B.A/B.Sc6th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMG6SEC41
(Boolean Algebra)

Time:2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any five questions $5 \times 2 = 10$

(a) Define a partially ordered Set and give an example. [2]

(b) In a Boolean Algebra B prove that; $a + x = a + y$ and $a' + x = a' + y$ implies $x = y$. [2]

(c) Let $S = \{1,2,3 \ldots, \ \ 100\}$. Let $x \leq y$ means x is a divisor of y. Find the maximal of S. [2]

(d) Give an example of a poset$(S, \leq)$which is not a Lattice. [2]

(e) In a Boolean Algebra B Prove that $x \leq y, x \leq z$ implies $x \leq y.z$ [2]

(f) Define a partial order relation $\leq$ on the set of all natural number N so that $(N, \leq)$ become a Lattice. [2]

(g) Let S be a finite set containing two elements. How many binary relation can be defined on S? [2]

(h) Using the principle of Duality prove that $x + (x.y) = x$ holds in a Boolean Algebra. [2]

Answer any two questions: $2 \times 5 = 20$

(a) Prove that there does not exist a Boolean Algebra containing exactly three elements. [5]

(b) Prove that in a poset$(S, \leq)$, if $a, b \in S$ have a least upper bound or greatest lower bound exists then it is unique. [5]

(c) Prove that a poset is a chain if and only if each of its subset is a sublattice. [5]

(d) Let $f(x, y, z) = yz + y'z'$ then find the conjunctive normal form of $f(x, y, z)$ [5]

Answer any two questions: $2 \times 10 = 20$

(a) (i) Find the complement of $x'yz + xy'z + xyz'$. [5]

(ii) Prove that there are exactly 2^{2^n} distinct functions of n variables in a Boolean Algebra when each variable is assigned the value 0 and 1. [5]

(b) (i) Show that the current will flow through the network represented by the Boolean function $[xy(x'y + xy')]$ irrespective of the states of x and y. [5]

(ii) Draw a circuit to realize the Boolean function with simplified form; $f(x,y,z) = yz + x'y' + xy'$ [5]

(c) (i) Prove that the zero element and unit element of a sub algebra are same as that of the Boolean algebra. [5]

(ii) Find all the Sub algebra of $B = \{1,2,5,7,10,14,35,70\}$, where $x + y, x.y$ and $\acute{x}$ are defined by you. [5]

(d) (i) Write down the steps of the Quuine McCluskey Method. Also write the advantage of the Quuine McCluskey Method. [5]

(ii) Draw a circuit to realize the Boolean function with simplified form; $f(x,y,z) = (x + y + z)\, z'\,(xy + x'z' + xz)$ [5]

B.A/B.Sc.6th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMG6SEC42
(Transportation and Game Theory)

Time:2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any five questions 5×2 = 10

(a) Show that the number of basic variables in a balanced transportation problem with m origins and n destinations is atmost $m + n - 1$. [2]

(b) What is the difference between pure strategy and mixed strategy for two persons zero sum game? [2]

(c) Find the optimal assignment and the corresponding cost of the following assignment problem: [2]

	I	***II***
P	2	5
Q	3	2

(d) Find the saddle point of the following two persons zero sum game: [2]

	Q_1	Q_2
P_1	2	3
P_2	1	5

(e) Find the value of k, when the given two persons zero sum game has strict solution. [2]

	P		
Q	k	7	3
	-2	k	-8
	-3	4	k

(f) Define fair game for two persons zero sum game. [2]

(g) Show that every transportation problem has a feasible solution. [2]

(h) Write down the optimality criterion for a transportation problem. Also, state the multiple optimal solution criteria. [2]

2. Answer any two questions $2\times5 = 10$

(a) (i) Find an initial basic feasible solution for the following transportation problem by North-West corner method: [5]

	I	*II*	*III*	*IV*	
X	5	2	1	8	10
Y	4	4	3	5	20
Z	3	6	4	3	30
	14	16	21	9	

Is this solution optimal? Give reason.

