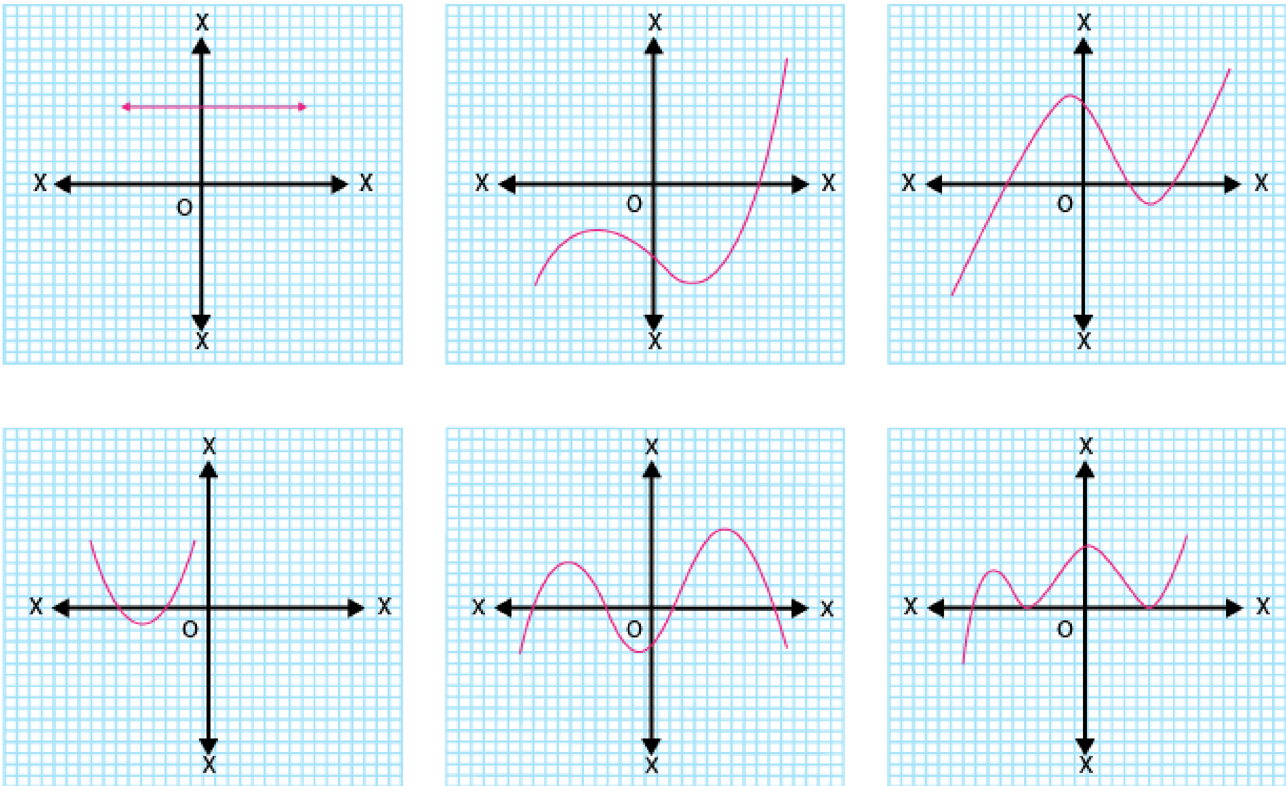
Access Answers to NCERT Class 10 Maths Chapter 2 – Polynomials Exercise 2.1

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



Solutions:

Graphical method to find zeroes:-

The total number of zeroes in any polynomial equation = the total number of times the curve intersects the x-axis.

(i) In the given graph, the number of zeroes of p(x) is 0 because the graph is parallel to the x-axis and does not cut it at any point.

(ii) In the given graph, the number of zeroes of p(x) is 1 because the graph intersects the x-axis at only one point.

(iii) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.

(iv) In the given graph, the number of zeroes of p(x) is 2 because the graph intersects the x-axis at two points.

(v) In the given graph, the number of zeroes of p(x) is 4 because the graph intersects the x-axis at four points.

(vi) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.

