MATHEMATICS FOR PROGRAMMING

Course about:-
⇒ math and logic basic

~~What is~~ a computer?
⇒ A machine that computes something.

⇒ Tasks:-
- Accept data/input
- Process data
- Produce output/result
- store result.

⇒ Usage:-
- writing documents
- Browse the internet
- send emails
- play games
- And many more---!

Components of a computer
⇒ ~~What is~~ software:-
- ~~set of~~ instruction in a way that the machine understands → computer program
- Associated Data

⇒ ~~What is~~ programming
- Building software
- writing instructions for the machine

- coding
- Telling the computer how to perform a task

⇒ How can you get in programming?
  - knowing a general purpose programming language like C, C++, Java, Python etc
  - Basic mathematics and logic
    - To solve a problem and to design the solution before writing code

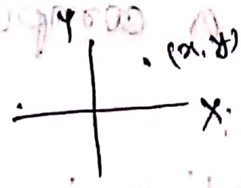
How much math should I know :-
- school level mathematics
- +, -, /, x
- % (mod), even-odd

$$\begin{array}{r|l} 2 & 7 \mid 3 \\ & \underline{6} \\ & 1 \text{ remainder} \end{array}$$

7 % 2 = 1

13 % 5 = 3

- percentage
- Co-ordinate system
- Basic geometry
- Number system



⇒ For special purpose:-
- Game development
  - 3D games and 3D graphics
    - Trigonometry, Linear Algebra, Matrices
- Cryptography

- Number theory
- Machine learning
  - Linear algebra, Calculus
  - Probability, statistics

⇒ How about libraries and black-boxes :-
  - using others solve or code

Number System :-

001
002
003
004
005
006
007
008
009
010

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 → 10 digits → Decimal number system

⇒ How about 2 Digits

0, 1 → 2 digits → Binary number system

000
001
010
011
100
101
110
111

1-20 decimal → binary practice

⇒ How many digits → base

- base 10: Decimal [0,1,2,3,4,5,6,7,8,9]
- base 2: Binary [0,1]
- base 8: Octal [0,1,2,3,4,5,6,7]
- base 16: Hexadecimal [0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F]

⇒ Convert Decimal to decimal

$$273$$

$$= 200 + 70 + 3$$

$$= 2 \times 100 + 7 \times 10 + 3 \times 1$$

$$= 2 \times 10^2 + 7 \times 10^1 + 3 \times 10^0$$

$$= 2 \times b^2 + 7 \times b^1 + 3 \times b^0$$

For decimal: $b = 10$

⇒ Convert Binary to Decimal

$$1101$$

$$= 1 \times b^3 + 1 \times b^2 + 0 \times b^1 + 1 \times b^0$$

$$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 1$$

$$= 13$$

CamScanner

⇒ Convert to Binary from Decimal :-

Decimal 13 ⟶ binary?

1101

2 | 13 | 6

12

1 (LSB: goes to the right)

2 | 6 | 3

6

0

2 | 3 | 1

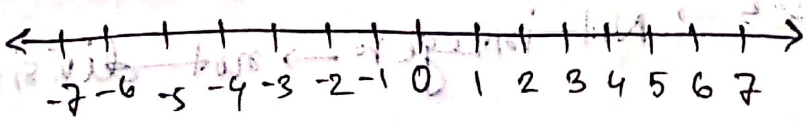
2

1

2 | 1 | 0

1 (MSB: goes to the left)

⇒ Types of Numbers:-



- Real number: any point on the number line
- Positive number: right to the zero
- Negative number: left to the zero
- Non-negative number: Positive and zero
- Natural Number: Counting number {1, 2, 3, ...}
- Whole Number: {0, 1, 2, 3, ...}
- Integers: {----, -3, -2, -1, 0, 1, 2, 3, ----}

- Rational and Irrational numbers

⇒ Let's learn about division:-

Divisor (ভাজক) ← 5) 17 → Dividend (ভাজ্য)

3 ← Quotient / ভাগফল

15

2 → Remainder / ভাগশেষ

⇒ Divisibility check

- check if the remainder is zero

$\boxed{\% \rightarrow \text{mod}}$

  - $12 \% 3 = 0 \rightarrow$ 12 is divisible by 3
  - $12 \% 5 = 2 \rightarrow$ 12 is not divisible by 5

- ~~just do the division~~
  - $12/3 = 4 \rightarrow$ Integer → divisible
  - ~~$12/5 = 2.5 \rightarrow$ Not integer → not divisible~~

⇒ Prime and composite numbers:-

- P is a factor of Q
  - P divides Q evenly (Remainder is 0).
    - 3 is a factor of 12

- Prime: Only two factors. 1, and itself
- Composite: There exists at least ~~one~~ factor

other than 1 and itself.

