Twyne V1

Whitepaper*

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Abstract

Twyne presents a noncustodial, risk-modular credit delegation protocol to overcome capital inefficiencies in DeFi lending markets, where the presence of conservative, risk-averse passive lenders leave significant borrowing capacity untapped. Built atop established DeFi lending protocols (e.g., Aave, Euler), our novel mechanism allows these passive lenders (Credit LPs) to delegate unused borrowing power to active borrowers, enabling higher leverage while maintaining overcollateralization. Borrowers access enhanced Twyne LTVs beyond underlying lending market limits, with positions secured via a dual-LTV framework and credit reservation invariant that keeps combined exposures within safe underlying thresholds. We employ liquidation-through-inheritance to transfer unhealthy positions, minimizing market impact and retaining capital; if this fails, fallback liquidations by the underlying protocol sanitize the combined position, potentially incurring Credit LP losses (while internal liquidations avoid them entirely). Extensive mathematical modeling, simulations, and smart contract verification quantify these loss scenarios under varying liquidator efficiencies, demonstrating substantial capital efficiency gains with extra yield for lenders by in exchange for quantifiable added risk. By operating as an overlay protocol that unifies borrowing capacity across existing markets without creating new fragmented liquidity pools, Twyne advances DeFi toward greater capital efficiency and granular risk exposure, while preserving the security guarantees of its established lending infrastructure.

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*DISCLAIMER: The outlined ideas for the Twyne V1 Protocol are for informational purposes only. The scope and implementation may change and should not be relied upon. Note that whenever the term Twyne is used, it refers to the protocol, not the company Twyne Devco OÜ.

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1 Introduction

Twyne is a noncustodial, risk-modular credit delegation protocol implemented for the Ethereum Virtual Machine. By integrating with established lending DeFi markets, Twyne unlocks levels of capital efficiency which currently are not possible due to both the balkanization of liquidity across lending protocols and the presence of unused borrow demand within them. Twyne creates a new trustless primitive that allows lenders across established lending markets to electively take on higher risks in exchange for higher yields without shouldering any new risks onto lending market participants that do not desire to interact with Twyne. By doing so, Twyne enables borrowers to access on-demand higher collateral factors and cross-lending market operations. We argue that this creates higher flexibility for borrowers, selective risk/yield trade-offs for credit lenders, as well as higher capital utilization rates for all the lending markets integrated through Twyne. We believe Twyne's democratic value-additive experience to be the next step towards demonstrating the potential for DeFi to unlock levels of credit liquidity currently unavailable in decentralized finance.

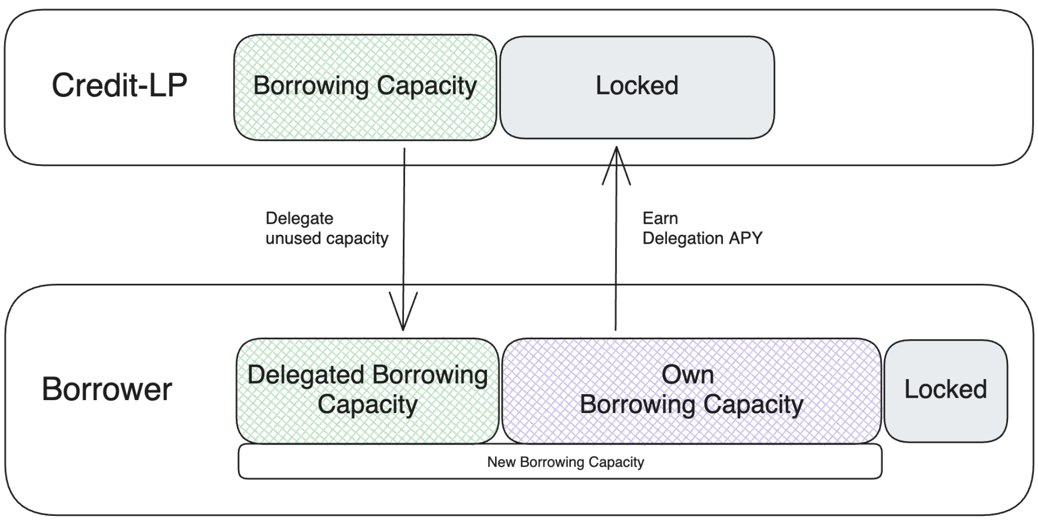

Figure 1: Comparison of borrowing limits for Borrowers operating through Twyne and established Lending Markets. By tapping into the unused borrowing power of other users (Credit LPs), a Borrower can boost their borrow limit while still accessing the same lending market liquidity, and satisfying its security constraints.

1.1 DeFi Lending Market Inefficiencies

The DeFi lending ecosystem is plagued by several structural inefficiencies that hinder optimal capital utilization and price discovery. Liquidity is fragmented across multiple protocols, preventing unified price discovery and efficient resource allocation. A significant portion of depositors do not utilize their available borrowing power, leaving substantial capital idle. Lending protocols enforce conservative loan-to-value (LTV) ratios to ensure solvency, which restricts credit access for users capable of managing higher leverage. Additionally, uniform risk parameters are applied across all participants, limiting the ability to tailor risk-return profiles to individual preferences. These inefficiencies present a substantial opportunity for innovation to enhance capital efficiency and user outcomes.

1.2 The Twyne Value Proposition

Twyne directly tackles these challenges through a novel credit delegation mechanism. This mechanism enables passive lenders, known as Credit LPs (CLPs), to monetize their unused borrowing power by delegating it to other users. Consequently, borrowers can access higher LTV ratios than those offered by external lending protocols, thereby unlocking additional liquidity while adhering to the risk constraints of the underlying markets. Crucially, Twyne's design preserves the security guarantees of the base lending protocols by layering its delegation logic on top. Lenders benefit from a new yield stream derived from borrowers utilizing their delegated credit, without incurring additional asset price exposure. By optimizing the utilization of existing capital, Twyne fosters a positive-sum system where borrowers gain enhanced leverage, lenders earn incremental returns, and external protocols experience increased liquidity utilization.

1.3 Protocol Participants

At its core, Twyne functions as a marketplace for unused borrowing power, built upon established DeFi lending protocols. It facilitates the rental of idle credit capacity by those who need it most. The protocol distinguishes between two primary participant types: Credit LPs (CLPs) and Borrowers.

1.3.1 Credit LPs (CLPs)

These users deposit assets into lending protocols to earn yield but do not intend to borrow against their deposits. By staking their lending market receipt tokens (e.g., aTokens, cTokens) into Twyne's Credit Vaults, CLPs authorize Twyne to delegate their borrowing power to others in exchange for additional interest. CLPs retain the base yield from the original protocol and earn extra yield from borrowers who rent their credit, thus creating a dual income stream. While they inherit the security assumptions of their chosen lending protocol, they also indirectly participate in backing higher-leverage borrowers, introducing a slight increase in risk.

1.3.2 Borrowers

These users lock collateral and take overcollateralized loans, similar to traditional lending protocols. However, through Twyne, they can reserve additional borrowing power from CLPs, enabling them to exceed the conservative LTV caps of the base protocols. To do this, borrowers deposit collateral into Twyne-managed Collateral Vaults and reserve credit from Credit Vaults. They pay interest to both the lending protocol for the borrowed funds and to CLPs for the use of their borrowing power. In return, they access superior leverage, often unattainable through conventional means.

1.4 Protocol Design Philosophy

Twyne's architecture is guided by five core principles:

- **Risk Modularity**: Users can customize their risk exposure according to individual preferences, allowing for tailored risk-return strategies.
- **Protocol Integration**: Twyne ensures compatibility with existing DeFi lending protocols without requiring modifications to their codebases, facilitating seamless operation.
- **Safety**: The protocol incorporates mechanisms to safeguard CLPs from undue losses, particularly under high-leverage scenarios, ensuring participant security.
- **Capital Efficiency**: By unlocking idle credit capacity, Twyne maximizes asset utilization across the ecosystem, enhancing overall efficiency.
- **Economic Sustainability**: Incentives are aligned to promote long-term protocol viability, encouraging behaviors that contribute to system stability and growth.

These principles underpin every aspect of Twyne's design, from its mathematical models to its risk buffers, interest rate curves, and liquidation mechanics.

1.5 Document Structure

The remainder of this document is structured as follows: Section 2 introduces the protocol's high-level design and outlines the key smart contract components and their interactions. Section 3 establishes the core mathematical framework used throughout the protocol. Section 4 delves into the mechanics of credit delegation, providing the central innovation behind Twyne. Section 5 explains the interest rate models used to determine the cost of credit for borrowers and returns for lenders. Section 6 covers the protocol's liquidation framework, with a focus on maintaining capital integrity under adverse conditions. Section 7 applies these theoretical concepts to quantify and simulate when and how Credit LPs might face capital impairment. Supporting materials, including proofs, algorithms, and numerical analysis, are provided in the appendices.

2 Protocol Architecture

The Twyne protocol represents a sophisticated financial system designed to enhance capital efficiency through credit delegation. This section describes the protocol's architectural components, their interactions, and the fundamental principles that govern the system's operation.

2.1 System Components

Twyne operates through three primary components that interact to form a cohesive credit delegation system. At the foundation are the External Lending Protocols, which consist of established DeFi lending markets such as Aave and Euler. These protocols provide the underlying liquidity, determine external liquidation loan-to-value (LTV) thresholds ($\tilde{\lambda}_e^X$) for various assets X (e.g., ETH, WBTC, USDC, etc) that serve as collateral, and specify liquidation policies that function as a backstop for Twyne.

Building upon this foundation, Credit Vaults serve as protocol-deployed smart contracts that hold assets from Credit LPs and manage the delegation of borrowing power. These vaults act as the critical bridge between liquidity providers and borrowers, enabling the efficient allocation of unused borrowing capacity across the system.

Finally, Collateral Vaults represent user-deployed smart contracts that hold *Borrower* collateral, reserve credit from Credit Vaults, and interact with external lending protocols. These vaults encapsulate individual borrower positions while maintaining the necessary relationships with both the Credit Vaults and external protocols.

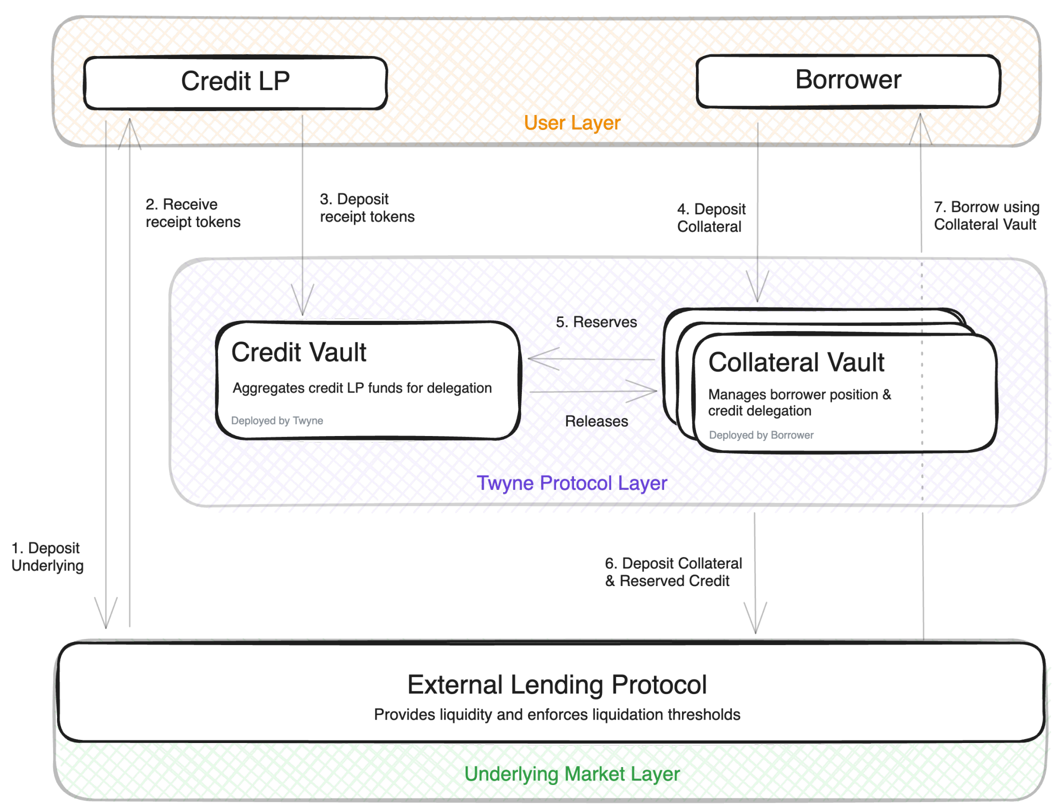

Figure 2: Architecture of the Twyne protocol showing the relationships between External Lending Protocols, Credit Vaults, Collateral Vaults, and user flows.

2.2 Protocol Operational Flow

The operation of the Twyne protocol can be understood through examining the distinct sequences of interactions for different participant types.

2.2.1 Credit LP Participation Flow

Credit LPs (CLPs) begin their participation by depositing assets into an external lending protocol, where they receive receipt tokens (such as aTokens or cTokens) representing their deposits. These receipt tokens are then staked in a Twyne Credit Vault, enabling the CLPs to begin earning additional yield as their borrowing power is delegated to borrowers seeking higher leverage.

2.2.2 Borrower Participation Flow

Borrowers initiate their participation by deploying a Collateral Vault through the Twyne Factory. After deployment, they deposit collateral assets (C) into their Collateral Vault, which automatically reserves credit (C_{LP}) from the appropriate Credit Vault. This credit reservation enables Borrowers to access loans (B) from the external lending protocol at a higher LTV than would otherwise be possible. Depending on the type/version of the Collateral Vault, the reserved credit asset may be of two types: correlated to the Borrower's collateral, or to the Borrower's loan asset.

2.2.3 Interest and Yield Flow

The protocol orchestrates multiple interest flows to ensure appropriate compensation for all participants. External protocols distribute yield to all lenders, including the Credit Vaults that hold CLP assets. Borrowers pay interest to the external protocol on their borrowed assets while simultaneously paying a siphoning rate to CLPs through a reduction in their collateral. This dual interest structure ensures that CLPs receive both the external protocol yield and additional compensation for providing their borrowing power to the system.

2.3 Credit Delegation Model

The core innovation of Twyne lies in its credit delegation model, which enables borrowers to access higher loan-to-value ratios while maintaining system safety.