Access answers to NCERT Class 10 Maths Chapter 2 – Polynomials Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i) x^2-2x-8

$\Rightarrow x^2-4x+2x-8 = x(x-4)+2(x-4) = (x-4)(x+2)$

Therefore, zeroes of polynomial equation x^2-2x-8 are (4, -2)

Sum of zeroes = 4–2 = 2 = -(-2)/1 = -(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = 4×(-2) = -8 =-(8)/1 = (Constant term)/(Coefficient of x^2)

(ii) $4s^2-4s+1$

$\Rightarrow 4s^2-2s-2s+1 = 2s(2s-1)-1(2s-1) = (2s-1)(2s-1)$

Therefore, zeroes of the polynomial equation $4s^2-4s+1$ are (1/2, 1/2)

Sum of zeroes = (½)+(1/2) = 1 = -(-4)/4 = -(Coefficient of s)/(Coefficient of s^2)

Product of zeros = (1/2)×(1/2) = 1/4 = (Constant term)/(Coefficient of s^2)

(iii) $6x^2-3-7x$

$\Rightarrow 6x^2-7x-3 = 6x^2-9x+2x-3 = 3x(2x-3)+1(2x-3) = (3x+1)(2x-3)$

Therefore, zeroes of the polynomial equation $6x^2-3-7x$ are (-1/3, 3/2)

Sum of zeroes = $-(1/3)+(3/2) = (7/6) = -$(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = $-(1/3)\times(3/2) = -(3/6) =$ (Constant term) /(Coefficient of x^2)

(iv) $4u^2+8u$

$\Rightarrow 4u(u+2)$

Therefore, zeroes of the polynomial equation $4u^2 + 8u$ are $(0, -2)$

Sum of zeroes = $0+(-2) = -2 = -(8/4) = = -$(Coefficient of u)/(Coefficient of u^2)

Product of zeroes = $0\times-2 = 0 = 0/4 =$ (Constant term)/(Coefficient of u^2)

(v) t^2-15

$\Rightarrow t^2 = 15$ or $t = \pm\sqrt{15}$

Therefore, zeroes of the polynomial equation $t^2 -15$ are $(\sqrt{15}, -\sqrt{15})$

Sum of zeroes $=\sqrt{15}+(-\sqrt{15}) = 0= -(0/1)= -$(Coefficient of t) / (Coefficient of t^2)

Product of zeroes = $\sqrt{15}\times(-\sqrt{15}) = -15 = -15/1 =$ (Constant term) / (Coefficient of t^2)

(vi) $3x^2-x-4$

$\Rightarrow 3x^2-4x+3x-4 = x(3x-4)+1(3x-4) = (3x - 4)(x + 1)$

Therefore, zeroes of the polynomial equation $3x^2 - x - 4$ are $(4/3, -1)$

Sum of zeroes = $(4/3)+(-1) = (1/3)= -(-1/3) = -$(Coefficient of x) / (Coefficient of x^2)

Product of zeroes$=(4/3)\times(-1) = (-4/3) =$ (Constant term) /(Coefficient of x^2)

2. Find a quadratic polynomial, each with the given numbers as the sum and product of its zeroes, respectively.

(i) 1/4 , -1

Solution:

From the formulas of sum and product of zeroes, we know,

Sum of zeroes = $\alpha+\beta$

Product of zeroes = $\alpha\beta$

Sum of zeroes = $\alpha+\beta = 1/4$

Product of zeroes = $\alpha\beta = -1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

$$x^2-(1/4)x+(-1) = 0$$

$$4x^2-x-4 = 0$$

Thus, $4x^2-x-4$ is the quadratic polynomial.

(ii) $\sqrt{2}, 1/3$

Solution:

Sum of zeroes = $\alpha + \beta = \sqrt{2}$

Product of zeroes = $\alpha\beta = 1/3$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

$$x^2-(\sqrt{2})x+(1/3) = 0$$

$$3x^2-3\sqrt{2}x+1 = 0$$

Thus, $3x^2-3\sqrt{2}x+1$ is the quadratic polynomial.

(iii) 0, $\sqrt{5}$

Solution:

Given,

Sum of zeroes = $\alpha+\beta = 0$

Product of zeroes = $\alpha\beta = \sqrt{5}$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly

as

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

$$x^2-(0)x+\sqrt{5} = 0$$

Thus, $x^2+\sqrt{5}$ is the quadratic polynomial.

(iv) 1, 1

Solution:

Given,

Sum of zeroes = $\alpha+\beta$ = 1

Product of zeroes = $\alpha\ \beta$ = 1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as

$\mathbf{x^2-(\alpha+\beta)x+\alpha\beta = 0}$

$\mathbf{x^2-x+1 = 0}$

Thus, $\mathbf{x^2-x+1}$ is the quadratic polynomial.

