3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2024 (CCFUP)

Subject : Mathematics

Course : MATH1051 (SEC)

(Graph Theory)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Distinguish between a trail and a path. Give an example of a trail which is not a path.

(b) When is a graph said to be a bipartite graph? Give an example.

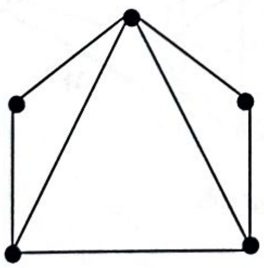
(c) Find the maximum and minimum number of edges of a simple graph with 14 vertices and 5 components.

(d) Let G be a connected 4-regular planar graph with 8 vertices. How many faces does G have?

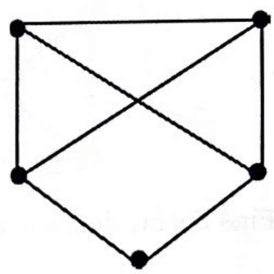
(e) Draw the graph whose adjacency matrix is given below:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(f) Is the following two graphs isomorphic? Justify your answer.



Graph-1



Graph-2

(g) How many spanning trees does the complete graph K_4 have?

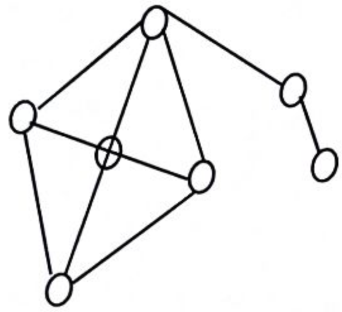
(h) Is there any graph with 6 edges and 5 vertices with degrees 1, 3, 3, 4, 5? Justify your answer.

2. Answer *any two* questions: 5×2=10

(a) State Hakimi-Hanel theorem regarding graphical vectors. Show that there exists a simple graph with 12 vertices and 28 edges such that the degree of each vertex is either 3 or 5. 1+4

(b) Prove that the degree of each vertex of a connected graph G will be even if it is an Eulerian graph. 5

(c) Define a planar graph and give an example. Find the faces and their sizes of the following graph:



Then check the validity of the Euler's formula. 2+2+1

(d) Define Isomorphism of two graphs. Are the graphs with adjacency matrix A_1 and A_2 isomorphic?

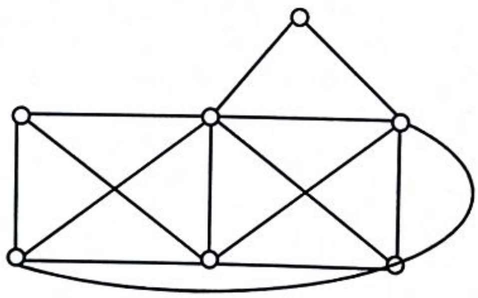
$$\underset{A_1}{\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}} ; \underset{A_2}{\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}}$$

2+3

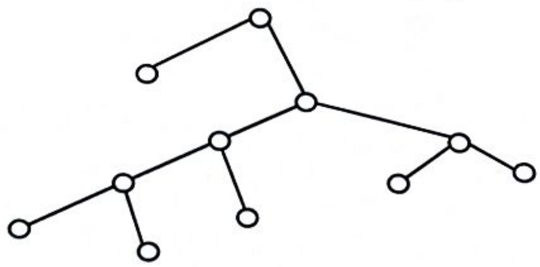
3. Answer *any two* questions: 10×2=20

(a) (i) Prove that in any tree with n vertices ($n \geq 2$), there are at least two pendent vertices.

(ii) Define a spanning tree of a connected graph. Find a spanning tree of the following graph:



(iii) Find the incidence matrix of the following graph:



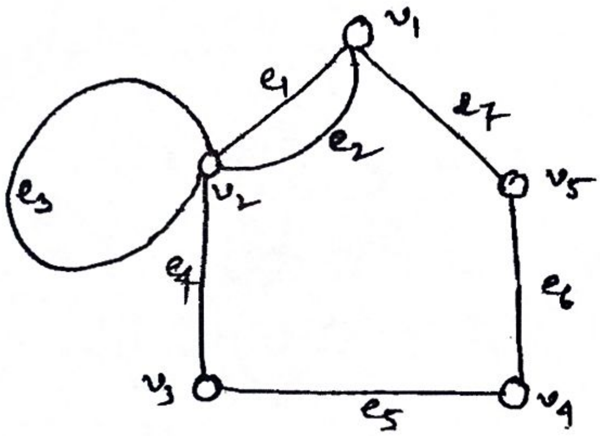
4+(1+2)+3

(b) (i) Apply Warshall algorithm to find the shortest distance matrix A_5 and the shortest path matrix P_5 of the undirected network with $V = \{1, 2, 3, 4, 5\}$ and $E = \{\{1, 2\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{4, 5\}\}$ with weights 1, 4, 5, 1, 2 and 1 respectively.

(ii) Prove that a simple graph with n vertices and k components can have at most $(n - k)(n - k + 1)/2$ edges. (3+3)+4

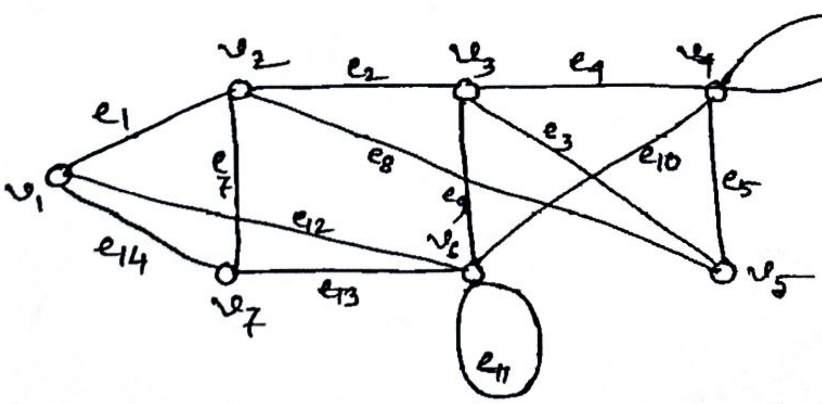
(c) (i) Show that there is a unique path between every pair of vertices in a tree.

(ii) Define incidence matrix of a graph. Find the incidence matrix of the following graph.



5+(2+3)

(d) (i) Define Hamiltonian cycle. In the following graph examine whether the Hamiltonian cycle exists or not. If exists, find the cycle.



(ii) Define chromatic number of a graph. Find the chromatic number of a graph with 7 vertices arranged in a cycle. (1+4)+(2+3)