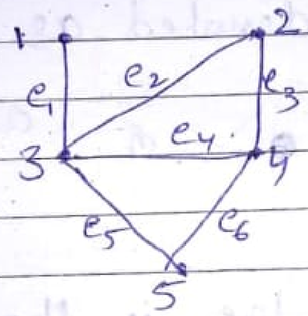
Module-5

What is a Graph?

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as Vertices, and the links that connect the vertices are called edges.

ie $G = (V, E)$ where V is the set of vertices and E is the set of edges.

(Eg)



$V = \{1, 2, 3, 4, 5\}$
$e = \{e_1\ e_2\ e_3\ e_4\ e_5\ e_6\}$

Applications of Graph Theory

Electrical Engineering: in designing circuit connections.

The type of connections are named as topologies. like, star, bridge, series, & parallel

Computer Science:- For the study of Algorithms like Kruskal's Algo, Prim's Algo, Dijkstra's Algo.

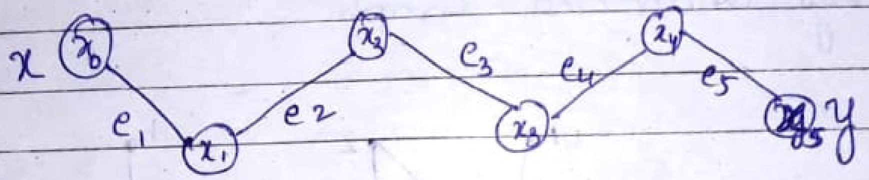
- Relation among interconnected computers in the n/w follows the principles of Graph Theory

Outdegree of vertex V is the number of edges which are going out from the vertex V.

Walk

A walk of length k in a graph G is a succe sequence of k edges of G, of the form.

$x = x_0 e_1 x_2 e, \cdots x_{n-1} e_n x_n = y$.



where $1 \le i \le n$.

or

A walk in G is a finite alternating sequence of vertices and edges from G.

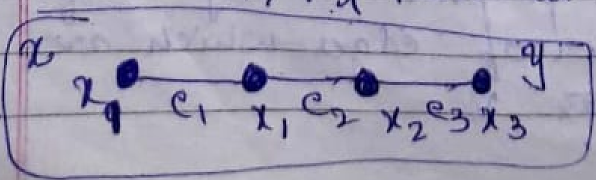
The length of this walk is n, the number of edges in the walk.

length = 5.

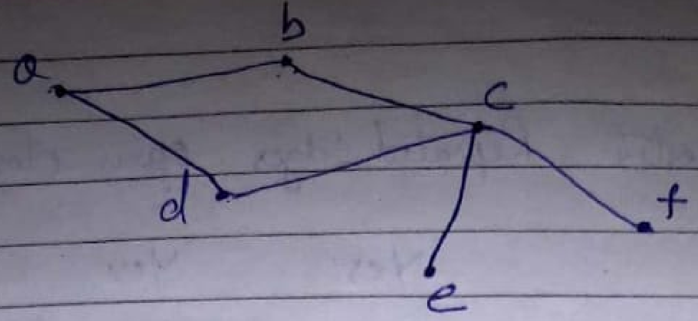
When the number of edges in the walk is 0 ie $n = 0$, then it is called trivial. ie. $x = y$.

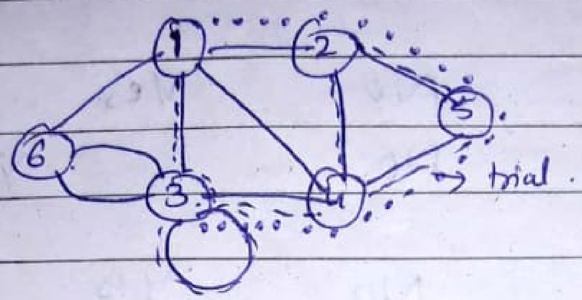
When $n > 1$, & edges & vertices are repeated then it is called closed walk

path



closed walk. (edges & vertices are repeated)



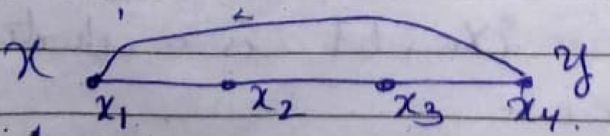


If all the edges of a walk are different, then the walk is called a trial.
If in addition all the vertices are different then the trial is called path.

Walk 1 3 3 4 2 5 4 is a trial
walk 1 2 5 4 3 is a path. (since no repeated vertices)

In an undirected gl.
If no edges in the walk is repeated, then the walk is called an trial. A closed trial is called Circuit.

If no vertex of the walk occurs more than once, then the walk is called an path
when $x = y$, the term cycle is used to describe such a closed path.



closed path

Table.

Repeated Vertex	Repeated Edges	open	closed	Name
Yes	Yes	Yes		Walk (open)
Yes	Yes		Yes	Walk (closed)
Yes	No	Yes		Trial
Yes	No		Yes	Circuit
No	No	Yes		path
No	No		Yes	Cycle

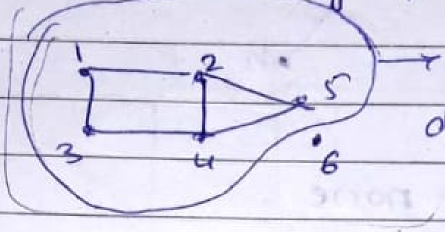
Theorem Let $G = \langle V, E \rangle$ be an undirected graph, with $a, b \in V$, $a \neq b$. If there exists a trial (in G) from a to b, then there is a path (in G) from a to b.

Proof: Since there is a trial from a to b, we select one of shortest length, say $\{a, x_1\}$ $\{x_1, x_2\} \dots \{x_n, b\}$. If this trial is not a path, we have the situation $\{a, x_1\}, \{x_1, x_2\}$ $\dots \{x_{k-1}, x_k\}, \{x_k, x_{k+1}\} \dots \{x_{m-1}, x_m\}$ $\dots \{x_m, x_{m+1}\} \dots \{x_n, b\}$, where $k < m$ and $x_k = x_m$, possibly with $k = 0$ and $a (= x_0) = x_m$ or $m = n+1$ and $x_k = b (= x_{n+1})$. But then we have a contradiction because $\{a, x_1\} \{x_1, x_2\} \dots \{x_{k-1}, x_k\}$ $\{x_m, x_{m+1}\} \dots \{x_n, b\}$ is a shorter trial from a to b.

