# B.A./B.Sc. 5th Semester (General) Examination, 2024 (CBCS)

## Subject : Mathematics

## Course : BMG5DSE1A1

## (Matrices)

**Time : 3 Hours** **Full Marks : 60**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Notation and symbols have their usual meaning.*

**1.** Answer *any ten* questions: 2×10=20

(a) Show that the subset $\{(1,0,0),(0,1,0),(8,-1,0)\}$ of $\mathbb{R}^3$ is linearly dependent over $\mathbb{R}$.

(b) Show that the subset $B=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2=1\}$ is not a subspace of $\mathbb{R}^3$.

(c) Examine whether in $\mathbb{R}^3$, the vector (1, 0, 7) is in the span of $S=\{(0,-1,2),(1,2,3)\}$.

(d) Find the rank of the matrix $\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}$.

(e) What do you mean by eigenvector and eigenvalue of a square matrix?

(f) Find the characteristic equation of the matrix: $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(g) Find the eigenvalue corresponding to the eigenvector (2, 3) of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.

(h) Find the solution(s) of the linear equation: $x+2y=0$

(i) Find all real values of $\lambda$ for which the rank of the matrix $A=\begin{pmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{pmatrix}$ is less than 3.

(j) Determine $k$ so that the set $S$ is linearly dependent in $\mathbb{R}^3$, where

$$S=\{(k,1,1),(1,k,1),(1,1,k)\}$$

(k) Write down the matrix representing rotation of axes through 45° anti-clockwise about origin $O$.

(l) Does the set $S=\{(1,1,2),(1,2,5),(5,3,4)\}$ of vectors form a basis of $\mathbb{R}^3$? Justify your answer.

(m) Show that the matrix $A=\begin{pmatrix} 2 & 8 \\ 0 & 2 \end{pmatrix}$ is not diagonalizable.

(n) Let two linear transformations $T:\mathbb{R}^3\to\mathbb{R}^2$ be defined by $T(x,y,z)=(4x,3y,-2z)$ and $S:\mathbb{R}^2\to\mathbb{R}^2$ be defined by $S(x,y)=(-2x,y)$. Find *ST*.

(o) Let $A$ be an $n\times n$ matrix over a field *F*. Show that the matrix $A$ is non-singular, if all its eigenvalues are non-zero.

**2.** Answer *any four* questions: 5×4=20

(a) Find a basis and the dimension of the subspace $W$ of $\mathbb{R}^3$, where $W = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$.

(b) Find the normal form of the following matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{pmatrix}$$

Hence find its rank. 4+1

(c) For what real value of $k$ the following system of equations have non-trivial solutions? Find the non-trivial solution. 2+3

$$\begin{aligned} x + 2y + 3z &= kx \\ 2x + 3y + z &= kz \\ 3x + y + 2z &= ky \end{aligned}$$

(d) Find the eigenvalues and eigenvectors of the following matrix:

$$A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$ 5

(e) Diagonalize the following matrix:

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$ 5

(f) If $\lambda$ be a eigenvalue of a real skew symmetric matrix, then show that $\left|\frac{1-\lambda}{1+\lambda}\right| = 1$. 5

**3.** Answer *any two* questions: 10×2=20

(a) (i) Diagonalize the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

(ii) Apply elementary row operations on $A$ to obtain $A^{-1}$, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$$

5+5

(b) (i) Show that the intersection of two subspaces of a vector space $V$ over the field $F$ is a subspace of $V$.

(ii) Examine whether $S$ is a subspace of $\mathbb{R}^3$, where $S = \{(x, y, z) \in \mathbb{R}^3 : x + 2y - z = 0, 2x - y + z = 0\}$. 5+5

(c) (i) Solve, by Matrix inversion method, the following system of equations:

$$\begin{aligned} 2x - 3y + 4z &= -4 \\ x + z &= 0 \\ -y + 4z &= 2. \end{aligned}$$

(ii) Find the condition so that the system of equations

$x + y + z = 1,$

$x + 2y - z = b,$

$5x + 7y + az = b^2$ admits of

(I) only one solution,

(II) no solution,

(III) many solutions. 5+5

(d) (i) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then show that $A^n = A^{n-2} + A^2 - I_3$ for every integer $n \geq 3$.

(ii) Find $P$ such that $P^{-1} AP$ is a diagonal matrix, where $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$. 5+5

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# B.A./B.Sc. 5th Semester (General) Examination, 2024 (CBCS)

## Subject : Mathematics

## Course : BMG5DSE1A2

## (Mechanics)

**Time : 3 Hours** **Full Marks : 60**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

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**1.** Answer *any ten* questions: 2×10=20

(a) Define the terms (i) Coplanar forces (ii) Concurrent forces. 1+1

(b) Two forces of magnitudes $3p$, $2p$ respectively have resultant $R$. If the first force is doubled, the magnitude of the resultant is doubled. Find the angle between the forces.

