A Uniform-Forcing Transceiver Design for Over-the-Air Function Computation

Li Chen, Xiaowei Qin, and Guo Wei

***Abstract*—The future Internet-of-Things network is expected to connect billions of sensors, which incurs high latency for data aggregation. To overcome this challenge, a new technique called over-the-air function computation was recently developed to enable fusion center to receive a desired function directly. It utilizes the superposition property of wireless channel to realize the uniform summation of the desired function. In order to compensate the non-uniform fading of different sensors, we propose a novel uniform-forcing transceiver design for over-the-air function computation. A corresponding min-max optimization problem is formulated to minimize the distortion of the computation which is measured by mean squared error. Due to the non-convexity of the problem, it is relaxed to semidefinite programming first. Then, the performance of the initial solution is improved through successive convex approximation. Simulation results show that the proposed design is able to achieve significant performance gain with low complexity.**

***Index Terms*—Function computation, IoT, transceiver design.**

I. Introduction

The future *Internet-of-Things* (IoT) network is predicted to connect billions of sensors [1]. For example, the 5th generation cellular system will interconnect up to 1 trillion devices, and a million connections per square kilometer. It makes the traditional "aggregate-then-compute" way impractical as the multiple access scheme for this large-scale network would result in high latency [2]. To overcome this challenge, a promising technique called "over-the-air function computation" was proposed which utilizes the superposition property of wireless channel to compute functions via concurrent transmission over a *multiple access channel* (MAC) [3].

The study about over-the-air function computation started from the information-theoretic point of view. Nazer and Gastpar [4] first pointed out that it was beneficial to utilize the superposition property of wireless channels to compute some target functions with a similar structure. Based on geometric random graph theory, the number of transmissions needed to compute specific functions was provided for Gaussian wireless network with linear network coding [5], [6]. Time and energy complexity of function computation over networks was further discussed in [7]. These works laid the information-theoretic foundation for over-the-air function computation. Applying nested lattice coding, the seminal work [8] exploited the wireless channel property to compute the noisy linear functions of transmitted messages. These works all adopted channel coding to achieve reliable over-the-air computation in digital way.

Over-the-air computation in analog way is more attractive for IoT network due to its low complexity and energy efficiency. An analog function computation scheme was proposed in [9] which was robust against synchronization errors utilizing random sequences. Considering a generalized IoT network composed of multiple clusters, over-the-air function computation was studied in [10], where each cluster aimed to independently compute the target function with an arbitrary subset of sensors. The work in [11] selected a subset of sensors in an opportunistic way to improve the computation performance. Various experiment platforms have been built to verify the idea of over-the-air computation in [12] and [13].

However, most of the existing works including the above mentioned ones did not consider the practical fading of MAC. It lacks the corresponding transceiver design to compensate the non-uniform fading of different sensors. In this letter, we propose a novel uniform-forcing transceiver design for over-the-air function computation. The computation performance is measured by the *mean squared error* (MSE). A corresponding min-max optimization is formulated and its duality with the downlink multicast beamforming is found. Due to the non-convexity of the problem, we relax it to *semidefinite programming* (SDP) and then improve its performance through *successive convex approximation* (SCA). According to the simulation results, the design can achieve significant performance gain with low complexity.

II. System Model

As illustrated in Fig. 1, there are K sensors in the network which are indexed by $k \in \{1, 2, \ldots, K\}$. The reading of sensor k is s_k. The sensors are all equipped with single antenna due to its size and power constraints. The *fusion center* (FC) has N antennas, and its target function that is computable over-the-air can be written in the form as

$$f = \psi\left[\sum_{k=1}^{K} \varphi_k(s_k)\right] \tag{1}$$

where $\varphi_k(\cdot)$ is the pre-processing function of the sensor k, and $\psi(\cdot)$ is the post-processing function of the FC. Given specific $\varphi_k(\cdot)$ and $\psi(\cdot)$, some common functions can be computed as illustrated in Table I.

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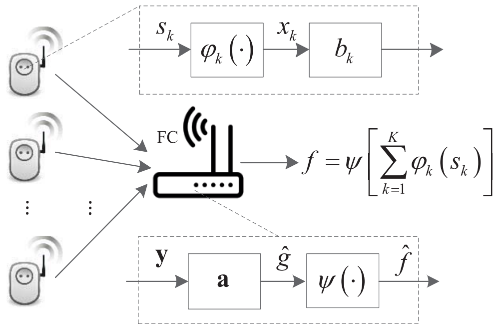

Fig. 1. System model.

