Combinational Logic Circuits

Sum of products Method

A	B	fundamental products
0	0	$\bar{A}\bar{B}$
0	1	$\bar{A}B$
1	0	$A\bar{B}$
1	1	AB

The fundamental products are also called minterms.

minterms are represented as m_i where i is the decimal equivalent of binary values

$m_0 \rightarrow \bar{A}\bar{B}$ } two bit inputs

$m_0 \rightarrow \bar{A}\bar{B}\bar{C}$ } three bit inputs

If $f=1$, we should consider that minterm which is defining the function f.

Example $Y = \bar{A}BC + A\bar{B}C + AB\bar{C}$

$= m_3 + m_5 + m_6$

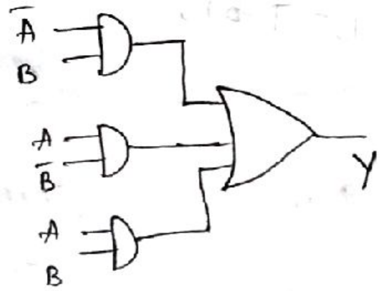
$Y = f(A.B.C) = \Sigma_m (3.5.6)$

Example

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$Y = \bar{A}B + A\bar{B} + AB$

$Y = f(A.B) = \Sigma_m (1,2,3)$



Karnaugh Map

2 variable

	0	1
0		
1		

3 variables

	0	1
00		
01		
11		
10		

	00	01	11	10
0				
1				

Steps :
- Representation
- grouping
- Adjacency
- overlapping conc

4 variables

	00	01	11	10
00				
01				
11				
10				

Grouping

PAIRS

QUADS

OCTETS

$\bar{A}$	$\bar{A}$ 0
A	A 1

(a) 1-Variable map

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}$ 0	$\bar{A}B$ 1
A	$A\bar{B}$ 2	AB 3

(b) 2-Variable map

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	$\bar{A}\bar{B}\bar{C}$ 0	$\bar{A}\bar{B}C$ 1	$\bar{A}BC$ 3	$\bar{A}B\bar{C}$ 2
A	$A\bar{B}\bar{C}$ 4	$A\bar{B}C$ 5	ABC 7	$AB\bar{C}$ 6

(c) 3-Variable map

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}CD$ 3	$\bar{A}\bar{B}C\bar{D}$ 2
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BCD$ 7	$\bar{A}BC\bar{D}$ 6
AB	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABCD$ 15	$ABC\bar{D}$ 14
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}CD$ 11	$A\bar{B}C\bar{D}$ 10

(d) 4-Variable map

① $f(a,b) = \Sigma m\,(0,2)$

A \ B	0	1
0	1_0	0_1
1	1_2	0_3

$Y = \bar{B}$

$f(a.b) = \Sigma m\,(0)$

	0	1
0	1	0
1	0	0

$Y = \bar{A}\bar{B}$

② $f(A.B.C) = \Sigma m\,(2,3,4,5$

	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$Y = A\bar{B} + \bar{A}B$

③ $f(A.B.C.D) = \Sigma m(0,2,3,4,9,11)$

AB \ CD	00	01	11	10
00	1	0	1	1
01	1	0	0	0
11	0	0	0	0
10	0	1	1	0

$Y = \bar{A}\bar{C}\bar{D} + A\bar{B}D + \bar{A}\bar{B}C$

④ $f(A,B,C) = \Sigma m(0,1,4,5)$

	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$Y = \bar{B}$

⑤ $f(ABC) = \Sigma m(0,1,2,3,4,5,6,7)$

	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$Y = 1$

⑥ $Y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

	00	01	11	10
0	1	1	1	0
1	1	1	0	0

$Y = \bar{A}C + \bar{B}$

⑦ $f(ABCD) = \Sigma m(2,4,5,9,12,13)$

	00	01	11	10
00	0	0	0	1
01	1	1	0	0
11	1	1	0	0
10	0	1	0	0

$Y = \bar{A}\bar{B}C\bar{D} + A\bar{C}D + B\bar{C}$

(8) $f(A,B,C,D) = \Sigma m(0, 2, 3, 8, 10, 11, 12, 14)$

	00	01	11	10
00	1	0	1	1
01	0	0	0	0
11	1	0	0	1
10	1	0	1	1

Prime implicants

$\bar{B}\bar{D}$

$\bar{B}C$

$A\bar{D}$

(9) $f(A,B,C,D) = \Sigma m(1, 5, 6, 7, 11, 12, 13, 15)$

	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1	1	0
10	0	0	1	0

Prime implicants	Essential PI
$AB\bar{C}$	$AB\bar{C}$
ACD	ACD
$\bar{A}BC$	$\bar{A}BC$
$\bar{A}\bar{C}D$	$\bar{A}\bar{C}D$
BD	—

