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Multiuser OFDM with Adaptive Sub-carrier, Bit, and Power Allocation

Cheong Yui Wong, Roger S. Cheng, Khaled Ben Letaief, and Ross D. Murch

Abstract— **Multiuser Orthogonal Frequency Division Multiplexing (OFDM) with adaptive multiuser sub-carrier allocation and adaptive modulation is considered. Assuming knowledge of the instantaneous channel gains for all users, we propose a multiuser OFDM sub-carrier, bit, and power allocation algorithm to minimize the total transmit power. This is done by assigning, to each user, a set of sub-carriers, and by determining, for each sub-carrier, the number of bits and the transmit power level. We obtain the performance of our proposed algorithm in a multiuser frequency selective fading environment for various time delay spread values and various numbers of users. The results show that our proposed algorithm outperforms multiuser OFDM systems with static TDMA or FDMA techniques which employ fixed and pre-determined time-slot or sub-carrier allocation schemes. We have also quantified the improvement in terms of the overall required transmit power, the bit error rate, or the area of coverage for a given outage probability.**

Keywords— **OFDM, Adaptive Modulation, Multiaccess Communication, Multiuser Channel, Frequency Selective Fading Channel, Resource Management**

I. Introduction

RECENTLY , intense interest has focused on modulation techniques which can provide broadband transmission over wireless channels for applications including wireless multimedia, wireless Internet access and future generation mobile communication systems. One of the main requirements on the modulation technique is the ability to combat inter-symbol interference (ISI) , a major problem in wideband transmission over multi-path fading channels There are lots of methods proposed to combat the ISI e.g. [1], [2], [3]. Multicarrier modulation techniques including Orthogonal Frequency Division Multiplex (OFDM) (e.g., [4]) are among the more promising solutions to this problem.

Assuming that the transmitter knows the instantaneous channel transfer functions of all users, many papers [5], [6], [7] have demonstrated that significant performance improvement can be achieved if adaptive modulation is used with OFDM. In particular, sub-carriers with large channel gains employ higher order modulation to carry more bits per OFDM symbol while sub-carriers in deep fade carry one or even zero bit per symbol. Integrated design of forward error correcting code and adaptive modulation has also been studied using BCH code and trellis coded modulation (TCM) in [8] and [9], respectively. Although both references considered only time-varying flat fading channels, the same coded adaptive modulation design can be easily applied to OFDM systems. As different sub-carriers experience different fades and transmit different numbers of bits, the transmit power levels must be changed accordingly. The problem of optimal power allocation has also been studied in [10].

In this paper, we consider extending OFDM with adaptive modulation to multiuser frequency selective fading environments. When OFDM with adaptive modulation is applied in a frequency selective fading channel, a significant portion of the sub-carriers may not be used. These are typically sub-carriers which experience deep fade and are not power efficient to carry any information bit. In multiuser systems using static TDMA or FDMA as multiaccess schemes, each user is allocated a pre-determined time-slot or frequency band to apply OFDM with adaptive modulation. Consequently, these unused sub-carriers (as a result of adaptive modulation) within the allocated time-slot or frequency band of a user are wasted and are not used by other users. However, the sub-carriers which appear in deep fade to one user may not be in deep fade for other users. In fact, it is quite unlikely that a sub-carrier will be in deep fade for all users as the fading parameters for different users are mutually independent. This motivates us to consider an adaptive multiuser sub-carrier allocation scheme where the sub-carriers are assigned to the users based on instantaneous channel information. This approach will allow all the sub-carriers to be used more effectively because a sub-carrier will be left unused only if it appears to be in deep fade to all users.

We consider a multiuser sub-carrier, bit, and power allocation where all users transmit in all the time slots. Our objective is to minimize the overall transmit power by allocating the sub-carriers to the users and by determining the number of bits and the power level transmitted on each sub-carrier based on the instantaneous fading characteristics of *all* users. In this paper, we formulate the multiuser sub-carrier, bit, and power allocation problem and propose an iterative algorithm to perform the multiuser sub-carrier allocation. Once the sub-carrier allocation is determined, the bit and power allocation algorithm can be applied to each user on its allocated sub-carriers. We also compare the performance of our proposed solution to various other static sub-carrier allocation schemes.

The results of the work can be applied, for instance, to the downlink transmission in a time division duplex (TDD) wireless communication system to improve the downlink capacity. In such a system, the base station (BS) can es-

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The authors are with the Department of Electrical and Electronic Engineering, The Hong Kong University of Science and Technology, Hong Kong. (e-mail: eeyui@ust.hk; eecheng@ust.hk; eekhaled@ust.hk; eermurch@ust.hk).

timate the instantaneous channel characteristics of all the BS-to-mobile links based on the received uplink transmissions. The multiuser sub-carrier, bit, and power allocation can then be used. It is clear that there is a certain amount of transmission overhead as the base station has to inform the mobiles about their allocated sub-carriers and the number of bits assigned to each sub-carrier.[1] However, this overhead can be relatively small, especially if the channels vary slowly (e.g., in an indoor low mobility environment) and the assignment is done once every many OFDM symbols. To further reduce the overhead, we can assign a contiguous band of sub-carriers with similar fading characteristics as a group instead of assigning each individual sub-carrier. In this paper, we will not focus on how the sub-carrier allocation information is transmitted. Instead, we will focus on how and by how much this new strategy can reduce the required transmit power; or how and by how much this new scheme can improve the bit error rate (BER) for a fixed transmit power. Alternately, we also consider how and by how much this new scheme can increase the area of coverage for a given transmit power and target BER.

While the bit allocation algorithm can be viewed as a practical implementation of the water-pouring interpretation for achieving the Shannon capacity of an ISI channel [13], the multiuser sub-carrier and bit allocation algorithm presented in this paper is the counterpart of the multiuser water-pouring solution given in [14]. In information theoretic studies, the usual approach is to maximize the capacity (or information rate) under the power constraint. In this study, we focus on deriving practical algorithms that can support real-time multimedia data whose bit rates are generally fixed by the compression algorithms. Hence, we assume a given set of users' data rates and attempt to minimize the total transmit power under a fixed performance requirement.

The organization of this paper is as follows. In Section II, we will first give the system model and formulate the minimum overall transmit power problem. The optimization problem seeks to minimize the overall transmit power using combined sub-carrier, bit, and power allocation schemes for multiuser OFDM systems. The bit and power allocation algorithm for a single-user system is studied in Section III. In Section IV, we derive a lower bound to the minimum overall transmit power by relaxing some of the constraints in the original problem. We also derive a sub-optimal sub-carrier allocation algorithm. In Section V, we compare the performance between our proposed method and other static approaches via Monte Carlo simulations. Finally, we conclude in Section VI.

