Ch 13 Fluids

Season 1 Episode 1 - HYDROSTATICS

In this episode of LARC Physics 3B, we're going to . . .

- Create a foundation to evaluate/predict fluid motion by first considering fluids at rest.
- Applying relevant concepts/equations, we can answer questions like:
  - Will this object sink or float? How deep can a submarine go? How many helium balloons does one need to fly? Is water wet?

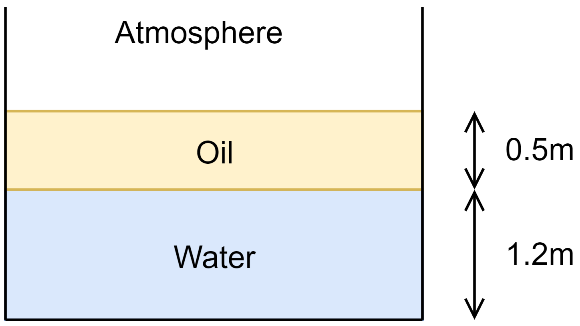
Guided Practice

Within an open container, a 50 cm thick layer of oil floats on a 120 cm layer of water. What is the pressure at the bottom of the water layer?

Useful info: $\rho_{oil} = 900\,\mathrm{kg/m^3}$

Answer: $P = 1.2 \times 10^5\,\mathrm{Pa} \approx 1.2\,\mathrm{atm}$

[SOLUTION]



The pressure at the bottom is the sum of the hydrostatic pressures of the oil and water, in addition to the pressure of the atmosphere since the container is open.

$$
\begin{aligned}
P &= P_{oil} + P_{H20} + P_0 \\
&= \rho_{oil}\, g\, h_{oil} + \rho_{H20}\, g\, h_{H20} + P_0 \\
&= (900)(9.8)(0.5) + (1000)(9.8)(1.2) + 10^5 \\
&\approx 1.2 \times 10^5\,\mathrm{Pa} \quad \text{or} \quad 1.2\,\mathrm{atm}
\end{aligned}
$$

A geologist finds that a Moon rock whose mass is 9.28 kg has an apparent mass of 6.18 kg when submerged in water. Find the

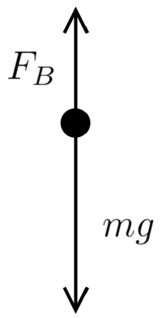
(a) Density of the rock ($\mathrm{kg/m^3}$)

(b) Specific gravity of the rock

Answer: $\rho_{rock} = 2990\,\mathrm{kg/m^3}$

[SOLUTION]

We want to find the density of the rock ρ_{rock}. Let's start with drawing a free-body diagram of the rock submerged underwater.



When the rock is submerged underwater, there are 2 forces acting on it: the buoyant force F_B and gravity F_g. The geologist measures the apparent mass $m_a = 6.18\,\mathrm{kg}$, which we can turn into apparent weight $w_a = m_a g = 60.6\,\mathrm{N}$.

Here is where I would like to ask you to stop and try to anticipate what the next few steps are. We want to find ρ_{rock}. How shall we move forward from here?

One approach is to work with the apparent weight. Here's one way: writing it as a sum of forces.

$$\sum F = F_B - mg = -w_a$$

$$\implies \rho_{H20}\, V_{rock}\, g - mg = -m_a g$$

Note: since we define upwards as positive, we need the negative sign on the right-hand side because the resultant force (w_a) is pointing downwards.

If the above way of describing the apparent weight doesn't click with you, check this other way. By definition, the apparent weight is the weight of the object minus the buoyant force

$$w_a = mg - F_B$$

$$m_a g = mg - \rho_{H20}\, V_{rock}\, g$$

In either of the two ways above, we end up with an equation with only 1 unknown: V_{rock}.

