Assignment #4

Steven Porretta Student # 100756494

Code can be found at https://github.com/0xSteve/learning_automata_simulator

1 Question 1

c1 = 0.05, c2 = 0.7, N = 13.

In this section we will examine some code snippets from the first question.

Listing 1: Testbench code for the Tsetlin.

```
c2 = 0.7
       c1 = 0.05
3
       for i in range (0, 7):
           print("c1 = " + str(c1) + ", c2 = " + str(c2) + ", N = 13.")
           a = la.Tsetlin(13, 2, [c1, c2])
           a.simulate(50, 30001)
7
           b = ala.Tsetlin.stationary_probability_analytic([c1, c2], 13)
           c = ala.Tsetlin.number_of_states_estimate([c1, c2])
           print("Tsetlin P1(infinity) = " + str(b) + "(Analytic)")
9
10
           print("Tsetlin P1(infinity) = " + str(a.action_average[0]) + "(Simulated)")
11
           print("Tsetlin # of states required = " + str(c) + "(Estimate)")
12
           c1 += 0.1
           c1 = round(c1, 2)
```

This excerpt of code generates the entire quantity of required code for this question. As can be seen in the following code-snippet.

Listing 2: Testbench output.

```
Tsetlin P1(infinity) = 0.9999999990725503(Analytic)
   Tsetlin P1(infinity) = 1.0(Simulated)
   Tsetlin # of states required = 3(Estimate)
   c1 = 0.15, c2 = 0.7, N = 13.
   Tsetlin P1(infinity) = 0.9999778874501014(Analytic)
   Tsetlin P1(infinity) = 1.0(Simulated)
   Tsetlin # of states required = 4(Estimate)
   c1 = 0.25, c2 = 0.7, N = 13.
10
   Tsetlin P1(infinity) = 0.9990142374252533(Analytic)
   Tsetlin P1(infinity) = 0.999998000067(Simulated)
12
   Tsetlin # of states required = 6(Estimate)
13
   c1 = 0.35, c2 = 0.7, N = 13.
   Tsetlin P1(infinity) = 0.9865794150962881(Analytic)
   Tsetlin P1(infinity) = 0.999797340089(Simulated)
15
   Tsetlin # of states required = 9(Estimate)
17
   c1 = 0.45, c2 = 0.7, N = 13.
   Tsetlin P1(infinity) = 0.9151468144874294(Analytic)
18
   Tsetlin P1(infinity) = 0.980783973868(Simulated)
20
   Tsetlin # of states required = 18(Estimate)
21
   c1 = 0.55, c2 = 0.7, N = 13.
   Tsetlin P1(infinity) = 0.7453193640776348(Analytic)
22
23
   Tsetlin P1(infinity) = 0.786514449518(Simulated)
24
   Tsetlin # of states required = O(Estimate)
25
   c1 = 0.65, c2 = 0.7, N = 13.
26
   Tsetlin P1(infinity) = 0.5680008401435711(Analytic)
27
   Tsetlin P1(infinity) = 0.570874970834(Simulated)
   Tsetlin # of states required = O(Estimate)
```

Now that it is seen working as one would expect, considering rounding errors from python 3.6, it is time to take a look at the useful snippets of code governing the functionality of the Tsetlin, and Krylov automata.

Listing 3: Tsetlin core code.

```
1
         def next_state_on_reward(self):
2
            ''', Find the next state of the learner, given that the teacher
3
               rewarded.''
4
            if (self.current_state mod (self.N / self.R) != 1):
5
                self.current_state -= 1
6
7
        def next_state_on_penalty(self):
            '''Find the next state of the learner, given that the teacher
9
               penalized. '''
10
            if(self.current_state mod (self.N / self.R) != 0):
11
                self.current_state += 1
            elif(self.current_state mod (self.N / self.R) == 0):
12
13
                # Don't really add states, just cycle through N, 2N, 4N, etc.
14
                if(self.current_state != self.N):
                    a = (self.N / self.R) mod self.N
15
16
                    self.current_state = a + self.current_state
17
18
                    self.current_state = self.N / self.R
19
20
        # Determine the next state as the teacher.
21
        def environment_response(self):
22
            '''Determine the next state of the learner from the perspective
23
            of the teacher.','
24
            response = uniform(0, 1)
25
            penalty_index = 1
26
            if(self.current_state <= self.n):</pre>
27
                self.actions[0] += 1
28
                penalty_index = 0
29
            else:
30
                self.actions[1] += 1
31
32
            if(response > self.c[penalty_index]):
33
                # Reward.
34
                self.next_state_on_reward()
35
            else:
36
                # Penalty.
37
                self.next_state_on_penalty()
```

The above is the core code of the Tsetlin machine, governing state translations and action choices. Essentially, whenever it is in the range 1 to N, it chooses action α_1 and α_2 otherwise. This code is essentially the same for the Krylov machine, which we will see in the next section.

