

Sinyaller ve Sistemler Final Ödevi

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Soru 1)

$$\mathcal{F}\left[\text{sinc}\left[\frac{\gamma t}{2\pi}\right]\right] \leftrightarrow 2\pi P_{\gamma}(\omega)$$

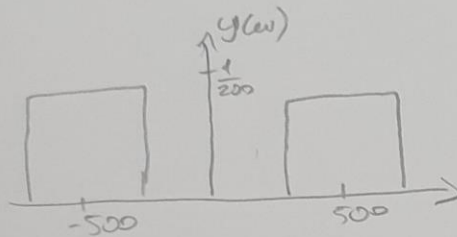
$\text{sinc } 100t$

$$\frac{\gamma t}{2\pi} = 100t \Rightarrow \gamma = 2\pi \times 100$$

$$x(t) = \text{sinc } 100t = \frac{\pi}{\pi} \text{sinc } 100t = \frac{2\pi \times 100}{2\pi \times 100} \text{sinc}(100t) = \frac{2\pi P_{2\pi \times 100}(\omega)}{2\pi \times 100}$$

$$x(t) = \frac{2\pi P_{2\pi \times 100}(\omega)}{2\pi \times 100} = \frac{1}{100} \cdot P_{2\pi \times 100}(\omega) \leftrightarrow x(\omega)$$

$$y(\omega) = \frac{1}{2} \cdot \frac{1}{100} [P_{2\pi \times 100}(\omega + 500) + P_{2\pi \times 100}(\omega - 500)]$$



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$$\text{Soru 2)} \quad 2y[n] - 3y[n-1] + \frac{1}{4}y[n-2] = x[n] - 2x[n-1]$$

$$2y[n] - \frac{3}{2}e^{j\omega}y[n] + \frac{1}{4}e^{j2\omega}y[n] = x[n] - 2e^{j\omega}x[n]$$

$$y[n]\left(2 - \frac{3}{2}e^{j\omega} + \frac{1}{4}e^{j2\omega}\right) = x[n](1 - 2e^{j\omega})$$

$$\frac{y[n]}{x[n]} = H(\omega) \Rightarrow \frac{1 - 2e^{j\omega}}{2 - \frac{3}{2}e^{j\omega} + \frac{1}{4}e^{j2\omega}} \quad e^{j\omega} = x \text{ olsun}$$

$$= \frac{1 - 2x}{2 - \frac{3x}{2} + \frac{1x^2}{4}} = \frac{4(1 - 2x)}{x^2 - 6x + 8} = \frac{4 - 8x}{x^2 - 6x + 8}$$

$$\frac{4 - 8x}{(x-4)(x-2)} = \frac{A}{(x-4)} + \frac{B}{(x-2)} \Rightarrow Ax - 2A + Bx - 4B = 4 - 8x$$

$$\begin{aligned} A + B &= -8 \\ -2A - 4B &= 4 \end{aligned}$$

$$-2B = -12$$

$$\boxed{B = 6}$$

$$A + 6 = -8$$

$$\boxed{A = -14}$$

$$H(\omega) = \frac{-14}{e^{j\omega} - 4} + \frac{6}{e^{j\omega} - 2}$$

$$a) \quad H(\omega) = \frac{14}{4} \left( \frac{1}{1 - \frac{1}{4}e^{j\omega}} \right) - \frac{6}{2} \left( \frac{1}{1 - \frac{1}{2}e^{j\omega}} \right)$$

$$b) \quad H[n] = \left(\frac{7}{2}\right) \cdot \left(\frac{1}{4}\right)^n u[n] - 3 \cdot \left(\frac{1}{2}\right)^n u[n]$$

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Soru 3)

(Gerilim bölücü kuralından)

$$y(s) = \frac{1/s}{R + Ls + 1/Cs} \cdot x(s)$$

$$x(t) = 2 \cdot e^{-t} \cdot u(t) \\ \mathcal{L}\{x(t)\} = 2 \cdot \frac{1}{s+1}$$

$$= \frac{20/s}{16 + 4s + 20/s} \cdot x(s) = \frac{20/s}{(16 + 4s + 20/s)} \cdot \frac{2}{s+1} = \frac{40/s}{(s+1)(16 + 4s + 20/s)} \cdot \frac{s}{s}$$

$$= \frac{40}{(s+1)(4s^2 + 16s + 20)} = \frac{10}{(s+1)(s^2 + 4s + 5)}$$

$$Y(s) = \frac{A}{(s+1)} + \frac{Bs+C}{s^2+4s+5} \Rightarrow A \cdot (s^2+4s+5) + (Bs+C)(s+1) = 10$$

$$s=1 \text{ için; } 2A=10 \Rightarrow \boxed{A=5}$$

$$s=0 \text{ için; } 5A+C=10 \Rightarrow \boxed{C=-15}$$

$$s=2 \text{ için; } 13A+6B+3C=10 \Rightarrow \boxed{B=-15}$$

$$Y(s) = \frac{5}{s+1} + \frac{-15s-15}{s^2+4s+5} = \frac{5}{s+1} - 15 \cdot \frac{s+1}{(s+2)^2+1}$$

$$= \frac{5}{s+1} - 15 \left[ \frac{s+2}{(s+2)^2+1} + \frac{5}{(s+2)^2+1} \right]$$

$$y(t) = 5e^{-t} u(t) - 15 \left( e^{-2t} \cos(t) u(t) - e^{-2t} \sin(t) u(t) \right)$$

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Soru 4)

$$y(0) = -y(-1) + 6y(-2) + u(0) + u(-1) = 5$$

$$y(1) = -y(0) + 6y(-1) + u(1) + u(0) = 1$$

$-5 + 6 \cdot 1 + 1 + 1$

$$y(n) = -y(n-1) + 6y(n-2) + x(n) + x(n-1)$$

$$y(2) = -y(1) + 6y(0) + u(2) + u(1) = 30$$

$-1 + 6 \cdot 5 + 1 + 1 =$

$$y(3) = -y(2) + 6y(1) + u(3) + u(1) = -22$$

$-30 + 6 \cdot 1 + 1 + 1$

$$y(w) + z^{-1}y(w) - yz^{-2}y(w) = x(w) + z^{-1}x(w)$$

$$y(w) \cdot [1 + z^{-1} - 6z^{-2}] = x(w) \cdot [1 + z^{-1}]$$

$$\frac{y(w)}{x(w)} = \frac{1 + z^{-1}}{1 + z^{-1} - 6z^{-2}} = \frac{1 + z^{-1}}{(1 + 3z^{-1})(1 - 2z^{-1})} = H(w)$$

$$z^{-1} = x$$

$$\Rightarrow \frac{1+x}{(1+3x)(1-2x)} = \frac{A}{1+3x} + \frac{B}{1-2x} \Rightarrow A = 2/5 \quad B = -3/5$$

$$H[n] = \frac{1}{5} (2(-3)^n + 3 \cdot 2^n)$$

$$y(n) = x(n) * h(n)$$

$$\frac{1 + \frac{1}{z}}{\left(1 + \frac{3}{z}\right)\left(1 - \frac{2}{z}\right)} = \frac{z+1}{(z+3)(z-2)}$$