Braeden Hall

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1.
2.
3.

Lowest number divisible by 11 is 55 (55/11 = 5)
Highest number divisible by 11 is 572 (572/11 = 52)

$$\sum_{i=5}^{52} 11i = 11\sum_{i=5}^{52} i$$
$$= 11\left[\sum_{i=1}^{52} i - \sum_{i=1}^{4} i\right]$$
$$= 11\left(\frac{52(52+1)}{2} - \frac{4(4+1)}{2}\right)$$
$$= 11\left(\frac{2756}{2} - \frac{20}{2}\right)$$
$$= 11(1368)$$
$$= 15048$$

4.

Lowest number divisible by 13 is 39 (39/13 = 3)
Highest number divisible by 13 is 806 (806/13 = 62)
Lowest number divisible by 5 is 30 (30/5 = 6)
Highest number divisible by 5 is 805 (805/5 = 161)
65 is the LCM between 5 and 13
Lowest number divisible by 65 is 65 (65/65 = 1)
Lowest number divisible by 65 is 780 (780/65 = 12)

$$= \sum_{i=30}^{809} i - \left(\sum_{i=3}^{62} 13i + \sum_{i=6}^{161} 5i\right) + \sum_{i=1}^{12} 65i$$
$$= \left[\sum_{i=1}^{809} i - \sum_{i=1}^{29} i\right] - \left[13\left[\sum_{i=1}^{62} i - \sum_{i=1}^{2} i\right] + 5\left[\sum_{i=1}^{161} i - \sum_{i=1}^{5} i\right]\right] + 65\sum_{i=1}^{12} i$$

$$= \left(\frac{809(810)}{2} - \frac{29(30)}{2}\right) - \left(13\left(\frac{62(63)}{2} - \frac{2(3)}{2}\right) + 5\left(\frac{161(162)}{2} - \frac{5(6)}{2}\right)\right) + 65\left(\frac{12(13)}{2}\right)$$

$$= (327645 - 435) - \big(13(1953 - 3) + 5(13041 - 15)\big) + 65(78)$$

$$= 327210 - (25350 + 65130) + 5070$$

$$= 332280 - 90480$$

$$= 241800$$

5.

a.

$\boldsymbol{p}$	$\boldsymbol{q}$	$\bar{\boldsymbol{q}}$	$\bar{\boldsymbol{q}} \cap \boldsymbol{q}$	$\overline{\bar{\boldsymbol{q}} \cap \boldsymbol{q}}$	$\boldsymbol{p} \cap \overline{(\bar{\boldsymbol{q}} \cap \boldsymbol{q})}$	$\overline{\left(\boldsymbol{p} \cap \overline{(\bar{\boldsymbol{q}} \cap \boldsymbol{q})}\right)}$	$\bar{\boldsymbol{p}}$	$\overline{(\bar{\boldsymbol{p}})}$	$\overline{\left(\boldsymbol{p} \cap \overline{(\bar{\boldsymbol{q}} \cap \boldsymbol{q})}\right)} \cap \overline{(\bar{\boldsymbol{p}})}$	$\overline{\left(\overline{\left(\boldsymbol{p} \cap \overline{(\bar{\boldsymbol{q}} \cap \boldsymbol{q})}\right)} \cap \overline{(\bar{\boldsymbol{p}})}\right)}$
1	1	0	0	1	1	0	0	1	0	**1**
1	0	1	0	1	1	0	0	1	0	**1**
0	1	0	0	1	0	1	1	0	0	**1**
0	0	1	0	1	0	1	1	0	0	**1**

b.

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{r}$	$\boldsymbol{p} \cup \boldsymbol{r}$	$(\boldsymbol{p} \cup \boldsymbol{r}) \cap \boldsymbol{r}$	$\boldsymbol{q} - \big((\boldsymbol{p} \cup \boldsymbol{r}) \cap \boldsymbol{r}\big)$	$\boldsymbol{p} \cup \boldsymbol{p}$	$\overline{(\boldsymbol{p} \cup \boldsymbol{p})}$	$\left(\boldsymbol{q} - \big((\boldsymbol{p} \cup \boldsymbol{r}) \cap \boldsymbol{r}\big) \cap \overline{(\boldsymbol{p} \cup \boldsymbol{p})}\right)$
1	1	1	1	1	0	1	0	**0**
1	1	0	1	0	1	1	0	**0**
1	0	1	1	1	0	1	0	**0**
1	0	0	1	0	0	1	0	**0**
0	1	1	1	1	0	0	1	**0**
0	1	0	0	0	1	0	1	**1**
0	0	1	1	1	0	0	1	**0**
0	0	0	0	0	0	0	1	**0**

c.

