Quick Notes

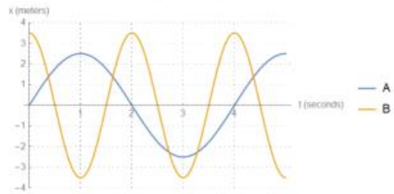
Monday, April 5, 2021 1:16 PM

Guided Practice

The following position vs. time graph depicts the SHM of two objects, labeled A and B.

(i) Determine the amplitude A, frequency f, and period T for both objects.

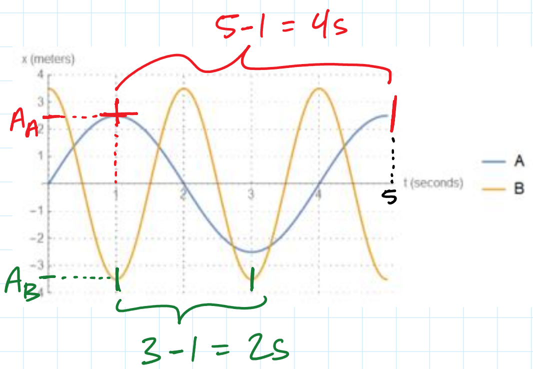
(ii) Write out the position function $x(t)$ for both objects.



Remark: This problem gives us a graph from which we extract the relevant values behind each variable. Other problems may provide the same information except in words instead of graphically.

This problem gives us a graph of the position function. Our job is to look at the graph and extract the relevant information.

= 1 WAVELENGTH OR 1 OSCILLATION



(i) FIND A, f, T

$$A: \begin{cases} A_A = 2.5\,m \\ f_A = 1/T = 0.25\,Hz \\ T_A = 4s \end{cases}$$

$$B: \begin{cases} A_B = 3.5\,m \\ f_B = 1/T_B = 0.5\,Hz \\ T_B = 2s \end{cases}$$

GENERAL FORM:

$$x(t) = A\cos(\omega t + \phi)$$

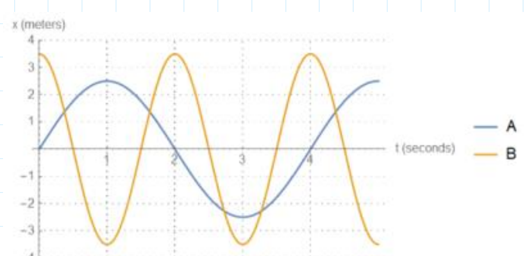
$$x_A(t) = A_A \cos(\overset{?}{\omega_A} t + \overset{?}{\phi_A})$$

phi = ϕ

$$\omega_A = \cancel{\sqrt{\frac{k}{m}}} = 2\pi f_A = 2\pi \cdot 1/4 = \pi/2$$

$$\phi_A = -\pi/2 \quad \text{OR} \quad +3\pi/2$$

NOTE: we can find the phase shift in 2 ways: graphically or algebraically.



$$x_A(t) = 2.5\cos\left(\frac{\pi}{2}t - \pi/2\right)$$

$$\text{OR } 2.5\cos\left(\frac{\pi}{2}t + 3\pi/2\right)$$

$$\text{OR } 2.5\sin\left(\frac{\pi}{2}t\right)$$

NOTE: cos() START AT MAX AMPLITUDE; sin() START AT ORIGIN

We can solve for the phase shift algebraically by picking out a point on the graph.
Let's choose a nice point like t = 1 since x would be equal to 2.5 at that point

$$x_A(t=1) = \cancel{2.5}\cos\left(\frac{\pi}{2}\cdot 1 + \phi_A\right) = \cancel{2.5}$$

