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Lecture 1 Worksheet

Task 1

1. What is the probability that at least 2 people in this room have the same birthday? It is easier to use the complement to find this probability. The complement would then be

$$\text{P(At Least 2)} = 1 - \text{P(None)}$$

where

$$\begin{aligned}\text{P(None)} &= \frac{365}{365} * \frac{364}{365} * \ldots * \frac{(365-n)}{365} \\ &= 1 - \frac{365!}{365^n\ (365-n)!}\end{aligned}$$

Given n = 32 people in our class

$$\begin{aligned}\text{P(At Least 2)} &= 1 - \frac{365!}{365^{32}\ (365-32)!} \\ &= 75.3\%\end{aligned}$$

2. How many people does it take for the probability of at least two of them sharing their birthday to exceed 50%? It takes n $\geq$ 23 for the probability to be at least 50%

Task 2

1. In how many ways can you order m distinct objects?

$$m!$$

2. In how many ways can you order m objects of n different types (n $\geq$ m)? Objects of the same type are indistinguishable.
Given n different types, we need to use the number of objects in each type, denoted as m_i for $i = 1, ..., n$ where $\sum_{i=1}^{n} m_i = m$
First we find the number of ways to reorder i "different" objects, which is $m!$, however we may be over-counting for objects that are of the same type (indistinguishable). We offset this by dividing $m_i!$ for all i, since for each type, we over-count $m_i!$ times.

$$\frac{m!}{\prod_{i=1}^{n} m_i}$$

3. How many **(sub)sets** of n elements can you generate from a set of m elements (n $\geq$ m)?

$$\binom{m}{n} = \frac{m!}{(m-n)!\; n!}$$

4. How many **sequences** of n elements can you generate from a set of m elements (n $\geq$ m)?

$$m^n$$

What is the difference between a **subset** *and a* **sequence***?* A subset is unordered whereas a sequence is ordered.

Task 3

1. If you randomly pick n objects from an infinite supply of objects of m different types, how many outcomes are possible? From Lecture Video 1, we are given that for m different types, we have $m-1$ number of partitions. Using the condition where each type has at least 1 object, we have $n-1$ possible spots to place the partition. Thus to compute the number of combinations we can just use

$$\binom{n-1}{m-1} = \frac{(n-1)!}{(n-m)!(m-1)!}$$

. However, if generalize the problem, then we may have zero objects for certain types. Referring to the extreme case of having all objects of just 1 type, we would have to place all the partitions back-to-back. To account for this case, we would need an additional m spots to the outer edge. Adding this feature to our original number of spots, we have $m + n - 1$ possible spots to put the partitions.

$$\binom{m+n-1}{m-1} = \frac{m+n-1}{n!\,(m-1)!}$$