B.A./B.Sc. 6th Semester (Honours) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMH6DSE31

(Mathematical Modelling)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) What are the key steps involved in the mathematical modelling?

(b) Write two assumptions of Malthusian model.

(c) What is Allee effect?

(d) What is the difference between stability and asymptotic stability of a system?

(e) What is density dependent growth model? Provide an example of it.

(f) Find the population doubling time for logistic growth model and mention under what condition it occurs.

(g) Discuss biotic and abiotic factors of an ecosystem.

(h) What are the basic equations of the Lotka-Volterra Model?

(i) A traffic model is given by $x_{n+1} = 0{\cdot}8\, x_n$. What happens to traffic flow over time?

(j) If arrivals follow a Poisson process with arrival rate $\lambda = 3$ per hour, what is the probability of 2 arrivals in 1 hour? (Given $e^{-3} = 0.0497$)

(k) What happens when $\rho \geq 1$ in an $M/M/1$ queue? (ρ = traffic intensity)

(l) What is the goal of the least squares method?

(m) Write down the general least square estimator formula.

(n) What kind of dynamics do predator-prey systems typically exhibit?

(o) For an $M/M/1$ queue with $\lambda = 2, \mu = 5$, find the expected waiting time in the queue.

2. Answer *any four* questions: 5×4=20

(a) Find the non-negative equilibrium of a population governed by $x_{n+1} = \frac{2x_n^2}{x_n^2+2}$. Then, investigate the stability. 3+2

(b) Show that the non-trivial equilibrium $(x^*, y^*)$of the predator prey model

$$\frac{dx}{dt} = x\left(1 - \frac{x}{K}\right) - \frac{\alpha xy}{x+A}$$

$$\frac{dy}{dt} = y\left(\frac{\alpha x}{x+A} - \frac{\alpha B}{A+B}\right)$$

is unstable if $K > A + \alpha B$ and asymptotically stable if $B < K < A + \alpha B$. 2+3

(c) Solve the exponential growth model $\frac{dN(t)}{dt} = 0{\cdot}7\ N(t), N(0) = 150$ where, $N(t)$ represents the population at time t and $N(0)$ is the initial population.

(d) Consider the Lotka-Volterra Model:

$$\frac{dx}{dt} = 2x - 0{\cdot}01xy$$

$$\frac{dy}{dt} = 0{\cdot}005xy - 1{\cdot}5y$$

(i) Identify the equilibrium points.

(ii) Interpret their biological meaning.

(iii) What happens to the predator population if the prey becomes extinct.

(e) By the method of least squares, find the straight line that best fits the following data:

x	2	3	4	5	6
y	11	15	24	27	36

(f) Arrival rate of telephone calls at a telephone booth is according to Poisson distribution, with an average time of 9 minutes between two consecutive arrivals. The length of a telephone call is assumed to be exponentially distributed with mean 3 minutes.

(i) Determine the probability that a person arriving at the booth will have to wait.

(ii) Find the average queue length that forms from time to time. 2½+2½

3. Answer *any two* questions: 10×2=20

(a) Obtain the maximum likelihood estimator of σ^2 where μ (known) and σ are mean and standard deviation of a normal population respectively. Show that this estimation is unbiased. 8+2

(b) Derive the steady state difference equations for the queuing model $(M / M / 1) : (N / FCFS / \infty)$ and then find the probability of n customers in the system. 6+4

(c) Find the inflexion point for logistic growth model. Analysing the sign of $\frac{dP}{dt}, \frac{d^2P}{dt^2}$, study the behaviour of logistic growth curve when $P >$ or $< K$ where, P is the population density at time t for a single species population and K is the carrying capacity. 2+8

(d) Find the equilibrium points and discuss asymptotic stability of the model $\frac{dx}{dt} = rx(1 - x/K)(x/A - 1)$, where $0 < A < K$ and r is the growth rate. 4+6

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B.A./B.Sc. 6th Semester (Honours) Examination, 2025 (CBCS)

Subject : Mathematics

Course : BMH6DSE32

(Industrial Mathematics)

Time: 3 Hours **Full Marks: 60**

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Candidates are required to give their answers in their own words as far as practicable.
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1. Answer *any ten* questions: 2×10=20

(a) Define attenuation coefficient.

(b) How do you identify a 3×3 or 4×4 grid uniquely?

(c) Define Radon transform of a function.

(d) Give an example on back projection of Radon transform.

(e) What is Shepp-Logan filter?

(f) Distinguish between ART and Fourier transform.

(g) State discrete Fourier transform and explain each terms.

(h) Define Dirac-delta function.

(i) State discrete version of the Rayleigh-Plancherel Theorem.

(j) Show that $R(f+g)(t,\theta) = Rf(t,\theta) + Rg(t,\theta)$ where, R denotes the Radon transform.

(k) Define the Hounsfield unit of a medium. Give its mathematical expression.

(l) For an integrable function f and fixed real number α, let $g(x) = f(x-\alpha)$. Then show that $Fg(\omega) = e^{i\omega\alpha}\, Ff(\omega)$.

(m) Define the width at the half height of the Lorentzian function.

(n) Define affine spaces. Give an example.

(o) What do you mean by Medical Imaging? Give an example.

2. Answer *any four* questions: 5×4=20

(a) Let $A(x) = 1 - |x|$, if $|x| \leq 1$ and 0 *otherwise* and

$$I(x) = \begin{cases} 1 & if\ x \leq -1 \\ e^{-x-\frac{1}{2}-\frac{1}{2}x^2} & if\ -1 \leq x \leq 0 \\ e^{-x-\frac{1}{2}+\frac{1}{2}x^2} & if\ 0 \leq x \leq 1 \\ e^{-1} & if\ x \geq 1 \end{cases}$$

(i) Evaluate $\int_{-1}^{1} A(x)dx$ and $\ln\left(\frac{I(-1)}{I(1)}\right)$.

