Group #5 @1pm
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Lecture Worksheet 5

Task 1

- Give at least 3 examples of experiments with finite sample spaces
  - *Shooting a basketball:*
    $S = \{Goes\ inside\ hoop,\ Misses\ hoop\}$
  - *Asking a person what their favorite sport is:*
    $S = \{Basketball,\ Soccer,\ Football,\ Ping\ Pong,\ldots\ \}$
  - *Patient's status after a surgery:*
    $S = \{Alive,\ Dead\}$
- Give at least 3 examples of experiments with finite sample space and equiprobable outcomes
  - *Buying a lottery ticket number out of several tickets:*
    $S = \{Lottery\ ticket\ numbers\}$
  - *Choosing a card out of a "well-shuffled" deck:*
    $S = \{Cards\ in\ a\ deck\}$
  - *Guessing an answer on a multiple-choice question:*
    $S = \{Answer\ Choices\ ie\ \{A,B,C,D,\ldots\}\}$
- Give at least 3 examples of experiments with finite sample space and non-equiprobable outcomes
  - *Recording the weather:*
    $S = \{Sunny,\ Rainy,\ Cloudy,\ldots\}$
  - *Ranking people who are competing in some contest:*
    $S = \{First,\ Second,\ Third,\ldots\ \}$
  - *Asking people what grade they got in Math 2B:*
    $S = \{A,B,C,D,F\}$

Task 2

- What is the probability that 4 cards of identical value be found in a hand of 5 cards randomly picked from a deck of 52 cards?

  Let E be the Event of having 4 cards of identical value out of a hand of 5 cards. There are various ways to try to find #E.
  One way is by doing:

$$\#E = \underbrace{\binom{13}{1} \cdot \binom{4}{4}}_{\text{13 values, choose 1 and 4 suits, choose all 4}} \cdot \overbrace{\binom{48}{1}}^{\text{48 cards left, choose 1}}$$

  Let S be the Sample Space of choosing a group of 4 out of a total of 52 objects.
  It follows that:

$$\#S = \binom{52}{5}$$

  Using the principle of $\frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$

$$P(E) = \frac{\#E}{\#S} = \frac{\binom{13}{1} \cdot \binom{4}{4} \cdot \binom{48}{1}}{\binom{52}{5}} = 0.00024 \approx 0.024\%$$

Task 3

- What is the probability that there will be no partners in a group of 5 people randomly picked from a group of 10 couples?

  Let E be the Event of there being no partners in a group of 5 people selected from a group of 10 couples
  It follows that:

$$\#E = \underbrace{\binom{10}{5}}_{\text{10 couples, choose 5}} \overbrace{\binom{2}{1}^{5}}^{\text{2 partners, choose 1, do this 5x}}$$

  Let S be the Sample Space of choosing a group of 5 from a total of 20 objects.

It follows that:

$$\#S = \binom{20}{5}$$

The probability of E occurring is given by:

$$P(E) = \frac{\#E}{\#S} = \frac{\binom{10}{5} \cdot \binom{2}{1}^5}{\binom{20}{5}} \approx 52\%$$

Task 4

Draw 3 balls from a bag containing 6 white balls and 5 black balls.

- What is the probability of obtaining 2 black and 1 white ball?

  Let S be the Sample Space of choosing 3 balls from the bag of 11 balls
  It follows that:

$$\#S = \binom{11}{3}$$

  Let E be the Event of obtaining 2 black and 1 white ball.
  It follows that:

$$\#E = \underbrace{\binom{5}{2}}_{\text{5 blacks, choose 2}} \cdot \overbrace{\binom{6}{1}}^{\text{6 whites, choose 1}}$$

  The probability is given by:

$$P(E) = \frac{\#E}{\#S} = \frac{\binom{5}{2} \cdot \binom{6}{1}}{\binom{11}{3}} \approx 36\%$$

- What is the probability of obtaining {black,white,black} in that order?

  Let F be the event of obtaining {black,white,black}
  It follows that:

$$\#F = \underbrace{\binom{5}{1}}_{\text{5 blacks, choose 1}} \cdot \overbrace{\binom{6}{1}}^{\text{6 whites, choose 1}} \cdot \underbrace{\binom{4}{1}}_{\text{4 blacks, choose 1}}$$

Here our Sample Space is now order-dependent. Since we're conducting 3 experiments (drawing a ball) we woudl multiply by the number of outcomes of each experiment.

$$\#S = 11 \cdot 10 \cdot 9$$

The probability is given by:

$$P(F) = \frac{\#F}{\#S} = \frac{\binom{5}{1} \cdot \binom{6}{1} \cdot \binom{4}{1}}{11 \cdot 10 \cdot 9} \approx 12\%$$