Ch 13 Fluids

Season 1 Episode 2 - HYDRODYNAMICS

In this episode of LARC Physics 3B, we're going to . . .

- Evaluate fluid motion by applying **Bernoulli's Equation** and the **Continuity Equation**, along with **Pascal's Principle** sprinkled on top.
- Applying these concepts/equations, we can answer questions like:
  - How fast water is flowing from the water tank to a house faucet? How much pressure does a syringe need to enter the blood stream? How does a vacuum cleaner create suction?

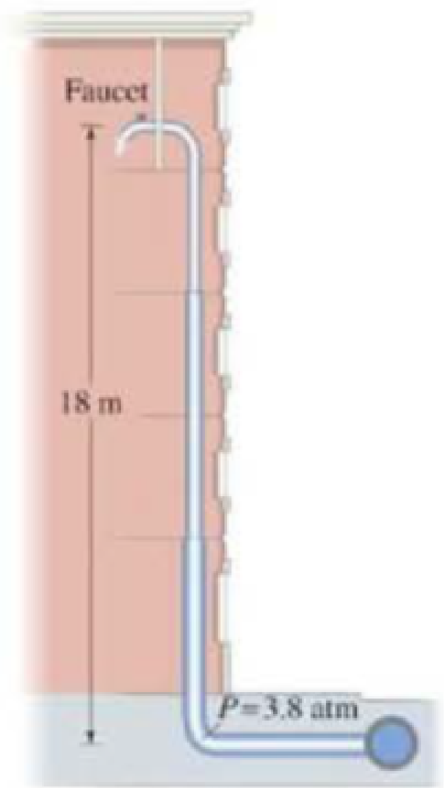
Guided Practice

Water at a gauge pressure of 3.8 atm at street level flows into an office building at a speed of 0.68 m/s through a 5 cm diameter pipe. The pipe tapers down to 2.8 cm in diameter by the top floor, which is located 18 m above the street level.

(a) Calculate the flow velocity of the water exiting the faucet on the top floor

(b) Calculate the gauge pressure in the pipe on the top floor

Answer: (a) $v = 2.2\,\mathrm{m/s}$ (b) $P = 2\,\mathrm{atm}$



[SOLUTION]

Let point 1 be inside the water pipe at street level and point 2 be inside the faucet on the top floor.

Part (a) is asking for the "flow velocity" (velocity of the fluid) inside the faucet. Since we're given the diameters of the pipes (and thus the cross-sectional area), we can use the Continuity Equation to find the velocity

$$A_1 v_1 = A_2 v_2 \quad \Longrightarrow \quad v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi(0.05/2)^2}{\pi(0.028/2)^2}(0.68) = 2.2\,\mathrm{m/s}$$

Part (b) is asking for the pressure on the top floor. We want to use Bernoulli's Equation to find the pressure. When using this equation, we want to be consistent in choosing to use either gauge pressure or absolute pressure. Keep in mind of their relationship: $P_G = P_{abs} - P_0$

Since we're given the gauge pressure inside the water pipe at street level, I'll just work with that. Plugging everything in, we get

$$\begin{aligned}
P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \\
\Longrightarrow P_2 &= P_1 + \rho g(h_1 - h_2) + \frac{1}{2}\rho(v_1^2 - v_2^2) \\
&= P_1 - \rho g(h_2 - h_1) - \frac{1}{2}\rho(v_2^2 - v_1^2) \\
&= P_1 - \rho g \Delta h - \frac{1}{2}\rho(v_2^2 - v_1^2) \\
&= (3.8 \times 10^5) - (1000)(9.8)(18) - \frac{1}{2}(1000)((2.2)^2 - (0.68)^2) \\
&= 201\,000\,\mathrm{Pa} \approx 2.0\,\mathrm{atm}
\end{aligned}$$

NOTE: This answer is already in gauge pressure form! (since it matches the answer)

