B.A./B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMH5DSE21

(Probability and Statistics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) The distribution function $F(x)$ of a random variable x is given by

$$F(x) = \begin{cases} 1-\frac{1}{2}e^{-x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find $P(X = 0)$ and $P(X > 1)$.

(b) If the random variable X is $\beta_2(m, n)$ distributed, prove that the random variable $Y = \frac{1}{X}$ is $\beta_2(n, m)$ distributed.

(c) For any random variable X, show that Var $(aX + b) = a^2$ Var $(X), \forall\, a, b \in \mathbb{R}$.

(d) Find the moment generating function of Poisson distribution.

(e) Let the joint distribution of X and Y be given by the probability density function

$$f(x,y) = \begin{cases} x+y & \text{if } 0 < x < 1,\ \ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find $E(XY)$.

(f) The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} 8xy & \text{if } 0 \leq x \leq y,\ \ 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Examine whether X and Y are independent or not.

(g) Show that s^2 is an unbiased estimate of σ^2.

(h) Define consistent estimate.

(i) If A and B are two independent events, then show that $\bar{A}$ and $\bar{B}$ are also independent, $\bar{A}$ denoting the complement of A.

(j) Determine the value of the constant k which makes $f(x,y) = kxy, (0 < x < 1, 0 < y < x)$ a joint probability density function.

(k) State central limit theorem in case of equal components.

(l) The random variable X is normal $(50, 20)$. Find $P(|X - 50| \leq 20)$, given that

$$\frac{1}{2\pi}\int_{-\infty}^{1} e^{-\frac{x^2}{2}}\,dx = 0{\cdot}8413.$$

(m) Write down the maximum likelihood function of the normal (m, σ) population.

(n) What is meant by Scatter diagram?

(o) Find the distribution function of $Y = \sin X$, if X is distributed with the probability density function $f(x) = \frac{1}{2}\cos x; -\frac{\pi}{2} < x < \frac{\pi}{2}$.

2. Answer *any four* questions: 5×4=20

(a) If $\{A_n\}$ is a monotonic sequence of events, show that

$$P\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} P(A_n).$$

(b) The joint probability density function of two random variates X, Y is given by

$$f(x,y) = \begin{cases} \dfrac{6 - x - y}{8} & if\ \ 0 < x < 2,\ \ 2 < y < 4 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate $P(X < 1, Y < 3)$ and $P(X + Y < 3)$. 2+3

(c) If a, b, c, d are constants such that $ac \neq 0$, then show that $\rho(aX + b, cY + d) = \frac{ac}{|a||c|}\rho(X, Y)$.

(d) Show that $\chi^2 = \frac{n\delta^2}{\sigma^2}$ has χ^2 distribution with $(n - 1)$ degrees of freedom.

(e) Prove that for Binomial distribution (n, p) the following relation holds good:

$$\mu_{r+1} = pq\left(\frac{d\mu_r}{dp} + nr\ \mu_{r-1}\right)$$

where $q = 1 - p$ and μ_r is the rth central moment.

(f) Find the maximum likelihood estimate of the parameter σ of the distribution.

$$f(x) = \sigma\alpha x^{\alpha-1}\, e^{-\sigma x^{\alpha}}, x > 0$$

using a sample of size n, assuming α is known.

3. Answer *any two* questions: 10×2=20

(a) (i) If X is normal (m, σ), then prove that

$$P(a < X < b) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right) \text{ and } P(|X - m| > a\sigma) = 2\{1 - \Phi(a)\},$$

where $\Phi(x)$ denotes the standard normal distribution function.

(ii) The density function of two dimensional random variables (X, Y) is given by $f(x, y) = k(1 - x^2 - y^2), 0 < x^2 + y^2 < 1$. Find k and the marginal density function of X. (2+4)+(2+2)

(b) (i) Two random variables X, Y have the least square regression lines with equations $3x + 2y = 26$ and $6x + y = 31$. Find $E(X), E(Y)$ and $\rho(X, Y)$.

