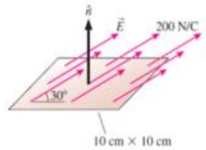
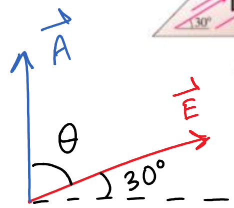
What is the electric flux through the surface shown below?

Answer: $\Phi = 1.0\,\mathrm{Nm^2/C}$





$\theta = 90^\circ - 30^\circ = 60^\circ$

FIND Φ

FORMULA: $\Phi = \oint \vec{E} \cdot d\vec{A}$

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$
$$= \oint E\, dA \cos\theta \qquad \because \vec{E} \cdot d\vec{A} = |\vec{E}||d\vec{A}|\cos\theta = E\, dA \cos\theta$$
$$= E\cos\theta \oint dA \qquad \because E \text{ \& } \cos\theta \text{ ARE CONST.}$$
$$= E\cos\theta\, A_{TOTAL} \qquad \because \oint dA = A_{TOTAL}$$
$$= (200)\cos 60^\circ\,(0.10)^2$$
$$= \boxed{1\ \mathrm{Nm^2/C}}$$

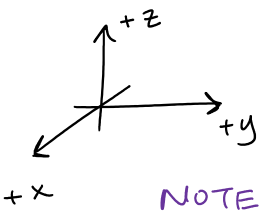
NEED TO ADD Φ PIECEWISE SINCE THERE ARE DIFFERENT SURFACES

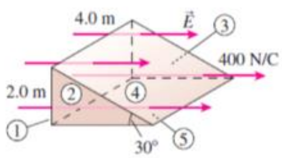
Find the net electric flux through the surface shown below

NOTE: Flux actually has pos/neg signs to it: incoming is negative while outcoming is positive.

Answer: $\Phi_{net} = 0$

AXES:

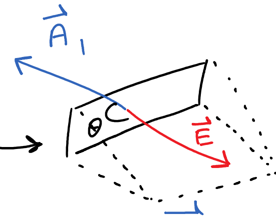




FIND $\underbrace{\Phi_{NET} = \Phi_1 + \Phi_2 + \dots + \Phi_6}$

FOR (1):

$\theta = 180^\circ$



$$\vec{A}_1 = A_1(-\hat{y})$$
$$= (4 \cdot 2)(-\hat{y})$$
$$= -8\hat{y}$$
$$\vec{E} = 400(+\hat{y})$$

NOTE: WE CAN FIND Φ BY TAKING THE DOT PRODUCT IN 2 WAYS:

USING $\cos\theta$

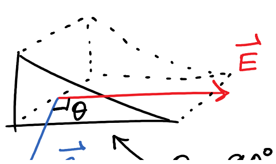
$$\Phi_1 = \oint \vec{E} \cdot d\vec{A}_1$$
$$= \oint E\, dA \cos\theta$$
$$= E A_1 \cos\theta$$
$$= (400)(8)\cos 180^\circ$$
$$= -3200\ \mathrm{Nm^2/C}$$

USING VECTORS

$$\Phi_1 = \oint \vec{E} \cdot d\vec{A}_1$$
$$= \vec{E} \cdot \oint d\vec{A}_1$$
$$= \vec{E} \cdot \vec{A}_1$$
$$= (0\hat{x} + 400\hat{y} + 0\hat{z}) \cdot (0\hat{x} - 8\hat{y} + 0\hat{z})$$
$$= (0)(0) + (400)(-8) + (0)(0)$$
$$= -3200\ \mathrm{Nm^2/C}$$

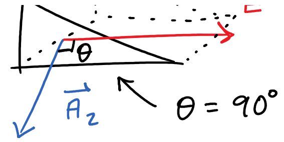
* FOR THE REST I'M JUST GOING TO USE $\cos\theta$ VERSION *

FOR (2):



$$\Phi_2 = \oint \vec{E} \cdot d\vec{A}_2$$
$$= \oint E\, dA_2 \cos\theta$$

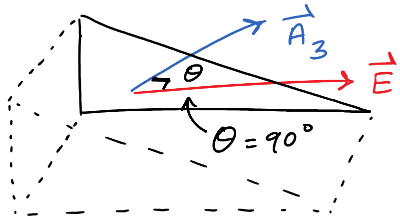
FOR (2):



$$\Phi_2 = \oint \vec{E} \cdot d\vec{A}_2$$
$$= \oint E\, dA_2 \cos\theta$$
$$= \oint E\, dA_2 \cancel{\cos 90°}^{\,0} = 0$$

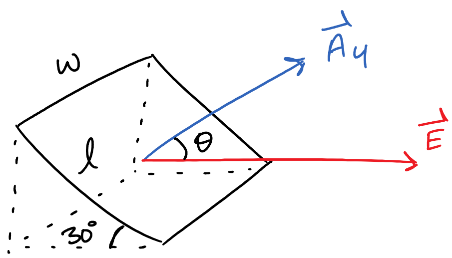
NOTE: INTUITIVELY, WE CAN SEE THAT THERE ARE NO $\vec{E}$ FIELD LINES FLOWING THROUGH THE AREA (2); THUS THERE IS NO FLUX.