$$\underbrace{\cos\left(\frac{\pi}{2}+\phi_A\right)}_{\parallel \atop 0} = 1$$

$$\Longrightarrow \frac{\pi}{2} + \phi_A = 0$$

$$\phi_A = -\frac{\pi}{2}$$

$$x_B(t) = A_B \cos(\omega_B t + \phi_B)$$

$$\omega_B = 2\pi f_B = 2\pi \cdot 1/2 = \pi \qquad \phi_B = 0$$

$$x_B(t) = 3.5\cos(\pi t)$$

$$\text{OR} \quad 3.5 \sin\left(\pi t + \frac{\pi}{2}\right)$$

Group Activity

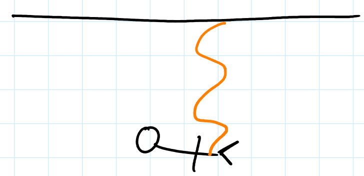
A 65.0 kg bungee jumper jumps from a high bridge. After reaching his lowest point, he oscillates up and down, hitting the low point 8 more times in 43.0 s. After a long time, he eventually comes to rest 25.0 m below the level of the bridge. Assuming friction is negligible,

(i) Estimate the spring stiffness constant k.

(ii) Determine the unstretched (natural) length ℓ_0 of the bungee cord assuming SHM.

Hint: we approximate the bungee cord as a spring, allowing us to apply the relevant equations in Ch 14

(i) FIND K



BRIDGE

$$\omega = \sqrt{\frac{K}{m}} \longrightarrow K = m\omega^2$$

?

FIND ω

$$\omega = \frac{2\pi}{T} \quad \text{OR} \quad 2\pi f \qquad \text{SINCE} \quad f = 1/T$$

FIND T OR f

$$T = \frac{[\#\ \text{OF s}]}{[1\ \text{osc}]} = \frac{43\ \text{s}}{8\ \text{osc}} = \frac{5.375\ \text{s}}{1\ \text{osc}} = 5.375\ \text{s}$$

$$f = \frac{[\#\ \text{OF osc}]}{[1\ \text{s}]} = \frac{8\ \text{osc}}{43\ \text{s}} = \frac{0.186\ \text{osc}}{1\ \text{s}} = 0.186\ \text{s}^{-1}$$

$$= 0.186\ \text{Hz}$$

$$\omega = \underbrace{2\pi}_{\text{rad}} \underbrace{f}_{1/s} = 1.17 \text{ rad/s}$$

$$\longrightarrow \quad k = m\omega^2 = (65)(1.17)^2 = \boxed{88.8 \text{ N/m}}$$

(ii)

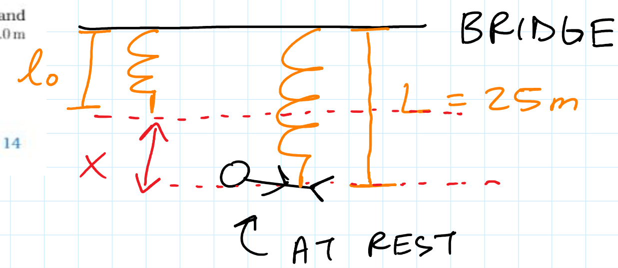
Group Activity

A $65.0\,\text{kg}$ bungee jumper jumps from a high bridge. After reaching his lowest point, he oscillates up and down, hitting the low point 8 more times in $43.0\,\text{s}$. After a long time, he eventually comes to rest $25.0\,\text{m}$ below the level of the bridge. Assuming friction is negligible,

(i) Estimate the spring stiffness constant k.

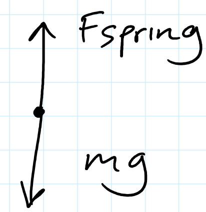
(ii) Determine the unstretched (natural) length ℓ_0 of the bungee cord assuming SHM.

Hint: we approximate the bungee cord as a spring, allowing us to apply the relevant equations in Ch 14



FIND ℓ_0

$x :=$ "DISPLACEMENT FROM EQUILIBRIUM"



$$F_{bungee} = F_{spring} = -kx$$

"AT REST" IMPLIES $\longrightarrow$ $|F_{bungee}| = |mg|$

HOW TO SUBSTITUTE x?

$$kx = mg$$

$\longrightarrow \quad x = L - \ell_0$

$$k(L - \ell_0) = mg$$

$$\ell_0 = L - \frac{mg}{k}$$

$$= 25 - \frac{(65)(9.8)}{(88.8)} \approx \boxed{17.8 \text{ m}}$$