B.Sc. 6th Semester (Honours) Examination, 2024 (CBCS)

Subject : Physics

Course : CC-XIV

(Statistical Mechanics)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer *any five* of the following questions: 2×5=10

(a) Differentiate between canonical and grand canonical ensembles.

(b) If the number of microstates of a system is doubled, by what amount does the entropy increase?

(c) Make a rough plot of the energy distribution at two different temperatures (T_1, T_2; $T_1 > T_2$) for a system of free particles governed by (i) M-B statistics and (ii) F-D statistics.

(d) Write down the expression for the specific heat of metals and sketch it as a function of the absolute temperature.

(e) Distinguish between ortho and para hydrogen.

(f) The molar mass of lithium is 0.00694 and its density 0.53×10^3 kg/m^3. Calculate the Fermi energy and Fermi temperature of the electrons.

(g) The spectral distribution from a celestial body shows a maximum at $\lambda = 500$ nm. What is the temperature of the body?

(h) Mention the physical conditions under which the Fermi-Dirac and the Bose-Einstein distributions reduce to the classical Maxwell-Boltzmann distribution law.

Group-B

2. Answer *any two* of the following questions: 5×2=10

(a) A system of four particles can occupy the energy levels with non degenerate energies 0, ϵ, 2ϵ and 3ϵ. If the total energy of the system is 3ϵ, find the number of microstates assuming that the particles are

(i) distinguishable

(ii) indistinguishable 2+3

(b) (i) Prove that for $N \to \infty$, $ln\ N! = N(lnN - 1)$

(ii) Explain the terms: (A) Phase space, (B) Ensemble, (C) Degeneracy 2+3

(c) (i) State and explain Maxwell-Boltzmann equipartition law of energy.

(ii) Determine the molar heat capacity of diatomic gases on the basis of equipartition theorem. 3+2

(d) What is Brownian motion? Derive Einstein's equation of translational Brownian motion. 1+4

Group-C

3. Answer *any two* of the following questions: 10×2=20

(a) (i) How is entropy defined in statistical mechanics? Use this to show that the difference in entropy between a state of volume V_i and a state of volume V_f (temperature and number of molecules remaining constant) is equal to $R\ ln \frac{V_f}{V_i}$. 1+3

(ii) What do you mean by partition function? In a canonical ensemble show that the relation between the partition function Z and the mean energy $\overline{\epsilon}$ is given by

$\overline{\epsilon} = -\frac{\partial}{\partial \beta}(ln\ Z)$, where $\beta = 1/kT$

Obtain an expression for $\overline{\epsilon^2}$ in terms of the derivatives of $ln\ Z$.

Also calculate $\overline{(\Delta\epsilon)^2} = \overline{\epsilon^2} - (\overline{\epsilon})^2$ 1+2+2+1

(b) Which of the three statistics is to be applied to explain thermionic emission? Deduce Richardson-Dushmann equation using the statistics you have suggested. 2+8

(c) (i) Define emissive power of a body. State Stefan-Boltzmann's law. 2

(ii) Derive Planck's radiation formula from Bose-Einstein statistics. Hence show that Wien's distribution law and Rayleigh-Jeans law can be obtained from Planck's law of black body radiation. 5+3

(d) (i) Write down an expression of the energy density of states of a non-interacting Fermi gas of electrons explaining each parameter. Sketch its variation with electronic energy. 2+1

(ii) Find an expression of the average energy per electron of such non-interacting Fermi gas in terms of Fermi energy (ϵ_f) at $T = 0$. Also calculate the pressure of the gas. 3+2

(iii) For sodium (*Na*) metal, there are approximately 2.6×10^{28} conduction electrons/m^3. Considering the metal as free electron gas, estimate the Fermi energy at $T = 0$.

$[m_e = 0.511\ \text{MeV}/c^2]$ 2

Given: Electron rest mass $= 9.1 \times 10^{-31}$ kg

Planck's constant $= 6.63 \times 10^{-34}$ JS

Boltzmann constant $= 1.38 \times 10^{-23}$ J/k

leV $= 1.6 \times 10^{-19}$ J