Quick Notes

Wednesday, April 7, 2021 5:06 PM

Objective:

I. Solve a problem that involves a **pendulum** in SHM

- ☐ By using the same approach for a **spring-mass** in SHM

II. Solve for the **velocity** and **acceleration** for an object in SHM

- ☐ By solving for $x(t)$ and then finding its **derivatives**.
- ☐ By using **Conservation of Energy**

Content Review: BUY 1 GET 2 FREE

- Recall: for an object in SHM, the **position function** $x(t)$ is generally given by

$$x(t) = A\cos(\omega t + \phi) \longrightarrow x_{MAX} = |x(t)| = A$$

where the variables are

A = amplitude ω = angular frequency ϕ = phase shift

- Taking the derivative (with respect to time) of $x(t)$ gives us the **velocity function** $v(t)$

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \longrightarrow v_{MAX} = |v(t)| = \omega A$$

- Taking the derivative of $v(t)$ gives us the **acceleration function** $a(t)$

$$a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi) \longrightarrow a_{MAX} = |a(t)| = \omega^2 A$$

NOTE: $-1 \le \sin, \cos \le +1$

- The angular frequency ω has 2 forms: spring-mass and pendulum

$$\omega_{spr} = \sqrt{\frac{k}{m}} \qquad \omega_{pen} = \sqrt{\frac{g}{\ell}} \sim \sqrt{\frac{[m]/[s^2]}{[m]}} = \sqrt{1/[s^2]}$$

where the variables are

k = spring constant, m = mass of object, ℓ = length of string, g = acc. due to gravity

$$\omega = \text{rad}/s = 1/s \;//$$

- The **total energy** of the oscillating spring-mass system is given by

$$E_{tot} = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

where the variables are

U = spring potential energy K = kinetic energy

Guided Practice

A pendulum has a period of 1.35 s on Earth. Suppose the same pendulum is now on Mars, where the acceleration of gravity is about 0.37 that on Earth.

(i) Determine its period on Mars.

(ii) How should the pendulum be modified so that its period is the same as on Earth?

NOTE: WE SHOULD IDENTIFY THIS PROBLEM AS A RATIO PROBLEM

(i) FIND T_M

$$\omega_{pen} = \sqrt{\frac{g}{\ell}} \overset{?}{\sim} T \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

$$\longrightarrow \sqrt{\frac{g}{\ell}} = \frac{2\pi}{T} \longrightarrow \boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

$$T_M = 2\pi\sqrt{\frac{\ell_M}{g_M}} \quad \text{AND} \quad T_E = 2\pi\sqrt{\frac{\ell_E}{g_E}}$$

CONSIDER

$$\text{RATIO} = \frac{T_M}{T_E} = \frac{\cancel{2\pi}\sqrt{\frac{l_M}{g_M}}}{\cancel{2\pi}\sqrt{\frac{l_E}{g_E}}} = \sqrt{\frac{\cancel{l_M}}{g_M}} \times \sqrt{\frac{g_E}{\cancel{l_E}}}$$

NOTE : "SAME PENDULUM" $\longrightarrow$ $l_M = l_E$

$$= \sqrt{\frac{g_E}{g_M}} = \sqrt{\frac{\cancel{g_E}}{0.37\,\cancel{g_E}}}$$

Guided Practice

A pendulum has a period of $1.35\,\mathrm{s}$ on Earth. Suppose the same pendulum is now on Mars, where the acceleration of gravity is about 0.37 that on Earth.

(i) Determine its period on Mars.

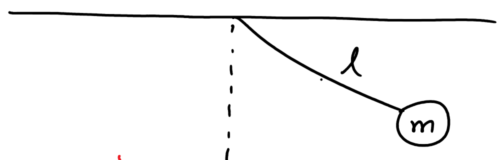
$g_M = 0.37\, g_E$

(ii) How should the pendulum be modified so that its period is the same as on Earth?

$$\text{RATIO} = \frac{T_M}{T_E} = \sqrt{\frac{1}{0.37}} \longrightarrow T_M = \sqrt{\frac{1}{0.37}}\, T_E$$

$$= \boxed{2.2\mathrm{s}}$$

(ii) $\boxed{T_{pen} = 2\pi\sqrt{\frac{l}{g}}}$



WE WOULD CHANGE l (DECREASE IT)

NOTE : CHANGING m DOES NOT AFFECT T

SINCE T_{pen} DOES NOT DEPEND ON m

Group Activity

An object with mass $2.7\,\mathrm{kg}$ is executing SHM, attached to a spring with $k = 280\,\mathrm{N/m}$. When the object is $0.020\,\mathrm{m}$ from its equilibrium position, it is moving with a speed of $0.55\,\mathrm{m/s}$.