Defn. Let $G = (V; E)$ be an undirected graph. We call G <u>connected</u> if there is a path between any two distinct vertices of G.

or

A graph is said to be connected if there is a path between every pair of vertex.

Eg 

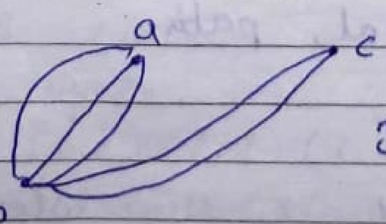
A graph that is not connected is called <u>disconnected</u>.

For any $G = (V, E)$, the number of components of G is denoted by $K(G)$

Eg: $K(G) = 1$. because the graph is connected.
$K(G) = 2$ because the graph is not connected (it has two components).

<u>Multigraph:</u>

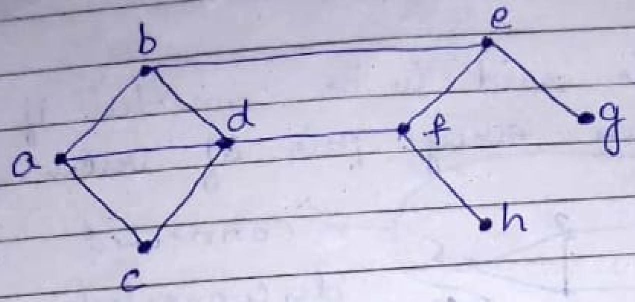
Let V be a finite nonempty set, ~~we say~~ the pair (V, E) determines a multigraph G with vertex set V and edge set E if there are two or more edges in E of the form ~~shown t~~ shown below.



The edge (b, c) has multiplicity 2.

The edge (a, b) has multiplicity 3.

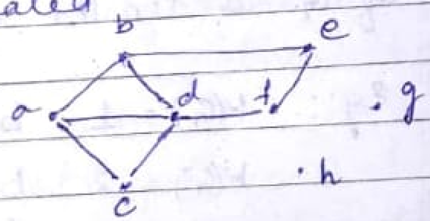
Problem: Determine which of the following sequences in the graph are walk, closed walk, closed trail, path and cycle.



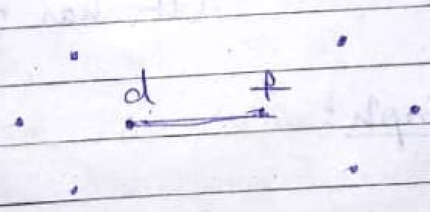
a) b, c, f, g — none.

b) a, b, e, f, d, a, c, d, b.

Vertices are repeated
edges are not repeated
it is open
∴ it is a walk, trial



c) d, f, d.



Vertices are repeated :
edges are repeated :
closed
} walk, closed walk.

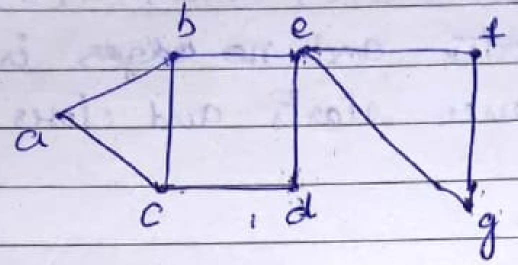
d) h. walk, trial, path

e) a, b, e, f, d, c, a
edges are not repeted
Vertices are repeated
closed walk.
Cycle

f) a, c, d, f, e, b, d, a   Walk
Closed walk.
trial
closed trial.

g) a, b, d, f, e, b, d, c   walk.

<u>Problem</u>



a) Determine a walk from b to d that is not a trail

<u>Ans:</u> (b, e), (e f), (f, g), (g, e), (e, b) (b, c) (c, d) is a walk but not a trail because the edge (b, e) is repeated.

b) a b-d trail that is not a path

<u>Ans:</u> (b, e) (e, f) (f, g) (g, e) (e d) is a trail because ~~edge~~ vertex e is repeated.

c) A path from b to d.

<u>Ans:</u> (b, c) (c, d) or (b, e) (e, d)
Since no vertex & no edges is repeated.

d) A closed walk from b to b that is not a cycle.

<u>Ans:</u> (b, e) (e, f) (f, g) (g, e) (e, b) is a closed walk (starting & ending at b) but is not a Circuit because the edges {b, e} is repeated.
Cycle

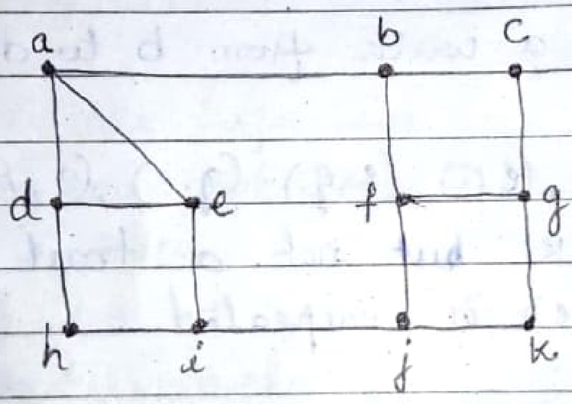
e) A Circuit from b to b. that is not a Cycle

Ans (b, e), (e, f), (f, g), (g, e), (e, d), (d, c), (c, b) is a circuit but not a cycle because the vertex e is repeated.

f) A cycle ~~from~~ from b to b.

Ans (b, e) (e, d) (d, c) (c, a) (a, b) is a Cycle where no. Vertex and no edges is repeated (and sequence starts and closes at b).

