Jeremy Kanovsky
ME-134 Robotics
Fall 2020 Midterm Exam
Professor Matthew Woodward

Short Answer

1. *Define kinematics*

    Kinematics examines the relationship between motion of an object or objects, and the forces imposed on the object(s). Kinematic equations, for example, describe the motion of an object given the forces acting upon it. Kinematic systems can mathematically represent the movement of multiple bodies in relation to one another as a result of the force interactions between them.

2. *Define differential kinematics*

    Differential kinematics examines and describes the relationship between the velocity of motion of an object or objects. Most commonly differential kinematics describe a *kinematic chain*, which is a series of bodies (or "links") interacting with one another and whose velocities are partially a result of the velocity of the body "before" it in a chain (i.e., the body driving the object being described or examined).

3. *Define inverse kinematics*

    Inverse kinematics is the mathematical inverse of (forward) kinematics in which the forces acting upon an object or objects can be determined as a result of it's displacement or velocity. Rearranging the variables in standard kinematic equations can yield the driving forces. In terms of robotics, inverse kinematics are used to back-calculate the required angles or velocities of a kinematic chain in order to achieve a desired position or velocity of an end-effector (or leg, or manipulator, etc). More generally inverse kinematics can be used to calculate the state of bodies in a kinematic chain for a desired state of the terminal body of the chain.

4. *What is a homogeneous transformation?*

    A homogeneous transformation is a transformation that can be applied to an arbitrary state (such as position, or velocity, or acceleration, etc) to change the reference frame of the state. More simply, a homogeneous transformation is a mapping from one reference frame to another. Transformation matrices, for example, can be used to map a set of coordinates (or other quantity) from one reference frame to another reference frame. These transformations may be translational, rotational, projective, or reflective in nature, among others..

5. *Given the <u>kinematics</u> equations for a manipulator and the joint positions*

    a. *How many possible solutions can exist for the position of the end-effector?*

        Given the forward kinematics equations and joint positions for a system there is only one possible solution for the end-effector (which is to say, there is only one possible position it can occupy). Unlike inverse kinematics, forward kinematics resolve to a single numerical solution describing the physical setup of the system.

b. *What limits exist for the joint positions? (think real robots)*

There are several physical factors that limit joint positions. The first and perhaps most obvious is the physical limitations of the actuator in question. For example, if a motor only has a range of $\pm 90°$, it cannot exceed that. Similarly, all joints are limited by the physical constraints with regards to interfering with itself. For example, two links of a robot cannot occupy the same physical space (which becomes a limiting factor while solving for end-effector possible solutions using inverse kinematics).

6. *Given the <u>inverse kinematic</u> equations for a manipulator and the desired end-effector position.*

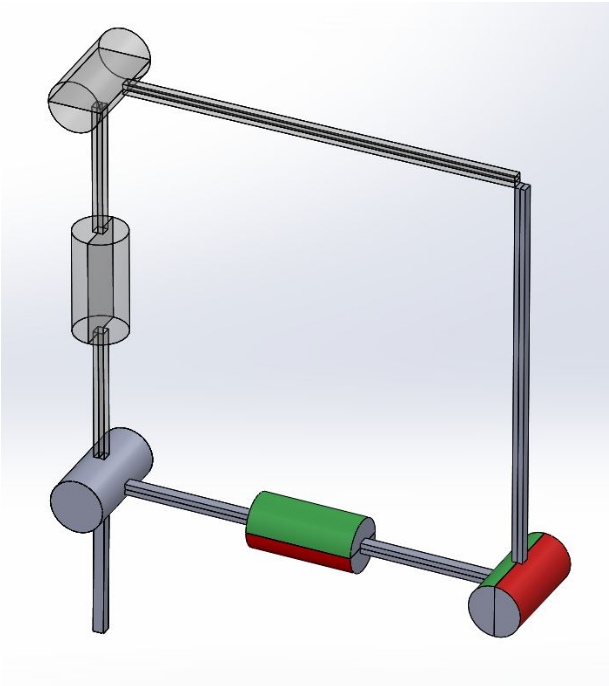
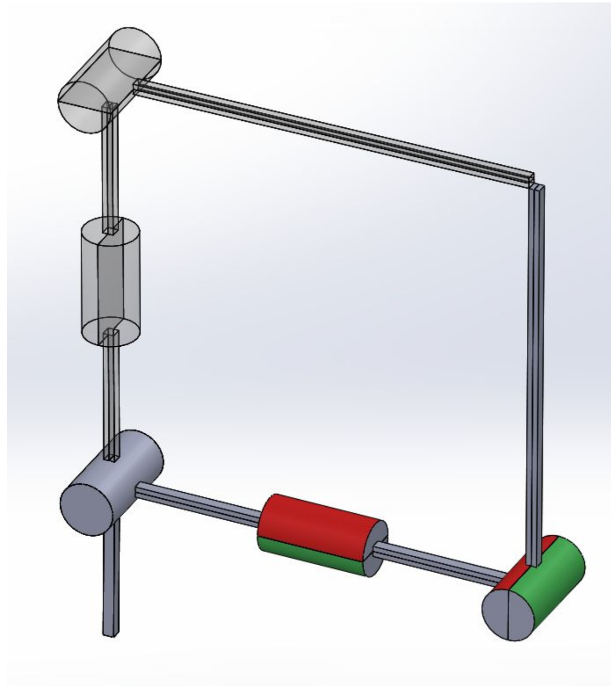
a. *How many possible solutions can exist for the joint positions?*

