Basic Geometry:-

- Points and lines
- Co-ordinate system
- Angles
- Some geometric shapes

Point

- Just a dot
- location only
- No length, width, shape, size

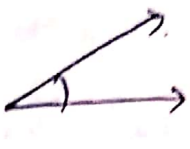
Line

- two different points
- Connect them
- take the straight path
  - Also the shortest
- This is called line segment
  - Finite length
- Line: extended in both ends

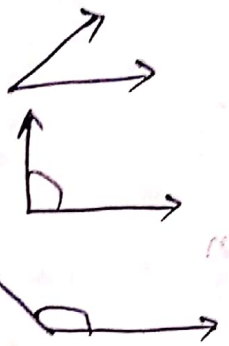
- Infinite length
- Ray : extended in one end ⟶

ANGLE :-

- Between two rays extending from the same point



- Circular arc to annotate
- Full rotation = 360 degrees
- Acute Angle ($< 90°$)
- Right Angle ($= 90°$)
- Obtuse Angle ($> 90°$)

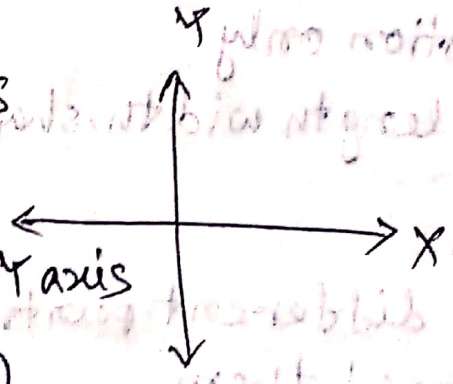


Co-ordinate System :-

- Reference point, directions, distance
- Number line → one dimensional co-ordinate system

⇒ 2D co-ordinate system

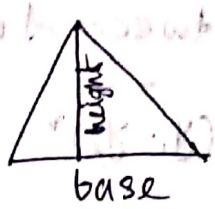
- take two perpendicular lines
- Right angle / 90 degree
- Horizontal : X axis, vertical : Y axis
- Intersecting point : Origin (0, 0)
- Any point can be identified as a pair of numbers



⇒ 3D co-ordinate system :-

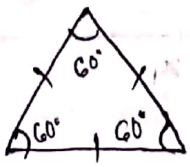
- X, Y, Z axis

TRIANGLE:-

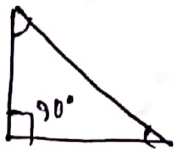


- 3 points and 3 line segments
- Sum of three angles = 180°
- Area = $\frac{1}{2}$ * base * height

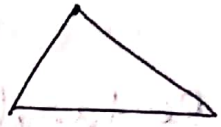
Isosceles triangle



Equilateral triangle



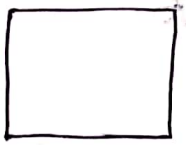
Right-angle triangle



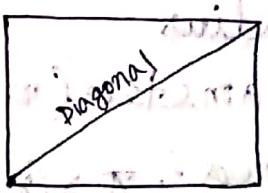
Scalence triangle

- Based on equality between sides
  - Equilateral
  - Isosceles
  - Scalene

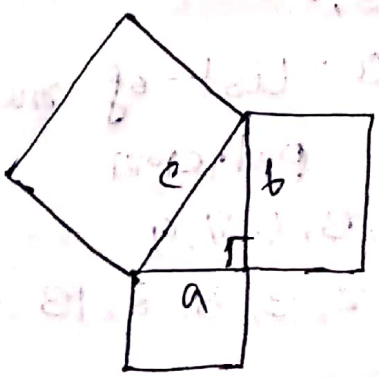
SQUARE



- 4 points and 4 line segments
- All sides are equal
- Each angle is 90°
- Sum of four angle is 360°
- Area $(\text{side-length})^2$
- Length of diagonal: $\sqrt{2} \times (\text{side\_length})$



PYTHAGOREAN THEOREM



$\Rightarrow c^2 = a^2 + b^2$

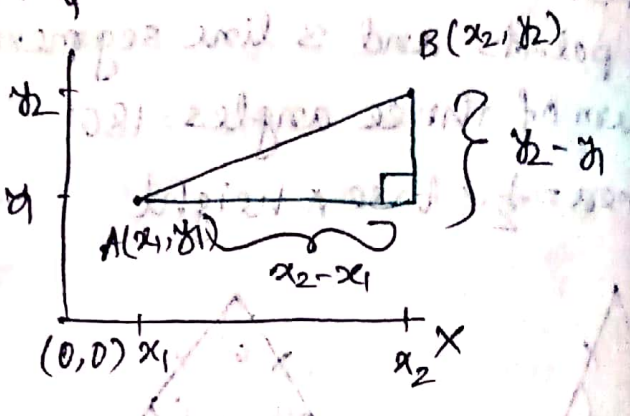
- Right-angle triangle and square
- Example:-

$5^2 = 3^2 + 4^2$

$13^2 = 5^2 + 12^2$

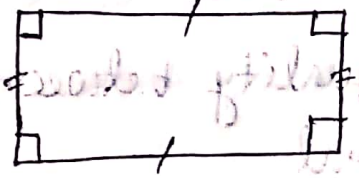
Distance between two points.