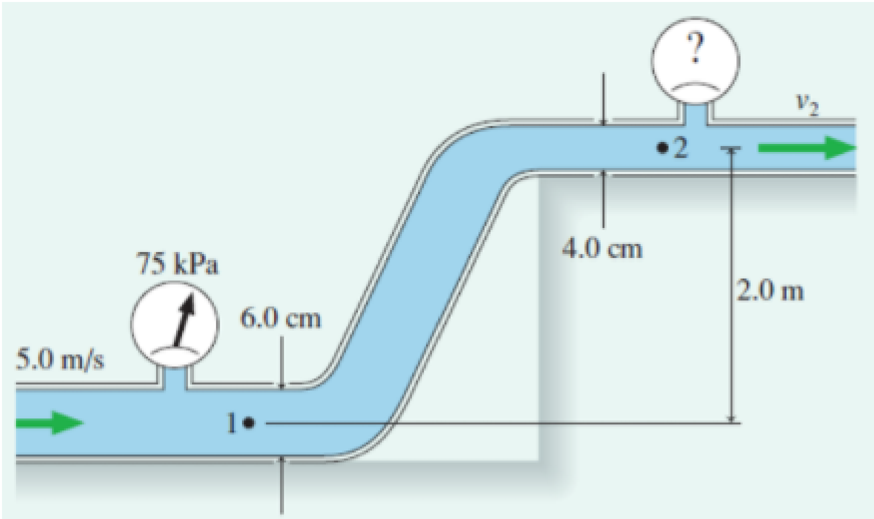
Try inputting the absolute pressure for P_1 and see what you get when solving for P_2. You'll find that it's already in absolute pressure! Whooaa trippy.

Breakout-Room Activity

Water flows through the pipes as shown in the figure below. The water's speed through the lower pipe is 5 m/s and a pressure gauge reads 75 kPa. What is the reading of the pressure gauge on the upper pipe, which is 2 m above?

NOTE: The pressure gauge device gives the gauge pressure, not the absolute pressure

Answer: $P = 4.6\,\text{kPa}$ (Gauge Pressure)



[SOLUTION]

The approach to this problem is identical to the Guided Practice problem: use Continuity Equation to solve for v_2 and then plug that into Bernoulli's Equation to solve for P_2. Keep in mind that we want the gauge pressure since that's the value that the pressure gauges (device) measures!

$$A_1 v_1 = A_2 v_2 \quad \Longrightarrow \quad v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi(0.06/2)^2}{\pi(0.04/2)^2}(0.5) = 11.25\,\text{m/s}$$

and then

$$\begin{aligned}
P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 &= P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \\
\Longrightarrow P_2 &= P_1 + \rho g(h_1 - h_2) + \frac{1}{2}\rho(v_1^2 - v_2^2) \\
&= P_1 - \rho g(h_2 - h_1) - \frac{1}{2}\rho(v_2^2 - v_1^2) \\
&= P_1 - \rho g \Delta h - \frac{1}{2}\rho(v_2^2 - v_1^2) \\
&= (75 \times 10^3) - (1000)(9.8)(2) - \frac{1}{2}(1000)((11.25)^2 - (5)^2) \\
&= 4600\,\text{Pa} = 4.6\,\text{kPa}
\end{aligned}$$

NOTE: Since we used the gauge pressure for P_1, when solving for P_2, we know this is also in gauge pressure; and thus it's the final answer. Bada bing bada boom.

Breakout-Room Activity

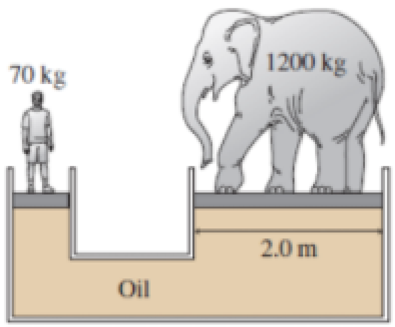


FIGURE P15.40

(a) A 70 kg student balances a 1200 kg elephant on a hydraulic lift as shown in the figure. What is the diameter of the piston the student is standing on?

(b) A second student joins the first, causing the elephant's platform to rise by 35 cm in order to return the system back to equilibrium. What is the mass of the second student?

Useful info: $\rho_{oil} = 900\,\mathrm{kg/m^3}$

Hint: the additional height gained on the elephant's side can be thought of as an additional "weight" or "pressure" due to the oil.

Answer: (a) $d = 0.5\,\mathrm{m}$ (b) $m = 53\,\mathrm{kg}$

[SOLUTION]

So this problem is actually a rewind back to Season 1 Episode 1 - HYDROSTATICS.

In this case, the fluid is at rest and in equilibrium, wherein all points within the fluid is the same. The student and the elephant both exert a force (and thus a pressure) on the confined fluid due to their weight.

For part (a), according to Pascal's Principle, these pressures must be the same.

$$P_{\text{left}} = P_{\text{right}} \implies \frac{F_s}{A_{\text{left}}} = \frac{F_e}{A_{\text{right}}} \implies \frac{mg}{\pi r_{\text{left}}^2} = \frac{Mg}{\pi r_{\text{right}}^2}$$

We can now solve the above equation for the radius on the left-side r_{left}.

$$\implies r_{\text{left}} = r_{\text{right}}\sqrt{\frac{m}{M}} = (1)\sqrt{\frac{70}{1200}} = 0.24\,\mathrm{m}$$

To get the diameter d_{left}, we simply multiply by 2

$$\implies d_{\text{left}} = 2r_{\text{left}} = 2(0.24) = 0.48 \approx 0.5\,\mathrm{m}$$

Now for part (b), another student hops onto the left platform, causing the system to momentarily go out of equilibrium. In response, the right platform (with the elephant) raises by a certain height to counteract this additional "pressure", bringing the system back into equilibrium.

