COMP 3803 Assignment 2

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Question 1.

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Question 2.

i) Regular Expression:

$$\Big((00)^*00\Big) \cup \Big((0 \cup 1)^*1(00)^*00\Big)$$

The first set of brackets before the union covers all binary strings that consist of an even number of zeroes and have no ones. The second set of brackets covers the other case, binary stings that containing 1s that have end in an even number of zeroes. The regular expression in the second set of brackets is very similar to the one in the second part of the question, however, this one has a 1 between $(0 \cup 1)^*$ and $(00)^*00$. This ensures that these string will truly end in an even number of zeroes as binary strings of the form $(0 \cup 1)^*$ may end in an odd number of zeroes and this makes it possible of binary strings of the form $(0 \cup 1)^*(00)^*00$ to end in an odd number of zeroes.

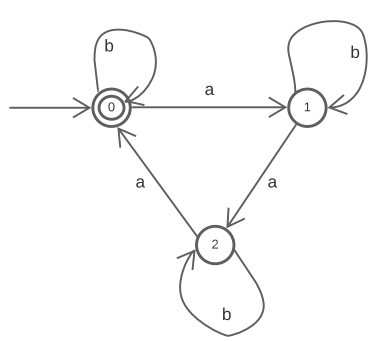
ii) Language:

$$\{w \in \{0,1\}^* : w \text{ ends in 2 or more zeores}\}$$

As I described in the previous part of the question, binary strings of the form $(0 \cup 1)^*(00)^*00$ can end in an odd number of zeroes. However, it also possible for binary strings of the form $(0 \cup 1)^*$ to end in a 1, which would lead to the final string having end in an even number of zeroes. Therefore, the given regular expression describes binary strings that end in 2 or more zeroes.

Question 3.

i) DFA:



Note that the given DFA accepts the language. Start in the start state, which is an accept state. If you read an a advance to the next state, 3 a's to get back to the accept state. If you read a b just stay in the current state.

I will use the construction seen in class to convert the given DFA into a regular expression.

For $\forall i = 0, 1, 2$:
Define L_i =language of DFA assuming that q_i is the start state.

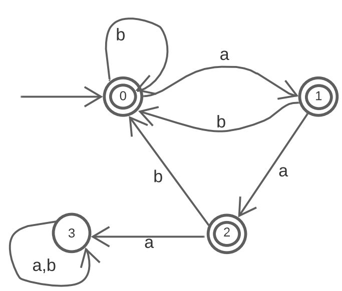
$$L_0 = \varepsilon \cup aL_1 \cup bL_0 \tag{1}$$
$$L_1 = aL_2 \cup bL_1 \Rightarrow L_1 = b^*aL_2 \tag{2}$$
$$L_2 = aL_0 \cup bL_2 \Rightarrow L_2 = b^*aL_0 \tag{3}$$

Note: notation (2) → (1) means sub equation 2 into equation 1.

$$\begin{aligned} L_0 &= \varepsilon \cup bL_0 \cup aL_1 \\ &= \varepsilon \cup bL_0 \cup a[b * aL_2] \qquad (2) \rightarrow (1) \\ &= \varepsilon \cup bL_0 \cup ab^*a[b^*aL_0] \qquad (3) \rightarrow (1) \\ &= (b \cup ab^*ab^*a)L_0 \cup \varepsilon \\ &= (b \cup ab^*ab^*a)^*\varepsilon = (b \cup ab^*ab^*a)^* \end{aligned}$$

Therefore, a regular expression that describes the given language is $(b \cup ab^*ab^*a)^*$.

ii) DFA:



Note that the given DFA accepts the language. Start in the start state. If you read an aaa you will advance to the bad state (state 3) and there are no edges leaving that state. If you read a or aa folowed by a b you will go back to the start state. All states except state 3 are accept states.

Again, I will use the construction seen in class to convert the given DFA into a regular expression.