(ii) Verify that the functions A and I satisfy the differential equation $\frac{dI}{dx} = -A(x)\cdot I(x)$.

(b) Evaluate the integral $\int_{s=-\sqrt{1-t^2}}^{\sqrt{1-t^2}}\left(1-\sqrt{t^2-x^2}\right)ds$.

(c) Establish a relation between the Radon transform and the Fourier transform of a function.

(d) Let A be an $M \times N$ matrix, $\vec{x}$ be a vector in $\mathbb{R}^N$ and $\vec{y}$ be a vector in $\mathbb{R}^M$. Then show that $A\vec{x}\cdot\vec{y} = \vec{x}\cdot A^T\vec{y}$.

(e) Demonstrate equiangular case of 3rd generation multi-slice CT.

(f) Find the Fourier transform of the Dirac-delta function.

3. Answer *any two* questions: 10×2=20

(a) (i) State Beer's Law.

(ii) Why Beer's Law is a plausible model for X-ray attenuation?

(iii) Explain the concept of convolution back projection. 2+3+5

(b) (i) For the system of two lines $x_1 - x_2 = 0$ and $x_1 + x_2 = 5$ and the starting point $x^0 = \begin{bmatrix}3\\1\end{bmatrix}$, apply Kaczmarz's method to compute $x^{0,1}$ and $x^{0,2}$. Show that the vector $x^{0,2}$ lies on both lines.

(ii) Draw a curve showing the intensity of the transmitted X-rays as a function of time following Beer's Law. Interpret the curve physically. 5+(3+2)

(c) (i) Let f be the function defined by $f(x) = \begin{cases} 1-\sqrt{x^2+y^2}, & if\ x^2+y^2 \le 1 \\ 0\ , & otherwise \end{cases}$

find the Radon transform of this function f.

(ii) Explain why $l_{t,\theta} = l_{-t,\theta+\pi}$ for all t and all θ. 6+4

(d) (i) Write an algorithm of CT scan.

(ii) State and prove the Nyquist's Theorem. 5+5

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(Group Theory–II)

Time: 3 Hours **Full Marks: 60**

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Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Let $X = \{1, 2, 3, \dots n\}$ and $G = A(X) = S_n$. Then prove that X is a G-set.

(b) Find the Automorphism group of $\mathbb{Z}_5$.

(c) Find the commutator subgroup of the group of order 99.

(d) Justify whether $V_4 = \{(1), (1,2), (3,4), (1,2)(3,4)\}$ is a characteristic subgroup of S_4.

(e) Find the number of Automorphisms of the group $\mathbb{Z}_7$.

(f) Show that $\frac{G}{Z(G)} \simeq Inn(G)$ for any group G.

(g) Verify: $\mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_6$.

(h) Let $G = \mathbb{Z}_6$ and define the action on itself by $a.x = a + x$ (mod 6). Find the orbit and stabilizer of $2 \in \mathbb{Z}_6$.

(i) Prove that every group of order $p^n (n > 1)$ is not simple, where p is prime.

(j) Let G be a non-commutative group of order 10. Find the class equation of G.

(k) Give an example of a group where all sylow 2-groups are normal.

(l) Let G be a finite group and H be a sylow p-subgroup of G. Then prove that H is unique sylow p-subgroup of G iff H is normal in G.

(m) Let G be a non-commutative group of order p^3, where p is a prime. Prove that $|Z(G)| = p$.

(n) Let G be a finite abelian group of order n. Let k be a divisor of n. Show that G has a subgroup H of order k.

(o) D_4 cannot be written as internal direct product of two proper subgroups. — Justify.

2. Answer *any four* questions: 5×4=20

(a) If G be a cyclic group of order n, show that $|\text{Aut}\,(G)| = \phi(n)$, where $\phi(n)$ denote the number of positive integers less than n and prime to n.

(b) Prove that $\text{Inn}\,(S_3) \cong S_3 \cong \text{Aut}\,(S_3)$.

(c) Let H be a subgroup of order 11 and index 4 of a group G. Prove that H is normal subgroup of G.

(d) Prove that commutator group G' is the smallest normal subgroup of G such that G/G' is abelian.

(e) Let $G = G_1 \times G_2$ be the direct product of two groups G_1 and G_2. Prove that G is commutative iff G_1 and G_2 are commutatives.

(f) Let G be a group and $a, b \in G$. Then prove that $cl(a) = cl(b)$ iff $b = gag^{-1}$ for some $g \in G$, where $cl(a)$ is conjugacy class of a and $cl(b)$ is conjugacy class of b.

3. Answer *any two* questions: 10×2=20

(a) (i) Show that A_5 is a simple group.

(ii) Show that a simple group of order 63 cannot contain a subgroup of order 21. 5+5

(b) (i) Let G be a group of order pn, p is a prime and $p \geq n$. If H is a subgroup of G of order p i.e prove that H is a normal in G.

(ii) If G be an infinite cyclic group, prove that $Aut\,(G)$ is a group of order 2 is $|Aut\,(G)| = 2$. 5+5

(c) (i) Find all Sylow 2-subgroups of group A_4.

(ii) Let G be a group such that $|G| = 2p$, where p is prime. Show that G is cyclic or dihedral. 5+5

(d) (i) Show that every group of order 35 is cyclic.

(ii) Let G be a group of order pq, where p, q are primes such that $p > q$ and q does not divide p-1. Then show that G is a cyclic group. 5+5