$\Rightarrow d_{AB} = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$



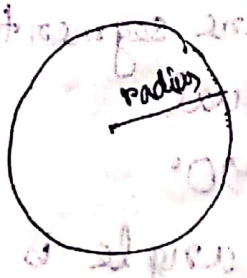
Rectangle:-

- 4 points and 4 line segments
- Opposite sides are equal
- Each angle is 90°
- Sum of four angles is 360°
- Area: length × height



Circle:-

- Center
- Radius
- Diameter (maximum distance)
- Area = $\pi \times (\text{radius})^2$



Series:-

- Sequence: List of numbers with some order or pattern
  - 1, 3, 5, 7, 9
  - 1, 1, 2, 3, 5, 8, 13 → Fibonacci series
- Series: Sum of elements in a sequence

$S_1 = 1+3+5+7+9$

$S_2 = 1+1+2+3+5+8+13+---$

- Finite vs Infinite series
- Arithmetic vs Geometric series

$1+6+11+16+21+---$ → arithmetic

$2+6+18+54+---$ → Geometric

$\Rightarrow 1+2+3+---+N = ?$

$\Rightarrow 1+2+3+4+5 = ?$

let, $S = 1+2+3+4+5$

$S = 5+4+3+2+1$

$2S = 6+6+6+6+6$

$2S = 5\times 6$

$S = \frac{5\times 6}{2}$

$\Rightarrow$ let, $S = 1+2+3+---+N$

$S = N+(n-1)+(n-2)+---1$

$2S = (N+1)+(N+1)+(N+1)----(N+1)$

$2S = N\times(N+1)$

$$\boxed{S = \frac{N\times(N+1)}{2}}$$

Series (Finite series sum)

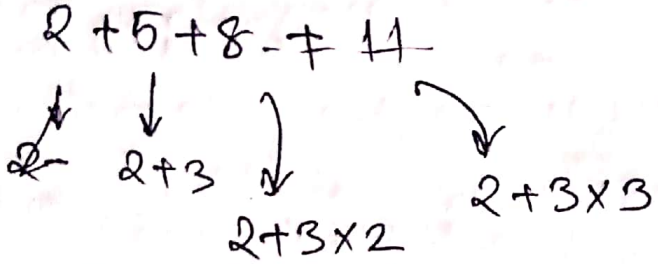
- Arithmetic series

$S = 2 + 5 + 8 + \cdots$

Find the 100th number in this series.

Find the sum of first 100 numbers.

- nth term in arithmetic series: $a + (n-1) \times d$
  - 100th number $= 2 + 99 \times 3 = 299$



- Sum of first n terms: $\frac{n}{2}\{2a + (n-1) \times d\}$

$S = a + (a+d) + \cdots + \{a + (n-1)d\}$

$S = \{a + (n-1)d\} + \cdots + a$

$2S = 2a + (n-1)d$

$2S = n\{2a + (n-1)d\}$

$S = \frac{n}{2}\{2a + (n-1) \times d\}$

→ sum of first 100 numbers = $\frac{100}{2}\{2\times2+(100-1)\times3\} = 15050$

Unitary method (একক নিয়ম)

- Find value for single unit
- Then multiply for necessary units

* Problem: You and your 6 friends went to a restaurant. All ordered the same meal, and the total bill was 1218 taka. Now if you go there with your 2 best friends and have the same meal, what would be the bill this time?

- Given: Cost for 7 persons
- Goal: Finding cost for 3 persons
- Unitary method: Find cost for 1 person first

cost for 7 persons = 1218

cost for 1 person = $\frac{1218}{7} = 174$

cost for 3 persons = $174\times3 = 522$

* Tom finishes his homework within 15 hours while Jerry takes 10 hours. How many hours will be the same homework take to be done if they work together?

- Idea: How is their performance per hour?
- Tom's 15 hours of work ≡ 1 homework
  ∴ Tom's 1 hour of work ≡ $\frac{1}{15}$ homework

- Jerry's 10 hours of work ≡ 1 homework
  - Jerry's 1 hour of work ≡ $\frac{1}{10}$ homework
- Tom and Jerry's 1 hour of work combined
- Answer : 6 hours $\equiv \left(\frac{1}{15} + \frac{1}{10}\right)$ homework

$= \frac{1}{6}$ homework

Percentage:-

- Per hundred quantity
- scaling reference to 100
  - You gave a test on 40 marks. Your score 32.
  - what would be the score if the test was taken with 100 marks?
  - may use unitary method.
    - out of 40 your score = 32
    - out of 1 your score = $\frac{32}{40}$
    - out of 100 your score = $\left(\frac{32}{40}\right) * 100 = 800$
  - this is per hundred quantity, therefore, percentage!
  - the mark is 80%.

