Ordinary Least Squares

For Simple Regression

- We have the Residual Sum of Squares and we use the OLS to find the Coefficients β_0, β_1 estimates $\hat{\beta}_0, \hat{\beta}_1$ to minimize The RSS
- Computing Partial Derivatives
- Finding the estimates for $\hat{\beta}_0$

$$egin{aligned} rac{\partial RSS}{\partial \hat{eta}_0} &= \sum (y_i - \hat{y})^2 = 2 \sum (y_i - (\hat{eta}_0 + \hat{eta}_1 x_i)) = 0 \ &rac{\partial RSS}{\partial \hat{eta}_0} &= \sum y_i - \hat{eta}_0 \sum 1 - \hat{eta}_1 \sum x_i = 0 \end{aligned}$$

With:

$$\sum_{i=1}^n y_i = n ar{y} \ \ ar{y} = rac{1}{n} \sum_{i=1}^n y_i$$

• \bar{y} is the mean on the y axis (same goes for \bar{x})

$$egin{aligned} ar{y}n-n\hat{eta}_0-\hat{eta}_1ar{x}n&=0\ \hat{eta}_0&=ar{y}-\hat{eta}_1ar{x} \end{aligned}$$

Now for $\hat{\beta}_1$:

$$egin{split} rac{\partial ext{RSS}}{\partial \hat{eta}_1} &= \sum_{i=1}^n (y_i - \hat{y}_i)^2 = -2 \sum_{i=1}^n (y_i - (\hat{eta}_0 + \hat{eta}_1 x_i)) x_i = 0 \ \sum_{i=1}^n x_i \left(\sum_{i=1}^n y_i - \hat{eta}_0 \sum_{i=1}^n 1 - \hat{eta}_1 \sum_{i=1}^n x_i
ight) = 0 \ \sum_{i=1}^n y_i x_i - \hat{eta}_0 \sum_{i=1}^n x_i - \hat{eta}_1 \sum_{i=1}^n x_i^2 = 0 \end{split}$$

• Now we substitute $\hat{eta}_0 = ar{y} - \hat{eta}_1 ar{x}$ in the equation

$$egin{aligned} \sum_{i=1}^n y_i x_i - (ar{y} - \hat{eta}_1 ar{x}) \sum x_i - \hat{eta}_1 \sum x_i^2 &= 0 \ ar{y} \sum x_i - \hat{eta}_1 ar{x} \sum x_i + \hat{eta}_1 \sum x_i^2 &= \sum_{i=1}^n y \sum x_i \ ar{y} - \hat{eta}_1 ar{x} + \hat{eta}_1 \sum x_i &= \sum y_i \ \hat{eta}_1 \left(\sum_{i=1}^n x_i - ar{x}
ight) &= \sum_{i=1}^n y_i - ar{y} \ \hat{eta}_1 &= rac{\sum_{i=1}^n y_i - ar{y}}{\sum_{i=1}^n x_i - ar{x}} \end{aligned}$$

• It can also be written as:

$$\hat{eta}_1 = rac{\sum_{i=1}^n (x_i - ar{x})(y_i - ar{y})}{\sum_{i=1}^n (x_i - ar{x})^2}$$