(b) Let $f(x,y)$ be a real valued function of x and y defined for $x \in A$ and $y \in B$ where A and B being two subsets of real numbers. Now if both $\max_{x\in A} \min_{y\in B}\{f(x,y)\}$ and $\min_{y\in B} \max_{x\in A}\{f(x,y)\}$ exist then [5]

$$\max_{x\in A} \min_{y\in B}\{f(x,y)\} \le \min_{y\in B} \max_{x\in A}\{f(x,y)\}$$

(c) Determine the optimal assignment and the corresponding cost of the following assignment problem: [5]

	I	***II***	***III***	***IV***	***V***
A	8	7	10	13	18
B	3	15	18	3	12
C	18	13	10	10	10
D	11	16	14	12	18
E	12	15	13	10	18

(d) Find an initial basic feasible solution of the following transportation problem by VAM and row minima method and compare their corresponding costs. [5]

	P	***Q***	***R***	
I	8	9	5	10
II	6	10	7	15
III	11	8	6	25
	16	14	20	

3. Answer any two questions 2×10 = 20

(a) Solve the following two persons zero sum game. Then find value of the game. [8+2]

	Y_1	Y_2	Y_3
X_1	3	2	4
X_2	4	3	1
X_3	-1	2	5

(b) Find an initial basic feasible solution by VAM for the following transportation problem: [8+2]

	D_1	D_2	D_3	D_4	
W_1	32	40	28	35	150
W_2	27	34	26	31	260
W_3	33	31	30	32	250
	120	170	190	180	

Is the solution optimal and unique? Give reason.

(c) Use row minima method to find an initial basic feasible solution for the following transportation problem: [2+8]

	D_1	D_2	D_3	D_4	
O_1	12	15	10	11	115
O_2	14	11	16	15	100
O_3	10	17	12	16	105
	95	75	90	60	

Hence, find the optimal solution.

(d) (i) A company has four workers and three jobs. The operating costscorresponding to different workers and jobs are given below: [5]

	M_1	M_2	M_3	M_4
W_1	14	20	24	28
W_2	4	9	13	15
W_3	6	11	15	18

Find the optimal assignment and the corresponding assignment cost.

(ii) Write down different steps of Hungarian method for solving assignment problem. [5]

B.A/B.Sc.6th Semester (General) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMG6SEC43
(Graph Theory)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any five questions $5\times2 = 10$

(a) Define loop and parallel edges of a graph. [2]

(b) Give an example of graph which is Hamiltonian but not Eulerian. [2]

(c) A graph has 8 vertices and 6 edges. Can it be connected? Explain. [2]

(d) Give an example of a graph which is both Eulerian and Hamiltonian. [2]

(e) Show by an example that a trail may not be a path. [2]

(f) Define a tree. [2]

(g) What is meant by shortest path between two vertices of a weighted graph? [2]

(h) Give an example of a graph which is Eulerian but not Hamiltonian. [2]

2. Answer any two questions $2\times5 = 10$

(a) Show that the sum of the degrees of a graph is twice its number of edges. [5]

(b) Let G be a graph with at most $2n$ vertices. If degree of each vertex is at least n, then show that G is connected. [5]

(c) Draw the graph $K_{2,3}$ and its complement. Are they isomorphic? [4+1]

(d) Describe the Travelling Salesman Problem. [5]

3. Answer any two questions $2\times10 = 20$

(a) (i) Prove that degree of every vertex in a connected Eulerian graph is even. [5]

(ii) Define isomorphism of graphs. Find two non-isomorphic graphs with the same degree sequence. [3+2]

(b) Find the adjacency matrix A and the incidence matrix I of the complete graph K_5 with respect to some labelling of your choice. [10]

(c) (i) Define the girth of a graph. Find the girth of the graph K_4. [2+2]

(ii) Show that a bipartite graph cannot have a cycle of odd length. [6]

(d) (i) What do you mean by a spanning tree? Does every graph contains a spanning tree? Justify your answer. [3+2]

(ii) Let T be a tree with exactly n vertices. Show that T has exactly n-1 edges. [5]