Örnek: $y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n]$

$IC's = 0$

a-) Sistemin Transfer fonksiyonu $H(z) = ?$

b-) Birim dürtü cevabı $h[n] = ?$

c-) Birim basamak cevabı ? $x[n] = u[n] \rightarrow y[n] = ?$

Çözüm: Denklemi Z-dönüşümü alınırsa;

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z)$$

$$\left(1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right) Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$\frac{z^2}{z^2}$ ile çarpılırsa

$$H(z) = \frac{z^2}{z^2 - \frac{3}{4} z + \frac{1}{8}}$$

$$\frac{H(z)}{z} = \frac{z}{z^2 - \frac{3}{4} z + \frac{1}{8}} \qquad \frac{A}{z - \frac{1}{2}} + \frac{B}{z - \frac{1}{4}}$$

$$(z - 1/2)(z - \tfrac{1}{4})$$

$A = 2 \qquad B = -1$

$$H(z) = \frac{2z}{z-\frac{1}{2}} - \frac{z}{z-\frac{1}{4}}$$

Ters z dönüşümü ile;

b.) $h[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$

c.) $x[n] = u[n]$ için

$$Y(z) = H(z) \cdot X(z)$$

$$Y(z) = \frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} \cdot \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)(z-1)}$$

$$\frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{4}} + \frac{C}{z-1}$$

$A = -2 \qquad B = 1/3 \qquad C = 8/3$

$$Y(z) = -\frac{2z}{z-1/2} + \frac{1}{3}\frac{z}{z-1/4} + \frac{8}{3}\frac{z}{z-1}$$

$$y[n] = -2\left(\frac{1}{2}\right)^n u[n] + \frac{1}{3}\left(\frac{1}{4}\right)^n u[n] + \frac{8}{3}u[n]$$

Örnek: $y[n] - \frac{1}{2} y[n-1] = x[n]$ $\quad y[-1] = 1$

$x[n] = \left(\frac{1}{3}\right)^n u[n]$ giriş için $y[n] = ?$

$$Y(z) - \frac{1}{2}\left(z^{-1} Y(z) + y[-1]\right) = X(z)$$

$$\left(1 - \frac{1}{2} z^{-1}\right) Y(z) = X(z) + \frac{1}{2} \qquad X(z) = \frac{z}{z - 1/3}$$

$$\left(1 - \frac{1}{2} z^{-1}\right) Y(z) = \frac{z}{z - \frac{1}{3}} + \frac{1}{2}$$

$$Y(z) = \frac{\left(\frac{z}{z - 1/3} + \frac{1}{2}\right) \cdot z}{z - \frac{1}{2}}$$

$$Y(z) = \frac{z\left(\frac{3z - 1/3}{2}\right)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)} = \frac{z\left(\frac{3}{2} z - \frac{1}{6}\right)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}$$

$$\frac{A}{z - \frac{1}{2}} + \frac{B}{z - 1/3}$$

$A = 7/2 \qquad B = -2$

$$Y(z) = \frac{7}{2} \frac{z}{z - 1/2} - 2 \frac{z}{z - 1/3}$$

$$y[n] = \frac{7}{2} \left(\frac{1}{2}\right)^n u[n] - 2 \left(\frac{1}{3}\right)^n u[n]$$

2.40c:

$$y[n] = y_{zi}[n] + y_{zs}[n]$$

$y_{zi}[n]$ için $x[n] = 0$ olacak.

$$Y_{zi}(z) - \frac{1}{2}\left(z^{-1} Y_{zi}(z) + y[-1]\right) = 0$$

$$Y_{zi}(z)\left(1 - \frac{1}{2}z^{-1}\right) = \frac{1}{2}$$

$$Y_{zi}(z) = \frac{1/2}{1 - \frac{1}{2}z^{-1}} = \frac{1}{2}\,\frac{z}{z - 1/2}$$

$$y_{zi}[n] = \frac{1}{2}\left(\frac{1}{2}\right)^{n} u[n]$$

$y_{zs}[n]$ için $IC's = 0$ olur

$$Y_{zs}(z) - \frac{1}{2}z^{-1} Y_{zs}(z) = X(z)$$

$$Y_{zs}(z)\left(1 - \frac{1}{2}z^{-1}\right) = \frac{z}{z - 1/3}$$

$$\frac{Y_{zs}(z)}{z} = \frac{z}{(z - \frac{1}{2})(z - 1/3)}$$

$$\frac{A}{z - 1/2} + \frac{B}{z - 1/3} \qquad A = 3 \quad B = -2$$

$$Y_{zs}(z) = 3\,\frac{z}{z - 1/2} - 2\,\frac{z}{z - 1/3}$$

$$y_{zs}[n] = 3\left(\frac{1}{2}\right)^{n} u[n] - 2\left(\frac{1}{3}\right)^{n} u[n]$$

$$y_{Total} = y_{zi}[n] + y_{zs}[n] = \frac{3}{2}\left(\frac{1}{2}\right)^{n} u[n] - 2\left(\frac{1}{3}\right)^{n} u[n]$$