Problem:



The length of the shortest path from a to b is the distance between two distinct vertices a, b in a connected graph. Find the distance from d to each of the other vertices.

e:

a : 1	c : 3
b : 1 ?	f : 3
e : 1	g : 4
h : 1	j : 3
i : 2	k : 4

Subgraphs, Complements and Graph Isomorphism.

If $G = \langle V, E \rangle$ is a graph (directed or undirected), then $G_1 = \langle V_1, E_1 \rangle$ is called a ~~and~~ subgraph of G.

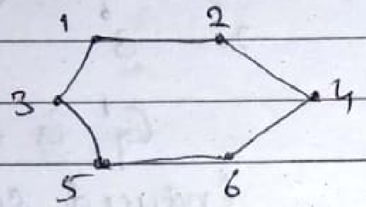
if $\phi \neq V_1$

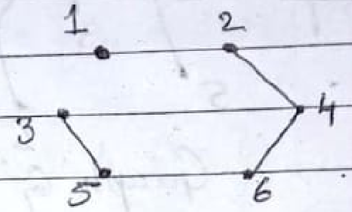
$V_1 \subseteq V$

$E_1 \subseteq E$ where V_1 is the Vertices

E_1 is the incident edges

Eg

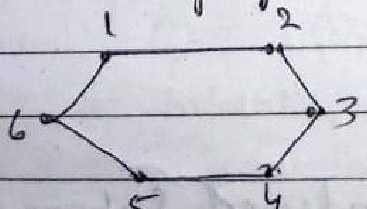


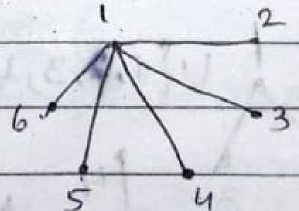


Defn Spanning Subgraph of G.

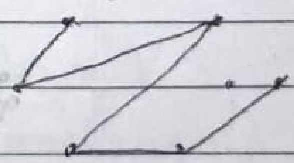
If $G = \langle V, E \rangle$ is a graph (directed | undirected), let $G_1 = \langle V_1, E_1 \rangle$ be a subgraph of G.

If $V_1 = V$ then G_1 is called a Spanning ~~tree~~ Subgraph of G.

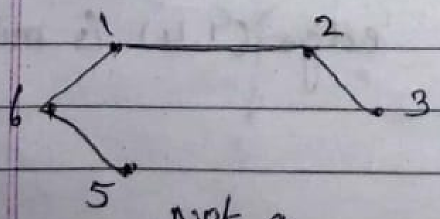




Spanning graph



Spanning graph.



Not a Spanning graph.

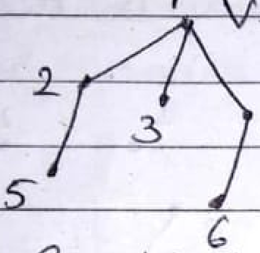
Defn **Induced Subgraph "U"**

Let $G = \langle V, E \rangle$ be a graph (directed or undirected)

If $\phi \neq U \subseteq V$, the subgraph of G induced by U is the subgraph whose vertex set is U and which contains all edges from G.

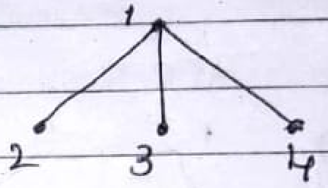
Eg

$V = \{1, 2, 3, 4, 5, 6\}$



Graph G

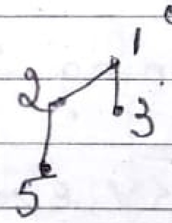
$U = \{1, 2, 3, 4\}$



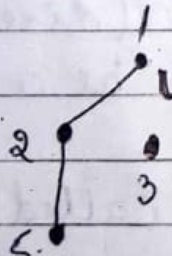
G' is called an induced subgraph of G.

or

$U = \{1, 2, 3, 5\}$

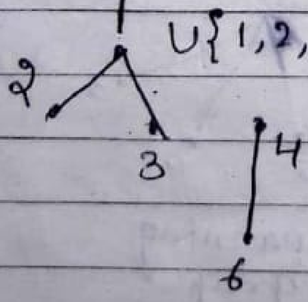


$U\{1, 2, 3, 5\}$



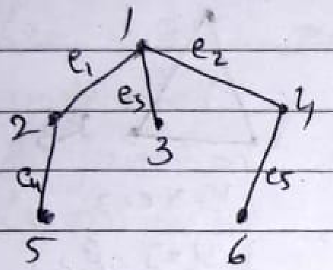
NOT induced subgraph of G because edge (1, 3) is missing.

$U\{1, 2, 3, 4, 6\}$



NOT induced subgraph of G because edge (1, 4) is missing.

Defn Let $G = \langle V, E \rangle$ (directed or undirected). The subgraph of G denoted by $G - v$ has the vertex set $V_1 = V - \{v\}$ and the edge set $E_1 \subseteq E$, where E_1 contains all the edges in E except for those that are incident with the vertex v.



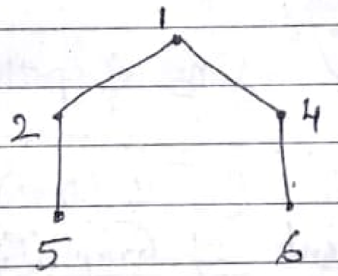
Graph G.

Eg. Let $V = \{1, 2, 3, 4, 5, 6\}$

$v = \{3\}$

$\therefore V_1 = \{V - \{v\}\}$

$\therefore V_1 = \{1, 2, 4, 5, 6\}$.

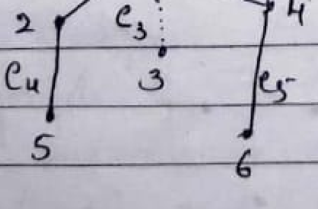


$E = \{e_1, e_2, e_3, e_4, e_5\}$

$E_1 = \{e_1\ e_2\ e_4\ e_5\}$

$\therefore E_1 \subseteq E$.