(c) Define limiting friction.

(d) Masses of 2, 3, 4, 5 lb are placed at the four angular points of a square. Find the centre of gravity of the system.

(e) State two laws of statical friction.

(f) Define moment of a force about a line.

(g) Distinguish between conservative and non-conservative forces.

(h) Define potential energy of a body.

(i) The speed $v$ of a point moving along the $X$-axis is given by $v^2 = 16 - x^2$. Prove that the motion is simple harmonic.

(j) Deduce $P = mf$ from Newton's second law of motion, where $P$ is the external force acting on a body of mass $m$ produces acceleration $f$.

(k) A particle describes a curve $r = ae^{\theta}$ with constant angular velocity. Find the transverse acceleration of its motion.

(l) State the principle of conservation of energy.

(m) Calculate the work done by gravity on a mass of $m$ lb during the $t$ th second of its fall.

(n) If the path of a particle be a circle, find its radial and cross radial accelerations.

(o) If a man can throw a ball $H$ meters vertically upwards, show that the greatest horizontal distance he can throw it is $2H$.

**2.** Answer *any four* questions: 5×4=20

(a) The straight line $4x + 3y = 5$ meets the rectangular axes $OX$ and $OY$ at the point $A$ and point $B$ respectively. If forces $P$, $Q$, $R$ act along the lines $OB$, $OA$ and $AB$, find the magnitude of the resultant and the equation of the line of action.

(b) Two equal uniform roads $AC$, $CB$ are freely joined at $C$ and rest in a vertical plane with ends $A$ and $B$ in contact with a rough horizontal plane. If the equilibrium is limiting and the coefficient of friction is μ, show that $\text{Sin}(\angle ABC) = \frac{4\mu}{1+4\mu^2}$.

(c) Find the centre of gravity of the area included between the curve $y^2(2a - x) = x^3$ and its asymptote.

(d) Find the tangential and normal components of acceleration of a particle moving along a plane curve.

(e) A projectile is launched at an angle α from a cliff of height $H$ above sea level. If it falls into the sea at a distance $D$ from the base of the cliff, prove that its maximum height above the sea level is $H + \frac{D^2 \tan^2 \alpha}{4(H+D\tan\alpha)}$.

(f) One end of an elastic string of unstretched length $a$ is tied to a point on the top of a smooth table, and a particle attached to the other end can move freely on the table. If the path be nearly circular of radius $b$, show that its apsidal angle is approximately $\pi\sqrt{\left(\frac{b-a}{4b-3a}\right)}$.

**3.** Answer *any two* questions: 10×2=20

(a) (i) A uniform rod $AB$ is in equilibrium at an angle α with the horizontal, with its upper end $A$ resting against a smooth peg and its lower end $B$ attached to a light cord which is fastened to a point $C$ on the same level as $A$. If the cord is inclined to the horizontal at an angle β, then $\tan\beta = 2\tan\alpha + \cot\alpha$.

(ii) A solid hemisphere of weight $W$ rests in limiting equilibrium with its curved surface on a rough inclined plane and its plane face is kept horizontal by a weight $P$ attached to a point in the rim. Prove that the coefficient of friction is $P/\sqrt{W(2P + W)}$. 5+5

(b) (i) A particle is allowed to slide down a smooth inclined plane under gravity alone. Show that the sum of the kinetic and potential energies is always constant throughout its motion.

(ii) In a simple harmonic motion, the distances of a particle from the middle point of its path at three consecutive seconds are observed to be $x$, $y$, $z$. Show that the time of a complete oscillation is $2\pi/\cos^{-1}\left(\frac{x+z}{2y}\right)$. 5+5

(c) (i) A particle of mass $m$ units is set sliding down the smooth curve $s = c\tan\psi$ which is in a vertical plane from the point for which $y = h$. Find an expression for the normal pressure.

(ii) A heavy uniform chain of length $2l$, hangs over a small smooth fixed pulley, the length $(l + c)$ being an one edge and $(l - c)$ at the other. If the end of the shorter portion be held and then let go, show by the principle of energy, that the chain will slip off the pulley in time $\sqrt{l/g}\log\{(l + \sqrt{l^2 - c^2})/c\}$. 4+6

(d) (i) A particle describes the equiangular spiral $r = ae^{\theta}$ in such a manner that the radial acceleration is zero. Prove that the magnitude of the acceleration is proportional to $r$.