TABLE I
SOME EXAMPLES OF f

Name	φ_k	ψ	f
Arithmetic Mean	$\varphi_k = s_k$	$\psi = \frac{1}{K}$	$f = \frac{1}{K}\sum_{k=1}^{K} s_k$
Weighted Sum	$\varphi_k = \omega_k s_k$	$\psi = 1$	$f = \sum_{k=1}^{K} \omega_k s_k$
Geometric Mean	$\varphi_k = \log(s_k)$	$\psi = \exp(\cdot)$	$f = \left(\prod_{k=1}^{K} s_k\right)^{\frac{1}{K}}$
Polynomial	$\varphi_k = \omega_k {s_k}^{\beta_k}$	$\psi = 1$	$f = \sum_{k=1}^{K} \omega_k {s_k}^{\beta_k}$
Euclidean Norm	$\varphi_k = {s_k}^2$	$\psi = (\cdot)^{\frac{1}{2}}$	$f = \sqrt{\sum_{k=1}^{K} {s_k}^2}$

The transmit signal after pre-processing at the sensor k is given as

$$x_k = \varphi_k(s_k). \tag{2}$$

And the received signal after concurrent transmissions of all sensors is

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{h}_k b_k x_k + \mathbf{z} \tag{3}$$

where $b_k \in \mathcal{C}$ is the transmitter scalar, $\mathbf{h}_k \in \mathcal{C}^N$ is the channel vector between the sensor k and the FC, and $\mathbf{z} \in \mathcal{C}^N$ is the noise vector with each element distributed as $\mathcal{CN}(0, \sigma_n^2)$.

The estimated function before post-processing at the FC is given as

$$\hat{g} = \mathbf{a}^T\mathbf{y} = \mathbf{a}^T \sum_{k=1}^{K} \mathbf{h}_k b_k x_k + \mathbf{a}^T\mathbf{z}, \tag{4}$$

where $\mathbf{a} \in \mathcal{C}^N$ is the receiver vector. Compared with the summation part of the target function in (1), i.e.,

$$g = \sum_{k=1}^{K} \varphi_k(s_k) = \sum_{k=1}^{K} x_k, \tag{5}$$

the distortion of $\hat{g}$ with respect to g, which quantifies the over-the-air computation performance, is measured by the MSE. That is

$$\mathrm{MSE}(\hat{g}, g) = \mathrm{E}\left(|\hat{g} - g|^2\right). \tag{6}$$

Remark 1: The Taylor series of the computed function $\hat{f} = \psi(\hat{g})$ at g can be expressed as $\hat{f} = \psi(g) + \psi'(g)(\hat{g} - g) + o(\hat{g} - g)$, where $o(\hat{g} - g)$ expresses the higher order terms. Then, with given $\psi'(g)$, the MSE of f can be approximated by the MSE of g, i.e., $\mathrm{MSE}(\hat{f}, f) \approx [\psi'(g)]^2\mathrm{MSE}(\hat{g}, g)$.

III. UNIFORM-FORCING TRANSCEIVER DESIGN

In this section, the transceiver b_k and $\mathbf{a}$ are designed and optimized to realize the uniform summation and minimize the MSE. It is assumed that the FC has global *channel state information* (CSI), and each sensor has its own CSI and the receiver vector $\mathbf{a}$.

The equivalent channel fading after receiver vector for the sensor k is given as $\mathbf{a}^T\mathbf{h}_k$. Then a uniform-forcing transmitter is designed as

$$b_k = \sqrt{\eta}\frac{\left(\mathbf{a}^T\mathbf{h}_k\right)^H}{\left\|\mathbf{a}^T\mathbf{h}_k\right\|^2}, \tag{7}$$

where η is the power control factor. Considering each sensor's transmit power constraint $\|b_k\|^2 \le P_0$, η is calculated as

$$\eta = P_0 \min_k \left\|\mathbf{a}^H\mathbf{h}_k\right\|^2, \tag{8}$$

which is determined by the minimum equivalent channel power gain $\|\mathbf{a}^H\mathbf{h}_k\|^2$ of all sensors.

