Ch 18 Kinetic Theory

Season 1 Episode 4 - VELOCITY & HUMIDITY

In this episode of LARC Physics 3B, we're going to . . .

- Evaluate ideal gases by looking at them in the molecular level.
- Create a connection between ideal gases and humidity.

Guided Practice

At 800 °C, Find the rms speed of the following:

(a) an Argon atom

(b) a Hydrogen molecule

NOTE: Here is a link to an online Periodic Table

Answer: (a) $v_{rms} = 820\,\text{m/s}$, (b) $v_{rms} = 3600\,\text{m/s}$

[SOLUTION]

When it comes to finding the v_{rms} of the particle (atom or molecule), there are usually 2 steps involved: finding the particle's mass m and the temperature T. In this case, we already have T.

One way to find the particle's mass is just by approximating the mass of the nucleus:

$$m = (\text{\# of protons \& neutrons}) \cdot (\text{mass of a proton})$$

where we assume protons and neutrons have roughly the same mass while ignoring the mass of all the electrons.

And the temperature is simply given by Ideal Gas Law: $PV = nRT = NkT$

For part (a), we find that the mass of a single Argon atom is given by

$$m = (39.9)(1.67 \times 10^{-27}) = 6.66 \times 10^{-26}\,\text{kg}$$

Plugging everything in, we get

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(1073)}{6.66 \times 10^{-26}}} = 820\,\text{m/s}$$

NOTE: I used the version of Boltzmann constant that involved degrees Kelvin which required me to convert the given temperature into Kelvin in order for the units to cancel out.

For part (b), it's the same exact process except with a Hydrogen molecule now. Because it's a *molecule* and not an *atom*, we're dealing with an H_2 particle.

$$m = 2(1.008)(1.67 \times 10^{-27}) = 3.37 \times 10^{-27}\,\text{kg}$$

$$\Longrightarrow v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(1073)}{3.37 \times 10^{-27}}} = 3600\,\text{m/s}$$

What is the mass of water (vapor) within a closed room with a $30\,\mathrm{m}^2$ floor and $2.4\,\mathrm{m}$ tall ceiling when the temperature is $24\,^\circ\mathrm{C}$ and the relative humidity is 65%?

Useful info: The Saturated Vapor Pressure of H2O at 24 degrees Celsius is $3000\,\mathrm{Pa}$

Answer: $m = 1\,\mathrm{kg}$

[SOLUTION]

To begin, let's identify: "what's the question asking for?"

They want to find the mass of the water vapor inside the room. Finding mass is like finding the # of moles n and then multiplying by the molar mass of water $M = 0.018\,\mathrm{kg/mol}$

We're told the relative humidity is 65%, which is basically telling us the (partial) pressure of the water vapor inside the room

$$\mathrm{RH} = \frac{\mathrm{PP}}{\mathrm{SVP}} \quad \Longrightarrow \quad \mathrm{PP} = \mathrm{RH} \cdot \mathrm{SVP} = (0.65)(3000) = 1950\,\mathrm{Pa}$$

Partial pressure is just pressure, so let $P = 1950\,\mathrm{Pa}$.

The water vapor is assumed to be an ideal gas so we may apply IGL to find the # of moles

$$PV = nRT \quad \Longrightarrow \quad n = \frac{PV}{RT} = \frac{(1950)(30)(2.4)}{(8.314)(297)} = 56.9\,\mathrm{mol}$$

$$\Longrightarrow m = n\,M = (56.9)(0.018) = 1.0\,\mathrm{kg}$$

Breakout-Room Activity

What is the rms speed of a nitrogen molecule N_2 contained in an $8.5\,\mathrm{m}^3$ volume at $3.1\,\mathrm{atm}$ if the total amount of nitrogen is $1800\,\mathrm{mol}$?

Answer: $v_{rms} = 400\,\mathrm{m/s}$

[SOLUTION]

If we attempt to plug in the given information into the v_{rms} equation, we will find that we are missing a temperature T value. Assuming that N_2 is an ideal gas, we can use IGL to find T

$$\Longrightarrow T = \frac{PV}{nR} = \frac{(3.1 \times 10^5)(8.5)}{(1800)(8.314)} = 176\,^\circ\mathrm{K}$$

The mass of 1 N_2 molecule is given by

$$m = 2(14)(1.67 \times 10^{-27}) = 4.68 \times 10^{-26}\,\mathrm{kg}$$

$$\Longrightarrow v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(176)}{4.68 \times 10^{-26}}} = 400\,\mathrm{m/s}$$

In humid climates, people constantly dehumidify their cellars to prevent rot and mildew. If a $322\,\mathrm{m}^3$ house cellar is kept at $20\,^\circ\mathrm{C}$, what is the mass of water (vapor) must be removed from the cellar in order to drop the humidity from 95% to a more natural 40%?

Useful info: The Saturated Vapor Pressure of H2O at 20 degrees Celsius is $2330\,\mathrm{Pa}$

Answer: $m = 3\,\mathrm{kg}$

[SOLUTION]

Let's take a moment to rephrase the question: "Find the mass of water vapor to remove in order to drop the humidity"

NOTE: When working with gases, finding mass is like finding # of moles. Dropping the humidity is simply lowering the pressure (from 95% to 40%).

If you prefer, you may convert these values of relative humidity into (partial) pressure values describing the gas' initial and final states.

$$\mathrm{RH}_1 = \frac{\mathrm{PP}_1}{\mathrm{SVP}} \quad \Longrightarrow \quad \mathrm{PP}_1 = \mathrm{RH}_1 \cdot \mathrm{SVP} = (0.95)(2330) = 2213\,\mathrm{Pa}$$

$$\mathrm{RH}_2 = \frac{\mathrm{PP}_2}{\mathrm{SVP}} \quad \Longrightarrow \quad \mathrm{PP}_2 = \mathrm{RH}_2 \cdot \mathrm{SVP} = (0.40)(2330) = 932\,\mathrm{Pa}$$

Now here's the cool part: we can find the change in pressure $\Delta P = |P_2 - P_1|$ and then find the corresponding # of moles of water vapor that would produce this pressure. That's how much we need to remove.

$$\Delta P = |P_2 - P_1| = 1280\,\mathrm{Pa}$$

Assuming water vapor is an ideal gas, we can use IGL to find the # of moles corresponding to this much pressure

$$PV = nRT \quad \Longrightarrow \quad n = \frac{PV}{RT} = \frac{(1280)(322)}{(8.314)(293)} = 169.2\,\mathrm{mol}$$

$$\Longrightarrow m = n\,M = (169.2)(0.0018) = 3.0\,\mathrm{kg}$$