RGB IMAGE PROCESSING BASED ON COMPRESSED SENSING

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Abstract

Compressed Sensing (CS) can project a high dimensional signal to a low dimensional signal by a random measurement matrix. In this paper, the signal reconstruction algorithm of Compressed Sensing is discussed and a new method is proposed to improve the speed of reconstruction and the quality of recovered images through the orthogonalization of measurement matrix based on proximate QR factorization row matrix. Here we use a $M \times N$ dimensional matrix Φ to complete the signal from high dimensional to low dimensional .In experiment, the RGB image is processed by the improved measurement matrix and OMP algorithm. The results show that the processing of image reconstruction would be fewer amounts of calculation and reducing the effect of the image reconstruction speed.

1 Introduction

Compressed Sensing (CS) has made significant impact to information theory and signal processing since the pioneering works of Candes,Romberg and Tao[1] and Donoho[2].It aims at reconstructing a signal from fewer linear measurements under a sparsity condition,which can be satisfied in most signals in practical applications,i.e.

Compressed Sensing [1, 2, 7, 8] have the following content:

A. Sparse representation of the signal

To simplify the statement, consider any signal $x \in R^N$,we should find an orthonormal basis $\Psi = [\psi_1, \psi_2, \cdots, \psi_N]$ that signal $x \in R^N$ can be expanded in it as follows:

$$x = \sum_{i=1}^{N} \psi_i \alpha_i \qquad (1)$$

Where $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_N]$ is the coefficient sequence of x , $\alpha_i = \langle x, \psi_i \rangle, i = 1,2,\cdots,N$ .when only K of the α_i coefficients are nonzero, $K << N$,the signal x is compressible and has a sparse representation,called $K-$ sparse.Although most of the natural signals are not sparse in a strict sense,they are always have concise representations when expressed in a proper sparsity basis Ψ ,in the sense that the sorted magnitudes of α_i decay quickly [3].Sparsity determines the efficiency of signal acquisition and decreases the needed resource for storage and transmission.

B. Sampling and Sensing Matrix

Via directly condensing signals into a small amount of data, the useful information in the compressible signals can be captured, which employs non-adaptive linear projections. The sampled values of x can be expressed as:

$$\left.\begin{array}{l} y = \Phi x \\ x = \Psi \alpha \end{array}\right\} = y = \Phi \Psi \alpha = A^{CS} \alpha \qquad (2)$$

Where A^{CS} is a operator of CS , y is an $M \times 1$ column vector and Φ is an $M \times N$ matrix that is fixed and independent of signal x .since, $M << N$, recovering x is $K-$sparse, signal recovery can be actually be made possible when matrix $A^{CS} = \Phi\Psi$ obeys the rule of restricted isometry property(RIP) or the measurement matrix Φ is in incoherence with the basis Ψ . The smaller the coherence the fewer samples are need[3].

C. Signal Reconstruction

If Φ content the RIP or incoherent to Ψ ,the signal can be recovered by l_0-norm minimalization, namely:

$$\min_{\alpha} \|\alpha\|_{l_0} \qquad s.t. \quad y = \Phi x = A^{CS} \alpha \qquad (3)$$

This is a non-convex optimization NP problem,in order to solve this problem,Candes and Donoho use l_1- norm replace l_0- norm ,change this problem to a convex optimization problem,namely:

$$\min_{\alpha} \|\alpha\|_{l_1} \qquad s.t. \quad y = \Phi x = A^{CS} \alpha \qquad (4)$$

It can exactly recover the sparse or compressible signal with high probability [2]. Tropp and Gilbert came up with matching pursuit (MP) algorithm and orthogonal matching pursuit (OMP) algorithm that can be fast and easily implemented. Recently, different kinds of new or improved compressed signal reconstruction algorithm have sprung up.

In this paperwork take OMP as reconstruction algorithm.To improve measurement matrix,thus, achieve to improve the quality of recovered and speed of reconstruction.

2 Improved Measurement matrix

CS theory have three factor:Sparsity; Nonlinear observe; Nonlinear optimization reconstruction. In the process of CS, measurement matrix play an important role in data sampling and signal reconstruction.The study of measurement matrix play important meaning. Here, we use Δ to express from y to x, so the reconstruction signal $\hat{x} = \Delta(\Phi x)$, use the equation:

$$\mathrm{E}(x, \Phi, \Delta) = \|x - \Delta(\Phi x)\|_p \quad (5)$$

Express the error of the primary signal to reconstruction signal. The error value as low as possible, the measurement matrix or reconstruction algorithm we choice is good. In this paper we will give detailed study of measurement matrix make the quality of reconstruction signal better than the primary.