[1]Note that the power level used does not need to be transmitted to the receiver in such a TDD system. As the sub-carrier gain is known to the transmitter, it can adjust the transmit power level to achieve a pre-determined receiver power level based on the number of bits allocated to that sub-carrier. However, in FDD systems, the transmit power levels determined by the receiver have to be sent back to the transmitter. In such systems, the additional performance gain achieved by power allocation may not justify the cost of sending the transmit power level information to the transmitter.

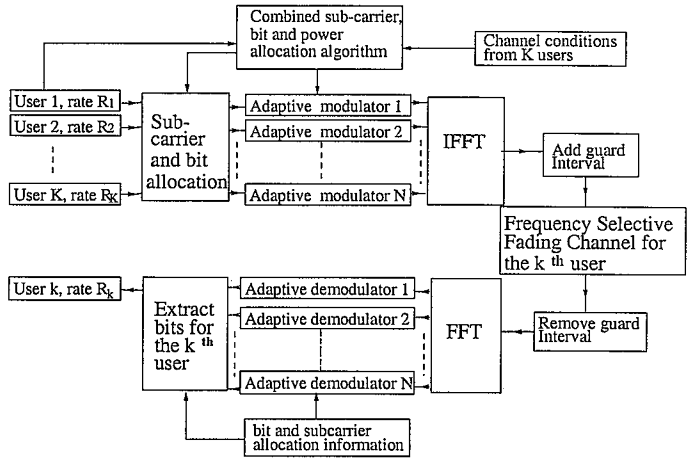

Fig. 1. Block diagram of a multiuser OFDM system with sub-carrier, bit and power allocation.

II. System Model

The configuration of our multiuser adaptive OFDM system is shown in Fig. 1. We assume that the system has K users and the k^{th} user has a data rate equal to R_k bit per OFDM symbol. In the transmitter, the serial data from the K users are fed into the sub-carrier and bit allocation block which allocates bits from different users to different sub-carriers. We assume that each sub-carrier has a bandwidth that is much smaller than the coherence bandwidth of the channel and the instantaneous channel gains on all the sub-carriers of all the users are known to the transmitter. Using the channel information, the transmitter applies the combined sub-carrier, bit, and power allocation algorithm to assign different sub-carriers to different users and the number of bits per OFDM symbol to be transmitted on each sub-carrier. Depending on the number of bits assigned to a sub-carrier, the adaptive modulator will use a corresponding modulation scheme and the transmit power level will be adjusted according to the combined sub-carrier, bit, and power allocation algorithm. We define $c_{k,n}$ to be the number of bits of the k^{th} user that are assigned to the n^{th} sub-carrier. As we do not allow more than one user to share a sub-carrier, it follows that for each n, if $c_{k',n} \neq 0$, $c_{k,n} = 0$ for all $k \neq k'$. We also assume that the adaptive modulator allows $c_{k,n}$ to take values in the set $\mathbf{D} = \{0, 1, 2, \ldots, M\}$ where M is the maximum number of information bits per OFDM symbol that can be transmitted by each sub-carrier.

The complex symbols at the output of the modulators are transformed into the time domain samples by inverse fast Fourier transform (IFFT). Cyclic extension of the time domain samples, known as the guard interval, is then added to ensure orthogonality between the sub-carriers, provided that the maximum time dispersion is less than the guard interval. The transmit signal is then passed through different frequency selective fading channels to different users.

We assume that the sub-carrier and bit allocation information is sent to the receivers via a separate control channel. At the receiver, the guard interval is removed to eliminate the ISI and the time samples of the k^{th} user are transformed by the FFT block into modulated symbols. The bit allocation information is used to configure the demodulators while the sub-carrier allocation information is used to extract the demodulated bits from the sub-carriers assigned to the k^{th} user.

In the frequency selective fading channel, different sub-carriers will experience different channel gains. We denote, by $\alpha_{k,n}$, the magnitude of the channel gain (assuming coherent reception) of the n^{th} sub-carrier as seen by the k^{th} user. We assume that the single-sided noise power spectral density (PSD) level, N_0, is equal to unity (i.e., $N_0 = 1$) for all sub-carriers and is the same for all users. Furthermore, we denote, by $f_k(c)$, the required received power (in energy per symbol) in a sub-carrier for reliable reception of c information bits per symbol when the channel gain is equal to unity. Note that the function $f_k(c)$ depends on k and this allows different users to have different quality of service (QOS) requirements and/or different coding and modulation schemes. In order to maintain the required QOS at the receiver, the transmit power, allocated to the n^{th} sub-carrier by the k^{th} user must equal to

$$P_{k,n} = \frac{f_k(c_{k,n})}{\alpha^2_{k,n}}. \tag{1}$$

Using these transmit power levels, the receiver can demodulate the modulated symbols at the output of the FFT processor and achieve the desired QOSs of all users.

The goal of the combined sub-carrier, bit, and power allocation algorithm is then to find the best assignment of $c_{k,n}$ so that the overall transmit power, the sum of $P_{k,n}$ over all sub-carriers and all users, is minimized for given transmission rates of the users and given QOS requirements specified through $f_k()$, $k = 1, \ldots, K$. In order to make the problem tractable, we further require that $f_k(c)$ is a convex and increasing function with $f_k(0) = 0$. This condition essentially means that no power is needed when no bit is transmitted and that the required additional power to transmit an additional bit increases with c (i.e., $f_k(c+1) - f_k(c)$ is increasing in c). Almost all popular coding and modulation schemes satisfy this condition.

It is important to note that even though the problem is formulated to minimize the overall transmit power for given QOS requirements, the same solution can be applied to improve the QOSs of the users for a given overall transmit power. The latter can simply be achieved by increasing the power proportionally for all the sub-carriers, while using the same set of $c_{k,n}$.

Mathematically, we can formulate the problem as

$$P_T^* = \min_{c_{k,n} \in \mathbf{D}} \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{1}{\alpha^2_{k,n}} f_k(c_{k,n}) \tag{2}$$

and the minimization is subjected to the constraints

$$\textbf{C1:} \text{ For all } k \in \{1, \ldots, K\},\ R_k = \sum_{n=1}^{N} c_{k,n}; \tag{3}$$

and

$$\textbf{C2:} \text{ For all } n \in \{1, \ldots, N\}, \text{ if there exists } k' \text{ with } c_{k',n} \neq 0, \text{ then } c_{k,n} = 0, \forall k \neq k'. \tag{4}$$

Note that constraint (3) is the data rate requirement and constraint (4) ensures that each sub-carrier can only be used by one user. Moreover, $\mathbf{D} = \{0, 1, 2, \ldots, M\}$ is the set of all possible values for $c_{k,n}$ and $c_{k,n} = 0$ means that the k^{th} user does not use the n^{th} sub-carrier to transmit any information.

III. Bit Allocation Algorithm for Single User Channel

Before we try to solve the multiuser allocation problem, we will first derive the bit allocation algorithm for the single-user environment. The single-user problem not only gives better understanding of the issues involved, but also provides a bit allocation algorithm that we will use in our multiuser solution.