IIIly if $G = \langle V, E \rangle$ (directed or undirected) then $G - e = \langle V_1, E_1 \rangle$ of G where $E_1 = E - \{e\}$ and $V_1 = V$ (unchanged).



$E = \{e_1\ e_2\ e_3\ e_4\ e_5\}$

$e = \{e_3\}$

$\therefore E_1 = E - \{e\}$

$= \{e_1\ e_2\ e_4\ e_5\}$

$V_1 = V = \{1, 2, 3, 4, 5, 6\}$

** A complete graph with 'n' vertices, namely K_n, has $\frac{1}{2}n(n-1)$ edges. or $\frac{1}{2}2^n \cdot n = n2^{n-1}$ edges

Defn. Complete Graph (K_n)

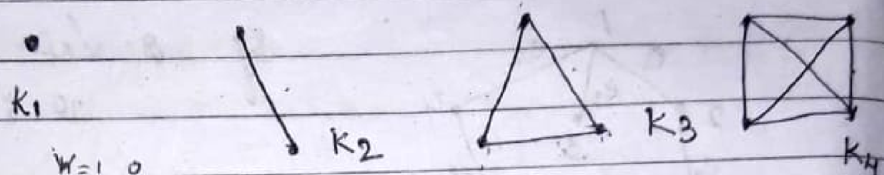
$n=3$

$\frac{4 \cdot \cancel{8} \cdot 3}{\cancel{2}}$ $\quad 3 \cdot 2^2 \;\; \cancel{?}\, 4$

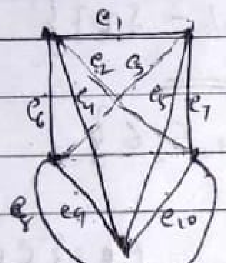
$12 = 12$

Let $G = \langle V, E \rangle$, where V is a set of n vertices. The complete graph is a loop free undirected graph, where for all $a, b \in V$, $a \neq b$ there is an edge $\{a, b\}$.

Eg



$V=1$ $e=0$ $\quad$ $V=2$ $e=1$ $\quad$ $V=3$ $e=3$ $\quad$ $V=4$ $e=6$



$\frac{1}{2} 3(2) = \frac{\cancel{6}}{\cancel{2}} 3 =$

$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$

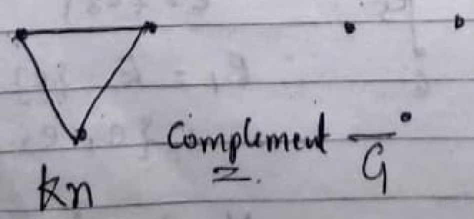
[no of paths of length m in K_n is $\frac{1}{2} n(n-1)(n-2) \ldots (n-m)$.

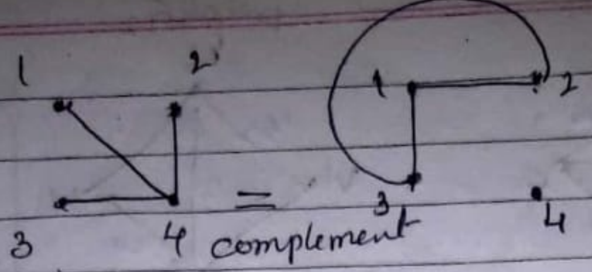
$\frac{1}{2}(2-1) = \frac{1}{2} = \quad \frac{1}{2}(3-\cancel{1}) = \frac{2}{2} = 1$]

Defn. Complement of Graph(G) : $\bar{G}$

The complement of G, denoted $\bar{G}$ is the subgraph of K_n consisting of the 'n' vertices in G and all edges that are not in G.

(if $G = K_n$, $\bar{G}$ is a graph consisting of n vertices and no edges. Such a graph is called a null graph).





Defn **Isomorphic graphs**

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two undirected graphs.
A function $f : V_1 \to V_2$ is called graph isomorphism if (a) f is one-to-one and onto.

(b) for all $a, b \in V_1$
$\{a, b\} \in E_1$ if & only if
$\{f(a), f(b)\} \in E_2$

When such a function exists, G_1 and G_2 are called isomorphic graphs.

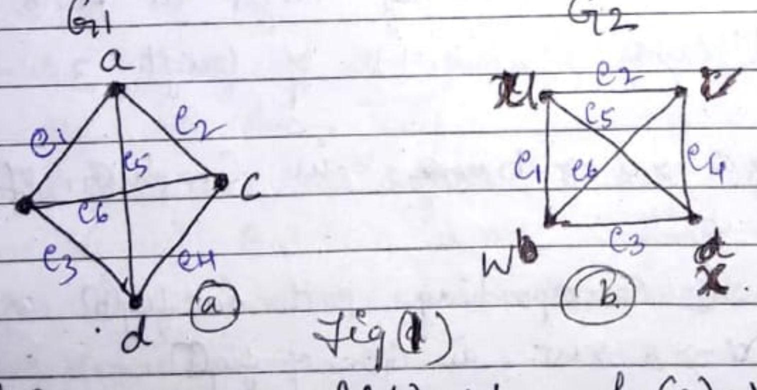


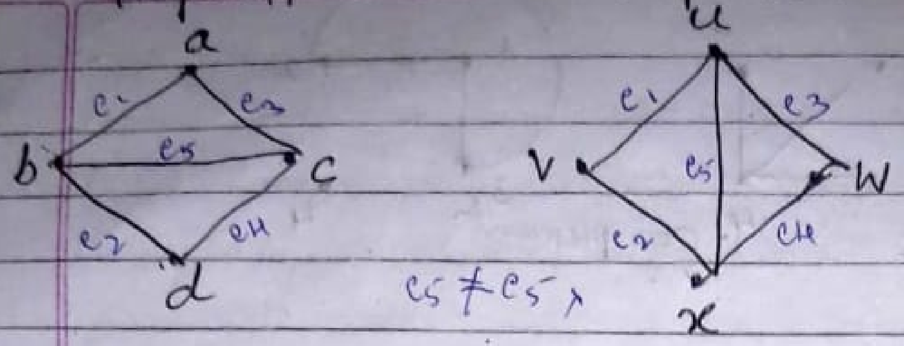
Fig (1)

$f(a) = u$ $f(b) = W$ $f(c) = V$ $f(d) = x$

$deg(a) = 3 = deg(u)$ $deg(b) = 3 = deg(w)$ |||ly ..

