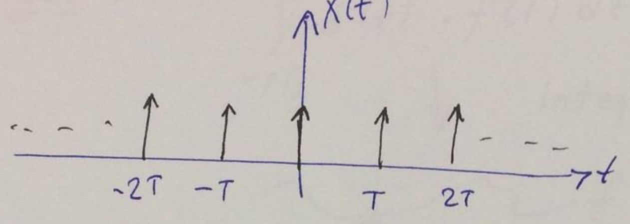
Örnek:



$C_k = ?$

C_k ifadesini türetin.

Verilen sinyal aşağıdaki şekilde ifade edilebilir.

$$X(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \qquad w_0 = \frac{2\pi}{T}$$

$$C_k = \frac{1}{T} \int_{t_0}^{t_0+T} X(t) \cdot e^{-jkw_0 t} \, dt \quad \text{idi.}$$

$$C_k = \frac{1}{T} \int_{t_0}^{t_0+T} X(t) \cdot e^{-jk\frac{2\pi}{T}t} \, dt$$

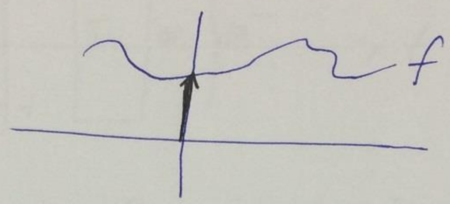
X(t) sinyali birim dürtü katarıdır. Birim dürtü işareti alt bileşenlerine ayrıştırılamadığından temel periyot aralığını $-T/2$ $T/2$ şeklinde almalıyız.

$$C_k = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) \cdot e^{-jk\frac{2\pi}{T}t} \, dt$$

Hatırlatma:

$$\int_{-T/2}^{T/2} \delta(t)\cdot f(t)\,dt = f(0)$$

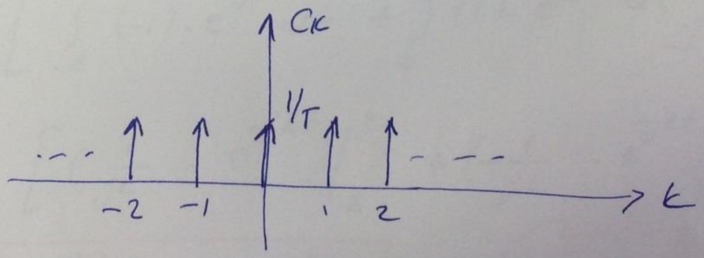
integrali $f(0)$'a eşittir.



integralin içerisi sadece 0 anında değer verir. O değerde $f(0)$'dır.

O halde;

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\cdot e^{-jk\frac{2\pi}{T}t}\,dt = \frac{1}{T}\left[e^{0}\right] = 1/T$$



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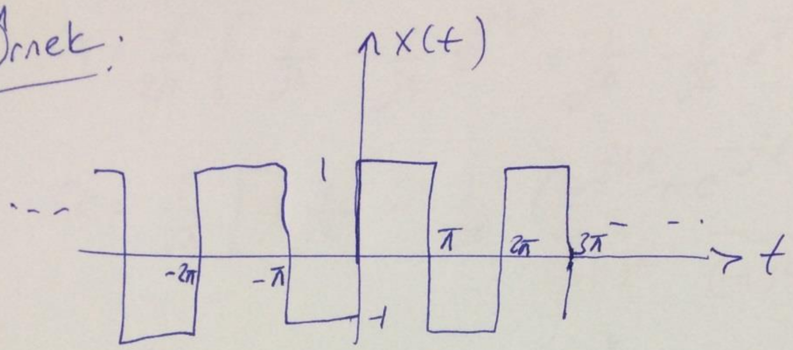

x(t) sinyali için C_k ifadesini türetin.

$$T = 2\pi \qquad w_0 = \frac{2\pi}{T} = 1 \text{ rad/sn}$$

$$C_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t).e^{-jkw_0t}\,dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t).e^{-jkt}\,dt$$

$[-\pi \quad \pi]$ aralığında sinyal ayrıştırılabilir

$$C_k = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(-1).e^{-jkt}\,dt + \int_{0}^{\pi}(1).e^{-jkt}\,dt\right]$$

$$C_k = \frac{1}{2\pi}\left[\left(\frac{1}{jk}.e^{-jkt}\right)\Big|_{-\pi}^{0} + \left(-\frac{1}{jk}.e^{-jkt}\right)\Big|_{0}^{\pi}\right]$$

$$\boxed{e^{\pm jk\pi} = (-1)^k}$$

$\cos k\pi \pm j\sin k\pi$

ya −1
ya +1
olur

$$C_k = \frac{1}{2\pi}\left[\frac{1}{jk} - \frac{1}{jk}e^{jk\pi} + \frac{1}{jk} - \frac{1}{jk}e^{-jk\pi}\right]$$

$$C_k = \frac{1}{2\pi}\left[\frac{2}{jk} - \frac{1}{jk}\left(\underbrace{e^{jk\pi}}_{(-1)^k} + \underbrace{e^{-jk\pi}}_{(-1)^k}\right)\right]$$

$$C_k = \frac{1}{jk\pi} - \frac{1}{jk2\pi}2(-1)^k$$

$$C_k = \frac{1}{jk\pi}\left(1-(-1)^k\right) \qquad k \neq 0$$

C_0 için tanımsız durum var

$$C_0 = \int_{-\pi}^{\pi} x(t)\,dt = \int_{-\pi}^{0}(-1)\,dt + \int_{0}^{\pi}(1)\,dt$$

$$C_0 = 0$$