(10) $f(A,B,C,D) = \Sigma m(1, 2, 3, 5, 6, 7, 11, 12, 13, 14, 15)$

	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	1	1	1	1
10	0	0	1	0

Prime Implicants	EPI
AB	AB
CD	CD
$\bar{A}D$	$\bar{A}D$
$\bar{A}C$	$\bar{A}C$
BD	—
BC	—

(11) $f(A.B.C.D) = \Sigma m(7, 9, 10, 11, 12, 13, 14, 15)$

	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	1	1	1	1
10	0	1	1	1

PI	EPI
AB	AB
AC	BCD
AD	AD
BCD	AC

(12) $f(A.B.C.D) = \Sigma m(0,1,2,3,4,6,7,8,9,10,11,12,13,14,15)$

$Y = \bar{A}BC + AD + B\bar{D} + C\bar{D} + A\bar{C} + \bar{A}\bar{B}$

	00	01	11	10
00	1	1	1	1
01	1	0	1	1
11	1	1	1	1
10	1	1	1	1

PI

A
C
$\bar{B}$
$\bar{D}$

(13) $f(ABCD) = \Sigma m(5, 7, 8, 12)$

	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	0	0	0
10	1	0	0	0

PI

$\bar{A}BC$
$A\bar{B}\bar{D}$

(14) $f(abcd) = \Sigma m(0, 3, 4, 5, 7, 9, 13, 14, 15)$

	00	01	11	10
00	1	0	1	0
01	1	1	1	0
11	0	1	1	1
10	0	1	0	0

PI → BD, $\bar{A}\bar{C}\bar{D}$, $\bar{A}B\bar{C}$, $\bar{A}CD$, ABC, $A\bar{C}D$

EPI → $\bar{A}\bar{C}D$, $\bar{A}B\bar{C}$, $\bar{A}CD$, ABC, $A\bar{C}D$

Don't Care Condition

① $f(a.b.c.d) = \Sigma(0,2,5,7.8.10.13.15) + \Sigma_d(1,4,11,14)$

	00	01	11	10
00	1	X	0	1
01	X	1	1	0
11	0	1	1	X
10	1	0	X	1

PI	EPI
$\bar{a}\bar{c}$	—
bd	bd
ac	—
$\bar{b}\bar{d}$	$\bar{b}\bar{d}$

② $f(abcd) = \Sigma_m(1,3.5,7.8.10,12.13.14) + \Sigma_d(4.6.15)$

	00	01	11	10
00	0	1	1	0
01	X	1	1	X
10	1	1	X	1
10	1	0	0	1

PI
$\bar{A}D$
B
$A\bar{D}$

③ $f(abcd) = \Sigma_m(6,7.9.10.13) + \Sigma_d(1,4.5,11.15)$

	00	01	11	10
00	0	X	0	0
01	X	X	1	1
11	0	1	X	0
10	0	1	X	1

P.I	EPI	or
$\bar{A}B$	$\bar{A}B$	$\bar{A}B$
$\bar{C}D$	—	$\bar{C}D$
AD	AD	—
$A\bar{B}C$	$A\bar{B}C$	$A\bar{B}C$

④ $f(abcd) = \Sigma_m(0,1,3.7.8.12) + d_c(5,10.13.14)$

	00	01	11	10
00	1	1	1	0
01	0	X	1	0
11	1	X	0	X
10	1	0	0	X

PI
$\bar{A}\bar{B}\bar{C}$
$\bar{A}D$
$A\bar{D}$

④ $f(a.b.c.d) = \bar{a}\bar{c}d + \bar{a}cd + \bar{b}\bar{c}d + a\bar{b}c + \bar{a}\bar{b}c\bar{d}$

	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	0	0
10	1	0	1	1

PI	EPI
$\bar{a}\bar{b}$	—
$\bar{a}d$	$\bar{a}d$
$\bar{b}c$	$\bar{b}c$
$\bar{b}\bar{d}$	$\bar{b}\bar{d}$

⑤ $f(a.b.c.d) = \Sigma(0.1.2,5.8.15) + \Sigma d(6,7.10)$

	00	01	11	10
00	1	1	0	1
01	0	1	X	X
11	0	0	1	0
10	1	0	0	X

PI	EPI
$\bar{A}\bar{C}D$	$\bar{A}\bar{C}D$
$\bar{B}\bar{D}$	$\bar{B}\bar{D}$
BCD	BCD
$\bar{A}C\bar{D}$	—

