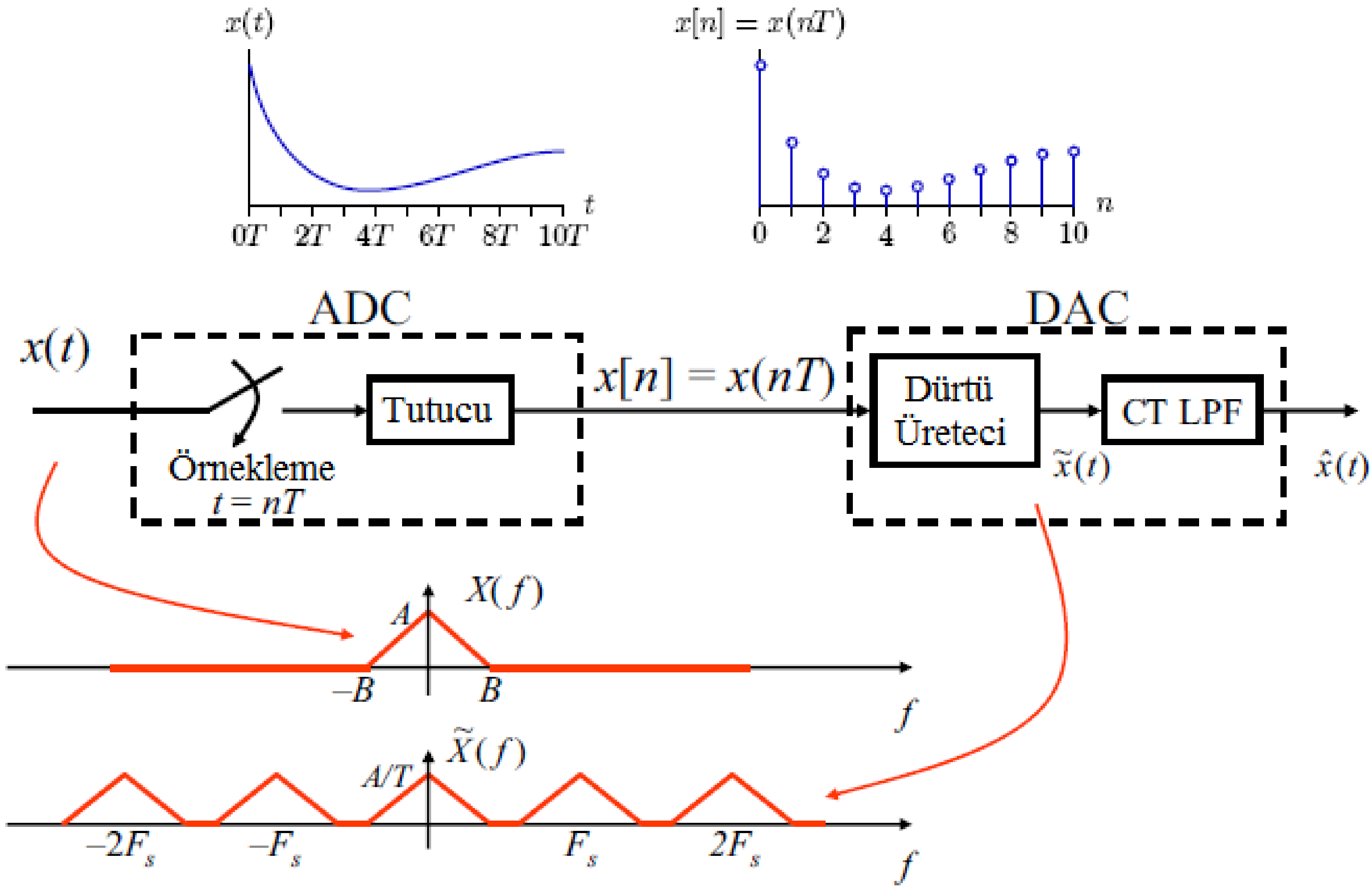
Sinyaller ve Sistemler

Örnekleme Analizi



$$F_s \geq 2\text{B}$$

Ayrık Zaman Fourier Dönüşümü
(Discrete-Time Fourier Transform-**DTFT**)

$$\tilde{x}(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = x(t)\delta_T(t)$$

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t-nT)$$

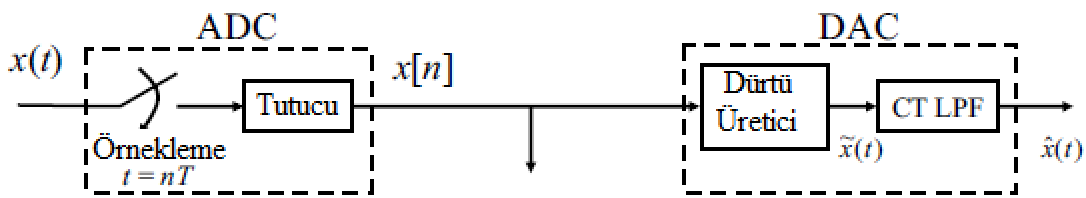
$$\tilde{X}(\omega) = \mathfrak{F}\left\{\sum_{n=-\infty}^{\infty} x[n]\delta(t-nT)\right\}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\mathfrak{F}\{\delta(t-nT)\}$$