- Is 15 a prime
    - 15 is divisible by: 1, 3, 5, 15, Not prime

- Is 19 a prime
    - 19 is divisible by: 1, 19. So prime

- What about 1?
    - special case. Neither!

⇒ Even and odd Numbers:-

- Even → জোড় → 0, 2, 4, 8, 10, - - -
- Odd → বিজোড় → 1, 3, 5, 7, 9, - - -
- How to check?
    - Divide the number with 2
    - If remainder is 0, even! Otherwise, odd!

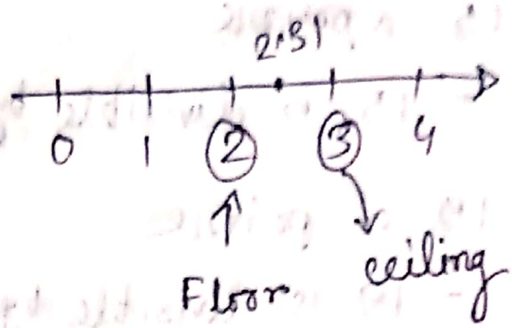
Convert the numbers to binary and see if you can find a pattern for even and odd numbers there

2 - 10	3 - 11
4 - 100	5 - 101
6 - 110	7 - 111

⇒ Floor: Nearest integer below

⇒ Ceiling: Nearest integer above

⇒ Round: Nearest integer



⇒ 12.65 → Floor: 12, Ceiling: 13, Round: 13

⇒ 9.21 → Floor: 9, ceiling: 10, Round: 9

⇒ 24 → Floor: 24, Ceiling: 24, Round: 24

Divisor Counting:-

* Problem statement: Given an integer n. How many divisor are there for n?

⇒ If n is a prime → divisor - 2
- Ex: 3, 11, 73

⇒ If n is not a prime
- Ex - 8, 15, 27

⇒ NAIVE approach:-
- take all the integers (1 to n), and test if remainder is 0
- Example - 12

1 - 12%1 = 0 ✓	4 - 12%4 = 0 ✓	7 - 12%7 = 5 ✗	11 - 12%11 = 1 ✗
2 - 12%2 = 0 ✓	5 - 12%5 = 2 ✗	8 - 12%8 = 4 ✗	12 - 12%12 = 0 ✓
3 - 12%3 = 0 ✓	6 - 12%6 = 0 ✓	9 - 12%9 = 3 ✗	
		10 - 12%10 = 2 ✗	

- the divisor: 1, 2, 3, 4, 6, 12
- Number of divisors: 6

Divisor Counting Better?

⇒ 1 and n must be in divisor

⇒ only need to check for 2 to $n/2$

Divisor counting Even better?

⇒ So we do not need to test any number greater than $\sqrt{n}$

⇒ or $\sqrt{n}$ floor value

$1 \times 16 = 16$
$2 \times 8 = 16$
$4 \times 4 = 16$
$8 \times 2 = 16$
$16 \times 1 = 16$

PRIMALITY TEST

* problem statement: Given an integer n. Determine if n is a prime number or not.

⇒ Is 77 is a prime number?

- $\sqrt{77} = 8.77$
- Check 2 to 8
- Not prime

$77 \% 2 = 1$
$77 \% 3 = 2$
$77 \% 5 = 2$
$77 \% 6 = 5$
$77 \% 7 = 0$

⇒ extra calculation we did?

77 % 2 = 1

77 % 3 = 2

$\boxed{77 \% 4 = 1}$

77 % 5 = 2

$\boxed{77 \% 6 = 5}$

77 % 7 = 0

⇒ Can we get a better method?

- Yes, Let's talk about, $\boxed{\text{Sieve of Eratosthenes}}$

SIEVE OF ERATOSTHENES

⇒ Identifies all the prime numbers in a given range (very fast)

⇒ Idea:-

- Step 1: take a number (start with 2) and find its multiple in the range
- Step 2: Those multiples must be composite, cross them out
- Step 3: Proceed to the next number
- Step 4: If it's prime, go to step 1, otherwise step 3

☐ Extract digits from an integer.

