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Measuring the rebound resilience of a bouncing ball

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Abstract
Some balls which are made of high-quality rubber (an elastomeric) material, such as tennis or squash balls, could be used for the determination of an important property of such materials called resilience. Since a bouncing ball involves a single impact we call this property ‘rebound resilience’ and express it as the ratio of the rebound height to the initial drop height of the ball. We determine the rebound resilience for three different types of ball by calculating the coefficient of restitution of the ball–surface combination from the experimentally measurable physical quantities, such as initial drop height and time interval between successive bounces. Using these we also determine the contact time of balls with the surface of impact. For measurements we have used audio, motion and surface-temperature sensors that were interfaced through a USB port with a computer.

Introduction

Resilience is the property of a body due to which it regains its original shape and size after suffering a substantial impact. When the deformation results from a single impact, the ratio of the energy after and before the impact expressed as a percentage is termed ‘rebound resilience’. In reference to a bouncing ball made of rubber or plastic polymer it is defined as the ratio of rebound height to the drop height [1].

If h is the rebound height and H is the initial height, the rebound resilience r can be related to the coefficient of restitution (CoR) e of the ball–surface combination as

$$r = h/H = e^2. \qquad (1)$$

As is well known, the value of CoR determines the nature of collision between the ball and the surface. The technique of dropping the balls to determine the value of CoR has been used for a long time [2–9]. Recent studies [8, 9] have shown that the CoR value depends on the initial drop height of the ball. The dependence of resilience on CoR or vice versa is significant for the manufacture and subsequent use of these balls in a sporting activity. For instance, the rebound resilience of a squash ball (yellow dot) of mass 24 g and diameter (outer) 4.0 cm has been specified [10] as 12% at 23 °C and 26% at 45 °C, as shown in table 1.

Table 1. Squash ball specifications.

WSF specifications of a standard yellow dot squash ball
Diameter (mm) 40.0 + or −0.5
Weight (g) 24.0 + or −1.0
Stiffness (N mm^{-1}) @ 23 °C 3.2 + or −0.4
Seam strength (N mm^{-1}) 6.0 minimum
Rebound resilience—from 100 in/254 cm
@ 23 °C 12% minimum
@ 45 °C 26%–33%

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This implies that resilience and hence CoR is a temperature-dependent quantity. Similar specifications [11] exist for balls of two other types used in the experiment. It is also known [6–9] that the time interval between the successive bounces varies with the initial drop height as well as the type of material the ball is made of. We can establish a relationship between this time interval and the CoR to determine the value of CoR as a function of initial height. The contact time of a rebounding ball with the surface on first impact can be determined as a function of initial height. The value of contact time can be used to further determine the magnitude of contact force experienced by the ball with surfaces of different types. In this study our aim is to establish a relationship between time intervals of successive bounces and CoR, rebound resilience and contact time of bouncing balls of different types and then describe an experimental arrangement to study their dependence on initial drop height at fixed surface temperature. The experiments have been performed with high-quality analogue (audio, surface temperature) and digital (motion detector) sensors through a computer interface. The audio sensor is an electret microphone that can detect a frequency range of 20 Hz–15 kHz. The maximum frequency is limited only by the sample data collection rate of the interface used with the sensor. We have used a computer interface that limits the maximum frequency at 10 kHz. The surface-temperature sensor is a thermistor with a measuring range of −25–125 °C and accuracy of ±0.2–±0.5 °C. The surface-temperature sensor [12] is specially meant for taking skin temperature measurements, human respiration studies etc. We have used it to measure the temperature of the surface of the balls. The motion detector sensor uses ultrasound waves to detect the position of the moving object, which in our case is the bouncing ball. These three sensors are shown in figure 1.

Figure 1. (From left) motion sensor, surface temperature sensor, audio sensor, interface.

Theory and methodology

Let a ball of mass m be dropped vertically down from a height H on a horizontal rigid cemented surface. Using the law of conservation of energy

$$mv^2/2 = mgH, \tag{2}$$

where v is the speed of the ball as it strikes the surface.

From (2) we get

$$v = \sqrt{2gH}. \tag{3}$$

After first impact the speed becomes

$$v' = ev, \tag{4}$$

where e is the coefficient of restitution (CoR) of the ball–surface.