2 Question 2

Listing 4: testbench for the Krylov 2-action.

```
for i in range(0, 7):
       print("c1 = " + str(c1) + ", c2 = " + str(c2) + ", N = 13.")
2
       a = la.Tsetlin(13, 2, [c1/2, c2/2])
       a.simulate(50, 30001)
       b = ala.Tsetlin.stationary_probability_analytic([c1, c2], 13)
       c = ala.Tsetlin.number_of_states_estimate([c1, c2])
7
       d = la.Krylov(13, 2, [c1, c2])
8
       d.simulate(10, 50000)
9
       e = ala.Tsetlin.stationary_probability_analytic([c1, c2], 13)
10
       f = ala.Tsetlin.number_of_states_estimate([c1, c2])
       print("Tsetlin P1(infinity) = " + str(b) + "(Analytic)")
11
       print("Tsetlin P1(infinity) = " + str(a.action_average[0]) + "(Simulated)")
12
       print("Tsetlin # of states required = " + str(c) + "(Estimate)")
13
       print("Krylov P1(infinity) = " + str(e) + "(Analytic)")
14
       print("Krylov P1(infinity) = " + str(d.action_average[0]) + "(Simulated)")
15
16
       print("Krylov # of states required = " + str(f) + "(Estimate)")
```

As can be seen from the code, this test bench looks very similar to the test bench of question 1, however, note the c vector for the Tsetlin automaton is now $c_1/2$, $c_2/2$. As is expected, both automata behave in the same manner. as can be seen from the output code snippet.

Listing 5: testbench output for the Krylov 2-action.

```
1
       c1 = 0.05, c2 = 0.7, N = 13.
2
       Tsetlin P1(infinity) = 0.9999999890725503(Analytic)
3
       Tsetlin P1(infinity) = 1.0(Simulated)
4
       Tsetlin # of states required = 3(Estimate)
5
       Krylov P1(infinity) = 0.9999999890725503(Analytic)
6
       Krylov P1(infinity) = 1.0(Simulated)
7
       Krylov # of states required = 3(Estimate)
       c1 = 0.15, c2 = 0.7, N = 13.
8
       Tsetlin P1(infinity) = 0.9999778874501014(Analytic)
9
       Tsetlin P1(infinity) = 0.999999333356(Simulated)
10
       Tsetlin # of states required = 4(Estimate)
11
12
       Krylov P1(infinity) = 0.9999778874501014(Analytic)
       Krylov P1(infinity) = 1.0(Simulated)
13
       Krylov # of states required = 4(Estimate)
14
15
       c1 = 0.25, c2 = 0.7, N = 13.
16
       Tsetlin P1(infinity) = 0.9990142374252533(Analytic)
17
       Tsetlin P1(infinity) = 1.0(Simulated)
18
       Tsetlin # of states required = 6(Estimate)
19
       Krylov P1(infinity) = 0.9990142374252533(Analytic)
20
       Krylov P1(infinity) = 1.0(Simulated)
21
       Krylov # of states required = 6(Estimate)
22
       c1 = 0.35, c2 = 0.7, N = 13.
23
       Tsetlin P1(infinity) = 0.9865794150962881(Analytic)
24
       Tsetlin P1(infinity) = 1.0(Simulated)
25
       Tsetlin # of states required = 9(Estimate)
26
       Krylov P1(infinity) = 0.9865794150962881(Analytic)
       Krylov P1(infinity) = 1.0(Simulated)
27
       Krylov # of states required = 9(Estimate)
28
29
       c1 = 0.45, c2 = 0.7, N = 13.
       Tsetlin P1(infinity) = 0.9151468144874294(Analytic)
30
       Tsetlin P1(infinity) = 1.0(Simulated)
31
32
       Tsetlin # of states required = 18(Estimate)
       Krylov P1(infinity) = 0.9151468144874294(Analytic)
33
       Krylov P1(infinity) = 1.0(Simulated)
34
35
       Krylov # of states required = 18(Estimate)
36
       c1 = 0.55, c2 = 0.7, N = 13.
37
       Tsetlin P1(infinity) = 0.7453193640776348(Analytic)
       Tsetlin P1(infinity) = 0.983473884204(Simulated)
38
39
       Tsetlin # of states required = O(Estimate)
       Krylov P1(infinity) = 0.7453193640776348(Analytic)
40
       Krylov P1(infinity) = 0.999998(Simulated)
41
       Krylov # of states required = 0(Estimate)
42
43
       c1 = 0.65, c2 = 0.7, N = 13.
       Tsetlin P1(infinity) = 0.5680008401435711(Analytic)
44
       Tsetlin P1(infinity) = 0.670734975501(Simulated)
45
46
       Tsetlin # of states required = 0(Estimate)
47
       Krylov P1(infinity) = 0.5680008401435711(Analytic)
48
       Krylov P1(infinity) = 0.870114(Simulated)
       Krylov # of states required = 0(Estimate)
49
```

Observing the code for the Krylov machine, one notices that most of the code is inherited from the Tsetlin machine, except the state translations. It is incredible, that the only major distinction is that a penalty is treated as a penalty with 50% probability and a success otherwise. Literally, all other code for the Krylov machine is inherited from the Tsetlin.

Listing 6: Krylov core code.

```
def next_state_on_penalty(self):
             \ref{eq:constraints} , \ref{eq:constraints} find the next state of the learner, given that the teacher
2
3
                penalized.'''
4
             \mbox{\tt\#} If this number is greater than 0.5, then penalize the learner.
5
6
             is_penalty = uniform(0, 1)
7
8
             if(is_penalty >= 0.5):
9
                  Tsetlin.next_state_on_penalty(self)
10
                  Tsetlin.next_state_on_reward(self)
```

3 Question 3