3 Yr. Degree/4 Yr. Honours 3rd Semester Examination, 2024 (CCFUP)

Subject : Mathematics

Course : MATH3051 (SEC)

(Mathematical Modeling)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) What is the basic difference between a deterministic and a stochastic mathematical modeling?

(b) What is the sum of residuals in a linear regression model? Give reason.

(c) Write down the logistic model of population growth mentioning the different terms involved in it.

(d) Mention two assumptions of the Malthus model of exponential population growth.

(e) What do you mean by steady state population in a logistic growth model?

(f) Define extreme point in connection with a linear programming problem.

(g) A call centre receives an average of 15 calls per hour. What will be the probability of receiving exactly 10 calls in the next hour?

(h) Define seasonality in time series analysis.

2. Answer *any two* questions: 5×2=10

(a) By the method of least square, find the straight line that best fits the following data:

x	0	5	10	15	20	25
y	12	15	17	22	24	30

(b) What is the primary difference between the exponential growth model and the logistic growth model. Explain graphically mentioning their shapes.

In which scenario would the exponential growth model be more realistic than the logistic growth model? 3+2

(c) Using graphical or any other suitable method solve the following LPP: 5

Maximize $z = 5x_1 + 10x_2$

subject to $12x_1 + 7x_2 \leq 42$

$5x_1 + 4x_2 \leq 20$

$2x_1 + 3x_2 \geq 6, x_1, x_2 \geq 0$

(d) The horse power I, required to drive a ship of displacement D tons is given by the following data. Find a formula of the form $I = aD^P$ which will fit the following data— 5

Displacement (D)	1720	2300	3200	4100
Horse Power (I)	655	789	1000	1164

3. Answer *any two* questions: 10×2=20

(a) Starting from the differential equation of the logistic model, find the population P of a single species population at any time t. Hence show that P approaches to the carrying capacity at large time whatever be the initial population P_0. Find the population doubling time of the logistic growth model and mention the condition when such doubling occurs. Mention two limitations of logistic growth model. 4+2+2+2

(b) (i) The weekly demand for a product over 10 weeks is given by the following table:

Week	1	2	3	4	5	6	7	8	9	10
Actual demands (in units)	800	1400	1000	1500	1500	1300	1800	1700	1300	1700

Calculate 3-week Simple Moving Average (3-SMA) and interpret the result. 4+1

(ii) Define random number. What is the difference between true random number and pseduo random number. What is a seed in random number generation? 2+2+1

(c) (i) State the principle of least square method. Describe how this principle is used to obtain a best fitted line for a given set of observations $(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$. 1+4

(ii) Give two examples of exponential decay models and describe their applications. 5

(d) (i) An agricultural farm has 180 tons of nitrogen fertilizers, 250 tons of phosphate and 220 tons of potash. It is able to sell 3 : 3 : 4 mixtures of these substances at a profit of Rs. 15 per ton and 1 : 2 : 1 mixtures at a profit of Rs. 12 per ton respectively. Formulate the above problem as a linear programming problem. 5

(ii) Write a brief note on Monte-Carlo method—mentioning its real life applications. 5