The ball again rises to a height h before falling down to suffer a second impact with the surface. The time interval between the first and second impacts is given by

$$\Delta t = 2v'/g. \tag{5}$$

Using (3) and (4) in (5) we get

$$\Delta t = e\sqrt{8H/g}.$$

The CoR is thus given by

$$e = \Delta t\sqrt{g/8H}. \tag{6}$$

Using (6) we can determine the value of rebound resilience from (1).

The value of Δt is determined experimentally by using the vernier computer interface and an audio sensor for different values of drop height and for three types of balls.

To determine the contact time of the ball on first impact with the floor we consider the initial and final momentum of the ball

$$p_{\rm i} = mv$$

$$p_{\rm f} = mv'.$$

Figure 2. Experimental arrangement.

Using (4) the change in momentum is given by

$$|\Delta p| = mv(1 + e). \tag{7}$$

If F_c is the contact force experienced by the ball and t_c is the contact time of the ball on impact the change of momentum can be written as

$$|\Delta p| = F_c t_c. \tag{8}$$

If p_e is the pressure inside the ball we can express contact force as

$$F_c = p_e A_c, \tag{9}$$

where A_c is the contact area of the ball with the surface on impact and is given by [5].

$$A_c = (2\pi a)x,$$

where a = radius (internal) of the ball and x = squashing distance on impact.

The squashing distance is the small distance through which the ball's surface gets compressed on impact.

From (7)–(9) we obtain contact time as

$$t_c = (1 + e)mv/p_e(2\pi a)x. \tag{10}$$

The squashing distance x is given by [5]

$$x = v\sqrt{m/p_e(2\pi a)}. \tag{11}$$

Using (11) in (10) we get

$$t_c = (1 + e)\sqrt{m/p_e(2\pi a)}. \tag{12}$$

In our analysis we have made use of three different types of ball:

(i) a squash ball (Dunlop, single yellow dot) of mass 23.5 g, size $a = 1.62$ cm (internal radius) and internal pressure $p_e = 14.5$ psi (=100 kPa);
(ii) a lawn tennis ball (Wilson, #1) of mass 56 g, size $a = 2.54$ cm (internal radius) and internal pressure $p_e = 26.4$ psi (=182 kPa);
(iii) a table-tennis ball (Shields) of mass 2.7 g, size $a = 2$ cm and internal pressure $p_e = 14.4$ psi (=100 kPa).

To measure the time interval between two successive bounces as a function of the initial drop height we have used high-quality motion and audio sensors using a computer interface. The experimental setup in figure 2 shows the clamp holding a ball at a height measured with a wall-mounted height-measuring scale. The picture on the right-most side of this figure shows the audio sensor with the interface connected to the computer.

Results and discussion

In figure 3 we show the variation of sound pressure as a function of time for a squash ball dropped from a height of 0.75, 1.25 and 1.75 m on a hard cemented surface. The sound peak at the first bounce represents the moment when the ball experiences its first impact with the surface and the sound peak at the second bounce represents the moment at second impact. We measured the time interval Δt between these two peaks for a given initial drop height H.

We recorded such audio signals for heights ranging from 0.25 to 2.0 m and obtained the graph plot of the time interval Δt as a function of drop height H shown in figure 4. The standard errors are indicated by capped-vertical bars.