Substituting b_k into (4) and scaling up with $1/\sqrt{\eta}$, the target function is estimated as

$$\hat{g} = \sum_{k=1}^{K} x_k + \frac{\mathbf{a}^H\mathbf{z}}{\sqrt{\eta}}. \tag{9}$$

The corresponding MSE is calculated as

$$\mathrm{MSE} = \frac{\left\|\mathbf{a}^H\right\|^2\sigma_n^2}{\eta} = \frac{\left\|\mathbf{a}^H\right\|^2\sigma_n^2}{P_0 \min_k \left\|\mathbf{a}^H\mathbf{h}_k\right\|^2} \tag{10}$$

Then the minimum MSE problem can be formulated as

$$(\mathrm{P1.1}) \ \ \min_{\mathbf{a}} \max_k \ \frac{\|\mathbf{a}\|^2}{\left\|\mathbf{a}^H\mathbf{h}_k\right\|^2}. \tag{11}$$

Proposition 1: The problem (P1.1) is the same as the following problem (P1.2).

$$\begin{aligned} (\mathrm{P1.2}) \ \ &\min_{\mathbf{a}} \ \ \|\mathbf{a}\|^2 \\ &\text{s.t.} \ \ \left\|\mathbf{a}^H\mathbf{h}_k\right\|^2 \ge 1, \ \forall k. \end{aligned} \tag{12}$$

Proof: By introducing an auxiliary variable $\tau = \min_k\|\mathbf{a}^H\mathbf{h}_k\|^2$, the problem (P1.1) is the same as

$$\begin{aligned} &\min_{\mathbf{a},\tau} \ \ \|\mathbf{a}\|^2/\tau \\ &\text{s.t.} \ \ \left\|\mathbf{a}^H\mathbf{h}_k\right\|^2 \ge \tau, \ \forall k \end{aligned} \tag{13}$$

Then introducing a new optimizing variable $\tilde{\mathbf{a}} = \mathbf{a}/\sqrt{\tau}$, the above problem can be rewritten as

$$\begin{aligned} &\min_{\tilde{\mathbf{a}}} \ \ \|\tilde{\mathbf{a}}\|^2 \\ &\text{s.t.} \ \ \left\|\tilde{\mathbf{a}}^H\mathbf{h}_k\right\|^2 \ge 1, \end{aligned} \tag{14}$$

which completes the proof. ■

Remark 2: The problem (P1.2) is a *quadratically constrained quadratic programming* (QCQP) problem with the non-convex constraints, which is NP-hard. It has the same form as the beamforming design for the downlink multicasting [14]. This reveals the duality between the receiver design for the uplink over-the-air computation and the beamforming design for the downlink multicasting.

IV. The Proposed Algorithm

In this section, we propose an efficient algorithm to get the receiver vector $\mathbf{a}$ that minimizes the MSE. The initial solution is calculated through *semidefinite relaxation* (SDR). Then its performance is improved through SCA.

Due to the non-convexity of (P1.2), it is relaxed to a convex SDP first. Assuming that $\mathbf{A} = \mathbf{a}\mathbf{a}^H$ and $\mathbf{H}_k = \mathbf{h}_k\mathbf{h}_k{}^H$, (P1.2) can be rewritten as

$$\begin{aligned}(\text{P2.1})\ \min_{\mathbf{A}}\ & \operatorname{tr}(\mathbf{A})\\ \text{s.t.}\ & \operatorname{tr}(\mathbf{A}\mathbf{H}_k) \geq 1,\ \forall k\\ & \mathbf{A} \succeq \mathbf{0}, \operatorname{rank}(\mathbf{A}) = 1. \end{aligned} \tag{15}$$

The only difficult non-convex constraint in (P2.1) is the rank constraint that rank$(\mathbf{A}) = 1$. By dropping it, the relaxed version of (P2.1) can be formulated as

$$\begin{aligned}(\text{P2.2})\ \min_{\mathbf{A}}\ & \operatorname{tr}(\mathbf{A})\\ \text{s.t.}\ & \operatorname{tr}(\mathbf{A}\mathbf{H}_k) \geq 1,\ \forall k,\ \mathbf{A} \succeq \mathbf{0}. \end{aligned} \tag{16}$$

which is convex and can be solved in polynomial time to get the optimal $\mathbf{A}^*$. If $\mathbf{rank}(\mathbf{A}^*) = 1$, the corresponding optimal equalizer can be given as $\mathbf{A}^* = \mathbf{a}^*\mathbf{a}^{*H}$.

While, there is no free lunch in turning a NP-hard problem into a polynomial-time solvable one [15]. $\mathbf{A}^*$ is not always rank-1. If $\mathbf{rank}(\mathbf{A}^*s) \neq 1$, a common rank-1 $\mathbf{A}^*_{ap}$ to approximate $\mathbf{A}^*$ is $\mathbf{A}^*_{ap} = \lambda_1\mathbf{u}_1\mathbf{u}_1{}^H$, where λ_1 is the maximum eigenvalue of $\mathbf{A}^*$, and $\mathbf{u}_1$ is its corresponding eigenvector. Then the sub-optimal equalizer can be approximated as $\mathbf{a}^*_{ap} = \sqrt{\lambda_1}\mathbf{u}_1$. SDR is capable of finding high quality solutions when the dimension of optimization variable is small. Its performance deteriorates quickly as the dimension of optimization variable increases [14]. The dimension variable $\mathbf{a}$ in (P1.2) is determined by the number of receive antennas N, which could be very large if the FC has many antennas.