In the literature [2], Donoho give the measurement matrix of CS should meet three feature, follows:

1, Column vector of measurement matrix should meet linear independence;

2, Column vector of measurement matrix reflect the independent random of some kind of noise;

3, The answer meet sparsity is a min-vector meet $l-$norm.

In this paper, in order to make the measurement matrix perfect, we give a method based on semi-QR factorization, the row vector of matrix after semi-QR factorization orthogonalization.Via test and theory come to a conclusion: the result of reconstruction of improved measurement matrix better than primary.

2.1 Semi-QR Factorization

In the literature [2], min-singular value of measurement matrix Φ must greater than a positive constant, it have affinity with linear correlation of matrix. The min-singular value more bigger, the independence of matrix more stronger. QR factorization is a good method, it can enlargement the singular value of matrix with don't change matrix nature.

Standard-QR factorization :if $A \in C^{N \times N}$, that A can sole factorization to $A = QR$,the size of Q, R is $N \times N$, matrix R is a upper triangular matrix.Compare with Standard-QR factorization, the Semi-QR factorization [4] processing matrix as follows:

1, QR factorization make measurement matrix Φ to:

$$\Phi^T = QR \Rightarrow \Phi = R^T Q^T \quad (6)$$

Among Q is a orthogonal matrix ,size $N \times N$; R is a upper triangular matrix,size $N \times M$.

2, We gain matrix R from equation (6),the main diagonal element of mat R much bigger than it off-diagonal element,So we reserve the element of main diagonal,other element of matrix R set to zero.Then, we acquire new diagonal matrix $\hat{R}$, so have :

$$\hat{\Phi} = \hat{R}^T Q^T \quad (7)$$

Now,we prove the singular value relationship of new matrix $\hat{\Phi}$ and primary Φ is :

$$\sigma_{\min}(\Phi) \le \sigma_{\min}(\hat{\Phi}), \sigma_{\max}(\Phi) \ge \sigma_{\max}(\hat{\Phi})$$

Prove:for measurement matrix Φ,have:

$$\begin{aligned}\sigma_{\min}(\Phi) &= \sqrt{\lambda_{\min}(\Phi \bullet \Phi^T)} = \sqrt{\min_v \frac{v^T \Phi \Phi^T v}{v^T v}} \\ &= \sqrt{\min_v \frac{v^T R R^T v}{v^T v}} \le \sqrt{\frac{\bar{v}^T R R^T \bar{v}}{\bar{v}^T \bar{v}}} \\ &= \sqrt{\lambda_{\min}(\hat{R}\hat{R}^T)} = \sqrt{\lambda_{\min}(\hat{\Phi}\hat{\Phi}^T)} \\ &= \sigma_{\min}(\hat{\Phi}) \end{aligned} \quad (8)$$

$$\begin{aligned}\sigma_{\min}(\Phi) &= \sqrt{\lambda_{\max}(\Phi \bullet \Phi^T)} = \sqrt{\max_v \frac{v^T \Phi \Phi^T v}{v^T v}} \\ &= \sqrt{\max_v \frac{v^T R R^T v}{v^T v}} \ge \sqrt{\frac{\hat{v}^T R R^T \hat{v}}{\hat{v}^T \hat{\hat{v}}}} \\ &= \sqrt{\lambda_{\max}(\hat{R}\hat{R}^T)} = \sqrt{\lambda_{\max}(\hat{\Phi}\hat{\Phi}^T)} \\ &= \sigma_{\max}(\hat{\Phi}) \end{aligned} \quad (9)$$

Among, $\bar{v}$, $\hat{v}$ is a column vector, weight of them correspond to max-element and min-element position of diagonal in matrix R set to one, other position element set to zero.

From the process of prove, we know: Semi-QR factorization reduce conditional number of primary measurement matrix very well; contract the interval of measurement matrix singular value; then, make new measurement matrix have a better constant of RIP.

2.2 Based on Semi-QR factorization row vector of measurement matrix orthogonalization

From equation(2) use simpleness knowledge of matrix,we can find: via measurement matrix observe sparse signal gain y, element of it gain from row vector of Φ and signal x multiply. we want to make the relevance of among element of y as low as better, so we give row vector of Φ to orthogonalization processing, make the row vector of it are mutual independence, thus, reduce the relevancy between elements of y. we use semi-QR factorization make matrix Φ to $\Phi = R^T Q^T$, matrix R improvement to $\hat{R}$ .last, we make the row vector of $\hat{R}$ orthogonalization, we gain a new matrix R'', so that we have new measurement matrix Φ'' :

$$\Phi'' = R''^T Q^T \quad (10)$$

Obviously, the matrix after we change is also content standard of RIP.

