Sinyaller ve Sistemler

Kompleks Sayılar

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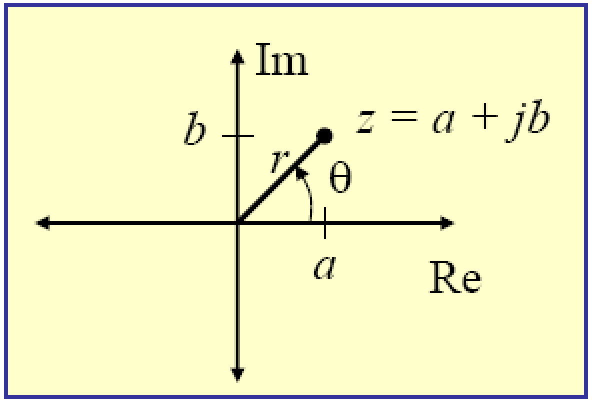
Rectangular kompleks sayılar:

$$j = \sqrt{-1} \Rightarrow j^2 = -1$$
$$\Rightarrow (-j)(j) = 1$$
$$\Rightarrow (-j)(-j) = -1$$

$$z = a + jb \qquad \begin{aligned} a &= \mathrm{Re}\{z\} \\ b &= \mathrm{Im}\{z\} \end{aligned}$$

Toplama: $(a + jb) + (c + jd) = (a + c) + j(b + d)$

Çarpma: $(a + jb)(c + jd) = (ac - bd) + j(ad + bc)$



$$\mathrm{Re}\{z\} = a = r\cos\theta$$
$$\mathrm{Im}\{z\} = b = r\sin\theta$$

Kompleks Sayılar

Açısal Form(Polar Form):

$$\boxed{z = re^{j\theta}}$$

$$|z| = r = \sqrt{a^2 + b^2}$$

$$\angle z = \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Çarpma:

$$\boxed{\left(r_1 e^{j\theta_1}\right)\left(r_2 e^{j\theta_2}\right) = r_1 r_2 e^{j(\theta_1+\theta_2)}}$$

$$z^n = \left(re^{j\theta}\right)^n = r^n e^{jn\theta}$$

$$z^{1/n} = r^{1/n} e^{j\theta/n}$$

Bölme:

$$\boxed{\frac{\left(r_1 e^{j\theta_1}\right)}{\left(r_2 e^{j\theta_2}\right)} = \frac{r_1}{r_2} e^{j(\theta_1-\theta_2)}}$$

$$\boxed{\frac{1}{z_2} = \frac{1}{r_2 e^{j\theta_2}} = \frac{1}{r_2} e^{-j\theta_2}}$$

Euler Denklemleri :

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \qquad \text{(A)}$$

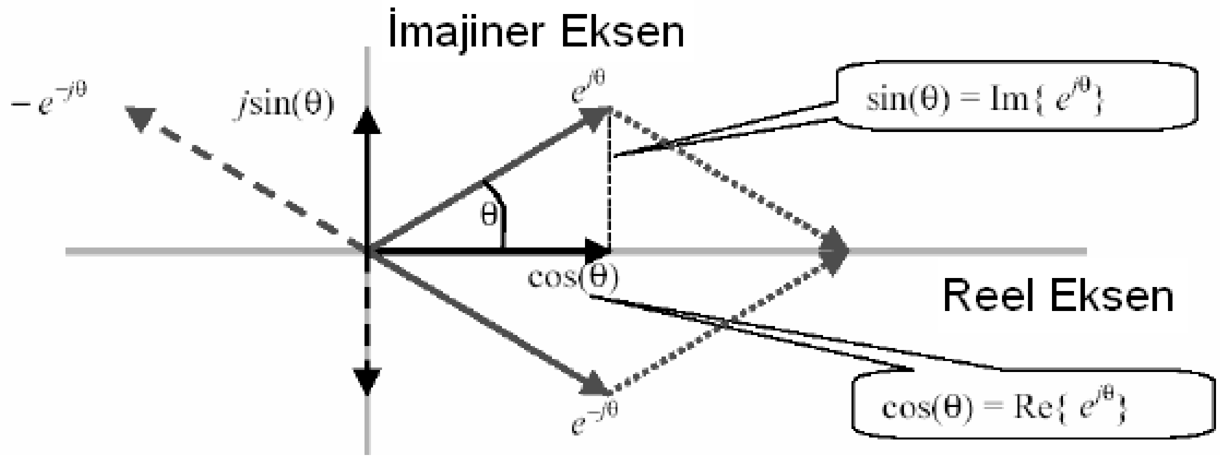
$$e^{-j\theta} = \cos(\theta) - j\sin(\theta) \qquad \text{(B)}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \text{(C)}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j} \qquad \text{(D)}$$

