

# Problem H

## Basic Tautologies

Time Limit: 1 second

Let  $A := \{=, -, a, b, c, \dots, z, A, B, C, \dots, Z\}$ .

We assume that  $*$  represent the operation of concatenation between strings.

We define the set of formulas over  $A$  recursively as follows:

- If  $X$  belongs to  $A \setminus \{=, -\}$  then  $X$  is formula (variable).
- IF  $X$  is a formula, so is  $X*-$ .
- If  $X$  and  $Y$  are formulas, so is:  $X*Y*=$ .

These formulas are understood as logical formulas with connectives  $-$  for negation,  $=$  for equivalence and  $A \setminus \{=, -\}$  as variables. That is  $=$  and  $-$  are not variables. Also, variables  $a$  and  $A$  are considered different. Similarly  $b$  is different to  $B$  and so on.

Of course our formulas are given in Reverse Polish Notation (RPN). We can evaluate a formula for a given boolean input  $\{0, 1\}$  and the output is either 0 or 1 as usual.

A formula is a tautology if it evaluates to 1 for every input. For example  $aa=$  is a tautology while  $aa=-$  is not. Note that  $aa=$  represents the formula  $a=a$  in the standard infix notation and  $aa=-$  represents the formula  $\neg[a=a]$ .

## Input

The first line is a natural number  $N$  less than 100. Then, there are  $N$  lines, each one is a string over  $A$ . Every string is of size less than 200 characters.

## Output

You must display  $N$  lines, each one with 3 possible answers: incorrect, tautology or formula. Answer number  $i$  gives the output of string number  $i$ . The output is "incorrect" if the input string is not a formula. The output is "formula" if the input string is a formula that is not a tautology. The output is "tautology" if the input string is a formula that is a tautology.

Sample Input	Sample Output
3 aa= aa=- ab	tautology formula incorrect

# Note:

Perhaps some students have no idea on how to evaluate a formula in RPN form. However I assume that she/he knows how to do it in the standard form, hence I need only to describe how to convert a RPN formula into a standard infix form. We define  $f(X)$  the translation of a RPN formula  $X$  by recursion as follows:

We assume that  $X, Y, Z$  represent formulas.

1. If  $X$  is a variable then  $f(X) := X$ .
2. If  $X$  is of the form  $Y^-$  then  $f(X) := [*-f(Y)*]$ .
3. If  $X$  is of the form  $Y*Z$  then  $f(X) := [*f(Y)*f(Z)*]$ .

where  $[$  and  $]$  are parenthesis symbols (not needed in a RPN formula).

Just in case, I include the truth tables for  $=$  and  $-$ .

The truth table for  $=$  is:

A	B	A=B
0	0	1
0	1	0
1	0	0
1	1	1

The truth table for  $-$  is:

A	-A
0	1
1	0

Good luck!

---

**Problemsetter: Dr. Mauricio Javier Osorio Galindo**