B.A./B.Sc. 6th Semester (Honours) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMH6DSE41

(Biomathematics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Define logistic growth model.

(b) What is intra-species competition? Explain with an example. 1+1

(c) What is a half-saturation constant in Michaelis Menten Kinetics?

(d) What are the assumption of the Nicholson Baily model?

(e) Define SIR epidemic model.

(f) Write a short note on Allee effect.

(g) Explain the concept of a continuous age structured model.

(h) Find the equilibrium points of $\dot{x} = x(2 - 3x)$.

(i) What is bifurcation of a dynamical system?

(j) Explain the self-crowding effect in the logistic growth model.

(k) What is the Jury stability condition? Explain with an example. 1+1

(l) What do you understand by insect out-break? Give example. 1+1

(m) Discuss Holling type growth with examples.

(n) Determine when the steady state of a differential equation is stable.

(o) Discuss a simple discrete prey-predator model.

2. Answer *any four* questions: 5×4=20

(a) Obtain the condition under which the following model have a positive interior equilibrium point.

$$\frac{dx}{dt} = x(1 - x - py)$$

$$\frac{dy}{dt} = y(1 - y - qx)$$

where p and q are positive constant.

(b) What is diffusion in mathematical model of an ecological system? Give an example of two species model with diffusion. 3+2

(c) Explain the concept of a single population harvesting model with logistic growth.

(d) Solve the following initial value problem of characteristics,

$$u_t + \vartheta u_x = 0, t \in (0, \infty), x \in (-\infty, \infty)$$

$$u(0, x) = \Phi(x), x \in (-\infty, \infty)$$

(e) Find the general solution to the following homogeneous difference equation $x_{t+3} + 5x_{t+2} - x_{t+1} - 5x_t = 0$

(f) Obtain the Routh-Hurwitz criteria for a cubic monic polynomial to have all real roots.

3. Answer *any two* questions: 10×2=20

(a) Consider the following Nicholson-Bailey model:

$$N_{t+1} = rN_t e^{-a P_t}$$

$$P_{t+1} = cN_t(1 - e^{-a P_t})$$

(i) Find the equilibrium points.

(ii) Prove that interior equilibrium point is unstable if $r > 1$. 3+7

(b) Consider the following system:

$$\frac{dx}{dt} = x\left(1 - \frac{x}{k}\right) - dxy$$

$$\frac{dy}{dt} = exy - py$$

where k, d, e and p are all positive constants. 5+5

(c) (i) Construct cobweb maps for $N_{t+1} = \frac{(1+p)Nt}{1+rNt}$ where 'r' is a positive constant. Discuss the global behaviour of the system.

(ii) Find the fixed point of the following system:

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right) - e\, N_t P_t$$

$$P_{t+1} = bN_t P_t + (1 - d)P_t$$

where r, k, e, b, d are all positive constants. 5+5

(d) State and derive the Malthusian growth model. Discuss the assumptions involved in the model. Solve the differential equation and analyse the behaviour of the solution. What are the advantages of this model in describing real biological populations? (2+2)+2+2+2

B.A./B.Sc. 6th Semester (Honours) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMH6DSE42

(Differential Geometry)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) State fundamental theorem of plane curves.

(b) Give an example of a surface whose each point is umbilic.

(c) When are two surfaces said to be equi-areal?

(d) Define diffeomorphism between two surfaces.

(e) Show that the surface of a unit sphere is a smooth surface.

(f) Show that any geodesic has constant speed.

(g) State Meusiner's theorem.

(h) What are the geodesics on a right circular cylinder?

(i) Define signed curvature of a unit speed plane curve.

(j) Show that if a curve on a surface has zero normal and geodesic curvature everywhere, then it is part of a straight line.

(k) Deduce the second fundamental form of a right circular cylinder.

(l) State Serret-Frenet formulae for a space curve.

(m) State a necessary and sufficient condition for unit speed curve with positive curvature and nowhere-vanishing torsion lies in the surface of a sphere.

(n) State third fundamental form of a surface.

(o) When is a point on a surface called hyperbolic point?

2. Answer *any four* questions: 5×4=20

(a) Deduce the torsion of a right circular helix.

(b) Prove that any tangent developable surface is isometric to a part of a plane.

(c) Give an example of a surface whose Gaussian curvature is zero but mean curvature is ½.

(d) Show that the torsion of a geodesic with nowhere vanishing curvature is equal to its geodesic torsion.

(e) Show that a curve on a surface is a line of curvature if and only if its geodesic torsion vanishes everywhere.

(f) Determine the asymptotic curves on the surface $\sigma(u, v) = (u\cos v, u\sin v, \log u)$.

3. Answer *any two* questions: 10×2=20

(a) Obtain a necessary and sufficient condition for a space curve to be a plane curve.

(b) State and prove Euler's theorem on a surface.