Solving for V_{rock}

$$\implies V_{rock} = \frac{mg - m_a g}{\rho_{H20} g} = \frac{m - m_a}{\rho_{H20}} = \frac{9.28 - 6.18}{1000} = 0.0031\,\mathrm{m^3}$$

We can find the rock's density just by using the simple definition of mass density (mass over volume)

$$\rho_{rock} = \frac{m}{V_{rock}} = \frac{9.28}{0.0031} = 2990\,\mathrm{kg/m^3}$$

And then for part (b), we're curious about the Specific Gravity of the moon rock. We can find this just using the formula

$$\mathrm{SG}_{rock} = \frac{\rho_{rock}}{\rho_{H20}} = \frac{2990}{1000} \approx 3$$

Breakout-Room Activity

A research submarine has a 20 cm diameter window. The manufacturer says the window can withstand forces up to 1.0×10^6 N. What is the submarine's maximum safe depth? Assume the pressure maintained inside the submarine is 1 atm.

Answer: depth ≈ 3 km

[SOLUTION]

This problem is quite juicy as it involves an application of hydrostatic pressure and forces.

Let's assign the variable d to represent the depth of the submarine underwater. The pressure at this level is given by

$$P = P_0 + \rho_{sea}\, g\, d$$

This is the pressure that the submarine's windows feel. We can obtain the force due to this pressure by doing the following

$$P = \frac{F}{A} \quad \Longrightarrow \quad F = P\,A, \quad \text{where the area is } A = \pi r^2$$

We are told that the window can withstand a max force of $F_{max} = 10^6$ N. This means that the window can tolerate a net force of this much.

We can see that the water's hydrostatic pressure exerts a force on the exterior surface of the window, F_{out}. However, there's another force to account for: the force of the submarine's internal pressure, which is 1 atm, acting on the window's interior surface.

These two forces fight against each other and produce a resultant force on the window, which should be no greater than F_{max}. Consider the following

$$\sum F = F_{out} - F_{in} = F_{max}$$

Plugging in the given information,

$$\begin{aligned} F_{out} - F_{in} &= F_{max} \\ P\,A - P_0\,A &= F_{max} \\ (P_0 + \rho_{sea}\, g\, d)A - P_0\,A &= F_{max} \\ \rho_{sea}\, g\, d\, A &= F_{max} \end{aligned}$$

Solving for the depth d, we get

$$\Longrightarrow d = \frac{F_{max}}{\rho_{sea}\, g\, A} = \frac{10^6}{(1030)(9.8)\pi(0.1)^2} = 3150\,\text{m} \approx 3.2\,\text{km}$$

And thus a safe maximum depth is like 3 km.

A crane lifts the 16 000 kg steel hull of a sunken ship out of the water. Accounting for the buoyant force in both cases, find the tension in the crane's cable when

(a) the hull is completely out of the water

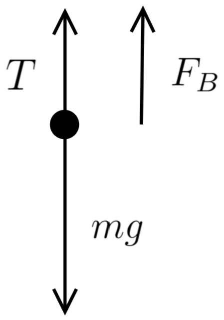
(b) the hull is fully submerged in the water.

Useful info: $\rho_{steel} = 7.8 \times 10^3 \text{kg/m}^3$, $\rho_{air} = 1.29\,\text{kg/m}^3$

Answer: (a) $T \approx 1.57 \times 10^5 \text{N}\,\text{N}$ (b) $T \approx 1.36 \times 10^5\,\text{N}$

[SOLUTION]

This is a classic buoyancy problem where we are trying to lift an object submerged in some fluid and the buoyant force gives us a helping hand. In this case, there are two fluids in which the object is submerged: air and water. Here is the free-body diagram



For part (a), the hull is out of the water and in the air. We can find the volume of the hull by doing some algebra

$$\rho_{steel} = \frac{m}{V} \quad \Longrightarrow \quad V = \frac{m}{\rho_{steel}} = 2.05\,\text{m}^3$$