$$\tilde{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\omega T}$$

Fourier Tablosu

$$\delta(t-c) \Rightarrow e^{-j\omega c}$$

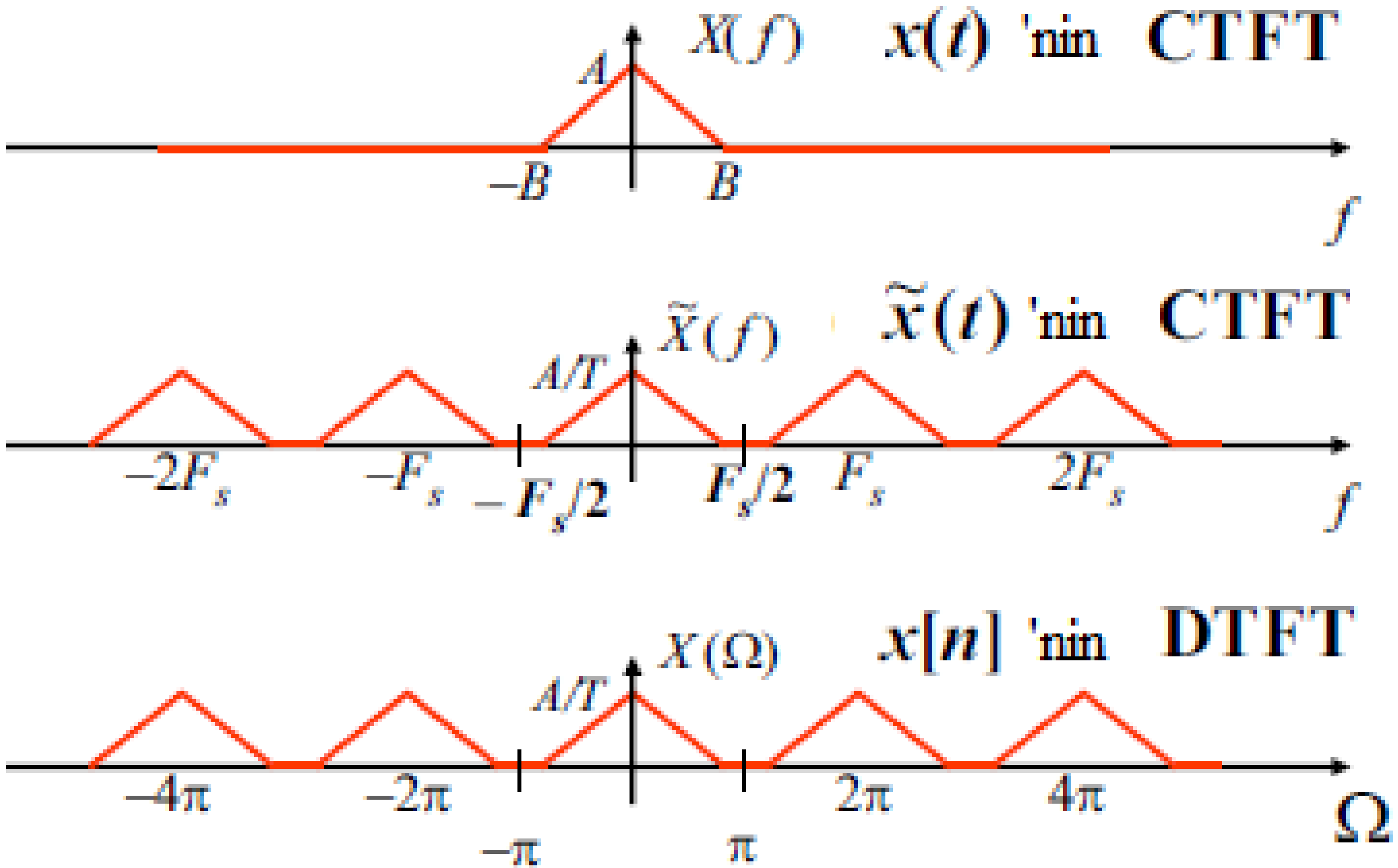


$\tilde{X}(f)$?????
Ayrık zaman frekens cevabı

$\Omega = \omega T$ dersek $T = 1/F_s$

$\Omega \Rightarrow$ **"D-T Frekansı"**

DTFT: $$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\Omega}$$


X(f)
x(t) 'nin CTFT
A
−B
B
f
X̃(f)
x̃(t) 'nin CTFT
A/T
−2Fs
−Fs
−Fs/2
Fs/2
Fs
2Fs
f
X(Ω)
x[n] 'nin DTFT
A/T
−4π
−2π
−π
π
2π
4π
Ω

DTFT 'nin Karakteristiği

DTFT $\boxed{X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}}$ **CTFT** $\boxed{X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt}$

Benzer yapıdalar !!!

1-Periyodikliği:

$X(\Omega)$ Ω 'un 2π periyotlu bir fonksiyonu ise

$\Rightarrow |X(\Omega)|$ periyodiktir 2π

$\angle X(\Omega)$ periyodiktir 2π

2-Genelde kompleks değerlidir:

$$X(\Omega)=\sum_{n} x[n]\underbrace{e^{-j\Omega n}}$$ kompleks

$$X(\Omega)=\underbrace{|X(\Omega)|}_{\text{genlik}}e^{j\underbrace{\angle X(\Omega)}_{\text{faz}}}$$

