

**B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)****Subject : Physics****Course : DSE-1 (1)****(Advanced Mathematical Physics)****Time: 2 Hours****Full Marks: 40***The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.***Group-A****1. Answer any five questions:****2×5=10**

- (a) Determine whether the transformation  $T(x, y, z) = (x + 2y - 3z, x + y + z, 7x - y + 5z)$  is linear or not.
- (b) Is the set  $\{1, -1, i, -i\}$  a group under multiplication? Explain with the help of Closure, Associativity, Identity and Inverse property of the elements.
- (c) If  $\vec{\alpha}$ ,  $\vec{\beta}$  and  $\vec{\gamma}$  are linearly independent vectors, determine whether the vectors  $\vec{\alpha}$ ,  $\vec{\alpha} + \vec{\beta}$  and  $\vec{\alpha} + \vec{\beta} + \vec{\gamma}$  are linearly independent or not.
- (d) Assume that  $A$ ,  $(I + A)$  and  $(I + A^{-1})$  are all non-singular matrices. Show that —  

$$(I + A)^{-1} + (I + A^{-1})^{-1} = I.$$
- (e) Evaluate  $H^{20}$ , where  $H = \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix}$ .
- (f) Show that  $A_\mu B^\mu$  is invariant.
- (g) Given  $ds^2 = 7(dx^1)^2 + 5(dx^2)^2 + 3(dx^3)^2 - 4dx^1 dx^2 + 2dx^2 dx^3$ . Find the matrix  $g_{pq}$ .
- (h) Write down the expression for distance  $dS$  between two adjacent points  $x^i$  and  $x^i + dx^i$  in Riemannian space of  $N$  dimensions. What is the expression of  $dS$  in Euclidean space?

**Group-B****Answer any two questions:****5×2=10**

2. Find all the eigenvalues of  $n \times n$  matrix  $A = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \cdots & \cdots & \ddots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}_{n \times n}$ .

3. If  $U^i = A^i + B^i$ , where  $A^i$  and  $B^i$  are two orthogonal unit vectors, show that the square of the length of the vector  $U^i$  is 2. 5
4. (a) Show that in  $R^3$  (i)  $\delta_{ii} = 3$  and (ii)  $\delta_{ij}\epsilon_{ijk} = 0$ , where  $\epsilon_{ijk}$  is Levi-civita symbol. 2+3
- (b) Prove that  $A_{pq}x^p x^q = 0$  if  $A_{pq}$  is skew-symmetric matrix.
5. Let us consider the moment of inertia matrix as:

$$I = \begin{bmatrix} \frac{1}{3}Ma^2 & \frac{1}{4}Ma^2 & 0 \\ \frac{1}{4}Ma^2 & \frac{1}{3}Ma^2 & 0 \\ 0 & 0 & \frac{2}{3}Ma^2 \end{bmatrix}$$

Prove that the direction of principal axes are  $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ ,  $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$  and  $\hat{k}$  irrespective of values of  $M$  and  $a$ . 5

### Group-C

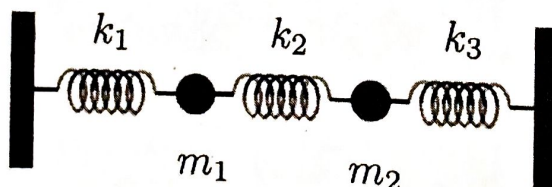
Answer any two questions:

10×2=20

6. If  $\hat{H}$  is a square matrix prove that  $\det(e^{\hat{H}}) = e^{Tr(\hat{H})}$ . Here  $\det$  and  $Tr$  are short forms of Determinant and Trace respectively. 10
7. The Euler-Lagrange's equations for the given spring mass system (See figure below) is given by,

$$m_1 \ddot{x}_1 = -(k_1 + k_2)x_1 + k_2 x_2$$

$$m_2 \ddot{x}_2 = k_2 x_1 - (k_2 + k_3)x_2$$



Here  $x_1$  and  $x_2$  are small displacement from equilibrium of the masses  $m_1$  and  $m_2$  respectively. Find the normal modes (i.e., eigenvectors of characteristics matrix) of vibration. Take spring constants  $k_1 = k_2 = k_3 = 1$  dyne/cm and masses  $m_1 = m_2 = 1$  gm. 10

8. (a) Use the antisymmetry of Levi-Civita symbol  $\epsilon_{ijk}$  to show that  $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$
- (b) If  $\Phi = a_{jk}A^j A^k$ , show that we can always write  $\Phi = b_{jk}A^j A^k$ , where  $b_{jk}$  is a symmetric tensor, whereas  $a_{jk}$  may or may not be symmetric. 5+5
9. (a) If  $g_{ij}$  denotes the components of a covariant tensor of rank two, show that the product  $g_{ij}x^i x^j$  is an invariant scalar.
- (b) If  $x^i$  and  $\bar{x}^i$  are independent coordinates of a point in two different coordinate system, show that—

$$\frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial \bar{x}^k}{\partial x^i} = \delta_i^j$$

5+5



( 3 ) SH-V/PHSH/DSE-1/25  
**B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)**

**Subject : Physics**

**Course : DSE-1 (2)**

**(Medical Physics)**

**Time: 2 Hours**

**Full Marks: 40**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**1. Answer any five questions:**

2×5=10

- (a) What is bremsstrahlung?
- (b) What is External Beam Radiotherapy?
- (c) Give examples of axial and appendicular skeletons in human body.
- (d) Define derived unit of equivalent dose of radiation in case of human body.
- (e) What is NMR imaging?
- (f) What is 'tachycardia'?
- (g) Define synovial joints in human body.
- (h) What is Respiratory cycle in case of human body?

**2. Answer any two questions:**

5×2=10

- (a) What is ultrasound imaging? The velocity of ultrasound in a transducer probe is 4000 m/s. If a vibration frequency of 5 MHz is desired, then find out the crystal thickness. 2+3
- (b) State the basic differences between X-ray imaging and MRI. Hence discuss the advantages of MRI. 3+2
- (c) What is Mechanical Ventilation? Describe PSV and SIMV in case of a mechanical ventilator. 1+2+2
- (d) Write short notes (on any two): 2½+2½
  - (i) Brachytherapy
  - (ii) Fluoroscopy
  - (iii) Electroencephalogram (EEG)

**3. Answer any two questions:**

10×2=20

- (a) Write down the main components of eye focussing system in case of human eye. Find out the focal lengths ( $f_{front}$  and  $f_{back}$ ) of cornea when,  $n_{ah} = n_{vh} = 1.33$ , Radius of cornea  $R = 7.5$  mm. What is corneal refractive power? 4+4+2

- (b) In case of human body obtain the expression of power (exerted) during a throw using right elbow motion. Calculate the average power generated during a throw, if the torque is  $36 \cdot 45$  N-m. The diameter of the muscle is  $7 \cdot 62$  c.m. 6+4
- (c) The mass of the pulmonary blood of a person is  $1 \cdot 5$  kg. Find the mass of this person (body mass). Also find the total mass of his systemic blood. Find the kinetic energy (KE) of 2 gm. of blood leaving aorta of radius  $1 \cdot 2$  c.m. 3+2+5
- (d) A hiker climbed a mountain of height 1000 m. He reached at the peak in 3 hr. Calculate the external work done by the climber if his weight is 70 kg. Assuming the work was done at a steady rate during the 3 hr. period, calculate the power generated during the climb. How much energy appeared as heat in his body? 4+3+3
-