Guided Practice

A 1.0 L volume of air, initially pressurized at 3.5 atm, is allowed to expand isothermally until the pressure reaches 1.0 atm. It is then compressed at constant pressure to its initial volume, and lastly is brought back to its original pressure by heating at constant volume.

(i) Draw the process on a PV-diagram, making sure to label all pressure & volume quantities.

(ii) Find the work done when the gas is compressed.

(iii) Find the heat gained by the gas in the last process.

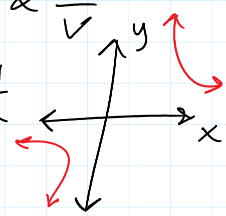
Answer: (ii) $W = 250\,\text{J}$ (iii) $Q = 375\,\text{J}$

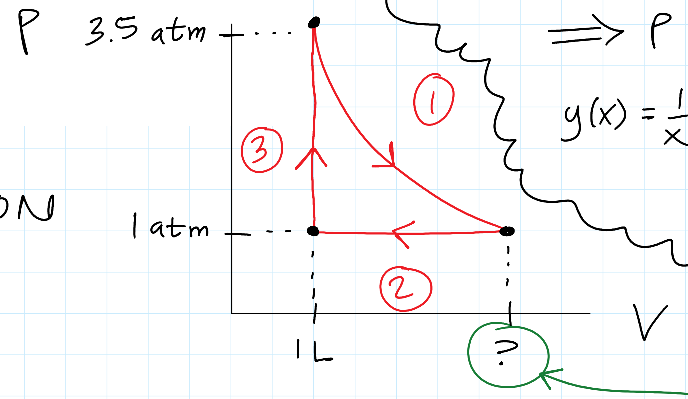
"ISOTHERMAL" $\longrightarrow$ T = CONST.

$PV = nRT$ = CONST.

$\Longrightarrow P \propto \frac{1}{V}$

$y(x) = \frac{1}{x}$





1) ISOTHERMAL EXPANSION
2) ISOBARIC COMPRESSION
3) ISOVOLUMETRIC HEATING

find V_f for 1

$P_f V_f = nRT$ = CONST. SINCE ISOTHERMAL & SEALED CONTAINER

$\hookrightarrow P_i V_i = P_f V_f \Longrightarrow V_f = \frac{P_i}{P_f} V_i = \frac{(3.5)}{(1.0)}(1.0) = 3.5\,\text{L}$

find W FOR PROCESS 2

$$W = \int P\,dV$$

$$= P \int_{V_i}^{V_f} dV \; ,$$ FACTOR OUT P B/C IT'S CONST. SINCE ISOBARIC

$$= P \left[V \right]_{V_i}^{V_f} = P \left[V_f - V_i \right] = P \Delta V$$

$$\Longrightarrow W = P\Delta V = (10^5)\underbrace{(1 - 3.5)}_{V_f - V_i} \times 10^{-3} = -2.5 \times 10^2 = -250\,\text{J}$$

WE FOUND W = -250 J ~ THIS IS THE WORK DONE BY THE GAS

THE QUESTION IS JUST ASKING FOR THE WORK IN GENERAL, SO

$$\Longrightarrow \boxed{W = 250\,\text{J}}$$

find Q (FOR PROCESS 3)

find Q (FOR PROCESS ③)

$$\Delta E = Q - \cancel{W}^{\,0}$$

$$W = \int P\,dV$$

$$= \int_{V_i}^{V_f} P\,dV$$

$$= 0$$, SINCE $V_i = V_f$, THE $\int$ IS STRAIGHT UP ZERO

$$\Longrightarrow \Delta E = Q \longrightarrow Q = \Delta E = E_f - E_i$$

$$= \frac{1}{2} f N K T_f - \frac{1}{2} f N K T_i$$

$$= \frac{5}{2} N K T_f - \frac{5}{2} N K T_i$$

pr0-gamer move : $NKT = PV$

$$= \frac{5}{2} P_f V_f - \frac{5}{2} P_i V_i$$

$$= \frac{5}{2} V \left[P_f - P_i \right] , \quad V_i = V_f$$

$$= \frac{5}{2} (10^{-3})(3.5 - 1)\, 10^5$$

$$Q = \boxed{625\ J}$$

NOTE : $E_{int} = \frac{1}{2} f N K T$ WHERE $f =$ DEGREES OF FREEDOM

AIR ~ N_2 & O_2 SO AIR IS PRETTY MUCH DIATOMIC

$$f = \begin{cases} 3, & \text{MONOATOMIC} \\ 5, & \text{DIATOMIC} \\ 7, & \text{DIATOMIC @ HIGH TEMP.} \end{cases}$$

IN THIS PROBLEM

* TYPO IN SOLUTION: THEY USED $f = 3$ ON ACCIDENT *

Breakout-Room Activity

A hot 0.40 kg iron horseshoe is dropped into 1.05 L of water in a 0.30 kg iron pot initially at 20 °C. If the final equilibrium temperature is 25 °C, what was the initial temperature of the horseshoe.