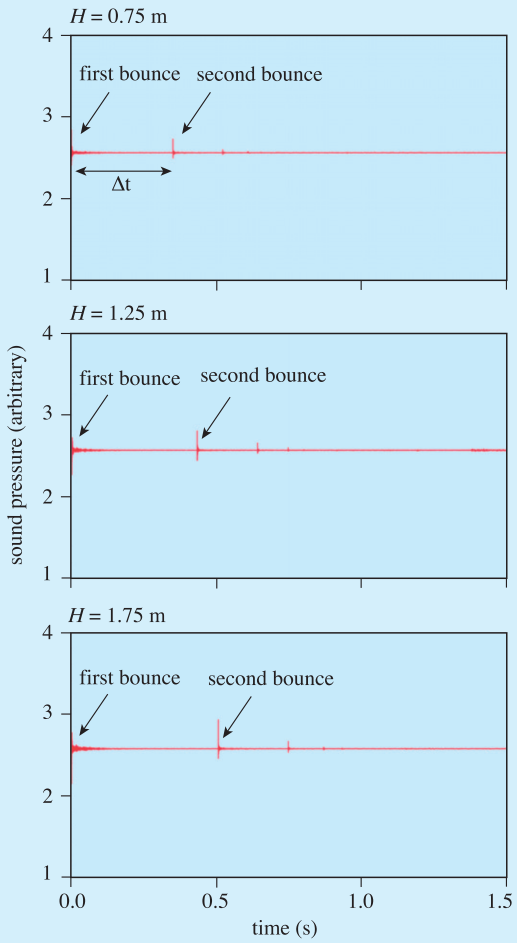

Figure 3. Audio signals of a squash ball for time-interval measurement.

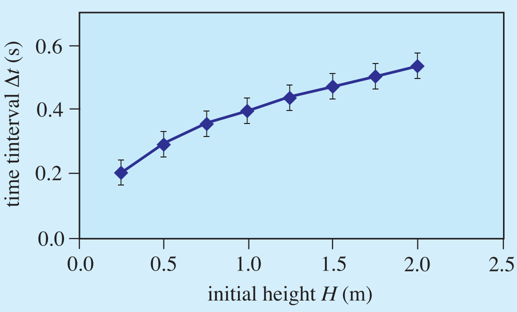

Figure 4. Squash ball Δt.

From figure 4 it is clear that Δt increases but non-linearly with H. The motion sensor has been used to detect the actual drop height of the ball and recording of acceleration at the moment of

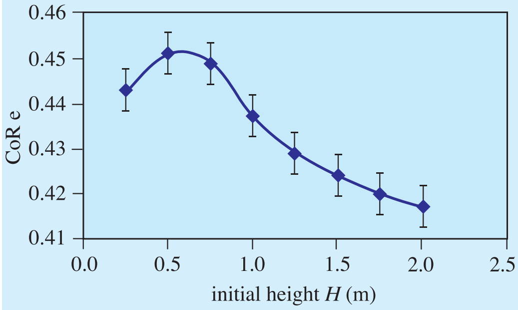

Figure 5. Squash ball 'e'.

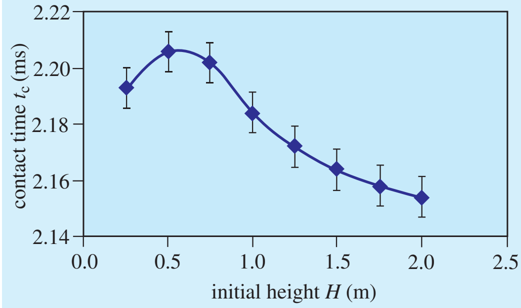

Figure 6. Squash ball 't_c'.

first impact. The coefficient of restitution e of the squash ball is determined from (6) and plotted as a function of drop height H in figure 5.

The value of e initially increases with H, becomes a maximum $e_{\max} = 0.451$ at $H = 0.5$ m and then decreases with increase in H. The behaviour of contact time t_c as calculated from (12), is quite similar to that of e, as shown in figure 6.

From figure 6 we see that the squash ball has maximum $t_{c\max} = 2.206$ ms at $H = 0.5$ m. The rebound resilience of the squash ball at this value of H is 20.34% at 28 °C ball–surface temperature.

In figure 7 we show the variation of sound pressure as a function of time for a table-tennis ball dropped from a height of 0.25, 0.75 and 1.25 m on a hard cemented surface.

We recorded such audio signals for heights ranging from 0.25 to 2.0 m and obtained the graph plot of the time interval Δt as a function of drop height H, as shown in figure 8.

From this figure 8 it is clear that Δt increases non-linearly with H.

The behaviour of the coefficient of restitution of the table-tennis ball as a function of H is shown in figure 9.