2.3.1 Credit Delegation Principles

The credit delegation system operates on four fundamental principles. First, CLPs provide unused borrowing power by staking their receipt tokens from external lending protocols. Second, borrowers reserve this credit in proportion to their collateral (or outstanding loan) and desired LTV parameters. Third, the combined position consisting of borrower collateral plus reserved credit maintains a safe buffer below external liquidation thresholds. Finally, interest flows from borrowers to CLPs in proportion to the delegated credit, ensuring fair compensation for the provided borrowing power.

2.3.2 Multi-Perspective LTV

A key architectural feature of Twyne is its dual perspective on loan-to-value ratios, which enables the protocol to offer enhanced leverage while maintaining safety. The Twyne LTV (λ_t) represents the ratio of borrowed assets to the borrower's own collateral, which can exceed external protocol limits ($\lambda_t > \tilde{\lambda}_e$). In contrast, External LTV (λ_e) measures the ratio of borrowed assets to total collateral including reserved credit, which remains within the limits of the external protocol with an appropriate safety buffer. These dual perspectives allow borrowers to operate at higher LTVs from Twyne's view while remaining safe from the external protocol's perspective.

2.4 Credit Vault Architecture

Credit Vaults serve as the cornerstone of Twyne's credit delegation system, functioning as the bridge between Credit LPs and Borrowers.

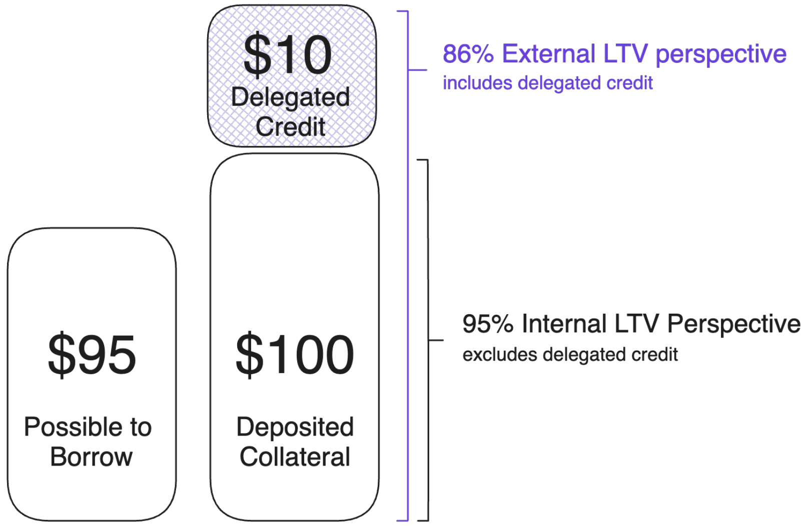

Figure 3: Comparison of Twyne LTV and External LTV perspectives

2.4.1 Credit Vault Functions

These vaults perform five critical functions within the protocol. They provide asset custody by safely holding CLP market receipt tokens, maintain credit tracking systems to monitor total credit capacity and utilization, process credit reservation requests from Collateral Vaults, accept excess credit released during rebalancing operations, and track and distribute yields to CLPs according to their proportional contribution to the system.

2.4.2 Credit Availability Management

Credit Vaults maintain several critical metrics to ensure proper credit allocation throughout the system. The Total Credit Existing (C_{LP}^{total}) represents the sum of all credit provided by Credit LPs to the system. Total Credit Reserved ($C_{LP}^{reserved}$) tracks the sum of all credit currently reserved by active borrowers. Credit Utilization (u) is calculated as the ratio of reserved credit to total credit, serving as a key parameter in interest rate determination. Available Credit ($C_{LP}^{available}$) indicates the amount of credit still available for reservation by new or existing borrowers. These metrics collectively govern both credit availability for new borrowers, user-specific deposit/borrow caps, and the interest rates paid to CLPs.

2.5 Collateral Vault Architecture

Collateral Vaults encapsulate individual borrower positions within the Twyne protocol, providing isolated risk management and position tracking.

2.5.1 Collateral Vault Functions

Each Collateral Vault performs multiple essential functions. It manages collateral by providing custody for borrower assets, handles credit reservation by securing appropriate credit from Credit Vaults based on position parameters, facilitates external borrowing by interacting with external lending protocols on behalf of the borrower, monitors position health by tracking key metrics and enforcing safety constraints including the maximum allowed liquidation LTV ($\tilde{\lambda}_t^{max}$) that a borrower can choose, and performs rebalancing operations to release excess credit back to Credit Vaults when position parameters change.

2.5.2 Position Parameters

Each Collateral Vault maintains several key parameters that define its state and behavior. The Collateral (C) represents the borrower's deposited collateral assets. Borrowed Amount (B) tracks assets borrowed from the external protocol. Reserved Credit (C_{LP}) indicates credit reserved from Credit Vaults to support the enhanced leverage. Liquidation LTV ($\tilde{\lambda}_t$) represents the borrower's chosen liquidation threshold within Twyne. External Protocol Parameters including $\tilde{\lambda}_e^C$, $\tilde{\lambda}_e^{C_{LP}}$, and β_{safe} govern the integration with external protocols and ensure safe operation within their constraints.

2.6 Protocol Invariants and Safety Mechanisms

Twyne implements comprehensive safety mechanisms to ensure protocol stability while maximizing capital efficiency. The credit reservation constraints ensure that borrowers never reserve less credit than required based on their collateral (and/or outstanding loans) and chosen liquidation parameters, preventing under-collateralization from the external protocol's perspective. Position health monitoring continuously verifies that positions remain healthy according to both Twyne and external protocol criteria, triggering appropriate actions when thresholds are approached. The safety buffer mechanism maintains a margin (β_{safe}) below external liquidation thresholds to prevent unexpected external liquidations that could disrupt the protocol's operation. Regular rebalancing operations release excess credit to maintain optimal position sizing as interest accrues and market conditions change. Finally, efficient liquidation mechanisms resolve unhealthy positions before they breach external protocol thresholds, protecting CLPs from adverse outcomes.

2.7 Protocol State Transitions

The Twyne protocol operates as a state machine with well-defined transition functions that maintain system integrity. Deposit operations add collateral to positions, potentially reserving additional credit to maintain proper ratios. Withdraw operations remove collateral from positions, potentially releasing excess credit back to the system. Borrow operations increase debt by accessing additional assets from external protocols, potentially requiring additional credit reservation. Repay operations reduce debt by returning borrowed assets, potentially releasing excess credit. Rebalance operations optimize credit utilization by releasing excess reserves after interest accrual. Liquidate operations transfer position ownership to resolve unhealthy states before they impact system stability. Each state transition enforces protocol invariants to maintain system integrity throughout the operation lifecycle.

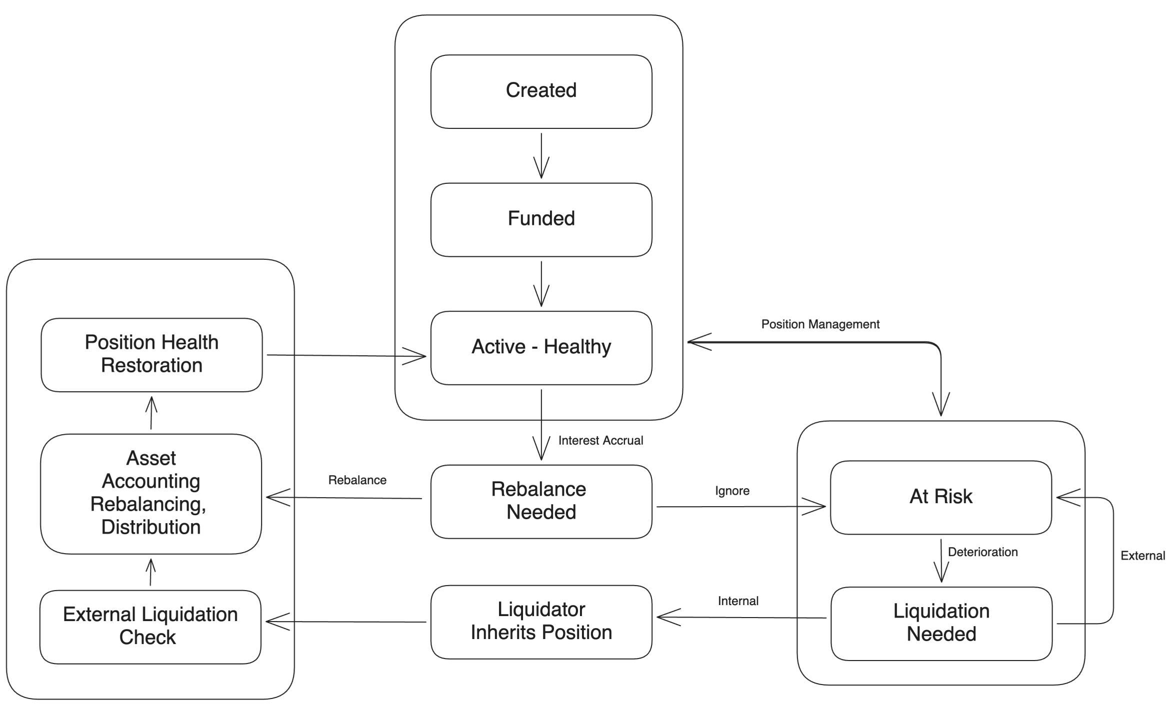

Figure 4: Operational flow of the Twyne protocol, showing lifecycle and sanitization of Borrower positions

2.8 User-Facing Interface and Operations

The protocol provides comprehensive interfaces tailored to each participant type's needs.

2.8.1 Credit LP Operations

CLPs interact with the protocol through four primary operations. The stake operation allows them to deposit receipt tokens and begin providing credit to the system. The unstake operation enables withdrawal of receipt tokens along with any accrued yield. Claim yield operations collect accumulated interest from borrower payments. Finally, rebalance operations allow CLPs to release excess credit from unoptimized Borrower positions, improving overall system efficiency.

2.8.2 Borrower Operations

Borrowers have access to a comprehensive suite of operations for managing their leveraged positions. They begin by creating a vault through the deployment of a new Collateral Vault. Deposit and withdraw operations manage collateral within the vault. Borrow and repay operations interact with external protocols to manage debt positions. Parameter adjustment capabilities allow modification of liquidation LTV and other settings to adapt to changing market conditions. Rebalance operations release excess credit to optimize position efficiency and reduce interest costs.

2.8.3 Liquidator Operations

Liquidators maintain protocol health through two key operations. The liquidate operation allows them to take over unhealthy positions before they breach safety thresholds. Additionally, they can handle external liquidation events by processing the aftermath when positions are liquidated by external protocols, ensuring proper redistribution of remaining assets between borrowers and CLPs. These operations provide a comprehensive interface for all protocol participants to manage their positions effectively while maintaining system stability.

3 Mathematical Overview

The Twyne protocol operates on a rigorous mathematical foundation that governs credit delegation, risk assessment, and liquidation mechanics. This section formalizes the mathematical models underpinning the protocol's operation and establishes the theoretical framework that ensures its security and efficiency.

3.1 Notation and Definitions

We begin by defining the fundamental variables and parameters that characterize the state of the protocol:

- C: User's collateral deposited in their Collateral Vault
- B: User's borrowed amount from the external lending protocol
- C_{LP}: Credit reserved from Intermediate Vault (the asset is either identical to the Borrower's collateral or loan asset)
- C_{total}: Total collateral as seen by the external protocol, $C_{total} = C + C_{LP}$
- λ_t: Twyne loan-to-value ratio, $\lambda_t = \frac{B}{C}$
- λ_e: External loan-to-value ratio, $\lambda_e = \frac{B}{C+C_{LP}}$
- $\tilde{\lambda}_t$: Twyne liquidation LTV threshold (user-selected parameter)
- $\tilde{\lambda}_t^{max}$: Maximum Twyne liquidation LTV threshold allowed by the protocol (protocol-enforced parameter)

- $\tilde{\lambda}_e^C$: External protocol's liquidation LTV threshold for the collateral asset
- $\tilde{\lambda}_e^{C_{LP}}$: External protocol's liquidation LTV threshold for the credit asset
- $\tilde{\lambda}_e$: External protocol's blended liquidation LTV threshold (fixed parameter), $\tilde{\lambda}_e = \frac{\tilde{\lambda}_e^C \cdot C + \tilde{\lambda}_e^{C_{LP}} \cdot C_{LP}}{C + C_{LP}}$
- β_{safe}: Safety buffer providing margin below external liquidation threshold ($\beta_{safe} = 1$ implies that positions become simultaneously liquidatable on both Twyne and the underlying lending market)

Quantities C, B, C_{LP}, and C_{total} are always to be interpreted as expressed in common unit of account. This allows comparing them even if they refer to different native assets.

3.2 Loan-to-Value Ratios

The Twyne protocol operates with two distinct LTV perspectives that govern borrower positions:

3.2.1 Twyne LTV

The Twyne LTV represents the borrower's leverage from Twyne's perspective:

$$\lambda_t = \frac{B}{C} \tag{1}$$

Conceptually, this ratio tracks a borrower's leverage with respect to their own collateral, ignoring the additional safety provided by credit delegation. This is the effective LTV the borrower operates at and forms the basis for Twyne's liquidation mechanics.

3.2.2 External LTV

The external LTV represents how the borrower's position appears to the external lending protocol:

$$\lambda_e = \frac{B}{C + C_{LP}} \tag{2}$$

This ratio captures the full collateralization of the borrowed position as seen by the external protocol, including both the borrower's collateral and the reserved credit from CLPs. For the position to remain safe from external liquidation, this ratio must stay below the external protocol's liquidation threshold with an additional safety buffer.

At any given moment, the following relationship holds between λ_e and λ_t:

$$\lambda_t \geq \lambda_e. \tag{3}$$

Twyne thus allows the Borrower to tap into underlying lending market liquidity as if they had λ_e, even when λ_t may already be liquidatable on that external lending market.

4 Credit Delegation

The credit delegation model constitutes the theoretical foundation of the Twyne protocol, enabling novel capital efficiency gains that distinguish it from traditional lending markets. It represents a fundamental innovation in the structure of overcollateralized lending. While traditional lending protocols enforce a single, protocol-wide liquidation threshold, Twyne introduces user-specific thresholds that enable more granular risk management and capital efficiency. The core insight underpinning Twyne's credit delegation model is that borrowing power can be disaggregated from collateral ownership. This disaggregation allows users with excess borrowing capacity to effectively "rent" this capacity to other users who desire higher leverage, creating a more efficient allocation of capital resources.

