Group #5 @1pm
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Lecture Worksheet 14

Task 1

- Find 5 distinct examples random variables in everyday life? Make sure that, at least two of them, do not assume real values.

  1. State of a traffic light
  2. Length of a TV show
  3. Time it takes to cook chicken
  4. The weather on a given day
  5. The price of a stock

- In the examples you just gave, is there a natural sample space on which you can think the random variables to be defined on?

  We tend to attribute the sample space to be defined on similar elements of the target space.
  However there is not always a natural sample space. If we did know the details of the sample space, then the source of the randomness would be clear.

- What do you think is the role of a sample space? Do you think that it is natural to think about a random variable starting with a sample space?

  The role of our sample space sort of shapes the outcomes and their probabilities.
  It is probably more natural to think about random variables and then try to derive where they come from and their sample space.

Task 2

A coin that comes up heads with probability p is flipped until heads showed up. When that happens, the number of tosses is recorded. Denote that number by X.

- What values can X assume?

  X may assume any number of the set of Natural Numbers ie $x \in \mathbb{N}$

- Is X a random variable?

  Yes because we cannot predict the amount of tosses to get heads.

- What are the probabilities that it assumes its various values?

  The probabilities can take on the form:

$$P(X = 1) = p$$
$$P(X = 2) = (1 - p) \cdot p$$
$$P(X = 3) = (1 - p) \cdot (1 - p) \cdot p = (1 - p)^2 \cdot p$$
$$P(X = n) = (1 - p)^{n-1} \cdot p$$

  Here we assume that each flip is independent, so we're able to multiply the probabilities.

- What is the sample space on which X is defined?

  The sample space takes on the form of the lists of outcomes of each flip
  For n-flips in total, the last flip has to be heads since we stop until heads shows up.

$$S = \{(\omega_1, \omega_2, ..., \omega_{n-1}, H) \mid \omega_i = \{H, T\}\}$$

- Do you see any use to knowing the sample space?

  The sample space is defined somewhat vaguely since in principle the coin tosses could go on forever resulting in infinity number of tosses, which may not be useful.
  However, knowing that the last element of the sample space is always heads is useful since it helps us calculate the probability.

Task 3

Five distinct natural numbers are randomly distributed to five Players called A,B,C,D,E. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially Players A and B compare their numbers; the winner then compares her number with that of Player C, and so on. Let X denote the number of times Player A is a winner.

- Find $P(X = i)$ for $i = 0, 1, ..., 4$

Let's assume that the randomly given numbers are $\{1, 2, 3, 4, 5\}$

For $P(X = 0)$*, this means Player A lost to Player B.*

$$\begin{aligned} P(X=0) &= P(A_1 \cap B_{2,3,4,5}) \\ &\quad + P(A_2 \cap B_{3,4,5}) \\ &\quad + P(A_3 \cap B_{4,5}) \\ &\quad + P(A_4 \cap B_5) \\ &= \frac{1}{5} \cdot \frac{4}{4} + \frac{1}{5} \cdot \frac{3}{4} + \frac{1}{5} \cdot \frac{2}{4} + \frac{1}{5} \cdot \frac{1}{4} \\ &= 50\% \end{aligned}$$

For $P(X = 1)$*, this means Player A won to B but lost to C.*

$$\begin{aligned} P(X=1) &= P(A_2 \cap B_1 \cap C_{3,4,5}) \\ &\quad + P(A_3 \cap B_{1,2} \cap C_{4,5}) \\ &\quad + P(A_4 \cap B_{1,2,3} \cap C_5) \\ &= \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{3}{3} + \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} \\ &\approx 16.7\% \end{aligned}$$

For $P(X = 2)$*, this means Player A won to B and C, but* *lost to D.*

$$\begin{aligned} P(X=1) &= P(A_3 \cap B_{1,2} \cap C_{1,2} \cap D_{4,5}) \\ &\quad + P(A_4 \cap (B_{1,2,3} \cap C_{1,2,3} \cap D_5) \\ &= \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{2} + \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \\ &\approx 8.33\% \end{aligned}$$

For $P(X = 3)$, this means Player A won to B,C, and D, but lost to E.

$$\begin{aligned} P(X=1) &= P(A_4 \cap B_{1,2,3} \cap C_{1,2,3} \cap D_{1,2,3} \cap E_5) \\ &= \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} \\ &= 5\% \end{aligned}$$

For $P(X = 4)$, this means Player A was unstoppable, beating B,C,D and E.

$$\begin{aligned} P(X=1) &= P(A_5 \cap B_{1,2,3,4} \cap C_{1,2,3,4} \cap D_{1,2,3,4} \cap E_{1,2,3,4}) \\ &= \frac{1}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1} \\ &= 20\% \end{aligned}$$