3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Physics

Course : PHYS1011 (MAJOR)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer *any five* questions: 2×5=10

(a) Write down the Taylor series expansion of a function $f(x)$ about $x = x_0$. Prove that:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots$$

(b) Solve the equation: $\frac{dy}{dx} + lnx^y = 0$

(c) The vector equation: $\vec{A} \times \vec{C} = \vec{B} \times \vec{C}$ does not necessarily imply $\vec{A} = \vec{B}$. Justify your answer.

(d) Prove the identity $\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B})$.

(e) A force $\vec{F}$ is defined as: $\vec{F} = (3x + 5y)\hat{\imath} + (5x + 2z)\hat{\jmath} + (2y + 3x + 4z)\hat{k}$. Is the force conservative?

(f) Write the expression of an elementary volume in terms of scale factors and co-ordinates. Hence calculate the volume of a sphere of radius a.

(g) $\phi(x, y, z) =$ Constant, describes a surface. Prove that $\vec{\nabla}\phi$ is perpendicular to the surface.

(h) The equation of motion of a particle of mass m is: $m\frac{d^2\vec{r}}{dt^2} = \hat{r}f(r)$. Prove that $\vec{r} \times \frac{d\vec{r}}{dt} =$ Constant.

Group-B

Answer *any two* questions: 5×2=10

2. Evaluate the integral: $\int_s \vec{A}.\hat{n}ds$ where $\vec{A} = 2xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and s are the surfaces of the unit cube $0 \le x, y, z \le 1$. 5

3. Solve the differential equation: $x\frac{dy}{dx} + y = x^2y^2$. 5

4. If $u = f(x - y, y - z, z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 5

5. Find the unit vectors in cylindrical co-ordinate system and prove that they are orthogonal to one another. 5

Group-C

Answer *any two* questions:

6. (a) Find the differential equation whose solution is $xy = Ae^{x} + Be^{-x} + x^2$.

(b) Solve the differential equation: $(x^3 - x)\frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$.

7. (a) Evaluate the integral: $\iiint \frac{dxdydz}{x^2+y^2+z^2}$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Apply Green's theorem in a plane to evaluate the integral: $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$.

5

8. (a) Divide 120 into three parts so that the sum of their products taken two at a time shall be the maximum.

5

(b) Define Jacobian, $J\left(\frac{u,v}{x,y}\right)$. Prove that $J\left(\frac{u,v}{x,y}\right) \times J\left(\frac{x,y}{u,v}\right) = 1$. 1+4

9. (a) A particle of mass m is falling under gravity. The resistive force from the medium is proportional to velocity of the particle. Set up the equation of motion of the particle and determine the velocity at a time t. Show its graphical variation. 1+3+1

(b) The position vector of a particle is: $\vec{r} = r\hat{e}_r + \hat{k}Z$. Determine the velocity and acceleration of the particle.

2½+2½