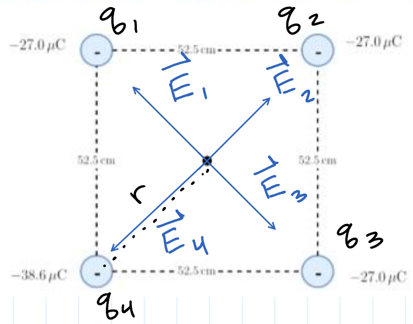
Wk6_S2_Key

Saturday, March 6, 2021 7:42 PM

Find the magnitude & direction of the $\vec{E}$ at the center of a square with sidelength 52.5 cm if one corner is occupied by a $-38.6\,\mu C$ charge and the other three are occupied by $-27.0\,\mu C$ charges.

Hint: Use symmetry to your advantage!

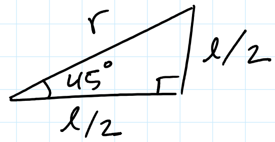
Answer: $|\vec{E}| = 7.6 \times 10^5$ N/C, towards the $-38.6\,\mu C$ charge i.e. 225° from the $+x$ axis



FIND $\vec{E}$ @ CENTER

- $\vec{E}_1$ & $\vec{E}_3$ CANCEL EACH OTHER OUT DUE TO SYMMETRY
- SINCE $E \propto q$, WE KNOW $|\vec{E}_4| > |\vec{E}_2|$ SO THEREFORE THE RESULTANT $\vec{E}_{NET}$ POINTS IN THE DIRECTION OF $\vec{E}_4$

LET $\ell = 52.5$ cm



$\longrightarrow \quad r = \dfrac{\ell\sqrt{2}}{2}$

$$|\vec{E}_{NET}| = |\vec{E}_4| - |\vec{E}_2| = k\frac{q_4}{r^2} - k\frac{q_2}{r^2}$$

* LOOKING AT MAGNITUDES ONLY *

$$= k\frac{1}{r^2}\left[q_4 - q_2\right] = k\frac{4}{\ell^2 2}\left[q_4 - q_2\right]$$

$$= (9\times 10^9)\frac{4}{(0.525)^2\,2}\left[38.6 - 27\right]$$

$$= \boxed{7.6\times 10^6 \text{ N/C}} \sim \text{MAGNITUDE}$$

- $\vec{E}_{NET}$ POINTS IN THE SAME DIRECTION AS $\vec{E}_4$

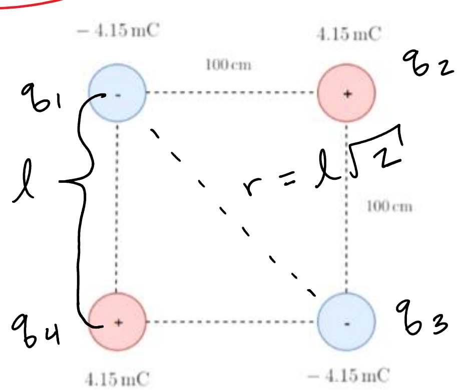
$\boxed{+225^\circ \text{ FROM } +x \text{ AXIS \quad OR \quad } 45^\circ \text{ SOUTH OF WEST}}$

NOTE: TYPO, SORRY, SHOULD SAY 1.4×10^5 N/C

Find the magnitude & direction of the force on each charge shown in the figure below.

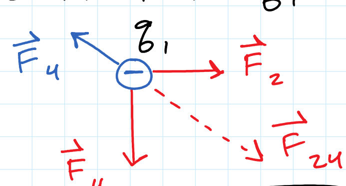
Hint: There's definitely some clever trick involving the symmetry . . . but what is it??!

Answer: $|\vec{F}| = 1.4 \times 10^7$ N and the direction for each charge is basically just towards the center of the square.

