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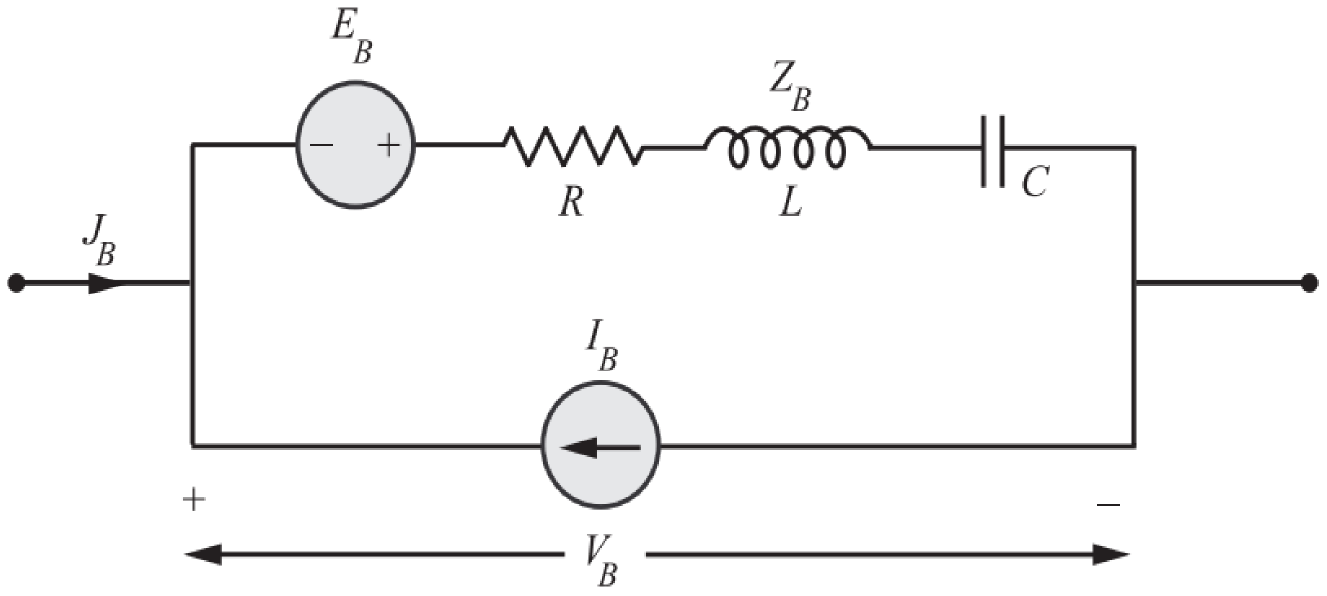
LECTURE 3

Network Equilibrium Equations

- Loop equations: A branch of a network can, in general, be represented as

shown in Figure



where $\mathbf{E_B}$ is the voltage source of the branch.
$\mathbf{I_B}$ is the current source of the branch
$\mathbf{Z_B}$ is the impedance of the branch
$\mathbf{J_B}$ is the current in the branch

Network Equilibrium Equations

The voltage-current relation is then given by

$$V_B = (J_B + I_B)\, Z_B - E_B$$

For a general network with many branches, the matrix equation is

$$\mathbf{V_B = Z_B\,(J_B + I_B) - E_B} \qquad \Rightarrow \qquad \mathbf{V_B = Z_B\,(I_B + I_s) - V_S} \quad \mathbf{(1)}$$

Where $\mathbf{V_B}$, $\mathbf{J_B}$, $\mathbf{I_B}$, and $\mathbf{E_B}$ are $B \times 1$ vectors and $\mathbf{Z_B}$ is the branch impedance matrix of $B \times B$

Network Equilibrium Equations

Each row of the tie-set matrix corresponds to a loop and involves all the branches of the loop. As per KV L, the sum of the corresponding branch voltages may be equated to zero. That is

$$\mathbf{B}\mathbf{V}_{\mathbf{B}} = \mathbf{0} \qquad \textbf{(2)}$$

where **B** is the tie-set matrix.

In the same matrix, each column represents a branch current in terms of loop currents. Trans- posed M is used to give the relation between branch currents and loop currents.

$$\mathbf{J}_{\mathbf{B}} = \mathbf{B}^{\mathbf{T}}\mathbf{I}_{\mathbf{L}} \qquad \textbf{(3)}$$

This equation is called loop transformation equation.

Network Equilibrium Equations

Substituting equation (1) in (2),

$$\mathbf{V_B = Z_B\ (J_B + I_B) - E_B} \qquad \mathbf{(1)}$$

$$\mathbf{BV_B = 0} \qquad \mathbf{(2)}$$

we get

$$\mathbf{BZ_B\ \{J_B + I_B\} - BE_B = 0} \qquad \mathbf{(4)}$$

$$\mathbf{J_B = B^T I_L} \qquad \mathbf{(3)}$$

Substituting equation (3) in (4)

we get

$$\mathbf{BZ_B\, B^T I_L + BZ_B\, I_B - BE_B = 0}$$

$$\mathbf{BZ_B\, B^T I_L = BE_B - BZ_B\, I_B} \Rightarrow \mathbf{BZ_B\, B^T I_L = BV_S - BZ_B\, I_S}$$

Network Equilibrium Equations

Get V_B branch voltage and J_B branch current using cut-set matrix

Each row of the cut-set matrix corresponds to a particular node pair

voltage and indicates different branches connected to a particular node. KCL can be applied to the node and the algebraic sum of the branch currents at that node is zero.

$$\mathbf{C J_B = 0} \qquad (5)$$

Each column of cut-set matrix relates a branch voltage to node pair voltages. Hence,

$$\mathbf{V_B = C^T E_N} \qquad (6)$$

Network Equilibrium Equations

Current voltage relation for a branch is.

$$J_B = Y_B \ (V_B + E_B) - I_B$$

For a network with many branches the above equation may be written in matrix form as,

$$\mathbf{J_B = Y_B V_B + Y_B E_B - I_B} \qquad \mathbf{(7)}$$

where Y_B is branch admittance matrix of B × B.

Network Equilibrium Equations

Substituting equation (7) in (5)

$$CJ_B = 0 \qquad (5)$$

$$J_B = Y_B V_B + Y_B E_B - I_B \qquad (7)$$

We get

$$CY_B V_B + CY_B E_B - CI_B = 0 \qquad (8)$$

Substituting equation (6) in (8)

$$V_B = C^T E_N \Rightarrow V_B = C^T V_T \qquad (6)$$

We get

$$CY_B C^T E_N = C(I_B - Y_B E_B) \Rightarrow CY_B C^T V_T = C I_S - C Y_B V_S)$$

Example

For the network shown in Figure write a tie-set matrix and then find all the branch currents and voltages.