# Super Stable Cycles of Quadratic functions

Osiris

April 2020

## 1 Abstract

The goal of this paper is to find the c-values to which the Quadratic function $Q_c(x) = x^2 + c$ has superstable cycles of period 1,2,4,8,16, and 32.

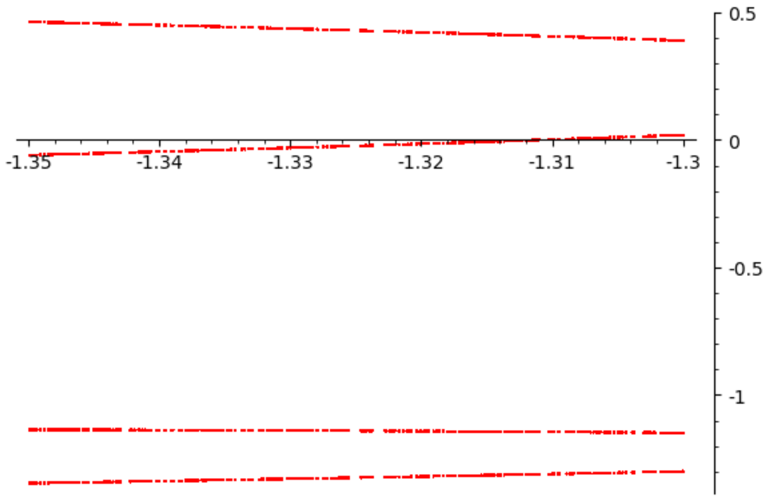
## 2 Method

In order to do this I plotted the orbit diagram for the quadratic function. I then magnified the sections to which the orbit diagram had each of the following periods and estimated the c-value to which the each period contains $x_0 = 0$ the critical value of the Quadratic function.

## 3 Results

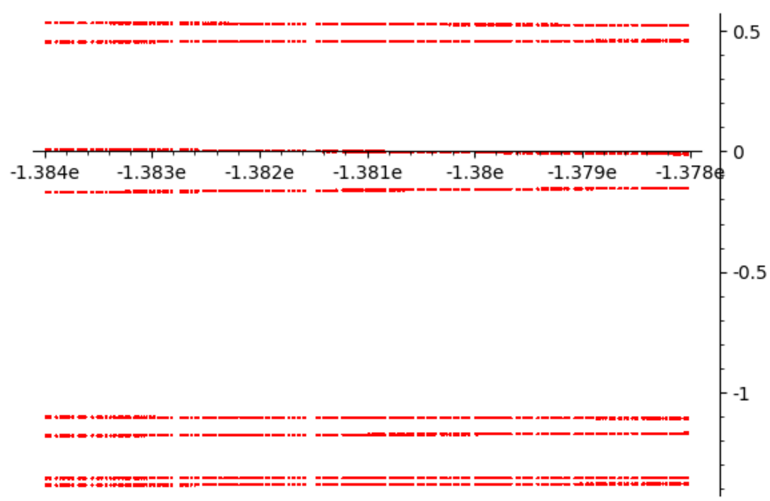
Let $c_n$ Be the $c-value$ to which the quadratic function has a superstable cycle of period $2^n$. We will calculate $c_n$ s.t $0 \leq n \leq 6$.
We have $Q_c(x) = x^2 + c$ and $Q'_c(x) = x_0 = 0$ To find $c_0$ we solve $Q^1_c(x_0) = x_0$ $\Rightarrow$c$_0 = 0$ To find $c_1$ we solve $Q^2_c(x_0) = x_0$ $\Rightarrow$(x$_0^2 + c_1)^2 + c_1 = 0$ $\Rightarrow$c$_1 = -1$

For the remaining 5 $c$ values I used the bifurcation plot results and zoomed in on the sections of the plot encapsulating period $2^n$ and estimated $c_n$ to be the value to which 0 belonged to the period. $c_2$ $\Rightarrow$Q$^4_{c}$2$(x_0) = x_0$. Estimating based on the magnification of the plot I obtained $c_2 = -1.312$.

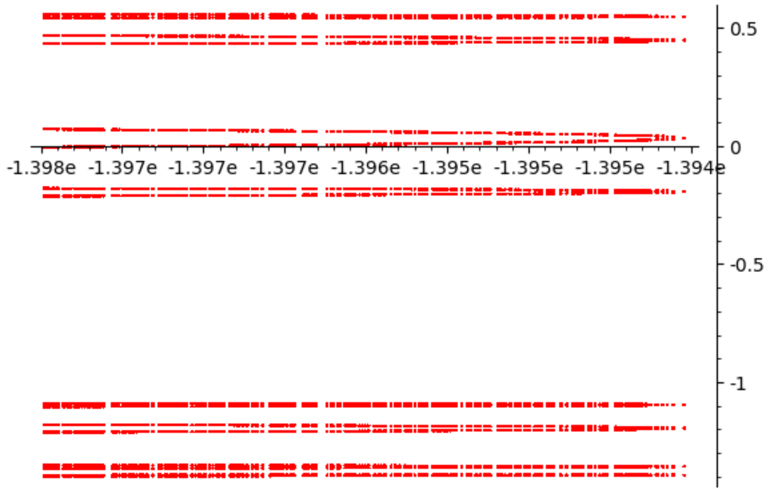



[.2 cm]

$c_3 \Rightarrow Q^8_{c3}(x_0) = x_0$. Estimating based on the magnification of the plot I obtained $c_3 = -1.381$.

