Sinyaller ve Sistemler

Fourier Serileri

$$x(t) = \sum_{k=-N}^{N} c_k e^{jk\omega_0 t} \qquad x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(k\omega_0 t + \theta_k)$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

k integers

Fourier Serisi
Kompleks Eksponensiyel Form

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)$$

k integers

Fourier Serisi
Trigonometrik Form

Zaman-düzlem modeli		Frekans-düzlem modeli
zaman düzleminde $x(t)$ verildiğinde	→	Fourier Serilerinin katsayılarını $\{c_k\}$ bulduk

Zaman-düzlemindeki sinyal modelini bir frekans-düzlemindeki sinyal modeline dönüştürdük

Fourier Serisi Katsayılarının Hesaplanması

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t) e^{-jk\omega_0 t}$$

Zaman-düzlemi $\Rightarrow$ Frekans-düzleminde

Periyodik bir sinyal $\Rightarrow$ Fourier Serisi katsayıları

$T = x(t)$ 'nin temel periyodu

$\omega_0 = x(t)$ 'nin temel frekansı

$= 2\pi/T$

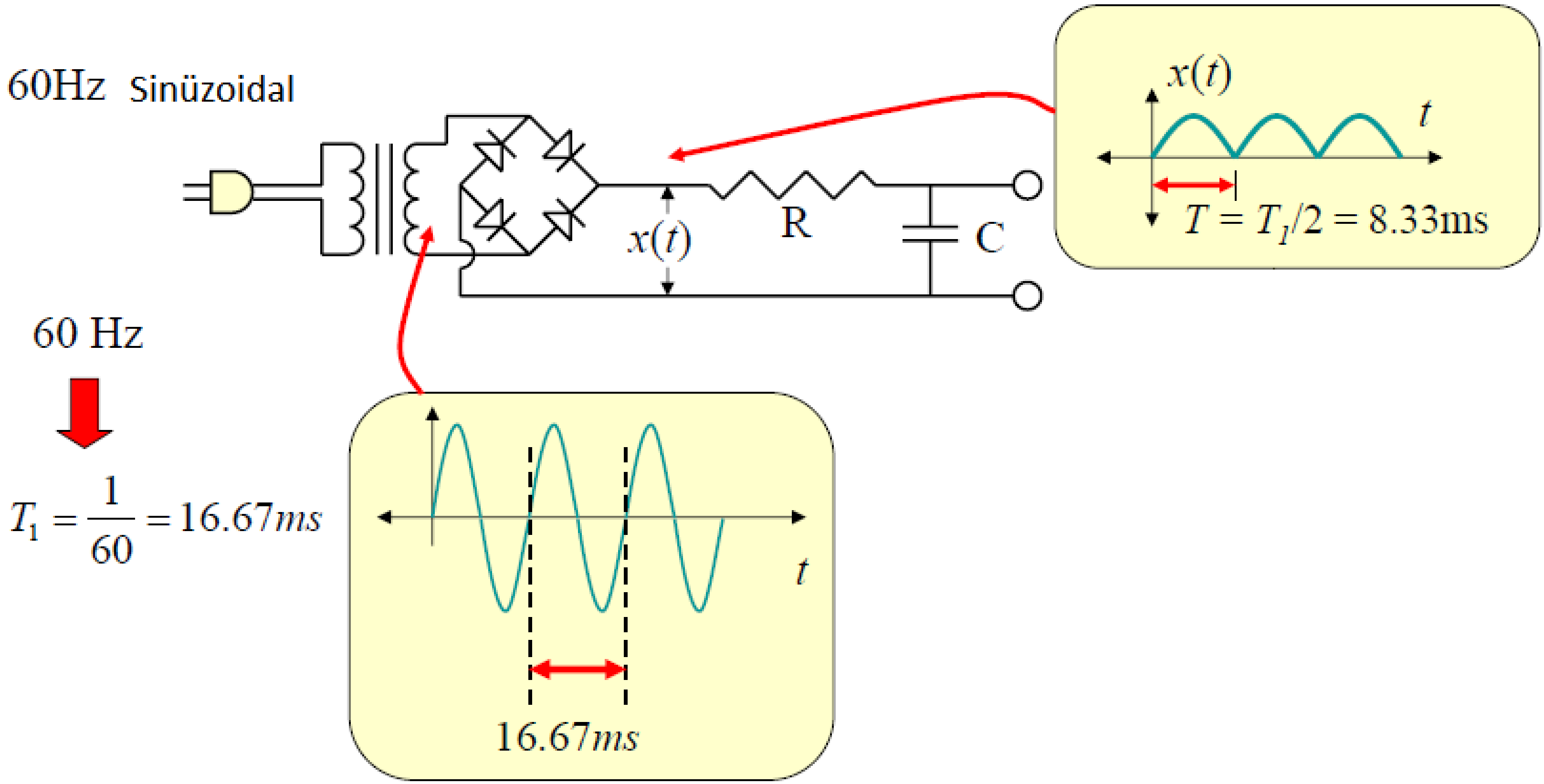
$t_0 =$ herhangi bir an