Depending on the target position for an end-effector, there can either be no solutions, one solution, or many solutions for the kinematic chain to result in a specific position. Consider a robotic arm. If the target position lies outside the reach of all the links of the arm (pointed in an optimal direction), then there is no solution for the joints to have the end-effector reach the target. The case with one solution is similar; if the target is at the extreme range of the kinematic chain (i.e., all joints are oriented optimally but cannot extend beyond the target), then there is only one solution. Finally, if the target position is within the range of the end-effector, and assuming there is more than one joint to the robot (the case of there being one joint would fall into the "single solution" case), then there are many joint configurations for the end-effector to reach the target position.

As an extension to the "many solutions" case, it is also possible for there to be infinitely many solutions for a given position. This is the case where the desired position is within the overall volume of influence the manipulator has, and two or more joints may make small adjustments in conjunction with one another to achieve the desired location.

b. *Given the presented manipulator (Fig. 1), draw 2 additional joint configuration which reach the same desired end-effector position (p_e).*

Consider the two orientations shown below. These are two possible configurations to achieve the same end-effector position as the original. By using the revolute joint in the middle of the arm the final joint can reverse itself to achieve the same position in two different ways. (Another possible option would have been to use the same "middle joint reversal" technique in the original orientation. Overall there are four possible ways to achieve the end-effector position.)