The graph G_1 & G_2 defined by fun f.
ie one-to-one correspondence between $\{a, b, c, d\}$ and $\{u, v, w, x\}$
and both graphs are complete graphs hence they are Isomorphic graphs.

Graph G_1 Graph G_2



$f(a) = u \quad f(b) = v \quad f(c) = w \quad f(d) = x$.

G_1 & G_2 are not isomorphic graphs.

Imp

* Isomorphic graphs preserves all adjacencies.
* It also preserves graph substructures such as paths and cycles.

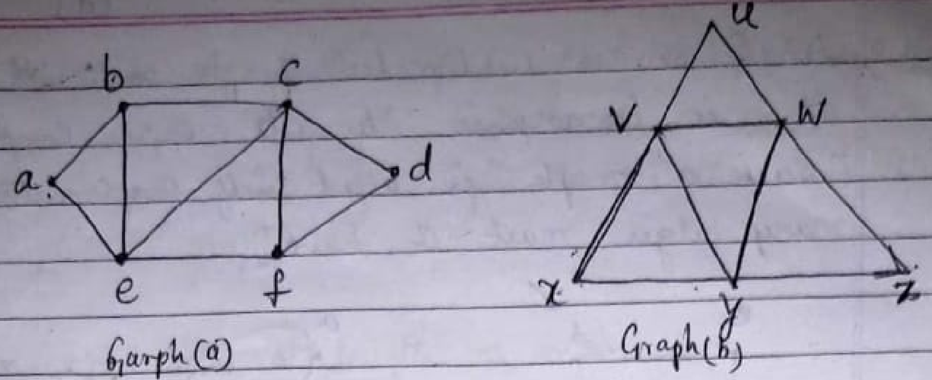
Eg (a,c) (c,d) (d,a) constitute a cycle of length 3 in figure 1

Illy its corresponding edges in fig (b) are (u,v) (v,x) (x,u) of length 3.

* a → c → d → b in Graph G_1 of figure (1) is a path

Illy its corresponding path in fig (b) are u → v → x → w. in G2 of fig (1).

Eg



Problem Each graph has 6 vertices and nine edges. Check whether they are isomorphic.

In graph (a) vertex a is adjacent to two other vertices of the graph. It can be associated with u of graph (b).
|||ly (d) can be associated with either x or z.

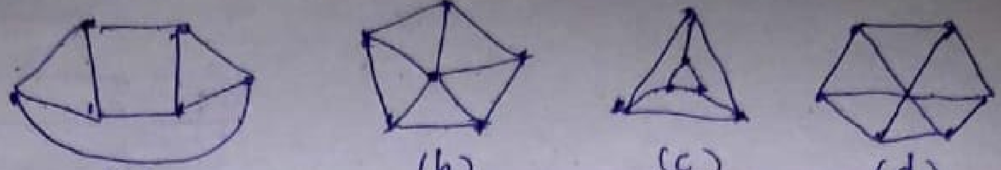
there is no other vertex to continue with one-to-one structure preserving correspondence.
∴ These graphs are not isomorphic.

Problem
Eg Which, if any, of the pairs of graphs shown in fig are isomorphic. Justify.

Ans (i) Graphs (a) (c) (d) are regular (ie each vertex has the same degree) But (b) is not regular since one vertex has degree 5, while the remaining vertices have degree 3.

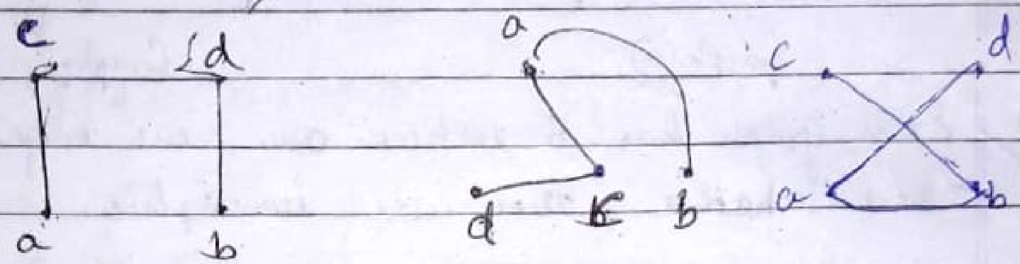
(ii) Graphs (a) and (c) have each two cycles of length 3 whereas (d) has no cycle of length 3.

(iii) (a) and (c) are isomorphic graphs.
(a) and (c) have the same number of six vertices and same number of nine edges.
The two cycles in each graph are {a, b, f} and {c, d, e} Also {b, c, e, f} is a cycle of length '4' Thus (a) & (c) are isomorphic.



(a) (b) (c) (d)

Problem a) Let G be an undirected graph with 'n' vertices. If G is isomorphic to its own complement $\overline{G}$ (such a graph is called self-complementary), how many edges must G have?



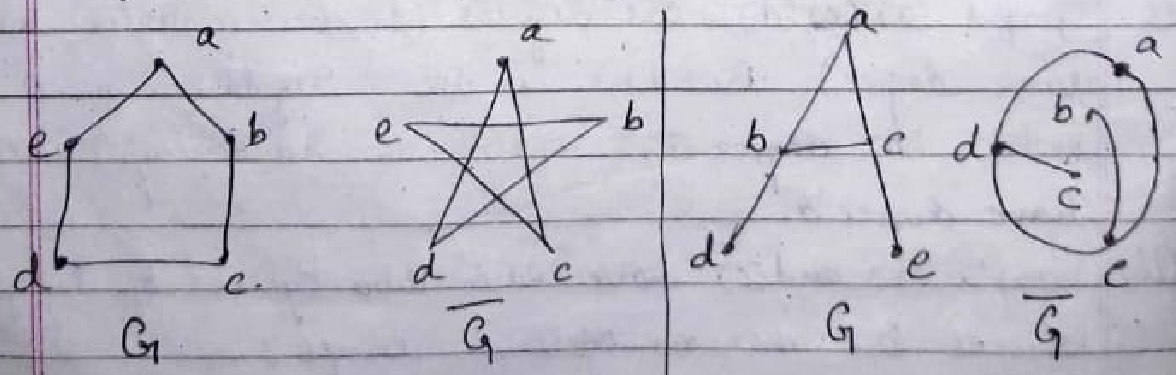
b) Give examples of self complementary graphs
i) of 4 vertices ii) five vertices.

