B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)

Subject : Physics

Course : CC-XI

(Quantum Mechanics & Applications)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* of the following questions: 2×5=10

(a) Find out the spectroscopic notation for the ground state configuration of Al (Z = 13) and Scandium (Z = 21).

(b) Why do all the alkali atoms have a qualitatively similar spectra?

(c) What is 'stationary' in stationary states of time independent Schrödinger equation?

(d) State the Larmor's theorem of precession. Write down the Larmor frequency.

(e) A wave function is given by $\psi(x) = A_n \sin\frac{2\pi nx}{L}$ in the region $0 \le x \le L$. Find the value of A_n using normalisation condition.

(f) Find out the magnetic moment of Fe^{2+} ions.

(g) For a particle inside a box of finite potential well, at what position (x) the particle is most stable?

(h) Differentiate between the expectation value and the eigenvalue of an observable.

2. Answer *any two* of the following questions: 5×2=10

(a) (i) Show that the momentum operator $\widehat{P_x}$ is Hermitian.

(ii) The normalised wave function of the ground state of the hydrogen atom is given by $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$. Find the distance from the nucleus at which the electron is most likely to be found. 2+3

(b) Show that —

(i) $[\hat{L}_x^2, \hat{L}_y^2] = [\hat{L}_y^2, \hat{L}_z^2] = [\hat{L}_z^2, \hat{L}_x^2] = 0$

(ii) $[\hat{L}_x, r^2] = [\hat{L}_y, r^2] = [\hat{L}_z, r^2] = 0$ 2½+2½

(c) The normalised ground state wave function of a hydrogen atom is given by,
$\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{\frac{3}{2}}} e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from nucleus, located at the origin. Find out the expectation value $\langle \frac{1}{r^2} \rangle$.

(d) The wave function of a particle moving in free space is given by, $\psi = (e^{ikx} + 2e^{-ikx})$. What will be the energy of the particle?

3. Answer *any two* of the following questions: 10×2=20

(a) Determine the transmission co-efficient of a particle having energy $E < V_0$ for a rectangular potential barrier defined by,

$$V(x) = 0 \quad \text{for } x < 0 \text{ and } x > a,$$

$$V(x) = V_0 \quad \text{for } 0 < x < a.$$

Explain the tunnel effect. 7+3

(b) Derive Schrödinger's time dependent wave equation and prove that $\frac{d\rho}{dt} + \nabla.\vec{S} = 0$ where,

ρ = Probability density

$\vec{S}$ = Probability current. 5+5

(c) Sketch a neat diagram of the experimental set up of Stern-Gerlach experiment, explaining the components. Mention the outcome of this experiment. Explain the experimental result with necessary theory. 3+2+5

(d) The ground state wave function of a one dimensional harmonic oscillator (mass = m, angular frequency = ω) is given by,

$$\psi_0(x) = \left(\frac{\beta^2}{\pi}\right)^{\frac{1}{4}} \exp\left(\frac{-\beta^2 x^2}{2}\right), -\infty \leq x \leq \infty.$$

and $\beta = \sqrt{\frac{m\omega}{\hbar}}$. Prove that in this state $\Delta x \,.\, \Delta P = \frac{\hbar}{2}$.

$\left[\text{note: } \int_0^\infty u^2 e^{-u^2} du = \frac{\sqrt{\pi}}{4}\right]$. 10

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