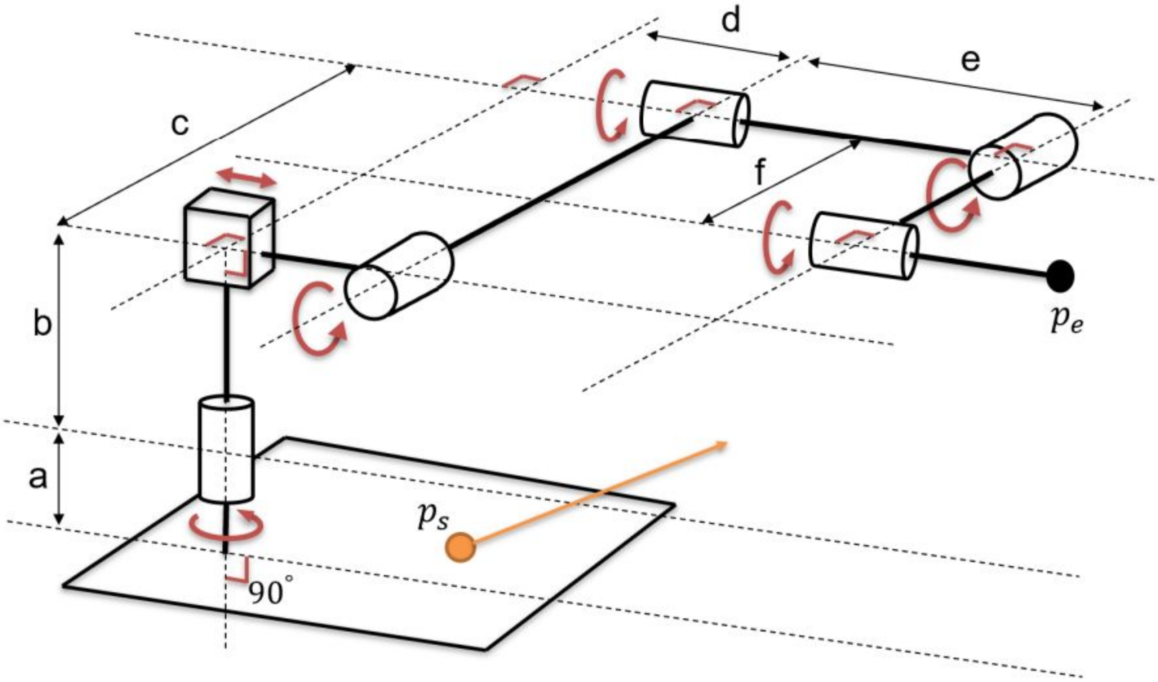



7. *What are the two general inverse kinematics methods? And what is the difference between them?*

   The two general inverse kinematics methods are analytical inverse kinematics, and numerical inverse kinematics. In analytical inverse kinematics, the forward kinematic equations can be mathematically inverted to yield a symbolic expression that, when solved, yields the possible configurations for the joint positions of a robot. Numerical inverse kinematics, however, rely on an initial guess of the desired configuration, and then find a number of possible solutions to achieve the desired result (with an error metric defined such that it stops searching once the desired number of configurations are found, or when the error of a given configuration goes below the defined threshold).

   Generally speaking, analytical methods are advantageous in that they find all solutions or prove that no solution exists, and they are extremely fast to compute once the necessary equations are solved. However, they are often much more difficult to solve initially and any change in physical robot configuration requires an update to the equations defining its motion.