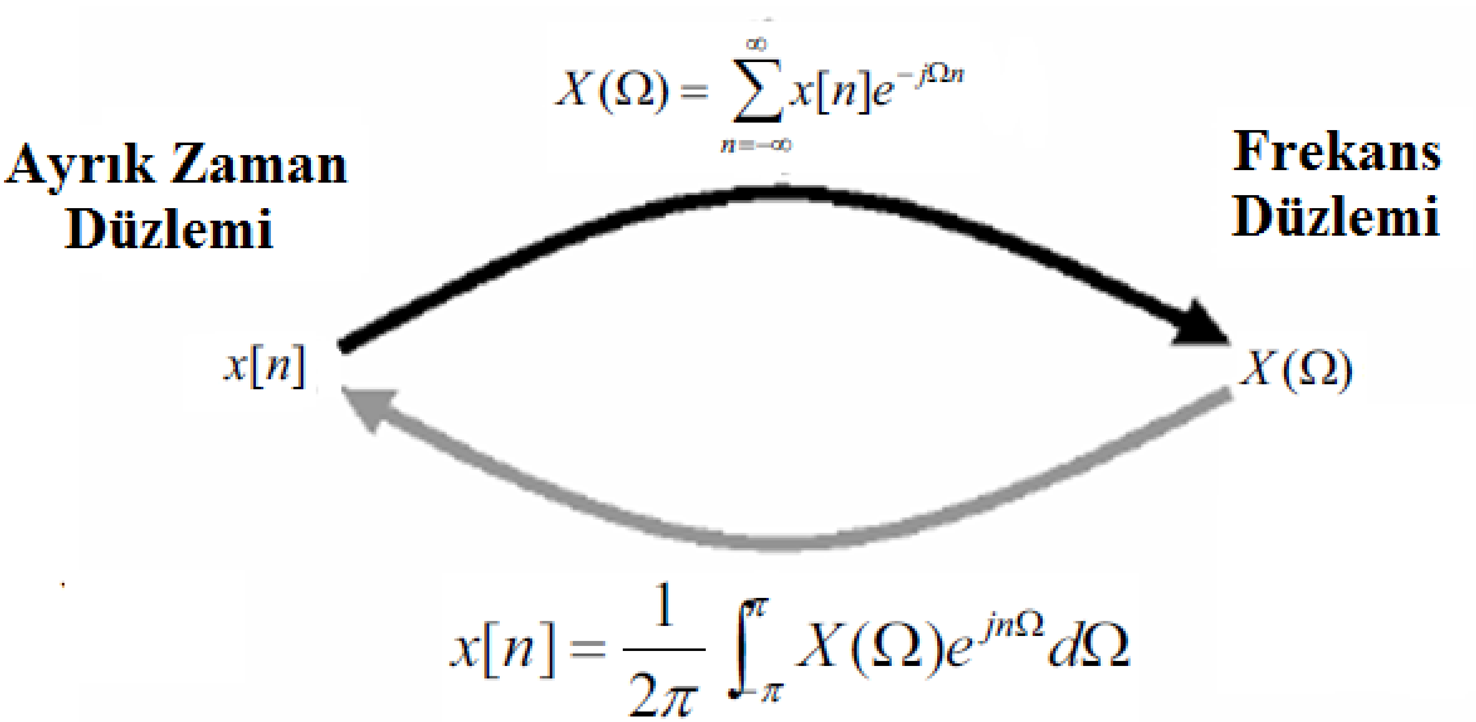
CTFT ile aynı

3-Simetri: $|X(-\Omega)|=|X(\Omega)|$

$$\angle X(-\Omega)=-\angle X(\Omega)$$

Ters Ayrık Zaman Fourier Dönüşümü (Inverse DTFT)

$$\boxed{x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\Omega)e^{jn\Omega}d\Omega}$$



Örnek-1

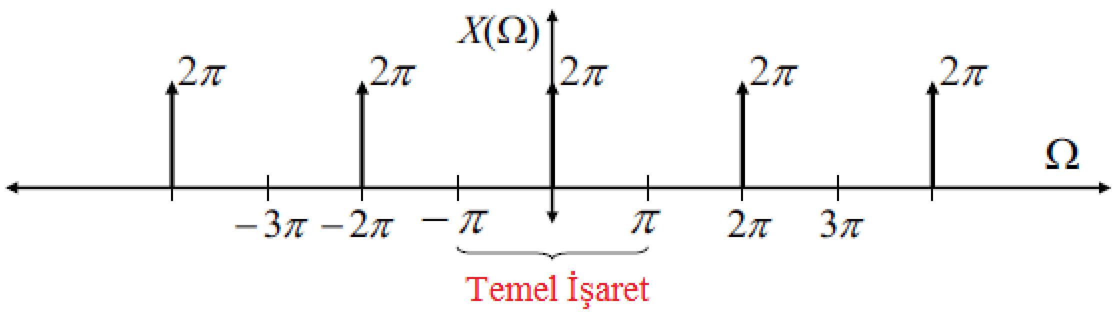
$$x[n]=1,\ \forall n \quad \leftrightarrow \quad X(\Omega)=\begin{cases} 2\pi\delta(\Omega), & -\pi<\Omega<\pi \\ \text{periyodik} & \text{diğer} \end{cases}$$

2π

Hatırlayın!

Fourier dönüşüm tablosu (sunu-6)

Time Signal	Fourier Transform
$1, \quad -\infty<t<\infty$	$2\pi\delta(\omega)$



$$X(\Omega)=2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-k2\pi)$$

Ters DTFT:

$$x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\Omega)e^{jn\Omega}d\Omega$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}2\pi\delta(\Omega)e^{jn\Omega}d\Omega$$

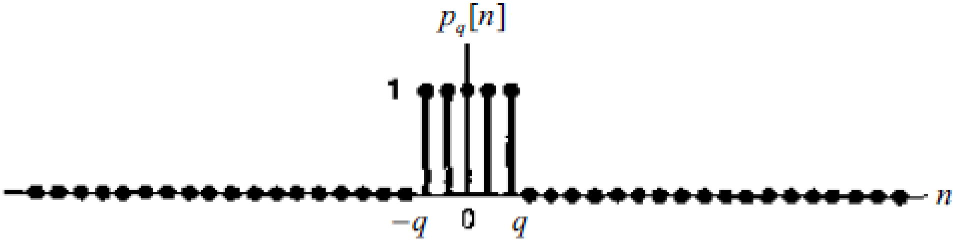
$$=e^{jn\cdot 0}$$

$$=1$$

Örnek-2

Rectangular darbe:

$$p_q[n] = \begin{cases} 1, & n = -q, \ldots, -1, 0, 1, \ldots, q \\ 0, & diğer \end{cases}$$