(v) -1/4, 1/4

Solution:

Given,

Sum of zeroes = $\alpha+\beta$ = -1/4

Product of zeroes = $\alpha\ \beta$ = 1/4

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as

$\mathbf{x^2-(\alpha+\beta)x+\alpha\beta = 0}$

$\mathbf{x^2-(-1/4)x+(1/4) = 0}$

$\mathbf{4x^2+x+1 = 0}$

Thus, $\mathbf{4x^2+x+1}$ is the quadratic polynomial.

(vi) 4, 1

Solution:

Given,

Sum of zeroes = $\alpha+\beta$ =4

Product of zeroes = $\alpha\beta$ = 1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-4x+1 = 0$

Thus, **x^2-4x+1** is the quadratic polynomia

Access answers of Maths NCERT Class 10 Chapter 2 – Polynomials Exercise 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following.

(i) $p(x) = x^3-3x^2+5x-3$, $g(x) = x^2-2$

Solution:

Given,

Dividend = $p(x) = x^3-3x^2+5x-3$

Divisor = $g(x) = x^2- 2$

$$
\begin{array}{llllll}
 & & x & -3 & & \\
\cline{3-6}
x^2-2 & \big) & x^3 & -3x^2 & +5x & -3 \\
 & - & & & & \\
 & & x^3 & +0x^2 & -2x & \\
\cline{2-6}
 & & & -3x^2 & +7x & -3 \\
 & & - & & & \\
 & & & -3x^2 & +0x & +6 \\
\cline{2-6}
 & & & & 7x & -9
\end{array}
$$

Therefore, upon division, we get

Quotient = $x-3$

Remainder = $7x-9$

(ii) $p(x) = x^4-3x^2+4x+5$, $g(x) = x^2+1-x$

Solution:

Given,

Dividend = p(x) = $x^4 - 3x^2 + 4x + 5$

Divisor = g(x) = $x^2 + 1 - x$

$$
\begin{array}{r|rrrrr}
 & x^2 & +x & -3 & & \\
\hline
x^2 - x + 1 & x^4 & +0x^3 & -3x^2 & +4x & +5 \\
 & - & & & & \\
 & x^4 & -x^3 & +x^2 & & \\
\hline
 & & x^3 & -4x^2 & +4x & +5 \\
 & & - & & & \\
 & & x^3 & -x^2 & +x & \\
\hline
 & & & -3x^2 & +3x & +5 \\
 & & & - & & \\
 & & & -3x^2 & +3x & -3 \\
\hline
 & & & & & 8
\end{array}
$$

Therefore, upon division, we get

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) p(x) =x^4-5x+6, g(x) = $2-x^2$

Solution:

Given,

Dividend = p(x) =$x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6$

Divisor = g(x) = $2-x^2 = -x^2+2$

$$
\begin{array}{r|rrrrr}
 & -x^2 & -2 & & & \\
\hline
-x^2 + 2 & x^4 & +0x^3 & +0x^2 & -5x & +6 \\
 & - & & & & \\
 & x^4 & +0x^3 & -2x^2 & & \\
\hline
 & & & 2x^2 & -5x & +6 \\
 & & & - & & \\
 & & & 2x^2 & +0x & -4 \\
\hline
 & & & & -5x & +10
\end{array}
$$

Therefore, upon division, we get

Quotient = $-x^2-2$

Remainder = $-5x + 10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) t^2-3, $2t^4+3t^3-2t^2-9t-12$

Solutions:

Given,

First polynomial = t^2-3

Second polynomial = $2t^4+3t^3-2t^2-9t-12$

$$
\begin{array}{lrrrrr}
 & 2t^2 & +3t & +4 & & \\
t^2-3 \;\big) & 2t^4 & +3t^3 & -2t^2 & -9t & -12 \\
 & - & & & & \\
 & 2t^4 & +0t^3 & -6t^2 & & \\
\hline
 & & 3t^3 & +4t^2 & -9t & -12 \\
 & & - & & & \\
 & & 3t^3 & +0t^2 & -9t & \\
\hline
 & & & 4t^2 & +0t & -12 \\
 & & & - & & \\
 & & & 4t^2 & +0t & -12 \\
\hline
 & & & & & 0
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$

(ii) x^2+3x+1 , $3x^4+5x^3-7x^2+2x+2$

Solutions:

