B.A./B.Sc. 6th Semester (Honours) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMH6DSE41

(Biomathematics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Define equilibrium point of a dynamical system.

(b) Define Limit sets and attractor.

(c) What is the environmental carrying capacity in logistic growth model?

(d) What do you understand by bifurcation of the dynamics of a biological system?

(e) What do you mean by phase portrait?

(f) Define Lotka-Volterra model with Allee effect.

(g) What is chemostat?

(h) Find the fixed point of the difference equation $x_{n+1} = rx_n(1 - x_n)$.

(i) What do you mean by reaction diffusion equation? Give an example. 1+1

(j) Define chaos.

(k) Sketch the bifurcation diagram for the equation $\frac{dx}{dt} = \mu - x^2$.

(l) What are the major limitation of Malthus growth model?

(m) What do you mean by basic reproduction number?

(n) State Poincare-Bendixson Theorem.

(o) What do you understand by maximum sustainable yield in the Harvested model?

2. Answer *any four* questions: 5×4=20

(a) Show that every solution of the following system with positive initial conditions is periodic:

$$\frac{dx}{dt} = x(1 - y), \frac{dy}{dt} = \propto y(x - 1)$$

where $\propto$ is a positive constant.

(b) Show that $u_{n+1} = (2 - r)\, u_n$ converges to 0 for $1 < r < 3$.

(c) Define SIS epidemic model and define basic reproduction number from it. 2+3

(d) Find the interior equilibrium point of the following model:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{axy}{b + x}$$

$$\frac{dy}{dt} = \frac{cxy}{b + x} - d_1 y$$

where r, k, a, b, c, d_1 are all positive constants.

(e) Explain the concept of a simple discrete predator-prey model.

(f) Explain the concept of single population harvesting model.

3. Answer *any two* questions: 10×2=20

(a) Reduce the following two species diffusion model into a linearised system around any specially-uniform steady state: (2+4)+4

$$\frac{\partial u}{\partial t} = f_1(u, v) + d_1 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = f_2(u, v) + d_2 \frac{\partial^2 v}{\partial x^2}$$

Finally state the conditions for diffusive instability of the system. 6+4

(b) Consider the following Nicholson-Bailey model:

$$N_{t+1} = rN_t e^{-a P_t}$$

$$P_{t+1} = cN_t(1 - e^{-a P_t})$$

(i) Find the interior equilibrium point.

(ii) Prove that interior equilibrium point is unstable if $r > 1$. 3+7

(c) (i) Explain the nature of the following model:

$$\frac{dN_1}{dt} = r_1 N_1 \left[1 - \frac{N_1}{K_1} - b_{12} \frac{N_2}{K_1}\right]$$

$$\frac{dN_2}{dt} = r_2 N_2 \left[1 - \frac{N_2}{K_2} - b_{21} \frac{N_1}{K_2}\right]$$

where $r_1, K_1, r_2, K_2, b_{12}$ and b_{21} are all positive constants and have their usual meaning.

(ii) Find the fixed point of the following system:

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right) - e\, N_t P_t$$

$$P_{t+1} = bN_t P_t + (1 - d)P_t$$

where r, k, e, b, d are all positive constants. 5+5

(d) Consider the following system:

$$\frac{dx}{dt} = x(4 - x - y)$$

$$\frac{dy}{dt} = y(8 - 3x - y)$$

representing the change in densities of two competing species x and y. Find the corresponding equilibrium points. Determine the stability of each equilibrium and state their nature. 3+5+2

B.A./B.Sc. 6th Semester (Honours) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMH6DSE42

(Differential Geometry)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) When is a surface said to be orientable? Only state an example of a non-orientable surface. 1+1

(b) Show by an example that the parametrization of a plane curve is not unique.

(c) When are two surfaces said to be isometric?

(d) State fundamental theorem of a space curve.

(e) Show that a twisted cubic is a regular curve.

(f) Give an example of a developable surface.

(g) Give an example of a quadric which is not a surface.

(h) Define geodesic curvature of a curve on a surface.

(i) Define normal curvature of a curve on a surface.

(j) Write only the expression among three fundamental forms of a surface.

(k) Define minimal surface and give an example of it. 1+1

(l) When is a point on a surface said to be parabolic point?

(m) Prove that straight line segments are geodesics on Euclidean plane.

(n) When is a point on a surface said to be elliptic point?

(o) Prove that any normal section of a surface is a geodesic.

2. Answer *any four* questions: 5×4=20

(a) Deduce the curvature of a circular helix.

(b) Show that the curve $Y(t) = ((1+t^2)/t, t+1, (1-t)/t)$ is planar.

(c) Show that surface of an ellipsoid is a smooth surface.

(d) Obtain a necessary and sufficient condition for parametric curves on a surface to be orthogonal.

(e) Find out the principal curvatures of a right circular unit cylinder.

