

Problem H

Prime Suspect

Input: Standard Input
Output: Standard Output

Let's define the function $Suspect(b, n)$, where b is an integer that is called the base, and n is an odd integer. It returns one of the boolean values TRUE or FALSE.

- Let t be the highest power of 2 so that 2^t divides $n-1$ and u be the biggest odd integer that divides $n-1$.
This means we can write $n-1 = 2^t u$.
- Let x_0 be $b^u \bmod n$.
- For all i from 1 to t , let x_i be $(x_{i-1})^2 \bmod n$.
- If, for any i from 1 to t , $x_i = 1$ and $x_{i-1} \not\equiv 1$ and $x_{i-1} \not\equiv n-1$, then return FALSE.
- else, if $x_t \not\equiv 1$, then return FALSE.
- else return TRUE.

The connoisseur will recognize this function as the essential part of the Miller-Rabin primality test, although it can appear in different forms throughout the literature. In *Cormen et. al.* Ch. 31 this function is called *Witness* and returns the opposite boolean value. We will call the odd number n **suspect in base b** if the above function returns TRUE. At the end of this description three examples are given.

It can be proved that whenever the function $Suspect(b, n)$ returns FALSE for some base b and an odd number n it is **sure** that n is not a prime number. The reverse, however, is not the case: whenever $Suspect(b, n)$ returns TRUE, there is a high probability that n is a prime number, but we can't be sure. We say that $Suspect(b, n)$ **fails** if it returns TRUE for an n that is not a prime number. Upto 1000000 there are only 46 failures in base 2, the first three being 2047, 3277 and 4033. In base 3 there are 73 failures, but all of them are different from the base 2 failures, so for every odd number $n < 1000000$ we have that if $Suspect(2, n)$ and $Suspect(3, n)$ we can be sure that n is a prime number. Upto 2^{32} we only need to calculate three bases: $Suspect(2, n)$, $Suspect(7, n)$ and $Suspect(61, n)$. If all three function calls return TRUE, it's sure that n is a prime number. This gives us a very quick primality test for all numbers within the range of current day integers.

In this problem we want you to calculate the failures of the function $Suspect(b, n)$ in a certain base and for a certain range of numbers n .

Input

The input consists of several lines, each containing three integers: *Base*, *Min* and *Max*. *Base* is an integer between 2 and 1024 (inclusive), *Min* and *Max* will be between 3 and 4294967295 (inclusive). *Max* will not be smaller than *Min*. *Max* will be at most 100000 bigger than *Min*. A line with three zeroes marks the end of the input; this line should not be processed.

Output

For each line of input, one line of output: "There are NUMBER1 odd non-prime numbers between NUMBER2 and NUMBER3."

If there are odd numbers within this range that fail as suspects in the given base, output an other line: "NUMBER4 suspects fail in base NUMBER5:", followed by all failures, in ascending order, each on a line by itself. Use the plural form, even if there is only one failure. If there are no failures in this range, output the line: "There are no failures in base NUMBER5.". NUMBER1 .. NUMBER5 are to be replaced by the appropriate values. Separate the cases by a blank line.

Sample Input

Output for Sample Input

186 800000 900000 2 4000000000 4000001000 3 121 121 0 0 0	There are 42677 odd non-prime numbers between 800000 and 900000. 4 suspects fail in base 186: 821059 840781 873181 876961 There are 457 odd non-prime numbers between 4000000000 and 4000001000. There are no failures in base 2. There are 1 odd non-prime numbers between 121 and 121. 1 suspects fail in base 3: 121
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Problem setter: Little Joey (Joachim)

Be Ware! This problem has no alternate solution

Suspect(2, 121):

$n-1 = 120 = 8 * 15 = 2^3 * 15$, therefore $t=3, u=15$

$x_0 = 2^{15} \bmod 121 = 32768 \bmod 121 = 98$

$x_1 = 98^2 \bmod 121 = 9604 \bmod 121 = 45$

$x_2 = 45^2 \bmod 121 = 2025 \bmod 121 = 89$

$x_3 = 89^2 \bmod 121 = 7921 \bmod 121 = 56$

None of the x_i is 1, so the loop test continues until the end. Since x_t is not 1, the function will return FALSE. This is a correct result, since $121 = 11*11$ is composite.

Suspect(3, 121):

$n-1 = 120 = 8 * 15 = 2^3 * 15$, therefore $t=3, u=15$

$x_0 = 3^{15} \bmod 121 = 14348907 \bmod 121 = 1$

$x_1 = 1^2 \bmod 121 = 1 \bmod 121 = 1$

$x_2 = 1^2 \bmod 121 = 1 \bmod 121 = 1$

$x_3 = 1^2 \bmod 121 = 1 \bmod 121 = 1$

All of the x_i are 1, so the loop test continues until the end. Since x_t is 1, the function will return TRUE. This is a failure!.

Suspect(3, 89):

$n-1 = 88 = 8 * 11 = 2^3 * 11$, therefore $t=3, u=11$

$x_0 = 3^{11} \bmod 89 = 177147 \bmod 89 = 37$

$x_1 = 37^2 \bmod 89 = 1369 \bmod 89 = 34$

$x_2 = 34^2 \bmod 89 = 1156 \bmod 89 = 88$

$x_3 = 88^2 \bmod 89 = 1 \bmod 89 = 1$

Here we see the loop test again continue to the end. Since x_t is 1, the function will return TRUE. This is correct, because 89 is a prime number.