We can rewrite the optimization problem in (2) for the single-user case as

$$P_T^* = \min_{c_n \in \mathbf{D}} \sum_{n=1}^{N} \frac{1}{\alpha^2_n} f(c_n) \tag{5}$$

and the minimization is under the constraint

$$R = \sum_{n=1}^{N} c_n. \tag{6}$$

Note that we have dropped the subscript k which denotes the user in all notations.

As the power needed to transmit a certain number of bits in a sub-carrier is independent of the numbers of bits allocated to other sub-carriers, it turns out that a greedy approach is optimal. A greedy algorithm assigns bits to the sub-carriers one bit at a time and in each assignment, the sub-carrier that requires the least additional power is selected. The bit allocation process will be completed when all R bits are assigned. Several papers (e.g., [15], [16]) have provided various algorithms for this problem and the basic structure of most algorithms are similar and can be described as follows:

Initialization:

For all n, let $c_n = 0$ and $\Delta P_n = [f(1) - f(0)] / \alpha_n^2$;

Bit Assignment Iterations:

Repeat the following R times:

$\hat{n} = \arg\min_n \Delta P_n$;

$c_{\hat{n}} = c_{\hat{n}} + 1$;

$\Delta P_{\hat{n}} = [f(c_{\hat{n}} + 1) - f(c_{\hat{n}})] / \alpha_{\hat{n}}^2$;

End;

Finish:

$\{c_n\}_{n=1}^N$ is the final bit allocation solution.

The initialization stage computes, for each sub-carrier, the additional power needed to transmit an additional bit. For each bit assignment iteration, the sub-carrier that needs the minimum additional power is assigned one more bit and the new additional power for that sub-carrier is updated. After R iterations, the final bit assignment gives the optimal bit allocation for each sub-carrier. It is important to note that the bit allocation is optimal only for the given function, $f(c)$, which depends on the selected modulation scheme. Different modulation schemes will lead to different $f(c)$, different bit allocation, and possibly lower transmit power, P_T^*.

The concept of this algorithm is fairly simple and many similar algorithms based on the same principle has been obtained before. In particular, there exist faster and less complex algorithms which can speed up the bit allocation process significantly (e.g., [15], [16]). In our simulations, we use the algorithm given in [16].

IV. Multiuser Sub-carrier and Bit Allocation

We have observed that, in the single-user case, a greedy approach which assigns one bit at a time to the sub-carrier that requires the least additional power gives the optimal allocation in the sense of minimizing the overall transmit power. Unfortunately, the problem becomes more difficult in the multiuser environment. As users cannot share the same sub-carrier, allocating bits to a sub-carrier essentially prevents other users from using that sub-carrier. This dependency makes any greedy algorithm a non-optimal solution. It turns out that the optimal solution may not assign any of a user's bits to the best sub-carrier seen by that user. This may happen when the best sub-carrier of a user is also the best sub-carrier of another user who happens to have no other good sub-carriers. Hence, the multiuser sub-carrier and bit allocation problem is much more complicated to solve than that of the single-user case.

It turns out that the optimization problem in (2) is a combinatorial optimization problem. To make the problem tractable, we consider a different but similar optimization problem. We relax the requirement $c_{k,n} \in \mathbf{D}$ to allow $c_{k,n}$ to be a real number within the interval $[0, M]$. Moreover, in order to deal with constraint (4), K variables, $\rho_{k,n}, k = 1, \ldots, K$, with values within the interval $[0,1]$, are introduced to the cost function as sharing factors of the n^{th} sub-carrier. The new optimization problem becomes

$$\underline{P}_T = \min_{\substack{c_{k,n} \in [0,M] \\ \rho_{k,n} \in [0,1]}} \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\rho_{k,n}}{\alpha_{k,n}^2} f_k(c_{k,n}) \tag{7}$$

where $c_{k,n}$ and $\rho_{k,n}$ have to satisfy

$$R_k = \sum_{n=1}^{N} \rho_{k,n} c_{k,n} \qquad \text{for all } k \in \{1, \ldots, K\}, \tag{8}$$

and

$$1 = \sum_{k=1}^{K} \rho_{k,n} \qquad \text{for all } n \in \{1, \ldots, N\}. \tag{9}$$

For any valid set of $c_{k,n} \in \mathbf{D}$ satisfying the constraints (3) and (4) in the original optimization problem, we can let

$$\rho_{k,n} = \begin{cases} 1 & \text{if } c_{k,n} \neq 0, \\ 0 & \text{if } c_{k,n} = 0. \end{cases} \tag{10}$$

Then, it is easy to show that the same set of $c_{k,n}$ and the corresponding $\rho_{k,n}$ defined in (10) satisfy the constraints (8) and (9) in the new optimization problem. Moreover, with $\rho_{k,n}$ defined in (10), the new cost function in (7) is equal to the cost function in (2). Hence, the minimization problem in (7) is the same as the original optimization problem, except that the minimization is done over a larger set. Consequently, the minimum power obtained in (7), $\underline{P}_T$, is a lower bound to the minimum power obtained in (2), P_T^*.

Another way to interpret the optimization in (7) is to consider $\rho_{k,n}$ as the time-sharing factor for the k^{th} user of the n^{th} sub-carrier. For example, in every L OFDM symbols (L being a very large number), user k uses the n^{th} sub-carrier in $L\rho_{k,n}$ symbols. Clearly, the *average* (over L symbols) information data rate and the *average* transmit power has to be scaled by the same factor $\rho_{k,n}$. Hence, we can consider (7) as the optimization problem when the users are allowed to time-share each sub-carrier over a large number of OFDM symbols. However, most wireless communication channels are time-varying and the channels may not stay unchanged long enough for time-sharing to be feasible. Hence, in this paper, we will continue to consider the original problem in (2) and use the optimization problem in (7) as a lower bound, even though it has its own physical interpretation.