There are many ways for us to find the tension in the cable. One way is to write out the sum of the forces (i.e. the net force). Let upwards be positive, we get

$$F_{net} = \sum F = T + F_B - mg$$

Now here's the "weird" part. We actually want to set the net force equal to zero. This gives us the minimum tension required to barely get the system ready to accelerate upwards. Technically, the tension has no upper limit; it could be 100 million Newtons and that would lift the hull. However, this is one of those "physics textbook" questions where we assume the question is asking for the minimum tension required to life the hull

$$\begin{aligned} F_{net} &= T + F_B - mg = 0 \\ \Longrightarrow T &= mg - F_B \\ &= mg - \rho_{air} V g \\ &= (16000)(9.8) - (1.29)(2.05)(9.8) \\ &\approx 1.57 \times 10^5\,\text{N} \end{aligned}$$

For part (b), the process is very similar, except this time our hull is submerged in the water so the density found in the buoyant force formula is ρ_{sea} instead of ρ_{air}.

We end up with something like

$$\begin{aligned} T &= mg - F_B \\ &= mg - \rho_{sea} V\, g \\ &= (16000)(9.8) - (1030)(2.05)(9.8) \\ &\approx 1.36 \times 10^5\,\text{N} \end{aligned}$$

Challenge Activity

Your LARC tutor weighs approximately 75 kg (on a good day :P). Suppose one helium balloon occupies volume $V = 15\,\mathrm{L}$ (liters). How many helium balloons would you buy so that your LARC tutor can fly? Assume the buoyant force acting on my body is negligible.

Useful info: $1\,\mathrm{m}^3 = 1000\,\mathrm{L}$, $\rho_{air} = 1.29\,\mathrm{kg/m^3}$, $\rho_{He} = 0.179\,\mathrm{kg/m^3}$

Answer: At least 4500 helium balloons to achieve human flight.

[SOLUTION]

With your newfound knowledge of fluid mechanics, you can help me achieve my dreams of being able to fly like in the Up movie. The question is to find the minimum number of balloons needed in order to lift my weight $F_g = Mg = 735\,\mathrm{N}$, where $M = 75\,\mathrm{kg}$.

Let's imagine myself and all the helium balloons as one big system. There's 3 forces to account for: my weight, the weight of the helium balloons, and the buoyant force of the air acting on our system. Similar to the crane problem on the previous page, we want to find the sum of the forces (the net force) on our system and set it equal to zero. This gives us the minimum buoyant force required to start accelerating upwards.

Let N represent the minimum number of helium balloons required. Consider the following

$$F_{net} = \sum F = F_B - m_{total}\, g - Mg = 0$$

$$\implies \rho_{air}\, V_{total}\, g - m_{total}\, g - Mg = 0$$

How do we find V_{total} and m_{total}? Given the volume of 1 balloon, we can find the total volume of all the balloons by simply multiplying by N. Similar logic applies to finding the total mass of all the balloons.

Consider the following

$$V_{total} = N\, V_{balloon} \qquad \text{and} \qquad m_{total} = N\, m_{balloon} = N\, \rho_{He}\, V_{balloon}$$

Plugging all this information in, we get

$$\implies \rho_{air}\, N\, V_{balloon}\, g - N \rho_{He}\, V_{balloon}\, g - Mg = 0$$

The only unknown in this equation is N. Solving for N, we get

$$\rho_{air}\, N\, V_{balloon}\, g - N \rho_{He}\, V_{balloon}\, g = Mg$$

$$N(\rho_{air}\, V_{balloon}\, g - \rho_{He}\, V_{balloon}\, g) = Mg$$

and thus

$$\implies N = \frac{Mg}{\rho_{air}\, V_{balloon}\, g - \rho_{He}\, V_{balloon}\, g} = \frac{(75)(9.8)}{(1.29)(0.015)(9.8) - (0.179)(0.015)(9.8)} = 4500\,\text{balloons}$$

REMARK: There's a neat balancing act going on here. Every additional balloon we add increases the weight of the cargo since the balloon itself has weight. However, the additional buoyant force on the system due to that balloon outweighs this cost, resulting in a net force upwards, and thus allowing us to keep adding balloons and eventually be able to achieve flight.