B.A./B.Sc. 6th Semester (General) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B1

(Numerical Methods)

Time: 3 Hours

Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) State the condition of convergence of Newton-Raphson method. When does Newton-Raphson method fail? 1+1

(b) Construct a backward difference table using the given values of x and $y = f(x)$.

x	1	2	3	4	5	6
y	5.0	5.4	6.0	6.8	7.5	8.1

(c) Explain the concept interpolation.

(d) Calculate $\sqrt{2}$ by using fixed point iteration method (Correct up to two decimal places).

(e) Evaluate $\int_0^1 x^2 dx$ by using trapezoidal rule with step length 0.2 (Correct up to one decimal place).

(f) Give geometrical interpretation of Euler's method for solving a differential equation $\frac{dy}{dx} = f(x)$, with initial condition $y(x_0) = y_0$.

(g) Prove that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$.

(h) Write down condition required for the convergence of the Gauss-Seidel iterative method for a system of linear algebraic equations.

(i) State one merit and one demerit of Lagrange's interpolation formula. 1+1

(j) If δ denotes the central difference operator for a function $y = f(x)$, then prove that $\Delta - \nabla = \delta^2$, where Δ and ∇ stand for forward and backward difference operators, respectively.

(k) Define degree of precision of a numerical quadrature formula.

(l) Compute $\int_0^2 (1+x)dx$ by applying trapezoidal rule with step length $h = 0.5$.

(m) "Simpson's rule yields exact results when second degree polynomial, $f(x) = ax^2 + bx + c$ is used". — Justify the statement.

(n) Given $\frac{dy}{dx} = x + y, y(0) = 1$. Find y (0.6) taking step length $h = 0.2$.

(o) Find $\Delta^n u_x$ where $u_x = e^{ax+b}$, where a and b are independent of x.

5×4=20

2. Answer *any four* questions:

(a) Explain the bisection method for finding a real root of an equation $f(x) = 0$ and show that the method is unconditionally convergent to the root, if it exists. 3+2

(b) Define nth order divided difference for a given function $f(x)$. Show that

$$f\,[x_0, x_1, x_2, \dots, x_n] = \sum_{i=0}^{n} \frac{f(x_i)}{(x_i - x_0)(x_i - x_1)\dots(x_i - x_{i-1})(x_i - x_{i+1})\dots(x_i - x_n)}.$$

(c) Prove that the sum of the coefficients in Lagrange interpolation formula is unity.

(d) Find the missing value of the following data:

x	1	2	3	4	5
$f(x)$	7	–	13	21	37

(e) Explain iterative method for computing a simple real root of an equation $f(x) = 0$.

(f) Let $y = ax^2 + bx + c$ be the equation of the parabola passing through $(-h, y_0), (0, y_1)$ and (h, y_2). Find the area of the given curve bounded by the x axis and two ordinates at $-h$ and h, using Simpson's $\frac{1}{3}$rd rule. Also compute $I = \int_{-h}^{h} y\, dx$ by actual integration. Find the error, if any.

10×2=20

3. Answer *any two* questions:

(a) (i) Find a real root of the equation $xe^x = \cos x$ by using secant method, correct up to four decimal places.

(ii) Prove that $e^x = \left(\frac{\Delta^2}{E}\right) e^x \frac{Ee^x}{\Delta^2 e^x}$. 6+4

(b) (i) Determine the polynomial $p(x)$ of degree 3 such that $y = p(x)$ holds for the following pairs of x, y:

x	0	1	2	3	4	5
y	-3	-5	-11	-15	-11	7

(ii) Derive Newton-Raphson iteration formula for computing a real root of the equation $f(x) = 0$. 5+5

(c) (i) Using Euler's method, calculate $y\,(0.1)$, $y(0.2)$ and $y(0.3)$, given that $\frac{dy}{dx} = 1 - y$, $y(0) = 0$.

(ii) Determine the constants a, b, c such that the quadrature formula

$\int_o^h f(x)\, dx \approx af(0) + bf(h) + cf(2h)$

is exact for polynomials of highest possible degree. 5+5

(d) (i) Write an algorithm to find H.C.F. (Highest Common Factor) of two positive integers.