⑥ $f(a.b.c.d) = \Sigma(7, 9.11.12.13.14) + \Sigma d(3,5.6.15)$

	00	01	11	10
00	0	0	X	0
01	0	X	1	X
11	1	1	X	1
10	0	1	1	0

PI	EPI
BC	BC
AD	AD
$AB\bar{C}$	$AB\bar{C}$
BD	—

⑦ $f(aBCd) = \Sigma(7, 9.12.13.14.15) + \Sigma d(4,11)$

	00	01	11	10
00	0	0	0	0
01	+	0	1	0
11	1	1	1	1
10	0	1	X	0

PI	EPI
AB	AB
AD	AD
BCD	BCD
$B\bar{C}\bar{D}$	—

Product of Sums Methods

A	B	fundamental products
0	0	$A+B$
0	1	$A+\bar{B}$
1	0	$\bar{A}+B$
1	1	$\bar{A}+\bar{B}$

M_i → maxterms, i is the decimal equivalent of binary values

$M_0 \rightarrow A+B$ } Two bit i/p's
$M_0 \rightarrow A+B+C$ } three bit i/p's

$Y = (A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

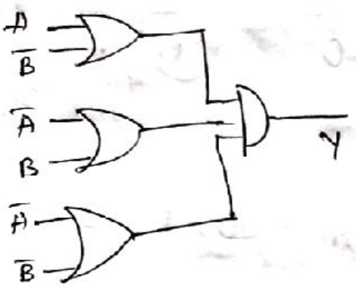
$Y = f(ABC) = \Pi_M(3,5,6)$

Example

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$Y = (A+\bar{B})(\bar{A}+B)(\bar{A}+\bar{B})$

$Y = f(A,B) = \Pi_M(0)$



① $f(abc) = \Pi_M(3,4,5,7)$

	00	01	11	10
0	1	1	0	1
1	0	0	0	1

PI	EPI
$\bar{A}+B$	$\bar{A}+B$
$\bar{B}+\bar{C}$	$\bar{B}+\bar{C}$
$\bar{A}+\bar{C}$	—

② $f(abc) = \Pi_M(0,1,4,6)$

	00	01	11	10
0	0	0	1	1
1	0	1	1	$\boxed{0}$

PI	EPI
$A+B$	$A+B$
$B+C$	—
$\bar{A}+C$	$\bar{A}+C$

$f(abcd) = \Pi_M(0, 2, 3, 8, 9, 10, 12, 14)$

	00	01	11	10
00	0	1	0	0
01	1	1	1	1
11	0	1	1	0
10	0	0	1	0

PI

$\bar{A}+D$

$\bar{A}+B+C$

$A+B+\bar{C}$

$B+D$

$f(abcd) = \Pi_M(1, 3, 6, 9, 11, 14, 15)$

	00	01	11	10
00	1	0	0	1
01	1	1	1	0
11	1	1	0	0
10	1	0	0	1

PI	EPI
$B+\bar{D}$	$B+\bar{D}$
$\bar{A}+\bar{C}+\bar{D}$	$\bar{A}+\bar{C}+\bar{D}$
$\bar{B}+\bar{C}+D$	$\bar{B}+\bar{C}+D$
$\bar{A}+\bar{B}+\bar{C}$	—