We can view the extra column of oil as an additional pressure acting on the right piston. This pressure can be derived from the hydrostatic pressure: $P = \rho g h$.

Consider the following

$$P_{\text{left}} = P_{\text{right}} \implies \frac{F_{s1} + F_{s2}}{A_{\text{left}}} = \frac{F_e}{A_{\text{right}}} + P_{oil} \implies \frac{m_1 g + m_2 g}{\pi r_{\text{left}}^2} = \frac{Mg}{\pi r_{\text{right}}^2} + \rho_{oil}\, gh$$

Solving for the second student's mass m_2, we get

$$\implies m_2 = \left(\frac{M}{\pi r_{\text{right}}^2} + \rho_{oil} h\right)(\pi r_{\text{left}}^2) - m_1 = \left(\frac{1200}{\pi(1)^2} + (900)(0.35)\right)(\pi(0.24)^2) - 70 = 56\,\mathrm{kg}$$

Challenge Activity

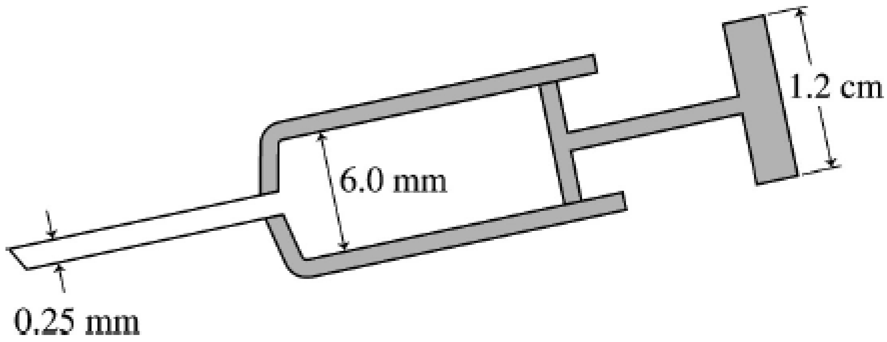
A 2.0 mL syringe has an inner diameter of 6.0 mm, a needle inner diameter of 0.25 mm, and a plunger pad diameter (where you place your finger) of 1.2 cm. A nurse uses the syringe to inject medicine into a patient whose blood pressure is 140/100.

(a) What is the minimum force the nurse needs to apply to the syringe?

(b) The nurse empties the syringe in 2.0 s. What is the flow speed of the medicine through the needle?

Useful info: $100\,\text{mmHg} = 1.33 \times 10^4\,\text{Pa}$, $1\,\text{m}^3 = 1000\,\text{L}$

Answer: (a) 1.5 N (b) $v = 20\,\text{m/s}$



[SOLUTION]

The pressure of the medicine exiting the syringe has to match the person's minimum blood pressure, which is 100 mmHg or 1.33×10^4 Pa

For part(a), let F represent the force of the nurse's hand. Let point 1 be at the needle, point 2 be at the inner middle tube, and point 3 be at the plunger. In this case, we want $P_1 = P_{blood} = 1.33 \times 10^4\,\text{Pa}$. The medicine is considered a confined fluid, so we may apply Pascal's principle, relating the pressures.

$$P_1 = P_2 = P_3 \implies P_1 = P_3 \implies P_{blood} = \frac{F}{A_3}$$

Solving for F, we get

$$F = P_{blood}\,A_3 = (1.33 \times 10^4)(\pi(0.012/2)^2) = 1.5\,\text{N}$$

For part(b), we want to find the velocity exiting the needle; in this case, we call it v_1.

We use the Continuity Equation but not in it's final version. Originally, the Continuity Equation is a statement of mass flow rate. The next step converts it to volume flow rate since mass is density times volume. Consider the following

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \implies \frac{\rho \Delta V_1}{\Delta t} = \frac{\rho \Delta V_2}{\Delta t} \implies A_1 v_1 = \frac{\Delta V_2}{\Delta t}$$

Plugging everything in and solving for v_1, we get

$$v_1 = \frac{1}{A_1}\frac{\Delta V_2}{\Delta t} = \frac{1}{\pi(0.00025/2)^2}\frac{2 \times 10^{-6}}{2} = 20.4 \approx 20\,\text{m/s}$$