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{r}$	$\boldsymbol{p} \cap \boldsymbol{r}$	$(\boldsymbol{p} \cap \boldsymbol{r}) \cup \boldsymbol{q}$	$\bar{\boldsymbol{r}}$	$\boldsymbol{p} \cap \bar{\boldsymbol{r}}$	$((\boldsymbol{p} \cap \boldsymbol{r}) \cup \boldsymbol{q}) \cup (\boldsymbol{p} \cap \bar{\boldsymbol{r}})$	$\boldsymbol{r} \cup \boldsymbol{p}$	$(((\boldsymbol{p} \cap \boldsymbol{r}) \cup \boldsymbol{q}) \cup (\boldsymbol{p} \cap \bar{\boldsymbol{r}})) \cap (\boldsymbol{r} \cup \boldsymbol{p})$
1	1	1	1	1	0	0	1	1	**1**
1	1	0	0	1	1	1	1	1	**1**
1	0	1	1	1	0	0	1	1	**1**
1	0	0	0	0	1	1	1	1	**1**
0	1	1	0	1	0	0	1	1	**1**
0	1	0	0	1	1	0	1	0	**0**
0	0	1	0	0	0	0	0	1	**0**
0	0	0	0	0	1	0	0	0	**0**

6.

A	B	C	$C-B$	$(C-B)-A$	$A\cup C$	$B-(A\cup C)$	$B-(B-(A\cup C))$	$((C-B)-A)\cup(B-(B-(A\cup C)))$	$B\cap C$	$A\cap(B\cap C)$	$((C-B)-A)$ $\cup(B-(B-(A\cup C)))$ $-(A\cap(B\cap C))$
1	1	1	0	0	1	0	1	1	1	1	**0**
1	1	0	0	0	1	0	1	1	0	0	**1**
1	0	1	1	0	1	0	0	0	0	0	**0**
1	0	0	0	0	1	0	0	0	0	0	**0**
0	1	1	0	0	1	0	1	1	1	0	**1**
0	1	0	0	0	0	1	0	0	0	0	**0**
0	0	1	1	1	1	0	0	1	0	0	**1**
0	0	0	0	0	0	0	0	0	0	0	**0**

A	B	C	$A\cap B$	$C\cap B$	$A\cap(C\cap B)$	$(A\cap B)-(A\cap(C\cap B))$	$C-A$	$(A\cap B)-A\cap(C\cap B)\cup(C-A)$
1	1	1	1	1	1	0	0	**0**
1	1	0	1	0	0	1	0	**1**
1	0	1	0	0	0	0	0	**0**
1	0	0	0	0	0	0	0	**0**
0	1	1	0	1	0	0	1	**1**
0	1	0	0	0	0	0	0	**0**
0	0	1	0	0	0	0	1	**1**
0	0	0	0	0	0	0	0	**0**

7.

8.

1. $\bar{C}\cap\left((C-(B\cap C))\cup((A\cap B)-(B\cap C))-A\right)$
2. $\bar{C}\cap\left((C\cap\overline{(B\cap C)})\cup((A\cap B)-(B\cap C))-A\right)$ by Difference Equivalence
3. $\bar{C}\cap\left((C\cap(\bar{B}\cup\bar{C}))\cup((A\cap B)-(B\cap C))-A\right)$ by De Morgan's Law
4. $\bar{C}\cap\left(((C\cap\bar{B})\cup(C\cap\bar{C}))\cup((A\cap B)-(B\cap C))-A\right)$ by Distribution
5. $\bar{C}\cap\left(((C\cap\bar{B})\cup\emptyset)\cup((A\cap B)-(B\cap C))-A\right)$ by Complement
6. $\bar{C}\cap\left((C\cap\bar{B})\cup((A\cap B)-(B\cap C))-A\right)$ by Identity
7. $\bar{C}\cap\left((C\cap\bar{B})\cup\left((A\cap B)\cap\overline{(B\cap C)}\right)-A\right)$ by Difference Equivalence
8. $\bar{C}\cap\left((C\cap\bar{B})\cup\left((A\cap B)\cap(\bar{B}\cup\bar{C})\right)-A\right)$ by De Morgan's Law
9. $\bar{C}\cap\left((C\cap\bar{B})\cup\left((A\cap(B\cap\overline{B}))\cup((A\cap B)\cap\bar{C})\right)-A\right)$ by Distributivity
10. $\bar{C}\cap\left((C\cap\bar{B})\cup\left((A\cap\emptyset)\cup((A\cap B)\cap\bar{C})\right)-A\right)$ by Complement
11. $\bar{C}\cap\left((C\cap\bar{B})\cup\left(((A\cap B)\cap\bar{C})\right)-A\right)$ by Identity

12. $\bar{C} \cap \left((C \cap \bar{B}) \cup \left((A \cap B) \cap \bar{C}\right) \cap \bar{A}\right)$ by Difference Equivalence
13. $\bar{C} \cap \left((C \cap \bar{B}) \cup \left((A \cap \bar{A}) \cap \bar{C} \cap B\right)\right)$ by Associativity
14. $\bar{C} \cap \left((C \cap \bar{B}) \cup (\emptyset \cap \bar{C} \cap B)\right)$ by Complement
15. $\bar{C} \cap \left((C \cap \bar{B}) \cup (\emptyset \cap B)\right)$ by Domination
16. $\bar{C} \cap \left((C \cap \bar{B}) \cup \emptyset\right)$ by Domination
17. $\bar{C} \cap (C \cap \bar{B})$ by Identity
18. $(\bar{C} \cap C) \cap B$ by Associativity
19. $\emptyset \cap B$ by Complement
20. $\emptyset$ by Domination