R1CS Programming

ZK0x04 Workshop Notes

Daniel Lubarov Brendan Farmer

October 18, 2019

Contents

1 Multiplicative inverse

Deterministically computing $1/x$ in an R1CS circuit would be expensive. Instead, we can have the prover compute $1/x$ outside of the circuit and supply the result as a witness element, which we will call x_{inv}. To verify the result, we enforce

$$(x)(x_{\mathrm{inv}}) = (1) \tag{1}$$

2 Zero testing

To assert $x = 0$, we simply enforce

$$(x)(1) = (0) \tag{2}$$

Asserting $x \neq 0$ is similarly easy: we compute $1/x$ (non-deterministically, as in Section 1). The result can be ignored; the mere fact that an inverse exists implies $x \neq 0$.

On the other hand, if we want to *evaluate*

$$y \coloneqq \begin{cases} 0 & \text{if } x = 0, \\ 1 & \text{otherwise,} \end{cases} \tag{3}$$

we can do so by introducing another variable, m, and enforcing

$$(x)(m) = (y), \tag{4}$$
$$(1 - y)(x) \ = (0). \tag{5}$$

Outside of the circuit, the prover generates y as in Equation 3, and generates m as

$$m \coloneqq \begin{cases} 1 & \text{if } x = 0, \\ y/x & \text{otherwise.} \end{cases} \tag{6}$$

This method is from [1].

3 Binary

To assert $b \in \{0, 1\}$, we enforce

$$(b)(b - 1) = (0). \tag{7}$$

To convert a field element x to its binary encoding, $(b_1, \ldots, b_n)$, we have the prover generate the binary encoding out-of-band. We then verify it by applying Equation 7 to each b_i, and enforcing

$$(x)(1) = \left(\sum_{i=0}^{n-1} 2^i b_i \right), \tag{8}$$

assuming a little-ending ordering of the bits.

Note that Equation 8 permits two encodings of certain field elements. In $\mathbb{F}_{13}$ for example, the element 1 can be represented as either 0001 or 1110. If a canonical encoding is required, we can prevent "overflowing" encodings by asserting that $(b_1, \ldots, b_n) < |F|$. Such binary comparisons are covered in Section 9.

4 Selection

Suppose we have a boolean value s, and we wish to compute

$$z \coloneqq \begin{cases} x & \text{if } s = 0, \\ y & \text{if } s = 1. \end{cases} \tag{9}$$

We can compute this as

$$z \coloneqq x + s(y - x). \tag{10}$$

This requires two constraints: one "is boolean" asertion (Equation 7), assuming s was not already known to be boolean, and another for the multiplication.

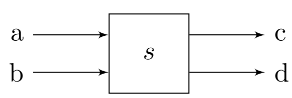
5 Random access

TODO: Discuss naive random access via index comparisons

TODO: Discuss binary tree method

6 2x2 switch

Suppose we wish to implement a switch with the following structure:



In particular, if $s = 0$ then the outputs should be identical the inputs: $(c, d) = (a, b)$. If $s = 1$ then the inputs should be swapped: $(c, d) = (b, a)$.

This requires two constraints: one "is boolean" assertion (Equation 7), and another for selecting the value of c (Equation 10). Once we have c, we can compute d "for free" as

$$d \coloneqq a + b - c. \tag{11}$$

7 Permutations

Say we want to verify that two sequences, $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$, are permutations of one another. This can be done efficiently using routing networks, which used a fixed (for a fixed n) network of 2x2 switches.

AS-Waksman networks [2] are a particularly useful construction, since they support arbitrary permutation sizes. They use about $n \log_2(n) - n$ switches, which is close to the theoretical lower bound of $\log_2(n!)$.

8 Sorting

Like permutation networks, sorting networks use a fixed network of gates. In particular, a sorting network is comprised of several 2x2 comparator gates, each of which takes two inputs and sorts them. It is theoretically possible to construct a sorting network for n elements using $\mathcal{O}(n \log n)$ gates [3], but practical constructions use $\mathcal{O}(n \log^2 n)$ gates.

TODO: Discuss permutation networks + comparisons to verify order

9 Comparisons

TODO: Describe basic comparison algorithm

TODO: Describe Ahmed's optimization

A couple other optimizations are possible in particular circumstances:

1. To assert (not evaluate) $x < y$, we can split x non-canonically and split y canonically. The prover is forced to use x's canonical representation anyway, otherwise $x_{\text{bin}} \geq |F| > y_{\text{bin}}$, making the assertion unsatisfiable.

2. To assert $x < c$ for some constant $c \ll |F|$, we can split x into just $\lceil \log_2 c \rceil$ bits.

10 Embedded curve operations

TODO: Discuss basic embedded curve operations

References

[1] B. Parno, J. Howell, C. Gentry, and M. Raykova, "Pinocchio: Nearly practical verifiable computation," in *2013 IEEE Symposium on Security and Privacy*, pp. 238–252, IEEE, 2013.

[2] B. Beauquier and E. Darrot, "On arbitrary size waksman networks and their vulnerability," *Parallel Processing Letters*, vol. 12, no. 03n04, pp. 287–296, 2002.

[3] M. Ajtai, J. Komlós, and E. Szemerédi, "An 0 (n log n) sorting network," in *Proceedings of the fifteenth annual ACM symposium on Theory of computing*, pp. 1–9, ACM, 1983.