B.A/B.Sc. 6th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMG6DSE1B1
(Numerical Methods)

Time:3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any ten questions $10\times2 = 20$

(a) Derive Newton-Raphson formula for finding $\sqrt{N}$ where N is a positive real number. [2]

(b) Define order of convergence. What is the order of convergence for Newton-Raphson method? [2]

(c) What is pivoting? [2]

(d) What are the disadvantages of fixed point iteration method in computing a real root of the equation $f(x) = 0$? [2]

(e) Construct a forward difference table from the following values of x and $y = f(x)$: [2]

x	0	5	10	15	20
$y = f(x)$	1.0	1.6	3.8	8.2	15.4

(f) Prove that $E = 1 + \Delta$ where E is shift operator and Δ is forward difference operator. [2]

(g) State Newton's formula for backward interpolation. [2]

(h) Evaluate: $\Delta^4(1-x)(1-2x)(1-3x)(1-4x)$ where the interval of difference being 1. [2]

(i) Define quadratic interpolation. [2]

(j) If $f(0) = 0, f(1) = 2, f(2) = 6, f(3) = 12, f(4) = 18$ then evaluate $\int_0^4 f(x)\,dx$ by Simpson's $\frac{1}{3}rd$ rule. [2]

(k) Given, $\frac{dy}{dx} = x^2 + y,\ y(0) = 1$; Obtain $y(0.02)$ using Euler's method (Taking step size $h = 0.02$). [2]

(l) Write down the condition for convergence of Gauss-Seidel iterative method. [2]

(m) Give geometrical interpretation of Trapezoidal rule for $\int_a^b f(x)dx$. [2]

(n) Write the fixed point iterative scheme for finding the root of $f(x) = 0$. [2]

(o) Write Lagrange interpolation formula for two equally spaced point (x_0, y_0) and (x_1, y_1). [2]

2. Answer any four questions $4\times5 = 20$

(a) Show that the error in approximating $g(x)$ by the interpolation polynomial using distinct interpolating points $x_0, x_1, x_2, \ldots, x_n$ is of the form [5]

$(x - x_0), (x - x_1)(x - x_2) \ldots (x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!}$.

(b) Find the missing term in the following table : [5]

x	0	1	2	3	4
$y = f(x)$	0	-	8	15	24

(c) Find $f(2.1)$ using following table: [5]

x	2	3	4	5
$y = f(x)$	4	5	7	10

(d) (i) Explain the Newton-Raphson's method for computing a simple real root of an equation $g(x) = 0$. [3]

(ii) Write down an algorithm for finding area of a rectangle. [2]

(e) Evaluate: $\int_0^1 x^3\, dx$ by Trapezoidal rule with five equal subintervals. [5]

(f) Show that the *n*-th order divided difference of a polynomial of degree *n* is constant. [5]

3. Answer any two questions $2\times10 = 20$

(a) (i) Solve the following system of equation by Gauss-Seidel iterative method: [5]

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

by taking four iteration only.

(ii) Establish Lagrange's interpolation and explain the difference from Newton's forward interpolation. [5]

(b) (i) The equation $x^3 - x - 0.1 = 0$ has a root in (1,2). Determine iterative function $\phi(x)$ such that the iterative formula $x_{n+1} = \phi(x_n)$ $n = 0,1,2 \ldots$ converges to the root. [5]

(ii) Prove that $\Delta^n \sin(a + bx) = \left(2\sin\frac{b}{2}\right)^n \sin\left(a + bx + \frac{n}{2}(b + \pi)\right)$ where 1 is the interval of differencing, $n \in \mathbb{N}$, Δ is forward difference operator. [5]

(c) (i) Determine a, b and c such that the formula [5]

$$\int_0^h f(x)\, dx = h\left[a\, f(0) + bf\left(\frac{h}{3}\right) + cf(h)\right]$$

is exact for polynomial of as high order as possible.

(ii) Find by Euler's method, the value of y for $x = 1.0$ from the differential equation $\frac{dy}{dx} = x + y$ with $y(0) = 0$ and taking step length 0.2. [5]

(d) (i) Find the values of $y(0.1)$ and $y(0.2)$ from the following differential equation by fourth order Runge-Kutta method: $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$. [2]

(ii) Show that Newton-Raphson method has quadratic rate of convergence for finding [8] a root of $f(x)=0$.

