Multiple Linear Regression

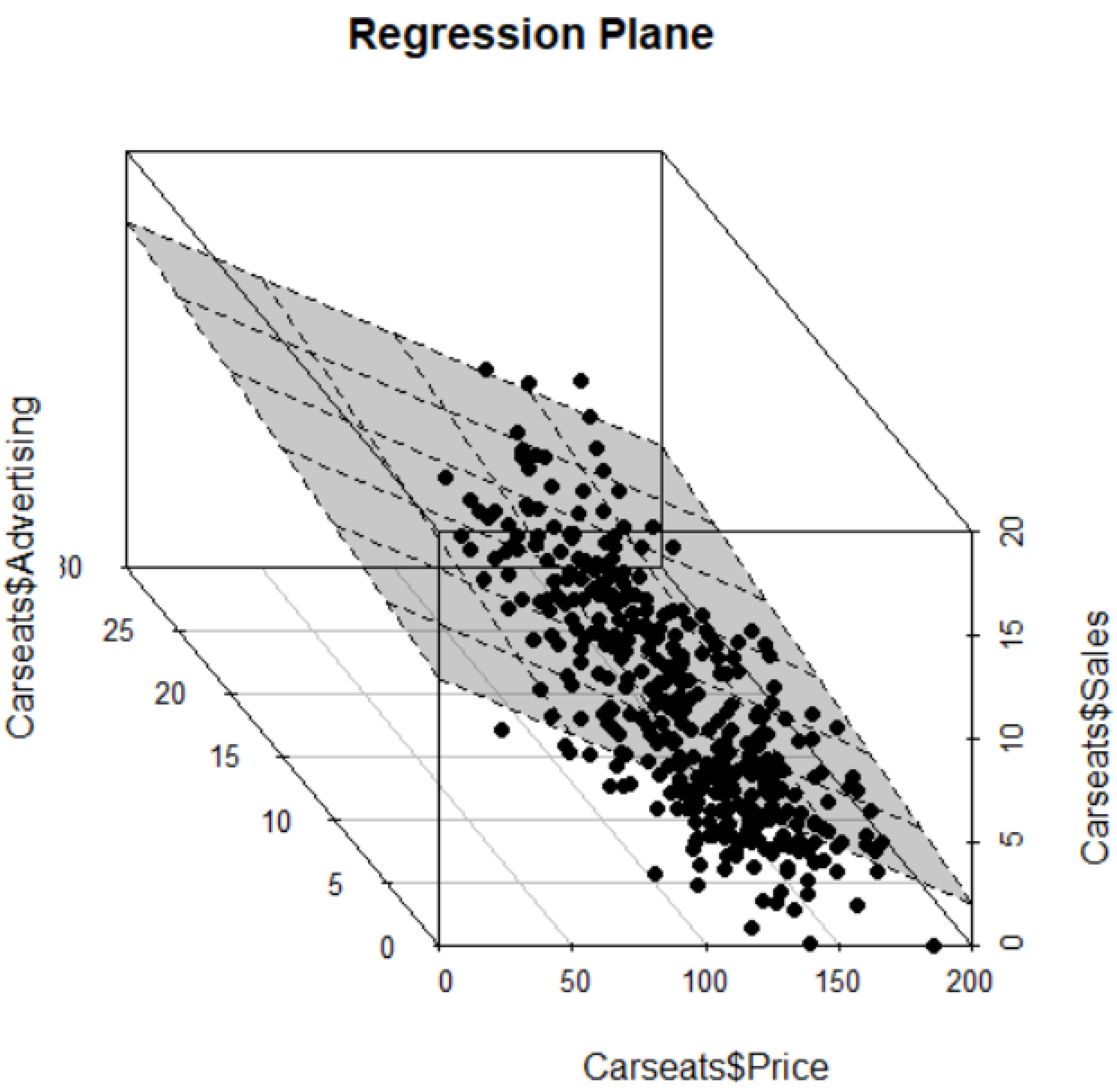
- Simple Linear Regression predict Response for a single predictor X, for example TV
- Multiple Linear Regression deals with multiple predictors, even fitting separate Simple regression to each predictor X this will make us miss some key correlations and associations between predictors and Response

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \varepsilon$$

For example :

$$\text{sales} = \beta_0 + \beta_1 TV + \beta_2 Radio + \beta_3 Newspaper$$

Geometric Interpretation (Regression Plane)



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- Unlike Simple Linear Regression Multiple Linear regression have multiple predictors which is draw as a hyperplan

Estimating The Regression Coefficients :

- The Coefficients in the multiple regression are unknown $\beta_0, \beta_1 \dots, \beta_p$
- So we estimate them as as in the Simple Linear Regression

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \cdots + \hat{\beta}_p x_p$$

Multiple Linear Regression Matrix Form :

- The multiple Regression formula can be written in a matrix form making it better to work with and derive the Coefficients

- $$Y = X\beta + \varepsilon$$

  With :
  - $Y \rightarrow$ Vector of dependent variables Response
  - $X \rightarrow$ Matrix of $n * p$ dimensions + **intercept** β_0
  - $\beta \rightarrow$ Vector of Coefficients (to be estimated)
  - $\varepsilon \rightarrow$ Vector of Error terms

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{21} & \dots & X_{p1} \\ 1 & X_{12} & X_{22} & \dots & X_{p2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{1n} & X_{2n} & \dots & X_{pn} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

- $E(\varepsilon) = 0$
- $var(\varepsilon) = \sigma^2 I_{n*n}$
- $\beta's$ are called partial regression coefficients cause β_1 is the expected change in Y per Unite change in X_1, While holding other $X's$ constant