The modified optimization problem in (7) is more tractable. However, even though the function $f_k(c)$ is convex in c, the terms in the cost function have the form $\rho f_k(c)$ and as a function of (ρ, c), $\rho f_k(c)$ is not convex in (ρ, c). To proceed further, we let $r_{k,n} = c_{k,n}\rho_{k,n}$ and rewrite the cost function in terms of $r_{k,n}$ and $\rho_{k,n}$. The constraint on $r_{k,n}$ becomes $r_{k,n} \in [0, M\rho_{k,n}]$ and it can be easily shown that $\rho f_k(c) = \rho f_k(r/\rho)$ is convex in (ρ, r) within the triangular region specified by $\rho \in [0,1]$ and $r \in [0, M\rho]$. In particular, the Hessian evaluated at any point within this region is a positive semidefinite matrix. Hence, we can reformulate the optimization problem in (7) as a convex minimization problem over a convex set. That is,

$$\underline{P}_T = \min_{\substack{r_{k,n} \in [0, M\rho_{k,n}] \\ \rho_{k,n} \in [0,1]}} \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\rho_{k,n}}{\alpha_{k,n}^2} f_k\left(\frac{r_{k,n}}{\rho_{k,n}}\right) \tag{11}$$

where $r_{k,n}$ and $\rho_{k,n}$ have to satisfy

$$R_k = \sum_{n=1}^{N} r_{k,n} \qquad \text{for all } k \in \{1, \ldots, K\}, \tag{12}$$

and

$$1 = \sum_{k=1}^{K} \rho_{k,n} \qquad \text{for all } n \in \{1, \ldots, N\}. \tag{13}$$

Using standard optimization techniques in [17], we obtain the Lagrangian,

$$\begin{aligned} L &= \sum_{n=1}^{N}\sum_{k=1}^{K}\frac{\rho_{k,n}}{\alpha_{k,n}^2}f_k\left(\frac{r_{k,n}}{\rho_{k,n}}\right) \\ &\quad -\sum_{k=1}^{K}\lambda_k\left(\sum_{n=1}^{N}r_{k,n}-R_k\right) \\ &\quad -\sum_{n=1}^{N}\beta_n\left(\sum_{k=1}^{K}\rho_{k,n}-1\right) \end{aligned} \tag{14}$$

where λ_k and β_n are the Lagrangian multipliers for the constraints (12) and (13), respectively.

After differentiating L with respect to $r_{k,n}$ and $\rho_{k,n}$, respectively, we obtain the necessary conditions for the optimal solution, $r^*_{k,n}$ and $\rho^*_{k,n}$. Specifically, if $\rho^*_{k,n} \neq 0$, we have

$$\begin{aligned} &\left.\frac{\partial L}{\partial r_{k,n}}\right|_{(r_{k,n},\rho_{k,n})=(r^*_{k,n},\rho^*_{k,n})} \\ &= \frac{1}{\alpha_{k,n}^2}f'_k\left(\frac{r^*_{k,n}}{\rho^*_{k,n}}\right)-\lambda_k \\ &\begin{cases} >0 & \text{if } r^*_{k,n}=0; \\ =0 & \text{if } r^*_{k,n}\in(0,M\rho^*_{k,n}); \\ <0 & \text{if } r^*_{k,n}=M\rho^*_{k,n}. \end{cases} \end{aligned} \tag{15}$$

and

$$\begin{aligned} &\left.\frac{\partial L}{\partial \rho_{k,n}}\right|_{(r_{k,n},\rho_{k,n})=(r^*_{k,n},\rho^*_{k,n})} \\ &= \frac{1}{\alpha_{k,n}^2}\left[f_k\left(\frac{r^*_{k,n}}{\rho^*_{k,n}}\right)-f'_k\left(\frac{r^*_{k,n}}{\rho^*_{k,n}}\right)\frac{r^*_{k,n}}{\rho^*_{k,n}}\right]-\beta_n \\ &\begin{cases} =0 & \text{if } \rho^*_{k,n}\in(0,1); \\ <0 & \text{if } \rho^*_{k,n}=1. \end{cases} \end{aligned} \tag{16}$$

On the other hand, if $\rho^*_{k,n}=0$, then $r^*_{k,n}=0$ and we have

$$\begin{aligned} &r_{k,n}\frac{\partial L}{\partial r_{k,n}}+\rho_{k,n}\frac{\partial L}{\partial \rho_{k,n}} \geq 0 \\ &\text{for all } \rho_{k,n}\in(0,1] \text{ and } r_{k,n}\in(0,M\rho_{k,n}]. \end{aligned} \tag{17}$$

These necessary conditions can be interpreted by the fact that if the minimum occurs within the constrained region ($(0,1)$ for $\rho_{k,n}$ and $(0, M\rho_{k,n})$ for $r_{k,n}$), then the derivative evaluated at the minimum point must be zero. On the other hand, if the optimal solution occurs at a boundary point, then the derivative must be positive along all directions pointing towards the interior of the constraint set. Equation (17) follows from considering the boundary point at $(r^*_{k,n},\rho^*_{k,n})=(0,0)$.

From (15) and (17), we can conclude that

$$r^*_{k,n}=\rho^*_{k,n}f_k'^{-1}(\lambda_{q,k}\alpha_{k,n}^2) \tag{18}$$

where

$$\lambda_{q,k}=\begin{cases} f'_k(0)/\alpha_{k,n}^2 & \text{if } f_k'^{-1}(\lambda_k\alpha_{k,n}^2)<0; \\ \lambda_k & \text{if } 0\leq f_k'^{-1}(\lambda_k\alpha_{k,n}^2)\leq M; \\ f'_k(M)/\alpha_{k,n}^2 & \text{if } f_k'^{-1}(\lambda_k\alpha_{k,n}^2)>M. \end{cases}$$

Moreover, from (16) and (17), it follows that

$$\rho^*_{k,n}=\begin{cases} 0 & \text{if } \beta_n<H_{k,n}(\lambda_{q,k}); \\ 1 & \text{if } \beta_n>H_{k,n}(\lambda_{q,k}). \end{cases} \tag{19}$$

where

$$H_{k,n}(\lambda)=\frac{1}{\alpha_{k,n}^2}[f_k(f_k'^{-1}(\lambda\alpha_{k,n}^2))-\lambda\alpha_{k,n}^2f_k'^{-1}(\lambda\alpha_{k,n}^2)]. \tag{20}$$

Since constraint (13) must be satisfied, we find from (19) that for each n, if $H_{k,n}(\lambda_{q,k})$ for $k=1,\ldots,K$ are all different, then only the user with the smallest $H_{k,n}(\lambda_{q,k})$ can use that sub-carrier. In other words, for the n^{th} sub-carrier, if $H_{k,n}(\lambda_{q,k})$ are different for all k, then

$$\rho^*_{k',n}=1,\ \rho^*_{k,n}=0, \qquad \text{for all } k\neq k' \tag{21}$$

where

$$k'=\arg\min_k H_{k,n}(\lambda_{q,k}). \tag{22}$$

Hence, it follows that, for a fixed set of Lagrange multipliers, λ_k, $k=1,\ldots,K$, we can use them to determine k' for each n using (22). The $r^*_{k,n}$ and $\rho^*_{k,n}$ obtained will then form an optimal solution for the optimization problem; however, the individual rate constraint (12) may not be satisfied.

