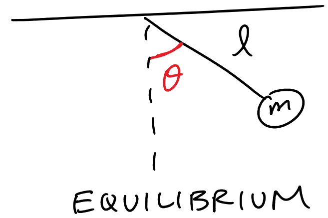
Quick Notes

Thursday, April 1, 2021 5:10 PM

CH 14 SIMPLE HARMONIC MOTION (SHM)

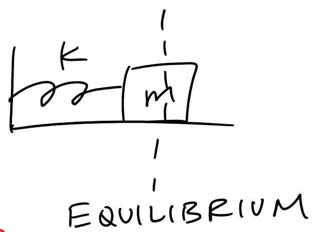
ex PENDULUM

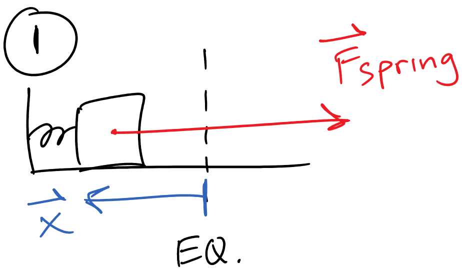


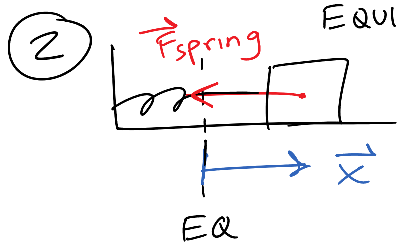
How do we know when an object is considered to be in SHM?

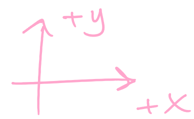
- If it's motion can be modeled by a cos() or sin() function. Simple as that. In our case, we'll just be looking at one-dimensional motion
  - For a pendulum, the variable that depicts motion is the angle (denoted by theta)
  - For a spring-mass, the variable that depicts motion is the horizontal displacement from equilibrium (denoted by x)

ex SPRING-MASS (HORIZONTAL)









$\vec{F}_{spring} = -k\vec{x}$, HOOKE'S LAW

↳ WHY NEG. SIGN?

MAKES $\vec{F}$ & $\vec{x}$ OPPOSITE

GOAL: IN MECHANICS, WE WANT TO FIND THE POSITION FUNCTION ~ $\underline{x(t)}$

NEWTON 2ND LAW: $F = ma \longrightarrow F_{NET} = ma_{NET}$

X-DIM: $F_{NET} = F_{spring} = ma$

$\longrightarrow -kx = ma$

$ma = -kx$

EQUATION OF MOTION; we would use this equation to solve for x(t) and thus obtain a function that depicts the block's position for all points in time. Amazing!! uwu →

$a = -\frac{k}{m}x$, NOTE: k, m ARE CONST.

WE WANT x(t), HOW DO WE REWRITE a IN TERMS OF x?

$$a = \frac{dv}{dt} = \frac{d}{dt}[v] = \frac{d}{dt}\left[\frac{dx}{dt}\right] = \underbrace{\frac{d}{dt}\left[\frac{d}{dt}[x]\right]}$$

$$a = \frac{d^2}{dt^2}[x]$$

AS A RESULT, WE NOW HAVE

$$\Longrightarrow \underbrace{\frac{d^2}{dt^2}[x] = -\cancel{\frac{k}{m}}\, x}_{\text{THE 2ND DERIV. OF x IS EQUAL TO NEGATIVE OF ITSELF}}$$

TEMPORARILY IGNORE

HOW DO WE SOLVE FOR x(t)??

LET'S TRY $x(t) = \cos(t)$

Using x(t) = sin(t) works equally well too. For some reason, the tradition is to use cosine to model oscillatory motion.

Recall: cos(x - pi/2) = sin(x), so we can always "adjust" our cosine function so that it may resemble the sine function by including a phase shift of -pi/2 or -90 degrees

$$\frac{d}{dt}[x(t)] = -\sin(t)$$

$$\frac{d^2}{dt^2}[x(t)] = -\cos(t) = -x(t)$$

Here, we did a pr0-gamer move by rewriting cos(t) as x(t). This is totally valid because we defined x(t) = cos(t) earlier. The reason why we want to rewrite this is so that it matches the form of our equation of motion earlier.