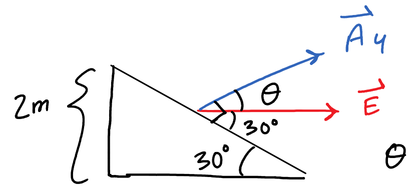
FOR (3):



$$\Phi = \oint \vec{E} \cdot d\vec{A}_3$$
$$= \oint E\, dA_3 \cos\theta$$
$$= \oint E\, dA_3 \cancel{\cos 90°}^{\,0} = 0$$

FOR (4):



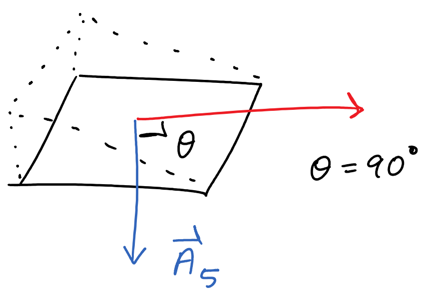


$$\theta = 90° - 30° = 60°$$

$$\Phi = \oint \vec{E} \cdot d\vec{A}_4$$
$$= E A_4 \cos\theta$$
$$= (400)(16)\cos 60°$$
$$= +3200\ Nm^2/C$$

$$A_4 = lw$$
$$= \left(\frac{2}{\sin 30°}\right)(4) = 16\,m^2$$

FOR (5):



$$\theta = 90°$$

$$\Phi_5 = \oint \vec{E} \cdot d\vec{A}_5$$
$$= \oint E\, dA_5 \cos\theta$$
$$= \oint E\, dA_5 \cancel{\cos 90°}^{\,0} = 0$$

PUTTING IT ALL TOGETHER:

$$\Phi_{NET} = \Phi_1 + \cancel{\Phi_2}^{\,0} + \cancel{\Phi_3}^{\,0} + \cancel{\Phi_4}^{\,0} + \Phi_5$$
$$= (+3200) + (-3200) = \boxed{0\ Nm^2/C}$$

"ALL THAT WORK JUST TO GET ZERO ?!?!"

"IT BE LIKE THAT FAM"

POSITIVE

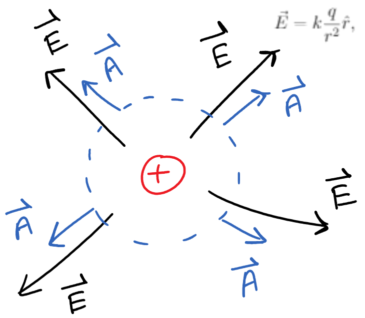
Using Gauss' Law, derive the following equation for the electric field of a point charge

GAUSS' LAW: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\ldots}$

POSITIVE

Using Gauss' Law, derive the following equation for the electric field of a point charge

$$\vec{E} = k\frac{q}{r^2}\hat{r}, \quad \text{radially outwards}$$



GAUSS' LAW: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

$\longrightarrow \oint E\, dA \cos\theta = \frac{Q_{enc}}{\epsilon_0}$ $\quad \because$ DOT-PRODUCT

$E\cos\theta \oint dA = \ldots$ $\quad \because$ $E\cos\theta$ IS CONST.

$E\cos\theta\, A_{TOTAL} = \ldots$ $\quad \because \oint dA = A_{TOTAL}$

$E\,\cancel{\cos 0^\circ}^{\,1}(4\pi r^2) = \frac{+q}{\epsilon_0}$ $\quad \because \theta = 0^\circ,\ A_{TOTAL} = 4\pi r^2$

$Q_{enc} = +q$

$E = \underbrace{\frac{1}{4\pi\epsilon_0}}_{=\,K} \cdot \frac{q}{r^2}$ MAGNITUDE

θ := "ANGLE B/W $\vec{E}$ & $\vec{A}$"

$= 0^\circ$

A_{TOTAL} := "SURFACE AREA OF GAUSSIAN SURFACE"

WE KNOW THE DIRECTION IS RADIALLY OUTWARD SO

$\longrightarrow \vec{E} = k\frac{q}{r^2}(+\hat{r})$ THIS MEANS "RADIALLY OUTWARD"

A spherical cavity of radius 4.50 cm is at the center of a metal sphere of radius 18.0 cm. A point charge $q = +5.5\,\mu C$ rests at the center of the cavity, whereas the metal sphere carries no net charge. Determine the electric field at a point

(a) 3.00 cm from the center of the cavity.

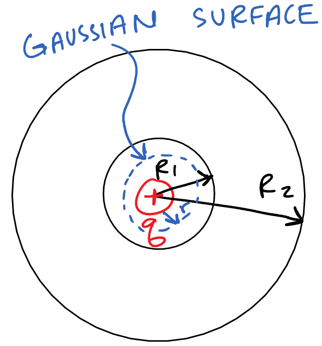
(b) 6.00 cm from the center of the cavity.