In order to find the set of λ_k such that the individual rate constraints are satisfied, we have obtained an iterative searching algorithm. Starting with some small values for all λ_k, this iterative procedure increases one of the λ_k until data rate constraint (12) for user k is satisfied. Then, we switch to another user and go through the users one at a time. This process repeats for all users until the data rate constraint for all users are satisfied. This algorithm converges because for a given k, as λ_k increases, $H_{k,n}(\lambda_{q,k})$ for all n decreases and more $\rho^*_{k,n}$ in (19) become 1 while $r^*_{k,n}$ in (18) increases for those n where $\rho^*_{k,n}>0$. Hence, $\sum_{n=1}^{N} r^*_{k,n}$ increases. During this process, some of the other $\rho^*_{k',n}$ may change from 1 to 0 and consequently decrease the total data rate for other users. However, as all the λ_k increase, $r^*_{k,n}$ increases accordingly. As long as the total data rate is less than MN bits per symbol which is the total number of bits possibly transmitted within an OFDM symbol, the algorithm will converge to a solution that satisfies all the constraints. Since the optimization problem is a convex optimization problem over a convex set, the set of necessary conditions is also sufficient and the solution that satisfies all the necessary conditions is the unique optimal solution.

In the process of adjusting λ_k for $k=1,\ldots,K$, the situation where, for a fixed n, more than one $H_{k,n}(\lambda_{q,k})$ has the same values cannot be ignored. In that case, $\rho^*_{k,n}$ has to take values within the interval $(0,1)$. This solution suggests that the sub-carrier should be shared by multiple users. In practice, this can be done by having these users with $\rho^*_{k,n} > 0$ time share the n^{th} sub-carrier, and the ratio of the symbols used by different users are set proportionally to $\rho^*_{k,n}$. The detailed flow chart of the algorithm is given in the Appendix.

Now, we have an algorithm to obtain the optimal values of $\rho^*_{k,n}$ and

$$c^*_{k,n} = \begin{cases} r^*_{k,n}/\rho^*_{k,n} & \text{if } \rho^*_{k,n} \neq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

This solution when substituted in (7) gives a lower bound to the minimum overall transmit power. However, we cannot use these results immediately in (2). One problem is that $c^*_{k,n}$ may not be in $\mathbf{D}$ and the other is that some $\rho^*_{k,n}$ may be within $(0,1)$ indicating a time-sharing solution. Furthermore, by simply quantizing $c^*_{k,n}$ and $\rho^*_{k,n}$ will not satisfy the individual rate constraints in (3).

To solve this problem, we propose a *Multiuser Adaptive OFDM* (MAO) scheme where the sub-carrier allocation follows essentially the solution to the lower bound in (7) and then the single-user bit allocation algorithm given in Section III is applied to each user on the allocated sub-carriers. Specifically, we modify $\rho^*_{k,n}$ for the optimization problem in (7) by letting, for each n, $\rho^*_{k',n} = 1$ where $k' = \arg\max_k \rho^*_{k,n}$, and $\rho^*_{k,n} = 0$ for $k \neq k'$. Then, we apply the single-user bit allocation algorithm on each user using the assigned sub-carriers. We denote the total transmit power (in energy per symbol) obtained using this MAO scheme by P_T. It is easy to see that $\underline{P}_T \leq P^*_T \leq P_T$, where P^*_T is the minimum power in the original problem and $\underline{P}_T$ is the minimum power for the modified problem with the relaxed constraints. More specifically, the difference between P_T and the minimum $\underline{P}_T$ gives an upper bound to how far our MAO scheme is away from the solution of our original optimization problem.

V. Performance Comparison

In this section, we obtain and compare the performance of the MAO scheme with other static sub-carrier allocation schemes. We consider a system that employs M-ary Quadrature Amplitude Modulation (MQAM) with $\mathbf{D} = \{0, 2, 4, 6\}$. Square signal constellations (4-QAM, 16-QAM and 64-QAM) are used to carry 2, 4, or 6 bits per symbol. The bit error probability is upper bounded by the symbol error probability which is tightly approximated by $4Q\left[\sqrt{d^2/(2N_0)}\right]$ [12, p. 281], where d is the minimum distance between the points in the signal constellation. Since the average energy of a M-QAM symbol is equal to $(M-1)d^2/6$, it follows that the required power for supporting c bits per symbol at a given BER, P_e, is

$$f(c) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_e}{4}\right)\right]^2 (2^c - 1),$$

where we recall that

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt.$$

It is easy to see that $f(c)$ is convex and increasing in c and that $f(0) = 0$.

To evaluate the performance of our scheme, we have simulated 1,000 sets of 5-path frequency selective Rayleigh fading channels with an exponential power delay profile. Each set of channels consists of K independent channels, one for each user. We use an OFDM system with 128 sub-carriers over a 5MHz band along with a total (over all users) transmission rate equal to 512 bits/symbol (or equivalently, an average of 4 bits/sub-carrier). Recall that the single-sided power spectral density level, N_0, is equal to unity and we assume that the average sub-carrier channel gain, $\mathbf{E}|\alpha_{k,n}|^2$, is equal to unity for all k and n.

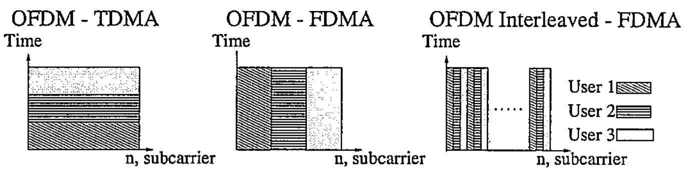

Fig. 2. Sub-carrier and time-slot allocations of OFDM-TDMA, OFDM-FDMA, and OFDM Interleaved-FDMA schemes.

For comparison purpose, we have also considered three other static multiuser sub-carrier allocation methods. Two of them are based on the multiple access methods described in [7]. These are:

OFDM-TDMA*:* Each user is assigned a pre-determined TDMA time slot and can use all the sub-carriers within that time slot exclusively.

OFDM-FDMA*:* Each user is assigned a pre-determined band of sub-carriers and can only use those sub-carriers exclusively in every OFDM symbol.

In a frequency selective fading channel, there is a high correlation between the channel gains of adjacent sub-carriers. In order to avoid the situation where all sub-carriers of a user are in deep fade, we propose an enhanced version of OFDM-FDMA, which we shall refer to as

OFDM Interleaved-FDMA*:* This is the same as OFDM-FDMA except that sub-carriers assigned to a user are interlaced with other users' sub-carriers in the frequency domain.

The time and sub-carrier assignment of these three multiuser OFDM schemes are illustrated in Fig. 2. Note that these static schemes have pre-determined sub-carrier allocations which are independent of the channel gains of the users. The main difference between the proposed MAO scheme and these static schemes is that MAO assigns sub-carriers adaptively based on the instantaneous channel gains. To ensure a fair comparison, we use the *Optimal single-user Bit Allocation* (OBA) for each user on the assigned sub-carriers. For comparison purpose, we also show the results when *Equal Bit Allocation* (EBA) is employed on the assigned sub-carriers for these three OFDM schemes. Notice that using EBA, all three schemes will have the same performance in an uncoded system. This is because the average bit SNR needed is a function of only the *marginal* probability density function of each sub-carrier gain.

