B.A./B.Sc. 6th Semester (General) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B1

(Numerical Methods)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Using Newton-Raphson formula, find $\sqrt{12}$, correct up to two decimal places.

(b) If $f(0) = 1, f(1) = 4, f(2) = 8, f(3) = 12, f(4) = 15$, then evaluate $\int_0^4 f(x)dx$ by Trapezoidal rule.

(c) Give geometrical interpretation of Simpson's 1/3 rule for $\int_a^b f(x)dx$.

(d) Write down the Newton-Raphson algorithm of the function $f(x) = x - x^2$ in the form $x_{n+1} = \phi(x_n)$.

(e) Evaluate: $\Delta^2(ka^{bx})$ where the interval of difference being 1.

(f) Mention one merit and one demerit of regula-falsi method. 1+1

(g) Compare Trapezoidal rule and Simpson's one-third rule for evaluating numerical integration.

(h) Prove that $\Delta - \nabla = \Delta\nabla$, where Δ and ∇ are the forward and backward difference operators, respectively.

(i) What do you mean by diagonally dominant matrix? Mention one numerical method where this condition is used.

(j) Find Lagrange interpolation polynomial for the following data:

x	5	6	9	11
y	12	13	14	16

(k) Using Newton's backward difference formula, write the formula for the first derivative up to 2nd order difference term.

(l) Find the first approximations $x^{(1)}, y^{(1)}, z^{(1)}$ of the system of simultaneous equations $2x - 5y + 3x = 7, x + 4y - 2z = 3, 2x + 3y + z = 2$ by applying the Gauss-Seidel method, with initial guesses: $x^{(0)} = 0, y^{(0)} = 0, z^{(0)} = 0$.

(m) Compare Gauss-Jacobi and Gauss-Seidel methods for solving a system of linear equations.

(n) Find $y(1 \cdot 2)$ using Euler's method from $\frac{dy}{dx} = x^2 + y^2, y(1) = 1$, taking step length $h = 0{\cdot}2$.

(o) Find the root of the equation $x^3 - 5x - 7 = 0$ between 2 and 3 by regula-falsi method after 1st iteration only. (Correct up to two decimal places.)

2. Answer *any four* questions: 5×4=20

(a) What is the lowest degree polynomial that takes the following values?

x	0	1	2	3	4	5
$f(x)$	1	4	9	16	25	36

Hence find the polynomial.

(b) Show that the error in approximating $f(x)$ by the interpolation polynomial using distinct interpolating points $x_0, x_1, x_2, \ldots, x_n$ is of the form

$$(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_n)\frac{f^{n+1}(\xi)}{(n+1)!}.$$

(c) Prove that $\Delta^n\left(\frac{1}{x}\right)=\frac{(-1)^n.\, n!\, h^n}{x(x+h).\,(x+2h)\ldots(x+nh)}$ for any positive integer n, where Δ is the forward difference operator and h is the step length.

(d) Explain Newton-Raphson's method for computing a simple real root of an equation $g(x)=0$. When the method fails. 4+1

(e) Evaluate $\int_0^1 \frac{x}{1+x}dx$ by using Trapezoidal rule with six equal sub-intervals (correct up to two decimal places).

(f) Given the matrix $A=\begin{pmatrix}3 & 12 & 9\\ 2 & 10 & 12\\ 1 & 12 & 2\end{pmatrix}$, find two matrices L (lower triangular matrix) and U (upper triangular matrix) such that $A=L.U$.

3. Answer *any two* questions: 10×2=20

(a) (i) Find the first two iterations of the Jacobi method for the following linear systems, using $x^{(0)}=0: 3x_1-x_2+x_3=1, 3x_1+6x_2+2x_3=0, 3x_1+3x_2+7x_3=4$.

(ii) Deduce the condition of convergence of fixed point iterative method. 5+5

(b) (i) State the Secant Method for finding a root of an equation $f(x)=0$. Derive the iterative formula used in the Secant Method, and explain the assumptions under which the method is applicable.

(ii) A curve is drawn to pass through the following points:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Using Simpson's 1/3 rule, estimate the area bounded by the curve, the x-axis, and the lines $x=1, x=4$.

(1+3+1)+5

(c) (i) Consider the initial-value problem $y'=y-x$, $y(0)=\frac{1}{2}$. Use Euler's method with $h=0{\cdot}1$ to obtain an approximation to $y(1)$. Given that the exact solution to the initial-value problem is, $y(x)=x+1-\frac{1}{2}e^x$. Compare the errors in the two approximations to $y(1)$.

