B.Sc. 4th Semester (Honours) Examination, 2023 (CBCS)

Subject : Physics

Course : CC-VIII

Time: 2 Hours

Full Marks: 40

The questions are of equals value.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words
as far as practicable.

Group-A

1. Answer *any five* questions: 2×5=10

   (a) Consider the function : $w = \tanh^{-1} z$. Prove that the principal branch of the function leads to $\tanh^{-1} z = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right)$.

   (b) Express $\frac{1+3i}{1-2i}$ in the form $re^{i\theta}$.

   (c) Prove that $\sin(ix) = i \sin hx$.

   (d) Evaluate $\oint_C (z-a)^n \, dz$ where C is the circle at centre a and of radius r. What will be the result of the integration for $n = -1$.

   (e) Find the Fourier sine transform of $\left(\frac{1}{x} \right)$.

   (f) If $F(S)$ is the Fourier transform of $f(x)$, prove that $Ff(ax) = \frac{1}{a} F \left(\frac{S}{a} \right)$.

   (g) Prove that $L(t^n) = \frac{n!}{S^{n+1}}$ where L stands for Laplace transform and both n and S are +ve.

   (h) Express $\sin^5 \theta$ in terms of sines of odd multiples of θ.

Group-B

2. Answer *any two* questions: 5×2=10

   (a) Determine the poles of the function: 1+4

   $$f(z) = \frac{1}{z^4 + 1}$$

   (b) What do you mean by analyticity of a function $f(z)$? Find the analytic function $f(z)$ such that $Ref'(z) = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$. 1+4

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(c) Find the Laplace transform of $f(t) = \sin 2t \cos 3t$. 5

(d) Write the Fourier cosine transform of a function $f(x)$. Determine $f(x)$ from the relation: 2+3

$$\int_0^\infty f(x) \cos \omega x \, dx = e^{-\omega}$$

Group-C
Answer *any two* questions. 10×2=20

3. (a) Write Cauchy integral formula. Evaluate the integral:

$$\oint_C \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz \text{ where } C \text{ is the circle } |z| = 1, \text{ using Cauchy integral formula.} \qquad 2+3$$

(b) Write Residue theorem. Evaluate the integral $\oint_C \dfrac{e^z}{\left(z^2 + \pi^2\right)^2} dz$ where C is the circle $|z| = 4$.

2+3

4. (a) Write Laurent's theorem for an analytic function. Expand the function in Laurent's series for, $1 < |z| < 3$ and $f(z) = \frac{1}{(z+1)(z+3)}$. 2+4

(b) Check whether the function $f(z) = \log z$ for $z > 0$ is analytic or not. 4

5. (a) Evaluate $\displaystyle\int_0^\infty te^{-3t} \sin t \, dt$. 3

(b) Find $L[f'(t)]$ where L stands for Laplace transform. 2

(c) Solve the equation: $y''(t) + 9y(t) = 6 \cos 3t$ using Laplace transform. Given: $y(0) = 2$ and $y'(0) = 0$. 5

6. (a) Find the Fourier transform of $f(x)$ defined as:

$$f(x) = x^2, \ |x| < a$$
$$= 0, \ |x| > a$$

5

(b) Find the complex form of Fourier integral representation of $f(x)$ defined as:

$$f(x) = e^{-kx}, \ x > 0, \ k > 0$$
$$= 0 \text{ otherwise}$$

5

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