AS A RESULT, WE FIND THAT

$$\boxed{\frac{d^2}{dt^2}[x(t)] = -x(t)}, \quad \text{IF } x(t) = \cos(t)$$

↳ CLOSE, BUT ...

WE WANT THE COEFFICIENT k/m

$$\frac{d^2}{dt^2}[x(t)] = -\frac{k}{m}x(t)$$

RECALL: $\frac{d}{dt}[\cos(at)] = -a\sin(at)$ where "a" is some constant

LET'S TRY $x(t) = \cos\left(\frac{k}{m} t\right)$

$$\frac{d}{dt}\left[x(t)\right] = -\frac{k}{m} \sin\left(\frac{k}{m} t\right)$$

$$\frac{d^2}{dt^2}\left[x(t)\right] = -\frac{k}{m}\left(\frac{k}{m} \cos\left(\frac{k}{m} t\right)\right) = -\frac{k^2}{m^2} x(t)$$

CLOSE!! BUT WE DONT WANT SQUARED

FINALLY, LET'S TRY: $x(t) = \cos\left(\sqrt{\frac{k}{m}}\, t\right)$

$$\frac{d}{dt}\left[x(t)\right] = -\sqrt{\frac{k}{m}} \sin\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$\frac{d^2}{dt^2}\left[x(t)\right] = -\frac{k}{m} \cos\left(\sqrt{\frac{k}{m}}\, t\right) = \underbrace{-\frac{k}{m} x(t)}_{}$$

NICE!! WE GOT OUR ORIGINAL EQN.

$$\Longrightarrow \quad x(t) = \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

LET $\omega = \sqrt{\frac{k}{m}}$, THEN $\boxed{x(t) = \cos(\omega t)}$

IN GENERAL,

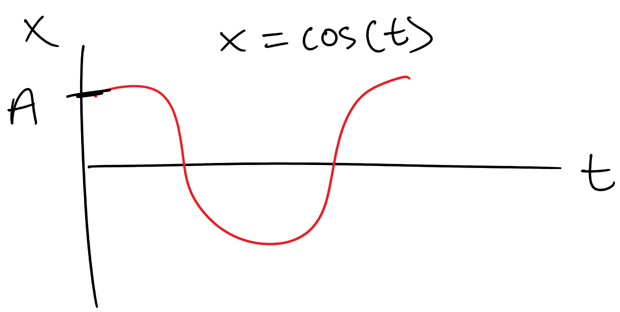
$$x(t) = A \cos(\omega t + \phi)$$

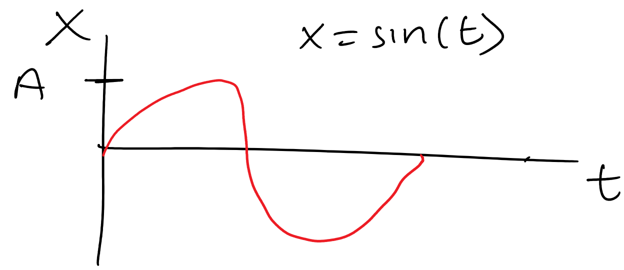
Where omega (curly w) represents the "angular frequency", basically how frequent the cosine function goes up and down in a given interval. This quantity is directly related to the phase and the period of oscillation of the spring-mass

phi represents the "phase shift", basically shifting the origin left or right so that the resulting cosine function correctly depicts the motion of the block.

A represents the "amplitude", which basically describes the maximum displacement of the block from equilibrium. Since cosine and sine have values from -1 to +1, multiplying by A is like scaling the function so that it fits the maximum displacement measured.

x $\quad x = \cos(t)$ $\qquad\qquad$ x $\quad x = \sin(t)$





The general form allows us to basically convert this cosine function into a sine function if need be.

- All we would have to do is set the phase shift phi = -pi/2 or -90 degrees in order for the cosine to resemble sine.

When do we know if we need cosine or if we need sine to model the motion of the block?

- It simply depends on where the block is when we start our timer at t = 0 i.e. the block's initial position.