Quick Notes

Sunday, April 25, 2021 6:13 PM

Content Review: [10mins]

Standing Waves

- What the heck is a standing wave??!

  Watch this 5-min YouTube video that illustrates how standing waves form.

  Check out the question in the Slido poll (link in Zoom chat)

- 2 common examples are: **mechanical waves on a string** and **sound waves in a tube**.

For **standing waves on a string**.

- ☐ We are looking at the **displacement of the string**.
- ☐ The endpoints are both **fixed**, corresponding to **displacement nodes**.
- ☐ **Displacement:** A **displacement antinode** occurs at points where the string is allowed to move widely; while a **displacement node** occurs at places such as the endpoints where the string is fixed and not moving freely.

For **standing sound waves in a tube**.

- ☐ We are looking at either the **pressure in the air** or the **displacement of air particles**.
- ☐ The tube's ends could be **open-open**, **closed-closed**, or **open-closed**.
- ☐ **Pressure:** A **pressure node** occurs at open-ends as that is where the air pressure equals that of the surrounding environmental pressure; while a **pressure antinode** occurs at closed-ends since the air pressure builds up against the inner walls.
- ☐ **Displacement:** A **displacement antinode** occurs at an open-end as air particles are allowed to move freely; while a **displacement node** occurs at closed-ends since the air particles' motion is restricted.

NODES ⟶ MINIMUM

ANTINODES ⟶ MAXIMUM

	pressure	displacement
open-end	node	antinode
closed-end	antinode	node

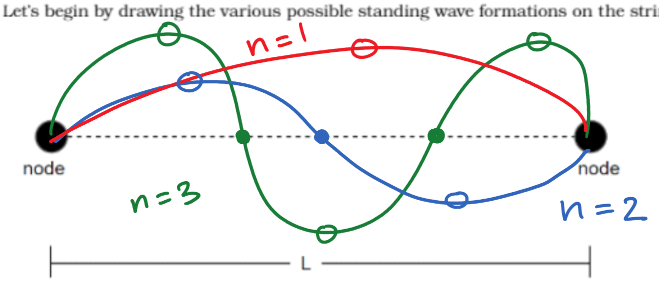
NOTE: In general, for sound waves within a tube:

pressure nodes ⟷ displacement antinodes   pressure antinodes ⟷ displacement nodes

Derivation: Standing Waves on a String

Consider a string of length L tied on both ends. Let's derive a mathematical expression for the **resonant frequencies** that would produce standing waves on the string.

Let's begin by drawing the various possible standing wave formations on the string.



o ANTINODE

● NODE

$$v = f\lambda \rightarrow \lambda = \frac{v}{f}$$

FOR $n=1$: $L = \frac{1}{2}\lambda = \frac{1}{2}\frac{v}{f} \longrightarrow f_1 = \frac{1}{2}\frac{v}{L}$

FOR $n=2$: $L = \frac{2}{2}\lambda = \frac{2}{2}\frac{v}{f} \longrightarrow f_2 = \frac{2}{2}\frac{v}{L}$

FOR $n=3$: $L = \frac{3}{2}\lambda = \frac{3}{2}\frac{v}{f} \longrightarrow f_3 = \frac{3}{2}\frac{v}{L}$

IN GENERAL,

$$f_n = n \cdot \frac{1}{2}\frac{v}{L}, \quad \text{FOR } n = 1, 2, 3, \ldots$$

We've just derived this equation for **a string tied on both ends** (e.g. guitar string or violin string)

It turns out, this equation applies to **open-open tubes** as well as **closed-closed tubes**!
(though I don't think there's such a situation as closed-closed tubes)

Oh and it also turns out, *n* corresponds to the number of antinodes as well!

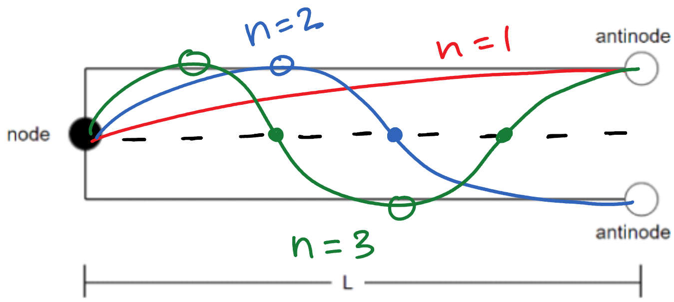
As long as the endpoints are the same (i.e. the endpoints are either both nodes or both antinodes), then the above equation is able to give the object's resonant frequencies.