Soln a) Let e_1 be the number of edges in G and e_2 be the number of edges in $\overline{G}$.
For any loop free undirected graph G,
we have the number of edges in K_n as $e_1 + e_2 = {}^nC_2$.

G is self complementary $e_1 = e_2$ so $e_1 = \frac{1}{2}({}^nC_2)$

$$= \frac{1}{2}\frac{n \cdot n-1}{2} = \frac{n(n-1)}{4}$$

b) Self complementary graph with five vertices



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Problem How many paths of length m are there in the Complete graph k_n with $m, n \in \mathbb{Z}^+$. How many paths of length 4 are there in the Complete graph k_7.

Soln:- no of paths of length m in k_n is

$$\frac{1}{2} n(n-1)(n-2)\cdots(n-m).$$

$7-4$:

In k_7, the ~~no~~ number of paths of length 4 is

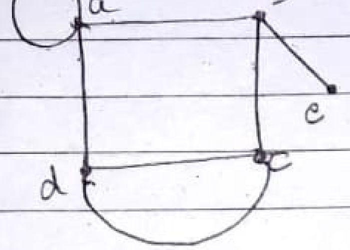
$$\frac{1}{\cancel{2}} 7 \cdot 6 \cdot 5 \cdot \cancel{4}^{2} \cdot 3 = \underline{\underline{1260}}.$$

Vertex Degree : Euler Trails and Circuits.

Defn Vertex Degree

Let G be an undirected graph or multigraph. For each vertex v of G, the degree of v, written $deg(v)$, is the number of edges in G that are incident with v. Here a loop at a vertex v is considered as two incident edges for v.

Eg:-



$deg(a) = 4$
$deg(b) = 3$
$deg(d) = 3$
$deg(c) = 3$
$deg(e) = 1$ (it is called Pendant Vertex)
$Total = 14$

$2(7) = 14$

Theorem If $G = \langle V, E \rangle$ is an undirected graph or multigraph, then $\sum_{v \in V} deg(v) = 2|E|$
(Handshaking Lemma)

Proof :- As we know that $\{a, b\}$ edge in graph G, we find that the edge contributes a count of 1 to $deg(a)$ and 1 to $deg(b)$, consequently a count of 2 to $\sum_{v \in V} deg(v)$

Thus $2|E|$.

$2|E| = 2.|7| = 14$

Note :

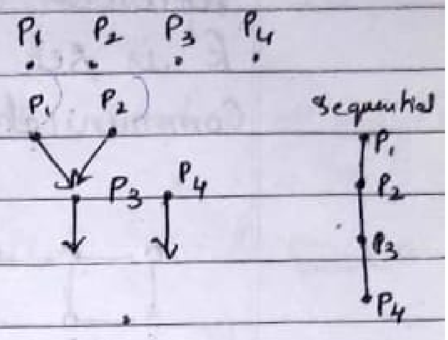
Defn Regular Graph

An undirected graph (or multigraph) where each vertex has the same degree is called a regular graph.
If $degree(v) = k$ for all vertices of v, then the graph is called k-regular.

ie: $2|E| = \deg|V|$.

Application of regular Graph in Computer Architecture.

Hyper Cube:



The Sequential model that we used is called Random Access Machine (RAM)

We assume that various operations such as addition, Subtraction, multiplication and various other operations can be performed in one unit of time.

Parallel Computational model :-

The main feature in parallel Computing is inter process Communication.

ie, given any problem, the processors have to communicate among themselves and agree on the Sub problems each will work on. The processors need to Communicate each other to see whether everyone has finished its task and so on.

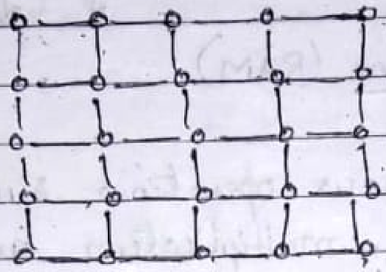
Each machine / processor in a parallel Computer can be assumed to be a Random Access Machine (RAM)

The parallel models are classified as

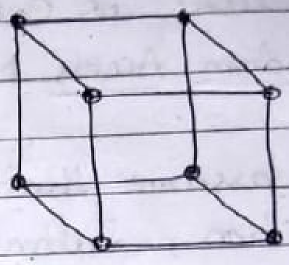
1. Fixed Connection Models
2. Shared memory Models

Fixed Connection Model.

Let $G = \langle V, E \rangle$
where V is set of vertices and each vertex represents a processor
E is set of edges and each edge represent Communication links b/n processors.



Mesh



Hyper Cube.

→ Communication is done through the communication ~~path~~ links.
Any two processors connected by an edge in G can communicate in one step.

→ In general, the two processors can communicate through any of the paths connecting them.

→ The Communication time depends on the lengths of these paths.

How can we decide on a model to speed up the processing time?