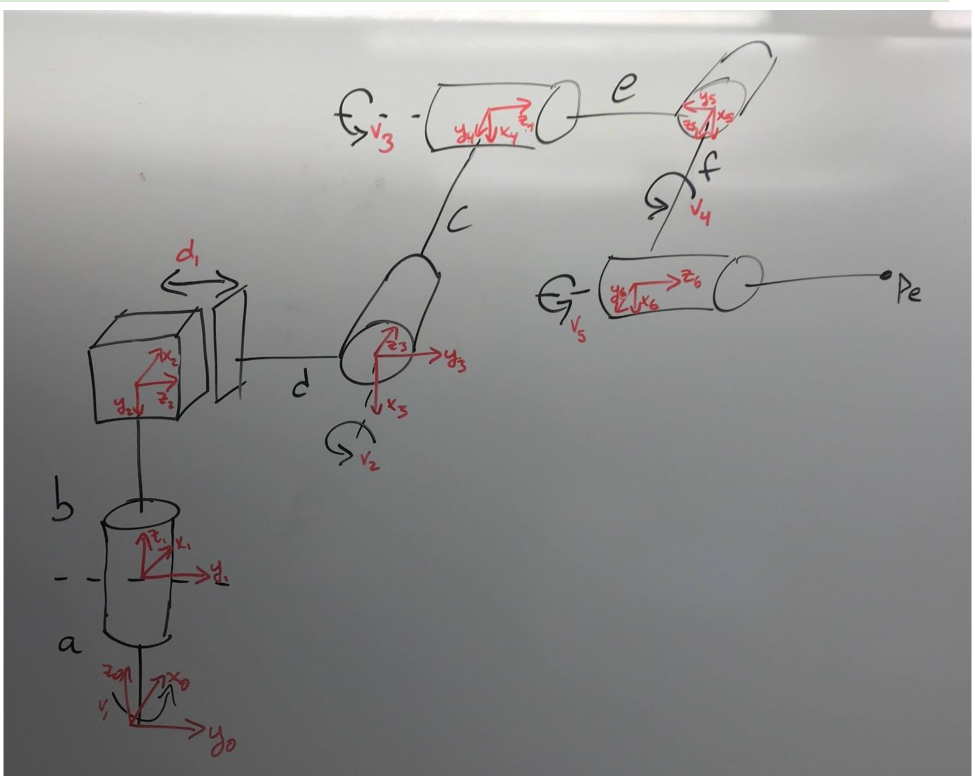
Manipulator Kinematics

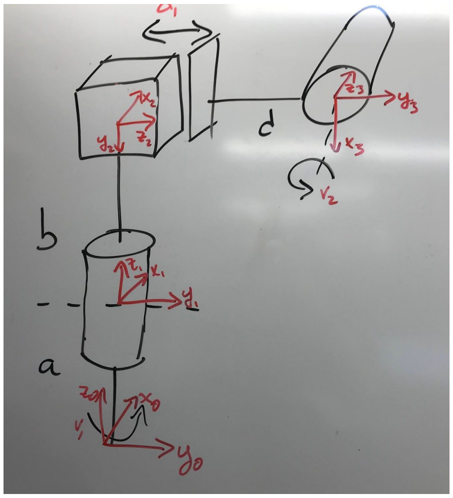


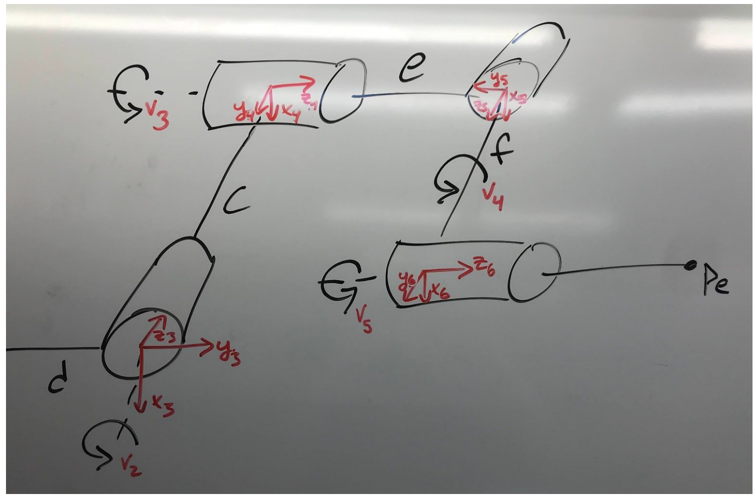
1. *How many links are in this manipulator?*

   This manipulator has 6 links.

2. *Draw the frames (x_i, y_i, z_i) using the Denavit-Hartenberg (DH) convention.*







3. *What are the 4 parameters which define the transformation between frames in the DH convention? And what do they represent?*

ϑ_i	Rotation around Z_{n-1} from X_{n-1} to X_n
α_i	Rotation around X_n from Z_{n-1} to Z_n
d_i	Displacement along Z_{n-1} from $Frame_{n-1}$ to $Frame_n$
a_i	Displacement along X_n from $Frame_{n-1}$ to $Frame_n$

4. *Write the DH table of parameters for this manipulator.*

Link	a_i	α_i	d_i	ϑ_i
1	0	0	a	v_i
2	0	0	b	0
3	0	$\pi/2$	$d + d_i$	0
4	0	0	c	v_2
5	0	0	e	v_3
6	0	0	f	v_4

5. *What does the first row of the DH table represent?*

The first row represents the Denavit-Hartenberg parameters for a transformation from the (x_0, y_0, z_0) frame to the (x_1, y_1, z_1) frame.

6. *If I have a sensor at position p_s that measures the distance to objects in my environment, how can I modify the transformation matrix to account for the sensor's position?*

To account for the sensor's position, an additional transformation matrix would need to be added to bring the sensor in the overall kinematic chain. Mathematically, this would be an additional multiplication of a transformation to the overall equation. Depending on the role of the sensor in the manipulator's actions, it would be conceivable to add it as a transformation that would come before Link 1 (i.e., a transformation from the sensor to what is currently considered reference frame 0). If the sensor were mounted on the end effector, however, it would be added to the end of the kinematic chain.

7. *If a new end-effector is added to the manipulator what should be modified / known to account for the changes it would cause to the kinematics?*

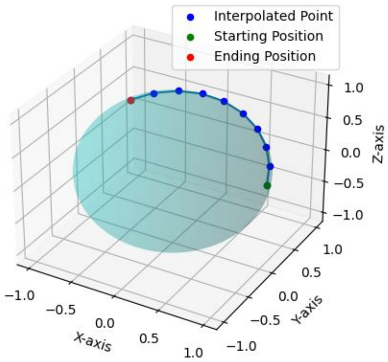
   If a new end-effector is added to the manipulator and there becomes a new link to the system, then an additional row of the DH parameter table must be added. In doing so, the distance from the previous joint and axes of motion of the new effector must be known. By finding the parameters the transformation matrix can be added to the overall kinematic model allowing for its position to be found.