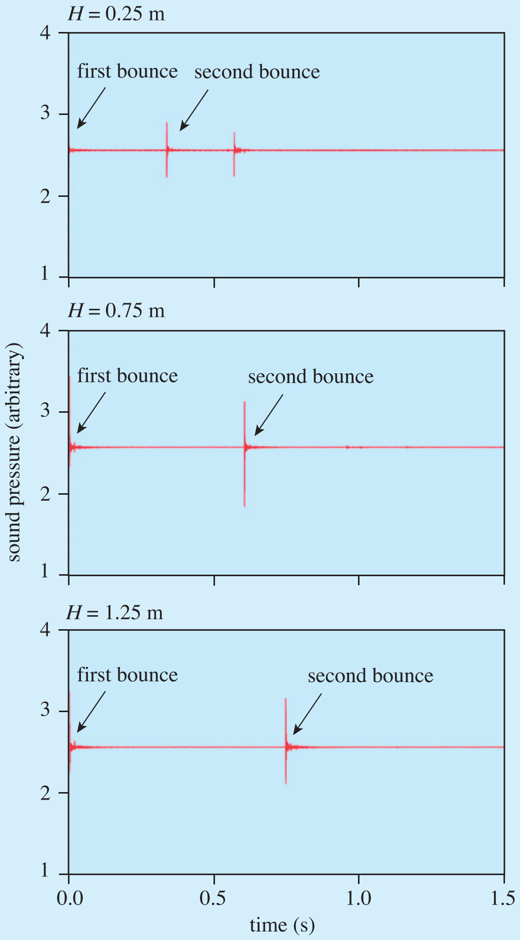

Figure 7. Audio signals of a table tennis ball for time-interval measurement.

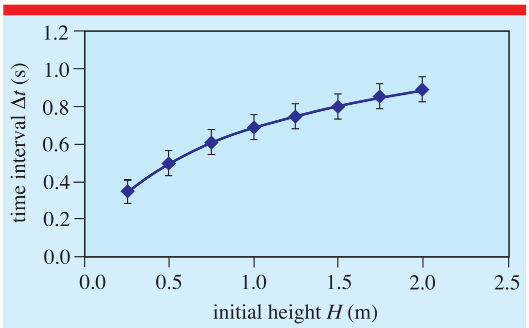

Figure 8. Table tennis ball Δt.

As expected, the values of CoR are much higher than those obtained for the squash ball. The value of e increases with H, becomes a maximum $e_{max} = 0.775$ at $H = 0.75$ m and

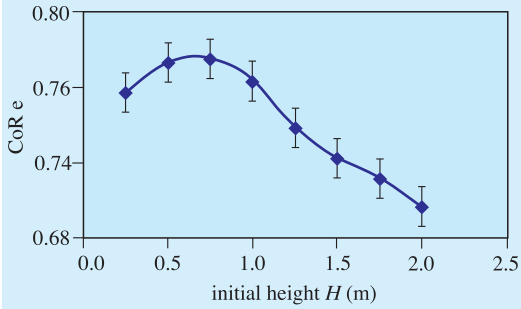

Figure 9. Table tennis ball 'e'.

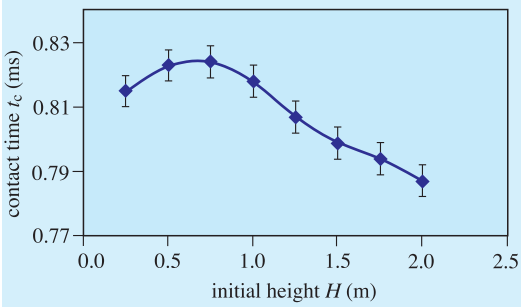

Figure 10. Table tennis ball t_c.

then gradually decreases. The contact times of the table-tennis ball are calculated from (12) and shown in figure 10.

As expected, the values of t_c are much lower than those for the squash ball. The table-tennis ball has maximum $t_{cmax} = 0.824$ ms at $H = 0.75$ m corresponding to the maximum value of e. The rebound resilience of the table-tennis ball at this value of H is 60% at 25 °C surface temperature of the ball.

Finally, for the lawn tennis ball, the audio signals are shown in figure 11 for three different drop heights of 0.25, 0.75 and 1.25 m.

We observe longer time intervals between successive bounces with increasing H, as shown in figure 12.

From figures 13 and 14 we observe higher values of e but longer contact times than for the squash ball at different values of drop heights.

The behaviour in the variation of e and t_c as a function of H is similar to that of the other balls but the maximum $e_{max} = 0.786$ and $t_{cmax} = 2.483$ ms occurs at $H = 1.5$ m. The rebound resilience of the tennis ball is 62% at $H = 1.5$ m at a ball–surface temperature of 28 °C.

