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1.

Let F be it is the fall semester.
Let P be my professor is teaching COMP1805.
Let Y be the year is 2019.
Let D be today's date is before December 7^{th}.

1) $F \rightarrow P$ Goal: D
2) P
3) $(F \wedge Y) \rightarrow D$
4) Y

To show he conclusion 'today's date is before December 7^{th} (D)' is invalid, show that the conclusion does not follow directly from the premises.

Assume it is not the fall semester $(\neg F)$ then statement $F \wedge Y$ is now false meaning D can be any value (true or false) and therefore it is not possible to determine if today's date is before December 7^{th} or not.

2.

1) $\forall x \neg A(x) \wedge \neg B(x)$ Goal: $\exists x\, E(x)$
2) $\exists x\, C(x) \rightarrow B(x)$
3) $\forall x\, D(x) \rightarrow C(x)$
4) $\exists x\, A(x) \rightarrow E(x)$
5) $\forall x\, D(x) \vee E(x)$
6) $\neg A(c) \wedge \neg B(c)$ by Universal Instantiation (1)
7) $\neg B(c)$ by Simplification (6)
8) $C(c) \rightarrow B(c)$ by Existential Instantiation (2)
9) $\neg C(c)$ by Modus Tollens (7,8)
10) $D(c) \rightarrow C(c)$ by Universal Instantiation (3)
11) $\neg D(c)$ by Modus Tollens (9,10)
12) $D(c) \vee E(c)$ by Universal Instantiation (5)
13) $E(c)$ by Disjunctive Syllogism (11,12)
14) $\exists x\, E(x)$ by Existential Generalization (13)

3.

$$(n \in \mathbb{Z} \land 5n + 2\ is\ odd) \rightarrow n\ is\ odd$$
$$\neg(n\ is\ odd) \rightarrow \neg(n \in \mathbb{Z} \land 5n + 2\ is\ odd) \text{ by Contrapositive}$$
$$Prove: n\ is\ even \rightarrow \neg(n \in \mathbb{Z}) \lor 5n + 2\ is\ even \text{ by De Morgan's Law}$$

1) $n\ is\ even$ by Assumption
2) $n = 2a$ by Definition
3) $5n + 2 = 5(2a) + 2$
4) $5n + 2 = 10a + 2$
5) $5n + 2 = 2(5a + 1)$

Let b be $5a + 1$

6) $5n + 2 = 2b$
7) $5n + 2\ is\ even$ by Definition

$$\therefore (n \in \mathbb{Z} \land 5n + 2\ is\ odd) \rightarrow n\ is\ odd$$

4.

$$(n \in \mathbb{Z} \land n^3 + 33\ is\ odd) \rightarrow n\ is\ even$$
$$\neg(n \in \mathbb{Z} \land n^3 + 33\ is\ odd) \lor n\ is\ even)\ by\ Implication\ Equivalence$$
$$\neg(\neg(n \in \mathbb{Z} \land n^3 + 33\ is\ odd) \lor n\ is\ even)\ by\ Contadiction$$
$$Prove: (n \in \mathbb{Z} \land n^3 + 33\ is\ odd) \land n\ is\ odd\ by\ De\ Morgan's\ Law$$

1) $n^3 + 33\ is\ odd$ by Assumption
2) $n^3 + 33 = 2a + 1$ by Definition
3) $n\ is\ odd$ by Assumption
4) $n = 2b + 1$ by Definition
5) $n^3 + 33 = (2b + 1)^3 + 33$
6) $n^3 + 33 = 8b^3 + 1 + 33$
7) $n^3 + 33 = 8b^3 + 34$
8) $n^3 + 33 = 2(4b^3 + 17)$

$Let\ c\ be\ 4b^3 + 17$

9) $n^3 + 33 = 2c$
10) $n^3 + 33\ is\ even$ by Definition
11) $n^3 + 33\ is\ even \land n^3 + 33\ is\ odd$ by Conjunction
12) $False$ by Negation

$$\therefore (n \in \mathbb{Z} \land n^3 + 33\ is\ odd) \rightarrow n\ is\ even$$

5. Unfinished

Prove: $\sqrt{4} + \sqrt{5}\ is\ irrational$

$\sqrt{4} + \sqrt{5}\ is\ rational$ by Contradiction

1) $\sqrt{4} + \sqrt{5}\ is\ rational$ by Assumption
2) $\left(\sqrt{4} + \sqrt{5} = \frac{a}{b}\right) \land (\frac{a}{b}\ is\ in\ lowest\ form)$ by Definition
3) $\sqrt{4} + \sqrt{5} = \frac{a}{b}$ by Simplification
4) $2 + \sqrt{5} = \frac{a}{b}$
5) $4 + 5 = \frac{a^2}{b^2}$

6) $(4+5)b^2 = a^2$
7) $4b^2 + 5b^2 = a^2$
8) $9b^2 = a^2$
9) $3b = a$

$Lemma$ 1

Prove: $a^2\ is\ divisble\ by\ 5 \rightarrow a\ is\ divisible\ by\ 5$

$a\ is\ not\ divisible\ by\ 5 \rightarrow a^2\ is\ not\ divisible\ by\ 5$ by Contrapositive

1) $a\%5 \neq 0 \rightarrow a^2\%5 \neq 0$

10)