UC
14.02.20

B.A./B.Sc. 3rd Semester (General) Examination, 2019 (CBCS)

Subject : Mathematics (General/Generic)

Paper : BMG3CC1C/MATH-GE3

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Group-A

1. Answer *any ten* questions from the following: 2×10=20

(a) Find the number of elements of the set $\{\omega^n : n \in \mathbb{N}\}$ where ω is a complex cube root of unity.

(b) Find supremum and infimum of the set $\{(-1)^n/n : n \in \mathbb{N}\}$.

(c) Prove that $\mathbb{Z}$ is countable.

(d) State Archimedean property of $\mathbb{R}$.

(e) Verify Bolzano-Weierstrass therorem for the set $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$.

(f) Show that the sequence $\left\{\frac{3n+1}{n+1}\right\}$ is bounded.

(g) State Cauchy's 1st theorem on limit.

(h) Show that the sequence $\left\{\frac{5n+3}{4n+1}\right\}$ is monotonically decreasing.

(i) State Cauchy's root test for a series of positive terms.

(j) Show that the series $\sum_{n=1}^{\infty} \frac{(2n+1)(3n-1)}{(3n+2)}$ is not convergent.

(k) Prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is convergent.

(l) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$.

(m) Show that $\sum_{n=1}^{\infty} \frac{\cos(nx)}{n^4}$ is uniformly convergent on $\mathbb{R}$.

(n) Define conditional convergence of a series.

(o) State Weierstrass's M-test for Uniform convergence.

Group-B

Answer *any four* questions. 5×4=20

2. (a) (i) Define supremum of a non-empty bounded above subset of $\mathbb{R}$.

(ii) Find supremum and infimum of the set $\{x \in \mathbb{R} : x^2 - 3x + 2 < 0\}$. 2+3=5

(b) Is the series $\frac{1+2}{2^3}+\frac{1+2+3}{3^3}+\frac{1+2+3+4}{4^3}+\cdots$ convergent? Support your answer. 5

(c) Test the convergence of the series $x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots$, for $x>0$. 5

(d) Using Cauchy's first theorem prove the followings: 2+3=5

(i) $\lim_{n\to\infty}\frac{1}{n}\left(1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+\cdots+n^{\frac{1}{n}}\right)=1$

(ii) $\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n^2+1}}+\frac{1}{\sqrt{2n^2+2}}+\cdots+\frac{1}{\sqrt{2n^2+n}}\right]=1$

(e) State Leibnitz's theorem for alternating series. Use it, to show that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\log(n+1)}$ is convergent. 2+3=5

(f) Show that the sequence $\{f_n\}$, where $f_n(x)=nxe^{-nx^2}$, is convergent pointwise, but not uniformly in $[0,k], k>0$. 5

Group-C

Answer *any two* questions. 10×2=20

3. (a) (i) Examine for convergence the infinite series $\sum\frac{1}{n^2+a^2}, a>0$.

(ii) Prove that every convergent sequence is bounded. Is the converse true? Support your answer. 2+(4+4)=10

(b) (i) Prove that the sequence $\left\{\frac{1}{n}\right\}_{n\in\mathbb{N}}$, is a Cauchy sequence.

(ii) Prove that every convergent sequence is Cauchy sequence.

(iii) Prove that every monotonically increasing bounded sequence is convergent. 3+3+4=10

(c) (i) Examine the convergence of the following series by using ratio test:

(I) $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

(II) $1+\frac{3}{2!}+\frac{5}{3!}+\frac{7}{4!}+\cdots$

(ii) Prove that, if a series $\sum_{n=1}^{\infty}x_n$ is convergent, then $\lim_{n\to\infty}x_n=0$. Does the converse hold? Support your answer. (3+3)+4=10

(d) (i) Prove that every subset of a countable set is countable.

(ii) Prove that the unit interval [0,1] is uncountable.

(iii) Prove that the set of all rational numbers is countable. 3+4+3=10