Vector Projections

First let's make briefly explain the **dot product**, simply it's a mathematical operation that takes two vectors of the same **length** and returns a scalar k number (just a way to multiply vectors) :

$$x =< x_1, x_2, x_3 >, y =< y_1, y_2, y_3 >$$

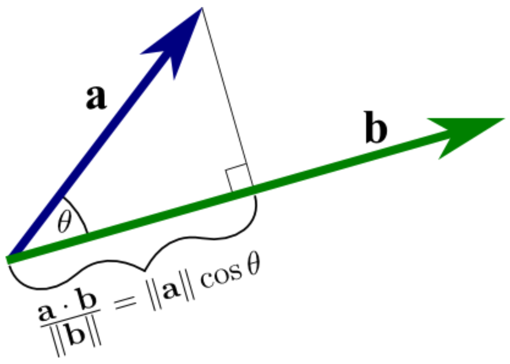
Component Form

$$x \cdot y = (x_1 \times y_1) + (x_2 \times y_2) + (x_3 \times y_3)$$

Magnitude-Angle Form

$$x \cdot y = \|x\|\|y\| \cos \theta$$

- $\|x\|, \|y\|$ is the magnitude (length) of a vector
- θ is the angle between the two vectors $\vec{x}, \vec{y}$



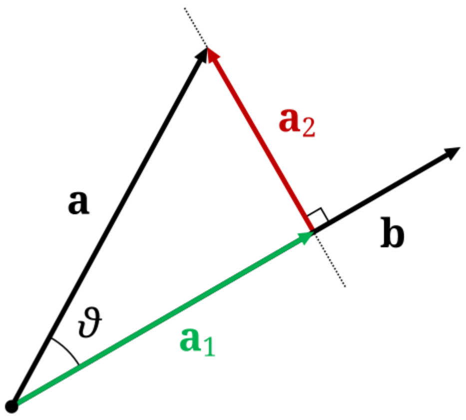
- Projection of a onto b
- The dot product of $b \cdot a$ can be thought as a **scalar projection** of a onto b, multiplied by the length of b
- The dot product is not the projection it self **but** the projection scaled by the length of the base vector, Hence $a \cdot b / \|b\|$

Property

- **Orthogonal** Vectors dot product equals **Zero** $x \cdot y = 0$

Vector Projection

As a motivation the practical use of the **vector Projection** is heavy used in Dimensionality Reduction, imagine having multiple vectors or data points and if we project them into a **Fitted line** we will reduce the dimensions from 2D $\rightarrow$ 1D



Projection is simply the *Shadow* of a on the vector b :

$$a_1 = proj_b\, a = \frac{a \cdot b}{\|b\|} \frac{a}{\|a\|}$$

- $\frac{a}{\|a\|} = 1 = u$
  Now on how we derived the formula for the **Projection** above, which will make stuff clear :

$$a = a_1 + a_1$$

- The vector a is simply the sum of two components a_1 and a_2

Our goal is to calculate the projection which is the *shadow* on the vector b, so :

$$a_1 = a - a_2$$

Now to make the calculations simpler and easier, we make an assumption of b being a **unite vector** :

$$u = \frac{b}{||b||} = 1$$

so the projected vector a_1 is simply a scalar λ times the **unite vector** u

$$a_1 = \lambda u$$

- we want to find λ

Now we have :

$$a_2 = a - a_1$$

$$a_2 = a - \lambda u$$

As shown in the graph above the projection is perpendicular, and the **dot product** of two perpendicular vectors is **zero**

$$a_1 \cdot a_2 = 0$$

$$(\lambda u) \cdot (a - \lambda u) = 0$$

- $a_1 = \lambda u$
- $a_2 = a - \lambda u$

$$\lambda au - \lambda^2 = 0$$

$$au - \lambda = 0$$

$$\lambda = au$$

So the projection vector $a_1$is given by :

$$a_1 = (au)u$$

$$a_1 = \frac{a \cdot b}{||b||} u$$

- Which is the $proj_b\ a$ onto vector b