$f(a, b, c, d) = \Pi_M(0, 1, 3, 8, 9, 10, 14, 15)$

	00	01	11	10
00	0	0	0	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	0

PI

$B+C$

$\bar{A}+\bar{B}+\bar{C}$

$\bar{A}+\bar{C}+D$

$A+B+\bar{D}$

$f(abcd) = \Pi(1,4,5,6,14)$

	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	1	1	1	D
10	1	1	1	1

PI

$A+\bar{B}+C$

$A+C+\bar{D}$

$\bar{B}+\bar{C}+D$

Dont Care Condition

$f(a.b.c.d) = \Pi M(0,3,4,5,6,7,11,15) + dc(2,14)$

	00	01	11	10
00	0	1	0	–
01	0	0	0	0
11	1	1	0	–
10	1	1	0	1

PI

$A+D$

$A+\bar{B}$

$\bar{C}+\bar{D}$

$f(a.b.c.d) = \Pi(0,1,4,5,8,9,11) + \Pi d(2,10)$

	00	01	11	10
00	0	0	1	–
01	0	0	1	1
11	1	1	1	1
10	0	0	0	–

PI	EPI
$A+C$	$A+C$
$\bar{B}+B$	$\bar{A}+B$
$b+c$	—
$b+d$	—

$f(abcd) = \Pi(2,8,11,15) + \Pi d(3,12,14)$

	00	01	11	10
00	1	1	–	0
01	1	1	1	1
11	–	1	0	–
10	0	1	0	1

PI

$a+b+\bar{c}$

$\bar{a}+c+d$

$\bar{a}+\bar{c}+\bar{d}$

$f(ABCD) = \Pi_M(1, 2, 3, 4, 9, 10) + \Pi_d(0, 14, 15)$

	00	01	11	10
00	–	0	0	0
01	0	1	1	1
11	1	1	–	–
10	1	0	1	0

PI

$A+B$

$B+C+\bar{D}$

$A+C+D$

$\bar{A}+\bar{C}+D$

Quine - Mccluskey method (Algorithm) for obtaining prime implicants

① Represent each minterm in its 1/0 notation

② Write down the minterm in increasing order of their index in one column.

③ Draw a line after each set of minterms with same index value.

④ Set $i = 0$

⑤ Pick up each term with index i & $i+1$ to see if they differ in exactly one position. If yes, write the signal term which results from the combination in 1, 0, – notation in a new column & place a ✓ besides the two terms that combined.

If no, proceed with other pairs until all the pairs with indices i & $i+1$ have been compared

Now draw a line under the last term in the new list.

⑥ Set $i = i+1$, repeat step 5 until all the terms have been covered.

7. Repeat step 4, 5 and 6 on the new list to form another list.

8. Terminate the process when no new lists are formed.

9. The prime implicants of the function are all those terms without a '✓' besides them.

① $f(a,b,c) = \Sigma m(0, 2, 3, 4)$

	a	b	c				
0	0	0	0 ✓	(0,2)	0-0	$\bar{a}\bar{c}$
2	0	1	0 ✓	(0,4)	-00	$\bar{b}\bar{c}$
4	1	0	0 ✓			
3	0	1	1 ✓	(2,3)	01-	$\bar{a}b$

Prime implicants: $\bar{a}\bar{c}$, $\bar{b}\bar{c}$, $\bar{a}b$

Essential prime implicants : $\bar{b}\bar{c}$, $\bar{a}b$

	0	2	3	4
$\bar{a}\bar{c}$	✓	✓		
✓ $\bar{b}\bar{c}$	✓			✓
✓ $\bar{a}b$		✓	✓	

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② $f(a,b,c) = \Sigma m(0, 1, 2, 3)$

	a	b	c					
0	0	0	0 ✓	(0,1)	00- ✓	(0,1)(2,3)	0--	$\bar{a}$
1	0	0	1 ✓	(0,2)	0-0 ✓	(0,2)(1,3)	0--	
2	0	1	0 ✓	(1,3)	0-1 ✓			
3	0	1	1 ✓	(2,3)	01- ✓			

Prime implicants : $\bar{a}$

③ $f(a,b,c) = \Sigma m(0,1,2,3,4,5,6)$

	a b c
0	0 0 0
1	0 0 1
2	0 1 0
4	1 0 0
3	0 1 1
5	1 0 1
6	1 1 0

(0,1)	00– ✓
(0,2)	0–0 ✓
(0,4)	–00 ✓
(1,3)	0–1 ✓
(1,5)	–01 ✓
(2,3)	01– ✓
(2,6)	–10 ✓
(4,5)	10– ✓
(4,6)	1–0 ✓

(0,1)(4,5)	–0–	$\}\ \bar{b}$
(0,1)(2,3)	0––	$\}\ \bar{a}$
(0,2)(1,3)	0––	
(0,2)(4,6)	––0	
(0,4)(1,5)	–0–	
(0,4)(2,6)	––0	$\bar{c}$

Prime implicants $\bar{b}, \bar{a}, \bar{c}$

Essential prime implicants are $\bar{a}, \bar{b}, \bar{c}$

	0	1	2	3	4	5	6
$\bar{a}$	✓	✓	✓	✓			
$\bar{b}$	✓	✓			✓	✓	
$\bar{c}$	✓		✓		✓		✓

④ $f(a,b,c,d) = \Sigma m(0,2,3,4,8,10,12,13,14)$

	a b c d
0	0 0 0 0 ✓
2	0 0 1 0 ✓
4	0 1 0 0 ✓
8	1 0 0 0 ✓
3	0 0 1 1 ✓
10	1 0 1 0 ✓
12	1 1 0 0 ✓
13	1 1 0 1 ✓
14	1 1 1 0 ✓

