Assume the pool has fee tier given by γ. For instance, for a 30 bps pool, $\gamma = 1.0 - 0.003$.

If we sell ΔX units of X into the pool, then we receive ΔY given by

$$(X + \gamma \Delta X) \cdot (Y - \Delta Y) = X \cdot Y, \tag{1}$$

which implies firstly that,

$$\Delta Y = \frac{Y \cdot (\gamma \Delta X)}{X + \gamma(\Delta X)}.$$

Secondly, the "touch price" of the pool defined by the invariant $X \cdot Y = k$ is given by

$$\left| \frac{\partial Y}{\partial X} \right| = \frac{Y}{X}.$$

So after the swap in Equation (1), the price of the pool is given by

$$P = \frac{Y - \Delta Y}{X + \gamma \Delta X} = \frac{X \cdot Y}{(X + \gamma \Delta X)^2}. \tag{2}$$

Re-writing this in terms of known quantities:

$$\gamma^2 (\Delta X)^2 + 2\gamma X (\Delta X) + \left[X^2 - \frac{k}{P} \right] = 0. \tag{3}$$

Solving this quadratic yields:

$$\Delta X = \frac{-2\gamma X \pm \sqrt{4\gamma^2 X^2 - 4\gamma^2 \left[X^2 - \frac{k}{P} \right]}}{2\gamma^2} = \frac{-2\gamma X \pm 2\gamma \sqrt{\frac{k}{P}}}{2\gamma^2} = \frac{-X \pm \sqrt{\frac{k}{P}}}{\gamma}. \tag{4}$$

In fact, a better way of writing this is:

$$\Delta X = \frac{-X \pm \sqrt{\frac{X^2 \left(\frac{Y}{X} \right)}{P}}}{\gamma} = \frac{-X \pm X \sqrt{\frac{\hat{P}}{P}}}{\gamma}. \tag{5}$$

where $\hat{P} = \frac{Y}{X}$ is the current price of the pool pre-swap.

If $P < \hat{P}$, then we need to sell $\Delta X > 0$ into the pool to move the price to P. Since $\frac{\hat{P}}{P} > 1$ in this case, we have

$$\Delta X = X \left[\frac{\sqrt{\frac{\hat{P}}{P}} - 1}{\gamma} \right]. \tag{6}$$

Now if $P > \hat{P}$, we need to sell ΔY into the pool. We need to re-orient our equations to be in the correct direction:

$$(Y + \gamma\Delta Y)(X - \Delta X) = X \cdot Y \tag{7}$$

which implies that

$$\Delta X = \frac{\gamma X(\Delta Y)}{Y + \gamma\Delta Y}. \tag{8}$$

Therefore, the new price P is given by

$$P = \frac{Y + \gamma\Delta Y}{X - \Delta X} = \frac{(Y + \gamma(\Delta Y))^2}{XY}. \tag{9}$$

Hence the solution is given by

$$\Delta Y = \frac{-Y \pm \sqrt{PXY}}{\gamma} \tag{10}$$

Since $P > \hat{P}$, and we know $\Delta Y > 0$, we take the positive root and get

$$\Delta Y = \frac{\sqrt{PXY} - Y}{\gamma}. \tag{11}$$

Observing similarly that $X = \frac{Y}{\hat{P}}$, we obtain a similar functional form:

$$\Delta Y = Y \left[\frac{\sqrt{\frac{P}{\hat{P}}} - 1}{\gamma} \right]. \tag{12}$$