Twyne is responsible for imposing overcollateralization of the Borrower's true position and making sure that any liquidations are triggered before the underlying lending market flags them for liquidation. This responsibility can be formalized through two conditions which represent the portfolio safety conditions that must hold on Twyne:

$$\begin{aligned} \lambda_t &\le \tilde{\lambda}_t & (4) \\ \lambda_e &\le \beta_{safe} \cdot \tilde{\lambda}_e . & (5) \end{aligned}$$

Equation 4 states that liquidations are not triggered on Twyne unless the Borrower's LTV λ_t exceeds their chosen liquidation LTV $\tilde{\lambda}_t$. Equation 5 states that the external LTV λ_e must, at all times, retain a buffer of safety β_{safe} (≤ 1) with respect to the external liquidation threshold $\tilde{\lambda}_e$.

The equality in **both** safety conditions is allowed to occur **solely** when the Borrower is liable to be liquidated on Twyne ($\lambda_t = \tilde{\lambda}_t$). When this occurs, the two safety conditions imply:

$$\tilde{\lambda}_t \cdot C = \beta_{safe} \cdot \tilde{\lambda}_e \cdot (C + C_{LP}) = \beta_{safe} \cdot \left(\tilde{\lambda}_e^{C} \cdot C + \tilde{\lambda}_e^{C_{LP}} \cdot C_{LP} \right), \tag{6}$$

which can be used to express the CLP asset amount C_{LP} that must be reserved by the Borrower at any given moment to ensure their desired liquidation LTV $\tilde{\lambda}_t$, given their instantaneous collateral C or loan B size:

$$\begin{aligned} C_{LP}^{ideal} &= C \cdot \left(\frac{\tilde{\lambda}_t}{\beta_{safe} \cdot \tilde{\lambda}_e^{C_{LP}}} - \frac{\tilde{\lambda}_e^{C}}{\tilde{\lambda}_e^{C_{LP}}} \right) & (7) \\ &= \frac{B}{\tilde{\lambda}_t} \cdot \left(\frac{\tilde{\lambda}_t}{\beta_{safe} \cdot \tilde{\lambda}_e^{C_{LP}}} - \frac{\tilde{\lambda}_e^{C}}{\tilde{\lambda}_e^{C_{LP}}} \right), & (8) \end{aligned}$$

where the specific choice of which of the two equations above to use depends on whether the CLP's credit asset provided to the Borrower's collateral vault, is of the same type[1] as the Borrower's collateral (7) or loan (8) assets. This ensures that, ignoring interest rate accrual effects [2], the reserved credit will be the right amount at all times regardless of the relative price dynamics between collateral and loan assets.

The above CLP invariant conditions 7 and 8 are verified by Twyne's smart contracts whenever an interaction affects a user's portfolio. If the necessary C_{LP} funds are not available to be reserved by the Borrower, either their total deposited collateral C or desired liquidation loan-to-value $\tilde{\lambda}_t$ must be capped to match available CLP funds.

4.1 Rebalancing Mechanics

An essential operational component of Twyne is the periodic rebalancing of collateral positions to maintain the optimal ratio between user collateral (C) and reserved credit (C_{LP}). As interest accrues, the collateral position of the borrower naturally decreases, thus increasing the proportion of reserved credit beyond what is required by the invariant conditions of the protocol. To address this imbalance, the protocol employs a rebalancing mechanism designed to release excess reserved credit C_{LP}^{excess} back into the intermediate vault. Mathematically, excess credit is calculated as:

$$C_{LP}^{excess} = C_{LP} - C_{LP}^{ideal}$$

If excess credit is positive, it indicates a surplus that can safely be released without compromising the position's health or triggering unintended liquidations. If the CLP assets are different from the Borrower's collateral, this excess credit (accumulated through accrual of interest paid on collateral) must be swapped for the CLP asset type upon rebalancing.

This mechanism not only preserves the integrity of individual borrower positions but also ensures optimal capital efficiency across the protocol and optimal siphoning rates for Borrowers. Releasing excess credit does not create any adverse risk for either Credit LPs or Borrowers, as such it can be triggered by anybody on demand. Some incentives for doing so can be listed:

[1] In the most general sense, the term "type" here refers to whether the C and C_{LP} assets are correlated or not. Equation 7 could be applicable in a scenario where, for example, ETH is the Borrower's collateral asset and stETH is the asset whose borrowing power is delegated. That said, scenarios such as this obviously come with price decorrelation risks, whose management goes beyond the scope of this manuscript.

[2] Interest rate accrual will always make the reserved assets exceed what is actually required by the borrower as long as the yield paid to Credit LPs is strictly greater than the external lending market's lending (credit+collateral asset case) or borrow (credit=loan asset case) rate differentials. This can be imposed in the interest rate model that Twyne uses to charge Borrowers (IR_{min} parameter discussed in Section 5).

- Borrowers wishing to minimize their credit reservation costs
- Third parties acting on behalf of Borrowers to optimize their collateral vault position
- CLPs wishing to release funds for the purpose of withdrawing or offering credit to new Borrowers during a high credit utilization period.

5 Interest Rate Model

Twyne integrates three distinct interest rates: lending market yield ($r_C^{underlying}$ and $r_{C_{LP}}^{underlying}$), lending market borrow $r_B^{underlying}$, and Twyne's CLP supply rates. The first three guarantee that all assets delegated onto Twyne continue to earn/pay whatever yield is paid/charged by whichever underlying lending market an entity's assets reside in/derive from. Twyne's CLP supply rate, on the other hand, is the rate at which a borrower's collateral is charged for reserving the borrowing power offered by CLPs. Twyne employs a curved interest rate model $IR(u)$ that depends solely on the asset-specific utilization rate of CLP assets accessible by the Borrower's collateral vault ($u \equiv C_{LP}^{reserved}/C_{LP}^{total}$), each with its own asset specific parameters IR_0, u_0, IR_{min}, IR_{max}, and γ:

$$IR(u) = IR_{min} + \frac{IR_0 - IR_{min}}{u_0} \cdot u + \left(IR_{max} - IR_{min} - \frac{IR_0 - IR_{min}}{u_0} \right) \cdot u^{\gamma}. \tag{9}$$

Figure 5 shows a couple of example curves plotted for various values of γ (with $IR_{min} = 0$ for simplicity).

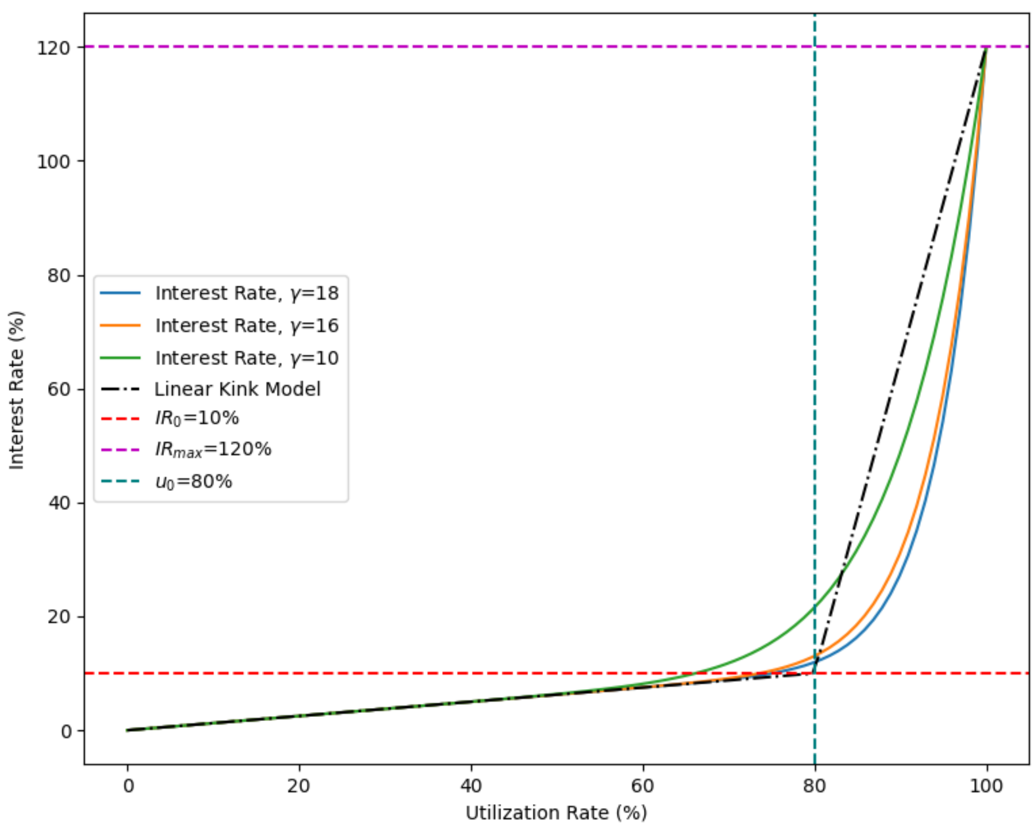

Figure 5: Sample Interest Rate curves for $IR_0 = 0.1$, $u_0 = 0.8$, $IR_{max} = 1.2$ and varying γ. Utilization and interest rates have been converted from decimals to percents.

The specific form of Equation 9 is designed to behave quasi-linearly for utilization rates $u_{asset} \leq u_0$, and polynomial ramp quickly to IR_{max} (with power γ) when the u_0 threshold is overcome. This interest rate curve, an evolution of that first deployed by the Keom Protocol [1], efficiently parametrizes the optimal 1D interest rate curves studied through Morpho's research efforts [2] without the need for complex on-chain operations or PID controllers. The parameters defining the precise shape of the interest rate model can be intuitively understood in relation to the more commonly used linear-kink model (black dot-dashed curve in Figure 5). The threshold utilization u_0 and the interest rate IR_0 represent the location and interest rate where the kink appears in the model. The choice of γ then controls how much the curved interest rate model overshoots IR_0 at the kink, as well as how much it lags the linear model at high utilization (see Appendix A for more details).

5.1 Borrower Siphoning Rate

From a borrower's perspective, their collateral experiences a yield that is the combination of a subtractive rate r_C paid to CLPs (termed the *siphoning rate*) and any positive lending yield accrued through the underlying lending market. Their borrow interest rate, on the other hand, coincides with whatever borrow rate is quoted by the underlying lending market for the assets being borrowed. The gross interest rate r_C paid to CLPs in units of collateral asset held by the Borrower, can be thought of as the ratio between the absolute interest payment owed on the CLP funds being reserved $C_{LP} \cdot IR(u)$ and the collateral C held by the borrower:

$$r_C = \frac{C_{LP} \cdot IR(u)}{C}. \tag{10}$$

However, a borrower will probably be more interested in the net rate r_C^{net} paid:

$$r_C^{net} = \frac{C_{LP} \cdot IR(u)}{C - B} = \frac{r_C}{1 - \lambda_{twyne}}. \tag{11}$$

If the Borrower's collateral vault can reserve credit from multiple credit vaults, Equation 10 should be modified to account for the weighted rate the Borrower experiences:

$$r_C = \frac{1}{C} \cdot \frac{\sum_{i \in [\text{reserved credit assets}]} C_{LP}^i \cdot IR_i(u_i)}{\sum_{i \in [\text{reserved credit assets}]} C_{LP}^i}. \tag{12}$$

At no cost to the generality of the exposition, to reduce the notational complexity, the rest of the manuscript will ignore multi-asset credit reservation and focus solely on cases where the Borrower reserves one asset type only to boost their collateral's borrowing power.

5.2 CLP credit provisioning rate

Similarly, CLPs experience a combined lending rate which is the sum of the interest rate paid to them by borrowers accessing their borrowing power, as well as the underlying lending market's asset rate. Whereas the gross CLP rate is identical to the utilization-based rate $IR(u)$ defined in Equation 9, the net rate r_{LP}^{net} perceived will be the ratio between the gross payment and the total CLP assets supplied:

$$r_{LP}^{net} = \frac{C_{LP}^{reserved} \cdot IR(u)}{C_{LP}^{total}} = u \cdot IR(u). \tag{13}$$

6 Liquidations

When allowing users to take out over-collateralized loan, the success of the liquidation process is extremely important [3]. In traditional DeFi lending markets, liquidations must ensure that enough Borrower collateral can be repossessed such that it can be swapped to repay some portion of the outstanding loan as well as incentivizing the liquidator for the effort. This operation must be executed before asset prices move so much that insolvencies or toxic liquidation spirals derail the entire lending market [4]. This is a particularly sensitive matter in Twyne, given that it allows borrowers to access higher LTVs than what is allowed on external lending markets. To avoid such matters as much as possible, Twyne primarily adopts a *liquidation-through-inheritance* for its internal liquidations.

6.1 Theoretical Advantages of Position Inheritance

The position inheritance model offers several theoretical advantages that stem from its fundamental design:

1. **Capital Retention**: By transferring positions rather than liquidating assets, the protocol retains capital within its ecosystem, enhancing overall systemic liquidity and stability.

2. **Reduced Market Impact**: Traditional liquidations can create selling pressure and exacerbate price declines during market stress. Position inheritance avoids this pro-cyclical effect by eliminating the need for immediate asset sales.

3. **Broader Liquidator Pool**: Ownership transfer allows Twyne users with existing positions to become liquidators without requiring additional external capital, potentially increasing liquidation efficiency. Positions could be inherited by groups of users collectively, further reducing stress on the individual liquidators.

4. **Position Recovery Potential**: Liquidators can gradually improve inherited positions by adding collateral or repaying debt, potentially creating value recovery opportunities not available in traditional liquidation models.

5. **Simplified Execution**: Position inheritance requires simpler computational logic and fewer external dependencies (such as price oracles or AMM integrations) for the core liquidation mechanism. Participation can be entirely intent-based.

6. **Capital Requirements**: Instead of supplying the full amount required to unwind the Borrower's debt, only the minimum amount of collateral required to make the position healthy is required. This enables flows where the liquidating entity has the option to slowly unwind the position after having inherited it.