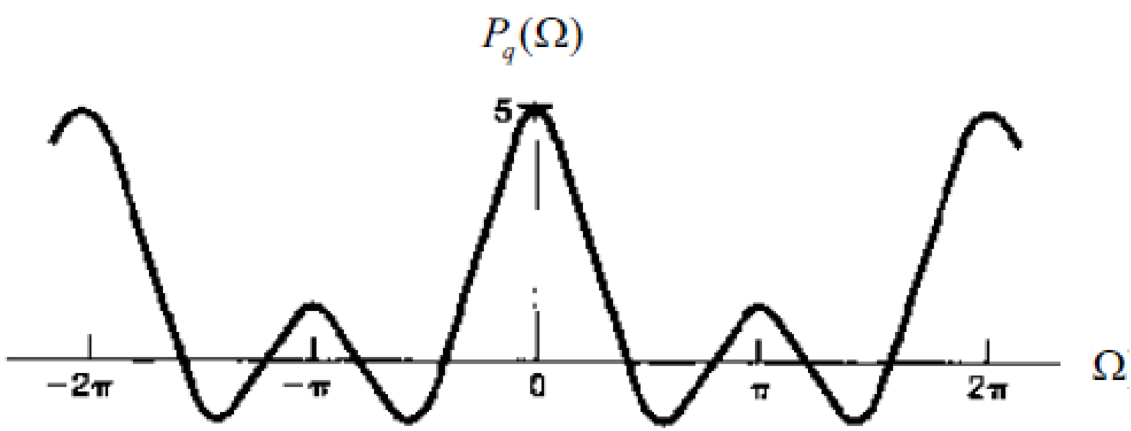
$$P_q(\Omega) = \sum_{n=-q}^{q} e^{-jn\Omega}$$

$$\sum_{n=q_1}^{q_2} r^n = \frac{r^{q_1} - r^{q_2+1}}{1-r}$$

$$P_q(\Omega) = \frac{e^{jq\Omega} - e^{-j(q+1)\Omega}}{1 - e^{-j\Omega}} = \frac{\sin\{(q+1/2)\Omega\}}{\sin\{\Omega/2\}}$$

Pay ve paydayı çarp $e^{j\ \Omega/2}$

$q = 2$ için



DTFT Table

Signal	DTFT
$1, \quad -\infty < n < \infty$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$
$\mathrm{sgn}[n] = \begin{cases} -1, & \ldots, -3, -2, -1 \\ 1, & 0, 1, 2, \cdots \end{cases}$	$\frac{2}{1 - e^{-j\Omega}}$
$u[n]$	$\frac{1}{1 - e^{-j\Omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$
$\delta[n]$	$1, \quad -\infty < \Omega < \infty$
$\delta[n-q], \quad q = \pm 1, \pm 2, \pm 3, \ldots$	$e^{-jq\Omega}, \quad q = \pm 1, \pm 2, \pm 3, \ldots$
$a^n u[n], \quad \lvert a \rvert < 1$	$\frac{1}{1 - ae^{-j\Omega}}, \quad \lvert a \rvert < 1$
$e^{j\Omega_o n}, \quad \Omega_o$ real	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - \Omega_o - 2\pi k), \quad \Omega_o$ real
$p_q[n] = \begin{cases} 1, & n = -q, -q+1, \ldots \\ & , -1, 0, 1, \ldots q \\ 0, & otherwise \end{cases}$	$\frac{\sin[(q + \frac{1}{2})\Omega]}{\sin(\Omega / 2)}$
$(n+1)\, a^n u[n], \quad \lvert a \rvert < 1$	$\frac{1}{(1 - ae^{-j\Omega})^2} \quad \lvert a \rvert < 1$
$\cos(\Omega_o n)$	$\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega + \Omega_o - 2\pi k) + \delta(\Omega - \Omega_o - 2\pi k)]$
$\cos(\Omega_o n + \theta)$	$\pi \sum_{k=-\infty}^{\infty} [e^{-j\theta} \delta(\Omega + \Omega_o - 2\pi k) + e^{j\theta} \delta(\Omega - \Omega_o - 2\pi k)]$
$\sin(\Omega_o n)$	$j\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega + \Omega_o - 2\pi k) - \delta(\Omega - \Omega_o - 2\pi k)]$
$\sin(\Omega_o n + \theta)$	$j\pi \sum_{k=-\infty}^{\infty} [e^{-j\theta} \delta(\Omega + \Omega_o - 2\pi k) - e^{j\theta} \delta(\Omega - \Omega_o - 2\pi k)]$

Ayrık Fourier Dönüşümü (Discrete Fourier Transform-**DFT**)

$$X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$ analitik olarak hesaplanabilir(bileşenlerine ayırma)

