B.Sc. 6th Semester (Honours) Examination, 2025 (CBCS)

Subject : Physics

Course : CC-XIV

(Statistical Mechanics)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer *any five* of the following questions: 2×5=10

(a) What is the difference between classical and quantum statistics?

(b) State the Equipartition theorem.

(c) Once upon a time, Magician Mandrake went to a planet in which the temperature was 0K. What was the entropy of the system consisting?

(d) One 5-D harmonic oscillator has an energy eigenvalue $7/2\ \hbar\omega$. Comment on the degeneracy of the energy level.

(e) What fraction of free electrons in a metal at 0K has a kinetic energy less than half of the maximum energy?

(f) Three particles are distributed in five energy states. Calculate all possible ways of this distribution when particles are (i) Fermions (ii) Bosons and (iii) Classical particles.

(g) Does the number of photon inside an enclosure remain constant? Justify.

(h) Define 'chemical potential'.

Group-B

2. Answer *any two* of the following questions: 5×2=10

(a) Define 'ensemble' and 'ensemble average'. Deduce an expression for the entropy of a perfect gas using microcanonical ensemble. 2+3

(b) What is Gibb's Paradox? Point out how this paradox was resolved? 3+2

(c) Using Planck distribution law, find the heat capacity C_v and the entropy S of black-body radiation enclosed within a cavity of volume V, kept at a temperature T. 2½+2½

(d) Obtain an expression of the pressure (P) of an ideal Fermi gas at $T = 0$ in terms of the number density of fermions and the Fermi temperature T_f. Compare it with the pressure of an ideal classical gas at a finite temperature T. 4+1

Group-C

3. Answer *any two* of the following questions: 10×2=20

(a) (i) Derive an expression of the number of vibrational modes of a crystaline solid in the frequency range γ and $\gamma + d\gamma$. Hence obtain an expression for the Debye temperature. Explain the significance of the Debye cut off frequency. Derive an expression of the specific heat at a very low temperature and a high temperature. Interpret the result.

(ii) The Debye temperature of diamond is 2000 K. Find the mean velocity of sound in diamond. Given the density of diamond as 3500 kg/m^3. 8+2

(b) (i) Write down the Bose-Einstein (B-E) distribution function and obtain an expression for the total energy of a B-E system. Explain the phenomenon of B-E condensation and show graphically how the condensate fraction varies with temperature.

(ii) Show that B-E condensation is a first order phase transition. 8+2

(c) (i) Write down the Saha ionization equation and explain the signification of each term. What are the assumptions of this equation? What role does electron pressure play in the Saha equation?

A star's atmosphere has a temperature of T = 9000 K and electron density $n_e = 1 \times 10^{17}\text{cm}^{-3}$. Find the ratio of ionized hydrogen atoms using the Saha ionization equation.

(ii) State which statistics (M-B, F-D, B-E) would be appropriate for the following cases and why.

Case-1 : Density of He^4 gas at room temperature and atmospheric pressure.

Case-2 : Density of electrons in copper at room temperature. (2+1+2+3)+2

(d) (i) Write down the expression for the energy density of states of an electron gas in a metal. Hence find the Fermi energy of a metal T = 0K and an expression for the zero point pressure.

(ii) Find the Fermi energy at 0K for metallic silver containing one free electron per atom. Also estimate the zero point pressure. [Given density = 10·5 × 10^3 kg/m^3, atomic weight = 107·87].

(iii) What prevents a white dwarf from collapsing under gravity? (1+3+2)+3+1

Given : Ionization energy of hydrogen = 13·6 eV

Electron rest mass = 9·1 × 10^{-31} kg

Planck's constant = 6·63 × 10^{-34} JS

Boltzmann constant = 1·38 × 10^{-23} J/k

1eV = 1·6 × 10^{-19} J

Statistical weight : $g_p = 1$, $g_H = 2$

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