Fig. 3 shows the average bit signal-to-noise ratio (SNR) needed to achieve a BER at $P_e = 10^{-4}$ for a 5-user system versus the *root mean square* (RMS) delay spread (for definition, see for example [18, p. 160]) for different multiuser OFDM schemes. The average required transmit power (in energy per bit) is defined as the ratio of the overall transmit energy per OFDM symbol (including all sub-carriers and all users) to the total number of bits transmitted per OFDM symbol. Moreover, we define the average bit SNR as the ratio of the average transmit power to the noise PSD level N_0. As we assume that the data rate is fixed and that N_0 is just a constant, the overall transmit power is proportional to the average bit SNR. For ease of comparison, we have used the average bit SNR for comparison. We find in Fig. 3 that the MAO scheme is never more than 0.6dB from the lower bound. Since the bit SNR of the optimal combined sub-carrier, bit, and power allocation algorithm must lie between the bit SNRs achieved by the lower bound and the MAO scheme, we find that the MAO scheme is never more than 0.6dB away from the optimal solution. On the other hand, we observe that our proposed MAO scheme is 3-5dB better than the static sub-carrier allocation schemes with OBA, which are in turn 5-10dB better than that with EBA. We also find that, when OBA is used, the OFDM Interleaved-FDMA scheme and the OFDM-TDMA scheme have very similar performance, and both of them outperform the OFDM-FDMA scheme.[2] A closer observation of Fig. 3 also indicates that the gains achieved by optimal bit allocation and optimal multiuser sub-carrier allocation increase with the RMS delay spread. This is mainly because the larger the RMS delay spread, the more the fading variation and hence higher gains can be obtained when the allocation is performed adaptively.

Fig. 4 shows the average bit SNR (in dB) needed to achieve the same BER versus the number of users when the RMS delay spread is 100ns. We find that the savings in the required bit SNR achieved by MAO when compared to other schemes are roughly the same, independent of the number of users in the system.

While these two figures show the improvement in the required bit SNR, the results can perhaps be more easily understood using the more familiar BER versus bit SNR curves. For each BER requirement, we compute $f(c)$ for all $c \in \mathbf{D}$, and then use our algorithm to calculate the sub-carrier allocation for the MAO case. For all other static sub-carrier allocation schemes, the allocations are independent of the BER. Once the sub-carrier allocation is fixed, we apply the optimal bit and power allocation algorithm to every user. The final average power per bit divided by the noise power spectral density level gives the average bit SNR. We repeat this procedure for different BER values

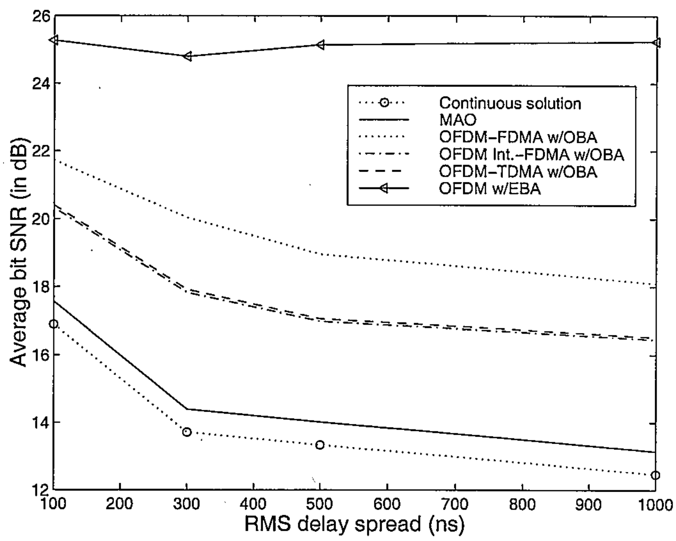

Fig. 3. Average bit SNR required by different schemes in various RMS delay spreads in a 5-user system with $P_e = 10^{-4}$.

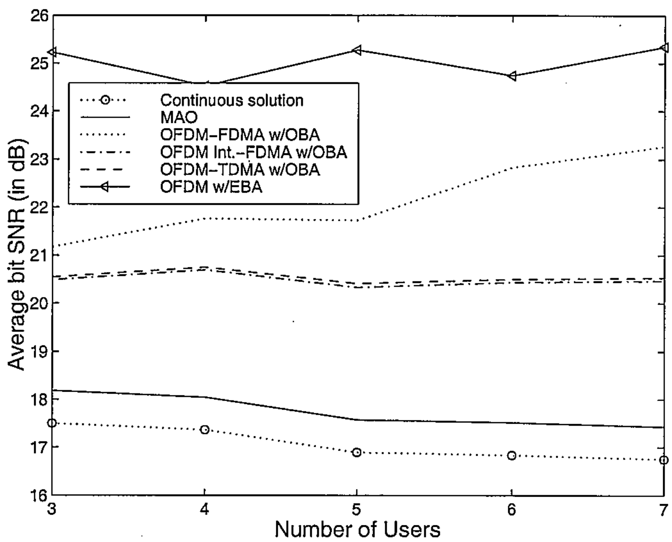

Fig. 4. Average bit SNR required by different schemes versus the number of users in a multiuser OFDM system with 100ns RMS delay spread and $P_e = 10^{-4}$.

and the results are plotted in Fig. 5 for a 5-user system with an RMS delay spread equal to 100ns. We find that our proposed MAO has at least 3-4dB advantage over all other schemes.

Another way to illustrate the impact of the bit and sub-carrier allocation is to consider the area of coverage for a given outage probability, assuming that the base station has a maximum transmit power. We consider a circular cell with 5 users, independently and uniformly distributed within the cell. A typical scenario is shown in Fig. 6 where the triangles represent the 5 users. In addition to frequency selective fading, path loss and log-normal shadowing are also included in simulating the actual channel gains seen by the users. Using these channel gains, subcarriers and

[2] OFDM-FDMA refers only to the specific FDMA scheme which assigns to each user a contiguous band of sub-carriers as shown in Fig. 2, but not the general FDMA schemes. In fact, both OFDM Interleaved-FDMA and MAO can be considered as different forms of FDMA and they are not outperformed by the OFDM-TDMA scheme.