DTFT

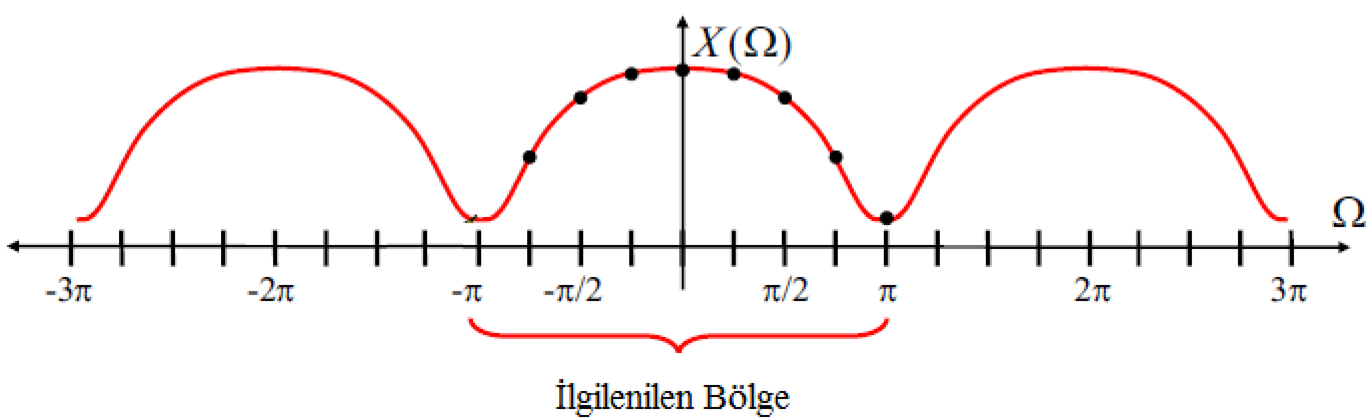
Gerekenler:

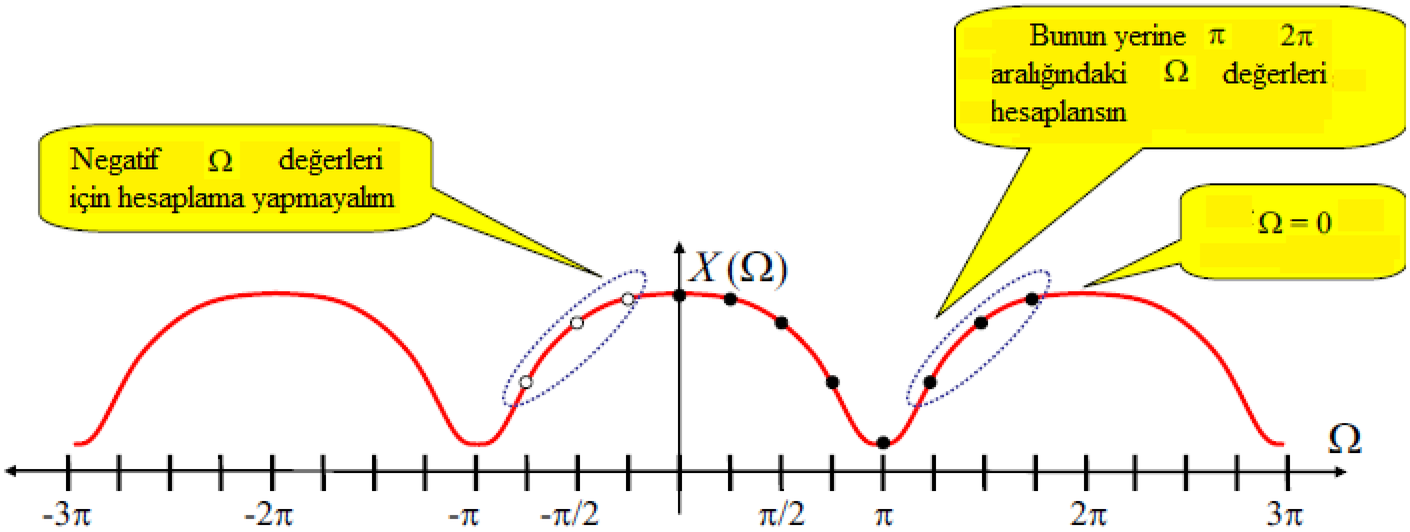
1-Sonlu sayıdaki terimlerin toplamı $n = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

2-Sonlu sayıda noktalar değerlendirilmelidir $\Omega \in (-\pi, \pi]$ aralığı

$x[n]$ $n = 0, \ldots, N\text{-}1$ adet veri

$$X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}=\sum_{n=0}^{N-1} x[n]e^{-j\Omega n}$$





M noktalı DTFT

$$\Omega_k = k\boxed{\frac{2\pi}{M}}, \qquad k = 0,1,2,\ldots,M-1$$

$$\boxed{\Omega_0 = 0,} \quad \boxed{\Omega_1 = \frac{2\pi}{M},} \quad \boxed{\Omega_2 = 2\frac{2\pi}{M},} \quad \ldots \quad \boxed{,\Omega_{M-1} = (M-1)\frac{2\pi}{M}}$$

$$X(\Omega_k) = \sum_{n=0}^{N-1} x[n]e^{-jn\Omega_k} = \sum_{n=0}^{N-1} x[n]e^{-jnk\frac{2\pi}{M}}, \quad for \quad k = 0, 1, 2, \ldots, M-1$$

frekans noktaları sinyal noktaları ile aynı olduğundan $M = N$

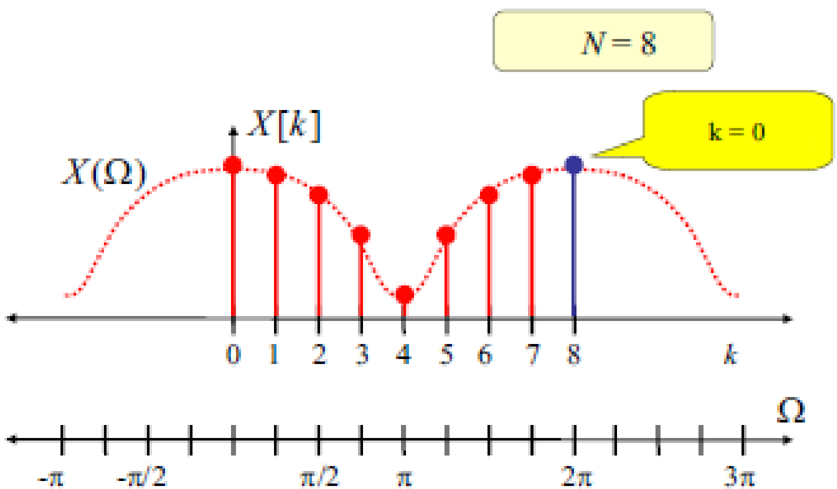
N adet veri noktası $x[n]$ $\quad n = 0, \ldots, N\text{-}1$