Given,

First polynomial = x^2+3x+1

Second polynomial = $3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{rllllll}
 & & 3x^2 & -4x & +2 & & \\
\cline{3-7}
x^2+3x+1 & \big) & 3x^4 & +5x^3 & -7x^2 & +2x & +2 \\
 & - & & & & & \\
 & & 3x^4 & +9x^3 & +3x^2 & & \\
\cline{2-7}
 & & & -4x^3 & -10x^2 & +2x & +2 \\
 & & - & & & & \\
 & & & -4x^3 & -12x^2 & -4x & \\
\cline{2-7}
 & & & & 2x^2 & +6x & +2 \\
 & & & - & & & \\
 & & & & 2x^2 & +6x & +2 \\
\cline{2-7}
 & & & & & & 0
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that $x^2 + 3x + 1$ is a factor of $3x^4+5x^3-7x^2+2x+2$

(iii) x^3-3x+1, $x^5-4x^3+x^2+3x+1$

Solutions:

Given,

First polynomial = x^3-3x+1

Second polynomial = $x^5-4x^3+x^2+3x+1$

$$
\begin{array}{rlllllll}
 & & x^2 & -1 & & & & \\
\cline{3-8}
x^3-3x+1 & \big) & x^5 & +0x^4 & -4x^3 & +x^2 & +3x & +1 \\
 & - & & & & & & \\
 & & x^5 & +0x^4 & -3x^3 & +x^2 & & \\
\cline{2-8}
 & & & & -x^3 & +0x^2 & +3x & +1 \\
 & & & - & & & & \\
 & & & & -x^3 & +0x^2 & +3x & -1 \\
\cline{2-8}
 & & & & & & & 2
\end{array}
$$

As we can see, the remainder is not equal to 0. Therefore, we say that x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$

3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$.

Solutions:

Since this is a polynomial equation of degree 4, there will be a total of 4 roots.

√(5/3) and – √(5/3) are zeroes of polynomial f(x).

∴ (x –**√(5/3)**) (x+**√(5/3)** = x^2–(5/3) = 0

(3x^2–5)=0, is a factor of given polynomial f(x).

Now, when we will divide f(x) by (3x^2–5), the quotient obtained will also be a factor of f(x), and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x^2-5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \quad\quad - 5x^2 \\
 & (-) \quad\quad\quad (+) \\
\hline
 & +6x^3 + 3x^2 - 10x - 5 \\
 & -6x^3 \quad\quad - 10x \\
 & (+) \quad\quad\quad (-) \\
\hline
 & 3x^2 \quad\quad -5 \\
 & 3x^2 \quad\quad -5 \\
 & (-) \quad\quad\quad (+) \\
\hline
 & 0
\end{array}
$$

Therefore, $3x^4+6x^3-2x^2-10x-5$ = ($3x^2$ –5)**(x^2+2x+1)**

Now, on further factorising (x^2+2x+1), we get

x^2+2x+1 = x^2+x+x+1 = 0

x(x+1)+1(x+1) = 0

(x+1)(x+1) = 0

So, its zeroes are given by: **x= –1** and **x = –1**

Therefore, all four zeroes of the given polynomial equation are

√(5/3),- √(5/3) , –1 and –1

Hence, the above-given solution is the answer.

4. On dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were x−2 and −2x+4, respectively. Find g(x).

Solution:

Given,

Dividend, $p(x) = x^3-3x^2+x+2$

Quotient = x-2

Remainder = −2x+4

We have to find the value of Divisor, g(x) =?

As we know,

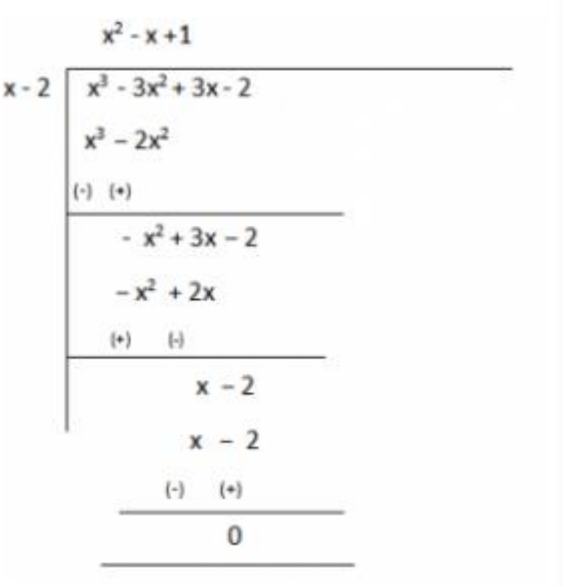
Dividend = Divisor × Quotient + Remainder

∴ $x^3-3x^2+x+2 = g(x)\times(x-2) + (-2x+4)$

$x^3-3x^2+x+2-(-2x+4) = g(x)\times(x-2)$

Therefore, $g(x) \times (x-2) = x^3-3x^2+3x-2$

Now, for finding g(x), we will divide x^3-3x^2+3x-2 with (x-2)



Therefore, $\mathbf{g(x) = (x^2-x+1)}$

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Solutions:

According to the division algorithm, dividend p(x) and divisor g(x) are two polynomials, where g(x)≠0. Then, we can find the value of quotient q(x) and remainder r(x) with the help of below-given formula.