(ii) If $y=a(3)^x+b(-2)^x$ with increment $h=1$, then evaluate $(\Delta^2+\Delta-6)y$. (5+2)+3

(d) (i) Rearrange the following system of linear equations so that Gauss-Seidel method can be applied and then find the solution by taking 3 iterations only:

$$9x - 3y + 2z = 23$$

$$6x + 3y + 14z = 38$$

$$4x + 12y - z = 35$$

(ii) Show that if Δ operates on n, then $\Delta\binom{n}{x+1} = \binom{n}{x}$ and hence

$\sum_{n=1}^{N}\binom{n}{x} = \binom{N+1}{x+1} - \binom{1}{x+1}$. Here $\binom{n}{x} = \frac{n!}{x!(n-x)!}$. 5+5

B.A./B.Sc. 6th Semester (General) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B2

(Complex Analysis)

Time: 3 Hours **Full Marks: 60**

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Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions from the followings: 2×10=20

(a) Show that $\underset{z\to 0}{\text{Lt}} \frac{\bar{z}}{z}$ does not exist.

(b) Let $f(z) = x^2 + iy^2$. Does it satisfy the Cauchy Riemann equation at origin?

(c) Find the value of $\int_{|z|=1} \frac{\cos z}{z(z-4)} dz$.

(d) If $\lim_{z\to z_0} f(x) = l$, then show that $f(z)$ is bounded in some deleted neighbourhood of z_0.

(e) Examine if $f(z) = \frac{|z|}{Re(z)}$, $Re(z) \neq 0$

$= 0$, $Re(z) = 0$

is continuous at $z = 0$.

(f) If the functions $f(z) = u + iv$ and $f(z) = u - iv$ are both analytic in a region G, then show that $f(z)$ in constant.

(g) Find the radius of convergence of the series $\frac{1}{2}z + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^3 + \cdots$.

(h) If $(z) = \frac{z^2}{z-4}$, then evaluate $\int_C f(z)dz$, where C is the positively oriented unit circle $|z| = 1$.

(i) Show that $f(z) = x^2 + iy^2$ is nowhere analytic in a region.

(j) Evaluate $\int_c f(z)\, dz$, where $f(z) = x^2 + iy^2$ and $c: z(t) = t + it,\ 0 \leq t \leq 1$.

(k) State the necessary and sufficient condition that a function $f(z) = u + iv$ is analytic in a region.

(l) $f(z) = z^2 - 2z + 3$. Prove that $f(z)$ is continuous everywhere in a finite plane.

(m) Evaluate $\oint_C \frac{dz}{z-a}$, where C is any simple closed curve and $z = a$ is inside C.

(n) State Cauchy Integral formula.

(o) Examine the convergence of the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$, $z \in \mathbb{C}$.

2. Answer *any four* questions from the following: 5×4=20

(a) Expand $f(z) = \sin z$ in a Taylor Series about $z = \frac{\pi}{4}$..

(b) Find Laurent series about the indicated singularity $(z-3)\sin\frac{1}{z+2}$, $z = -2$.

(c) Prove that an absolute convergent series is convergent.-

(d) Find the value of $\oint_C \frac{\sin^6 z}{z-\frac{\pi}{6}} dz$, where C is the circle $|z| = 1$.

(e) (i) Let $f(z) = \frac{1}{z^2}$ and Γ be the straight line joining the points i and $2+i$, then show that $\left|\int_\Gamma f(z)\, dz\right| \leq 2$.

(ii) Evaluate $\int_\Gamma \frac{dz}{z-\alpha}$, where Γ denotes a circle $|z-\alpha| = r$ traversed in counter clockwise sense. 2+3

(f) Prove that every bounded entire function is constant. Is $\sin z$ bounded? 3+2

3. Answer *any two* questions: 10×2=20

(a) (i) Show that the function $f(z)$, where $f(z) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & if\ z \neq 0, \quad z = x+iy \\ 0\ , & if\ z = 0 \end{cases}$

is continuous at the origin.
Also show that $f'(0)$ does not exist though $C-R$ equations are satisfied at the origin.

(ii) If f and g are analytic in a region D of $\mathbb{C}$ and $f'(z) = g'(z)$, $\forall z \in D$, then show that $f-g$ is a constant in D. (2+2+2)+4

(b) (i) State and prove Fundamental Theorem of Algebra.

(ii) Evaluate the integral $\int_\Gamma \frac{e^{2z}}{(z-1)(z-2)} dz$, where $\Gamma\colon |z| = 3$. (2+5)+3

(c) (i) Define absolute convergence of the series $\sum_{n=1}^{\infty} z_n$ of complex numbers. Prove that the series $\sum_{n=1}^{\infty} u_n(z)$ of complex variable, where $u_n(z) = \frac{z^n}{n(n+1)}$, converges absolutely for $|z| \leq 1$.

