B.A. 6th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BAMATH6GE2 (Generic Elective)
(Geometry and Vector Calculus)

Time : 3 hours   Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any ten questions $10\times2 = 20$

(a) Find the centre of the hyperbola $4x^2-9y^2-16x+54y-101=0$ [2]

(b) Find the vertex of the parabola $3x^2-9x-5y-2=0$. [2]

(c) Show that the conic $x^2+y^2-2xy-x-y-1=0$ represents a parabola. [2]

(d) Find the eccentricity of the ellipse $9x^2+4y^2+18x-16y=11$. [2]

(e) Find the foci of the hyperbola $4x^2-9y^2=3$. [2]

(f) Find the equation of the sphere through the circle $x^2+y^2+z^2=9,\ x+y-2z=4$ and the origin. [2]

(g) Find the equation of the sphere whose centre is (2,3,1) and radius 4. [2]

(h) Give the definitions of a cylinder and guiding curve of a cylinder [1+1]

(i) If $\vec{\alpha}=t^2\vec{\imath}-t\vec{\jmath}+(2t+1)\vec{k}$ and $\vec{\beta}=(2t-3)\vec{\imath}+\vec{\jmath}-t\vec{k}$ then find $\frac{d}{dt}(\vec{\alpha}\times\vec{\beta})$. [2]

(j) If $\vec{r}=(x^2y-x^3)\vec{\imath}+(e^{xy}-y\cos x)\vec{\jmath}+(x^2cosy)\vec{k}$then find $\frac{\partial^2\vec{r}}{\partial x\partial y}$ [2]

(k) Find $grad\ f$ at $(1,1,-2)$ if $f=x^3-y^3+xz^2$. [2]

(l) Define $curl$ and $divergence$ of a vector point function (statingthe necessary property thatthe functionshould possess) [1+1]

(m) If $\vec{f}$=$x^2z\vec{\imath}-3y^3z^2\vec{\jmath}+xyz^2\vec{k}$, then find $div\vec{f}$. [2]

(n) If $\vec{v}=e^{xyz}(\vec{\imath}+\vec{\jmath}+\vec{k})$, find$curl\vec{v}$. [2]

(o) If $\vec{r}=t\vec{\imath}+t^2\vec{\jmath}+2t^3\vec{k}$, then find $\frac{d^2\vec{r}}{dt^2}\times\frac{d\vec{r}}{dt}$ [2]

2. Answer any four questions $4\times5 = 20$

(a) Find the equation of the parabola whose focus is $(3,-2)$ and directrix is the straight line $2x-y+3=0$. [5]

(b) Find the equation of a conic which passes through the point of intersection of the straight lines $x-3y-4=0$ and $x+y=0$ and the intersection of the conics, $x^2-3xy+y^2-6x-4y+5=0$ and $3x^2+7xy-3y^2-14x-2y+23=0$. [5]

(c) Find the equation of the sphere which passes through the points $(2,7,-4)$ and $(4,5,-1)$ and has centre on the line joining these two points as diameter. [5]

(d) Find the directional derivative of $\phi = xy^2z - 4xz^2$ at the point$(2,1,-1)$in the direction $(2\vec{i} - 2\vec{j} + \vec{k})$.. [5]

(e) Show that $curl\, \vec{u} = \vec{0}$, if $\vec{u} = (y^2 + z^3)\vec{i} + (2xy - 5z)\vec{j} + (3xz^2 - 5y)\vec{k}$ [5]

(f) If r is the distance of $P(x, y, z)$from the origin and $\vec{r}$ is the position vector of P relative to the origin then find$\nabla^2 \left(\frac{1}{r}\right)$. [5]

3. Answer any two questions 2×10 = 20

(a) (i) Prove that the necessary and sufficient condition for a vector $\vec{r}$=$\vec{f}(t)$to have a constant direction is $\vec{f} \times \frac{d\vec{f}}{dt} = \vec{0}$. [8]

(ii) If $\vec{\alpha} = 3t^2\vec{i} + t\vec{j} - t^3\vec{k}$ and $\vec{\beta} = sin\ t\ \vec{i} - 2cos\ t\ \vec{j}$ then find$\frac{d}{dt}(\vec{\alpha}.\vec{\beta})$. [2]

(b) Find $div\vec{F}$ and $curl\ \vec{F}$ where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.. [5+5]

(c) Reduce the equation $x^2 - 6xy + y^2 - 4x - 4y + 12 = 0$ to the normal form and then find the nature of the conic [10]

(d) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$. [10]