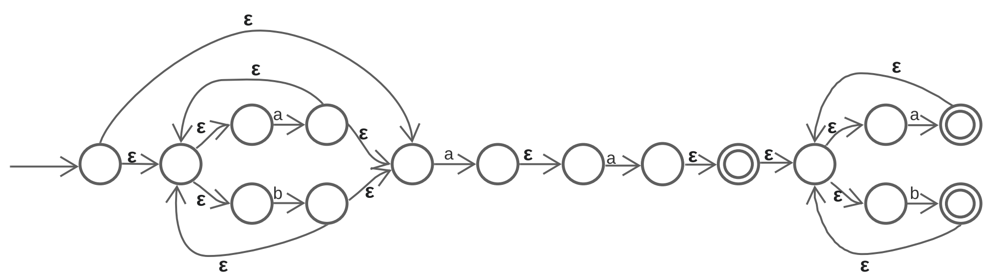
For $\forall i = 0, 1, 2, 3$:
Define L_i =language of DFA assuming that q_i is the start state.

$$L_0 = \varepsilon \cup aL_1 \cup bL_0 \tag{4}$$
$$L_1 = \varepsilon \cup aL_2 \cup bL_0 \tag{5}$$
$$L_2 = \varepsilon \cup aL_3 \cup bL_0 \tag{6}$$
$$L_3 = (a \cup b)L_3 \Rightarrow L_3 = (a \cup b)L_3 \cup \emptyset = (a \cup b)^*\emptyset = \emptyset \tag{7}$$

$$
\begin{aligned}
L_0 &= \varepsilon \cup bL_0 \cup a[aL_2 \cup bL_0 \cup \varepsilon] \qquad (5) \to (4) \\
&= \varepsilon \cup bL_0 \cup abL_0 \cup a \cup aa[aL_3 \cup bL_0 \cup \varepsilon] \qquad (6) \to (4) \\
&= \varepsilon \cup bL_0 \cup abL_0 \cup a \cup aabL_0 \cup aa \cup aaa[\emptyset] \qquad (7) \to (4) \\
&= (b \cup ab \cup aab)L_0 \cup \varepsilon \cup a \cup aa \cup \emptyset \\
&= (b \cup ab \cup aab)^*(\varepsilon \cup a \cup aa \cup \emptyset) = (b \cup ab \cup aab)^*(\varepsilon \cup a \cup aa)
\end{aligned}
$$

Therefore, a regular expression that describes the given language is $(b \cup ab \cup aab)^*(\varepsilon \cup a \cup aa)$.

Question 4.



Question 5.

For $\forall i = 1, 2, 3$:
Define L_i =language of DFA assuming that q_i is the start state.

$$
\begin{aligned}
L_1 &= aL_1 \cup bL_2 && (8) \\
L_2 &= \varepsilon \cup bL_2 \cup aL_3 \Rightarrow L_3 = b^*(aL_3 \cup \varepsilon) = b^*aL_3 \cup b^* && (9) \\
L_3 &= \varepsilon \cup aL_3 \cup bL_1 \Rightarrow L_2 = a^*(bL_1 \cup \varepsilon) = a^*bL_1 \cup a^* && (10)
\end{aligned}
$$

$$
\begin{aligned}
L_1 &= aL_1 \cup b[b^*aL_3 \cup b^*] \qquad (9) \to (8) \\
&= aL_1 \cup bb^* \cup bb^*a[a^*bL_1 \cup a^*] \qquad (10) \to (8) \\
&= (a \cup bb^*aa^*b)L_1 \cup bb^* \cup bb^*aa^* \\
&= (a \cup bb^*aa^*b)^*(bb^* \cup bb^*aa^*)
\end{aligned}
$$

Therefore, a regular expression that describes the given language is $(a \cup bb^*aa^*b)^*(bb^* \cup bb^*aa^*)$.

Question 6.

A is a regular language. This means there exists a regular language that describes the language A, call this regular expression R_1.

Define the regular expression $R_2 = (a \cup b)(a \cup b)$.

Note that u is a string described by R_1 and v is a string described by R_2. Thus, the string uv is described by R_1R_2. We have seen in class that the concatenation of 2 regular expressions is also a regular expression. So R_1R_2 is a regular expression that describes the language B. Therefore, the language B is a regular language.