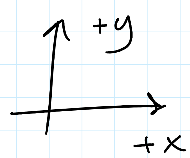


FIND $\vec{F}_{NET}$ FOR EACH POINT CHARGE

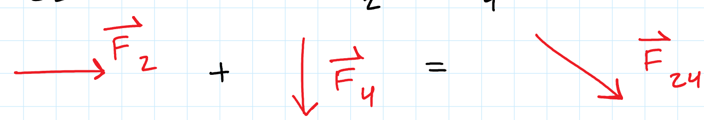
LOOKING AT q_1:



AXES:



LET'S COMBINE $\vec{F}_2$ & $\vec{F}_4$



q^2

WE SEE THAT $|\vec{F}_2| = |\vec{F}_4| = k\dfrac{q^2}{\ell^2}$, WHERE $q = 4.15\,mC$

$$\vec{F}_{24} = \vec{F}_2 + \vec{F}_4 = k\frac{q^2}{\ell^2}(+\hat{\imath}) + k\frac{q^2}{\ell^2}(-\hat{\jmath})$$

$$|\vec{F}_{24}| = \sqrt{\left(k\frac{q^2}{\ell^2}\right)^2 + \left(k\frac{q^2}{\ell^2}\right)^2} = \sqrt{2}\,k\frac{q^2}{\ell^2}$$

SIMILAR TO THE PREVIOUS PROBLEM, WE CAN STRAIGHT UP ADD/SUBTRACT MAGNITUDES SINCE $\vec{E}_3$ & $\vec{E}_{24}$ LIE ON THE SAME LINE

$$|\vec{F}_{NET}| = |\vec{F}_{24}| - |\vec{F}_3| = \sqrt{2}\,k\frac{q^2}{\ell^2} - k\frac{q^2}{r^2}$$

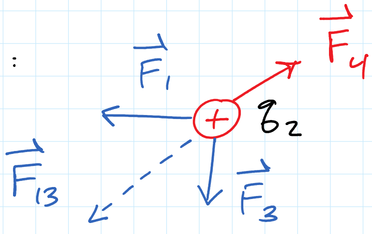
$$= kq^2\left[\frac{\sqrt{2}}{\ell^2} - \frac{1}{r^2}\right] = (9\times10^9)(4.15\times10^{-3})^2\left[\frac{\sqrt{2}}{(1)^2} - \frac{1}{(1)^2 2}\right]$$

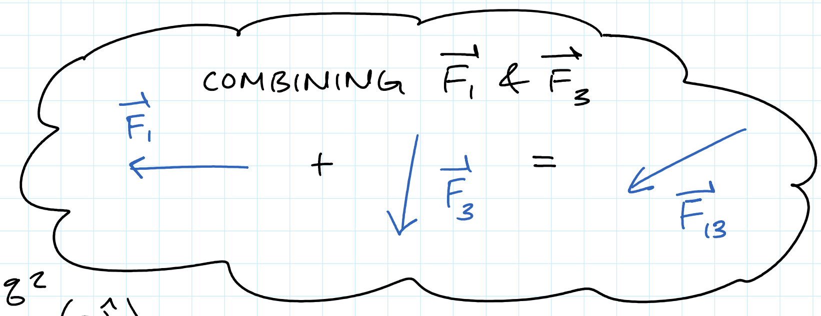
$$= \boxed{1.42\times10^5\ N/C}$$

BY LOOKING AT THE SIGN OF THE DIFFERENCE $|\vec{F}_{24}| - |\vec{F}_3|$ WE CAN FIND OUT WHICH FORCE "WINS", WHICH ALSO TELLS US THE RESULTANT DIRECTION

→ IN THIS CASE, WE SEE THAT $|\vec{F}_{24}| > |\vec{F}_3|$ SO $\vec{F}_{NET}$ POINTS IN THE DIRECTION OF $\vec{F}_{24}$ i.e. $\boxed{\text{45° SOUTH OF EAST}}$