⇒ Division is all need

⇒ $n = 237$

- last digit
- $237 \% 10 = 7$

- Divide 237 with 10 and get the quotient
- $\text{Floor}(237 \div 10) = 23$
- Repeat the same process with 23

* Find the i-th digit ~~of a~~ number (say $n = 91408$) from right. $[i = 0, 1, 2, 3, 4]$

$i=4 \quad i=2 \quad i=1 \quad i=0$

$i=3$

- $i = 0 \rightarrow n \% 10 = 8$
- $i = 1 \rightarrow \text{Floor}(n/10) \% 10 = 0$
- $i = 2 \rightarrow \text{Floor}(n/100) \% 10 = 4$
- $i = 3 \rightarrow \text{Floor}(n/1000) \% 10 = 1$
- $i = 4 \rightarrow \text{Floor}(n/10{,}000) \% 10 = 9$

$$\boxed{\text{Floor}(n/10^i) \% 10}$$

⇒ General Formula: $\text{Floor}\left(\frac{n}{10^i}\right) \% 10 = \left\lfloor \frac{n}{10^i} \right\rfloor \% 10$

$\lfloor \ \rfloor \rightarrow$ Floor

$\lceil \ \rceil \rightarrow$ ceiling

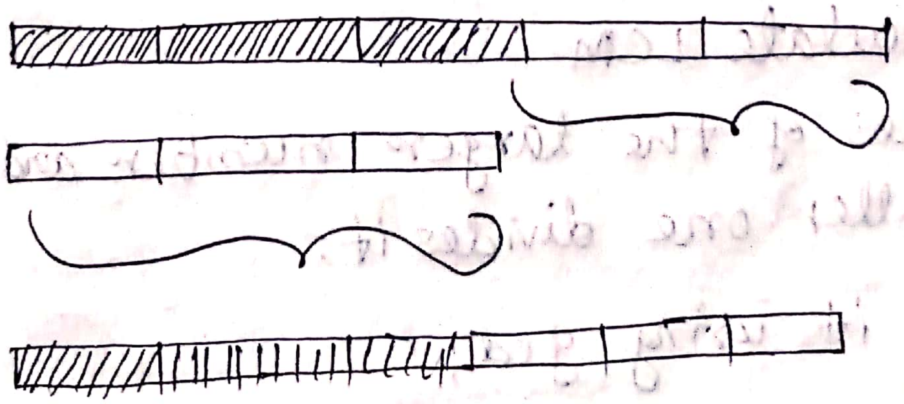
GCD - Greatest Common Divisor

- স.সা.গু. - গরিষ্ঠ সাধারণ গুণনীয়ক
- Divisor
- Common divisor
- Greatest common divisor
  - Example: Find the gcd of 15 and 6
    - Divisors of 15: 1, 3, 5, 15
    - Divisors of 6: 1, 2, 3, 6
    - Common divisors: 1, 3
    - GCD: 3
- Co-prime: If gcd(a,b) = 1, a and b are co-prime
  - Ex: 8, 15

⇒ How to calculate GCD

- Simple way to find gcd(a,b)
  - Check if i divides both a and b
  - Largest such i is the gcd
- The Euclidean Algorithm to find GCD easily

GCD - The Euclidean algorithm



- gcd (a, b) = gcd (b, a % b)
- gcd (p, 0) = gcd (0, p) = p
  - Ex: gcd (15, 6) = gcd (6, 3) = gcd (3, 0) = 3
- Changing order does not matter
- gcd (15, 8) = gcd (8, 7) = gcd (7, 1) = gcd (1, 0) = 1
  ↳ co-prime

LCM - least common multiple

- ল.সা.গু = লঘিষ্ঠ সাধারণ গুণিতক
- multiple
- common multiple
- least common multiple
- Ex: Find the lcm of 15 and 6
  - multiples of 15: 15, 30, 45, 60, - - -
  - multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 6
  - common multiples: 30, 60 - - -
  - LCM: 30

⇒ How to calculate LCM
- take multiples of the larger number and see if the smaller one divides it.
- We can find it using gcd;

- $\gcd(a,b) \times \text{lcm}(a,b) = a \times b$ [Proof: try yourself]
- Ex- $\text{lcm}(15,6) = \frac{15 \times 6}{3} = 30$