Question 7.

1.
$$A = \{a^n ba^m ba^{n+m} : n \geq 0, m \geq 0\}$$

Suppose that A is regular. This means, by the Pumping Lemma (PL), $\exists$ pumping length p such that we can pump all strings $s \in A$ where $|s| \geq p$.

Take string $s = a^p baba^{p+1}$, $s \in A$ and $|s| = 2p + 3 \geq p$.

By the PL, we can split the string s into xyz such that $s = xyz$ and for $\forall i \geq 0$ $xy^iz \in A$. We know that for the given string s, y exists in the first section of p a's and $|y| \geq 1$.

If we pump y up, we know that the number of a's before the first b is some number q where $q > p$. However, since pumping y has no effect on the last section of a's, there will still be $p + 1$ a's in that section. $p + 1 < q + 1$, so the string $xyyz \notin A$. But the PL says that $xyyz \in A$ so we have a contradiction. Therefore, A is not a regular language.

2.
$$A = \{w : w \text{ is not a palindrome}\}$$
$$\bar{A} = \{w : w \text{ is a palindrome}\}$$

Suppose that A is regular. then, as seen in class, $\bar{A}$ is a regular language. This means, by the Pumping Lemma (PL), $\exists$ pumping length p such that we can pump all strings $s \in \bar{A}$ where $|s| \geq p$.

Take string $s = a^p ba^p$, $s \in \bar{A}$ and $|s| = 2p + 1 \geq p$.

For $i = 0$, by the PL $xy^0z = xz \in \bar{A}$. Since y exists in the first section of p a's and $|y| \geq 1$, we know that in the string xz there are less a's before the b than there are after. This means that xz is not a palindrome and thus, $xz \notin \bar{A}$. This is a contradiction, and therefore, $\bar{A}$ is not a regular language. Thus, A is not a regular language.

3.
$$A = \{ucu : u \in \{a, b\}^*\}$$

Suppose that A is regular. This means, by the Pumping Lemma (PL), $\exists$ pumping length p such that we can pump all strings $s \in A$ where $|s| \geq p$.

Take string $s = a^p ca^p$, $s \in A$ and $|s| = 2p + 1 \geq p$.

For $i = 0$, by the PL $xy^0z = xz \in A$. Since y exists in the first section of p a's and $|y| \geq 1$, we know that in the string xz there are less a's before the c than there are after. This means that the string before the c is not the same as the string after. Thus, $xz \notin A$. This is a contradiction, and therefore, A is not a regular language.

4.

$$A = \{aba^2ba^3b \dots a^nb : n \geq 0\}$$

Let us note that strings in A have n b's and $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ a's. This means that the length of strings in A is $n + \frac{n(n+1)}{2}$ where $n \geq 0$.

Suppose that A is regular. This means, by the Pumping Lemma (PL), $\exists$ pumping length p such that we can pump all strings $s \in A$ where $|s| \geq p$.

Take string $s = aba^2ba^3b \dots a^pb$, $s \in A$ and $|s| = p + \frac{p(p+1)}{2} \geq p$.

For $i = 2$, by the PL $xy^2z = xyyz \in A$. $|xyyz| = |xyz| + |y|$.

$$\begin{aligned}
|xyyz| &= p + \frac{p(p+1)}{2} + |y| \\
&\geq p + \frac{p(p+1)}{2} + 1 \\
&> p + \frac{p(p+1)}{2}
\end{aligned}$$

$$\begin{aligned}
|xyyz| &= p + \frac{p(p+1)}{2} + |y| \\
&\leq p + \frac{p(p+1)}{2} + p \\
&< (p+1) + \frac{(p+1)(p+2)}{2}
\end{aligned}$$

Since $p + \frac{p(p+1)}{2} < |xyyz| < (p+1) + \frac{(p+1)(p+2)}{2}$, we know that $xyyz \notin A$. Thus, we have a contradiction, and therefore, A is not a regular language.