$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N} \qquad k = 0,1,2,\ldots,N-1}$$

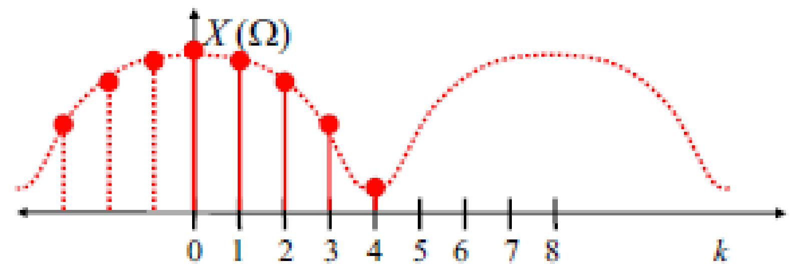
Ayrık Fourier Dönüşümü: **DFT**

DFT ‘nin Özellikleri

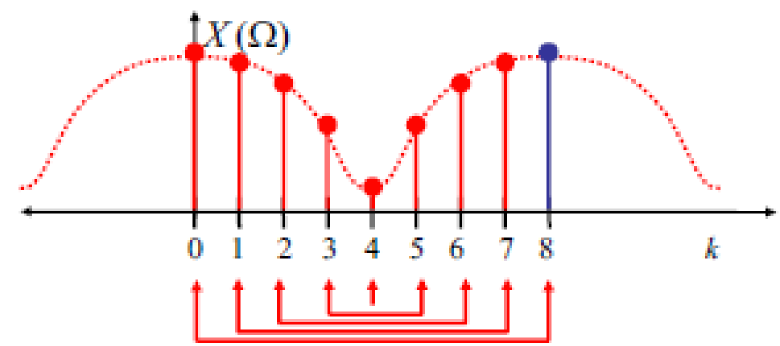
1-Simetri: $$X[N-k]=\overline{X}[k], \quad k=0,1,2,\ldots,N-1$$



$$\frac{2\pi}{N} \Rightarrow \frac{2\pi}{8}=\frac{\pi}{4}$$



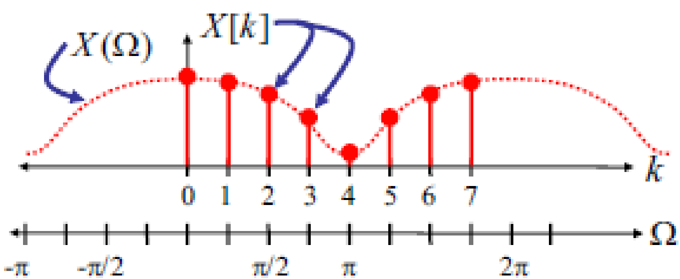
Temel frekansa ötelenirse

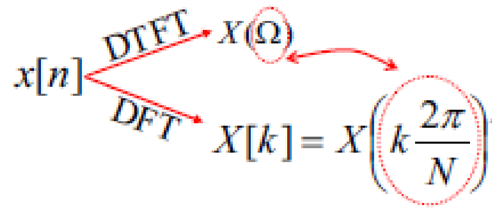


2-Ters DFT: $$x[n]=\frac{1}{N}\sum_{n=0}^{N-1}X[k]e^{j2\pi kn/N} \qquad n=0,1,2,\ldots,N-1$$

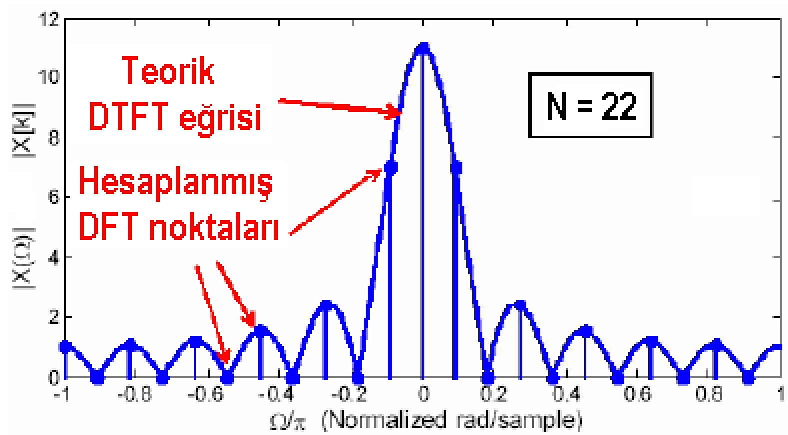
Inverse DFT (IDFT)