Percentage problem:-

* A clothing store is selling one of their most popular products at 870 Tk after 40% discoun

What was the original price?

- Let it be x
  - The discounted price $= x \times (100-40)\% = x \times \frac{60}{100} = 870$
  - The original price

$$x = 870 \times \frac{100}{60} = 1450$$

* The price of oil increased by 25% and then decreased by 15%. What is the net percentage of increase or decrease in oil price?

- Initial value is unknown
- Let's assume the initial price was 100
  - Increase by 25%. : Updated price $= 125$
  - Decrease by 15%
    - Price gets reduced to 85%.
    - Updated price $= 125 \times 85\% = 125 \times \frac{85}{100} = 106.25$
  - Final price is more than 100
  - Net increase $(106.25-100) = 6.25$ if the original price was ৳100 (per hundred)
  - Increased by 6.25%.

Capital and Interest:-

- Interest rate
  - Per 100 taka
  - Per 1 year (3 months, 6 months are also common)

- Simple interest vs compound interest
- Simple interest :-
  - $I = P \times n \times r$
  - Interest = Capital × Time unit × Interest rate

* Your friend deposits 7000 tk in sonali bank for 3 years which earn him an interest of 8%. What is the amount he gets after 3 years?
  - $I = ?, \quad P = 7000, \quad n = 3, \quad r = 8\%$
  - $I = \frac{7000 \times 3 \times 8}{100} = 1680$

* You deposit 5400 tk and got back an amount of 6000 tk after 2 years. Find the simple interest rate of the bank.
  - $r = ?, \quad P = 5400, \quad n = 2, \quad I = 6000 - 5400 = 600$

$$I = \frac{P \times n \times r}{100}$$

$$r = \frac{I \times 100}{P \times n} = \frac{600 \times 100}{5400 \times 2} = 5.56\%$$

Mean & Median :-

- Mean
  - Give some numbers
  - take the sum of them
  - Divide by the count of numbers

- You get the mean or average
- Can you interpret the formula on right?

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

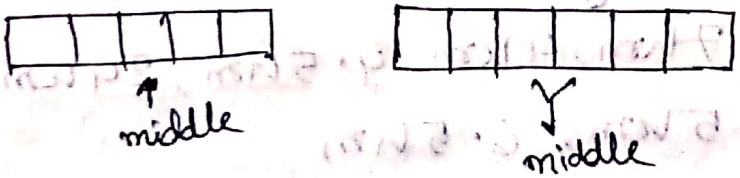
5, 4, 10 → x

x_1 x_2 x_3 $n = 3$

* You and your friends went to a restaurant. Everyone puts their money on the table and others the same meal for all within budget. How much is the maximum budget for each person?
  - money collected from 6 friends: 103, 210, 57, 85, 500, 180
  - Ans; Everyone has a budget of at most the average amount.
  - $\frac{103+210+57+85+500+180}{6} = 189.167$

Median

- Given some numbers
- Sort them in ascending order
- take the number found at the middle.



- If count is odd
  - middle one is unique

- If count is even
  - There are two elements in the middle
  - take average of those two.

- What does it mean to take the middle number?
  - Say your median value is X
  - Then ~50% of the values are less than X
  - And ~50% of the values are greater than X.

- 8, 7, 3, 2, 10
  - Sort: 2, 3, 7, 8, 10
  - median: 7
- 8, 7, 3, 2, 10, 1
  - sort: 1, 2, 3, 7, 8, 10
  - Median: (3+7)/2 = 5

- Why median?
  - Because sometimes mean can be misleading
  - Example: You try to walk every day around 4km. Here is the history of ten days of your walking:
    - 3, 5 km, 7 km, 4 km, 4.5 km, 24 km, 5 km, 5.5 km, 5 km, 6.5 km
    - mean 7 km
    - leave out the outlier 24 km and

mean would be 5.11 km

- Let's look at the median:
  - 3.5, 4, 4.5, 5, 5, 5, 5.5, 6.5, 7, 24 (sorted)
  - Median $(5+5)/2 = 5$
  - Even if we leave out 24
    - 3.5, 4, 4.5, 5, 5, 5, 5, 5.5, 6.5, 7
    - Median = 5
- Takeaway: Median can not be deviated as much as mean by some outliners.