(ii) A particle of mass $m$ is projected into the air with velocity $u$ in a direction making an angle $\theta$ with the horizontal. Find its motion and the path described. 5+5

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# B.A./B.Sc. 5th Semester (General) Examination, 2024 (CBCS)

## Subject : Mathematics

## Course : BMG5DSE1A3

## (Linear Algebra)

**Time : 3 Hours** **Full Marks : 60**

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*Notation and symbols have their usual meaning.*

**1.** Answer *any ten* questions: 2×10=20

(a) Find the condition among $x, y, z$ such that the vector $(x, y, z)$ belongs to the space generated by $\alpha = (2,1,0), \beta = (1,-1,2), \gamma = (0,3,-4)$.

(b) Find the dual basis of the set $A = \{(1,2),(3,2)\}$ of $\mathbb{R}^2$.

(c) Use Cayley–Hamilton theorem to find $A^{-1}$, where $A = \begin{pmatrix} 2 & -7 \\ -1 & 3 \end{pmatrix}$.

(d) Prove that the product of the eigenvalues of a matrix $A$ is $|A|$.

(e) Find the eigen-vectors of the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(f) Find the values of $x$ such that the vectors $(1,2,1), (x,3,1)$ and $(2,x,0)$ are linearly dependent.

(g) Check whether the linear mapping $T(a+ib) = (a,b)$ is an isomorphism or not, where $T : \mathbb{C} \longrightarrow \mathbb{R}^2$.

(h) Prove that the eigenvalue of the idempotent matrix is either 1 or 0.

(i) If $T : \mathbb{R}^3 \to \mathbb{R}$ defined by $T(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$, then show that $T$ is not a linear transformation.

(j) Find the dimension of the subspace $S$ of $\mathbb{R}^3$ defined by $S = \{(x,y,z) \in \mathbb{R}^3 : 2x + y - z = 0\}$.

(k) Define null space of a linear transformation.

(l) What do you mean by dual space of a vector space $V$ over a field $F$?

(m) If $A$ is any square matrix, prove that the eigenvalues of $A$ and $A^t$ are the same.

(n) Is the union of two subspaces of a vector space $V$ a subspace of $V$? Justify your answer.

(o) Write down a basis for $\mathbb{C}$ over $\mathbb{R}$.

**2.** Answer *any four* questions: 5×4=20

(a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$. 1+4

(b) Show that $S = \{(x,y,z) \in \mathbb{R}^3 : x + y - z = 0 \text{ and } 2x - y + z = 0\}$ is a subspace of $\mathbb{R}^3$. Find the dimension of $S$. 2+3

(c) Determine the matrix of $T$, where $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x, y, z) = (x + 3y + 3z, 2x + y + 3z, 2x + 2y)$ relative to the ordered basis $\{(2,1,1), (1,2,1), (1,1,2)\}$ of $\mathbb{R}^3$. Is $T$ invertible? Justify your answer. 4+1

(d) Prove that two eigenvectors of a square matrix $A$ over a field $F$, corresponding to two distinct eigenvalues of $A$ are linearly independent.

(e) Let $V$ and $W$ be vector spaces over a field $F$. Let $T : V \to W$ be a linear mapping. Then prove that $T$ is injective if and only if $\text{Ker}\, T = \{\theta\}$.

(f) Define Kernel of a linear transformation. Prove that Kernel of a linear transformation $T : V \to W$ is a sub-space of $V$. 1+4

**3.** Answer *any two* questions: 10×2=20

(a) (i) Find the co-ordinates of an arbitrary vector $v = (a, b, c)$ relative to the basis

$$S = \{(1,1,0), (0,1,1), (1,3,3)\}.$$

(ii) Express $v = (3, 7, -8)$ in $\mathbb{R}^3$ as a linear combination of the vectors

$u_1 = (1,2,3), u_2 = (2,3,5), u_3 = (3,5,7)$. 5+5

(b) (i) Determine whether the vectors (1, 1, 2), (3, 4, 7), (5, 3, 1) form a basis of $\mathbb{R}^3$ or not.

(ii) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation given by $T(x, y, z) = (x + y + 3z, x + 2y + z)$. Find the dimension of Ker $T$ and Im $T$.

(iii) Let $T : \mathbb{R}^2 \to \mathbb{R}$, where $T(2,1) = 3$ and $T(1,0) = -2$. Find $T(x, y)$. 3+4+3

(c) (i) Prove that any linear transformation on a finite dimensional vector space onto itself is an isomorphism.

(ii) Show that the set of all matrices of order 2 × 2 over the field of real numbers is a vector space over the field of real numbers. 5+5

(d) (i) Find the eigenvalues and the corresponding eigenvectors of the following real matrix:

$$\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 3 & -1 & 3 \end{pmatrix}.$$

(ii) Prove that the eigenvalues of a real symmetric matrix are real. 5+5