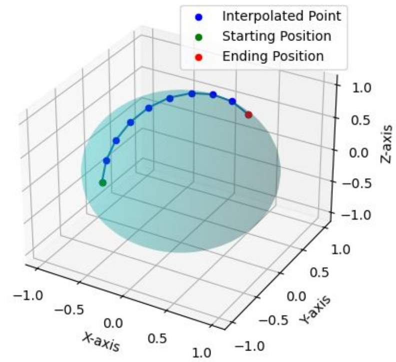
   If the new end-effector is replacing the last joint in the system, then the Link 6 row must be modified to account for it. The same information (distance from previous joint and axis of motion) must be known. In that case it would simply replace the Link 6 transformation matrix in the overall kinematic model.

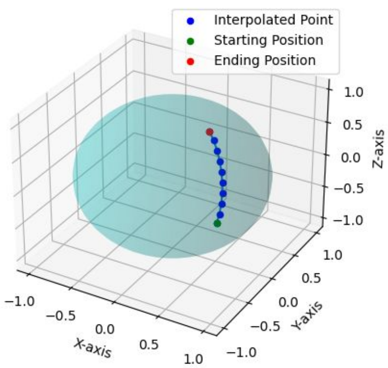
Mini-HW Problem *Remember homework 3 (robotic arm that wrote your initials). You will now have the opportunity to develop one that writes in 3-dimensions. This question could also just as easily be replaced with a swimming or flying robot stroke, or a legged robot foot trajectory.*

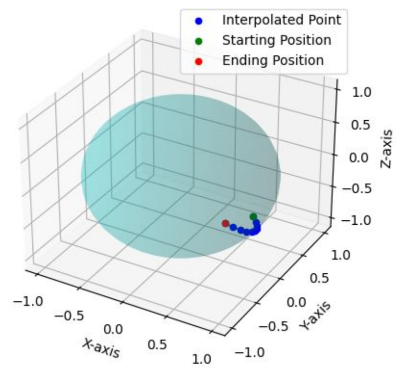
1. *Assume you have the inverse kinematics of a robotic arm capable of 3D positioning. Write code (Python or MATLAB) that takes two arbitrary 3-dimensional points **P1[x, y, z]** and **P2[x, y, z]** on a sphere of radius **R**, and draws a line of the sphere between them. The code should output a 3D plot showing the 3D line. Upload the code file (make sure to include comments) with your midterm (we will change both the points and radius and run it -- it must work).*

   [See attached files. README.md includes instructions to run the program. Use `'python main.py --test'` to test the program automatically using randomly generated points.]



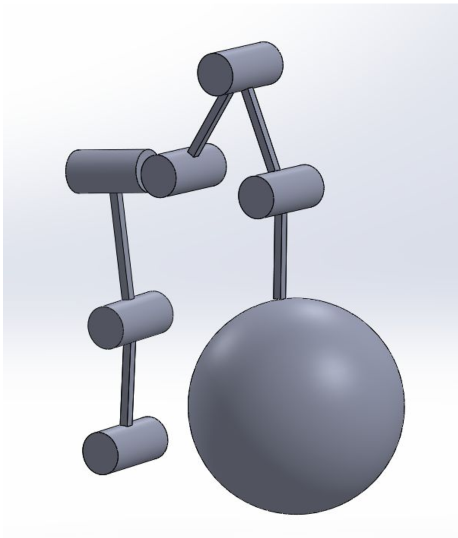
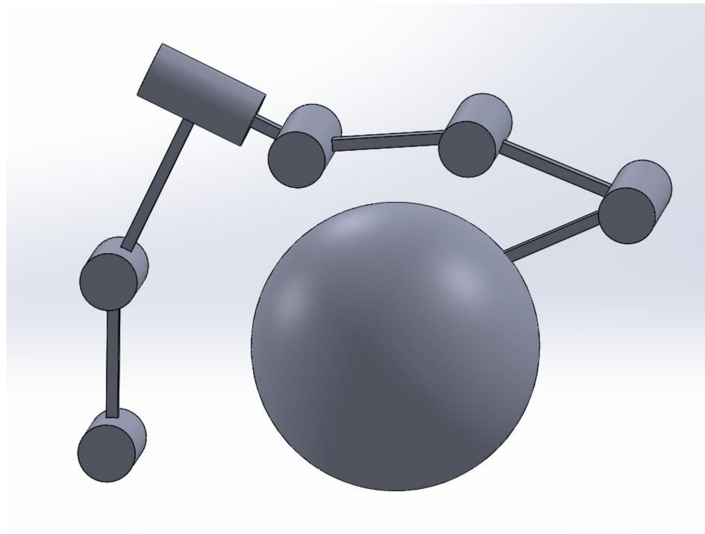
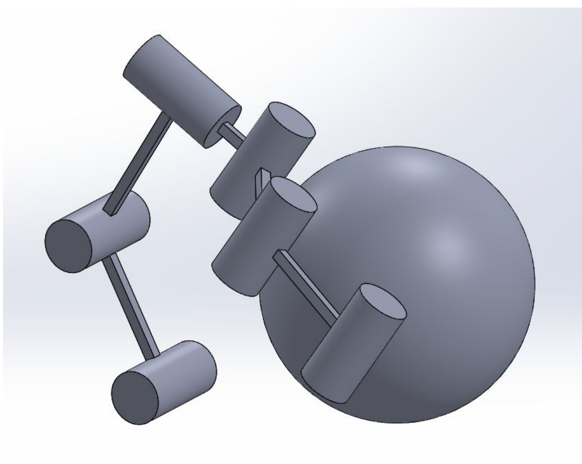
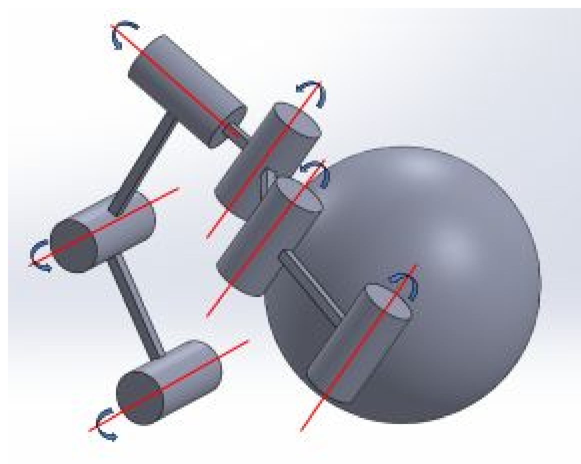