- Determine the amplitude of the motion.
- Find the maximum speed of the object as it's oscillating.

GIVEN INFO : m, K

" WHEN $x = 0.02\mathrm{m} \rightarrow v = 0.55\ \mathrm{m/s}$ "

(i) FIND A

2 WAYS TO APPROACH THIS PROBLEM

(1) USING CONS. OF ENERGY

(2) USING x (t) , v (t)

$$\begin{cases} x(t) = A\cos(\omega t + \phi) \\ v(t) = -\omega A \sin(\omega t + \phi) \end{cases}$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi)$$

$$\omega_{spr} = \sqrt{\frac{K}{m}}$$

$\longrightarrow \{\ x = A\cos(\omega t + \phi) = 0.02$

$$\rightarrow \begin{cases} x = A\cos(\omega t + \phi) = 0.02 \\ v = -\omega A \sin(\omega t + \phi) = 0.55 \end{cases}$$

NOTE: WE ARE HARD-STUCK B/C 3 UNKNOWNS AND CANT ISOLATE SIN & COS

↳ APPROACH #2 IS A DEAD-END :(

NOW, USING CONS. OF ENERGY:

Group Activity

An object with mass $2.7\,\mathrm{kg}$ is executing SHM, attached to a spring with $k = 280\,\mathrm{N/m}$. When the object is $0.020\,\mathrm{m}$ from its equilibrium position, it is moving with a speed of $0.55\,\mathrm{m/s}$.

- Determine the amplitude of the motion.
- Find the maximum speed of the object as it's oscillating.

$$E_{TOTAL} = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$E_{TOTAL} = U_{MAX} \sim \frac{1}{2}kx^2 = \frac{1}{2}kA^2, \quad x = A$$

$$\rightarrow \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$A = \sqrt{x^2 + \frac{m}{k}v^2} = \sqrt{(0.02)^2 + \frac{(2.7)}{(280)}(0.55)^2} = \boxed{0.058\,m}$$

(ii) FIND v_{MAX}

Group Activity

An object with mass $2.7\,\mathrm{kg}$ is executing SHM, attached to a spring with $k = 280\,\mathrm{N/m}$. When the object is $0.020\,\mathrm{m}$ from its equilibrium position, it is moving with a speed of $0.55\,\mathrm{m/s}$.

- Determine the amplitude of the motion.
- Find the maximum speed of the object as it's oscillating.

$$v_{MAX} = \omega A = \sqrt{\frac{k}{m}}\,A = \sqrt{\frac{280}{2.7}}(0.058) = \boxed{0.59\,m/s}$$

ALTERNATIVELY,

$$E_{TOTAL} = \frac{1}{2}k\underset{0.02}{x^2} + \frac{1}{2}m\underset{0.55}{v^2} = \frac{1}{2}mv_{MAX}^2$$

NOTE: E_{TOTAL} IS CONST.

$$\hookrightarrow \boxed{v_{MAX} = \sqrt{\frac{k}{m}x^2 + v^2}}$$

$$E_{TOTAL} = K_{MAX} \sim \frac{1}{2}mv^2 = \frac{1}{2}mv_{MAX}^2$$

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$$E_{TOTAL} = U_{MAX} = K_{MAX} \longrightarrow \cancel{\tfrac{1}{2}}kA^2 = \cancel{\tfrac{1}{2}}mv_{MAX}^2$$

These are all valid ways of solving for the maximum speed!

$$v_{MAX} = \boxed{\sqrt{\frac{k}{m}A^2} = \omega A}$$

happy Week 2!
hope you find this helpful :)