6.2 Internal Liquidations

Whenever a Borrower (*Alice*) is found to have an LTV greater than their chosen liquidation threshold ($\lambda_t^{Alice} > \tilde{\lambda}_t^{Alice}$), ANY user with sufficient spare borrowing power on Twyne can trigger an inheritance of the Borrower's entire collateral vault. This includes the loan position B^{Alice}, CLP assets C_{LP}^{Alice} (becoming liable for accrued and ongoing siphoning costs), and sufficient collateral to cover the loan plus a liquidation incentive i_t^{liq}. The liquidation incentive is calculated as a percentage on the unencumbered portion of the borrower's portfolio such that the liquidator's profit p_t can be written as:

$$p_t = i_t^{liq} \cdot (C^{Alice} - B^{Alice}) = i_t^{liq}(\lambda_t) \cdot C^{Alice}, \tag{14}$$

where the liquidation incentive can generally be set as a function of the Borrower's health $i_t^{liq}(\lambda_t) \leq 1 - \lambda_t$ such that an internal liquidation never risks the reserved CLP funds[3] (liquidation incentive details are discussed in Section 8).

Thus, after a generic borrower (*Bob*) triggers the liquidation and inherits Alice's position, their position on Twyne will become:

$$\begin{aligned}
C^{Bob,new} &= C^{Bob} + \Big[\lambda_t + (1-\tau) \cdot i_t^{liq}(\lambda_t)\Big] \cdot C^{Alice} && (15)\\
B^{Bob,new} &= B^{Bob} + B^{Alice} && (16)\\
\lambda_t^{Bob,new} &= \frac{B^{Bob} + B^{Alice}}{C^{Bob} + \Big[\lambda_t + (1-\tau) \cdot i_t^{liq}(\lambda_t)\Big] \cdot C^{Alice}} && (17)\\
C_{LP}^{Bob,new} &= C_{LP}^{Bob} + C_{LP}^{Alice} && (18)\\
\tilde{\lambda}_t^{Bob,new} &= \beta_{safe} \cdot \tilde{\lambda}_e^{C_{LP}} \cdot \left(\frac{C_{LP}^{Bob,new}}{C^{Bob,new}} + \frac{\tilde{\lambda}_e^{C}}{\tilde{\lambda}_e^{C_{LP}}} \right), && (19)
\end{aligned}$$

[3] As we will see in the following section, potential CLP losses only occur when liquidations are executed by the underlying lending protocol.

where we have also included a potential liquidation tax τ charged by the protocol on the liquidator profits.

If the user executing the liquidation is a CLP (Charlie), they agree to become a borrower. In doing so, they engage some amount of their unutilized[4] assets C_{LP}^{extra} as collateral, thus allowing the execution of a successful liquidation call. Charlie's position following the inheritance would become:

$$
\begin{aligned}
C^{Charlie,new} &= C_{LP}^{extra} + \Big[\lambda_t + (1-\tau)\cdot i_t^{liq}(\lambda_t)\Big]\cdot C^{Alice} && (20)\\
B^{Charlie,new} &= B^{Alice} && (21)\\
\lambda_t^{Charlie,new} &= \frac{B^{Alice}}{C_{LP}^{extra} + \Big[\lambda_t + (1-\tau)\cdot i_t^{liq}(\lambda_t)\Big]\cdot C^{Alice}} && (22)\\
C_{LP}^{Charlie,new} &= C_{LP}^{Bob} + C_{LP}^{Alice} && (23)\\
\tilde{\lambda}_t^{Charlie,new} &= \beta_{safe}\cdot\tilde{\lambda}_e^{C_{LP}}\cdot\left(\frac{C_{LP}^{Charlie,new}}{C^{Charlie,new}} + \frac{\tilde{\lambda}_e^{C}}{\tilde{\lambda}_e^{C_{LP}}}\right). && (24)
\end{aligned}
$$

In both the Bob and Charlie scenarios, we have also recomputed the newly updated liquidation LTVs $\tilde{\lambda}_t^{Bob,new}$ and $\tilde{\lambda}_t^{Charlie,new}$ ignoring any subsequent modifications. If these recomputed values exceed $\tilde{\lambda}_t^{max}$, this implies that excess CLP funds were inherited and, thus, must be released back to the Credit vault. In general, Bob and Charlie may choose to further adjust their liquidation LTVs in a later step, thus choosing to reserve(release) some or all their CLP funds.

Alice has thus been cleared of her entire loan position and will retain whatever collateral is leftover from the above process:

$$
C^{Alice,end} = \Big[1 - \lambda_t - i_t^{liq}(\lambda_t)\Big]\cdot C^{Alice} \tag{25}
$$

In all cases, Twyne smart contracts check ahead of time that a liquidator has enough collateral on Twyne to safely execute the inheritance ($\lambda_t^{liquidator} < \tilde{\lambda}_t^{liquidator}$). The advantage of this model is two-fold. First, it maximizes the amount of capital retained on the protocol by attempting to make liquidators act as users first and foremost. Secondly, since the inheritance does not necessarily require a swap[5], a liquidation should be more easily executable by a wider array of entities than the *just-in-time* (JIT) liquidators whose objective is to immediately unwind the position.

6.3 Fallback Liquidations

In case of delay in executing the liquidation call and further worsening of Alice's collateral vault's health beyond the external safety threshold ($\lambda_e > \tilde{\lambda}_e$), the underlying lending market will trigger a liquidation which may in whole or in part affect Alice as well as any CLPs helping to back Alice's loan. Since these are the last line of defense against toxic debt accumulating on Twyne, we call these *fallback liquidations*.

From the underlying lending market's perspective there is no notion of CLP and Borrower funds since they are an accounting abstraction introduced by Twyne. Furthermore, given the predominant usage of JIT liquidations by DeFi lending markets, it may not be possible to deterministically say how much of the combined CLP+Borrower collateral vault position will be unwound by the liquidation. Some lending markets set a maximum closing factor allowed and fixed liquidation incentives, whereas others offer liquidators full freedom to choose their closing factor while constraining the liquidation incentives as a function of the Borrower's health[6].

From Twyne's perspective, once a fallback liquidation takes place, the position is merely defined by whatever collateral C_{left} and outstanding loan B_{left} amounts are left behind by the sanitization process. In these circumstances, Alice may still gain access to her collateral vault, but her share of collateral ownership C_{Alice}^{new} relative to the CLP's share C_{LP}^{new}, must be defined such that Alice remains liquidatable on Twyne while maximizing the amount of funds retrievable by the CLPs.

[4]'Unutilized' in the sense that they can be withdrawn from the Credit vault they were deposited into.
[5]Twyne's inheritance mechanism attempts to draw in fresh borrowers willing to bet on price mean reversions.
[6]This effectively auctions the liquidation to whomever bids for the lowest liquidation incentive

6.3.1 Post-Fallback Accounting

At every interaction with a borrower position on Twyne, the total collateral assets in the collateral vault (C and C_{LP}) is cached. With the exception of the withdraw/rebalancing actions, this quantity (in native asset terms) can only increase due to interest rates set on the underlying lending market[7]. As such, if this quantity is found to have suddenly decreased, it can only be due to a liquidation triggered by the underlying lending market. When this occurs, Twyne imposes a specific accounting transformation on the position and flags it for expiration [8] at the very next interaction that any entity performs on it.

First, the Borrower's chosen liquidation LTV $\tilde{\lambda}_t^{old}$ is ignored and assumed to be the maximum liquidation LTV $\tilde{\lambda}_t^{max}$ allowed by Twyne. The purpose of this is to immediately upgrade Alice to the highest-risk cohort on the protocol. Using this new $\tilde{\lambda}_t^{max}$ as baseline, the contracts check for any remaining collateral C_{left}, and whether any outstanding liabilities B_{left} still encumber her position. If any are found to exist, they reserve the minimal amount of collateral required to make Alice's position health match $\tilde{\lambda}_t^{max}$:

$$C_{temp} = \min\left(\frac{B_{left}}{\tilde{\lambda}_t^{max}}, C_{left}\right), \tag{26}$$

thus ensuring that a minimally sufficient collateral is reserved to be assigned back to the borrower/liquidator [9]. As we're about to see these are not necessarily the only funds accounted back to the borrower (hence the $\cdot_{temp}$ subscript).

Given the value C_{temp} computed above, the protocol can resolve how many funds C_{LP}^{new} can be returned to the CLPs depending on whether there is enough collateral leftover, after subtracting C_{temp}, to pay back the outstanding credit C_{LP}^{old} reserved by the collateral vault prior to the fallback liquidation:

$$C_{LP}^{new} = \min\Big(C_{left} - C_{temp},\, C_{LP}^{old}\Big). \tag{27}$$

Lastly, if anything remains, it is accounted to the Borrower[10], alongside C_{temp}.

$$C^{new} = \max\Big(C_{temp}, C_{left} - C_{LP}^{old}\Big). \tag{28}$$

The Borrower's post-fallback Twyne LTV can now be recomputed:

$$\lambda_t^{new} = \frac{B_{left}}{C^{new}}, \tag{29}$$

guaranteeing that the post-fallback accounting results in $\lambda_t^{new} \leq \tilde{\lambda}_t^{max}$.

Assuming Bob [11] triggers the post-fallback accounting logic instead of liquidating Alice on Twyne (as done in the previous section), Bob's new position becomes:

$$C^{Bob,new} = C^{Bob} + \Big[\lambda_t^{new} + (1-\tau)\cdot i_t^{liq}(\lambda_t^{new})\Big]\cdot C^{new} \tag{30}$$

$$B^{Bob,new} = B^{Bob} + B_{left} \tag{31}$$

$$\lambda_t^{Bob,new} = \frac{B^{Bob} + B_{left}}{C^{Bob} + \Big[\lambda_t^{new} + (1-\tau)\cdot i_t^{liq}(\lambda_t^{new})\Big]\cdot C^{new}} \tag{32}$$

$$C_{LP}^{Bob,new} = C_{LP}^{Bob} + C_{LP}^{new} \tag{33}$$

$$\tilde{\lambda}_t^{Bob,new} = \beta_{safe}\cdot\tilde{\lambda}_e^{C_{LP}}\cdot\left(\frac{C_{LP}^{Bob,new}}{C^{Bob,new}} + \frac{\tilde{\lambda}_e^{C}}{\tilde{\lambda}_e^{C_{LP}}}\right) \tag{34}$$

[7] We ignore exotic effects such as negative lending rates (whose instance the authors know not of an exmaple) and socialization of losses from underlying lending market insolvencies (which would impact all Twyne users in a matter which would require individual assessment).

[8] Collateral vault is not allowed to perform any further operations aside from repaying loans, releasing credit assets, withdrawing any remaining collateral, and being liquidated.

[9] If the accounting is triggered by a liquidator unwinding the position, C_{temp} is the total collateral the liquidator can collect.

[10] If the accounting is being executed as the result of a liquidator unwinding the position, this remaining quantity is the only amount of funds that the borrower actually recovers from the liquidation.

[11] We leave the Charlie example to the reader.

whereas Alice is left with:

$$C^{Alice,new} = \left[1 - \lambda_t^{new} - i_t^{liq}(\lambda_t^{new})\right] \cdot C^{new}. \tag{35}$$

7 Credit LP Loss Analysis

As discussed in the previous section, internal liquidations will always make CLPs whole. This is the preferred avenue for liquidations and Twyne should strive to incentivize the maturation of its liquidator ecosystem as much as possible precisely for this reason. The same is not true for fallback liquidations where the post-fallback accounting never resets the Borrower's LTV to a value greater than the maximum liquidation LTV allowed on Twyne $\lambda_t^{new} \leq \tilde{\lambda}_t^{max}$ independently of the state of health of the position when it was actually liquidated by the underlying lending market. This decoupling is a necessary requirement to guarantee that incentives exist for market actors to trigger execution of the post-fallback accounting on Twyne.

As a result of these considerations, CLP funds may be impacted by losses depending on a non-trivial interaction of:

- the protocol's choice of β_{safe} and $\tilde{\lambda}_t^{max}$.
- the underlying lending market's $\tilde{\lambda}_e^{C}$ and $\tilde{\lambda}_e^{C_{LP}}$.
- the Borrower's choice of liquidation LTV $\tilde{\lambda}_t^{old}$ prior to the fallback liquidation being executed.
- *fallback liquidator efficiency*: the (potentially unpredictable) closing factor c and liquidation incentive i at which the collateral vault's liquidation was processed by the underlying lending market's liquidators.

These losses can be expressed in fractional terms as (see Appendix B for details):

$$l_{CLP} = \min\left(0, \frac{\beta_{safe} \cdot \tilde{\lambda}_e^{C_{LP}}}{\tilde{\lambda}_t^{old} - \beta_{safe} \cdot \tilde{\lambda}_e^{C}} \cdot \left[1 - \frac{\rho}{\beta_{safe}} \cdot \Big((1+i) \cdot c \cdot \tilde{\lambda}_t^{max} + (1-c)\Big)\right]\right), \tag{36}$$

where we identify:

$$\rho \equiv \tilde{\lambda}_t^{old} / \tilde{\lambda}_t^{max}, \tag{37}$$

representing the ratio between Borrower's chosen liquidation LTV and the maximum allowed by Twyne, as a key variable impacting such losses.

The above result is either zero or negative depending on the variable tuple (c,i) (subject to the other parameters). We will hereon denote the c-vs-i curve where the transition from zero to non-zero losses as the CLP *loss frontier*. It is expressed mathematically as:

$$c(i) = \frac{1}{\rho} \cdot \frac{\beta_{safe} - \rho}{\tilde{\lambda}_t^{max} \cdot (1+i) - 1} \tag{38}$$

7.1 Loss Frontier Analysis

7.1.1 $\rho < \beta_{safe}$

We plot Equation 36 for some nominal values of $\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^{C}$, c_0, i_0, β_{safe}, and ρ (with $\rho < \beta_{safe}$) in Figure 6, where we show the full shape of the expected CLP loss surface and highlighting the loss frontier 38 where losses will begin to occur. We notice how losses only commence to occur after $i > i_0$. The key observation in this regime $\rho < \beta_{safe}$ is that the CLP-loss frontier has a negative slope. This implies that, all else being equal, smaller closing factors amplify the liquidation incentive range where CLPs can expect not to lose money.

