B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE11

(Linear Programming)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Define basic feasible solution of an LPP.

(b) What do you mean by convex hull and convex polyhedron?

(c) Is the following set A in R^2 is convex? Justify with your reason. 1+1
$A = \{(x, y): x > 0, y > 0 \text{ and } xy \leq 1\}$

(d) Find the extreme points of the set $A = \{(x, y): |x| \leq 2, |y| \leq 2\}$.

(e) What do you mean by alternative optima of an LPP?

(f) Find a basic feasible solution of the following system of equations:

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

(g) Test whether the following set of vectors are linearly dependent or not.
$\{(3, 0, 2), (7, 0, 9), (4, 1, 2)\}$

(h) Find the condition under which the following game problem will be a fair game.
$\begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$ where a, b, c, d are all ≥ 0.

(i) Determine the value of θ so that the game with following payoff matrix is strictly determinable.

	Player B		
Player A	θ	6	2
	− 1	θ	− 7
	−2	4	θ

(j) Give an example of symmetric game and find its value.

(k) Prove that the solution of a transportation problem with 2 origins and 3 destinations is bounded. 1+1

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(l) Find the dual of the primal problem given by
Minimize $Z = -6x_1 - 8x_2 + 10x_3$
subject to
$x_1 + x_2 - x_3 \geq 2,$
$2x_1 - x_3 \geq 1,$
$x_1, x_2, x_3 \geq 0.$

(m) State complementary slackness theorem.

(n) "All boundary points are not necessarily extreme points."— Justify this statement with example.

(o) Prove that if a linear programming problem has two feasible solutions, then it has an infinite number of feasible solution.

2. Answer *any four* questions: 5×4=20

(a) Use Simplex method to obtain inverse of the matrix $\begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}$.

(b) Solve the following linear programming problem:
Maximize $Z = 3x_1 + 4x_2$
Subject to $x_1 - x_2 \geq 0,$
$-x_1 + 3x_2 \leq 3$
and $x_1, x_2 \geq 0,$

(c) Use Dual simplex method to solve the LPP:
Minimize $Z = 10x_1 + 6x_2 + 2x_3$
subject to $-x_1 + x_2 + x_3 \geq 1,$
$3x_1 + x_2 - x_3 \geq 2$
and $x_1, x_2, x_3 \geq 0.$

(d) Use dominance property to reduce the following payoff matrix to 2×2 matrix and hence solve the problem:

		Player A					
		A_1	A_2	A_3	A_4	A_5	A_6
Player B	B_1	4	2	0	2	1	1
	B_2	4	3	1	3	2	2
	B_3	4	3	7	−5	1	2
	B_4	4	3	4	−1	2	2
	B_5	4	3	3	−2	2	2

(e) Prove that any points of a convex polyhedorn can be expressed as a convex combination of its extreme points.

(f) Prove that the number of basic variables in a transportation problem with 2 origins and 3 destinations is at most 4.

3. Answer *any two* questions: 10×2=20

(a) (i) Use Vogel's Approximation Method to find the initial B.F.S. of the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	10	

(ii) Solve graphically the game whose payoff matrix is given below: 5+5

		Player B	
		B_1	B_2
Player A	A_1	2	7
	A_2	3	5
	A_3	11	2

(b) (i) Prove that the set of optimal strategies for each player in an $m \times n$ matrix game is a convex set.

(ii) Solve the travelling salesman problem: 5+5

		To				
		A	B	C	D	E
From	A	∞	6	12	6	4
	B	6	∞	10	5	4
	C	8	7	∞	11	3
	D	5	4	11	∞	5
	E	5	2	7	8	∞

(c) (i) Find the maximum value of $Z = 6x + 8y$.
subject to $5x + 2y \leq 20$
$x + 2y \geq 10$
$x, y \geq 0$
by solving its dual problem.

(ii) Solve the following assignment problem: 5+5

	A	B	C	D	E
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

(d) (i) Solve the following LPP by two phase method:

Maximize $z = 3x_1 - x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_1 \leq 4$

and $x_1, x_2 \geq 0$

(ii) Is

	D_1	D_2	D_3	D_4
O_1			50	20
O_2	55			
O_3	30	35		25

an optimal solution of the following transportation problem? 5+5

	D_1	D_2	D_3	D_4	a_i
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	90
b_j	85	35	50	45	

If not, modify it to obtain a better feasible solution.

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B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE12

(Number Theory)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: $2 \times 10 = 20$

(a) Show that Goldbach conjecture implies that every even integer greater than 5 is a sum of three primes.

(b) For any integers a, b, c prove that $a|b$ and $b|a$ iff $a = \pm b$.

(c) Prove that $(n^2 + 2)$ is not divisible by 4 for any integer n.

(d) Find the remainder when 3^{100} is divided by 5.

(e) State Fermat's Little Theorem.

(f) Show that $19^{20} \equiv 1 \pmod{181}$.

(g) Prove that if $8 \times 7 \equiv 2 \times 7 \pmod 6$ and $(7, 6) = 1$, then $8 \equiv 2 \pmod 6$.

(h) Solve $x^2 + 3x + 11 \equiv 0 \pmod{13}$.

(i) If p is prime, prove that $2(p-3)! + 1 \equiv 0 \pmod p$.

(j) Find the missing digit in the number 287*932 if it is divisible by 13.

(k) Prove that $2^n < n!$ for $n \in \mathbb{N}$ and $n \geq 4$.

(l) If $d_1, d_2, \ldots, d_r$ be the list of all positive divisors of a positive integer n, prove that $\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_r} = \frac{\sigma(n)}{n}$.

(m) Solve the linear congruence: $28x \equiv 63 \pmod{105}$.

