

Background

◆ A Mobile Server

- ❖ Real-time inference applications
- ❖ Collect features from sensors

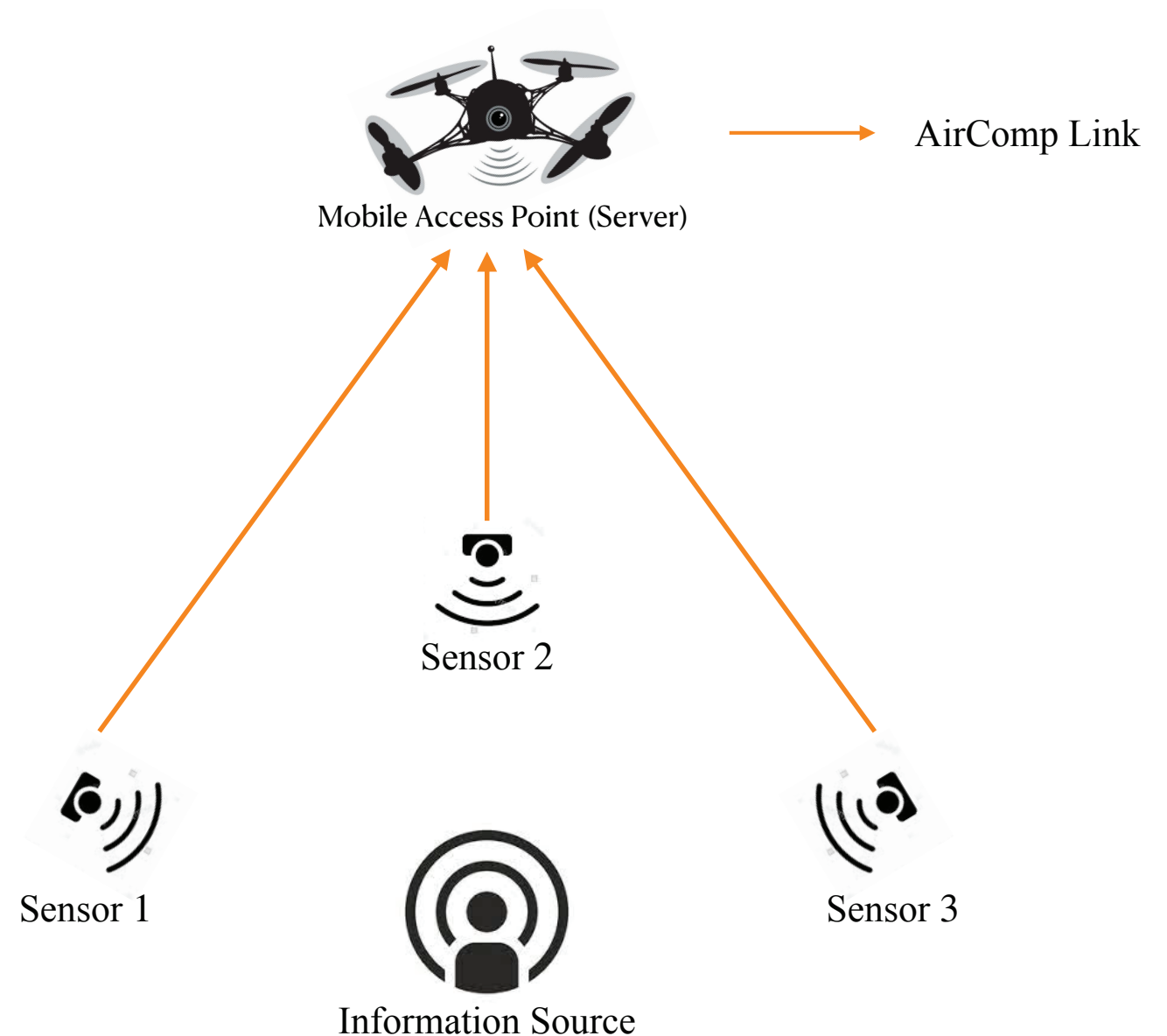
◆ Multiple Sensors

- ❖ Sensing **the same source**
- ❖ **Each observation is a corrupted version of ground-true data [1],**

◆ AirComp

- ❖ Aggregate the features to average the corruptions.