(0,2)	00–0 ✓	
(0,4)	0–00 ✓	
(0,8)	–000 ✓	
(2,3)	001–	$\bar{a}\bar{b}c$
(2,10)	–010 ✓	
(4,12)	–100 ✓	
(8,10)	10–0 ✓	
(8,12)	1–00 ✓	
(10,14)	1–10 ✓	
(12,13)	110–	$ab\bar{c}$
(12,14)	11–0 ✓	

(0,2)(8,10)	–0–0	$\bar{b}\bar{d}$
(0,4)(8,12)	––00	$\bar{c}\bar{d}$
(0,8)(2,10)	–0–0	
(0,8)(4,12)	––00	
(8,10)(12,14)	1––0	$a\bar{d}$
(8,12)(10,14)	1––0	

Prime implicants $\bar{a}\bar{b}c$, $ab\bar{c}$, $\bar{b}\bar{d}$, $\bar{c}\bar{d}$, $a\bar{d}$

Essential prime implicants $\bar{c}\bar{d}$, $a\bar{d}$, $\bar{a}\bar{b}c$, $ab\bar{c}$

	0	2	3	4	8	10	12	13	14
* $\bar{b}\bar{d}$	X	X			X	X			
* $\bar{c}\bar{d}$	X			X	X		X		
* $a\bar{d}$					X	X	X		X
* $\bar{a}\bar{b}c$		X	X						
* $ab\bar{c}$							X	X	
	✓	✓	✓	✓	✓		✓	✓	✓

Q) $f(a,b,c,d) = \Sigma m(0,1,2,3,10,11,12,13,14,15)$

	abcd
0	0000 ✓
1	0001 ✓
2	0010 ✓
3	0011 ✓
10	1010 ✓
12	1100 ✓
11	1011 ✓
13	1101 ✓
14	1110 ✓
15	1111 ✓

(0,1)	000– ✓
(0,2)	00–0 ✓
(1,3)	00–1 ✓
(2,3)	001– ✓
(2,10)	–010 ✓
(3,11)	–011 ✓
(10,11)	101– ✓
(10,14)	1–10 ✓
(12,13)	110– ✓
(12,14)	11–0 ✓
(11,15)	1–11 ✓
(13,15)	11–1 ✓
(14,15)	111– ✓

(0,1) (2,3)	00– –	$\bar{a}\bar{b}$
(0,2) (1,3)	00– –	
(2,3) (10,11)	–01–	$\bar{b}c$
(2,10) (3,11)	–01–	
(10,11) (14,15)	1–1–	ac
(10,14) (11,15)	1–1–	
(12,13) (14,15)	11– –	ab
(12,14) (13,15)	11– –	

Prime implicants $\bar{a}\bar{b}$, $\bar{b}c$, ac, ab

	0	1	2	3	10	11	12	13	14	15
* $\bar{a}\bar{b}$	✓	✓	✓	✓						
$\bar{b}c$			✓	✓	✓	✓				
* ac					✓	✓			✓	✓
ab							✓	✓	✓	✓

essential PI

$\bar{a}\bar{b}$

ac

ab

⑥ $f(abcd) = \Sigma_m(7, 9, 12, 13, 14, 15) + \Sigma_d(4, 11)$

	a b c d
4	0 1 0 0 ✓
9	1 0 0 1 ✓
12	1 1 0 0 ✓
7	0 1 1 1 ✓
11	1 0 1 1 ✓
13	1 1 0 1 ✓
14	1 1 1 0 ✓
15	1 1 1 1 ✓

(4,12)	– 1 0 0	$b\bar{c}\bar{d}$
(9,11)	1 0 – 1 ✓	
(9,13)	1 – 0 1 ✓	
(12,13)	1 1 0 – ✓	
(12,14)	1 1 – 0 ✓	
(7,15)	– 1 1 1	bcd
(11,15)	1 – 1 1 ✓	
(13,15)	1 1 – 1 ✓	
(14,15)	1 1 1 – ✓	

(9,11)(13,15)	1 – – 1	ad
(9,13)(11,15)	1 – – 1	
(12,13)(14,15)	1 1 – –	ab
(12,14)(13,15)	1 1 – –	

Prime implicants $b\bar{c}\bar{d}$, bcd, ad, ab

	7	9	12	13	14	15	4	11
bcd	×					×		
ad		×		×		×		×
ab			×	×	×	×		
$b\bar{c}\bar{d}$			×				×	

essential prime implicants are bcd, ad, ab, $b\bar{c}\bar{d}$

⑦ $f(abcd) = \Sigma_m(3, 4, 5, 7, 10, 12, 14, 15) + \Sigma_d(0)$

prime implicants $\bar{a}\bar{b}c$, $\bar{b}c\bar{d}$, $\bar{a}b\bar{c}$, $b\bar{c}\bar{d}$