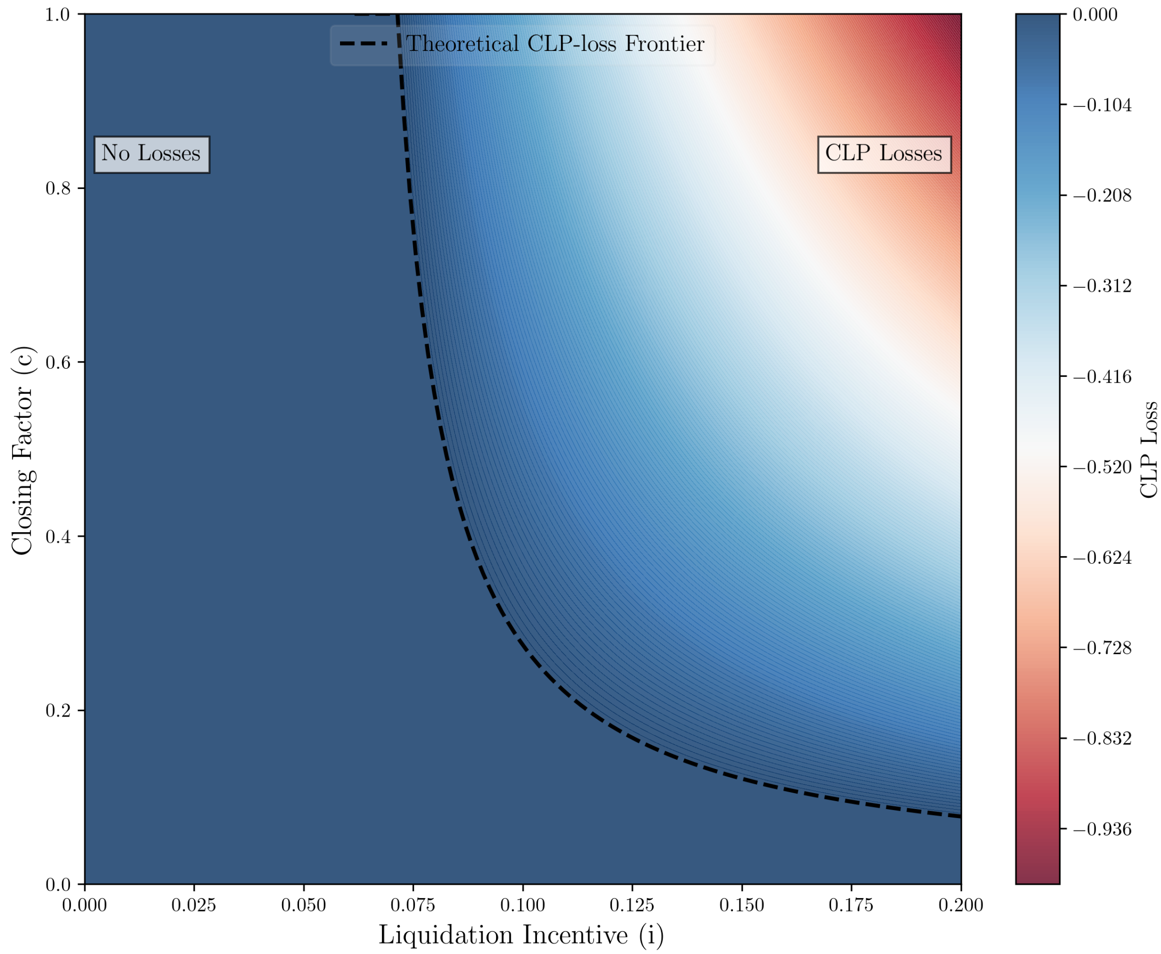

Figure 6: Expected CLP losses as a function of liquidation incentive i and closing factor c with $\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^C = 0.85$, $\beta_{safe} = 0.99$, $\tilde{\lambda}_t^{old} = \tilde{\lambda}_t^{max} \simeq 0.943$ (by using $c_0 = 1$, $i_0 = 0.05$ in 58), and $\rho = 0.98$. Losses are expressed in decimals ($l_{CLP} = -1 \Leftrightarrow 100\%$ loss). The loss frontier (black dashed line) corresponds to Equation 38. Interactive plot can be found at this Desmos link.

7.1.2 $\rho > \beta_{safe}$

We plot Equation 36 for the same nominal values of $\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^C$, c_0, i_0, β_{safe}, but with $\rho > \beta_{safe}$ this time in Figure 7. Again, we notice that losses generally begin after $i > i_0$. However, unlike in the previously discussed regime, the CLP-loss frontier now has **positive** slope. This implies that, all else being equal, smaller closing factors reduce the liquidation incentive range where CLPs can expect not to lose money. A very small partial liquidation will lead to losses for CLPs. This can be intuitively understood in the limit $\rho = 1$. In this limit, the Borrower's $\tilde{\lambda}_t$ already equals the maximum allowed $\tilde{\lambda}_t^{max}$ value. When post-fallback accounting is performed, Twyne reassigns the Borrower the same liquidation LTV they had before the fallback liquidation took place (see Equation 26). As such, the fallback liquidation could only have sanitized the collateral vault by repossessing collateral contributed by the CLPs. In the limit of very small closing factors ($c \to 0$), CLPs are left with a limiting loss equal to:

$$\lim_{c \to 0} l_{CLP} = \frac{\beta_{safe} \cdot \tilde{\lambda}_e^{C_{LP}}}{\tilde{\lambda}_t^{old} - \beta_{safe} \cdot \tilde{\lambda}_e^C} \cdot \left(1 - \frac{\rho}{\beta_{safe}}\right)$$

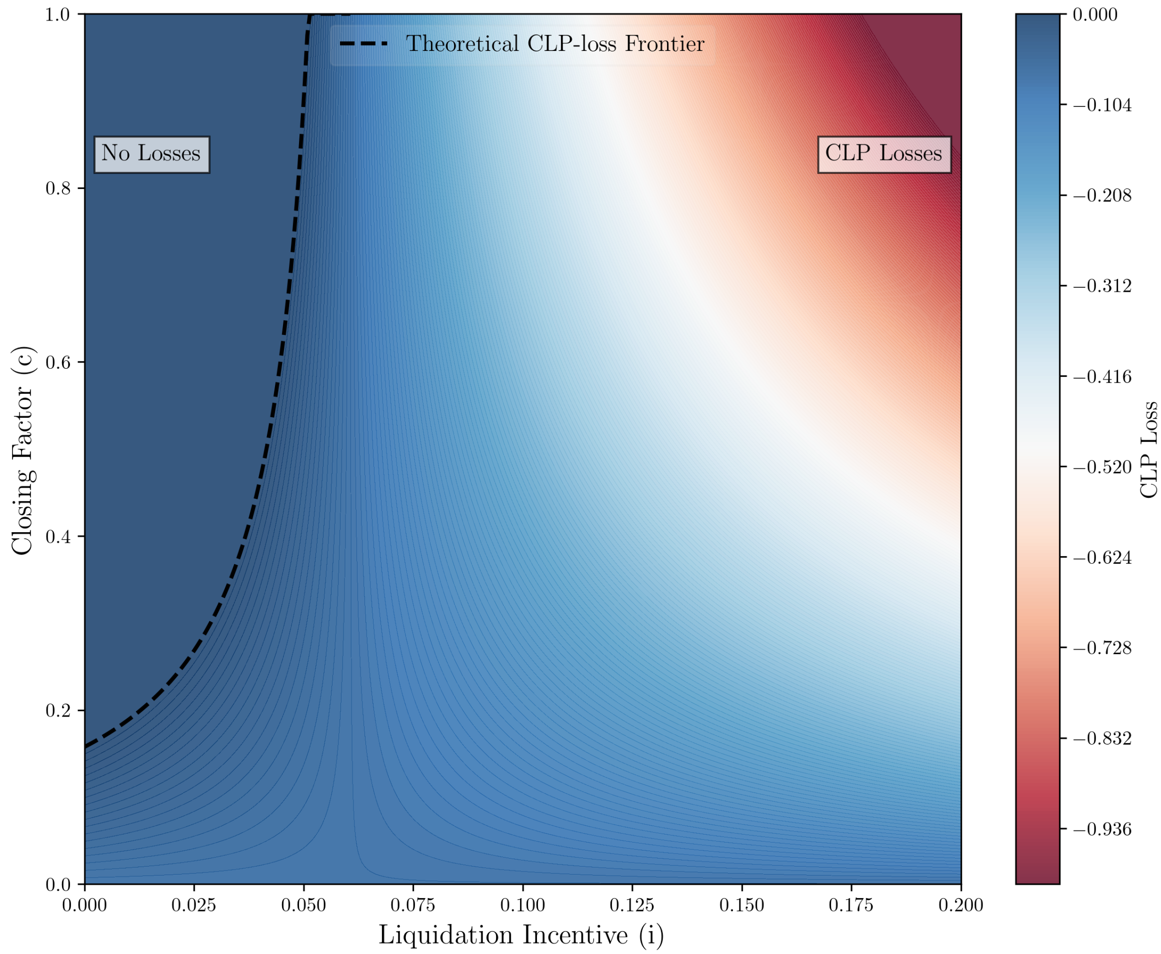

Figure 7: Expected CLP losses as a function of liquidation incentive i and closing factor c with $\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^{C} = 0.85$, $\beta_{safe} = 0.99$, $\tilde{\lambda}_t^{old} = \tilde{\lambda}_t^{max} \simeq 0.943$ (by using $c_0 = 1$, $i_0 = 0.05$ in 58), and $\rho = 1$. Losses are expressed in decimals ($l_{CLP} = -1 \Leftrightarrow 100\%$ loss). The loss frontier (black dashed line) corresponds to Equation 38. Interactive plot can be found at this Desmos link.

Whereas the worsening of the CLP losses in the $\rho > \beta_{safe}$ case can somewhat be improved by introducing a dynamic safety buffer β_{safe}^{dyn} which depends on the Borrower's chosen liquidation LTV $\tilde{\lambda}_t$, any exceedance in the Borrower's LTV beyond their chosen liquidation threshold [12] will have the effect of artificially inflating ρ. As such, the results of this section should be taken as a general qualitative framework for understanding how losses are affected by the (c,i) tuple. Any further quantitative assessment should proceed by collecting historical liquidator performance of the lending markets Twyne chooses to deploy on in order to assess and set protocol parameters.

8 Liquidation Incentive

In Section 6 we introduced the policy by which Twyne liquidators are incentivized to process liquidations, thus guaranteeing full retrieval of funds owed to the CLPs. The profit perceived by a liquidator on Twyne was defined as

$$p_t = (1 - \tau) \cdot i_t^{liq}(\lambda_t) \cdot C, \tag{39}$$

where we include the effect of a liquidation tax τ on the liquidator's profits. The purpose of this section is to refine how the dependency of the liquidation incentive on the Borrower's health can be tuned to satisfy three key principles that Twyne liquidations should follow.

[12] as a result of adverse price movements in the liquidator's reaction time window

- Twyne can be used as a proxy for the underlying lending market
- Healthy internal liquidations must never create bad debt.
- Maximize the initial profitability of liquidating on Twyne versus liquidating on the underlying lending market.

8.1 Twyne as a proxy

As a result of the freedom to choose any desired liquidation LTV $\tilde{\lambda}_t \leq \tilde{\lambda}_t^{max}$, a Borrower on Twyne could choose not to reserve any CLP funds for an extended period of time. They would still, however, retain the optionality of doing so at any moment without worrying about suddenly having to onboard their position onto Twyne. This could be crucial when, facing imminent liquidation risk, the Borrower may need to tap into higher $\tilde{\lambda}_t$ values than the underlying lending market allows at a moment's notice.

By setting their desired liquidation LTV close to the underlying lending market's liquidation LTV they should be able to freely operate on Twyne as if it were the underlying lending market. If the Borrower were to be liquidated on Twyne in such a circumstance, their experience should be as similar as possible to being liquidated on the underlying market. This is obviously not exactly possible in general, least of which because Twyne enforces full closing of liquidated user positions through its liquidation-by-inheritance process. However, we can tune the liquidation incentive $i_t^{liq}(\lambda_t)$ such that the financial consequences of an internal liquidation triggered by Twyne is not significantly more incentivized than a liquidation taking place on the underlying lending market.

We do so by setting the liquidation incentive to zero whenever Borrowers are not reserving CLP funds. This occurs whenever $\tilde{\lambda}_t \leq \beta_{safe} \cdot \tilde{\lambda}_e$.

8.2 Healthy Internal Liquidations

As discussed in Section 6, internal liquidations should never create bad debt. This is guaranteed by defining the liquidation incentive as a percentage on the excess collateral owned by the borrower. This is done by ensuring, in Equation 39, that $i_t^{liq}(\lambda_t) \leq 1 - \lambda_t$. We wish for this incentive to be maximized precisely when the Borrower's LTV reaches the maximum allowed by the protocol: $i_t^{liq}(\lambda_t) = 1 - \lambda_t$, for $\lambda_t \geq \tilde{\lambda}_t^{max}$. Beyond this point, the profits of the liquidator in Twyne are designed to decrease as the fallback liquidation process takes over.

8.3 Internal vs Fallback liquidation profitability

In the interest of user experience, even when fallback liquidations first become possible, there should still be a buffer incentivizing liquidators to trigger an internal liquidation on Twyne vs sanitizing the entire collateral vault through the underlying lending market. Ensuring this in general cannot be guaranteed as it depends on the underlying lending market's liquidator ecosystem. However, we can outline a high level analysis that aids liquidators to understand the trade-offs involved.

8.4 Dynamic Liquidation Incentive

The above considerations can be summarized and formalized into a dynamic liquidation incentive expressed as:

$$i_t^{liq}(\lambda_t) = \begin{cases} 0 & \lambda_t \leq \beta_{safe} \cdot \tilde{\lambda}_e \\ (1 - \tilde{\lambda}_t^{max}) \cdot f\left(\frac{\lambda_t - \beta_{safe} \cdot \tilde{\lambda}_e}{\tilde{\lambda}_t^{max} - \beta_{safe} \cdot \tilde{\lambda}_e}\right) & \beta_{safe} \cdot \tilde{\lambda}_e \leq \lambda_t \leq \tilde{\lambda}_t^{max} \\ 1 - \lambda_t & \lambda_t \geq \tilde{\lambda}_t^{max} \end{cases} \tag{40}$$

where $f(x) : [0,1] \to [0,1]$ can theoretically be any monotonic interpolation function, and β_{safe} can in principle be substituted with its dynamic cognate β_{safe}^{dyn} if desired.

In Figure 8, we plot Equation 40 for two different interpolation functions (linear and quadratic) to give the reader intuition for their overall behavior. For the purpose of constructing the figure, the parameter values $\beta_{safe} = 0.98$, $\tilde{\lambda}_e = 0.85$, and $\tilde{\lambda}_t = 0.95$ were chosen. We note that for these choices, a Borrower will not need to reserve any CLP funds as long as their liquidation LTV satisfies $\tilde{\lambda}_t \leq \beta_{safe} \cdot \tilde{\lambda}_e \simeq 0.833$. In such a regime, were a Borrower to be liquidated, their collateral vault would likely be liquidated directly by the underlying lending market. As the LTV of the Borrower at liquidation enters the regime $\tilde{\lambda}_t > \beta_{safe} \cdot \tilde{\lambda}_e$, Twyne increasingly incentivizes internal liquidators up to a maximum incentive $i_t(\tilde{\lambda}_t) = 1 - \tilde{\lambda}_t = 0.05$, obtained when the LTV of the Borrower reaches $\lambda_t = \tilde{\lambda}_t$[13]. Beyond this threshold, incentives for internal liquidations decline linearly in the Borrower's LTV. It is during this regime that a transition occurs where fallback liquidations become more profitable and, thus, likely to occur.

