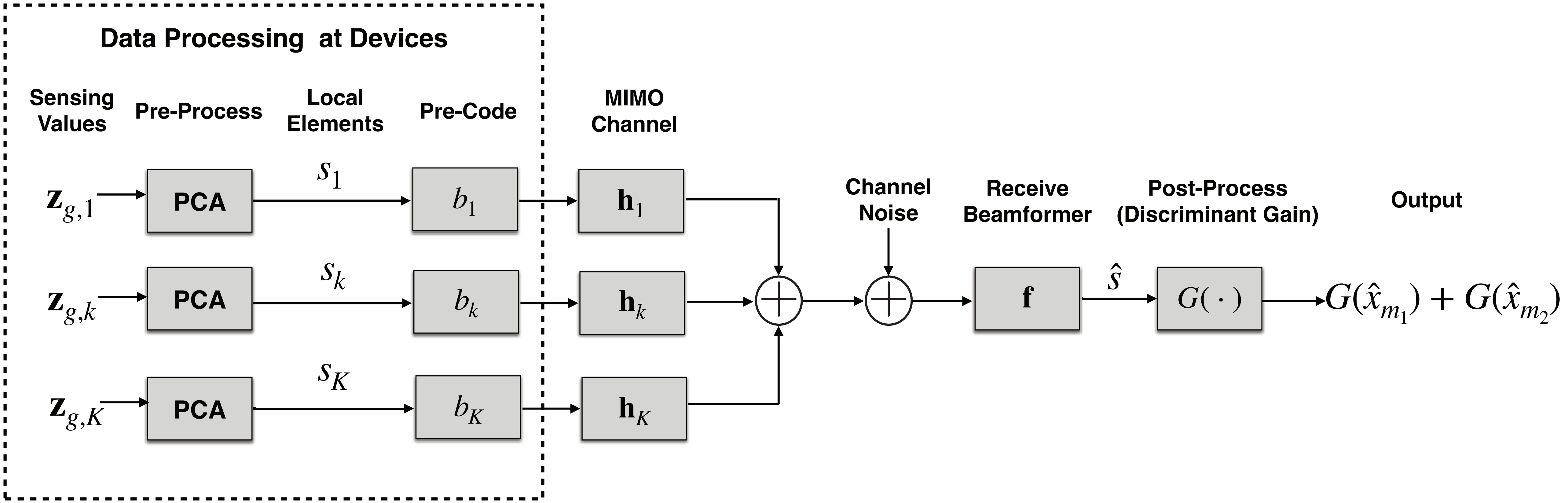
- Discriminant Gain
  - Consider the $m-$th feature dimension, its discriminant gain is

$$G(x_m) = \frac{2}{L(L-1)} \sum_{\ell'=1}^{L} \sum_{\ell<\ell'} \frac{\left(\mu_{\ell,m} - \mu_{\ell',m}\right)^2}{\sigma_m^2},$$

  - The total discriminant gain can be written as

$$G(\mathbf{x}) = \sum_{m=1}^{M} G(x_m).$$

AirComp Model



- Transmit Symbols
  - s_k is a complex scalar:

$$s_k = x_{k,m_1} + jx_{k,m_2},$$

    - j represents the imaginary unit.
    - x_{k,m_1} and x_{k,m_2} are the m_1-th and m_2-th elements of the k-th observation.

AirComp Model

- At the server, the receive signal can be written as

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{h}_k b_k s_k + \mathbf{n},$$

  - $\mathbf{h}_k \in \mathbb{C}^N$ is the channel vector of device k,
  - $b_k \in \mathbb{C}$ is the pre-coding scalar of sensor k,
  - $\mathbf{n}$ is the noise with the following distribution,

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \delta_0^2 \mathbf{I}\right).$$

- Symbol estimation (receive beamforming)

$$\hat{s} = \mathbf{f}^H \mathbf{y} = \mathbf{f}^H \sum_{k=1}^{K} \mathbf{h}_k b_k s_k + \mathbf{f}^H \mathbf{n},$$

  - $\mathbf{f} \in \mathbb{C}^N$ is the receive beamforming.

AirComp Model

✦ Estimated Elements

$$\begin{cases} \hat{x}_{m_1} = \mathrm{Re}\left(\hat{s}\right) = \mathrm{Re}\left(\mathbf{f}^H \sum_{k=1}^{K} \mathbf{h}_k b_k s_k + \mathbf{f}^H \mathbf{n}\right), \\ \hat{x}_{m_2} = \mathrm{Im}\left(\hat{s}\right) = \mathrm{Im}\left(\mathbf{f}^H \sum_{k=1}^{K} \mathbf{h}_k b_k s_k + \mathbf{f}^H \mathbf{n}\right). \end{cases}$$

Problem Formulation

- Objective: Maximize the Receive Discriminant Gain

$$\max_{\{b_k\},\mathbf{f}} \quad G = G(\hat{x}_{m_1}) + G(\hat{x}_{m_2}),$$

- Power Constraint of Each Sensor

$$b_k s_k s_k^H b_k^H \leq P_k, \quad 1 \leq k \leq K,$$

  - $\{s_k\}$ have different variance, as the distortion of different devices is different.
  - $s_k s_k^H$ is known to device k. Therefore,

$$b_k b_k^H \leq \hat{P}_k, \quad 1 \leq k \leq K,$$

    - $\hat{P}_k = P_k / s_k s_k^H$

Objective Simplification

- Zero-forcing (ZF) pre-coding

$$\mathbf{f}^H \mathbf{h}_k b_k = c_k, \quad 1 \le k \le K,$$

  - c_k is a real number and represents the receive signal strength from device k.

