Group #5 @1pm
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Lecture Worksheet 8

Task 1

We know that $P(E \cap F) = P(E)P(F|E)$ for any events $E, F \subset S$ of a probability space.

- Explain the meaning of the formula in plain English. You may argue in terms of frequency of occurrence

  We can obtain the number of occurrences of E and F by finding the ratio of F occurrences per E occurrences; then multiply this by the number of E occurrences.
  Mathematically, this looks like:

  $$\frac{P(E \cap F)}{P(E)} \cdot P(E)$$

  The ratio represents how much E has in common with F.

  Alternatively, we may first try to reason that the probability of 2 events happening ought to be less than or equal to the probability of 1 event.
  Then we can put together the formula by getting the probability of 1 event, P(E), and then "modify" the quantity so that we limit the outcomes to just F given E.
  Otherwise, we would be counting elements of F where E did not occur, which is irrelevant.

- Find a similar formula which is valid for 3 sets and prove it

  $$P(E \cap F \cap G) = P(E)P(F|E)P(G|E \cap F)$$

  Again, by the above reasoning, we find that the probability of 3 events occurring ought to be less than the probabilities of 1, and 2 events occurring.
  So we can start by getting the probability of E first.

  $$P(E \cap F \cap G) = P(E)...$$

To incorporate the next event, let's say F, we have isolate the outcomes of F that only occur when E does ie F intersect E. Then we find this probability and multiply.

$$P(E \cap F \cap G) = P(E)P(F|E)...$$

To include G, we have to isolate the outcomes where F and E do not occur.

$$P(E \cap F \cap G) = P(E)P(F|E)P(G|E \cap F)$$

Now, to double check if this holds, we can plug in the definition of Conditional Probability

$$\begin{aligned} P(E)P(F|E)P(G|E \cap F) &= P(E)\frac{P(E \cap F)}{P(E)}\frac{P(E \cap F \cap G)}{P(E \cap F)} \\ &= P(E \cap F \cap G) \end{aligned}$$

- Find the general formula for $4 \leq n$ sets and prove it

  We find that the general pattern is such that we multiply by the Conditional Probabilities until we've accounted for all events $E_1, E_2, ..., E_n$

  At first the equation resembles the case of $n = 3$

  $$P(E_1 \cap E_2 \cap ... \cap E_n) = P(E_1)P(E_2|E_1)P(E_3|E_1 \cap E_2)...$$

  As add the i-th term, we have to isolate the terms from 1 up to i-1. Otherwise we would be counting events that should not have occurred.

  The equation can be generalized to:

  $$P(E_1 \cap E_2 \cap ... \cap E_n) = P(E_1)P(E_2|E_1)P(E_3|E_1 \cap E_2)...P(E_n|E_1 \cap ... \cap E_{n-1})$$

Task 2

Consider a regular deck of 52 cards with 4 suits and 13 values.

- What is the probability of drawing 4 consecutive diamonds?
  Consider the event D_j that the j^{th} draw is diamonds for $j = 1, 2, 3, 4$

  The event of drawing 4 consecutive diamonds can be denoted as the union $D_1 \cup D_2 \cup D_3 \cup D_4$. The probability is given by:

$$
\begin{aligned}
P(D_1 \cap D_2 \cap D_3 \cap D_4) &= P(D_1) \cdot P(D_2|D_1) \cdot P(D_3|D_1 \cap D_2) \cdot P(D_4|D_1 \cap D_2 \cap D_3) \\
&= \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \\
&\approx 0.26\%
\end{aligned}
$$

We can check our answer by doing the old-school method of multiplying the number of outcomes per experiment (event) and applying the principle of favorable over total outcomes.

Since order doesn't matter, the probability is given by:

$$P(D_1 \cap D_2 \cap D_3 \cap D_4) = \frac{\binom{13}{4}}{\binom{52}{4}} \approx 0.26\%$$

Task 3

Consider the following table that records pet ownership percentages according to biological gender

	Pet	**No Pet**
Male	0.41	0.08
Female	0.45	0.06

- What is the probability that a randomly selected person is Male, given that they own a pet?

  Using the events M,F, Pe, and noPe, the probability is given by:

$$P(M|Pe) = \frac{P(M \cap Pe)}{P(Pe)} = \frac{0.41}{0.41 + 0.45} \approx 48\%$$