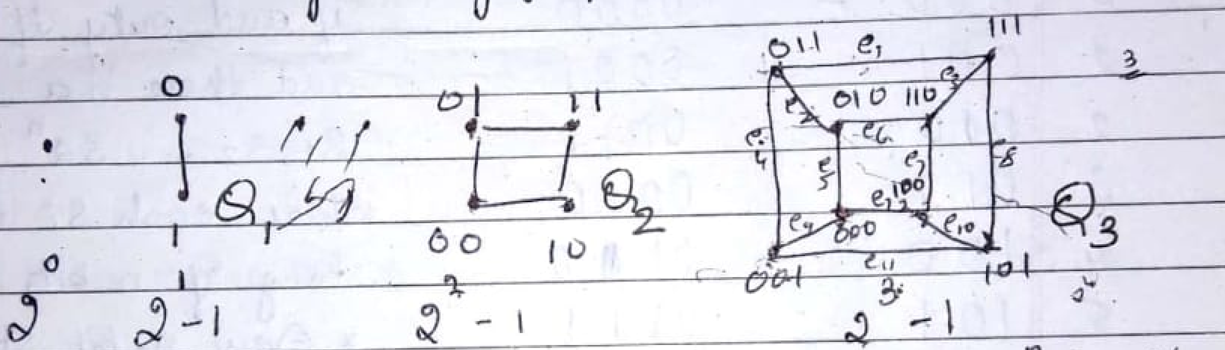
→ The Complete graph would be ideal - but expensive because of all the necessary Connections.

→ On the other hand one can connect 'n' processors along a path with n-1 edges, or on a cycle with 'n' edges. P_1 P_2 ... P_{n-1}

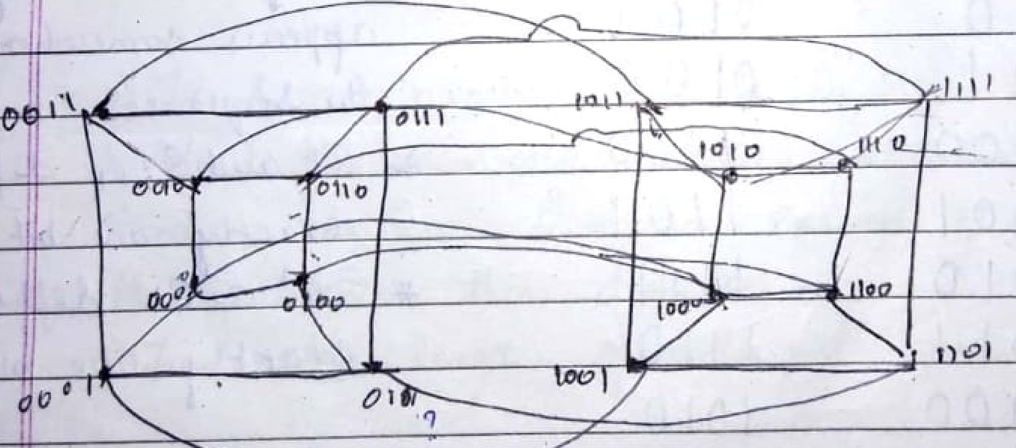
Date
Page

→ Another possible model is grid or mesh. But in these models the distances b/n pairs of processors get longer and longer as the number of ~~processing~~ processors increases.

→ A compromise that weighs the number of edges against the distance b/n pairs of vertices is the regular ~~gr~~graph called the hypercube.



2^n n-bit sequence.



$4 : 2^{4-1}$

$4 \cdot 2^3$

Summary: ① for $n \in N$, the hypercube Q_n is an n-regular ~~loop free undirected graph with~~ 2^n vertices.

② the distance b/n any two vertices is at most n

③ Q_n has $(\frac{1}{2}) n\, 2^n = n \cdot 2^{n-1}$ edges.

$\frac{1}{2} \cdot 3 \cdot 2^3 (8) = 3 \cdot 2 \cdot 2$
$12 = 12$

If we use Grey code to label the vertices of this diagram, we have ~~hypercu~~ hypercube.

Binary 1 ⊕ 0 0 1 0 1

1 1 0 1 1 1

1 ⊕ 1 ⊕ 1 ⊕ 0

1 0 0 1

Ex-OR ⊕		
0	0	0
0	1	1
1	0	1
1	1	0

$(100101)_2 = (110111)$ Gray code

2^4

	Binary	Gray code
0	000	0000
1	001	0001
2	010	0011
3	011	0010
4	100	0110
5	101	0111
6	110	0101
7	111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

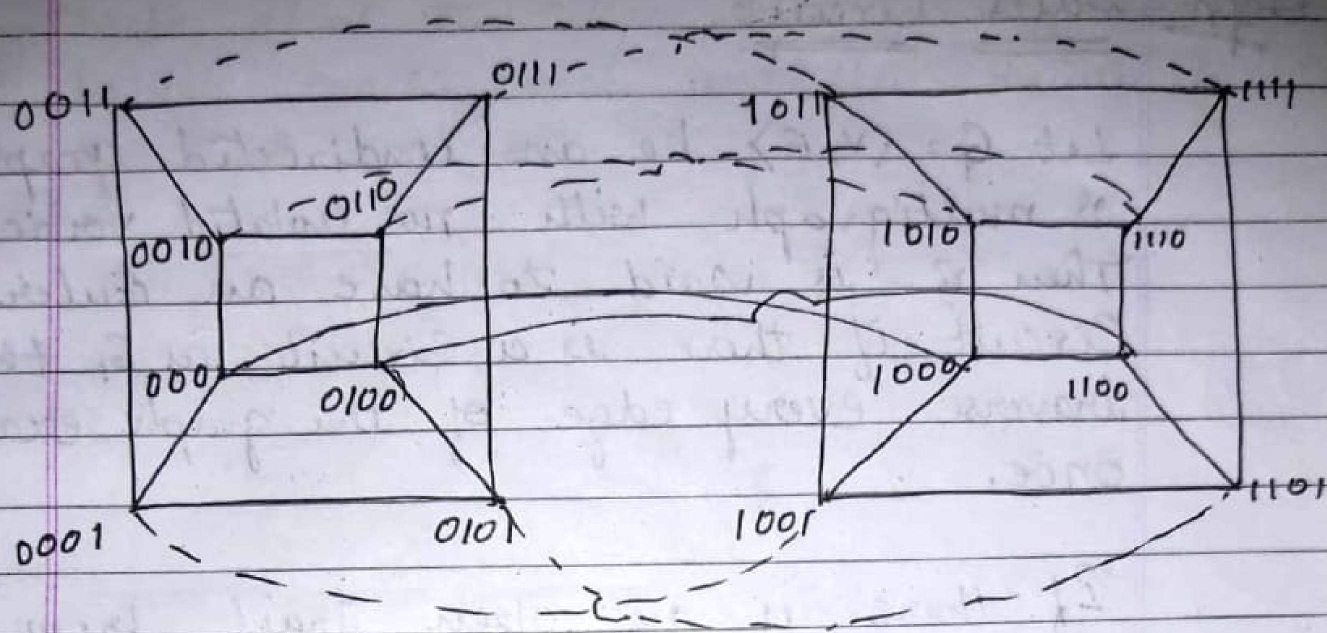
0 to $2^n - 1$ vertices.

