Quick Notes

Tuesday, April 13, 2021 5:06 PM

CH 14: OSCILLATIONS
CH 15: WAVE MOTION

} BOTH INVOLVE WAVES

Q: IN CH 14, WHAT WAS ASSOCIATED WITH WAVES?

A: THE OBJECT'S MOTION ~ $\underbrace{x(t) = A\cos(\omega t)}_{\text{SHM}}$

Q: IN CH 15, WHAT WAS ASSOCIATED WITH WAVES?

A: THE OBJECT ITSELF IS A WAVE

Objective:

I. Analyze the **Traveling Wave equation $y(x,t)$**

- ☐ By examining the equation $y(x,t)$ in its different forms.
- ☐ By creating 2 graphs: **position-graph** (y vs x) & **time-graph** (y vs t)

II. Solve problems that involve **a traveling wave**

- ☐ By constructing our own equation $y(x,t)$ to fit the situation.

RECALL: FROM CH 14

$$x(t) = A\cos(\omega t) = A\cos(2\pi f t) = A\cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

Content Review: [10mins]

- The **Traveling Wave equation $y(x,t)$** is a function of 2 variables: x and t

$$y(x,t) = A\sin(kx + \omega t + \phi)$$

where the variables are:

$y =$ VERTICAL POSITION ("DISP. FROM EQBRM. ($y=0$)") $A =$ AMP. $k =$ WAVE NUMBER $x =$ HORIZONTAL POSITION

$\omega =$ ANG. FREQ. $t =$ time $\phi =$ PHASE-SHIFT

- For a wave traveling **on a string**, the velocity is given by

$$v_{string} = \sqrt{\frac{T}{\mu}} \sim \sqrt{\frac{[N]}{[kg/m]}} = \sqrt{\frac{[\cancel{kg}\cdot m/s^2]}{[\cancel{kg}/m]}} = \sqrt{[m^2/s^2]}$$

where T is the tension and μ is the string's mass density (units of kg/m)

$v = [m/s] \longleftrightarrow = \sqrt{[m^2/s^2]}$

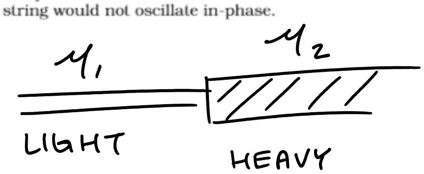
- For traveling waves **in general**, the velocity is given by

$$v = \lambda \cdot f = \frac{\cancel{2\pi}}{k} \cdot \frac{\omega}{\cancel{2\pi}} = \frac{\omega}{k}$$

where ω is the angular frequency and k is the wave number

$$\begin{cases} \omega = 2\pi f \\ k = \dfrac{2\pi}{\lambda} \end{cases}$$

- For a string with 2 sections of **differing mass density** (μ_1 and μ_2), the tension T and frequency f throughout the string is constant.
  - ☐ If tension was NOT the same, then the joint between the 2 sections would accelerate and disrupt the string's motion.
  - ☐ If frequency was NOT the same, then the wave would not be a smooth wave i.e. different points of the string would not oscillate in-phase.



$T_1 = T_2$ & $f_1 = f_2$

$\downarrow$

$\omega_1 = \omega_2$

A sinusoidal wave traveling on a string in the negative x direction has amplitude $1.00\,\text{cm}$, wavelength $3.00\,\text{cm}$, and frequency $245\,\text{Hz}$. At $t = 0$, the particle of string at $x = 0$ is displaced a distance 0.80 cm above the origin (equilibrium) and moving upward.

$\rightarrow y(x=0, t=0)$

(i) Construct the equation representing this traveling wave as a function of x and t ~ IN CENTIMETERS & SECONDS

(ii) Plot the following 2 equations and describe their physical representation:

- ☐ Holding $t = 0\,\text{s}$, graph $y(x,t)$ over distance x.
- ☐ Holding $x = 5.00\,\text{cm}$, graph $y(x,t)$ over time t.

