Ch 14 Oscillations

Key words: simple harmonic motion (SHM), Hooke's Law, spring constant, spring-mass, pendulum, position function, displacement from equilibrium, frequency, period.

Note: Ch 14 MasteringPhysics homework due on Monday 4/5 @11:59pm

Objective:

- Familiarize ourselves with the position function $x(t)$ in its generalized form.
  - By briefly checking out the PDF with SHM derivation.
- Craft our own handmade position function $x(t)$ to model the object's motion.
  - By fitting the variables A, k, ω, ϕ to the situation.

Content Review:

- The overarching goal is to cook up a position function $x(t)$ to model the object's motion, so that at any time t, we know the position x.

  In its general form,

  $$\text{position function:} \quad x(t) = A\cos(\omega t + \phi)$$

  where the variables are

  $$A = \text{amplitude} \qquad \omega = \text{angular frequency} \qquad \phi = \text{phase shift}$$

- For a spring-mass system, the angular frequency has this identity

  $$\omega = \sqrt{\frac{k}{m}} \qquad \text{where } k \text{ is the spring constant}$$

- In general, the oscillation frequency has 2 forms: "normal" frequency f and angular frequency ω.

  The two variables are related by

  $$\omega = 2\pi f$$

  Contextualizing this,

  $$f = \frac{\text{\# of oscillations}}{\text{1 second}} \qquad \omega = \frac{\text{\# of oscillations}}{\text{1 second}} \times \frac{2\pi \text{ radians}}{\text{1 oscillation}} = \frac{\text{\# of radians}}{\text{1 second}}$$

- **Important ideas/remarks**:
  - The amplitude A is the maximum displacement from equilibrium.
  - The angular frequency ω is identical to angular velocity; that is, how many radians the object is traversing per 1 second.

    e.g. If the object makes 1 revolution (2π radians) per second, then $\omega = 2\pi \,\text{rad/s}$
  - The phase shift ϕ serves as a mathematical adjustment to the initial value of the cosine function, which is related to the initial position of the object

    e.g. Let $\phi = 0$ so that the position function simplifies to $x(t) = A\cos(\omega t)$.

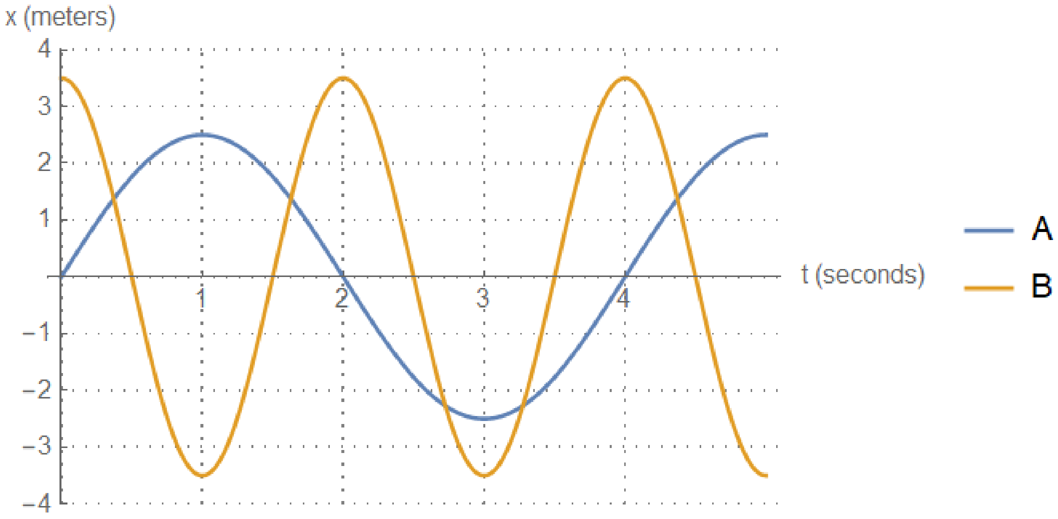
    This entails that at $t = 0$, $x = +A$, implying that the object's initial position is at its amplitude in the positive direction. If we are explicitly told that the object starts at the origin, then we need to shift the phase of the cosine function by $\pi/2$ so that we have at $t = 0$, $x = 0$. Alternatively, this is identical to using a sine function instead of cosine.

Guided Practice

The following position vs. time graph depicts the SHM of two objects, labeled A and B.

(i) Determine the amplitude A, frequency f, and period T for both objects.

(ii) Write out the position function $x(t)$ for both objects.



Remark: This problem gives us a graph from which we extract the relevant values behind each variable. Other problems may provide the same information except in words instead of graphically.

Solution

(i) $A_A = 2.5\,\mathrm{m},\ f_A = 0.25\,\mathrm{Hz},\ T_A = 4.0\,\mathrm{s}$ $\qquad A_B = 3.5\,\mathrm{m},\ f_B = 0.50\,\mathrm{Hz},\ T_B = 2.0\,\mathrm{s}$

(ii) $x_A(t) = 2.5 \sin\left(\frac{1}{2}\pi t\right)$ $\qquad x_B(t) = 3.5 \cos\left(\pi t\right)$

Group Activity

A $65.0\,\mathrm{kg}$ bungee jumper jumps from a high bridge. After reaching his lowest point, he oscillates up and down, hitting the low point 8 more times in $43.0\,\mathrm{s}$. After a long time, he eventually comes to rest $25.0\,\mathrm{m}$ below the level of the bridge. Assuming friction is negligible,

(i) Estimate the spring stiffness constant k.

(ii) Determine the unstretched (natural) length ℓ_0 of the bungee cord assuming SHM.

Hint: we approximate the bungee cord as a spring, allowing us to apply the relevant equations in Ch 14

Solution

(i) $k = 88.8\,\mathrm{N/m}$

(ii) $\ell_0 = 17.8\,\mathrm{m}$