$\bar{a}cd$, $\bar{a}bd$, $ac\bar{d}$, $ab\bar{d}$, bcd, abc

Essential PI : $b\bar{c}\bar{d}$, $\bar{a}cd$, $\bar{a}bd$, $ac\bar{d}$, abc

or

$\bar{a}b\bar{c}$, $b\bar{c}\bar{d}$, $\bar{a}cd$, $ac\bar{d}$, bcd

	a b c d	
2	0 0 1 0	✓
4	0 1 0 0	✓
3	0 0 1 1	✓
5	0 1 0 1	✓
10	1 0 1 0	✓
12	1 1 0 0	✓
7	0 1 1 1	✓
14	1 1 1 0	✓
15	1 1 1 1	✓

(2,3)	001–
(2,10)	–010
(4,5)	010–
(4,12)	–100
(3,7)	0–11
(5,7)	01–1
(10,14)	1–10
(12,14)	11–0
(7,15)	–111
(14,15)	111–

$\bar{a}\bar{b}c$

$\bar{b}c\bar{d}$

$\bar{a}b\bar{c}$

$b\bar{c}\bar{d}$

$\bar{a}cd$

$\bar{a}bd$

$ac\bar{d}$

$ab\bar{d}$

bcd

abc

	3	4	5	7	10	12	14	15
$\bar{a}\bar{b}c$	X							
$\bar{b}c\bar{d}$					X			
$\bar{a}b\bar{c}$		X	X					
$b\bar{c}\bar{d}$		X				X		
$\bar{a}cd$	X			X				
$\bar{a}bd$			X	X				
$ac\bar{d}$					X		X	
$ab\bar{d}$						X	X	
bcd				X				X
abc							X	X

(8) $f(abcd) = \prod_M (0, 6, 7, 8, 9, 13)$.

$m_i = \overline{M_i}$

$f = M_0 \cdot M_6 \cdot M_7 \cdot M_8 \cdot M_9 \cdot M_{13}$

$\bar{f} = \overline{M_0 \cdot M_6 \cdot M_7 \cdot M_8 \cdot M_9 \cdot M_{13}}$

$= \overline{M_0} + \overline{M_6} + \overline{M_7} + \overline{M_8} + \overline{M_9} + \overline{M_{13}}$

$= m_0 + m_6 + m_7 + m_8 + m_9 + m_{13}$

	a	b	c	d
0	0	0	0	0 ✓
8	1	0	0	0 ✓
6	0	1	1	0 ✓
9	1	0	0	1 ✓
7	0	1	1	1 ✓
13	1	1	0	1 ✓

(0,8)	-000	$\bar b \bar c \bar d$
(8.9)	$100-$	$a\bar b \bar c$
(6.7)	$011-$	$\bar a b c$
(9.13)	$1-01$	$a \bar c d$

Prime implicants are $b+c+d$, $\bar a+b+c$, $a+\bar b+\bar c$, $\bar a+c+\bar d$

	0	6	7	8	9	13
$\bar b \bar c \bar d$	X			X		
$a \bar b \bar c$				X	X	
$\bar a b c$		X	X			
$a \bar c d$					X	X

Essential prime implicants are $b+c+d$, $a+\bar b+\bar c$, $\bar a+c+\bar d$

(9) $f(abcd) = \Pi_M(0, 6.7.8.9.13) + \Pi_d(5,15)$

	a	b	c	d
0	0	0	0	0 ✓
8	1	0	0	0 ✓
5	0	1	0	1 ✓
6	0	1	1	0
9	1	0	0	1 ✓
7	0	1	1	1 ✓
13	1	1	0	1 ✓
15	1	1	1	1 ✓

(0.8)	-000	$\rightarrow \bar b \bar c \bar d$
(8.9)	$100-$	$\rightarrow a\bar b \bar c$
(5,7)	$01-1$ ✓	
(5.13)	-101 ✓	
(6,7)	$011-$	$\rightarrow \bar a bc$
(9,13)	$1-01$	$\rightarrow a\bar c d$
(7.15)	-111 ✓	
(13,15)	$11-1$ ✓	

