

Time: 2 Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers in their
own words as far as practicable.
Abbreviations and symbols have their usual meaning.*

Full Marks: 40

Group-A

Answer any five questions.

1. (a) Describe the locus of a particle whose position vector is $\vec{r}(\theta, \varphi) = (a \cos \theta \cos \varphi)\hat{i} + (b \sin \theta \cos \varphi)\hat{j} + (c \sin \varphi)\hat{k}$. $2 \times 5 = 10$
- (b) Find the normal vector and equation of the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at $(3, 4, 5)$.
- (c) A water tank having height H and cross-sectional area A is completely filled. There is an orifice of cross-sectional area α at the bottom. Calculate the time the tank will be empty when the orifice is opened.
- (d) Find the directional derivative of $\Phi(x, y, z) = x^3 + y^3 + z^3$ at $(1, -1, 2)$ in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$.
- (e) Find the unit vector \hat{e}_r and \hat{e}_θ in polar co-ordinate system.
- (f) Find the Taylor series expansion of e^x about $x = 0$.
- (g) Two players A and B play a game such that A has probability $\frac{2}{3}$ of winning. If A plays 4 games, find the probability that A wins exactly 2 games.
- (h) Evaluate the integral $\int (x^3 + 1) \delta(x^2 - 9) dx$.

Group-B

Answer any two questions.

$5 \times 2 = 10$

2. (a) Show that $\int_C \frac{xdx+ydy}{\sqrt{x^2+y^2}}$ is independent of the path of integration which does not pass through the origin. Find the value of the integral from the point $P(-1, 2)$ to $Q(2, 3)$. $2+3$

- (b) Solve the differential equation:

$$\frac{dy}{dx} - y = y^2(\sin x + \cos x)$$

- (c) Find the expression of $\vec{\nabla} \cdot \vec{V}$ in curvilinear co-ordinate system. Hence write the expression for spherical co-ordinates. $4+1$

Please Turn Over

- (d) If $f(x) = \frac{1}{2}(x+1)$ for $-1 < x < 1$ and
 $= 0$ otherwise

is a density function, calculate $E(x)$ and $Var(x)$.

2+3

Group-C

Answer any two questions.

10×2=20

3. (a) Evaluate the integral $\int_C [(x^2 - y^3)dx + (x + y)dy]$, where C is the contour bounded by $y = x$, $y = -x$ and $x^2 + y^2 = 16$ for $x \geq 0$.

- (b) Solve the differential equation:

5+5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

4. (a) A boat is rowed with a velocity u directly across a stream of width a . If the velocity of the current is directly proportional to the product of distances from the two banks, find the path of the boat and the distance where it will land.

- (b) Evaluate $\iint \Phi \hat{n} ds$ where $\Phi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

5+5

5. (a) Represent the vector $\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in spherical co-ordinates.

- (b) Prove that $\int_{-\infty}^{\infty} f(x) \delta(x - a) dx = f(a)$.

5+5

6. (a) Prove that Poisson distribution is the limiting case of binomial distribution for very large value of n (no. of trials) and small value of p (probability of occurrence) such that $n.p$ is finite and equals to λ .

- (b) A set of observations is as:

$x: 0 \quad 1 \quad 2 \quad 3 \quad 4$

$f: 30 \quad 63 \quad 45 \quad 10 \quad 2$ where f = frequency

Construct the distribution function. Hence calculate the theoretical frequency for the third observation ($x = 2$). Given $e^{-1.3} = 0.273$.

6+2+2