B.A/B.Sc. 6th Semester (General) Examination, 2022 (CBCS)

Subject: Mathematics

Course: BMG6DSE1B2

(Complex Analysis)

Time: 3 Hours — Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any ten questions: $10\times2 = 20$

(a) Define the continuity of the function of complex variable at a point. [2]

(b) Prove that $f(z)=|z|^2$ is continuous for all $z\in\mathbb{C}$. [2]

(c) Show that the function $u=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic and find its harmonic conjugate. [2]

(d) State Taylor's theorem. [2]

(e) Define power series. [2]

(f) Define conformal transformation. [2]

(g) If $f(z)=\sin(2z-\bar{z})$, then show that $f'(0)$ does not exist. [2]

(h) Show that the function $f(z)=z^2$ is analytic in a domain *D* of the complex plane $\mathbb{C}$. [2]

(i) Define entire function. [2]

(j) Let f be analytic in a region *G*. If $f'(z)$ on *G* then prove that *f* is constant. [2]

(k) Find the fixed points of the transformation $w=\frac{2z-5}{z+4}$. [2]

(l) If the complex number number $\frac{z-i}{z+i}$ is purely imaginary, then show that the points *z* lies on a circle with centre at origin and radius 1. [2]

(m) Find the radius of convergence of the power series $\sum\left(\frac{z^n}{n!}\right)$. [2]

(n) Justify with proper example that the continuity of a function of complex variable does not necessarily imply that the function is derivable, [2]

(o) Give an example of a continuous function which is not analytic. [2]

2. Answer any four questions: $4\times5 = 20$

(a) Obtain a necessary condition for a complex valued function to be analytic. [5]

(b) Let $f:\mathbb{C}\to\mathbb{C}$ bedefined by

$f(z) = \frac{|z|^2}{z}$,when $z \neq 0$ [5]

=0, when $z=0$.

Show that the function f satisfies the Cauchy-Riemann equations at $z=0$ but the derivative of f fails to exists thereat.

(c) Find the condition that the transformation $w = \frac{az+b}{cz+d}$ transform the unit circle in the w-plane into straight line in the z-plane. [5]

(d) Use the method of contour integration to find the value of $\int_0^{2\pi} \left(\frac{\cos 2\theta}{5+4\cos\theta} \right) d\theta$ [5]

(e) Prove that a real valued function of a complex variable either has derivative zero or the derivative does not exist. [5]

(f) Prove that the analytic function with constant modulus is constant. [5]

3. Answer any two questions: $2 \times 10 = 20$

(a) (i) If a function of complex variable f is differentiable at a given point, then give an example to support that $|f|$ may not be differentiable at the same point. [5]

(ii) Find the Taylor's series expansion of the function $f(z) = \frac{2z^3+1}{z^2+z}$, valid in the neighborhood of the point $z=i$. [5]

(b) (i) If a function $f(z)$ is analytic and one valued inside and on a simple closed contour C, then prove that $\int f(z)dz = 0$ over $\mathbb{C}$. [5]

(ii) Show that, if a power series has a non-zero radius of convergence then its sum is an analytic function within its circle of convergence. [5]

(c) (i) If a complex function f is differentiable and $|f|$ is constant in a rectangular region D, then prove that f is constant in D. [5]

(ii) Define analytic function in a domain D. A function f *is* differentiable for all z except $z=0$ and the real part of f is $\frac{x+y}{x^2+y^2}$. Find f. [2+3]

(d) (i) If $f(z) = \frac{z^2+5z+6}{z-2}$, does Cauchy's theorem apply [5]

(1) when the path of integration C is a circle of radius 3 with centre at origin.

(2) when C is a circle of radius 1 with centre at origin.

(ii) Show that the harmonic conjugates of a harmonic function is differ by a constant. [5]

B.A./B.Sc. 6th Semester (General) Examination, 2022 (CBCS)

Subject: Mathematics

Course: BMG6DSE1B3

(Linear Programming)

Time: 3 Hours — Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any ten questions: $10\times2= 20$

(a) Define feasible solution and optimal solution of an L. P. P. [2]

(b) Verify graphically the following problem has an unbounded solution : [2]

Maximize $Z= 3x_1 + 4x_2$

subject to $x_1 - 3x_2 \leq 3$, $x_2 - x_1 \leq 1$, $x_1 + x_2 \geq 4$ and $x_1, x_2 \geq 0$.