(ii) Let $X_n \xrightarrow[in\,p]{} a$ and $Y_n \xrightarrow[in\,p]{} b$ as $n \to \infty$. Then show that $X_n \pm Y_n \xrightarrow[in\,p]{} a \pm b$ as $n \to \infty$ and $X_n Y_n \xrightarrow[in\,p]{} ab$ as $n \to \infty$. 4+(2+4)

(c) (i) Obtain the moment generating function of a gamma random variable with density function

$$f(x) = \frac{e^{-x}x^{m-1}}{\Gamma(m)}, 0 < x < \infty, m > 0.$$
$$= 0, \qquad \text{elsewhere.}$$

Hence find mean.

(ii) Prove that the first absolute moment about any point is minimum when taken about the median (for continuous case only). 5+5

(d) (i) Find the approximate confidence interval for p for a binomial (n, p) population.

(ii) Show that, sample mean is asymptotically normal $N(m, \sigma/\sqrt{n})$, where $m =$ population mean and $\sigma =$ population standard deviation. 5+5

B.A./B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMH5DSE22

(Portfolio Optimization)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Define geometric mean return.

(b) Discuss Marshal Blumis formula.

(c) Explain the three well known risk premiums.

(d) What do you mean by time value money?

(e) What is the difference between compound and simple interest?

(f) State the formula for the future value of an annuity.

(g) What are the different types of financial ratios?

(h) Discuss the important turnover ratios.

(i) Discuss the key valuation ratios.

(j) What guidelines would you follow in financial statement analysis?

(k) What is the expected return on a portfolio of risky assets?

(l) Distinguish the three levels of market efficiency.

(m) How is a portfolio study done? Discuss the evidence of portfolio studies.

(n) What is the relationship between risk and return for efficient portfolios?

(o) What is the empirical evidence on the CAPM (Capital Asset Pricing Model)?

2. Answer *any four* questions: 5×4=20

(a) What is the equilibrium risk return relationship according to the APT?

(b) What do you mean by beta? What is the impact on stock of beta? 2+3

(c) Give the brief description about cash flow.

(d) What is an efficient portfolio? Why is it called efficient? 4+1

(e) What are the risks of an n-security portfolio?

(f) Write a short note about investment constraint.

3. Answer *any two* questions: 10×2=20

(a) During the past five years, the returns of a stock were as follows:

Year	Return	Year	Return	Year	Return
1	0·07	3	– 0·09	5	0·10
2	0·03	4	0·06		

Compute the following: (i) cumulative wealth index, (ii) arithmetic mean, (iii) geometric mean, (iv) variance and (v) standard deviation. 2+2+2+2+2

(b) Mention the assumptions underlying the standard capital asset pricing model. Discuss the procedure commonly used in practice to test the CAPM. Despite its limitations, why is the CAPM widely used? 4+3+3

(c) (i) A finance company advertises that it will pay a lumpsum of Rs. 44,650 at the end of five years to investors who deposit annually Rs. 6,000 for 5 years. What is the interest rate implicit in this offer?

(ii) What is the present value of Rs. 10,00,000 receivable 60 years from now, if the discount rate is 10 per cent?

(iii) You want to take a world tour which costs Rs. 10,00,000 — the cost is expected to remain unchanged in nominal terms. You are willing to save annually Rs. 80,000 to fulfill your desire. How long will you have to wait if your savings earn a return of 14 per cent per annum? 4+2+4

(d) The rates of return on stock A and market portfolio for 15 periods are given below:

Period	Return on stock A (%)	Return on market portfolio (%)	Period	Return on stock A (%)	Return on market portfolio (%)
1	24	12	9	−8	1
2	13	14	10	13	12
3	17	13	11	14	−11
4	15	10	12	−15	16
5	14	9	13	25	8
6	18	13	14	9	7
7	16	14	15	−9	10
8	6	7			

(i) Calculate the beta for stock A.

