Ordinary Least Squares

For Simple Regression

- We have the Residual Sum of Squares and we use the OLS to find the Coefficients β_0, β_1 estimates $\hat{\beta}_0, \hat{\beta}_1$ to minimize The RSS
- Computing Partial Derivatives
- Finding the estimates for $\hat{\beta}_0$

$$\frac{\partial RSS}{\partial \hat{\beta}_0} = \sum (y_i - \hat{y})^2 = 2 \sum (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)) = 0$$
$$\frac{\partial RSS}{\partial \hat{\beta}_0} = \sum y_i - \hat{\beta}_0 \sum 1 - \hat{\beta}_1 \sum x_i = 0$$

With :

$$\sum_{i=1}^{n} y_i = n\bar{y} \;\; \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

- $\bar{y}$ is the mean on the y axis (same goes for $\bar{x}$)

$$\begin{aligned} \bar{y}n - n\hat{\beta}_0 - \hat{\beta}_1 \bar{x} n = 0 \\ \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \end{aligned}$$

Now for $\hat{\beta}_1$:

$$\begin{aligned} \frac{\partial \text{RSS}}{\partial \hat{\beta}_1} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = -2 \sum_{i=1}^{n} (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)) x_i = 0 \\ \sum_{i=1}^{n} x_i \left(\sum_{i=1}^{n} y_i - \hat{\beta}_0 \sum_{i=1}^{n} 1 - \hat{\beta}_1 \sum_{i=1}^{n} x_i \right) = 0 \\ \sum_{i=1}^{n} y_i x_i - \hat{\beta}_0 \sum_{i=1}^{n} x_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i^2 = 0 \end{aligned}$$

- Now we substitute $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ in the equation

$$\begin{aligned} \sum_{i=1}^{n} y_i x_i - (\bar{y} - \hat{\beta}_1 \bar{x}) \sum x_i - \hat{\beta}_1 \sum x_i^2 = 0 \\ \bar{y} \sum x_i - \hat{\beta}_1 \bar{x} \sum x_i + \hat{\beta}_1 \sum x_i^2 = \sum_{i=1}^{n} y \sum x_i \\ \bar{y} - \hat{\beta}_1 \bar{x} + \hat{\beta}_1 \sum x_i = \sum y_i \\ \hat{\beta}_1 \left(\sum_{i=1}^{n} x_i - \bar{x} \right) = \sum_{i=1}^{n} y_i - \bar{y} \\ \hat{\beta}_1 = \frac{\sum_{i=1}^{n} yi - \bar{y}}{\sum_{i=1}^{n} x_i - \bar{x}} \end{aligned}$$

- It can also be written as :

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i - \bar{x} \sum_{i=1}^{n} y_i - \bar{y}}{\sum_{i=1}^{n} x_i - \bar{x}}$$