◆ Metric: Discriminant Gain



[1] J.-J. Xiao, S. Cui, Z.-Q. Luo and A. J. Goldsmith, "Power scheduling of universal decentralized estimation in sensor networks", IEEE Trans. Signal Processing, vol. 54, no. 2, pp. 413-422, Feb. 2006.

System Model

◆ Network Model

- ❖ One mobile server (access point) with N_r receive antennas
- ❖ K sensors with each having $N_t \leq N_r$ transmit antennas

◆ Sensing Data (Features)

- ❖ The sensed data (feature) vector, denoted as \mathbf{x} , has $M = 2N_t$ dimensions.
- ❖ For the k -th sensor, its observed feature vector is \mathbf{x}_k , given as

$$\mathbf{x}_k = \mathbf{x} + \mathbf{d}_k,$$

- ❖ \mathbf{x} is the ground-true data (feature).
- ❖ \mathbf{d}_k is the observation distortion with the following distribution,

$$\mathbf{d}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{D}_k).$$

- $\mathbf{D}_k = \text{diag}\{\delta_{k,1}^2, \delta_{k,2}^2, \dots, \delta_{k,M}^2\}$ is the diagonal covariance matrix.

Discriminant Gain Model

- ◆ The ground-true feature \mathbf{x} has the following distribution

$$\mathbf{x} \sim \sum_{\ell=1}^L \mathcal{N}(\boldsymbol{\mu}_{\ell}, \boldsymbol{\Sigma}_{\ell}),$$

- ✿ L is the total number of classes.

- ✿ $\boldsymbol{\mu}_{\ell} = [\mu_{\ell,1}, \mu_{\ell,2}, \dots, \mu_{\ell,M}]^T$, is the mean vector of the ℓ -th class.

- ✿ $\boldsymbol{\Sigma}_{\ell} = \text{diag} \{ \sigma_1^2, \sigma_2^2, \dots, \sigma_M^2 \}$, is the covariance matrix.

- ✦ $\boldsymbol{\Sigma}$ is a **diagonal matrix**, as **PCA** is used to pre-process the features and different feature elements are independent.

- ✦ All classes are assumed to have the same variance.

Discriminant Gain Model

◆ Discriminant Gain

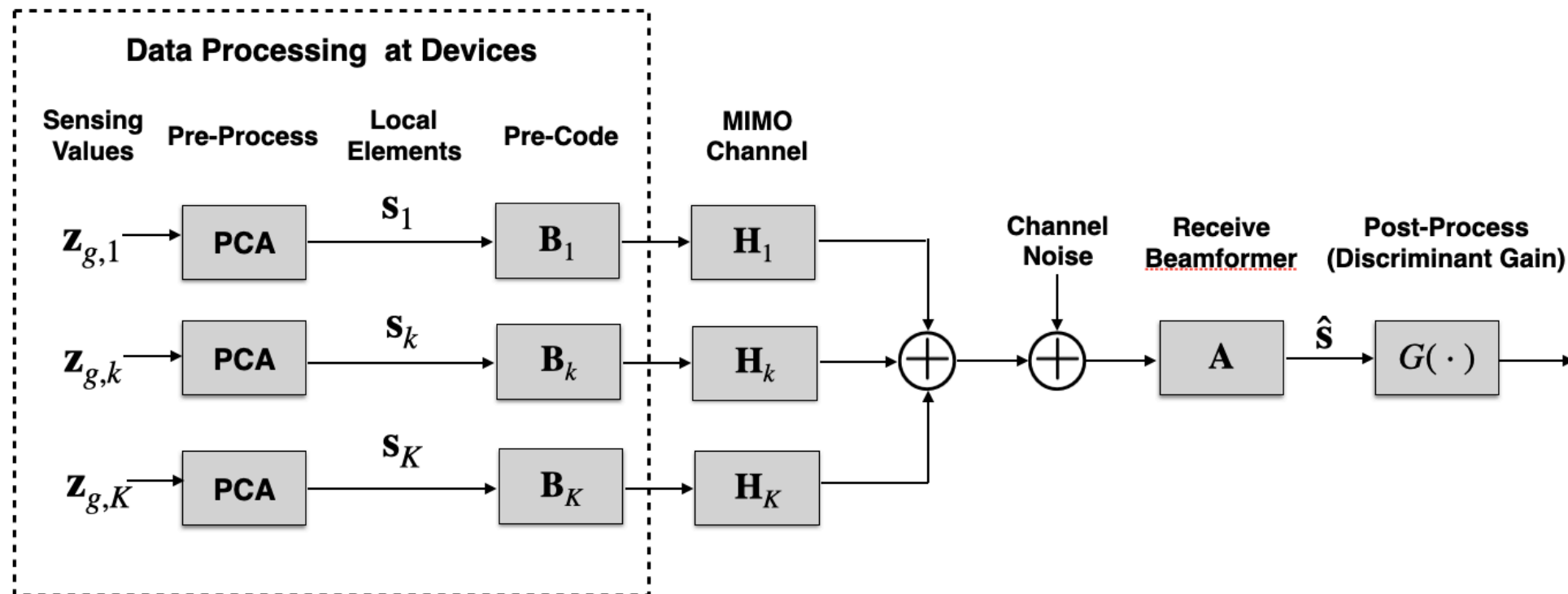
♣ Consider the m —th feature dimension, its discriminant gain is