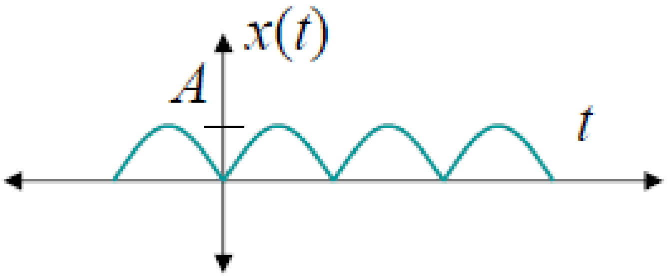
$k = 0$

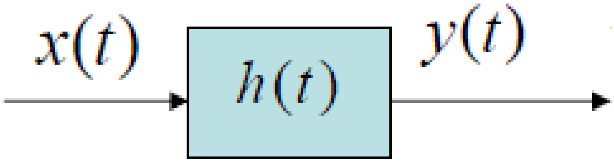
$$c_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)dt$$

DC offset bileşeni

DC Güç Kaynağı







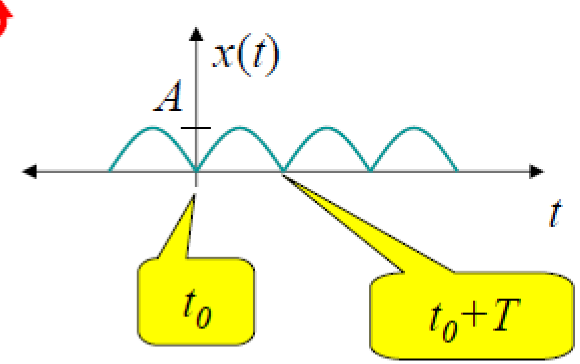
Not: x(t) için Fourier serisini buluyoruz, sistem frekans cevabı(Laplace dönüşümleri) daha sonra anlatılacaktır.

FS katsayı denklemi:

$$\boxed{c_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}}$$

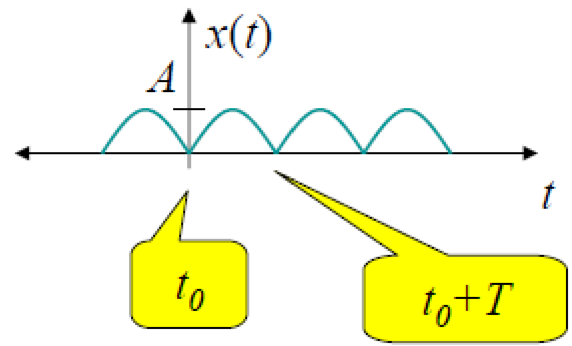
$$c_k = \frac{1}{T}\int_{0}^{T} x(t)e^{-jk\omega_0 t}dt \qquad \omega_0 = \frac{2\pi}{T}$$

$t_0 = 0$



$x(t)$ 'yi belirleyelim $\quad t \in [0,T]$?

$$\Rightarrow x(t) = A\sin\left(\frac{\pi}{T}t\right) \quad 0 \le t \le T$$



$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$$

$$= \frac{1}{T}\int_0^T A\sin\left(\frac{\pi}{T}t\right)e^{-jk\left(\frac{2\pi}{T}\right)t}dt$$

$$\omega_0 = \frac{2\pi}{T}$$

$$c_k = \frac{1}{T}\int_0^T A\sin\left(\frac{\pi}{T}t\right)e^{-jk\left(\frac{2\pi}{T}\right)t}dt$$