NOW FOR q_2:





$$\vec{F}_{13} = \vec{F}_1 + \vec{F}_3 = k\frac{q^2}{\ell^2}(-\hat{\imath}) + k\frac{q^2}{\ell^2}(-\hat{\jmath})$$

$$|\vec{F}_{13}| = \sqrt{\left(k\frac{q^2}{\ell^2}\right)^2 + \left(k\frac{q^2}{\ell^2}\right)^2} = \sqrt{2}\,k\frac{q^2}{\ell^2}$$

$$|\vec{F}_{NET}| = |\vec{F}_{13}| - |\vec{F}_4| = \sqrt{2}\,k\frac{q^2}{\ell^2} - k\frac{q^2}{r^2} = kq^2\left[\frac{\sqrt{2}}{\ell^2} - \frac{1}{r^2}\right]$$

$$|F_{NET}| = |F_{13}| - |F_4| = \sqrt{2}\,k\frac{q^2}{\ell^2} - k\frac{q^2}{r^2} = kq^2\left[\frac{\sqrt{2}}{\ell^2} - \frac{1}{r^2}\right]$$

$$= kq^2\left[\frac{\sqrt{2}}{\ell^2} - \frac{1}{2\ell^2}\right] = (9\times10^9)(4.15\times10^{-3})^2\left[\frac{\sqrt{2}}{(1)^2} - \frac{1}{2(1)^2}\right]$$

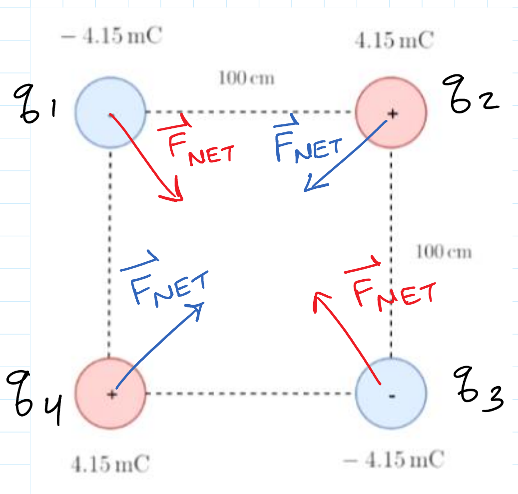
$$= \boxed{1.42\times10^5\ N/C}$$ ~ MAGNITUDE

DIRECTION: $\boxed{45^\circ \text{ SOUTH OF WEST}}$

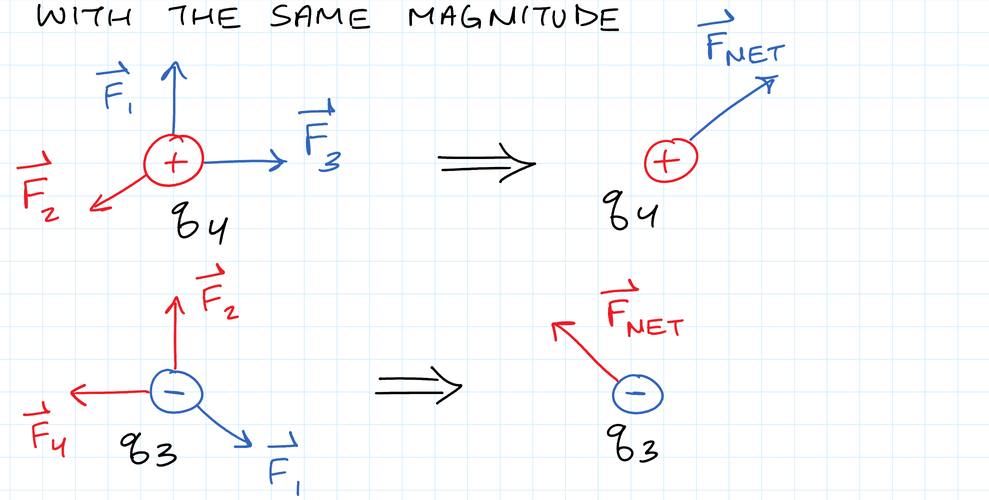
LET'S STOP HERE TO REFLECT. SO FAR, WE FOUND

q_1 : $|\vec{F}_{NET}| = 1.42\times10^5\ N/C$ & POINTS 45° SOUTH OF EAST
q_2 : $|\vec{F}_{NET}| = 1.42\times10^5\ N/C$ & POINTS 45° SOUTH OF WEST

LET'S DRAW OUT THE NET FORCE VECTORS:



SIMILARLY, BY SYMMETRY, WE SEE THAT q_3 & q_4 HAVE A NET FORCE GOING TOWARDS THE CENTER WITH THE SAME MAGNITUDE



Thank you for signing up for my LARC sessions!