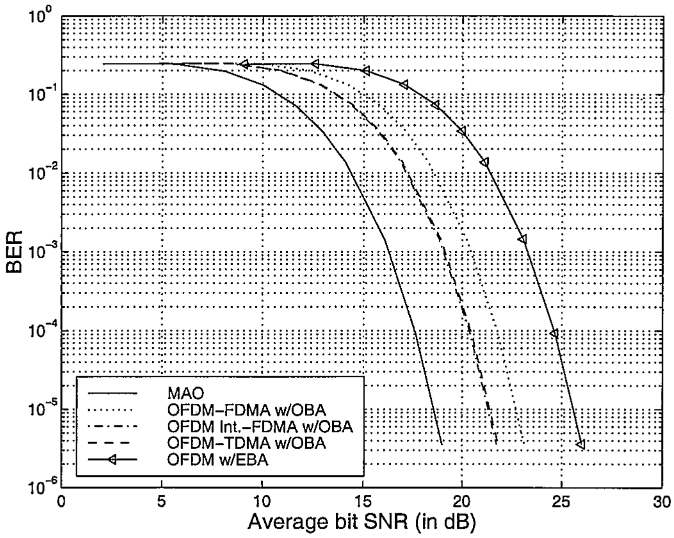

Fig. 5. BER versus average bit SNR for various sub-carrier allocation schemes.

bits assigned to each user are determined by the various multiple access schemes and the total required transmit power is calculated. If the total power for all five users exceeds the maximum power of the base station, the user requiring the largest transmit power (in this case, the black one) is dropped and counted as one outage event occurring at a distance equal to the distance between the base station and the dropped user. This process continues until the transmit power is smaller than the maximum power of the base station. In this example, the maximum transmit power is set to the transmit power required for all five users assuming that they are all located at the boundary of the cell, taking into account the path loss effect and a 17dB fading margin for shadowing.

The cumulative outage probabilities at various normalized distances, normalized to the cell radius, are plotted in Fig. 7. A cumulative outage probability of 5% at a normalized distance of 0.8 means that there is a 5% chance of outage for a mobile located more than $0.8R$ away from the base station where R is the radius of the cell. We observe that MAO outperforms others with a large reduction in the outage probability at all distances. Alternatively, if the same outage probability is maintained, say at 1%, the coverage are provided by MAO is 36% larger than the best of all other schemes.

VI. Conclusions

In this paper, we considered OFDM transmission in a multiuser environment and formulated the problem of minimizing the overall transmit power by adaptively assigning sub-carriers to the users along with the number of bits and power level to each sub-carrier. In particular, we derived a multiuser adaptive sub-carrier and bit allocation algorithm. Given the instantaneous channel information, the algorithm obtains a sub-optimal sub-carrier allocation and then single-user bit allocation is applied on the allocated sub-carriers. Using this scheme, the overall required transmit power can be reduced by about 5-10dB from the conventional OFDM without adaptive modulation. Likewise, the transmit power can be reduced by about 3-5dB from the conventional OFDM with adaptive modulation and adaptive bit allocation, but without adaptive sub-carrier allocation. The reduction in transmit power can also be translated to a significant reduction in the required bit SNR for a given BER. Moreover, the same improvement can also be translated to a reduction in the outage probability or to an increase in the area of coverage.

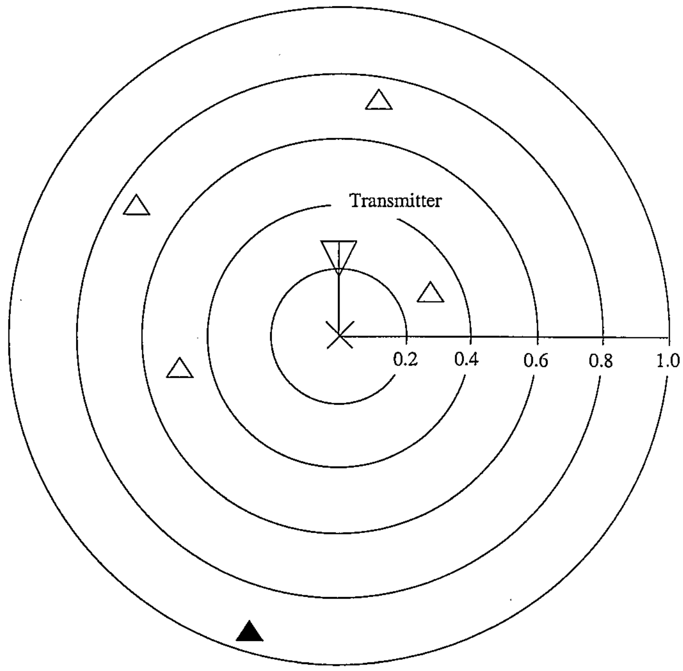

Fig. 6. Cell for analyzing the outage probability.

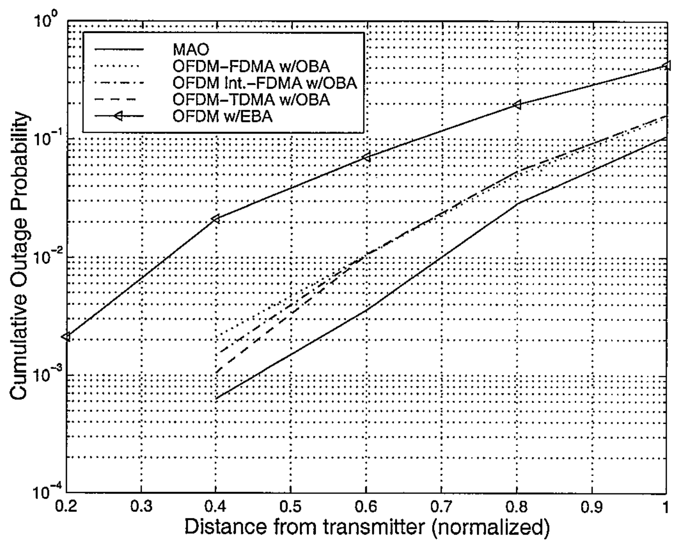

Fig. 7. Outage probability at 17dB fading margin.

The results in this paper assume perfect channel estimation and we have not considered issues related to imperfect implementation such as imperfect synchronization. As channel estimation in wireless fading channels is in general not very accurate, the effect of non-ideal channel information on the performance of our proposed MAO scheme is a very important issue. We have started looking at this issue and our preliminary results have indicated that the MAO scheme is not very sensitive to channel estimation errors. Nevertheless, detailed sensitivity studies will be needed before the algorithm can be applied to practical systems.

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Appendix

A flow chart providing the detailed description of the multiuser sub-carrier allocation algorithm is shown in Fig. 8.