(5,7) (13.15) $-1-1 \rightarrow bd$

(5,13) (7.15) $-1-1$

Prime implicants $\bar{b}+\bar{d}$, $b+c+d$, $\bar{a}+b+c$, $a+\bar{b}+\bar{c}$

$\bar{a}+c+\bar{d}$

	0	6	7	8	9	13
bd			X			X
$\bar{b}\bar{c}\bar{d}$	X					
$a\bar{b}\bar{c}$				X	X	
$\bar{a}bc$		X	X			
$a\bar{c}d$					X	X

Essential prime implicants $b+c+d$, $\bar{a}+b+c$, $a+\bar{b}+\bar{c}$

$\bar{a}+c+\bar{d}$

(10) $f(a.b.c.d) = \Pi M(0.2.3.4.5, 12.13) + \Pi d(8.10)$

PI - $a+\bar{b}+\bar{c}$, $b+d$, $c+d$, $\bar{b}+c$ EPI = $a+b+\bar{c}$, $b+d$, $\bar{b}+c$

Hazards & Hazard Covers

Simplification Techniques gives minimal expression for a logic equation. Simplified equations can be realised using minimum hardware. But to overcome some practical problems, in certain cases we may prefer to include more terms in the simplified equation. practical logic circuit do not generate o/ps instantaneously. There is a finite propogation delay This propogation delay gives rise to several Hazards

Hazard Covers are additional term in a logic equation that prevents hazards.

For combinational logic circuits hazards may go unnoticed but in sequential logic ckts it may cause major malfunctioning.

Types of Hazards

Static Hazards

A Static Hazards exists if an o/p Signal is Supposed to remain at a particular logic value when an i/p variable changes its values, but instead the signal undergoes a momentary change in its required value.

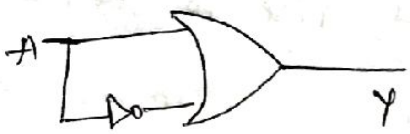
Static-1 Hazard

This Type of hazard occurs when $Y = A + A'$ Type of situation appears for a logic circuit for certain combination of other i/ps & A makes a transition $1 \rightarrow 0$.

An $A + A'$ condition should always generate logic 1 at the o/p, ie Static-1

A	A'	Y = A+A'
1	0	1+0 = 1
0	1	0+1 = 1

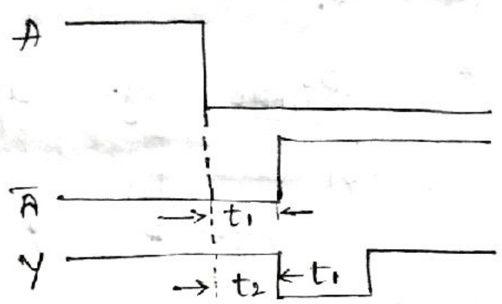
consider simple logic circuit



The NOT gate o/p takes finite time to become logic following $1 \rightarrow 0$ transition at the i/p A. The OR gate o/p goes to logic 0 for a small duration which is unwanted

A	A'	$Y = A + A'$
1	0	$1+0=1$
0	0	$0+0=0$
0	1	$0+1=1$

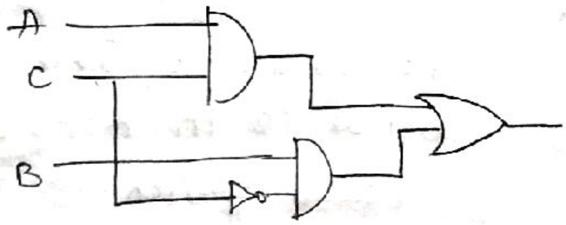
The width of this logic 0 o/p is in nanoseconds and called a glitch.



t_1 = NOT gate delay
t_2 = OR gate delay

Static -1 Hazard cover

Consider $Y = AC + BC'$, Corresponding Logic Ckt



Consider i/p $A=1$, $B=1$ & C makes a transition $1 \to 0$.

The o/p has a glitch & hence the circuit has Static 1 hazard.

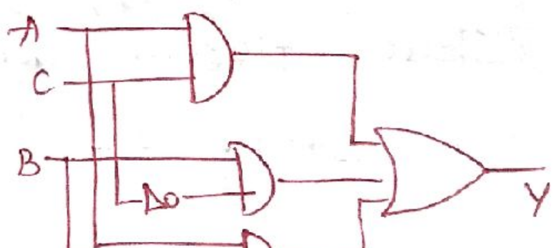
K-map $C + C'$

A \ BC	00	01	11	10
0	0	0	0	1
1	0	1	1	1

→ $B'C$

→ Additional Term AB

AC



$Y = B'C + AC + AB$

The additional term AB ensures $Y=1$ for $A=1$ & $B=1$ & a $1 \to 0$ transition at C does not affect the o/p.