(ii) Solve the following system of equations by LU decomposition method with all diagona elements 1 of the U matrix:

$4x + y + z = 4, x + 4y - 2z = 4, 3x + 2y - 4z = 6$ 3+

B.A./B.Sc. 6th Semester (General) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B2

(Complex Analysis)

Time: 3 Hours **Full Marks: 60**

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Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) For any complex number z, show that $-|z| \le RL(z) \le |z|$, where $RL(z)$ = Real part of z.

(b) Verify $\lim_{z \to \infty} \frac{1}{z} = 0$, where z is a complex number.

(c) If $f(z)$ is continuous at $z = \alpha$, then show that $\overline{f(z)}$ is also continuous at α.

(d) Show that the function $2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate.

(e) State fundamental theorem of Algebra.

(f) Prove that if $f(z)$ is differentiable at $z = a$, then it is continuous at $z = a$.

(g) Evaluate $\int_c \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve $c : z = t^2 + i\, t$.

(h) Evaluate $\oint_c \frac{dz}{z-a}$, where c is any simple closed curve and $z = a$ is outside c.

(i) Evaluate $\int_c \frac{z^2 - z + 1}{z-1}\, dz$, where c is the circle $|z| = \frac{1}{2}$.

(j) Show that the series $\sum_{n=1}^{\infty} \frac{z^n}{n(n+1)}$ converges absolutely for $|z| \le 1$.

(k) Given $(z) = \frac{1}{z-3}$. Find its Laurent Series expansion for $|z| < 3$.

(l) Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.

(m) State Cauchy-Goursat theorem.

(n) Show that the function $f(z) = x^2 + i\, y^3$ is not analytic anywhere.

(o) Prove that $f(z) = |z|^4$ is differentiable but not analytic at $z = 0$.

2. Answer *any four* questions: 5×4=20

(a) If $f(z)$ is continuous on the rectifiable path Γ of length L and if there exists an $M > 0$ such that $|f(z)| \le M$ for all z on Γ, then show that $\left|\int_\Gamma f(z)dz\right| \le ML$.

(b) If C is the circle $|z| = 2$ described in the counter Clock-wise direction and if $g(z_0) = \oint_C \frac{2z^2-z+1}{z-z_0}\,dz$, then show that $g(1) = 4\pi i$. What is the value of $g(z_0)$ when $|z_0| > 2$? 4+1

(c) If $f(z)$ be analytic and $|f(z)|$ be constant, then show that $f(z)$ itself is constant.

(d) (i) Define radius of convergence of a complex power series.

(ii) Show that $1 + \frac{a.b}{1.c}z + \frac{a(a+1)b(b+1)}{1.2.c\,(c+1)}z^2 + \cdots$ has unit radius of convergence. 1+4

(e) If $f(z)$ be analytic then show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)[RL\, f(z)]^2 = 2|f'(z)|^2$.

(f) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent Series for $1 < |z| < 3$ and $|z| > 3$. 3+2

3. Answer *any two* questions: 10×2=20

(a) (i) State Lionville's theorem and using it, show that if $f(z)$ is analytic for all $z \in \mathbb{C}$ and if $|f(z)| \ge 1$, then $f(z)$ is constant.

(ii) Let $f'(z) = 0$ in a region D, then prove that $f(z)$ must be constant in D. Does the result hold if the word 'region' is dropped. Justify your answer. (1+3)+(4+2)

(b) (i) State Taylor's theorem. Expand $f(z) = \frac{1}{1-z}$ in a Taylor's series near $z = 0$ and determine the region of its convergence.

(ii) Find the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 \cdot 4^n}$. (2+2+1)+5

(c) (i) If $f(z)$ is analytic function, then show that $\nabla^2|f(z)|^2 = 4|f'(z)|^2$.

(ii) If $f(z)$ is analytic in a region bounded by two simple curves C and C_1, where C_1 lies inside C. Then show that $\oint_C f(z)\,dz = \oint_{C_1} f(z)dz$.

(iii) Show that $\sin z^2 = z^2 - \frac{z^6}{3!} + \frac{z^{10}}{5!} - \cdots$; in finite complex plane 4+3+3

(d) (i) If $f(z)$ and $\overline{f(z)}$ are analytic in a domain D, then show that $f(z)$ is constant in D.