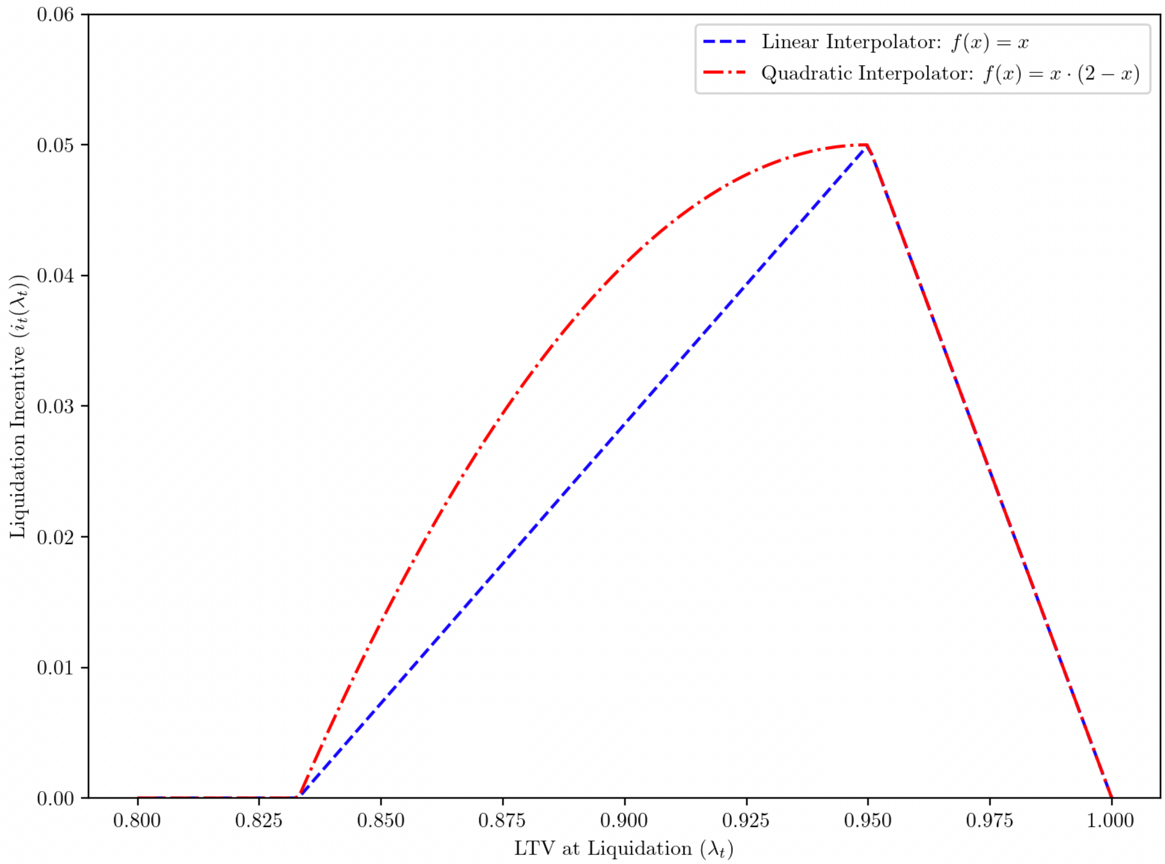

Figure 8: Twyne's dynamic liquidation incentive (Equation 40) is plotted as a function of the Twyne LTV λ_t for linear (blue) and quadratic (red) interpolation functions subject to parameter choices $\beta_{safe} = 0.98$, $\tilde{\lambda}_e = 0.85$, and $\tilde{\lambda}_t = 0.95$. The linearly interpolated liquidation incentive shown is equivalent to Equation 41.

Upon choosing a linear interpolation function $f(x) = x$ for definiteness, we note that the liquidation incentive can also be written explicitly as:

$$i_t^{liq}(\lambda_t) = \frac{1 - \max\left(\tilde{\lambda}_t^{max}, \lambda_t\right)}{\tilde{\lambda}_t^{max} - \beta_{safe} \cdot \tilde{\lambda}_e} \cdot \left[\max\left(\beta_{safe} \cdot \tilde{\lambda}_e, \min\left(\lambda_t, \tilde{\lambda}_t^{max}\right)\right) - \beta_{safe} \cdot \tilde{\lambda}_e\right] \tag{41}$$

8.5 Relative Profitability of Liquidating on Twyne

Equations 39 and 40 together quantify the profit p_t a liquidator can expect to make when performing an internal liquidation in Twyne. To understand whether this incentive is sufficient it must be compared to the equivalent profit p_e they stand to make if they were to execute the liquidation on the

[13] In this regime, the interpolator function controls how quickly or slowly the liquidation incentives rise to the maximum incentive offered.

underlying lending market instead. To do so we must account for the underlying lending market's liquidation incentive $i_e^{liq}(\lambda_e)$ and closing factor $c_e^{liq}(\lambda_e)$ policies which can, generally, be health-dependent themselves.

DeFi lending markets incentivize liquidators by allowing them to repossess excess collateral from a liquidated borrower, computed as a percentage the of the loan repaid by the liquidator. As such, the profit for a liquidator on the underlying lending market can be written as:

$$\begin{aligned} p_e &= i_e^{liq}(\lambda_e) \cdot c_e^{liq}(\lambda_e) \cdot B \\ &\leq i_e^{liq}(\lambda_e) \cdot \lambda_t \cdot C, \end{aligned} \tag{42}$$

where we have assumed the closing factor policy to be 100% as an upper bound, and remind the reader that λ_e can always be written in terms of λ_t whenever deemed necessary[14].

The relative profitability of liquidating on Twyne versus liquidating on the underlying lending market (when possible) is thus simply $p_{ratio} = p_t/p_e$. A rational liquidator will then necessarily choose to liquidate on Twyne whenever:

$$p_{ratio}(\lambda_t) = \frac{p_t}{p_e} = \frac{1-\tau}{\lambda_t} \cdot \frac{i_t^{liq}(\lambda_t)}{i_e^{liq}(\lambda_e)} > 1. \tag{43}$$

To exemplify the behavior of $p_{ratio}(\lambda_t)$ we must first choose a liquidation incentive policy $i_e^{liq}(\lambda_e)$ for the underlying lending market. To this end, we choose the following fixed $i_e^{liq,fix}$ and dynamic $i_e^{liq,dyn}$ incentive policies to compare to:

$$i_e^{liq,fix} = i_0 \tag{44}$$

$$i_e^{liq,dyn} = \max\left(0, \beta_{safe} \cdot \frac{\lambda_t}{\tilde{\lambda}_t} - 1\right), \tag{45}$$

where the dynamic incentive policy corresponds to a linear growth of the incentive as the collateral vault's LTV exceeds past its liquidation LTV. The reader should keep in mind that the LTV range where these underlying lending market incentives become available corresponds to $\lambda_t \geq \tilde{\lambda}_t/\beta_{safe}$.

Plugging Equations 41, 44, and 45 into Equation 43 we get that liquidating on Twyne will be relatively more profitable for Borrower LTVs λ_t up to some safe threshold λ_t^{safe}:

$$\lambda_t \leq \lambda_t^{safe} = \begin{cases} \frac{1-\tau}{1-\tau+i_0} & \text{static policy} \\ \\ \sqrt{(1-\tau)\frac{\tilde{\lambda}_t}{\beta_{safe}}} & \text{dynamic policy} \end{cases} \tag{46}$$

8.5.1 Upper bound to Twyne's imposable liquidation tax

The analysis just performed offers one final bonus in the form of a limit on the maximum tax that Twyne can charge on liquidations. By imposing $\lambda_t^{safe} > \tilde{\lambda}_t^{max}$ such that Twyne's relative profitability remains true across all possible choices of liquidation LTV $\tilde{\lambda}_t$, we obtain an upper bound to the tax rate that the protocol can charge:

$$\tau \leq \begin{cases} 1 - i_0 \cdot \frac{\tilde{\lambda}_t^{max}}{1-\tilde{\lambda}_t^{max}} & \text{static policy} \\ \\ 1 - \beta_{safe} \cdot \tilde{\lambda}_t^{max} & \text{dynamic policy} \end{cases} \tag{47}$$

Taxing Twyne liquidators beyond these thresholds would make fallback liquidations more profitable to execute the moment they become available to liquidators.

[14] $\lambda_e = \frac{\beta_{safe}}{1+(\beta_{safe}/\tilde{\lambda}_t^{max})\cdot\left(\tilde{\lambda}_e^{CLP}-\tilde{\lambda}_e^{C}\right)} \cdot \lambda_t$

9 Theory to Smart Contract Equivalence

The previous sections have discussed in depth the theoretical framework for quantifying the loss risk to CLPs on Twyne. The main avenue of validation for the theory was to compare it with results obtained using a Python simulation framework to efficiently emulate the behavior of the protocol logic. The effort has demonstrated great agreement between theory and simulation, thus justifying further use of simulations to explore more subtle aspects of Twyne's mechanics which may not be directly accessible via theoretical means. What the effort still has not shown, however, is that the Twyne smart contracts [5] truly implement the mechanistic logic introduced in this manuscript.

We do so by creating Borrower-CLP positions, on-chain, via forked Twyne smart contracts deployed on top of a forked instance of Euler V2 [6]. The simulated position consists of WETH collateral and credit assets ($\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^{C} = 0.85$), and USDC loan assets. Our objective is to create a position which is liquidatable on Euler to emulate a scenario where liquidations on Twyne fail. This then allows us to liquidate the artificial positions directly through the Euler smart contracts with varying closing factors, and subsequently evaluating how much money the CLPs would lose as a result of the post-fallback accounting on the Twyne contracts.

To establish the artificial borrower position, we set the max Twyne liquidation LTV $\tilde{\lambda}_t^{max} = 0.931$, impose the chosen borrower liquidation LTV to be maxed out $\tilde{\lambda}_t = \tilde{\lambda}_t^{max}$, and set $\beta_{safe} = 1$. This automatically sets the ratio C_{LP}/C according to Equation 7. We are then free to adjust the total value of the loan asset B to create whichever $\lambda_t \geq \tilde{\lambda}_t$ desired. Since $\beta_{safe} = 1$, any such value will qualify the constructed position for liquidation on the Euler smart contracts since $B/(C + C_{LP}) \geq \tilde{\lambda}_e$.

All that remains is to choose the closing factor and liquidation incentive we wish to trigger the liquidation for. In practice however, Euler implements a dynamic liquidation incentive mechanism [7] which is entirely dependent the LTV exceedance of the position $\lambda_e/\tilde{\lambda}_e$ at the moment of liquidation, and a protocol constant called *maxLiquidationDiscount*. For the purpose of our experiment, we employ the *maxLiquidationDiscount* value set on Base (*maxLiquidationDiscount*=0.15). The liquidation incentive i can thus be written as:

$$i(\rho_{eff}) = \frac{1}{\max\left(\frac{1}{\rho_{eff}}, 0.85\right)} - 1, \tag{48}$$

where $\rho_{eff} = \lambda_t/\tilde{\lambda}_t^{max} > 1$ mathematically represents the user "choosing to be liquidated" at liquidation LTV values larger than $\tilde{\lambda}_t^{max}$ even though this is not technically voluntary [15].

The purpose of introducing ρ_{eff} is to allow using our theoretical expression for the CLP loss frontier (Equation 38). More specifically, inverting Equation 48 to obtain $\rho_{eff}(i) = 1 + i$, we can write the theoretical CLP loss frontier for a Twyne deployment on Euler as:

$$c(i) = \frac{1}{\rho_{eff}(i)} \cdot \frac{\beta_{safe} - \rho_{eff}(i)}{\tilde{\lambda}_t^{max} \cdot (1+i) - 1} = \frac{i}{(1+i) \cdot \left[1 - \tilde{\lambda}_t \cdot (1+i)\right]}, \tag{49}$$

where $\beta_{safe} = 1$ was used to obtain the second equality.

In Figure 9 we plot this result against simulated CLP losses extracted directly from smart contract operations. The excellent agreement between smart contract and theory finally closes the circle demonstrating full validity across the theory -> simulation -> smart contract stack. To guide the eye, the Figure also highlights the relative profitability region (vertical purple line) obtained by converting Equation 46 consistently with the incentive axis shown[16]. Although fallback liquidations are possible to execute for all the liquidation incentives shown, it is only once they fall beyond this region that liquidating on the underlying lending market becomes more profitable than liquidating directly on Twyne.

