Sinyaller ve Sistemler Final Ödevi

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Soru 1)

$\tau \, \text{sinc}\left[\frac{\tau t}{2\pi}\right] \longleftrightarrow 2\pi P_\tau(w)$

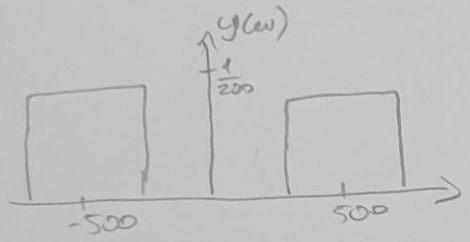
$\text{sinc}\ 100t$

$\frac{\tau t}{2\pi} = 100t \Rightarrow \tau = 2\pi \times 100$

$x(t) = \text{sinc}\ 100t \Rightarrow \frac{\tau}{\tau}\ \text{sinc}\ 100t = \frac{2\pi \times 100}{2\pi \times 100}\ \text{sinc}(100t)$ ($2\pi P_{2\pi\times 100}(w)$)

$x(t) = \frac{2\pi\, P_{2\pi\times 100}(w)}{2\pi \times 100} = \frac{1}{100} \cdot P_{2\pi\times 100}(w) \longleftrightarrow X(w)$

$y(w) = \frac{1}{2} \cdot \frac{1}{100}\left[P_{2\pi\times 100}(w+500) + P_{2\pi\times 100}(w-500)\right]$



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Soru 2) $2y[n] - 3y[n-1] + \frac{1}{4}y[n-2] = x[n] - 2x[n-1]$

$2 - y(\Omega) - \frac{3}{2}e^{j\Omega}y(\Omega) + \frac{1}{4}e^{-j2\Omega}y(\Omega) = x(\Omega) - 2e^{j\Omega}x(\Omega)$

$y(\Omega)(2 - \frac{3}{2}e^{-j\Omega} + \frac{1}{4}e^{-j2\Omega}) = x(\Omega)(1 - 2e^{-j\Omega})$

$\frac{y(\Omega)}{x(\Omega)} = H(\Omega) \Rightarrow \frac{1 - 2e^{-j\Omega}}{2 - \frac{3}{2}e^{-j\Omega} + \frac{1}{4}e^{-j2\Omega}}$ $\quad e^{-j\Omega} = x$ olsun

$= \frac{1-2x}{2 - \frac{3x}{2} + \frac{1x^2}{4}} = \frac{4(1-2x)}{x^2 - 6x + 8} = \frac{4-8x}{x^2-6x+8}$

$\frac{4-8x}{(x-4)(x-2)} = \frac{A}{(x-4)} + \frac{B}{(x-2)} \Rightarrow Ax - 2A + Bx - 4B = 4 - 8x$

$A + B = -8$
$-2A - 4B = 4$

$-2B = -12$
$\boxed{B = 6}$

$A + 6 = -8$
$\boxed{A = -14}$

$H(\Omega) = \frac{-14}{e^{-j\Omega} - 4} + \frac{6}{e^{-j\Omega} - 2}$

a) $H(\Omega) = \frac{14}{4}\left(\frac{1}{1 - \frac{1e^{-j\Omega}}{4}}\right) - \frac{6}{2}\left(\frac{1}{1 - \frac{1}{2}e^{-j\Omega}}\right.$

b) $H[n] = \left(\frac{7}{2}\right) \cdot \left(\frac{1}{4}\right)^n u[n] - 3 \cdot \left(\frac{1}{2}\right)^n u[n]$

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Soru 3)

Gerilim bölücü kuralından;

$$y(s) = \frac{1/Cs}{R+Ls+1/Cs} \cdot x(s) \qquad x(t) = 2 \cdot e^{-t} u(t)$$

$$L\{x(t)\} = 2 \cdot \frac{1}{s+1}$$

$$= \frac{20/s}{16+4s+20/s} \cdot x(s) = \frac{20/s}{(16+4s+20/s)} \cdot \frac{2}{s+1} = \frac{40/s}{(s+1)(16+4s+20)} \cdot \frac{s}{s}$$

$$= \frac{40}{(s+1)(4s^2+16s+20)} = \frac{10}{(s+1)(s^2+4s+5)}$$

$$Y(s) = \frac{A}{(s+1)} + \frac{Bs+C}{s^2+4s+5} \Rightarrow A \cdot (s^2+4s+5) + (Bs+C)(s+1) = 10$$

$s=1$ için; $2A = 10 \Rightarrow \boxed{A=5}$

$s=0$ için; $5A + C = 10 \Rightarrow \boxed{C=-15}$

$s=2$ için; $13A + 6B + 3C = 10 \Rightarrow \boxed{B=-15}$

$$Y(s) = \frac{5}{s+1} + \frac{-15s-15}{s^2+4s+5} = \frac{5}{s+1} - 15\,\frac{s+1}{(s+2)^2+1}$$

$$= \frac{5}{s+1} - 15\left[\frac{s+2}{(s+2)^2+1} + \frac{5}{(s+2)^2+1}\right]$$

$$y(t) = 5e^{-t} u(t) - 15\left(e^{-2t}\cos(t)\, u(t) - e^{-2t}\sin(t)\, u(t)\right)$$

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Soru 4)

$y(0) = -y(-1) + 6y(-2) + u(0) + u(-1) = 5$

$y(1) = -y(0) + 6y(-1) + u(1) + u(0) = 1$

$-5 + 6.1 + 1 + 1$

$y(n) = -y(n-1) + 6y(n-2) + x(n) + x(n-1)$

$y(2) = -y(1) + 6y(0) + u(2) + u(1) = 30$

$-1 + 6.5 + 1 + 1 =$

$y(3) = -y(2) + 6y(1) + u(3) + u(1) = -22$

$-30 + 6.1 + 1 + 1$

$y(w) + z^{-1} y(w) - 6z^{-2} y(w) = x(w) + z^{-1} x(w)$

$y(w) \cdot [1 + z^{-1} - 6z^{-2}] = x(w) \cdot [1 + z^{-1}]$

$$\frac{y(w)}{x(w)} = \frac{1+z^{-1}}{1+z^{-1}-6z^{-2}} = \frac{1+z^{-1}}{(1+3z^{-1})(1-2z^{-1})} = H(w)$$

$z^{-1} = x$

$$\Rightarrow \frac{1+x}{(1+3x)(1-2x)} = \frac{A}{1+3x} + \frac{B}{1-2x} \Rightarrow A = 2/5 \quad B = -3/5$$

$$\frac{A(1-2x) + B(1+3x)}{(1-2x)(1+3x)}$$

$$H[n] = \frac{1}{5}\left(2(-3)^n + 3.2^n\right)$$

$y(n) = x(n) * h(n)$

$$\frac{1+\frac{1}{z}}{\left(1+\frac{3}{z}\right)\left(1-\frac{2}{z}\right)} = \frac{z+1}{(z+3)(z-2)}$$