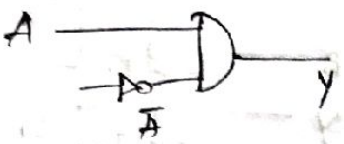
The circuit free from Static-1 hazard is shown above.

Static 0 Hazard

This type of hazard occurs when $Y = A \cdot A'$ type of situation appears for a logic circuit for certain combination of other inputs & A makes a transition $0 \to 1$. An $A \cdot A'$ condition should always generate logic 0 at o/p, i.e Static 0.

A	A'	$Y = A \cdot A'$
0	1	$0 \cdot 1 = 0$
1	0	$1 \cdot 0 = 0$

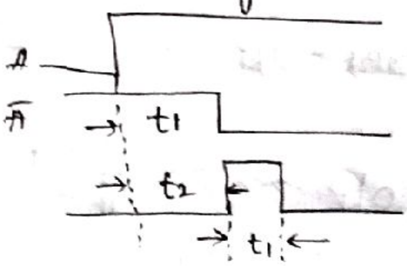
Simple Logic Circuit



The NOT gate o/p takes finite time to become logic 0 following $0 \to 1$ transition at the input A. The AND gate output goes to logic 1 for a small duration which is unwanted.

A	A'	$Y = A \cdot A'$
0	1	$0 \cdot 1 = 0$
1	1	$1 \cdot 1 = 1$
1	0	$1 \cdot 0 = 0$

The width of this logic 1 output is in nano sec called glitch.

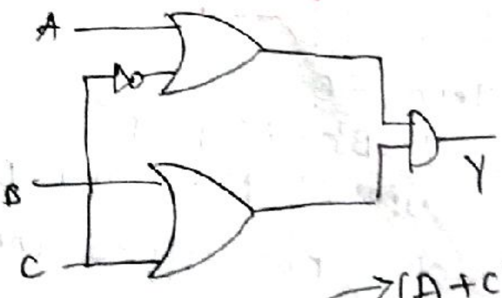


$t_1 \to$ NOT gate delay

$t_2 \to$ OR gate delay

Static 0 Hazard cover

$$Y = (B+C) \cdot (A+C')$$



$\to (A+C')$

	00	01	11	10
0	0	0	0	1
1	0	1	1	1

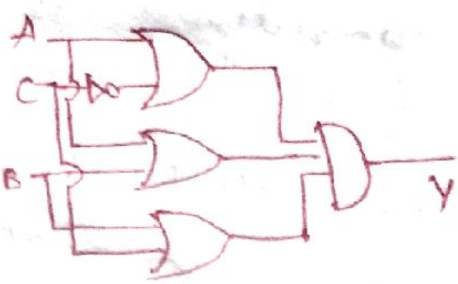
(A+B)

(B+C)

$$Y = (A+C')(A+B)(B+C)$$

Additional term (A+B) ensures Y=0 for A=0, B=0 and $0 \to 1$ transition at C does not affect the o/p

The hazard free ckt is



Dynamic Hazard

Dynamic hazard causes glitches on 0→1 or 1→0 transitions of an o/p signal. When only one transition is required, the output makes multiple transitions. It is caused by the structure of the logic circuit where there exists multiple paths for a given signal change to propogate along.

$\left.\begin{matrix} 1\to 0 \\ 0\to 1 \end{matrix}\right\} \to 1 : \begin{matrix} 1\to 0\to 1\to 0 & Y = A+\bar{A}\cdot A \\ 0\to 1\to 0\to 1 & Y=(A+\bar{A})\cdot A \end{matrix}$

Dynamic hazards are encounterd in multiple level circuits. They are not easy to detect. They can be avoided simply by using two level circuits and ensuring that there are no static hazards.

HDL implementation of Combinational

- ✓ Data flow modeling
- ✓ Behavioral modeling

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Table 3.12 ***A Partial List of Verilog Operator***

Relational Operation	*Symbol*	*Bit-wise Operation*	*Symbol*
Less than	<	Bit-wise NOT	~
Less than or equal to	<=	Bit-wise AND	&
Greater than	>	Bit-wise OR	|
Equal to	==	Bit-wise Ex-OR	^
Not equal to	!=		
Logical Operation (for expressions)	**Symbol**	**Arithmetic Operation**	**Symbol**
		Binary addition	+
Logical NOT	!	Binary subtraction	−
Logical AND	&&	Binary multiplication	*
Logical OR	||	Binary division	/