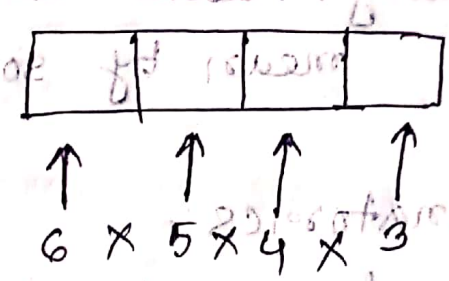
Combinatorics:

- Number of ways to choose or arrange objects
- Arranging some objects in all possible orders.
  - Remember factorial
- Choose some objects out of a collection of objects
  - Combination problem
  - How many ways are there to choose r object from a collection of n objects?
    - ${}_nC_r = ?$
- Choose and order some objects out of a collection of objects
  - Permutation problem
  - How many ways are there to arrange r objects from a collection of n objects?
    - ${}_nP_r = ?$

$\Rightarrow {}^{n}P_{r}$

- You have n objects in hand (all different)
- You have r slots to put them ($r \leq n$) in order
- Example: Batting order of a 11 player cricket match from a squad of 15 players.
- Answering order in an exam to answer 4 out of 6 questions.
  - Number of ways: $6 \times 5 \times 4 \times 3$

    $= 360$

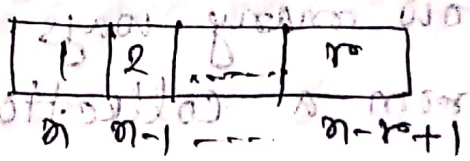


  - But could we write

$$\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

$$= \frac{6!}{2!}$$

- Generalize the formula



- Number of permutation: $n(n-1) \times (n-2) \times \cdots (n-r+1)$

$$= \frac{n \times (n-1) \times (n-2) \times (n-r+1) \times (n-r) \times \cdots \times 1}{(n-r) \times \cdots \times 1}$$

$$= \frac{n!}{(n-r)!}$$

- Batting order of an 11-player cricket match from a squad of 15 players?

$r = 11, \quad n = 15$

$$\frac{15!}{(15-11)!}$$

$\Rightarrow {}^{n}C_{r}$

- You have n objects in hand (all different)
- You have r slots to put them ($r \leq n$) [order does not matter]
- Choosing objects instead of arranging them
- Example:-
  - Choosing 11 players from a squad of 15 players.
  - Answering combinations in an exam to answer 4 out of 6 questions.
  - We already know how many ways are there if the order is considered too (${}^{n}P_{r} = {}^{6}P_{4}$)
  - Here: 2->3->1->6 is same as 1->2->6->3 as we only need the choice (without order)
  - How many orders are possible with a fixed set of 4 questions?

    $= 4!$

    - All of them should be count as one combination

- So, we can just divide the permutation by 4!.
- Generalize the formula
- Number of permutation:-

$$^{n}P_{r} = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$$

- Each r! corresponds to the same choice

$$- \; ^{n}C_{r} = \frac{^{n}P_{r}}{r!} = \frac{n!}{r!\,(n-r)!}$$

* Selection ~~Batting order~~ of 11-player cricket match from a squad of 15 players

$$\frac{15!}{11!\,(15-11)!}$$

4] Probability :-

- Probability is a number to indicate the chance or likehood of a particular event.
- Event: outcome of an experiment or process
- For example-
  - Tossing a coin is an experiment.
  - Event would be the appearance of Head or Tail
- The number is between always between 0 to 1

- Impossible event: 0
- Certain event: 1
- Everything in between

- $P(\text{event}) = \frac{\text{Number of ways it can happen}}{\text{Number of all possible outcomes}}$

- What is the probability of getting a prime number if you throw a 6-faced dice?
  - All possible outcomes: 1, 2, 3, 4, 5, 6
  - Target outcomes / prime numbers: 2, 3, 5
  - $P = \frac{3}{6} = 0.5$

- What is the probability of getting exactly two heads if you throw three coins?
  - Outcomes of single coin: H, T
  - Two coins: HH, HT, TH, TT

	H	T
H	HH	HT
T	TH	TT

  - What about three coins?
  - $P = \frac{3}{8}$

	HH	HT	TH	TT
H	HHH	HHT	HTH	HTT
T	THH	THT	TTH	TTT

- Can you do it without building table?
- Number of possible outcomes:-

$$2 \times 2 \times 2 = 8$$

- Outcomes with exactly 2 heads out of 3 trials
  - Choose 2 positions out of 3 for H
  - $^3C_2 = \frac{3!}{1!\,2!} = 3$

- $P = \frac{B}{8}$

- Binary <-> Octal
  - 8 and 16 are powers of 2, 10 is not
    - $8 = 2^3$ and $16 = 2^4$

- 000 to 111 => 0 to 7 (Octal)
- 0000 to 1111 => 0 to F (Hexadecimal)
- Binary 10111011 to octal
  - 010 111 011 => 273

- Octal 314 to binary?
  - 314 => 011 001 100