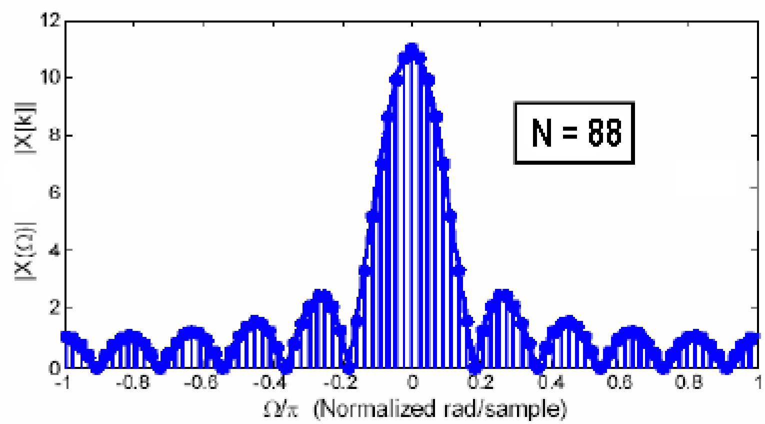
DTFT - DFT İlişkisi

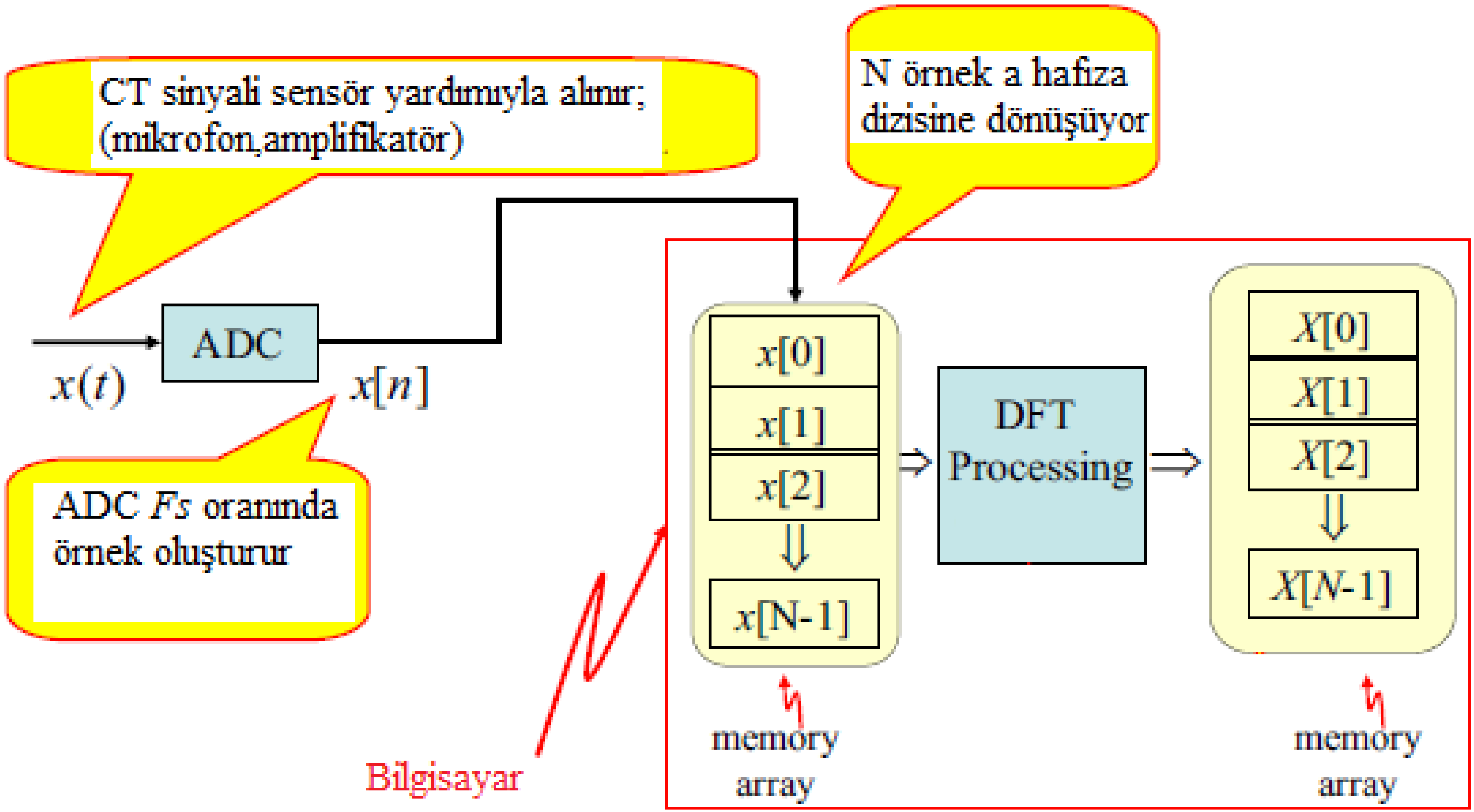




DFT noktaları
x[n] DTFT 'nin örnekleridir






CT sinyali sensör yardımıyla alınır;
(mikrofon,amplifikatör)
N örnek a hafiza
dizisine dönüşüyor
ADC
$x(t)$
$x[n]$
ADC Fs oranında
örnek oluşturur
$x[0]$
$x[1]$
$x[2]$
$x[N-1]$
DFT
Processing
$X[0]$
$X[1]$
$X[2]$
$X[N-1]$
Bilgisayar
memory
array
memory
array

DTFT Özellikleri

1-Doğrusallık:

$$x_1[n] \overset{\mathfrak{F}}{\leftrightarrow} X_1(\Omega)$$

$$x_2[n] \overset{\mathfrak{F}}{\leftrightarrow} X_2(\Omega)$$

$$a x_1[n] + b x_2[n] \overset{\mathfrak{F}}{\leftrightarrow} a X_1(\Omega) + b X_2(\Omega)$$