- ZF pre-coders

$$b_k = \frac{c_k \mathbf{h}_k^H \mathbf{f}}{\mathbf{h}_k^H \mathbf{f}\mathbf{f}^H \mathbf{h}_k}, \quad 1 \le k \le K\,.$$

- Estimated Elements

$$\begin{cases} \hat{x}_{m_1} = \mathrm{Re}\left(\sum_{k=1}^{K} c_k s_k + \mathbf{f}^H \mathbf{n} \right) = \underline{\sum_{k=1}^{K} c_k x_{k,m_1} + \mathrm{Re}(\mathbf{f}^H \mathbf{n})}, \\ \hat{x}_{m_2} = \mathrm{Im}\left(\sum_{k=1}^{K} c_k s_k + \mathbf{f}^H \mathbf{n} \right) = \underline{\sum_{k=1}^{K} c_k x_{k,m_2} + \mathrm{Im}(\mathbf{f}^H \mathbf{n})}\,. \end{cases}$$

Objective Simplification

- Receive Elements Distribution

$$\hat{x}_{m_i} \sim \frac{1}{L}\mathcal{N}\left(\hat{\mu}_{\ell,m_i}, \hat{\sigma}^2_{m_i}\right), \quad i = 1,2,$$

where

$$\begin{cases} \hat{\mu}_{\ell,m_1} = \sum_{k=1}^{K} c_k \mu_{\ell,m_1}, \\ \hat{\sigma}^2_{m_1} = \sigma^2_{m_1}\left(\sum_{k=1}^{K} c_k\right)^2 + \sum_{k=1}^{K} c_k^2 \delta^2_{k,m_1} + \frac{\delta_0^2}{2}\left(\mathbf{f}_1^T\mathbf{f}_1 + \mathbf{f}_2^T\mathbf{f}_2\right), \\ \hat{\mu}_{\ell,m_2} = \sum_{k=1}^{K} c_k \mu_{\ell,m_2}, \\ \hat{\sigma}^2_{m_2} = \sigma^2_{m_2}\left(\sum_{k=1}^{K} c_k\right)^2 + \sum_{k=1}^{K} c_k^2 \delta^2_{k,m_2} + \frac{\delta_0^2}{2}\left(\mathbf{f}_1^T\mathbf{f}_1 + \mathbf{f}_2^T\mathbf{f}_2\right). \end{cases}$$

$$\mathbf{f}_1 = \mathrm{Re}(\mathbf{f})$$

$$\mathbf{f}_2 = \mathrm{Im}(\mathbf{f})$$

Objective Simplification

- Receive Discriminant Gains

$$G(x_{m_i}) = \frac{2}{L(L-1)} \sum_{\ell'=1}^{L} \sum_{\ell<\ell'} \frac{\left(\hat{\mu}_{\ell,m_i} - \hat{\mu}_{\ell',m_i}\right)^2}{\hat{\sigma}_{m_i}^2}, \quad i = 1,2,$$

- Power Constraint

$$c_k^2 \le P_k \mathbf{h}_k^H \mathbf{f}\mathbf{f}^H \mathbf{h}_k, \quad 1 \le k \le K.$$

Objective Simplification

- Simplified Problem

$$\max_{\{c_k\},\mathbf{f}_1,\mathbf{f}_2} \quad G = \frac{2}{L(L-1)} \sum_{i=1}^{2} \sum_{\ell'=1}^{L} \sum_{\ell<\ell'} \frac{\left(\hat{\mu}_{\ell,m_i} - \hat{\mu}_{\ell',m_i}\right)^2}{\hat{\sigma}_{m_i}^2},$$

$$\text{s.t.} \quad c_k^2 \le P_k \mathbf{h}_k^H \left(\mathbf{f}_1\mathbf{f}_1^T + \mathbf{f}_2\mathbf{f}_2^T\right) \mathbf{h}_k, \quad 1 \le k \le K,$$

$$\begin{cases} \hat{\mu}_{\ell,m_1} = \sum_{k=1}^{K} c_k \mu_{\ell,m_1}, \\ \hat{\sigma}_{m_1}^2 = \sigma_{m_1}^2 \left(\sum_{k=1}^{K} c_k\right)^2 + \sum_{k=1}^{K} c_k^2 \delta_{k,m_1}^2 + \frac{\delta_0^2}{2} \left(\mathbf{f}_1^T\mathbf{f}_1 + \mathbf{f}_2^T\mathbf{f}_2\right), \\ \hat{\mu}_{\ell,m_2} = \sum_{k=1}^{K} c_k \mu_{\ell,m_2}, \\ \hat{\sigma}_{m_2}^2 = \sigma_{m_2}^2 \left(\sum_{k=1}^{K} c_k\right)^2 + \sum_{k=1}^{K} c_k^2 \delta_{k,m_2}^2 + \frac{\delta_0^2}{2} \left(\mathbf{f}_1^T\mathbf{f}_1 + \mathbf{f}_2^T\mathbf{f}_2\right). \end{cases}$$