We adopt SCA to improve the receiver performance. The basic idea of SCA is using iterative successive approximation to tackle the non-convexity of the constraints [16]. The non-convex constraints in problem (P1.2) are $\|\mathbf{a}^H\mathbf{h}_k\|^2 \geq 1,\ \forall k$. By introducing auxiliary variables $\mathbf{c}_k = [\operatorname{Re}(\mathbf{a}^H\mathbf{h}_k), \operatorname{Im}(\mathbf{a}^H\mathbf{h}_k)]^T$, (P1.2) can be rewritten as

$$\begin{aligned}(\text{P3.1})\ \min_{\mathbf{c}_k}\ & \|\mathbf{a}\|^2\\ \text{s.t.}\ & \|\mathbf{c}_k\|^2 \geq 1,\ \forall k\\ & \mathbf{c}_k = \left[\operatorname{Re}\left(\mathbf{a}^H\mathbf{h}_k\right), \operatorname{Im}\left(\mathbf{a}^H\mathbf{h}_k\right)\right],\ \forall k \end{aligned} \tag{17}$$

The non-convex constraints in (P3.1) is $\|\mathbf{c}_k\|^2 \geq 1, \forall k$, which can be approximated by the iterative relaxed linear constraint as

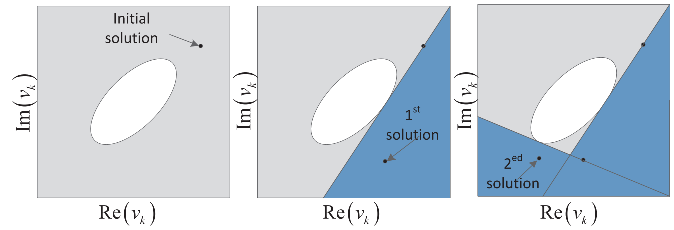

Fig. 2. The illustration of the proposed algorithm. gray area: feasible set; white are: infeasible set; colored area: approximated set.

Algorithm 1 The Receiver Optimization With SDA and SCA

Initialize $l = 0$, ε.
Solve (P2.2) to obtain $\mathbf{A}^*$
if rank$\mathbf{A}^* \neq 1$ **then**
  The initial solution $\tilde{\mathbf{a}}^* = \sqrt{\lambda_1}\mathbf{u}_1$.
  Set $\mathbf{c}_k{}^{(0)} = \left[\operatorname{Re}\left(\tilde{\mathbf{a}}^{*H}\mathbf{h}_k\right), \operatorname{Im}\left(\tilde{\mathbf{a}}^{*H}\mathbf{h}_k\right)\right], \forall k$
  repeat
    Solve (P3.2) to obtain $\mathbf{a}$ and $\left\{\mathbf{c}_k^{(l+1)}\right\}, \forall k$
  until $\sum_{k=1}^{K}\left\|\mathbf{c}_k^{(l+1)} - \mathbf{c}_k^{(l)}\right\| \leqslant \varepsilon$
else
  $\tilde{\mathbf{a}}^* = \sqrt{\lambda_1}\mathbf{u}_1$
end if

$$\|\mathbf{c}_k\|^2 \geq \left\|\mathbf{c}_k^{(l)}\right\|^2 + 2\left(\mathbf{c}_k^{(l)}\right)^T\left(\mathbf{c}_k - \mathbf{c}_k^{(l)}\right) \geq 1, \forall k \tag{18}$$

where $\mathbf{c}_k^{(l)}$ is the solution after the lth iterative optimization. By approximating the non-convex constraints with convex ones in (18), the problem during each iteration is

$$\begin{aligned}(\text{P3.2})\ \min_{\mathbf{a},\{\mathbf{c}_k\}}\ & \|\mathbf{a}\|^2\\ \text{s.t.}\ & \left\|\mathbf{c}_k^{(l)}\right\|^2 + 2\left(\mathbf{c}_k^{(l)}\right)^T\left(\mathbf{c}_k - \mathbf{c}_k^{(l)}\right) \geq 1, \forall k\\ & \mathbf{c}_k = \left[\operatorname{Re}\left(\mathbf{a}^H\mathbf{h}_k\right), \operatorname{Im}\left(\mathbf{a}^H\mathbf{h}_k\right)\right], \forall k \end{aligned} \tag{19}$$

An initial solution $\mathbf{c}_k^{(0)}$ is found through SDP (P2.2). Then its performance is improved by solving (P3.2) in an iterative way. The corresponding algorithm is summarized in Algorithm 1 with an illustration given in Fig. 2.