(c) 30.0 cm from the center of the cavity.

Answers: (a) $E_1 = 5.49 \times 10^7$ N/C, (b) $E_2 = 0$, (c) $E_3 = 5.49 \times 10^7$ N/C

TYPO $\sim 5.49\times10^5$

$\begin{cases} R_1 = 0.045m \\ R_2 = 0.18m \end{cases}$



(a) AT $r = 0.03m$ ~ INSIDE CAVITY ($r < R_1$)

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$E\,A_{TOTAL} = \ldots$

$E(4\pi r^2) = \frac{q}{\epsilon_0} \longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi(8.85\times10^{-12})}\frac{(5.5\times10^{-6})}{(0.03)^2}$

$= \boxed{5.5\times10^7\ N/C}$

(b) FOR $r = 0.06m$ ~ WITHIN THE SPHERICAL SHELL'S THICKNESS ($R_1 < r < R_2$)

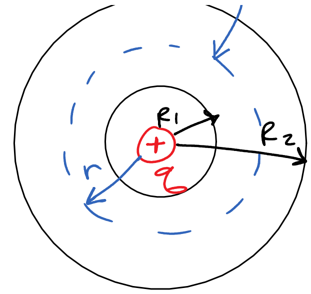
0 SINCE CONDUCTOR

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\ldots}$ $\quad \frac{Q_{enc}}{\ldots}$

0 SINCE CONDUCTOR

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \longrightarrow \cancel{E}(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$\longrightarrow \boxed{E=0}$$

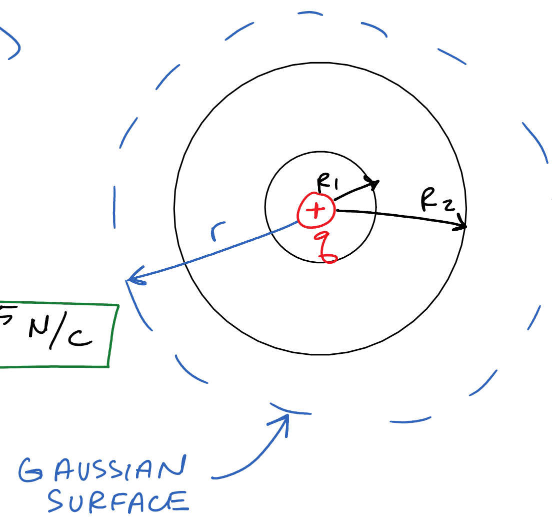


(c) FOR $r = 0.30\,m$ ~ OUTSIDE SHELL $(r > R_2)$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \longrightarrow E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\longrightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$= \frac{1}{4\pi(8.85\times10^{-12})}\frac{(5.5\times10^{-6})}{(0.30)^2} = \boxed{5.5\times10^{5}\ N/C}$$



A spherical cavity of radius 4.50 cm is at the center of a metal sphere of radius 18.0 cm. A point charge $q = +5.5\,\mu C$ rests at the center of the cavity, whereas the metal sphere carries a net charge of $Q = -5.5\,\mu C$.

Determine the electric field at a point

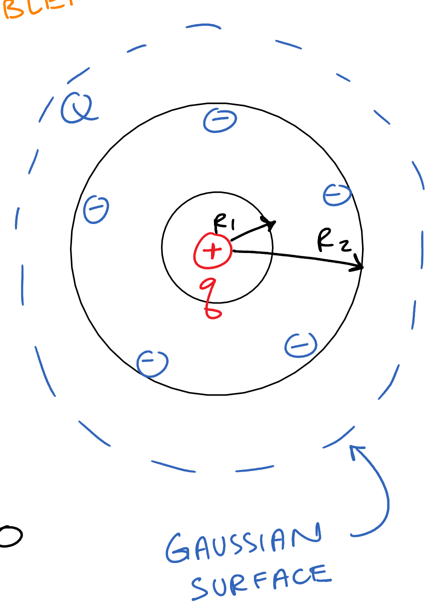
(a) 3.00 cm from the center of the cavity.

(b) 6.00 cm from the center of the cavity.

$\}$ IDENTICAL TO PREVIOUS PROBLEM

(c) 30.0 cm from the center of the cavity.

Answer: (a) $E_1 = 5.49\times10^{7}$ N/C, (b) $E_2 = 0$, (c) $E_3 = 0$



(c) FOR $r = 0.30\,m$ ~ OUTSIDE SHELL $(r > R_2)$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \longrightarrow E(4\pi r^2) = \frac{(Q+q)}{\epsilon_0}$$

$$\ldots = \frac{(-5.5+5.5)}{\epsilon_0} = 0$$

$$\longrightarrow \boxed{E=0}$$

NOTE: $Q_{enc} = 0$ SINCE $Q = -5.5\,\mu C$ & $q = +5.5\,\mu C$ CANCELS EACH OTHER OUT