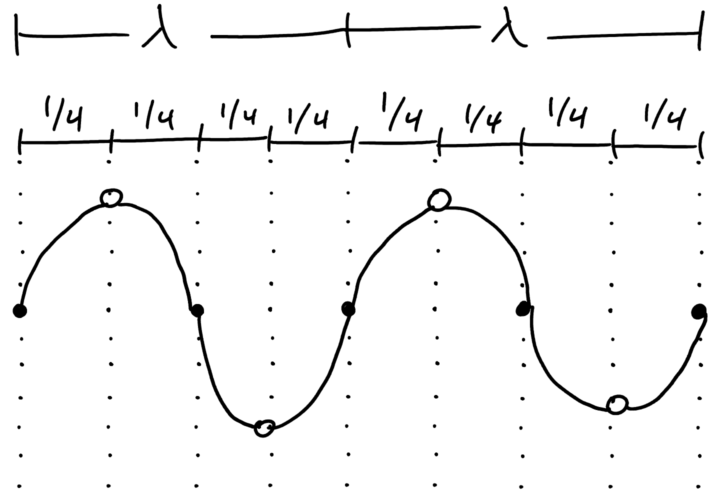
You can check for yourself and see what the standing waves would look like for n = 1, 2, 3, ... for open-open tubes

Derivation Standing Sound Waves in a Open-Closed Tube

Consider an open-closed tube; that is, a tube that is open on one end and closed on the other. Let's derive a mathematical expression for the **resonant frequencies** that would produce standing sound waves within the tube.

Let's begin by drawing the various possible standing wave formations within the tube.





FOR $n=1$: $L = \frac{1}{4}\lambda = \frac{1}{4}\frac{v}{f} \longrightarrow f_1 = \frac{1}{4}\frac{v}{L}$

FOR $n=2$: $L = \frac{3}{4}\lambda = \frac{3}{4}\frac{v}{f} \longrightarrow f_2 = \frac{3}{4}\frac{v}{L}$

FOR $n=3$: $L = \frac{5}{4}\lambda = \frac{5}{4}\frac{v}{f} \longrightarrow f_3 = \frac{5}{4}\frac{v}{L}$

IN GENERAL,

$$f_n = n \cdot \frac{1}{4}\frac{v}{L} \quad \text{FOR } n = 1, 3, 5, \ldots$$

(ODD VALUES ONLY)

Organ Pipe

An organ pipe is 124-cm long. Determine the fundamental and first 3 audible overtones if the pipe is

(a) closed at one end (and open at the other end)

(b) open at both ends

$\hookrightarrow$ $20\,Hz \le f \le 20\,kHz$ ~ HUMAN HEARING

a) OPEN-CLOSED TUBE: $f_n = n \cdot \frac{1}{4}\frac{v}{L}$, $n = 1, 3, 5, \ldots$

SPEED OF SOUND

FUNDAMENTAL FREQUENCY: $f_1 = 1 \cdot \frac{1}{4}\frac{v}{L} = \frac{1}{4}\frac{343}{1.24} = \boxed{69.15\ Hz}$

FREQUENCY $\cdot$ $\frac{1}{4}\frac{}{L}$ $\frac{}{1.24}$

1ST OVERTONE: $f_3 = 3 \cdot \frac{1}{4}\frac{v}{L} = 3 \cdot f_1 = \boxed{207.45\ Hz}$

2ND OVERTONE: $f_5 = 5 \cdot f_1 = \boxed{345.75\ Hz}$

3RD OVERTONE: $f_7 = 7 \cdot f_1 = \boxed{484.05\ Hz}$

b) OPEN-OPEN TUBE: $f_n = n \cdot \frac{1}{2}\frac{v}{L}, \quad n = 1, 2, 3, \ldots$

FUNDAMENTAL FREQUENCY: $f_1 = 1 \cdot \frac{1}{2}\frac{v}{L} = \frac{1}{2}\frac{343}{1.24} = \boxed{138\ Hz}$

1ST OVERTONE: $f_2 = 2 \cdot \frac{1}{2}\frac{v}{L} = 2 \cdot f_1 = \boxed{277\ Hz}$

2ND OVERTONE: $f_3 = 3 \cdot \frac{1}{2}\frac{v}{L} = 3 \cdot f_1 = \boxed{414\ Hz}$

3RD OVERTONE: $f_4 = 4 \cdot f_1 = \boxed{552\ Hz}$