

Ordinary Least Squares

For Simple Regression

- We have the [Residual Sum of Squares](#) and we use the OLS to find the Coefficients β_0, β_1 estimates $\hat{\beta}_0, \hat{\beta}_1$ to minimize The RSS
- Computing Partial Derivatives
- Finding the estimates for $\hat{\beta}_0$

$$\frac{\partial RSS}{\partial \hat{\beta}_0} = \sum (y_i - \hat{y})^2 = 2 \sum (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)) = 0$$

$$\frac{\partial RSS}{\partial \hat{\beta}_0} = \sum y_i - \hat{\beta}_0 \sum 1 - \hat{\beta}_1 \sum x_i = 0$$

With :

$$\sum_{i=1}^n y_i = n\bar{y} \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

- \bar{y} is the mean on the y axis (same goes for \bar{x})

$$\begin{aligned} \bar{y}n - n\hat{\beta}_0 - \hat{\beta}_1 \bar{x}n &= 0 \\ \hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} \end{aligned}$$

Now for $\hat{\beta}_1$:

$$\begin{aligned} \frac{\partial RSS}{\partial \hat{\beta}_1} &= \sum_{i=1}^n (y_i - \hat{y}_i)^2 = -2 \sum_{i=1}^n (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)) x_i = 0 \\ \sum_{i=1}^n x_i \left(\sum_{i=1}^n y_i - \hat{\beta}_0 \sum_{i=1}^n 1 - \hat{\beta}_1 \sum_{i=1}^n x_i \right) &= 0 \\ \sum_{i=1}^n y_i x_i - \hat{\beta}_0 \sum_{i=1}^n x_i - \hat{\beta}_1 \sum_{i=1}^n x_i^2 &= 0 \end{aligned}$$

- Now we substitute $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ in the equation
- And get $\sum y_i x_i$ to the left side of the equation
- Divide both sides with $\sum x_i$
- Factor by $\hat{\beta}_1$

$$\begin{aligned} \sum_{i=1}^n y_i x_i - (\bar{y} - \hat{\beta}_1 \bar{x}) \sum_{i=1}^n x_i - \hat{\beta}_1 \sum_{i=1}^n x_i^2 &= 0 \\ \bar{y} \sum_{i=1}^n x_i - \hat{\beta}_1 \bar{x} \sum_{i=1}^n x_i + \hat{\beta}_1 \sum_{i=1}^n x_i^2 &= \sum_{i=1}^n y_i \sum_{i=1}^n x_i \\ \bar{y} - \hat{\beta}_1 \bar{x} + \hat{\beta}_1 \sum_{i=1}^n x_i &= \sum_{i=1}^n y_i \\ \hat{\beta}_1 \left(\sum_{i=1}^n x_i - \bar{x} \right) &= \sum_{i=1}^n y_i - \bar{y} \\ \hat{\beta}_1 &= \frac{\sum_{i=1}^n y_i - \bar{y}}{\sum_{i=1}^n x_i - \bar{x}} \end{aligned}$$

- It can also be written as : (For ease of calculation)

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$