İntegrali 3 adımda hesaplanır:

1-Yeni değişken tanımlama

2- Yeni değişkenin türevini alma

3- integralin sınırlarını değiştirme

1. Adım: $\boxed{\tau = \frac{\pi}{T}t}$ $\boxed{\sin(\tau)e^{-jk2\tau}}$

2. Adım: $\boxed{d\tau = \frac{\pi}{T}dt}$ $\boxed{\Rightarrow dt = \frac{T}{\pi}d\tau}$

3. Adım: $t = 0 \Rightarrow \boxed{\tau = \frac{\pi}{T}0 = 0}$

$t = T \Rightarrow \boxed{\tau = \frac{\pi}{T}T = \pi}$

$$c_k = \frac{1}{T}\int_0^T A\sin\left(\frac{\pi}{T}t\right)e^{-jk\left(\frac{2\pi}{T}\right)t}dt$$

$$= \frac{1}{T}\int_0^{\pi} A\sin(\tau)e^{-jk2\tau}\left(\frac{T}{\pi}d\tau\right)$$

$$= \frac{A}{\pi}\int_0^{\pi} \sin(\tau)e^{-jk2\tau}d\tau$$

$$\boxed{c_k = \frac{A}{\pi}\int_0^{\pi} \sin(\tau) e^{-jk2\tau} d\tau}$$

İntegral Tablosu

$$\int e^{ax}\sin(bx)dx = \frac{e^{ax}[a\sin(bx) - b\cos(bx)]}{a^2 + b^2}$$

$a = -j2k \quad b = 1$

$$c_k = \frac{A}{\pi}\left[\frac{e^{-j2k\tau}[-j2k\sin(\tau) - \cos(\tau)]}{1-4k^2}\right]_0^{\pi}$$

$\sin(0) = \sin(\pi) = 0$ olduğundan ifadeyi sadeleştirirsek;

$$c_k = \frac{-A}{\pi(1-4k^2)}\left[e^{-j2k\tau}\cos(\tau)\right]_0^{\pi}$$

$$c_k = \frac{-A}{\pi(1-4k^2)}\underbrace{\left[\underbrace{e^{-j2\pi k}}_{=1}\underbrace{\cos(\pi)}_{=-1} - \underbrace{e^{-j2k0}}_{=1}\underbrace{\cos(0)}_{=1}\right]}_{=-2}$$

