UMDCTF 2017

Forensics 15 Points

The Lost Flag

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Prove: $\sqrt{3} \notin \mathbb{Q}$.

- (i) Take the negation

$$\sqrt{3} \in \mathbb{Q}$$

- (ii) By definition of rationality:

$$\exists a, b \in \mathbb{Z}, b \neq 0 : \ \sqrt{3} = \frac{a}{b}$$

- (iii) Squaring both sides and multiplying all by b^2

$$3b^2 = a^2$$

- (iv) By the Fundamental Theorem of Arithematic (FTA):

$$b = p_1^{e_1} p_2^{e_2} ... p_k^{e_k}$$
$$b^2 = (p_1^{e_1} p_2^{e_2} ... p_k^{e_k})(p_1^{e_1} p_2^{e_2} ... p_k^{e_k})$$

- (v) From (iv) and (iii) we get:

$$\exists n_1 \in \mathbb{Z}^{even} : 3n_1 = b^2$$

- (vi) From (iii) and (v):

$$\exists n_2 \in \mathbb{Z}^{odd} : 3n_2 = a^2$$

- (vii) We also have by the FTA that $3 \mid a$, meaning:

$$\exists n_3 \in \mathbb{Z}^{even} : 3n_3 = a^2$$

- (viii) (vii) and (vi) contradict each other as a^2 cannot be simultaneously divisible by 3 an even and odd number of times

$$c$$

A contradiction was reached, therefore $\sqrt{3} \notin \mathbb{Q}$.

$\square$