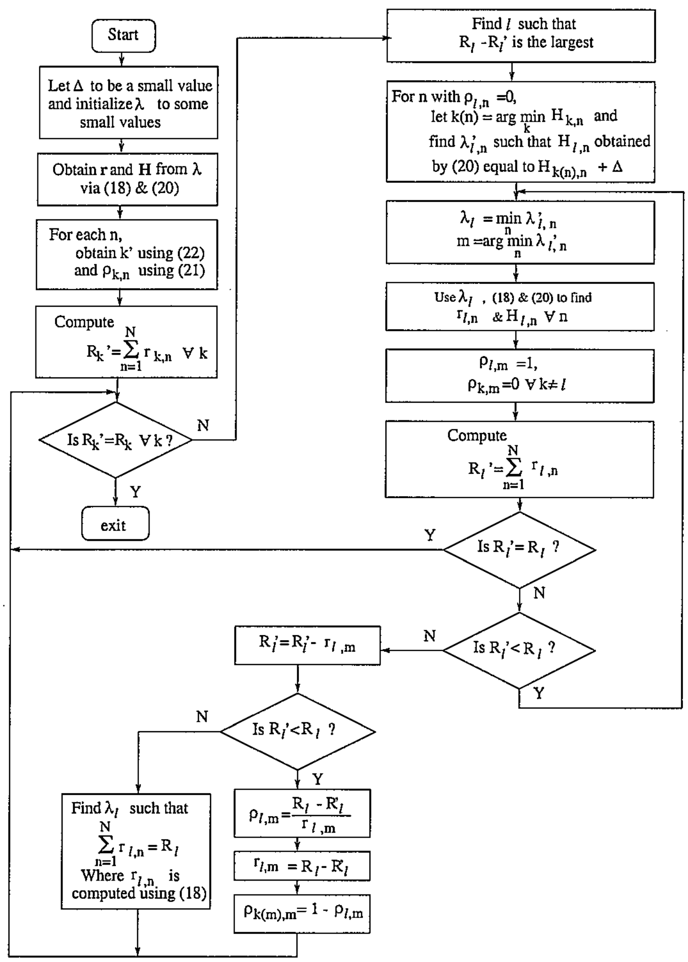

Fig. 8. Flow chart of the multiuser sub-carrier allocation algorithm.

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Cheong Yui Wong was born in Hong Kong, China in 1975. He received the B.Eng. degree in electronic engineering from the Hong Kong University of Science and Technology (HKUST), Clear Water Bay, Hong Kong in 1997. He is currently pursuing the M.Phil. degree at HKUST and expect to graduated in 1999.

During his master studies, he has joined a project that aim at achieving wireless multimedia communication using OFDM.

His research interests include adaptive modulation, OFDM and optimization.

Roger S. Cheng (S'86-M'92) received the B.S. degree from Drexel University, Philadelphia, PA, in 1987, and both the M.A. and Ph.D. degrees from Princeton University, Princeton, NJ, in 1988 and 1991, respectively, all in electrical engineering.

From 1987 to 1991, he was a Research Assistant in the Department of Electrical Engineering, Princeton University, Princeton, NJ. From 1991 to 1995, he was an Assistant Professor in the Electrical and Computer Engineering Department of University of Colorado at Boulder. In June 1995, he joined the Faculty of Hong Kong University of Science and Technology, where he is currently an Assistant Professor in the Department of Electrical and Electronic Engineering. He also held visiting positions with Qualcomm, San Diego, in the summer of 1995, and with Institute for Telecommunication Sciences, NTIA, Boulder, CO, in the summers of 1993 and 1994. His current research interests include wireless communications, multiuser communications, spread spectrum CDMA, digital implementation of communication systems, wireless multimedia communications, information theory, and coding.

Dr. Cheng is the recipient of the Meitec Junior Fellowship Award from the Meitec Corporation in Japan, the George Van Ness Lothrop Fellowship from the School of Engineering and Applied Science in Princeton University, and the Research Initiation Award from the National Science Foundation. He was a member of the Technical Program Committees of GLOBECOM98 and IEEE Wireless Communications and Networking Conference (WCNC'99). He has served as Guest Editor of the special issue on Multimedia Network Radios in the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, Associate Editor of the IEEE TRANSACTION ON SIGNAL PROCESSING, and membership chair for of the IEEE Information Theory Society.

Khaled Ben Letaief received the BS degree with distinction in Electrical Engineering from Purdue University at West Lafayette, Indiana, USA, in December 1984. He has also received the MS and Ph.D. Degrees in Electrical Engineering from Purdue University, in August 1986, and May 1990, respectively. Since January 1985 and as a Graduate Instructor in the School of Electrical Engineering at Purdue University, he has taught courses in communications and electronics and in 1990 he has been awarded at Purdue University the Mangoon Teaching Award. From 1990 to September 1993, he has been an academic staff at the Department of Electrical and Electronic Engineering at the University of Melbourne, Australia where he has been also a member of the Center for Sensor Signal and Information Systems. Since September 1993, Dr. Letaief has been with the Department of Electrical & Electronic Engineering at the Hong Kong University of Science & Technology (HKUST) where is currently an Associate Professor. From 1995 to 1998, he served as the Director of the Undergraduate Studies program in the Department and in Spring 1995, Fall 1996, and Fall 1997 he was awarded the Teaching Excellence Appreciation Award by the School of Engineering at HKUST. He is also the 1998 university recipient of the Michael G. Gale Medal for Distinguished Teaching (Highest University Teaching Award). His research interests include Wireless personal and mobile communications, Spread spectrum systems, Optical fiber networks, Multiuser detection, Wireless multimedia communications, and CDMA systems.

He is a Senior member of the IEEE and has been an active member of various professional societies and has published papers in several journals and conference proceedings.

Dr. Letaief is the Editor for Wireless Systems of the IEEE Transaction on Communications, a Technical Editor of the IEEE Communications Magazine, and an Editor of (Kluwer) Wireless Personal Communications Journal. He has been a guest Editor of the 1997 IEEE Journal on Selected Areas in Communications Special Issue on Computer-Aided Modeling, Analysis and Design of Communications Links. He is also a guest Editor of the 1999 (Kluwer) Wireless Personal Communications Journal Special Issue on Intelligent Multimedia Systems, Terminals, and Components. In addition, he served as the Technical Program Chair of the 1998 IEEE Mini-Conference on Communications Theory (CTMC'98) which was held in Sydney, Australia, November 1998.

Professor Letaief is currently serving as an Officer in the IEEE Communications Society Technical Committee on Personal Communications.

Ross D. Murch received his PhD degree in Electrical and Electronic Engineering from the University of Canterbury in 1990 for research involving electromagnetic inverse scattering. From 1990-92 he was a post doctorate fellow at the Department of Mathematics and Computer Science at Dundee University, Scotland where he worked on approximate approaches to electromagnetic scattering from three-dimensional objects.

Currently he is an Associate Professor in the Department of Electrical and Electronic Engineering at the Hong Kong University of Science and Technology. His most recent research activities center around the application of electromagnetics and signal processing to wireless communications. Research interests include: antenna design for mobile telephone handsets, smart antenna algorithms and propagation prediction for wireless communications. He is also the founding Director of the Center for Wireless Information Technology, which was begun in August 1997.

He is an editor of IEEE Journal on Selected Areas in Communications, Wireless Series and has been involved in the organisation of several international conferences. From August-December 1998 he was on sabbatical leave at Allgon Mobile Communications, Sweden (manufactures approximately one million handset antennas per week) and AT & T Research Labs, New Jersey, USA. He also acts as a consultant for various industrial projects on wireless communications and has two US patents related to wireless communication. He is a senior member of IEEE, a Chartered Engineer, a member of IEE, and also an URSI correspondent. In 1993 and 1996 he won an URSI young scientist and engineering teaching excellence appreciation awards respectively.