(n) If $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_r^{\alpha_r}$ where $p_1, p_2, \ldots, p_r$ are prime to one another, find $\phi(n)$ ($\alpha_1, \alpha_2, \ldots, \alpha_r$ are positive integers).

(o) Prove that $2^n - 1$ has at least n distinct prime factors.

2. Answer *any four* questions: 5×4=20

(a) (i) Find $\sigma(360)$ and $\sigma(900)$.

(ii) Let $k > 1$ and $2^k - 1$ is a prime. If $n = 2^{k-1}(2^k - 1)$, then show that n is a perfect number. 2+3

(b) Prove that Möbius μ-function is a multiplicative function.

(c) State and prove Euclid's Theorem.

(d) Prove that $an \equiv bn \pmod{m}$ if and only if $a \equiv b \left(\bmod \frac{m}{(m,n)}\right)$, where a, b, m, n are integers.

(e) Show that $3^{4n+2} + 5^{2n+1} \equiv 0 \pmod{14}$.

(f) Find the primitive roots of 41.

3. Answer *any two* questions: 10×2=20

(a) (i) Prove that every integer $(n > 1)$ can be expressed as a product of finite number of primes.

(ii) Find the remainder when $2^{73} + 14^3$ is divided by 11. 8+2

(b) (i) Find the digit in unit place of 3^{400}.

(ii) State and prove Chinese Remainder Theorem. 2+8

(c) (i) Prove that $7 \mid (2222^{5555} + 5555^{2222})$.

(ii) Solve the linear Diophantine equation: $221x + 35y = 11$ 5+5

(d) (i) Find the least natural number which when divided by 7, 10 and 11 leaves in order the remainders 1, 6 and 2.

(ii) Let p be an odd prime. Then prove that the congruence $x^2 \equiv -1 \pmod{p}$ has a solution if and only if $p \equiv 1 \pmod{4}$. 5+5

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B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE13

(Point Set Topology)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) State the axiom of choice.

(b) Define product topology.

(c) Define addition and multiplication of two cardinal numbers. 1+1

(d) If u, v and w are cardinal numbers then prove that $u(vw) = (uv)w$.

(e) Let $(A, \leq)$ be a totally ordered set and $x \leq y$, $x, y \in A$. Prove that $A_x \subset A_y$, where A_x and A_y denote the initial segments determined by x and y respectively.

(f) Let (X, τ) be a topological space. Prove that a subset G of X is open if and only if it is a neighbourhood of each of its points.

(g) Let $(X,\tau), (Y,\tau')$ and (Z,τ'') be three topological spaces. Let $f\colon (X,\tau) \to (Y,\tau')$ and $g\colon (Y,\tau') \to (Z,\tau'')$ be two continuous functions. Prove that $gof\colon (X,\tau) \to (Z,\tau'')$ is continuous.

(h) Define a cofinite topological space.

(i) Prove that a topological space (X, τ) is connected if and only if it has no non-empty proper subset which is both open and closed.

(j) Let A and B be two connected subsets in a topological space (X, τ) with $A \cap B \neq \emptyset$. Prove that $A \cup B$ is connected in (X, τ).

(k) Let Q be the set of rational numbers equipped with subspace topology of usual topology of $\mathbb{R}$. Examine if Q is connected.

(l) Prove that each cofinite space is compact.

(m) Let X be a compact space and Y be a T_2 space and $f: X \to Y$ be continuous. Prove that f is a closed map. 2

(n) Show that a circle or a line or a parabola in $\mathbb{R}^2$ is not homeomophic to a hyperbola. 2

(o) Using the definition of a compact set, prove that the open interval (0, 1) is not a compact subset of $\mathbb{R}$, the real number space equipped with usual topology. 2

2. Answer *any four* questions: 5×4=20

(a) Prove that a sequentially compact metric space is compact.

(b) Let $f:(X,\tau) \to (Y,\tau')$ be a function. When is f called closed? Prove that f is closed map if and only if $f(\overline{A}) \supset \overline{f(A)}$ for every $A \subset X$. 1+2+2

(c) If u, v, w are cardinal numbers show that $(uv)^w = u^w \cdot v^w$.

(d) Let $(A, \leq)$ be a well ordered set. Prove that

(i) A is order isomorphic to no initial segment of A,

(ii) if $A_x \cong A_y$, then $x = y$. 3+2

(e) Define a path connected space. Prove that every path connected space is connected. 1+4

(f) Prove that a topological space is locally connected if and only if each component of an open set is open. 2½+2½

3. Answer *any two* questions: 10×2=20

(a) (i) Prove that for any two cardinal numbers u and v, either $u \leq v$ or $v \leq u$.

(ii) If u, v and w are cardinal numbers, then prove that $u^v u^w = u^{v+w}$. 5+5

(b) (i) Prove that a compact subset in a metric space is closed and bounded.

(ii) When is a topological space said to be locally connected? Is every connected space locally connected? Support your answer. (2+3)+(1+4)

(c) (i) Let (X,τ) and (Y,τ') be two topological spaces and $f: X \to Y$ be a mapping. Let $\{x_n\}$ be a sequence in X converging to x. If f is continuous then show that the sequence $\{f(x_n)\}$ converges to $f(x)$ in Y. Does the converse hold? Support your answer.

(ii) Let C be a connected subset of a topological space (X,τ). Show that $\bar{C}$ is connected. Hence or otherwise show that component in a topological space is closed. (2+4)+(3+1)

(d) (i) Define a locally compact space. Prove that a closed subset of a locally compact space is locally compact.

(ii) If u, v, w are cardinal numbers, prove that $(u^v)^w = u^{vw}$. (1+4)+5

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