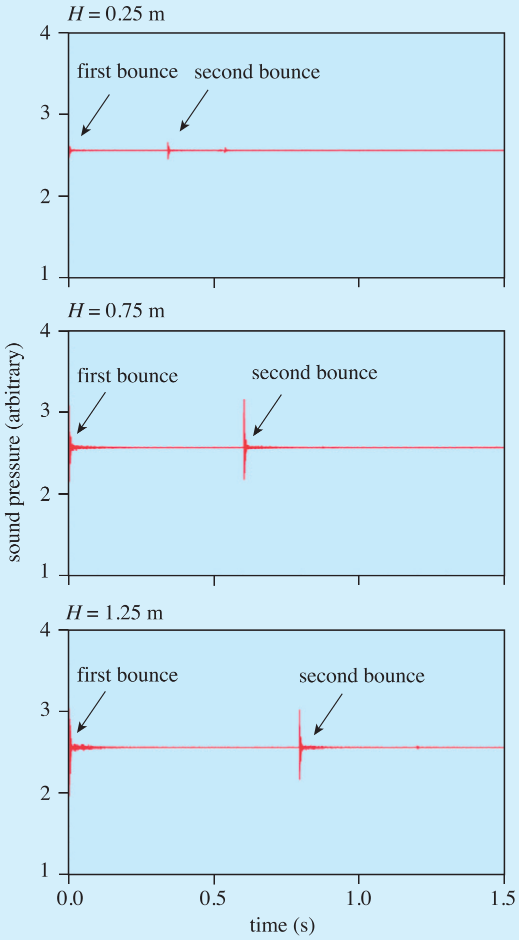

Figure 11. Audio signals for a lawn tennis ball for time-interval measurement.

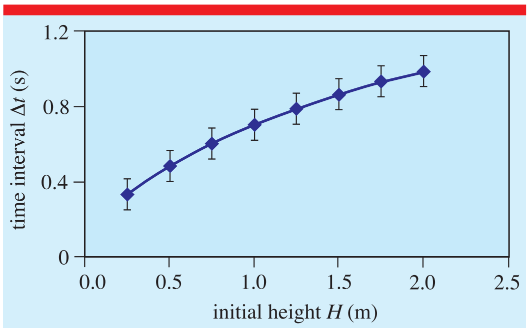

Figure 12. Lawn tennis ball Δt.

The above results can be interpreted from expression (6) in which the value of e is shown to depend on Δt and H. With increase in H initially the value of e increases in proportion to Δt. After

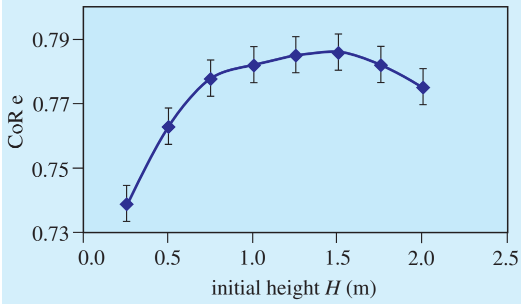

Figure 13. Lawn tennis ball 'e'.

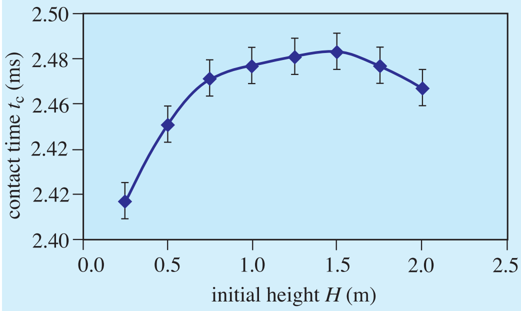

Figure 14. Lawn tennis ball 't_c'.

reaching a maximum, the value of e decreases with increase in H in inverse proportion to $\sqrt{H}$. This is because increase in Δt is not significant enough to oppose the decrease due to $\sqrt{H}$.

The fall in e and t_c with H is delayed in the case of the lawn tennis ball because unlike the other two balls the tennis ball is pressurized and therefore its Δt is longer and dominates for larger values of H.

The significance of our study is to present a novel method for students of physics to determine an important elastic property of a commonly used material such as rubber. The experiment can be repeated on different types of surfaces and the results compared. An even more interesting exercise would be to determine the dependence of rebound resilience on temperature. Work in this direction is in progress.

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