2. *Assume we now want to specify our end-effector position and orient it perpendicular to the tangent plane of the sphere at the point of contact. Draw (similar to Fig. 1 and 2) a manipulator that can achieve this behavior; remember to label joints and indicate rotation / translation axes. Discuss any important considerations.*

   Consider the following manipulator (shown below in four images). Using 6 degrees of freedom (all revolute joints), it is capable of placing an end-effector tangent to the plane of a sphere at any given point around the surface. All axes (denoted on the last image) are rotation axes; there are no translation axes. When building such a manipulator it would be important to take into account the size of the sphere in question. For example, it must be capable of reaching the far side of the sphere meaning that the middle three links, when extended, have a reach further than the diameter of the sphere, while still capable of reaching all the way down to the bottom.

3. *Assume the robotic arm is autonomous, and we do not know the size of the sphere. Discuss what sensors are necessary, and how they would be used, within the arm configuration, to complete the task in question 1.*

   There exist several methods for determining the size of the sphere, some more suited to the task than others, and some considerably more precise.

   Beginning with vision, the robotic arm might be equipped with stereo cameras. Using pixel differencing, the relative distance to any point in the field of view may be determined. Using this, a point cloud may be generated of the environment, and specifically the sphere in front of it. By moving the camera view around the sphere, a reasonably high-definition point cloud may be obtained, and then it is a matter of calculating the sphere size (or, in the event of an imperfectly shaped object, simply tracing a path along the surface of the object).

   This same effect can be achieved with LIDAR systems. Using a LIDAR scanner, the robot is also capable of generating a point cloud of its environment, and from there the steps are the same as a stereo camera system.

Switching paradigms, the robot arm end effector may include a limit switch. Starting naively, the arm could begin exploring its environment, making sure to lead with the limit switch. Assuming the arm is aware the object it is trying to find is a sphere, it could be programmed to explore along surfaces that appear spherical until it locates the sphere. By keeping track of the arm position where the limit switch is activated, it could generate a 3D mapping of its environment (with regards to the sphere), and do the same calculations regarding size mentioned above.

To achieve a better resolution more quickly than using a limit switch, a digital dial indicator may be used instead. The steps following the exploration phase will be the same, but it will be able to achieve better precision more quickly than a limit switch.

Once a model of the sphere has been created, the program from part (1) can be used to generate a path along the sphere. Because the radius and the center of the sphere is now known, the program has all the required parameters to generate the points along its surface.