[15] See discussion of LTV exceedance effects in Appendix C

[16] $\lambda_t^{safe} = \sqrt{(1-\tau)\frac{\tilde{\lambda}_t}{\beta_{safe}}} \Leftrightarrow i_{safe} = \sqrt{\frac{1-\tau}{\beta_{safe} \cdot \tilde{\lambda}_t}} - 1$

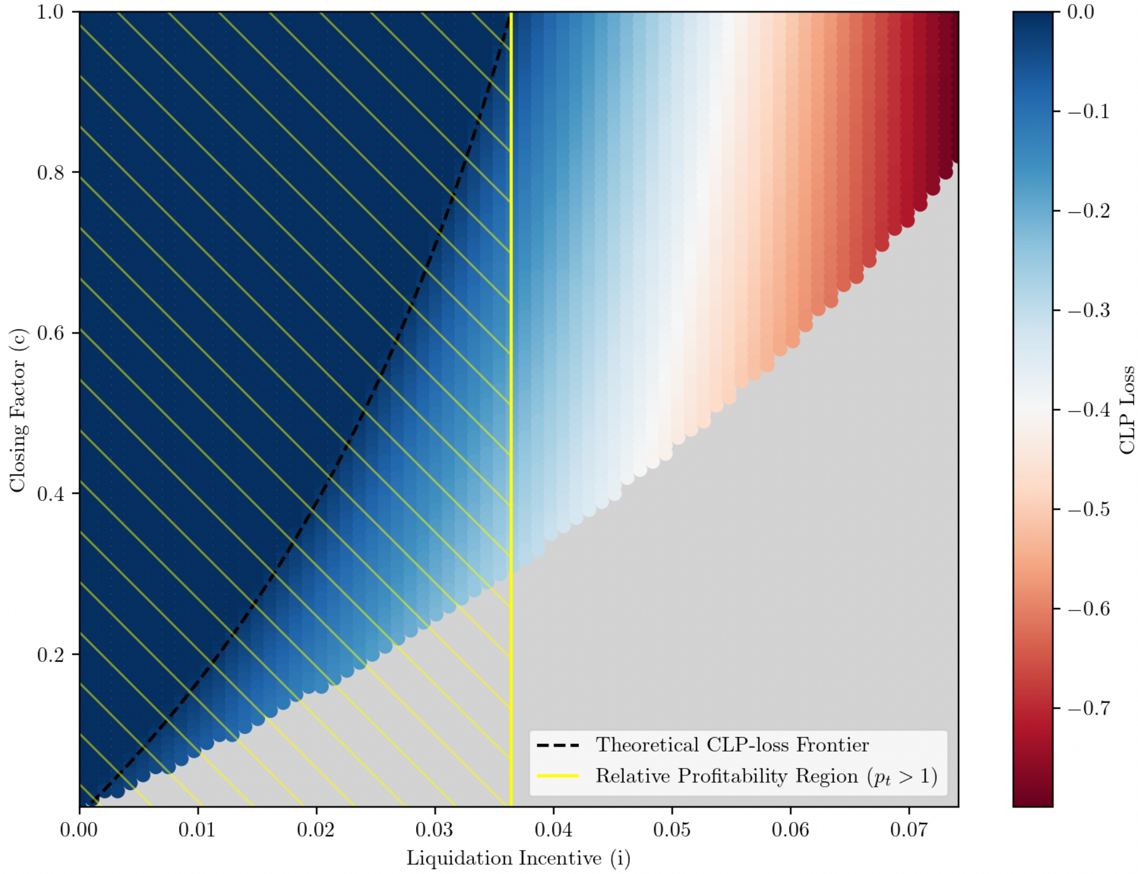

Figure 9: Expected CLP losses as a function of liquidation incentive i and closing factor c with $\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^C = 0.85$, $\beta_{safe} = 1.0$, $\tilde{\lambda}_t^{old} = \tilde{\lambda}_t^{max} = 0.931$ for a Twyne deployment on top of Euler V2 on Base. Losses are expressed in decimals ($l_{CLP} = -1 \Leftrightarrow 100\%$ loss). The loss frontier (black dashed line) corresponds to Equation 38 where $\rho = 1 + i$ has been set to account for Euler's dynamic liquidation incentive mechanism. Similarly we highlight the relative profitability region (yellow thatched) where rational liquidators can be expected to liquidate on Twyne regardless of whether fallback liquidations are available (see Section 8.5 for discussion). Data was generated by constructing arbitrary user positions on the Twyne contracts with $\rho \geq 1$ (to model a position whose liquidation fails to take place on Twyne) and liquidating them through the Euler contracts with varying closing factors such that they become sanitized. Subsequently, Twyne's post-fallback accounting was executed on the Twyne contracts and relevant CLP losses extracted.

10 Economical Exploit Risk

The previous sections described in detail how liquidations are processed on Twyne and how fallback liquidations are accounted for. While the former does not lead to any losses for Credit LPs (CLPs) given that the liquidation incentive is entirely extracted from the Borrower's excess collateral, the latter creates a dynamic context which may at times lead to losses for CLPs. Given that Twyne cannot create bad debt since it inherits the security assumptions of the underlying market it deploys on, the main attack vector leading to loss of user funds in the context of a Twyne-specific exploit must be found in the realm of processes which can systematically trigger a controlled fallback liquidation.

An actor capable of doing so could proceed to liquidate the unhealthy collateral vault by choosing some combination of closing factor and liquidation incentive such that losses attributed by Twyne's post-fallback accounting (see 6.3.1) are guaranteed to maximally hit the CLPs. The attack would proceed roughly as follows:

1. Create a collateral vault and deposit an amount of flashloaned collateral such that the credit delegation invariant (Equation 7) allows reserving all the available credit in the intermediate

vault ($C_{LP} = C_{LP}^{available}$).

2. Max borrow from the underlying lending market to get as close as possible to liquidation ($\lambda_t \simeq \tilde{\lambda}_t$)

3. Manipulate asset prices to make the position eligible for liquidation via fallback in the same block as it becomes eligible on Twyne ($\lambda_t \to \tilde{\lambda}_t/\beta_{safe}^{dyn}$, $\lambda_e \to \tilde{\lambda}_e$)

4. Self-liquidate with judiciously chosen closing factor and liquidation incentive such that CLP losses will be generated by Twyne's post-fallback accounting.

5. Trigger post-fallback accounting and unwind position on Twyne before anyone can attempt to.

6. Collect the CLP losses as profit.

The above scenario is by no means simple to execute as it requires the ability to trigger some form of controlled price manipulation due to the risk that someone else might trigger the liquidation before the attacker, thus causing him to incur significant losses. Setting $\beta_{safe} < 1$ forces the attacker to manipulate the price by a specific percentage amount to execute the exploit. In theory, if $\beta_{safe} = 1$ the attacker could simply wait for the collateral vault to drift unhealthily and compete to push the final price oracle update themselves, creating the possibility of sandwiching the rest of the exploit into the same transaction.

Waiting for a collateral vault position to drift into liquidation territory could however be very expensive depending on the siphoning rates charged by the interest rate model. Especially so given that the vault would accumulate excess credit rather quickly, thus making it even harder to hit the fallback trigger $\lambda_e \to \tilde{\lambda}_e$. The one way to avoid this would be to borrow from the underlying up to the trigger point and hope in an immediate price move that allows proceeding with the chain of events. This however can be mitigated by not allowing Borrowers to borrow up to their liquidation threshold. After all, this is standard practice across lending markets and should be prescribed for any Twyne deployments.

11 Conclusion

This paper introduced Twyne, a novel noncustodial and risk-modular credit delegation protocol for the Ethereum Virtual Machine, designed explicitly to enhance capital efficiency in decentralized finance (DeFi) lending markets. By leveraging unused borrowing power from passive lenders (Credit LPs), Twyne provides a robust solution to existing inefficiencies including fragmented liquidity, conservative loan-to-value (LTV) ratios, and limited risk customization.

Twyne's innovative credit delegation mechanism uniquely allows borrowers to access higher leverage ratios than those available directly from external lending markets while preserving underlying lending market security assumptions. This mechanism generates a mutually beneficial ecosystem in which credit LPs earn additional returns without incurring significant additional risks beyond their initial exposure, borrowers gain flexible and higher liquidation LTV thresholds, and underlying lending protocols experience increased capital utilization.

The mathematical foundations of the protocol, clearly outlined through precise invariants and rigorous bounds, provide a structured framework quantifying system safety and operational integrity. Critical functions, such as credit reservation and liquidation checks, have been meticulously defined and supported by accompanying Python specifications to facilitate straightforward verification by auditors. The adoption of a curved interest rate model further optimizes the dynamic responsiveness of yield and borrowing costs, effectively aligning incentives across different protocol participants.

Moreover, Twyne incorporates robust safeguards against liquidation risks through dual internal and external (fallback) liquidation strategies. The liquidation-through-inheritance model notably reduces market impact and capital outflow risks, providing a novel and efficient liquidation method that enhances systemic stability. Fallback liquidation mechanisms ensure resilience against adverse market movements, safeguarding Credit LP interests even in most extreme scenarios.

Finally, Twyne's design philosophy of risk modularity, seamless integration with existing lending protocols, capital efficiency maximization, and economic sustainability positions it as a transformative advancement in DeFi. Future iterations of Twyne could explore extensions such as multi-asset collateral support, support for collateral assets which have no borrowing power whatsoever $\tilde{\lambda}_e^C = 0$, dynamic liquidation LTV adjustments via governance/curator mechanisms, and further integration across a broader spectrum of lending markets. Overall, Twyne stands as a substantial progression towards democratizing financial services, offering unparalleled flexibility and capital efficiency in the decentralized credit space.

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Appendix

A Interest Rate Model Parametric Consistency

To guarantee that Equation 9 is a convex and monotonically increasing function of utilization, the choice of parameters must satisfy the following consistency conditions:

$$\begin{aligned}
IR_{min} &\geq 0 \\
IR_0 &\geq IR_{min} \\
IR_{max} &> \frac{IR_0 - (1-u_0)\cdot IR_{min}}{u_0} \\
\gamma &> 1 \\
1 &> u_0 > 0.
\end{aligned} \tag{50}$$

B CLP Loss Analysis

We define the Borrower's initial position (prior to the fallback liquidation) with the tuple C^{old}, B^{old}, C_{LP}^{old}, and $\tilde{\lambda}_t^{old}$, representing the Borrower's owned collateral, their outstanding loan, the reserved credit amount, and their chosen liquidation LTV respectively. Specifically, C_{LP}^{old} and $\tilde{\lambda}_t^{old}$ are related by the Twyne invariant 7:[17]

[17] If the credit asset is of the same type as the loan asset, the exposition proceeds identically with $\beta_{safe} \cdot B^{old}/\tilde{\lambda}_t^{old}$ replacing C^{old}. This difference does not affect the analysis' main result expressed in Equation 36 and Equation 61.

$$C_{LP}^{old} = C^{old} \cdot \left(\frac{\tilde{\lambda}_t^{old}}{\beta_{safe} \cdot \tilde{\lambda}_e^{C_{LP}}} - \frac{\tilde{\lambda}_e^{C}}{\tilde{\lambda}_e^{C_{LP}}} \right) \tag{51}$$

Assuming the fallback liquidation of the collateral vault occurs promptly at $\tilde{\lambda}_e$, such that the borrower's LTV at the time of the fallback liquidation is $\lambda_t = \tilde{\lambda}_t^{old}/\beta_{safe}$, we can express the value of their outstanding loan B^{old} in terms of their collateral C^{old}:

$$B^{old} = C^{old} \cdot \lambda_t = \frac{C^{old} \cdot \tilde{\lambda}_t^{old}}{\beta_{safe}}. \tag{52}$$

We can then define what C_{left} and B_{left} are leftover in the position following the fallback liquidation assuming a closing factor c, and liquidation incentive i charged by the underlying lending market:

$$\begin{aligned} B_{left} &= (1-c) \cdot B^{old} = (1-c) \cdot \frac{C^{old} \cdot \tilde{\lambda}_t^{old}}{\beta_{safe}} && (53) \\ C_{left} &= C^{old} + C_{LP}^{old} - (1+i) \cdot c \cdot B^{old} \\ &= C^{old} \left[1 - (1+i) \cdot c \cdot \frac{\tilde{\lambda}_t^{old}}{\beta_{safe}} \right] + C_{LP}^{old}, && (54) \end{aligned}$$

where Equation 52 has been used to write everything in terms of C^{old}.

We can now rigorously compute the accounting prescription of Equation 27 and obtain the funds that CLPs can expect to receive back:

$$C_{LP}^{new} = C_{LP}^{old} + C^{old} \cdot \min\Big(0,\, 1 - \frac{\tilde{\lambda}_t^{old}}{\beta_{safe} \cdot \tilde{\lambda}_t^{max}} \cdot \Big[(1+i) \cdot c \cdot \tilde{\lambda}_t^{max} + (1-c)\Big]\Big). \tag{55}$$

Furthermore, by inspecting the expression for C_{LP}^{new} in Equation 55, we can define a first condition that guarantees that CLPs do not lose any money as a result of the fallback liquidation, namely:

$$\rho \equiv \tilde{\lambda}_t^{old}/\tilde{\lambda}_t^{max} \leq \frac{\beta_{safe}}{(1+i) \cdot c \cdot \tilde{\lambda}_t^{max} + (1-c)}, \tag{56}$$

where we take the opportunity to define the variable $\rho = \tilde{\lambda}_t^{old}/\tilde{\lambda}_t^{max}$[18] representing how much of the maximum allowable liquidation LTV $\tilde{\lambda}_t^{max}$ the Borrower actually chose to use.

[18] It would be fair to use ρ^{old} given that this value transforms into $\rho^{new} = 1$ following a JIT liquidation, but we suppress it to avoid encumbering the mathematical notation. In passing, one can state that ρ always satisfies the bounds $\rho \in \left\{\beta_{safe} \cdot \tilde{\lambda}_e^{C}/\tilde{\lambda}_t^{max}, 1\right\}$.

