

R1CS Programming

ZK0x04 Workshop Notes

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Contents

1	Multiplicative inverse	2
2	Zero testing	2
3	Binary	2
4	Comparisons	3
5	Permutations	3
6	Sorting	3
7	Random access	3
8	Embedded curve operations	3

1 Multiplicative inverse

Deterministically computing $1/x$ in an R1CS circuit would be expensive. Instead, we can have the prover compute $1/x$ outside of the circuit and supply the result as a witness element, which we will call x_{inv} . To verify the result, we enforce

$$(x)(x_{\text{inv}}) = (1) \quad (1)$$

2 Zero testing

To assert $x = 0$, we simply enforce

$$(x)(1) = (0) \quad (2)$$

Asserting $x \neq 0$ is similarly easy: we compute $1/x$ (non-deterministically, as in [Section 1](#)). The result can be ignored; the mere fact that an inverse exists implies $x \neq 0$.

On the other hand, if we want to *evaluate*

$$y := \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{otherwise} \end{cases} \quad (3)$$

we can do so by introducing another variable, m , and enforcing

$$(x)(m) = (y) \quad (4)$$

$$(1 - y)(x) = (0) \quad (5)$$

Outside of the circuit, the prover generates y as in [Equation 3](#), and generates m as

$$m := \begin{cases} 1 & \text{if } x = 0 \\ y/x & \text{otherwise} \end{cases} \quad (6)$$

This method is from [\[1\]](#).

3 Binary

To assert $b \in \{0, 1\}$, we enforce

$$(b)(b - 1) = (0) \quad (7)$$

TODO: Discuss splits and joins

4 Comparisons

TODO: Describe basic comparison algorithm

TODO: Describe Ahmed’s optimization

A couple other optimizations are possible in particular circumstances:

1. To assert (not evaluate) $x < y$, we can split x non-canonically and split y canonically. The prover is forced to use x ’s canonical representation anyway, otherwise $x_{\text{bin}} \geq |F| > y_{\text{bin}}$, making the assertion unsatisfiable.
2. To assert $x < c$ for some constant $c \ll |F|$, we can split x into just $\lceil \log_2(c) \rceil$ bits.

5 Permutations

Say we want to verify that two sequences, (x_1, \dots, x_n) and (y_1, \dots, y_n) , are permutations of one another.

6 Sorting

TODO: Discuss sorting networks

TODO: Discuss permutation networks + comparisons to verify order

7 Random access

TODO: Discuss naive random access via index comparisons

TODO: Discuss binary tree method

8 Embedded curve operations

TODO: Discuss basic embedded curve operations

References

- [1] B. Parno, J. Howell, C. Gentry, and M. Raykova, “Pinocchio: Nearly practical verifiable computation,” in *2013 IEEE Symposium on Security and Privacy*, pp. 238–252, IEEE, 2013.