2.3 Experimental result of using measurement matrix

In this section,we choice shipwrecks image from sonar observe as test image,size of it is 256*256.Use Gaussian random matrix as primary measurement matrix, OMP algorithm.Here, we use PSNR of image to express equation (5)

Reconstruction error.The PSNR bigger,the quality of reconstruction image better.due to random matrix uncertainty, so we choice mean from ten times experimental data.

		Primeval	Semi-QR factorization	New method
N=256	M=80	16.4583	18.2724	23.7146
N=256	M=110	27.9023	27.9440	28.4606
N=256	M=140	30.4110	30.5999	31.9405
N=256	M=170	32.2341	32.3090	34.6057
N=256	M=200	33.4728	33.5872	36.0048
N=256	M=230	34.3327	34.3879	36.7411

Table1. The PSNR of image reconstruction compared improved measurement matrix with primeval.

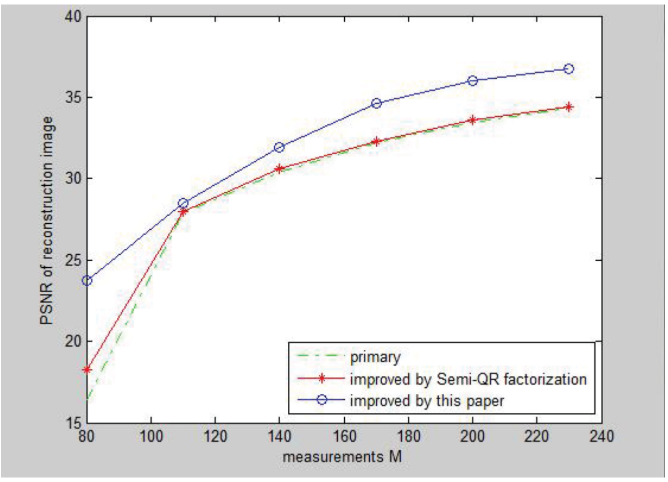
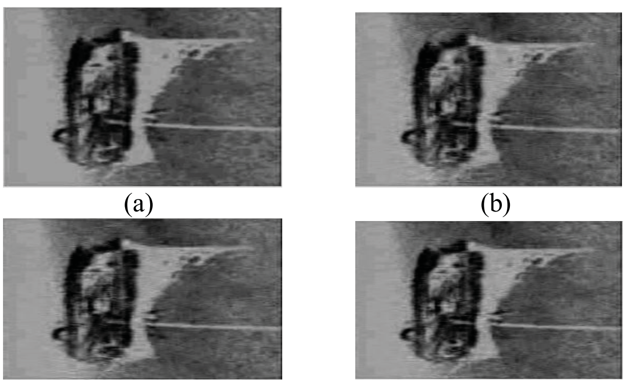


Figure 1. The PSNR of image reconstruction by primary, Semi-QR factorization and improved by this paper measurement matrix.

According to table 1 and Figure 1, we can know: the result of Semi-QR factorization and method in this paper improve the measurement matrix,the PSNR of reconstruction image is increased. we also know the method from this paper in PSNR spending increase bigger than use Semi-QR factorization. So this result explain this paper give method to improve matrix can enhance the quality of reconstruction image.



(a) (b) (c) (d)

Figure 2. Result of reconstruction image measurement matrix improved compared with primeval.

(a),(b),(c),(d) is result of primeval,Semi-QR factorization and method in this paper structure measurement matrix, when M=140.

3 To RGB image processing

Now CS theory mostly use gray image as a test image,this paper try to make a deep research of CS theory, and apply theory to the image processing of RGB.

To RGB image processing.First, we make RGB image into the gray image; Second,we choose OMP algorithm as the image reconstruction algorithm,choose Gaussian random matrix and the improvement of the method presented in this paper improved Gaussian random matrix as measurement matrix for image compression,reconstruction get reconstruction images, compare the reconstruction effect; Finally,we in according to the formula:

$$p * R + q * G + t * B = g \tag{11}$$

Here, p , q , t is a constant, $p + q + t = 1$. g express the gray value of reconstruction gray image. According to formula (11) we choose among R , G , B any two component with the original RGB image is the same,that we gain another component of RGB, so we receive a new RGB group. And thus make the gray image into RGB image.Here we use a scenery RGB image as test image, size 512×512 .