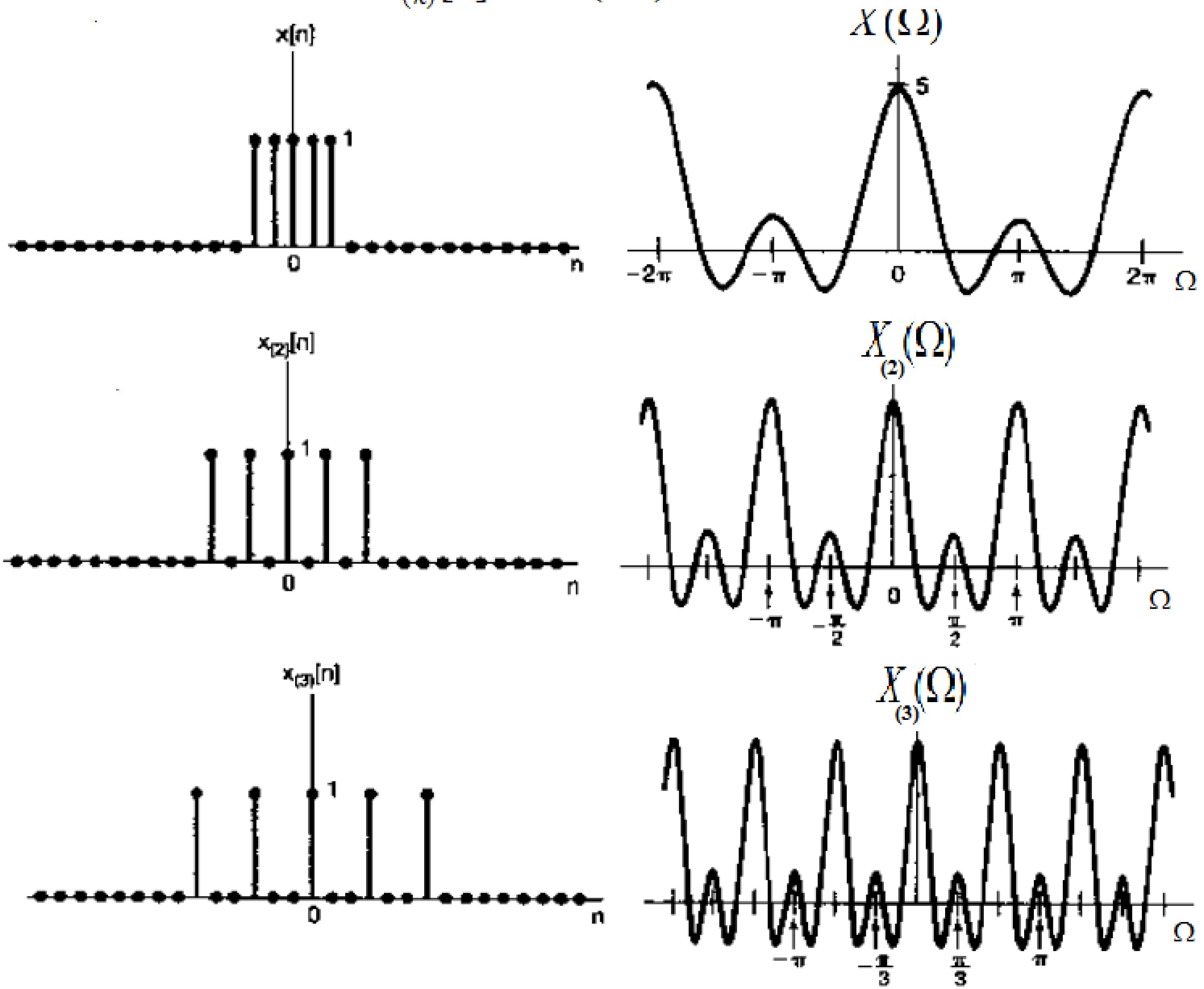
2-Zamanda Öteleme:

$$x[n] \overset{\mathfrak{F}}{\leftrightarrow} X(\Omega)$$

$$x[n - n_0] \overset{\mathfrak{F}}{\leftrightarrow} e^{-j\Omega n_0} X(\Omega)$$

3-Zaman Genişlemesi (Ölçeklendirme):

$$x_{(k)}[n] \overset{\mathfrak{F}}{\leftrightarrow} X(k\Omega)$$



4-Konvolüsyon:

$$y[n] = x[n] * h[n]$$

$$\boxed{Y(\Omega) = X(\Omega)H(\Omega)}$$

Örnek: Dürtü cevabı $h[n] = \alpha^n u[n]$ olan LTI sistemin girişi $x[n] = \beta^n u[n]$ ise

$$y[n] = ?$$

$$H(\Omega) = \frac{1}{1-\alpha e^{-j\Omega}} \qquad X(\Omega) = \frac{1}{1-\beta e^{-j\Omega}}$$

$$Y(\Omega) = H(\Omega)X(\Omega) = \frac{1}{(1-\alpha e^{-j\Omega})(1-\beta e^{-j\Omega})}$$

$$Y(\Omega) = \frac{A}{1-\alpha e^{-j\Omega}} + \frac{B}{1-\beta e^{-j\Omega}} \qquad A = \frac{\alpha}{\alpha-\beta} \qquad B = -\frac{\beta}{\alpha-\beta}$$

$$y[n] = \frac{\alpha}{\alpha-\beta}\alpha^n u[n] - \frac{\beta}{\alpha-\beta}\beta^n u[n]$$

$$= \frac{1}{\alpha-\beta}\left[\alpha^{n+1}u[n] - \beta^{n+1}u[n]\right]$$