(c) Deduce the Gaussian curvature of a surface of revolution.

(d) Obtain a necessary and sufficient condition for a space curve to be a general helix.

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ASH-VI/MTMH/DSE-4/25

B.A./B.Sc. 6th Semester (Honours) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMH6DSE43

(Mechanics–II)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: $2\times10=20$

(a) Write down the dimension of Gravitational potential.

(b) If a parallelogram be immersed in a homogeneous liquid. Prove that the sum of the pressures at the extremities of each diagonal is the same.

(c) What is the degree of freedom of a particle moving along x-direction in three-dimension. Give explanation in support of your answer.

(d) What happens to the position of the centre of pressure if the plane area is lowered infinitely?

(e) Define 'force of buoyancy' and 'centre of buoyancy' when a body floating wholly immersed in a fluid.

(f) What is the relation between pressure and temperature in an adiabetic process?

(g) Write two properties of stress quadric.

(h) Write down the stress matrix when the fluid is at rest, where the symbols used are to be explained by you. 1+1

(i) A container of gas has a volume of 10 liters at 25°C and 1 atm. If the temperature is increased to 45°C and pressure is increased to 2 atm, what is the new volume of gas?

(j) Write down the pressure volume relation for a perfect gas in

(i) isothermal change of state

(ii) adiabatic change of state. 1+1

(k) Distinguish between inertial frame of reference and non-inertial frame of reference.

(l) Prove that the distance between two points remains invariant under Galilean transformation.

(m) Define Rheonomic constraints in a dynamical system with an example. 1+1

(n) Examine whether the forces (X,Y,Z) per unit mass of a fluid of (x,y,z) parallel of the axes are in equilibrium or not: $X = y(a-z), Y = x(a-z), Z = xy$.

(o) If $L(x,\dot{x},t) = \frac{1}{2}e^{at}(\dot{x}^2 - p^2x^2)$ where a and p are constants. Show that the Lagrange's equation is $\ddot{x} + a\dot{x} + p^2x = 0$

2. Answer *any four* questions: 5×4=20

(a) (i) Write down three limitations of Newton's laws in solving problems.

(ii) Define the concept of absolute time and absolute length. 3+(1+1)

(b) A particle of mass m is constrained to move on the plane xy=c under the action of gravity. Show that the Lagrange's equation of motion is of the form

$$m(x^5 + c^2x)\ddot{x} - 2mc^2\dot{x}^2 - mgcx^3 = 0$$

(c) A gas satisfying Boyle's law $p = k\rho$ is acted upon by forces $X = \frac{-y}{x^2+y^2}, Y = \frac{x}{x^2+y^2}$. Show that the density varies as $e^{\theta/k}$, where $\theta = \tan^{-1}\frac{y}{x}$.

(d) Find the centre of pressure of a triangular area, depth of whose vertices are given.

(e) In regard to the change in temperature with aspect to the height, reduce the relation $\frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma}, \frac{z}{H}$, where the symbols used are to be explained by you.

(f) A right circular cone has a plane base in the form of an ellipse; the cone floats in a fluid with its longest generator horizontal; if 2α be the vertical angle of the cone and β be the angle between the plane base and the shortest generating lines. Show that $\cot\beta = \frac{1}{5}\operatorname{cosec} 4\alpha - \cot 4\alpha$

3. Answer *any two* questions: 10×2=20

(a) (i) Derive a necessary and sufficient condition for equilibrium of a fluid under the action of a force whose components are X, Y, Z along the axes.

(ii) Show that the system of the forces per unit muss given by $\lambda y(a^2 + z^2)$, $-\lambda x(a^2 + z^2)$, $\mu(x^2 + y^2)$ where μ, λ, a are constants can keep a fluid be equilibrium. Determine the density and pressure of the fluid. 5+5

(b) (i) ABC is a triangular lamina with the site AB in the surface of a heavy homogeneous liquid. A point D is taken in AC, such that the thrusts on the areas ABD and DBC are equal. Then prove that $AD: AC = 1: \sqrt{2}$.

(ii) Find the conditions of equilibrium of a body floating in a liquid with one point of the body fixed. 5+5

(c) (i) Prove that for holonomic, conservative bilateral dynamical system, having n degrees of freedom, the Lagrange's equations of motion is

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_i}\right) - \frac{\delta L}{\delta q_i} = 0, \; i = 1, 2, \ldots, n.$$

where the symbols have their usual meanings.

(ii) A particle of mass m is constrained to move on the inner surface of a cone of semi-vertical angle α under gravity. Find the Lagrangian and the equation of motion.

5+(3+2)

(d) (i) Prove that the length of a rod remains invariant under Gallilean transformation.

(ii) The stress tensor at a point in a continuum is given by

$$(\tau_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

Determine the principal stresses and the corresponding principal directions. 7+3