(c) Distinguish between extreme point and boundary point with suitable example. [2]

(d) Define convex set. Give an example of a convex set in which all boundary points are vertices. [2]

(e) Write all the characteristics for the standard form of an L. P. P. [2]

(f) Construct the dual of the following L. P. P. [2]

Maximize $Z = 4x_1 + 9x_2 + 2x_3$

subject to $2x_1 + 3x_2 + 2x_3 \leq 7$, $3x_1 - 2x_2 + 4x_3 = 5$ and $x_1, x_2, x_3 \geq 0$.

(g) Determine the convex hull of the points (0, 0), (0, 1), (1, 2), (1, 1) and (4, 0). [2]

(h) Obtain one basic feasible solution of the system of equations [2]

$x_1 + 4x_2 - x_3 = 5$, $2x_1 + 3x_2 + x_3 = 8$

(i) Does a basis contain a null vector ? Give reasons for your answer. [2]

(j) When artificial variables are used for solving an L. P. P. by simplex method? [2]

(k) Show that the dual of the dual of an L.P.P. is the primal itself. [2]

(l) State the fundamental theorem of duality. [2]

(m) Define separating and supporting hyperplanes. [2]

(n) Under what condition an L.P.P. will have unbounded solution ? [2]

(o) Prove that a hyperplane and a closed half space in E^n are unbounded closed convex sets. [2]

2. Answer any four questions: $4\times5 = 20$

(a) Solve the following L. P. P. by graphical method: [5]

Minimize $Z = x_1 + 2x_2$

subject to $5x_1 + 9x_2 \leq 45$, $x_1 + x_2 \geq 2$, $x_1 \leq 4$ and $x_1, x_2 \geq 0$.

(b) Food X contains 7 units of vitamin A and 5 units of vitamin B per gram and costs 20p/gm. Food Y contains 12 units and 15 units of A and B per gram respectively and [5] costs 50p/gm. The daily requirements of vitamin A and vitamin B are at least 200 units and 320 units respectively. Formulate this problem as an L.P.P to minimize the cost.

(c) $x_1 = 1,\ x_2 = 1,\ x_3 = 2$ is a feasible solution of the equations [5]

$x_1 + 2x_2 + 3x_3 = 9$

$2x_1 - x_2 + x_3 = 3$ and $x_1,\ x_2, x_3 \geq 0$

Reduce the feasible solution to a basic feasible solution of the above system of equations.

(d) Show that the set given by $X = \left\{(x_1,\ x_2) : 9x_1^2 + 16x_2^2 \leq 144\right\}$ is a convex set. [5]

(e) Show that if either the primal or dual problem has a finite optimal solution, then the other problem also has a finite optimal solution and the values of the two objective functions are equal. [5]

(f) Solve the following L.P.P. : [5]

Maximize $Z = 2x_1 + x_2 + x_3$

subject to $4x_1 + 6x_2 + 3x_3 \leq 8$, $3x_1 - 6x_2 - 4x_3 \leq 1$, $2x_1 + 3x_2 - 5x_3 \geq 4$

and $x_1, x_2, x_3 \geq 0$.

3. Answer any two questions: 2×10=20

(a) (i) Solve the following L.P.P. by using two-phase simplex method [7]

Minimize $Z = x_1 + x_2$

subject to $2x_1 + 4x_2 \geq 4$, $x_1 + 7x_2 \geq 7$ and $x_1,\ x_2 \geq 0$.

(ii) Show that the set of all feasible solutions to an L. P. P. is a closed convex set. [3]

(b) Obtain the dual of the following L.P.P. and hence solve it [2+8]

Maximize $Z = 3x_1 + 4x_2$

subject to $x_1 + x_2 \leq 10$, $2x_1 + 3x_2 \leq 18$, $x_1 \leq 8$, $x_2 \leq 6$ and $x_1, x_2 \geq 0$.

(c) (i) Use Big-M method to [8]

Minimize $Z = 2x_1 + 9x_2 + x_3$

subject to $x_1 + 4x_2 + 2x_3 \geq 5$, $3x_1 + x_2 + 2x_3 \geq 4$ and $x_1, x_2, x_3 \geq 0$.

(ii) State complementary slackness theorem of duality. [2]

(d) (i) Using simplex method, find the inverse of the following matrix [7]

$$A = \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$$

(ii) Show that the feasible solution $x_1 = 1$, $x_2 = 0$, $x_3 = 1$ and $x_4 = 6$ to the system [3]

$x_1 + x_2 + x_3 = 2$, $x_1 - x_2 + x_3 = 2$, $2x_1 + 3x_2 + 4x_3 = x_4$ is not basic.