Factorial:-

- A function defined for non-negative integers
  - 0, 1, 2, 3. . .
- $n! = n \times (n-1) \times (n-2) \times (n-3) \times \cdots \times 3 \times 2 \times 1$
  - $4! = 4 \times 3 \times 2 \times 1 = 24$
- Exception: $0! = 1$
  - consider how other factorials are related
  - $5! = 120 \rightarrow$ divide it by $5 \rightarrow 24$
  - $4! = 24 \rightarrow$ divide it by $4 \rightarrow 6$
  - $3! = 6 \rightarrow$ divide it by $3 \rightarrow 2$
  - $2! = 2 \rightarrow$ divide it by $2 \rightarrow 1$
  - $0! = 1$

⇒ Factorial why?

- Consider a cricket team 11 players
- How many batting orders are possible?
- what if we had 2 players
  - Ans: 2
  - AB or BA
- what about 3 players:-
  - Ans: 6
  - ABC, ACB, BCA, BAC, CAB, CBA

X	X	.

$\underbrace{3 \times 2 \times 1}_{3!} = 6$

- Now for 11 players

11	10	9	8	7	6	5	4	3	2	1

- 11!
- These type of problems are part of "combinatorics"
  - A branch of mathematics
  - Deals with permutation and combination

Intro to matrix/matrices

- collection of numbers
- Arranged in rows and columns

$$A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix} \leftarrow 2 \text{ rows}$$

3 columns

- value/element = 2×3 = row x column = 6
- $A_{ij} = 9 \rightarrow$ row: i, column: j
- $A_{23} = 9 \longrightarrow$ row: 2, column - 3
- How many rows and columns

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \quad \begin{bmatrix} 1 & 9 & -3 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$$

3×4 — 1×4 row matrix — 3×1 column matrix

⇒ Adding matrices:-

$$\begin{bmatrix} 40 & 36 \\ 28 & 32 \\ 30 & 27 \end{bmatrix} + \begin{bmatrix} 19 & 35 \\ 48 & 12 \\ 33 & 48 \end{bmatrix} = \begin{bmatrix} 59 & 71 \\ 76 & 44 \\ 63 & 75 \end{bmatrix}$$

Power and Roots

- Power: b^x
  - $b \rightarrow$ base
  - $x \rightarrow$ exponent
  - $b^x = \underbrace{b \times \cdots\cdots \times b}_{x\ \text{times}}$
  - $5^3 = 5 \times 5 \times 5 = 125$
- Root: $\sqrt[n]{a}$ (nth root of a)
  - Assume: $\sqrt[n]{a} = x$
  - then, $x^n = a$
  - $\sqrt[3]{27} = 3$

Intro to sets:-

- A collection of objects
  - No specific order or index [unlike matrix]
- Objects are called elements
- Example
  - Natural numbers set: $\{1, 2, 3 \ldots\ldots\}$ → infinite set

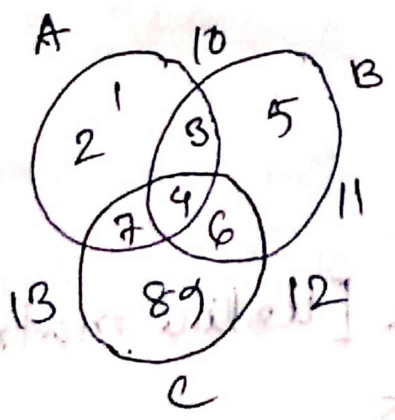
- Name of favourite sports: {cricket, football, hockey, volleyball}
  ↳ finite set

- subset:-
  - A is a subset of B if all the elements of A is in B
  - A = {10, 12, 29} and B = {101, 12, 10, 29, 32}
- Universal Set:
  - Depends on context
  - Example:-
    - when talking about numbers, universal set might be the set of Real Numbers

- Empty / Null set:-
  - set with zero elements: { } or ϕ

⇒ U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

A = {1, 2, 3, 4, 7}   B = {3, 4, 5, 6}   C = {4, 6, 7, 8, 9}



⇒ Three set operations:

- Union
  - $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$
- Intersection
  - $A \cap B = \{3, 4\}$
- Complement
  - $A' = U - A = \{4, 5, 6, 8, 9, 10, 11, 12, 13\}$