$$G(\mathbf{x}_m) = \frac{2}{L(L-1)} \sum_{\ell'=1}^L \sum_{\ell < \ell'} \frac{(\mu_{\ell,m} - \mu_{\ell',m})^2}{\sigma_m^2},$$

♣ The total discriminant gain can be written as

$$G(\mathbf{x}) = \sum_{m=1}^M G(\mathbf{x}_m).$$

AirComp Model



◆ Transmit Symbols $\{\mathbf{s}_k\}$

$$\mathbf{s}_k = [s_{k,1}, s_{k,2}, \dots, s_{k,M}]^T,$$

where

$$s_{k,i} = x_{k,(2i-1)} + jx_{k,2i}, \quad 1 \leq i \leq N_t.$$

♣ Note: The order of the elements can be re-scheduled.

AirComp Model

◆ At the server, the received signal can be written as

$$\hat{\mathbf{s}} = \mathbf{A} \sum_{k=1}^K \mathbf{H}_k \mathbf{B}_k \mathbf{s}_k + \mathbf{n},$$

- ❖ $\mathbf{s}_k \in \mathbb{C}^{N_t}$ is the transmit symbol,
- ❖ $\mathbf{B}_k \in \mathbb{C}^{N_t \times N_t}$ is the pre-coding matrix of sensor k ,
- ❖ $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix of sensor k ,
- ❖ $\mathbf{A} \in \mathbb{C}^{N_t \times N_r}$ is the receive beam-forming matrix at the receiver,
- ❖ $\mathbf{n} \in \mathbb{C}^{N_r}$ is the noise with the following distribution,

$$\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \delta_0^2 \mathbf{I}),$$

- ❖ δ_0^2 is the noise variance.

Problem Formulation

- ◆ Received Feature Vector

$$\hat{\mathbf{x}} = [\text{Re}(\hat{\mathbf{s}})^T, \text{Im}(\hat{\mathbf{s}})^T]^T.$$

- ◆ Objective: Maximize the Receive Signal's Discriminant Gain

$$\max_{\mathbf{A}, \{\mathbf{B}_k\}} G(\hat{\mathbf{x}})$$

- ◆ Power Constraint of Each Sensor

$$\text{tr}(\mathbf{B}_k \mathbf{B}_k^H) \leq P_k,$$

- ♣ P_k is the maximum transmit power of sensor k .

Objective Simplification

◆ Zero-Forcing (ZF) Pre-Coding

$$\mathbf{A}\mathbf{H}_k\mathbf{B}_k\mathbf{x}_k = \mathbf{C}_k,$$

$$\clubsuit \mathbf{C}_k = \text{diag}\{c_{k,1}, c_{k,2}, \dots, c_{k,N_t}\},$$

$$\clubsuit c_{k,i} \in \mathbb{R} \text{ is non-negative.}$$

★ Based on ZF precodes, simplify the objective, i.e., discriminant gain.

★ Note: One symbol for two elements may be too difficult. Could revise the assumption that using one symbol for transmitting one element.