

# Ordinary Least Squares

## For Simple Regression

- We have the [Residual Sum of Squares](#) and we use the OLS to find the Coefficients  $\beta_0, \beta_1$  estimates  $\hat{\beta}_0, \hat{\beta}_1$  to minimize The RSS
- Computing Partial Derivatives
- Finding the estimates for  $\hat{\beta}_0$

$$\frac{\partial RSS}{\partial \hat{\beta}_0} = \sum (y_i - \hat{y})^2 = 2 \sum (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)) = 0$$

$$\frac{\partial RSS}{\partial \hat{\beta}_0} = \sum y_i - \hat{\beta}_0 \sum 1 - \hat{\beta}_1 \sum x_i = 0$$

With :

$$\sum_{i=1}^n y_i = n\bar{y} \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

- $\bar{y}$  is the mean on the  $y$  axis (same goes for  $\bar{x}$ )

$$\begin{aligned} \bar{y}n - n\hat{\beta}_0 - \hat{\beta}_1 \bar{x}n &= 0 \\ \hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} \end{aligned}$$

Now for  $\hat{\beta}_1$ :

$$\begin{aligned} \frac{\partial RSS}{\partial \hat{\beta}_1} &= \sum_{i=1}^n (y_i - \hat{y}_i)^2 = -2 \sum_{i=1}^n (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)) x_i = 0 \\ \sum_{i=1}^n x_i \left( \sum_{i=1}^n y_i - \hat{\beta}_0 \sum_{i=1}^n 1 - \hat{\beta}_1 \sum_{i=1}^n x_i \right) &= 0 \\ \sum_{i=1}^n y_i x_i - \hat{\beta}_0 \sum_{i=1}^n x_i - \hat{\beta}_1 \sum_{i=1}^n x_i^2 &= 0 \end{aligned}$$

- Now we substitute  $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$  in the equation

$$\begin{aligned} \sum_{i=1}^n y_i x_i - (\bar{y} - \hat{\beta}_1 \bar{x}) \sum_{i=1}^n x_i - \hat{\beta}_1 \sum_{i=1}^n x_i^2 &= 0 \\ \bar{y} \sum_{i=1}^n x_i - \hat{\beta}_1 \bar{x} \sum_{i=1}^n x_i + \hat{\beta}_1 \sum_{i=1}^n x_i^2 &= \sum_{i=1}^n y_i \sum_{i=1}^n x_i \\ \bar{y} - \hat{\beta}_1 \bar{x} + \hat{\beta}_1 \sum_{i=1}^n x_i &= \sum_{i=1}^n y_i \\ \hat{\beta}_1 \left( \sum_{i=1}^n x_i - \bar{x} \right) &= \sum_{i=1}^n y_i - \bar{y} \\ \hat{\beta}_1 &= \frac{\sum_{i=1}^n y_i - \bar{y}}{\sum_{i=1}^n x_i - \bar{x}} \end{aligned}$$

- It can also be written as :

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$