The convergence of Algorithm 1 can be guaranteed. Firstly, $\|\mathbf{a}\|^2$ is lower-bounded. Secondly, the iterative optimization of (P3.2) can promise that $\|\mathbf{a}\|^2$ is non-increasing. It can be also explained through Fig. 2. The $(l-1)$th solution is still in the search space of lth optimization. And the lth optimization in (P3.2) will find the optimal solution of this new search space due to its convexity. Thus, the lth solution will not be worse that the $(l-1)$th one. The worst case complexity of solving *semidefinite programming* (SDP) during the initialization of the algorithm is $\mathcal{O}((K+N^2)^{3.5})$ [15]. And the worst case complexity of solving *second-order cone programming* (SOCP) during each iteration of the algorithm is $\mathcal{O}(N^3)$ [16].

V. Numerical Results

In this section, we provide some numerical results to illustrate the performance of the proposed transceiver design

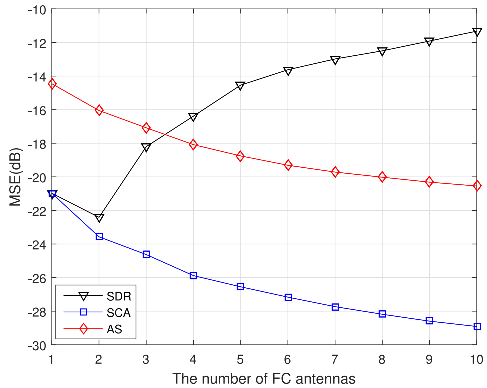

Fig. 3. The MSE versus different numbers of receive antennas.

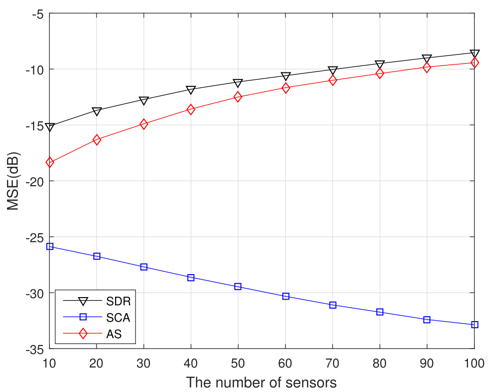

Fig. 4. The MSE versus different numbers of sensors.

(SCA for short) for over-the-air computation. The proposed algorithm is compared with SDR with maximum eigenvalue approximation (SDR for short) and antenna selection which selects the best antenna of the FC (AS for short). The transmit signal to noise ratio P_0/σ_n^2 is fixed at 20dB, and all numerical results are given based on 10^4 Monte-Carlo simulations.

The MSE versus different numbers of the FC's antennas N is shown in Fig. 3. The number of sensors is $K = 20$. With the increase in the number of receive antennas, the MSE of SCA and AS both decrease. That is because the diversity gain of receiver increases with the number of receive antennas. It can be seen that the MSE of SDR and SCA is the same when $N = 1$, and SDR can achieve the optimal solution of (P1.2). While, the MSE of SDR deteriorates as the number of receive antennas increases. That is because the rank-1 approximation of the SDR solution causes great distortion when the dimension of optimization variable increases. Thus, the proposed design can achieve great performance gain especially when the number of receive antennas is large.

The MSE versus different numbers of sensors K is shown in Fig. 4. The number of the FC's antennas is $N = 4$. The increase in the number of sensors will cause two effects to the MSE performance. The one is negative, because the uniform-forcing transceiver will force the non-uniform fading of different sensors to the same which is determined by the minimum equivalent channel power gain. The other is positive, because the combined receive signal will increase with the number of sensors. The MSE of SDR and AS increases with K because the negative effect dominates. Satisfyingly, the MSE performance of SCA decreases with K because Algorithm 1 makes the positive effect dominate. Thus, the proposed design can achieve performance gain when the number of sensors increases.

VI. Conclusion

In this letter, we proposed a uniform-forcing transceiver design problem for over-the-air computation to compensate the non-uniform fading of different sensors. A corresponding min-max optimization was formulated and its duality with multicast beamforming was discussed. We adopted SDR to relax the formulated non-convex problem. Then the solution was improved through SCA. According to numerical results, the proposed transceiver design can achieve great performance gain with an acceptable complexity.

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