Useful info: $c_{\text{iron}} = 450\,\text{J/kg°C}$ and $c_{\text{water}} = 4186\,\text{J/kg°C}$

Hint: How many objects are there in the system?

Answer: $T = 150\,°\text{C}$

* 3 objects to account for *

find T_i^{HS} (HS := HORSESHOE)

$$\sum Q = 0$$

$$Q_{HS} + Q_{H2O} + Q_{POT} = 0$$

$$m_{HS}\, c_{Fe} \left[T_f - T_i^{HS}\right] + m_{H2O}\, c_{H2O} \left[T_f - T_i^{H2O}\right] + m_{POT}\, c_{Fe} \left[T_f - T_i^{POT}\right] = 0$$

$$m_{HS}\, c_{Fe} \left[T_f - T_i^{HS}\right] = - m_{H2O}\, c_{H2O} \left[T_f - T_i^{H2O}\right] - m_{POT}\, c_{Fe} \left[T_f - T_i^{POT}\right]$$

$$T_i^{HS} = \frac{m_{H2O}\, c_{H2O} \left[T_f - T_i^{H2O}\right] + m_{POT}\, c_{Fe} \left[T_f - T_i^{POT}\right]}{m_{HS}\, c_{Fe}} + T_f$$

$$= \boxed{151^\circ C}$$

NOTE: you can choose to plug in #'s earlier to lessen the algebra

Breakout-Room Activity

A bicycle pump is a cylinder 22 cm long and 3.0 cm in diameter. The pump contains air at 20 °C and 1.0 atm. If the outlet at the base of the pump is blocked and the handle is pushed in very quickly, compressing the air to half of its original volume, how hot does the air in the pump become?

NOTE: Since this process occurs very rapidly, you may assume it is adiabatic. Also, air is made of mostly N2 and O2 which are diatomic molecules, so $\gamma = C_P/C_V = 7/5 = 1.4$

Answer: $T = 387\,^\circ K$ or $114\,^\circ C$

(SEALED CONTAINER)

GIVEN: $V_1 = \ldots$ $\quad V_2 = \frac{1}{2} V_1$ $\quad n_1 = n_2$

$T_1 = 293^\circ K$ $\quad T_2 = ?$

$P_1 = 10^5\ Pa$ $\quad P_2 = ?$

find T_2

ADIABATIC $\Longrightarrow$ $\boxed{P_1 V_1^\gamma = P_2 V_2^\gamma}$ (1) & $Q = 0$

$PV = nRT \Longrightarrow \frac{PV}{T} = nR = \text{CONST.} \Longrightarrow \boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$ (2)

WE USE EQN (1) & (2) IN CONJUNCTION TO FIND T_2

(2) $\longrightarrow$ $T_2 = \boxed{P_2} V_2 \ldots = \left[\frac{V_1^\gamma}{\ldots} \right] \cdot \frac{V_2}{\ldots}$

(2) $\longrightarrow$ $$\frac{T_2}{T_1} = \boxed{\frac{P_2 V_2}{P_1 V_1}} = \left[\frac{V_1^{\gamma}}{V_2^{\gamma}}\right] \cdot \frac{V_2}{V_1}$$

(1) $\longrightarrow$ $$\boxed{\frac{P_2}{P_1}} = \frac{V_1^{\gamma}}{V_2^{\gamma}}$$

$$= \frac{V_1^{\gamma} \cdot V_1^{-1}}{V_2^{\gamma} \cdot V_2^{-1}}$$

$$= \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} = \left[\frac{V_1}{V_2}\right]^{\gamma-1}$$

$$\Longrightarrow \frac{T_2}{T_1} = \left[\frac{V_1}{V_2}\right]^{\gamma-1} = \left[\frac{\cancel{V_1}}{\frac{1}{2}\cancel{V_1}}\right]^{\gamma-1} = 2^{\gamma-1}$$

$$\Longrightarrow T_2 = 2^{\gamma-1} T_1 = 2^{1.4-1}(293)$$

$$= \boxed{387^\circ K}$$

P.S. thank you for letting me be your LARC tutorial leader! :-)