(ii) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n\sqrt{2}+i}{1+2in} z^n$. (2+3)+5

(d) (i) Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 3$.

(ii) Prove that every polynomial equation $p(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n = 0$, where the degree $n \geq 1$ and $a_n \neq 0$, has exactly n roots. 5+5

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B.A./B.Sc. 6th Semester (General) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B3

(Linear Programming)

Time: 3 Hours **Full Marks: 60**

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Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Verify graphically the following problem has an unbounded solution:

Max $Z = 3x_1 + 4x_2$,

subject to $x_1 - 3x_2 \leq 3, x_2 - x_1 \leq 1,$

$x_1 + x_2 \geq 4$ and $x_1, x_2 \geq 0$.

(b) Write all the characteristics for the standard form of an L.P.P.

(c) Show that the dual of the dual of an L.P.P. is the primal itself.

(d) Define separating and supporting hyperplanes. 1+1

(e) State the fundamental theorem of L.P.P.

(f) Define hyperplane and half-space with examples.

(g) Examine whether the set $X = \{(x_1, x_2) : x_1 \leq 4, x_2 \geq 5, x_1, x_2 \geq 0\}$ is convex *or* not.

(h) Are all the boundary points of a convex set necessarily extreme points? Justify.

(i) Find a basic feasible solution of the system of equations:

$x_1 + x_2 + x_3 = 5,$

$2x_1 + 2x_2 + x_3 = 7.$

(j) Write down the dual problem of the given problem:

Min $Z = 4x_1 + 5x_2$

subject to $4x_1 + 5x_2 \geq 7$

$3x_1 - 2x_2 \geq 2$

$x_1, x_2 \geq 0$

(k) State the criterion for existence of infinite solutions for an L.P.P.

(l) Express $(9, 5, 9)$ as a linear combination of $\{(1, 2, 0), (3, 0, 7), (4, 1, 2)\}$.

(m) Express the following L.P.P. in the matrix form:

Max $Z = 2x_1 + 3x_2 + 4x_3$

subject to $x_1 + x_2 + x_3 \geq 5,$

$x_1 + 2x_2 = 7,$

$5x_1 - 2x_2 + 3x_3 \leq 9,$

$x_1, x_2, x_3 \geq 0.$

(n) When artificial variable(s) is used for solving an L.P.P. by simplex method?

(o) State two fundamental properties of duality.

2. Answer *any four* questions: 5×4=20

(a) Solve the following system of linear simultaneous equations by simplex method.

$x_1 + x_2 = 1,\ 2x_1 + x_2 = 4.$

(b) Define basic feasible solution. Obtain the basic feasible solutions of the following system of equations:

$x_1 + 4x_2 - x_3 = 5$ and $2x_1 + 3x_2 + x_3 = 8$ 2+3

(c) What do you mean by 'unrestricted in sign' of a variable? If any variable of the primal problem is unrestricted in sign, then prove that the corresponding constraint of the dual problem turns into equality. 1+4

(d) Solve the following L.P.P. graphically:

Min $Z = 4x + 2y$

subject to $3x + y \geq 27$

$-x - y \leq -21$

$x + 2y \geq 30$

$x, y \geq 0$

(e) Prove that the set of all feasible solutions of an L.P.P. is a convex set.

(f) Solve using simplex method:

Max $Z = 6x_1 - 2x_2$

subject to $2x_1 - x_2 \leq 2,$

$x_1 \leq 4,$

$x_1, x_2 \geq 0.$

3. Answer *any two* questions: 10×2=20

(a) Solve the following L.P.P. by using two-phase Simplex method.

Min $Z = x_1 + x_2$

subject to $2x_1 + 4x_2 \geq 4$

$x_1 + 7x_2 \geq 7;\ x_1, x_2 \geq 0.$

(b) Use duality to solve the following problem:

Max $Z = 2x_1 + 3x_2$

subject to $-x_1 + 2x_2 \leq 4$

$x_1 + x_2 \leq 6$

$x_1 + 3x_2 \leq 9$

$x_1, x_2 \geq 0$

(c) Solve the following L.P.P. by Big-M method:

Max $Z = 3x_1 - x_2$

subject to $-x_1 + x_2 \geq 2$

$5x_1 - 2x_2 \geq 2;\ x_1, x_2 \geq 0.$

(d) (i) State Complementary Slackness theorem of duality.

(ii) Using Simplex method, find the inverse of the following matrix: 2+8

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$$