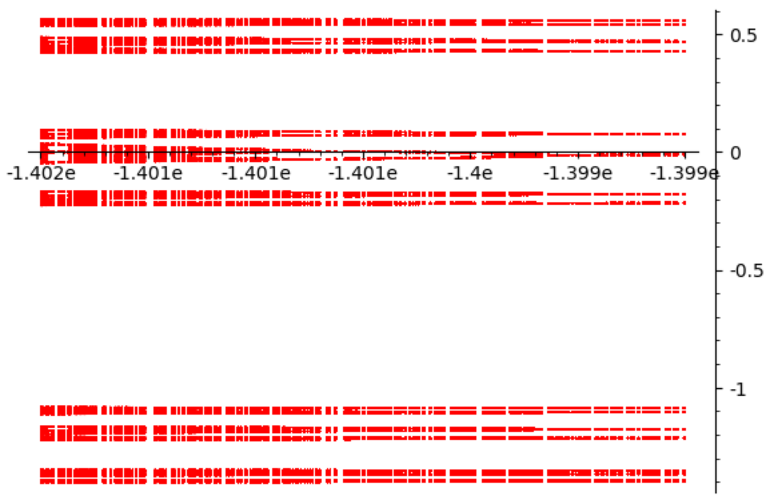



$c_4 \Rightarrow Q^1 6_{c4}(x_0) = x_0$. Estimating based on the magnification of the plot I obtained $c_4 = -1.397$.



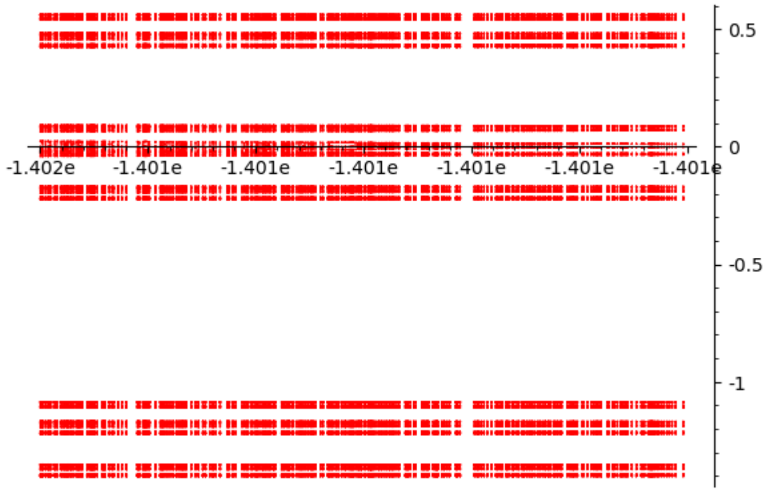

[.2 cm]

$c_5 \Rightarrow \mathrm{Q}^{3}2_{c5}(x_0) = x_0$ Estimating based on the magnification of the plot I obtained $c_5 = -1.4008$.




[.2 cm]

$c_6 \Rightarrow \mathrm{Q}^{6}4_{c6}(x_0) = x_0$ Estimating based on the magnification of the plot I obtained $c_6 = -1.4017$ which is a very rough estimate.




# 4 Discussion

Let
$f_0 = \frac{c_0 - c_1}{c_1 - c_2}$
$f_1 = \frac{c_1 - c_2}{c_2 - c_3}$
$f_2 = \frac{c_2 - c_3}{c_3 - c_4}$
$f_3 = \frac{c_3 - c_4}{c_4 - c_5}$
$f_4 = \frac{c_4 - c_5}{c_5 - c_6}$
$\Rightarrow$f$_0 = 3.2051$ $f_1 = 4.5217$ $f_2 = 4.3125$ $f_3 = 4.210526$ $f_4 = 4.6666667$

To my suprise the values do seem to be converging to "Feigenbaum's constant" which is pretty cool. Because of this I can assume that the c values are fairly accurate in being the c values of superstable periods for the quadratic function. So, in conclusion we have found all of the $c_n$ values to which satisfy the quadratic function having superstable period $n$, and implemented these values ratios to show convergence to "Feigenbaum's constant".