B.A./B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMH5CC12

(Mechanics-I)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols bear usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) State Kepler's third law regarding motion of planet.

(b) A particle describes the parabola $p^2 = ar$ in pedal form (p, r) under a force which is always directed towards its focus. Find the law of force.

(c) The speed v of a point moving along the x-axis is given by $v^2 = 16 + x^2$. Prove that the motion is simple harmonic and find its amplitude.

(d) Prove that moment of inertia of a right circular cylinder of height h, radius a and mass M is $\frac{1}{2}Ma^2$.

(e) Find the centre of gravity of a circular arc making an angle 2α at the centre.

(f) State the principle of conservation of energy.

(g) Define impressed forces and effective forces with examples.

(h) State D'Alembert's Principle.

(i) For a system of coplanar forces acting on a rigid body, find the conditions of astatic equilibrium.

(j) State the energy test of stability.

(k) Prove that a central orbit is a plane curve.

(l) Define limiting friction.

(m) Define Wrench and Pitch. What is the relation between them?

(n) A particle is constrained to move along the inner surface of a fixed hemispherical bowl. What is the number of degrees of freedom of the particle?

(o) A uniform cubical box of edge a is placed on the top of a fixed sphere. Show that the least radius of the sphere for which the equilibrium will be stable is $\frac{a}{2}$.

2. Answer *any four* questions from the following: 5×4=20

(a) A force parallel to the axis of z acts at the point $(a, 0, 0)$ and an equal force perpendicular to the axis of z acts at the point $(-a, 0, 0)$. Show that the central axis of the system lies on the surface $z^2(x^2 + y^2) = (x^2 + y^2 - ax)^2$.

(b) Four equal rods each of weight W form a rhombus $ABCD$, with smooth hinges at the joints. The frame is suspended by the end A and a weight w' is attached at C. A stiffening rod of negligible weight joins the middle points of AB, AD keeping in these inclined at an angle α to AC. Show that the thrust in the stiffening rod is $(4W + 2W')\tan\alpha$.

(c) A plank of mass M, and length $2a$, is initially at rest along a line of greatest slope, of a smooth plane inclined at an angle α to the horizon, and a man of mass M', starting from the upper end walks down the plank so that it does not move. Show that he will reach the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$, where a is the length of the plank.

(d) A particle is moving in a straight line with an acceleration n^2x towards a fixed origin in the line, in a medium which offers a resistance proportional to the velocity and is simultaneously acted on by a periodic disturbing forces $F\cos pt$ per unit mass. Determine the motion.

(e) Show that the differential equation of the central orbit in pedal form is $\frac{h^2}{p^3}\frac{dp}{dr} = F$, where the symbols have their usual meaning.

(f) A bead moves along a rough curved wire which is such that it changes its direction of motion with constant angular velocity. Show that a possible form of the wire is equiangular spiral.

3. Answer *any two* questions from the following: 10×2=20

(a) (i) If A, B and C are the moments of inertia and D, E, F are the products of inertia of a rigid body with respect to rectangular axes OX, OY and OZ. Prove that the moment of inertia about a line through O having direction cosines (l, m, n) is given by $Al^2 + Bm^2 + Cn^2 - 2Dmn - 2Enl - 2Flm$.

(ii) If A, B and F are respectively the moments and products of inertia of a plane lamina about rectangular axes OX and OY in its plane. Show that the principal axes at O are inclined to OX at an angle α and $\frac{\pi}{2} + \alpha$ when $\tan 2\alpha = \frac{2F}{B-A}$.

(b) (i) A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination to the horizon is α. Show that the least co-efficient of friction between it and the plane so that it may roll and not slide, is $\frac{1}{3}\tan\alpha$.

(ii) A solid hemisphere rests on a plane inclined to the horizon at an angle $\alpha < \sin^{-1} 3/8$, and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable.

(c) (i) A particle is projected from an apse at a distance c with a velocity $\sqrt{\frac{2\mu}{3}}\, c^3$. If the force to the centre is $\mu(r^5 - c^4 r)$, then find the path. (r being the distance of the particle from the centre of force).

(ii) What is escape velocity?

A circular orbit of radius a is described under central attractive force $f(r) = \mu\left[\frac{b}{r^2} + \frac{c}{r^4},\right] \mu > 0$. Prove that the motion is stable if $a^2 b - c > 0$. 5+(1+4)

(d) (i) Show that the moment of momentum of a rigid body of mass M about a fixed point O, moving in two dimensions is equal to $Mvp + MK^2 \frac{d\theta}{dt}$, where the symbols have their usual meanings.

(ii) A homogeneous sphere of radius a, rotating with angular velocity ω about a horizontal diameter is placed on a plane table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $\frac{2\omega a}{7\mu g}$ and the sphere will roll with angular velocity $\frac{2\omega}{7}$. [g being the acceleration due to the gravity]. 5+5

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