3 UNKNOWN CONST.

■ Here is a link to Desmos online graphing calculator

(i) GENERAL FORM: $y(x,t) = A \sin(\underset{?}{k}x + \underset{?}{\omega} t + \underset{?}{\phi})$

$$y(x,t) = A \sin\left(\frac{2\pi}{\lambda} x + 2\pi f\, t + \underset{?}{\phi}\right)$$

$$= (1.00\,\text{cm}) \sin\left((2.09\,\text{cm}^{-1})\, x + (1540\,\text{rad/s})\, t + \underset{?}{\phi}\right)$$

$$y(x=0, t=0) = (1.00\,\cancel{\text{cm}}) \sin(0 + 0 + \phi) = 0.80\,\cancel{\text{cm}}$$

$$\sin(\phi) = 0.80 \rightarrow \phi = \sin^{-1}[0.80]$$

$$= 0.93\,\text{rad}$$

$$\rightarrow \boxed{y(x,t) = (1.00) \sin(2.09\, x + 1540\, t + 0.93)}$$

↳ PLUG THIS EQN INTO DESMOS GRAPHING CALCULATOR

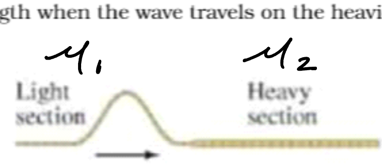
More info in this Desmos Graphing page: https://www.desmos.com/calculator/exk0pji3zn

A cord has 2 sections with linear mass densities of $0.10\,\text{kg/m}$ and $0.20\,\text{kg/m}$. An incident wave given by $D = (0.05\,\text{m}) \sin(7.5x - 12.0t)$, where x is in meters and t in seconds, travels along the lighter cord.

(i) Determine wavelength on the lighter section of cord

(ii) Determine the tension in the cord

(iii) Determine the wavelength when the wave travels on the heavier section



(i) FIND λ_1 (LIGHT)

GIVEN: $D = 0.05 \sin(7.5x - 12t)$

GENERAL FORM: $D = A \sin(kx - \omega t)$

GENERAL FORM : $D = A\sin(kx - \omega t)$

$$k = \frac{2\pi}{\lambda} \longrightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{(7.5)} = \boxed{0.84\,m}$$

(ii) FIND T

- For a string with 2 sections of **differing mass density** (μ_1 and μ_2), the tension T and frequency f throughout the string is constant.
  - If tension was NOT the same, then the joint between the 2 sections would accelerate and disrupt the string's motion.
  - If frequency was NOT the same, then the wave would not be a smooth wave i.e. different points of the string would not oscillate in-phase.

$\equiv$ MEANS "DEFINE"

$$\begin{cases} T_1 = T_2 \equiv T \\ f_1 = f_2 \equiv f \longrightarrow \omega_1 = \omega_2 \equiv \omega \end{cases}$$

$$v = \frac{\omega}{k} \quad \text{AND} \quad v = \sqrt{\frac{T}{\mu}},$$

NOTE : CAREFUL!!

$\mu_1 \neq \mu_2$ $\quad v_1 \neq v_2$

$k_1 \neq k_2$

$$\frac{\omega}{k} = \sqrt{\frac{T}{\mu}} \longrightarrow T = \mu\frac{\omega^2}{k^2}$$

FOR THIS PROBLEM,

$$T = T_1 = \mu_1\frac{\omega^2}{k_1^2} = (0.10)\frac{(12)^2}{(7.5)^2} = \boxed{0.26\ N}$$

THESE VALUES ARE PULLED FROM

$D(x,t) = 0.5\sin(7.5x - 12t)$ WHICH HOLDS ONLY FOR THE <u>LIGHT</u> SECTION

(iii) FIND λ_2

SINCE WE KNOW λ_1, WE CAN DO SOME RATIO

CONSIDER:

$$v = f\lambda \longrightarrow \lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$$

$$\text{RATIO} = \frac{\lambda_2}{\lambda_1} = \frac{\frac{1}{f_2}\sqrt{\frac{T_2}{\mu_2}}}{\frac{1}{f_1}\sqrt{\frac{T_1}{\mu_1}}} = \frac{\cancel{\frac{1}{f}}\sqrt{\frac{\cancel{T}}{\mu_2}}}{\cancel{\frac{1}{f}}\sqrt{\frac{\cancel{T}}{\mu_1}}} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\longrightarrow \lambda_2 = \lambda_1 \cdot \sqrt{\frac{\mu_1}{\mu_2}} = (0.84)\sqrt{\frac{(0.10)}{(0.20)}}$$

$$= \boxed{0.59\,\text{m}}$$