

B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)**Subject : Physics****Course : DSE-1 (1)****(Advanced Mathematical Physics)****Time: 2 Hours****Full Marks: 40***The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words
as far as practicable.***Group-A****1. Answer any five questions:** $2 \times 5 = 10$

- (a) Determine whether the transformation $T(x, y, z) = (x + 2y - 3z, x + y + z, 7x - y + 5z)$ is linear or not.
- (b) Is the set $\{1, -1, i, -i\}$ a group under multiplication? Explain with the help of Closure, Associativity, Identity and Inverse property of the elements.
- (c) If $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$ are linearly independent vectors, determine whether the vectors $\vec{\alpha}$, $\vec{\alpha} + \vec{\beta}$ and $\vec{\alpha} + \vec{\beta} + \vec{\gamma}$ are linearly independent or not.
- (d) Assume that A , $(I + A)$ and $(I + A^{-1})$ are all non-singular matrices. Show that —

$$(I + A)^{-1} + (I + A^{-1})^{-1} = I.$$
- (e) Evaluate H^{20} , where $H = \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix}$.
- (f) Show that $A_\mu B^\mu$ is invariant.
- (g) Given $ds^2 = 7(dx^1)^2 + 5(dx^2)^2 + 3(dx^3)^2 - 4dx^1dx^2 + 2dx^2dx^3$. Find the matrix g_{pq} .
- (h) Write down the expression for distance dS between two adjacent points x^i and $x^i + dx^i$ in Riemannian space of N dimensions. What is the expression of dS in Euclidean space?

Group-B**Answer any two questions:** $5 \times 2 = 10$

2. Find all the eigenvalues of $n \times n$ matrix $A = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \cdots & \cdots & \ddots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}_{n \times n}$

3. If $U^i = A^i + B^i$, where A^i and B^i are two orthogonal unit vectors, show that the square of the length of the vector U^i is 2. 5

4. (a) Show that in R^3 (i) $\delta_{ii} = 3$ and (ii) $\delta_{ij}\epsilon_{ijk} = 0$, where ϵ_{ijk} is Levi-civita symbol.
 (b) Prove that $A_{pq}x^p x^q = 0$ if A_{pq} is skew-symmetric matrix. 2+3

5. Let us consider the moment of inertia matrix as:

$$I = \begin{bmatrix} \frac{1}{3}Ma^2 & \frac{1}{4}Ma^2 & 0 \\ \frac{1}{4}Ma^2 & \frac{1}{3}Ma^2 & 0 \\ 0 & 0 & \frac{2}{3}Ma^2 \end{bmatrix}$$

Prove that the direction of principal axes are $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$, $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$ and \hat{k} irrespective of values of M and a . 5

Group-C

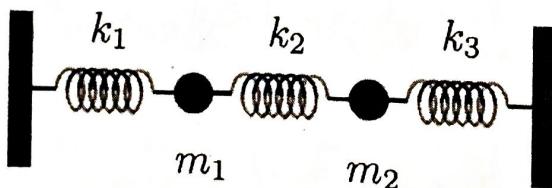
Answer any two questions: 10x2=20

6. If \hat{H} is a square matrix prove that $\det(e^{\hat{H}}) = e^{Tr(H)}$. Here \det and Tr are short forms of Determinant and Trace respectively. 10

7. The Euler-Lagrange's equations for the given spring mass system (See figure below) is given by,

$$m_1\ddot{x}_1 = -(k_1 + k_2)x_1 + k_2x_2$$

$$m_2\ddot{x}_2 = k_2x_1 - (k_2 + k_3)x_2$$



Here x_1 and x_2 are small displacement from equilibrium of the masses m_1 and m_2 respectively. Find the normal modes (i.e., eigenvectors of characteristics matrix) of vibration. Take spring constants $k_1 = k_2 = k_3 = 1$ dyne/cm and masses $m_1 = m_2 = 1$ gm. 10

8. (a) Use the antisymmetry of Levi-Civita symbol ϵ_{ijk} to show that $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$
 (b) If $\Phi = a_{jk}A^j A^k$, show that we can always write $\Phi = b_{jk}A^j A^k$, where b_{jk} is a symmetric tensor, whereas a_{jk} may or may not be symmetric. 5+5
9. (a) If g_{ij} denotes the components of a covariant tensor of rank two, show that the product $g_{ij}x^i x^j$ is an invariant scalar.
 (b) If x^i and \bar{x}^i are independent coordinates of a point in two different coordinate system, show that—

$$\frac{\partial x^i}{\partial \bar{x}^k} \frac{\partial \bar{x}^k}{\partial x^l} = \delta_i^l$$

5+5

Subject : Physics

Course : DSE-1 (2)
(Medical Physics)

Time: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

1. Answer any five questions:

2×5=10

- (a) What is bremsstrahlung?
- (b) What is External Beam Radiotherapy?
- (c) Give examples of axial and appendicular skeletons in human body.
- (d) Define derived unit of equivalent dose of radiation in case of human body.
- (e) What is NMR imaging?
- (f) What is 'tachycardia'?
- (g) Define synovial joints in human body.
- (h) What is Respiratory cycle in case of human body?

2. Answer any two questions:

5×2=10

- (a) What is ultrasound imaging? The velocity of ultrasound in a transducer probe is 4000 m/s. If a vibration frequency of 5 MHz is desired, then find out the crystal thickness. **2+3**
- (b) State the basic differences between X-ray imaging and MRI. Hence discuss the advantages of MRI. **3+2**
- (c) What is Mechanical Ventilation? Describe PSV and SIMV in case of a mechanical ventilator. **1+2+2**
- (d) Write short notes (on any two):
 - (i) Brachytherapy
 - (ii) Fluoroscopy
 - (iii) Electroencephalogram (EEG)**2½+2½**

3. Answer any two questions:

10×2=20

- (a) Write down the main components of eye focussing system in case of human eye. Find out the focal lengths (f_{front} and f_{back}) of cornea when, $n_{ah} = n_{vh} = 1.33$, Radius of cornea $R = 7.5$ mm. What is corneal refractive power? **4+4+2**

- (b) In case of human body obtain the expression of power (exerted) during a throw using right elbow motion. Calculate the average power generated during a throw, if the torque is $36 \cdot 45$ N-m. The diameter of the muscle is $7 \cdot 62$ c.m. 6+4
- (c) The mass of the pulmonary blood of a person is $1 \cdot 5$ kg. Find the mass of this person (body mass). Also find the total mass of his systemic blood. Find the kinetic energy (KE) of 2 gm. of blood leaving aorta of radius $1 \cdot 2$ c.m. 3+2+5
- (d) A hiker climbed a mountain of height 1000 m. He reached at the peak in 3 hr. Calculate the external work done by the climber if his weight is 70 kg. Assuming the work was done at a steady rate during the 3 hr. period, calculate the power generated during the climb. How much energy appeared as heat in his body? 4+3+3