$$\boxed{c_k = \frac{2A}{\pi(1-4k^2)}}$$

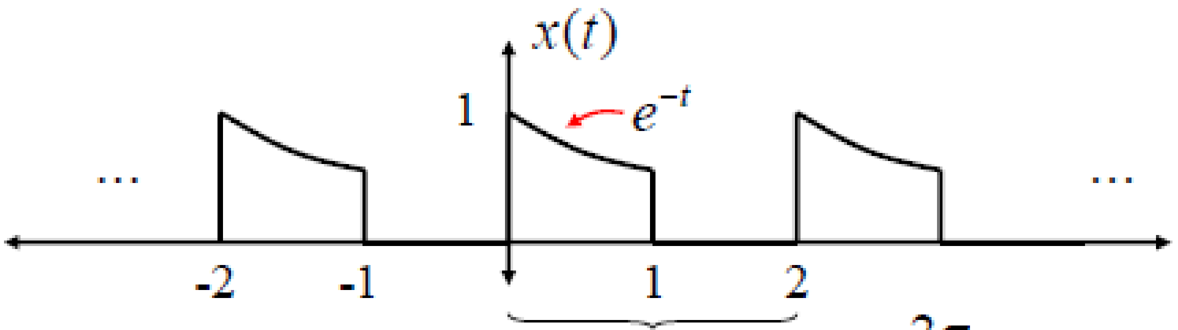
Trigonometrik Dönüşüm Tablosu

$e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$
$\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
$\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$
$\cos(\theta \pm \pi/2) = \mp\sin(\theta)$
$\sin(\theta \pm \pi/2) = \pm\cos(\theta)$
$2\sin(\theta)\cos(\theta) = \sin(2\theta)$
$\sin^2(\theta) + \cos^2(\theta) = 1$
$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$
$\cos^2(\theta) = \frac{1}{2}\left[1 + \cos(2\theta)\right]$
$\sin^2(\theta) = \frac{1}{2}\left[1 - \cos(2\theta)\right]$
$\cos^3(\theta) = \frac{1}{4}\left[3\cos(\theta) + \cos(3\theta)\right]$
$\sin^3(\theta) = \frac{1}{4}\left[3\sin(\theta) - \sin(3\theta)\right]$
$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$
$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$
$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$
$a\cos(\theta) + b\sin(\theta) = C\cos(\theta + \phi)$ $C = \sqrt{a^2 + b^2} \quad \phi = \tan^{-1}(-b/a)$

İntegral Tablosu

$\int u\,dv = uv - \int v\,du$
$\int f(x)\dot{g}(x)dx = f(x)g(x) - \int \dot{f}(x)g(x)dx$
$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)$
$\int \cos(ax)dx = \frac{1}{a}\sin(ax)$
$\int \sin^2(ax)dx = \dfrac{x}{2} - \frac{1}{4a}\sin(2ax)$
$\int \cos^2(ax)dx = \dfrac{x}{2} - \frac{1}{4a}\sin(2ax)$
$\int x\sin(ax)dx = \frac{1}{a^2}\left[\sin(ax) - ax\cos(ax)\right]$
$\int x\cos(ax)dx = \frac{1}{a^2}\left[\cos(ax) + ax\sin(ax)\right]$
$\int x^2\sin(ax)dx = \frac{1}{a^3}\left[2ax\sin(ax) + 2\cos(ax) - a^2x^2\cos(ax)\right]$
$\int x^2\cos(ax)dx = \frac{1}{a^3}\left[2ax\cos(ax) - 2\sin(ax) + a^2x^2\sin(ax)\right]$
$\int \sin(ax)\sin(bx)dx = \frac{1}{2(a-b)}\sin((a-b)x) - \frac{1}{2(a+b)}\sin((a+b)x)$ $a^2 \neq b^2$
$\int \sin(ax)\cos(bx)dx = -\frac{1}{2(a-b)}\cos((a-b)x) - \frac{1}{2(a+b)}\cos((a+b)x)$ $a^2 \neq b^2$
$\int \cos(ax)\cos(bx)dx = \frac{1}{2(a-b)}\sin((a-b)x) + \frac{1}{2(a+b)}\sin((a+b)x)$ $a^2 \neq b^2$
$\int e^{ax}dx = \frac{1}{a}e^{ax}$
$\int xe^{ax}dx = \frac{1}{a^2}e^{ax}(ax-1)$
$\int x^2e^{ax}dx = \frac{1}{a^3}e^{ax}\left(a^2x^2 - 2ax + 2\right)$
$\int e^{ax}\sin(bx)dx = \frac{1}{a^2+b^2}e^{ax}\left(a\sin(bx) - b\cos(bx)\right)$
$\int e^{ax}\cos(bx)dx = \frac{1}{a^2+b^2}e^{ax}\left(a\cos(bx) + b\sin(bx)\right)$

Örnek



$$T = 2 \;\Rightarrow\; \omega_0 = \frac{2\pi}{2} = \pi \text{ rad/sn}$$

$$c_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{2}\int_{0}^{2} x(t)e^{-jk\pi t}\,dt$$

$$= \frac{1}{2}\left[\int_{0}^{1} e^{-t}e^{-jk\pi t}\,dt + \int_{1}^{2} 0\times e^{-jk\pi t}\,dt\right]$$

$$= \frac{1}{2}\int_{0}^{1} e^{-(1+jk\pi)t}\,dt$$

$$= \frac{1}{2}\left[\frac{-1}{1+jk\pi}e^{-(1+jk\pi)t}\right]_{0}^{1}$$

$$= \frac{-1}{2(1+jk\pi)}\left[e^{-(1+jk\pi)} - 1\right]_{0}^{1}$$

$$\boxed{c_k = \frac{1-e^{-1}e^{jk\pi}}{2(1+jk\pi)}}$$