(f) Calculate the mean curvature of the surface $\sigma(u,v) = (u+v, u-v, uv)$ at the point (2, 0, 1).

3. Answer *any two* questions: 10×2=20

(a) Deduce Serret Frenet formulae for a space curve.

(b) If every point of a connected surface is an umbilic point, then prove that it is either a part of a plane or a part of a sphere.

(c) Show that the Gaussian curvature of a ruled surface is negative or zero.

(d) Using the geodesic equations, deduce the geodesics on the unit sphere.

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B.A./B.Sc. 6th Semester (Honours) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMH6DSE43

(Mechanics–II)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Distinguish between body force and surface force.

(b) Write down the dimensions in terms of fundamental units M, L and T of the following quantities: 1+1

(i) Tangential stress at a point

(ii) Angular momentum

(c) Define the normal and shearing stress at a point in a medium. Are they unique for a given element of surface? 1+1

(d) State the Archimedes' Principle.

(e) Show that the Cauchy stress quadric for a state of stress represented by

$\sigma_{ij} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ is an ellipsoid, when a, b, c are of same sign.

(f) Determine the number of degrees of freedom in the following cases:

(i) A particle moving on the circumference of a circle

(ii) A rigid body moving freely in three dimensional space

(g) Determine the type of constraint that emerges in the motion of a simple pendulum with rigid support.

(h) Determine the Lagrange's equation when Lagrangian has the form

$L = q_k\dot{q}_k - \sqrt{1-\dot{q}_k^2}$, q_k is the generalized co-ordinate.

(i) Write down the conditions of equilibrium of a body (a) floating in a liquid and (b) being supported by a string attached to some point on it.

(j) State the First law of Thermodynamics.

(k) Prove that the free surface of a homogeneous liquid at rest under gravity is horizontal.

(l) Does the velocity of light remain invariant under Galilean transformation? Justify your answer.

(m) Define scleronomic constraint. Give an example of it.

(n) Can a deformable body have a purely rigid body motion? Illustrate your answer.

(o) Show that the surface of separation of two liquids of different densities which do not mix, at rest under gravity, is a horizontal plane.

2. Answer *any four* questions: 5×4=20

(a) Prove that the pressure at a point in a fluid in equilibrium is the same in every direction.

(b) If a given volume of fluid is at rest under forces whose components per unit mass are $\lambda y(a-z), \lambda x(a-z)$ and μxy where λ and μ are constants. Show that the density must be proportional to $\frac{1}{xy(a-z)}$. Hence obtain the expression of pressure. 3+2

(c) A fine glass tube in the shape of an equilateral triangle is filled with equal volumes of three liquids which do not mix, whose densities are in arithmetical progression. The tube is held in a vertical plane and the side that contains portion of the heaviest and lightest liquids makes and angle θ with the veritcal. Show the surfaces of separation divide the side in the ratio $\cos\left(\frac{\pi}{6}-\theta\right) : \cos\left(\frac{\pi}{6}+\theta\right)$. 1+1+3

(d) If the law connecting the pressure p and density ρ of the air is $p = k\rho^2$, prove that neglecting variations of gravity and temperature, the height of the atmosphere would be $\frac{n}{n-1}$ times the homogenous atomosphere.

(e) Is it possible to obtain Newton's equation of motion from Lagrange's equation for a given dynamical system? Justify your answer.

(f) A particle of mass m moves in one dimension such that the Lagrangian is given by

$L(x,\dot{x}) = \frac{1}{2}m^2\dot{x}^4 + m\dot{x}^2v(x) - v^2(x)$,

where v is a differentiable function of x. Find the equation of motion for $x(t)$ and interpret it.

3. Answer *any two* questions: 10×2=20

(a) (i) Discuss Gibbs-Appell's principle of least constraint.

(ii) Show that kinetic energy is not an invariant under Galilean transformation but acceleration remains invariant under this transformation. 5+5

(b) (i) Deduce the differential equation of pressure.

(ii) A cone of given weight and volume floats in a given liquid with its vertex downwards; show that the surface of the cone in contact with the liquid is least when the vertical angle of the cone is $2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$. 5+5

(c) (i) A bead slides on a wire in the shape of a cycloid described by equations:

$x = a(\theta - \sin\theta)$

$y = a(1 + \cos\theta),\ 0 \leq \theta \leq 2\pi$

Find (A) the Lagrangian function,

(B) the equation of motion.

(ii) Write down the Lagrange's equation of motion for a particle of mass m falling freely under gravity near the surface of earth.

(3+3)+4

(d) (i) A uniform rod capable of turning about one of its ends, which is out of the water, rests inclined to the vertical with one third of its length in some water, prove that its specific gravity is $\frac{5}{9}$.

(ii) Prove that the temperature of the atmosphere diminishes inversely as the square of the distance from the centre of the earth.

5+5