Eq. C = (Eq. A+ Eq. B)/2 D = (A – B)/2

A = C + jD B = C – jD



Konjugeyt

Conjugate

$z^{*} \quad \bar{z}$

$$z = a + jb \quad \Rightarrow \quad z^{*} = a - jb$$

$$z = re^{j\theta} \quad \Rightarrow \quad z^{*} = re^{-j\theta}$$

$$1.\ z + z^{*} = 2\,\mathrm{Re}\{z\}$$

$$2.\ z \times z^{*} = (a + jb)(a - jb) = a^2 + b^2 = |z|^2$$

Genel Sonuçlar

$$z = re^{j\theta}$$

$$z = r\cos\theta + jr\sin\theta$$

$$z = a + jb$$

$$z = \sqrt{a^2 + b^2}\, e^{j\tan^{-1}(b/a)}$$

Açısal form çarpma:

$$\left(r_1 e^{j\theta_1}\right)\left(r_2 e^{j\theta_2}\right) = r_1 r_2 e^{j(\theta_1+\theta_2)}$$

$$z^n = \left(re^{j\theta}\right)^n = r^n e^{jn\theta}$$

$$z^{1/n} = r^{1/n} e^{j\theta/n}$$

$$z = a + jb$$

$$|z| = \sqrt{a^2 + b^2} \qquad \angle z = \tan^{-1}(b/a)$$

Açısal form bölme:

$$\frac{\left(r_1 e^{j\theta_1}\right)}{\left(r_2 e^{j\theta_2}\right)} = \frac{r_1}{r_2} e^{j(\theta_1-\theta_2)}$$

$$\frac{1}{z_2} = \frac{1}{r_2 e^{j\theta_2}} = \frac{1}{r_2} e^{-j\theta_2}$$

$$|z_1 z_2| = |z_1||z_2|$$

$$\angle\{z_1 z_2\} = \angle\{z_1\} + \angle\{z_2\}$$

$$|z_1 / z_2| = |z_1| / |z_2|$$

$$\angle\{z_1 / z_2\} = \angle\{z_1\} - \angle\{z_2\}$$

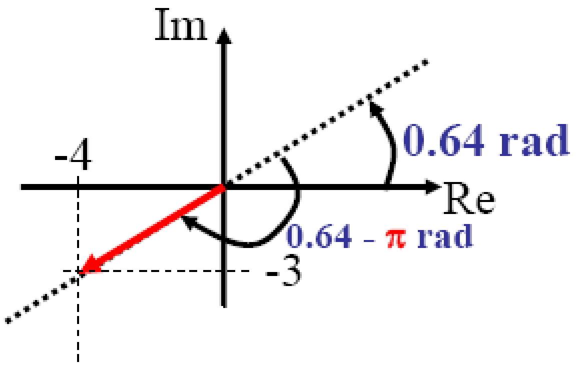
Örnek-1

$z = -4 - j3$ sayısını açısal forma çevirin.

$$|z| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\angle z = \tan^{-1}\left(\frac{-3}{-4}\right) = \tan^{-1}(0.75) - \pi = 0.64 - \pi \approx -2.5\ rad$$

$\boxed{z = -4 - j3} \Leftrightarrow \boxed{z = 5e^{-j2.5}}$



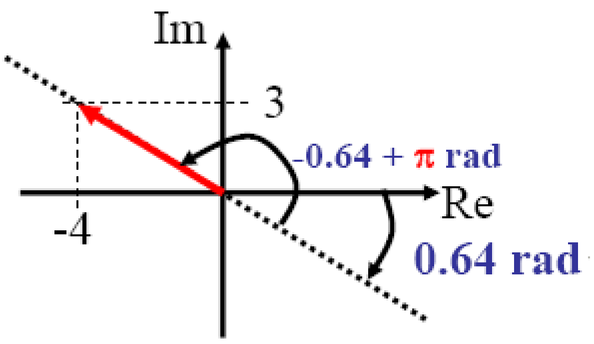
Örnek-2

$z = -4 + j3$ sayısını açısal forma çevirin.

$$|z| = \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\angle z = \tan^{-1}\left(\frac{3}{-4}\right) = \tan^{-1}(-0.75) + \pi = -0.64 + \pi \approx 2.5\ rad$$

$$\boxed{z = -4 + j3} \Leftrightarrow \boxed{z = 5e^{j2.5}}$$



Örnek-3

$z = 3e^{j\pi/4}$ sayısını rectangular forma çevirin.

$$|z| = 3 \quad \angle z = \pi/4$$

$$\cos(\pi/4) = 1/\sqrt{2} \qquad \sin(\pi/4) = 1/\sqrt{2}$$

$$z = |z|\cos(\angle z) + j\,|z|\sin(\angle z)$$

$$\boxed{z = 3e^{j\pi/4}} \Leftrightarrow \boxed{z = \frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}}}$$