(ii) Let $(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$, when $z \ne 0$

$= 0$, when $z = 0$.

Show that $f(z)$ is continuous at the origin. Also show that $f'(0)$ does not exist though C-R equations are satisfied at the origin. 4+(2+2+2)

B.A./B.Sc. 6th Semester (General) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B3

(Linear Programming)

Time: 3 Hours **Full Marks: 60**

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Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Define feasible solution and optimal solution of an L.P.P. 1+1

(b) Verify graphically the following problem has an unbounded solution:
Maximize $Z = 3x + 4y$
subject to $x - 3y \le 3; y - x \le 1; x + y \ge 4$ and $x, y \ge 0$.

(c) Distinguish between extreme point and boundary point with suitable example.

(d) Show that the set $X = \{(x, y, z) \in E^3 : 2x + 3y + z \le 5\}$ is a convex set.

(e) Find the extreme points, if any, of the set $S = \{(x, y) \in E^2;\ |x| \le 1, |y| \le 1\}$.

(f) State the fundamental theorem of duality.

(g) Construct the dual of the following L.P.P.
Minimize $Z = 3x_1 - 2x_2$
subject to $2x_1 + x_2 \le 1, -x_1 + 3x_2 \ge 4; x_1, x_2 \ge 0$.

(h) Express (5, 2, 1) as a linear combination of (1, 4, 0), (2, 2, 1) and (3, 0, 1).

(i) Define convex polyhedron. Give an example of convex set in which all boundary points are vertices. 1+1

(j) Find the basic feasible solutions of the system of equations $x_1 + x_2 + x_3 = 8, 3x_1 + 2x_2 = 18$ and $x_1, x_2, x_3 \ge 0$.

(k) Give examples of simplex in zero and one dimension.

(l) Write down the dual to the following L.P.P.
Maximize $Z = x_1 - x_2 + 3x_3$
subject to $x_1 + x_2 + x_3 \le 10$,
$2x_1 - x_2 \le 2, x_1, x_2 \ge 0$.

(m) Determine the convex hull of the points {(0, 0), (0, 1), (1, 2), (1, 1), (4, 0)}.

(n) Under what condition an L.P.P. will have unbounded solution?

(o) What is the difference between a degenerate and a non-degenerate basic solutions of a system of linear equations?

2. Answer *any four* questions: 5×4=20

(a) Solve the following LPP by simplex method:
Maximize $Z = 6x_1 - 2x_2$
subject to $2x_1 - x_2 \leq 2$,
$x_1 \leq 4$ and $x_1, x_2 \geq 0$.

(b) Show that if either the primal or dual problem has a finite optimal solution, then the other problem also has a finite optimal solution.

(c) Show that all the basic feasible solutions of the system
$2x + 6y + 2z + w = 3$,
$6x + 4y + 4z + 6w = 2$
are degenerate.

(d) Solve the following LPP graphically:
Minimize $Z = 4x + 2y$
subject to $3x + y \geq 27$,
$-x - y \leq -21$,
$x + 2y \geq 30$,
$x, y \geq 0$

(e) Show that a hyperplane is a convex set.

(f) Write down the algorithm of simplex method.

3. Answer *any two* questions: 10×2=20

(a) Use two phase method to solve the following problem:
Maximize $Z = 3x_1 - x_2$
subject to $2x_1 + x_2 \geq 2$,
$x_1 + 3x_2 \leq 2$,
$x_1 \leq 4$; $x_1, x_2 \geq 0$

(b) Using simplex method, show that the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$.

(c) (i) Given that $x_1 = x_2 = x_3 = 1$ is a feasible solution of the system of equations
$x_1 + x_2 + 2x_3 = 4$
$2x_1 - x_2 + x_3 = 2$.
Reduce the given feasible solution to a basic feasible solution.

(ii) Prove that intersection of two convex sets is also convex. 5+5

(d) (i) Use duality to solve the following LPP :
Maximize $Z = 3x_1 + x_2$
subject to $x_1 + x_2 \leq 1$, $2x_1 + 2x_2 \geq 2$ and $x_1, x_2 \geq 0$.

(ii) Prove that if any of the constraints in the primal be a perfect equality, the corresponding dual variable is unrestricted in sign. 5+5

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