Dividend = Divisor × Quotient + Remainder

∴ p(x) = g(x)×q(x)+r(x)

Where r(x) = 0 or degree of r(x)< degree of g(x)

Now, let us prove the three given cases, as per the division algorithm, by taking examples for each.

(i) deg p(x) = deg q(x)

The degree of dividend is equal to the degree of the quotient only when the divisor is a constant term.

Let us take an example: p(x) = $3x^2+3x+3$ is a polynomial to be divided by g(x) = 3

So, $(3x^2+3x+3)/3 = x^2+x+1$ = q(x)

Thus, you can see the degree of quotient q(x) = 2, which is also equal to the degree of dividend p(x).

Hence, the division algorithm is satisfied here.

(ii) deg q(x) = deg r(x)

Let us take an example: p(x) = $x^2 + 3$ is a polynomial to be divided by g(x) = x – 1

So, $x^2 + 3 = (x - 1)\times(x) + (x + 3)$

Hence, quotient q(x) = x

Also, remainder r(x) = x + 3

Thus, you can see the degree of quotient q(x) = 1, which is also equal to the degree of remainder r(x).

Hence, the division algorithm is satisfied here.

(iii) deg r(x) = 0

The degree of remainder is 0 only when the remainder left after the division algorithm is constant.

Let us take an example: $p(x) = x^2 + 1$ is a polynomial to be divided by $g(x) = x$.

So, $x^2 + 1 = (x)\times(x) + 1$

Hence, quotient $q(x) = x$

And, remainder $r(x) = 1$

Clearly, the degree of remainder here is 0.

Hence, the division algorithm is satisfied here.

Access Answers to NCERT Class 10 Maths Chapter 2 – Polynomials Exercise 2.4

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2$; -1/2, 1, -2

Solution:

Given, $p(x) = 2x^3+x^2-5x+2$

And zeroes for p(x) are = 1/2, 1, -2

$\therefore p(1/2) = 2(1/2)^3+(1/2)^2-5(1/2)+2 = (1/4)+(1/4)-(5/2)+2 = 0$

$p(1) = 2(1)^3+(1)^2-5(1)+2 = 0$

$p(-2) = 2(-2)^3+(-2)^2-5(-2)+2 = 0$

Hence, proved 1/2, 1, -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, comparing the given polynomial with the general expression, we get;

$\therefore ax^3+bx^2+cx+d = 2x^3+x^2-5x+2$

a=2, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = c/a$

$\alpha\beta\gamma = -d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha+\beta+\gamma = ½+1+(-2) = -1/2 = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = (1/2\times1)+(1\times-2)+(-2\times1/2) = -5/2 = c/a$

$\alpha\beta\gamma = ½\times1\times(-2) = -2/2 = -d/a$

Hence, the relationship between the zeroes and the coefficients is satisfied.

(ii) x^3-4x^2+5x-2 ;2, 1, 1

Solution:

Given, $p(x) = x^3-4x^2+5x-2$

And zeroes for p(x) are 2,1,1.

$\therefore p(2)= 2^3-4(2)^2+5(2)-2 = 0$

$p(1) = 1^3-(4\times1^2)+(5\times1)-2 = 0$

Hence proved, 2, 1, 1 are the zeroes of x^3-4x^2+5x-2

Now, comparing the given polynomial with the general expression, we get;

$\therefore ax^3+bx^2+cx+d = x^3-4x^2+5x-2$

a = 1, b = -4, c = 5 and d = -2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$

$\alpha\beta\gamma = -d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha+\beta+\gamma = 2+1+1 = 4 = -(-4)/1 = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = 2\times1+1\times1+1\times2 = 5 = 5/1= c/a$

$\alpha\beta\gamma = 2\times1\times1 = 2 = -(-2)/1 = -d/a$

Hence, the relationship between the zeroes and the coefficients is satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, −7, −14 respectively.