		Primeval matrix	Improved matrix
N=512	M=150	19.6910	20.2411
N=512	M=200	21.3792	21.8991
N=512	M=250	22.8555	23.7377
N=512	M=300	24.1526	25.5677
N=512	M=350	25.4667	27.6851
N=512	M=400	26.8271	29.5784
N=512	M=450	28.0037	31.2892

Table 2. The PSNR of image reconstruction compared improve measurement matrix with primeval.

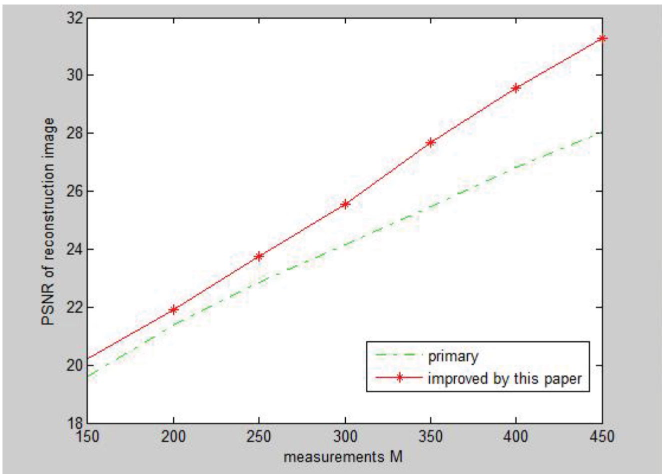


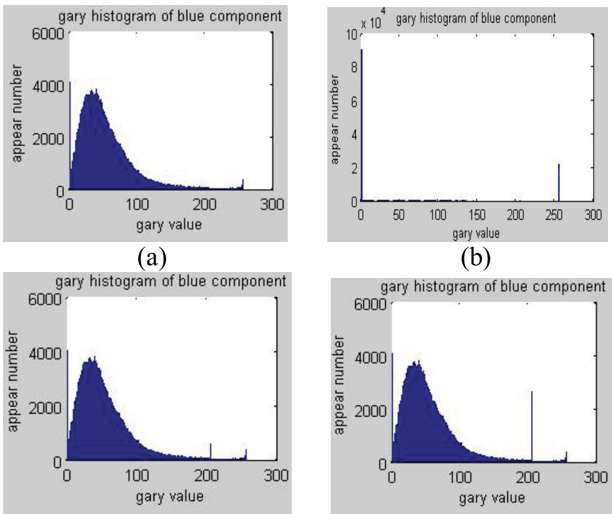
Figure 3. The PSNR of image reconstruction by primeval and improved measurement matrix in this paper.

According to table 2 and Figure 3, we can know:the result of method in this paper improved the measurement matrix, PSNR of reconstruction image is increased. So this result explain this paper give method to improve matrix can enhance the quality of reconstruction RGB image.

The next, wo make the gray image into RGB image.In this process, we according formula (11) make gray image which we gain from improved matrix as measurement matrix reconstruction image into RGB image.

Figure 4. From the reconstruction of the gray image back into a different RGB image effect.

According to Figure 4, we can found in keep R , G component stay the same with primeval RGB image to get new component B , obvious the effects of obtains RGB image is not good. why? we get analysis to the different RGB image, that we find: keep in R , G component unchanged, the recovery of each component of RGB image gray histogram of blue components is almost zero.



(a) (b) (c) (d)

Figure 5. Gary histogram of return to RGB image blue components.

(a) is blue component of RGB image that primeval image; (b) is R and G stay the same, the blue component of new RGB image; (c) is R and B stay the same, the blue component of new RGB image; (d)is B and G stay the same, the blue component of new RGB image.

Via use method from this paper processing more RGB image, we found in the process of compression and reconstruction lost almost all of the blue components. This also explains the blue components includes RGB in RGB image high frequency part. So, in the process of we restore RGB image as long as the guarantee blue component unchanged, will be have better able to turn to RGB image.

4 Conclusion

In this paper, we discussed some basic theory, reconstruction algorithm of CS theory. Put forward a new method improved the quality of image reconstruction. Like Semi-QR factorization, new method can contraction value interval of measurement matrix signal value and have a good RIP constant. It also can make the data we gain from observation have low correlation. So, new method to enhance the quality of reconstruction image is present. Through the experiment, we get to the same conclusion.

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