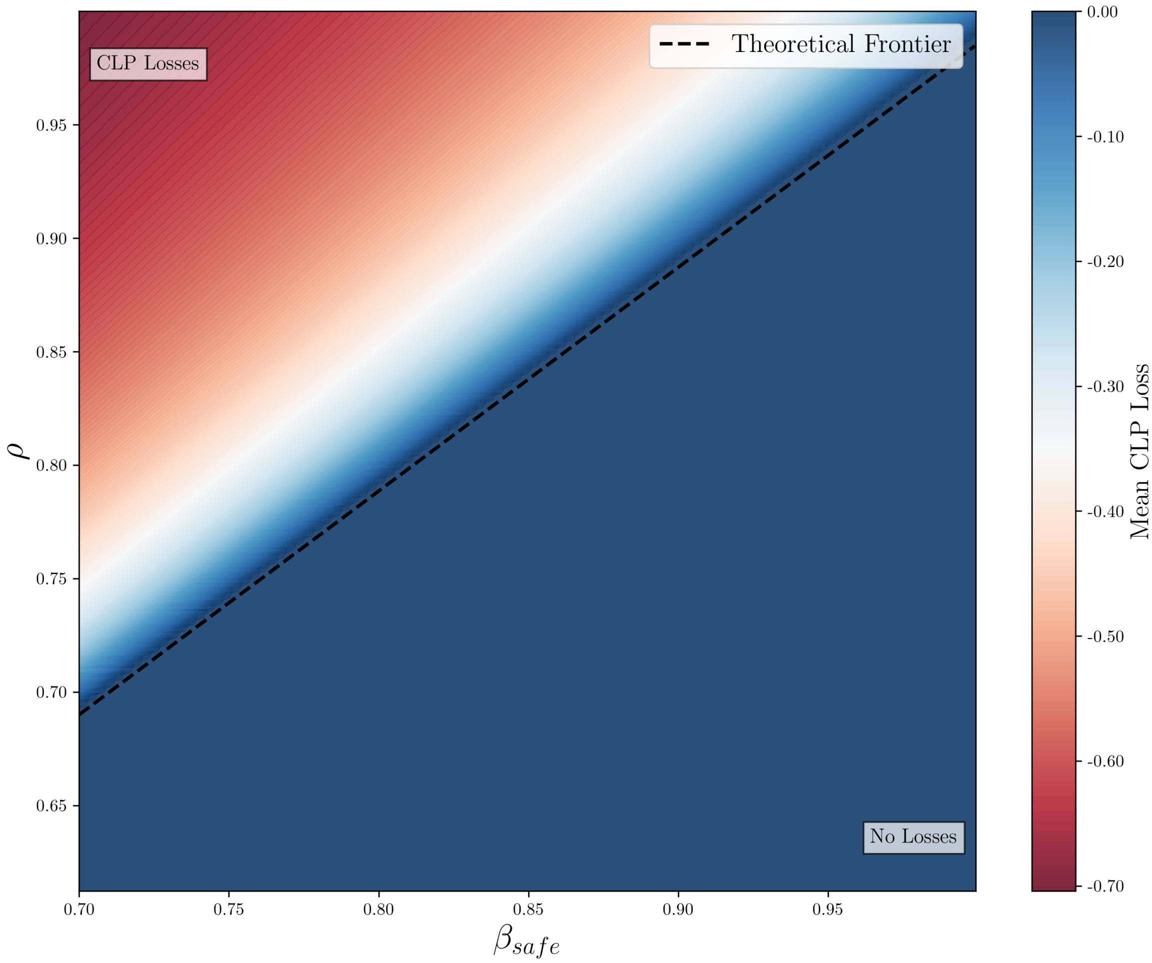


Figure 10: A random ensemble of static Borrowers is simulated with varying ρ and β_{safe}, but fixed $i = 0.05$, $c = 0.5$, $\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^C = 0.85$, and $\tilde{\lambda}_t^{old} = \tilde{\lambda}_t^{max} = 0.97$) under adverse price dynamics aimed at purposely triggering fallback liquidations. Observed CLP losses (expressed in decimals) are plotted via color scale as a function of ρ and β_{safe}, demonstrating good accordance with theory (see Equation 56).

In Figure 10, where a random ensemble of static Borrowers is simulated to trigger fallback liquidations, extracted CLP losses are plotted as a function of ρ and β_{safe}, demonstrating good accordance with Equation 56. Said differently, CLPs are guaranteed to not lose funds as long as Twyne's max allowable LTV $\tilde{\lambda}_t^{max}$ satisfies:

$$\tilde{\lambda}_t^{max} \leq \frac{\beta_{safe} - (1-c)}{(1+i) \cdot c} \leq \frac{\beta_{safe} - (1-c) \cdot \rho}{(1+i) \cdot c \cdot \rho} \tag{57}$$

where we take advantage of the fact that $\rho = 1$ (i.e. $\tilde{\lambda}_t^{old} = \tilde{\lambda}_t^{max}$) creates the worst case scenario for CLPs.

By fixing liquidation incentives and closing factors to specific values i_0, and c_0, one can define $\tilde{\lambda}_t^{max}$ unambiguously on Twyne:

$$\tilde{\lambda}_t^{max} = \frac{\beta_{safe} - (1-c_0)}{(1+i_0) \cdot c_0}. \tag{58}$$

To guarantee $\tilde{\lambda}_t^{max} > \tilde{\lambda}_e^C$ (otherwise Twyne would be pointless) and $i_0 > 0$ (otherwise all fallback liquidations would lead to losses), the quantities i_0 and c_0 are further constrained according to:

$$
\begin{aligned}
c_0 &\geq \frac{1-\beta_{safe}}{1-\tilde{\lambda}_e} && (59)\\
i_0 &< \frac{c_0\cdot(1-\tilde{\lambda}_e)-(1-\beta_{safe})}{c_0\cdot\tilde{\lambda}_e} && (60)
\end{aligned}
$$

To guide some hand-waving intuition, a $\tilde{\lambda}_t^{max}$ defined in this way protects CLPs from losses as long as the actual fallback incentive $i \leq i_0$ and closing factor $c \geq c_0$. However, this level of detail for defining $\tilde{\lambda}_t^{max}$ might be completely unnecessary and other considerations could instead be used to select the maximum allowed LTV in applications. This does not affect the rest of the exposition as it assumes that $\tilde{\lambda}_t^{max}$ has somehow been chosen.

Without any loss in generality, assume that $\tilde{\lambda}_t^{max}$ has been set in Twyne. How much, in relative terms, can CLPs expect to lose (l_{CLP}) if the underlying lending market liquidates the Borrower at incentive i and closing factor c? Plugging 51 into 55 one obtains, after some algebra:

$$
\begin{aligned}
l_{CLP} &= \frac{C_{LP}^{new}}{C_{LP}^{old}} - 1\\
&= \min\left(0, \frac{C^{old}}{C_{LP}^{old}}\cdot\left[1-\frac{\rho}{\beta_{safe}}\cdot\Big((1+i)\cdot c\cdot\tilde{\lambda}_t^{max}+(1-c)\Big)\right]\right)\\
&= \min\left(0, \frac{\beta_{safe}\cdot\tilde{\lambda}_e^{C_{LP}}}{\tilde{\lambda}_t^{old}-\beta_{safe}\cdot\tilde{\lambda}_e^{C}}\cdot\left[1-\frac{\rho}{\beta_{safe}}\cdot\Big((1+i)\cdot c\cdot\tilde{\lambda}_t^{max}+(1-c)\Big)\right]\right),
\end{aligned}
\tag{61}
$$

whence we see again (as in Equation 57) that losses, when they occur, are always maximal in the case $\rho = 1$ (Borrower utilizing max leverage). Most importantly, the result applies independently of the CLP asset type. This is the result presented in Equation 36 of Section 7.

The above result is either zero or negative depending on the variable tuple (c,i) (subject to the other parameters). We will hereon denote the c-vs-i curve where the transition from zero to non-zero losses as the CLP *loss frontier*. It is expressed mathematically as:

$$
c(i) = \frac{1}{\rho}\cdot\frac{\beta_{safe}-\rho}{\tilde{\lambda}_t^{max}\cdot(1+i)-1},
\tag{62}
$$

which was already presented in Equation 38.

C Dynamic Safety Buffer

Sections 7.1.1 and 7.1.2 have highlighted the non-trivial relationship that exists between the CLPs worst-case expected losses and the key parameters $\tilde{\lambda}_e^C$, $\tilde{\lambda}_e^{C_{LP}}$, c_0, i_0, β_{safe}. Whereas the the current setup suffices to handle Twyne deployments on top of underlying lending markets with consistently large closing factors, it lacks robustness in handling the fallback liquidations of Borrowers with $\rho > \beta_{safe}$ by underlying lending markets where small partial liquidations are allowed. This is especially important given that the entire exposition up to this point has assumed that the collateral vault is liquidated promptly at the threshold $\lambda_e = \tilde{\lambda}_e$. In reality, there will always be some exception to this condition that leads to a LTV $\lambda_e > \tilde{\lambda}_e$. Such an exceedance has the effect of artificially lowering β_{safe} to an effective value $\beta_{safe}^{effective} = (\tilde{\lambda}_e/\lambda_e)\cdot\beta_{safe}$ implying that even if Twyne were to set $\beta_{safe} = 1$, it may still be possible to observe $\rho > \beta_{safe}^{effective}$ leading to CLP loss behavior analogous to that discussed in subsection 7.1.2.

The scope of this section is to introduce a dynamic safety buffer $\beta_{safe}^{dyn}(\rho, \beta_{safe})$ that ramps up as a function of a Borrower's ρ when it exceeds the statically defined β_{safe}. While this does not offer a silver bullet to the LTV exceedance effect, it does minimize the discrepancy between ρ and $\beta_{safe}^{effective}$.

Requiring that $\beta_{safe}^{dyn}(\beta_{safe}, \beta_{safe}) = \beta_{safe}$ and $\beta_{safe}^{dyn}(1, \beta_{safe}) = 1$ such that $\rho \leq \beta_{safe}^{dyn}$ at all times, one has:

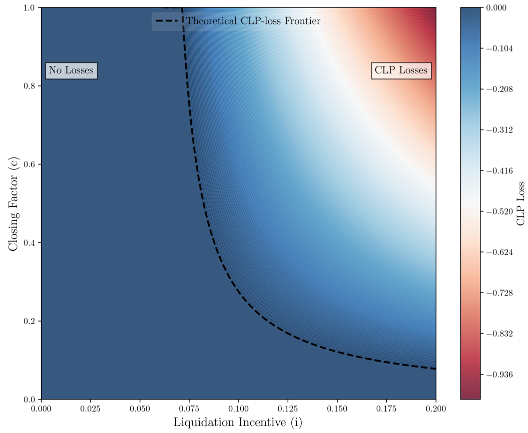
$$\beta_{safe}^{dyn}(\rho, \beta_{safe}) = \max(0, \rho - \beta_{safe}) + \beta_{safe} \tag{63}$$

The dynamic safety buffer just defined is a user-specific quantity affecting all the pre-fallback quantities quantities defined in this exposition. Among these we note the expressions for the protocol invariant C_{LP}^{old} (Equation 51), the post-fallback accounting C_{LP}^{new} (Equation 55), and the CLP loss surface which, from Equation 61, can be rewritten as:

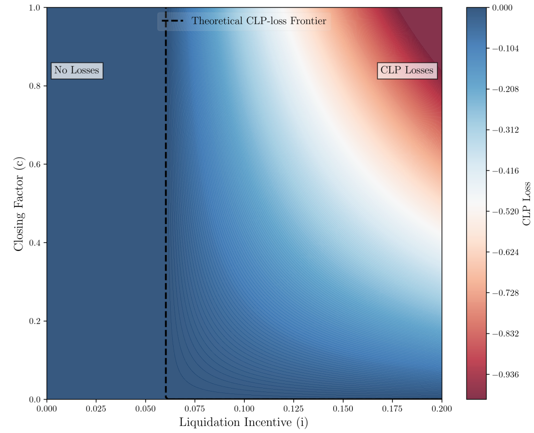
$$l_{CLP}^{dyn} = \min\left(0, \frac{\beta_{safe}^{dyn} \cdot \tilde{\lambda}_e^{C_{LP}}}{\tilde{\lambda}_t^{old} - \beta_{safe}^{dyn} \cdot \tilde{\lambda}_e^{C}} \cdot \left[1 - \frac{\rho}{\beta_{safe}^{dyn}} \cdot \Big((1+i) \cdot c \cdot \tilde{\lambda}_t^{max} + (1-c)\Big)\right]\right), \tag{64}$$

In Figure 11 we show the benefits of employing a dynamic safety buffer by elaborating the identical parametric setup of Figure 7. One can see how the use of the dynamical safety buffer β_{safe}^{dyn} guarantees protection against CLP losses even within the limit of small closing factors for scenarios where $\rho > \beta_{safe}$. Furthermore, we test the theoretical results by simulating an ensemble of static borrowers whose positions are artificially driven to liquidation by imposing an exponentially declining price on the collateral and credit assets (assumed to be identical in type) over the course of a year. At each liquidation, closing factors and incentives are chosen randomly, and available swap liquidity is assumed to be large compared to absolute position sizes to ignore slippage effects.

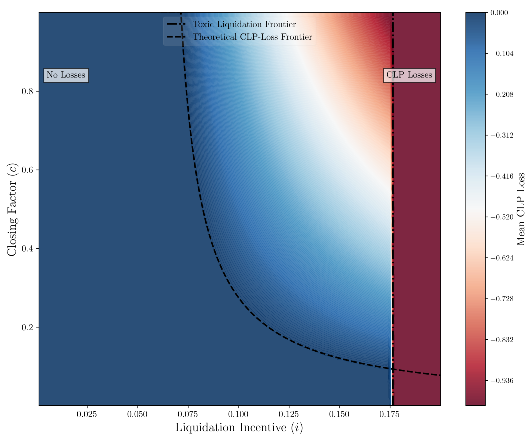
The numerical results quantitatively reproduce the theoretical results across a wide range of realistic liquidation incentive and closing factors. The one deviation observed is at large liquidation incentives where toxic liquidation spirals [4] are expected to occur, implying that fallback liquidations will not sanitize collateral vaults. In such a scenario, barring loss socialization measures by the underlying lending market, CLPs can expect to lose all their funds.



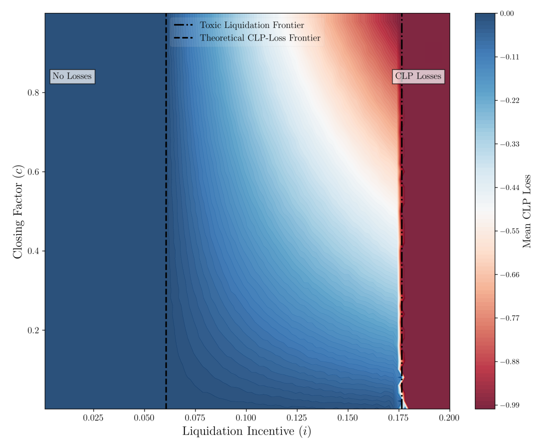
(a) $\rho = 0.98$



(b) $\rho = 1$



(c) $\rho = 0.98$ (simulation)



(d) $\rho = 1$ (simulation)

Figure 11: Expected CLP losses as a function of liquidation incentive i and closing factor c with $\tilde{\lambda}_e^{C_{LP}} = \tilde{\lambda}_e^{C} = 0.85$, $\beta_{safe} = 0.99$, and $\tilde{\lambda}_t^{old} = \tilde{\lambda}_t^{max} \simeq 0.943$ (using $c_0 = 1$, $i_0 = 0.05$ in 58), for $\rho = 0.98$ (a) and $\rho = 1$ (b) respectively. Losses are expressed in decimals ($l_{CLP} = -1 \Leftrightarrow 100\%$ loss). The loss frontier (black dashed line) corresponds to Equation 64. Figures (c) and (d) are experimental reproduction of Figures (a) and (b) respectively, obtained by generating $100k$ users with random initial LTVs, but identical ρ, which were subsequently methodically liquidated by simulating an exponentially decreasing price trajectory which makes the collateral worthless over the course of one year (timestep=). At each liquidation, liquidation incentives i and closing factors c where randomly chosen, and the CLP loss extracted numerically. Discrepancies between theory and simulation arise for large liquidation incentive values where toxic liquidation spirals occur (leading to bad debt) which are not considered in the theoretical exposition. Interactive plots are available at this Desmos link.