(ii) What is the characteristic line for stock A? 5+5

B.A/B.Sc. 5th Semester (Honours) Examination, 2024 (CBCS)

Subject : Mathematics

Course : BMH5DSE23

(Boolean Algebra and Automata Theory)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Consider the set Z of all integers. Define $a \leq b$ if there is a positive integer r such that $b = a^r$. Prove that $(Z, \leq)$ is a partially ordered set.

(b) Prove that in a distribution lattice, complement of an element is unique.

(c) Let $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and the relation '$\leq$' is defined by "$a \leq b$ if and only if a divides b". The operations $\wedge$ and $\vee$ are G.C.D and L.C.M respectively. The zero element is 1. Then find the set of all atoms of this Boolean algebra.

(d) Write the duals of $a + ((b + a)\cdot b') = 1$ and $a + (b\cdot a) = a$.

(e) What is a 'grammar'? Give an example of a grammar.

(f) For $\Sigma = \{0,1\}$, give a regular expression R such that $L(R) = \{W \in \Sigma^*;\ W$ has atleast one pair of consecutive zeros$\}$.

(g) Karnaugh map for a Boolean polynomial $f(x, y, z)$ in three Boolean variables x, y, z is given below. Determine $f(x, y, z)$ from the map and then express

z \ xy	00	01	11	10
0	0	1	1	1
1	0	0	0	1

it in DNF in the variables x, y, z.

(h) Define ID of a Non-deterministic Pushdown Automaton (NPDA).

(i) Simplify the regular expression $01^*1 + 11^*01^*1 + (0 + 01^*1)1^*1$ on $\Sigma = \{0, 1\}$.

(j) Why is pushdown automata more powerful than deterministic finite automata?

(k) State the Pumping Lemma for regular languages.

(l) What is a Context free Grammar?

(m) What is Chomsky Normal Form?

(n) What is recursively enumerable language?

(o) Define a Turing Machine.

2. Answer *any four* questions: 5×4=20

(a) A light in a room is to be controlled independently by three wall switches, located at the three entrances of the room. This means that flicking any one of the wall switches changes the state of the light (on to off and off to on). Design a simple circuit which allows current to flow to the light under the required conditions.

(b) Truth table for a Boolean polynomial $f(x, y, z)$ in three variables x, y, z is given below:

x	y	z	f
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

From this table, form Karnaugh map for the polynomial $f(x, y, z)$, taking x along the row and yz along the column of the map. From the Karnaugh map determine $f(x, y, z)$. Then, construct the logic circuit (of the logic-gates) for $f(x, y, z)$.

(c) Show that $L = \{a^p : p \text{ is prime}\}$ is not regular.

(d) Prove that any context-free language is generated by a context-free grammar is Chomsky normal form.

(e) If L is a Turing-decidable language, then it's complement L is also Turing-decidable. Prove it.

(f) Prove that for each non-deterministic finite automata, there is an equivalent deterministic finite automata.

3. Answer *any two* questions: 10×2=20

(a) (i) Construct a grammar which generates all even integers up to 998.

(ii) Design a Turing Machine that accept the language $L = \{a^n b^n : n \geq 1\}$ over $\Sigma = \{a, b\}$. 5+5

(b) (i) Prove that the normal subgroups of a group form a modular lattice, under set inclusion. How many minimal Boolean polynomials are there, in n Boolean variables?

(ii) Construct a Pushdown Automata to accept the language $L = \{a^n b^{2n} : n \geq 1\}$ over $\Sigma = \{a, b\}$. (3+2)+5

(c) (i) Construct Finite Automata equivalent the Regular Expression

$L = (a + b)^*(aa + bb)(a + b)^*$.

(ii) Prove that for every non-deterministic finite automata there is an equivalent deterministic finite automata. 5+5

(d) (i) Write the following Boolean expression in conjunctive normal form:

$f(x_1, x_2, x_3) \equiv x_1x_2 + x_1x_3 + x_2' x_3$, x_2' denotes the complement of x_2.

(ii) Show that in a bounded distributive lattice, the elements which have complements, form a sub-lattice. 5+5

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