Solution:

Let us consider the cubic polynomial is ax^3+bx^2+cx+d, and the values of the zeroes of the polynomials are α, β, γ.

As per the given question,

$\alpha+\beta+\gamma = -b/a = 2/1$

$\alpha\beta+\beta\gamma+\gamma\alpha = c/a = -7/1$

$\alpha\beta\gamma = -d/a = -14/1$

Thus, from the above three expressions, we get the values of the coefficients of the polynomial.

$a = 1, b = -2, c = -7, d = 14$

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

3. If the zeroes of the polynomial x^3-3x^2+x+1 are a – b, a, a + b, find a and b.

Solution:

We are given the polynomial here,

$p(x) = x^3-3x^2+x+1$

And zeroes are given as $a - b, a, a + b$

Now, comparing the given polynomial with the general expression, we get;

$\therefore px^3+qx^2+rx+s = x^3-3x^2+x+1$

$p = 1, q = -3, r = 1$ and $s = 1$

Sum of zeroes = $a - b + a + a + b$

$-q/p = 3a$

Putting the values q and p.

$-(-3)/1 = 3a$

$a=1$

Thus, the zeroes are $1-b, 1, 1+b$.

Now, product of zeroes = 1(1-b)(1+b)

-s/p = $1-b^2$

-1/1 = $1-b^2$

$b^2 = 1+1 = 2$

$b = \pm\sqrt{2}$

Hence,$1-\sqrt{2}$, 1 ,$1+\sqrt{2}$ are the zeroes of x^3-3x^2+x+1.

4. If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2 \pm\sqrt{3}$, find other zeroes.

Solution:

Since this is a polynomial equation of degree 4, there will be total of 4 roots.

Let $f(x) = x^4-6x^3-26x^2+138x-35$

Since $2 +\sqrt{\mathbf{3}}$ and $2-\sqrt{\mathbf{3}}$ are zeroes of given polynomial f(x).

$\therefore [x-(2+\sqrt{\mathbf{3}})]\ [x-(2-\sqrt{\mathbf{3}})] = 0$

$(x-2-\sqrt{\mathbf{3}})(x-2+\sqrt{\mathbf{3}}) = 0$

After multiplication, we get,

x^2-4x+1, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x), and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+x^2 \\
 & (-)\ (+)\ (-) \\
\hline
 & -2x^3-27x^2+138x-35 \\
 & -2x^3+8x^2-2x \\
 & (+)\ (-)\ (+) \\
\hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & (+)\ (-)\ (+) \\
\hline
 & 0
\end{array}
$$

So, $x^4-6x^3-26x^2+138x-35 = (x^2-4x+1)(x^2 -2x-35)$

Now, on further factorizing $(x^2-2x-35)$ we get,

x^2**$-(7-5)x -35$** $= x^2- 7x+5x+35 = 0$

$x(x -7)+5(x-7) = 0$

$(x+5)(x-7) = 0$

So, its zeroes are given by:

$x= -5$ and $x = 7$.

Therefore, all four zeroes of the given polynomial equation are $2+\sqrt{3}$, $2-\sqrt{3}$, **-5 and 7.**

Q.5: If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Solution:

Let's divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$.

$$
\begin{array}{l}
x^2-2x+k\ )\ x^4-6x^3+16x^2-25x+10\ (x^2-4x+(8-k) \\
\qquad\qquad\quad x^4-2x^3+kx^2 \\
\hline
\qquad\qquad\qquad -4x^3+(16-k)x^2-25x \\
\qquad\qquad\qquad -4x^3+8x^2-4kx \\
\hline
\qquad\qquad\qquad\qquad (8-k)x^2+(4k-25)x+10 \\
\qquad\qquad\qquad\qquad (8-k)x^2-2(8-k)x+k(8-k) \\
\hline
\qquad\qquad\qquad\quad (4k-25+16-2k)x+[10-k(8-k)]
\end{array}
$$

Given that the remainder of the polynomial division is $x + a$.

$(4k - 25 + 16 - 2k)x + [10 - k(8 - k)] = x + a$

$(2k - 9)x + (10 - 8k + k^2) = x + a$

Comparing the coefficients of the above equation, we get;

$2k - 9 = 1$

$2k = 9 + 1 = 10$

$k = 10/2 = 5$

And

$10 - 8k + k^2 = a$

$10 - 8(5) + (5)^2 = a$ [since $k = 5$]

$10 - 40 + 25 = a$

$a = -5$

Therefore, $k = 5$ and $a = -5$.