if and only if $n \geq 2$ and there is a sequence $S_1, S_2 \ldots S_{2^n}$ where each S_i is a string of n bits satisfying

* Every n-bit String appears somewhere in the Sequence
* S_i and S_{i+1} differ in exactly one bit
* S_{2^n} & S_1 differ in exactly one bit

This type of Sequence is Called a Gray Code and Corresponds to

1) Hyper Cube
2) Hamiltonian Cycle



$n = 3$.

0 - 7 are allocated to nodes 0, 1, 3, 2, 6, 7, 5 4.

Problem: a) Find the number of edges in Q_{10}.
b) Find the maximum distance between pairs of Vertices in Q_{10}. Give an example of one pair that achieves this distance.
c) Find the length of a longest path in Q_{10}.

Soln Q_n has $\frac{1}{2} 2^n n = n \cdot 2^{n-1}$ edges.

a) $\therefore Q_{10}$ has $10 \cdot 2^9 = 5120$ edges.

b) The distance b/n any two vertices is at most $n = 10$.

c) A longest path in Q_{10} ~~Contai~~ Contains all of the 2^{10} Vertices. Such a path has length $2^{10} - 1 = 1023$

Defn **Euler Circuit**

Let $G = \langle V, E \rangle$ be an undirected graph or multigraph with no isolated vertices. Then G is said to have an Euler Circuit if there is a circuit in G that traverse every edge of the graph exactly once.

If there is an open trail from a to b in G and this trial traverses each edge in G exactly once, the trail is called an Euler ~~unit~~ trail.

Circuit : Vertex can be repeated, edges are not repeated, closed. } Circuit.

Trial : Vertex can be repeated, edges are not repeated, open } trail

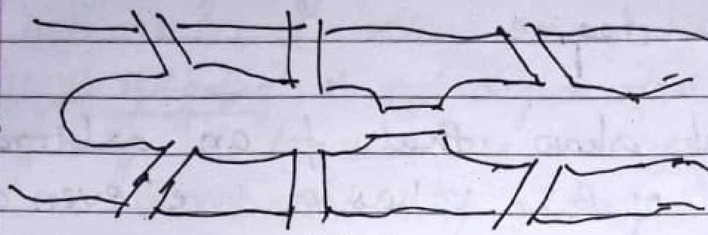
Konigsberg problem.

The Seven bridges of Konigsberg :-

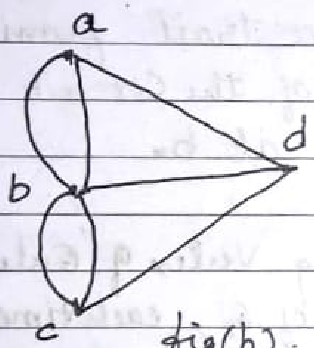
Problem During the eighteenth century, the city of Konigsberg (in East Prussia) was divided into four sections by the Pregel River.

Seven bridges connected as shown below.

It was said that residents spent their Sunday walks trying to find a way to walk about the City so as to cross each bridge exactly once and then return to the Starting point.



fig(a)



a

(a-d) (d,c) (c,b) (b,a

fig(b).

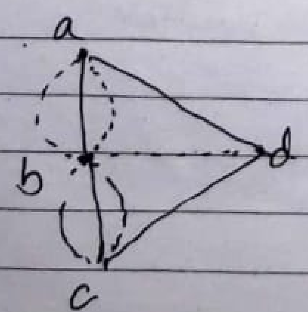
Deg(a) = 3 = Deg(c) = Deg(d)

Deg(b) = 5

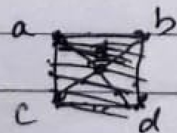
All the degree is odd number.

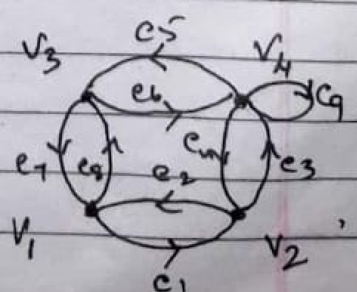
∴ Euler Circuit does not exist.

It is not possible to walk over each of the 7 bridges exactly once and return to the Starting point.



(ab)(bc) (cd) (d,b) (ba)





Theorem: Let $G = \langle V, E \rangle$ be an undirected graph or multigraph with no isolated vertices. Then G has an Euler Circuit if and only if G is Connected and every vertex in G has even degree.

Proof: We must show that for an arbitrary Vertex v of G, v has a +ve even degree.

If G has an Euler Circuit, then for all $a, b \in V$ there is a trail from a to b - namely, that part of the Circuit that starts at a and terminates at b.

Let s be the starting Vertex of Euler circuit. For any other vertex v of G, each time the circuit comes to v it then departs from the vertex. Thus the circuit has traversed either two edges that are incident with v or loop at v. In either case a count of 2 is contributed to $deg(v)$. Since v is not the starting point i.e each edge incident to v is traversed only once a count of 2 is obtained each time the circuit passes through v. So $deg(v)$ is even.

~~As for the~~

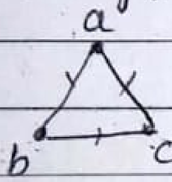
Defn Let $G = \langle V, E \rangle$ be a directed graph or multigraph For each $v \in V$

a) The incoming, OR in degree of v is the number of edges in G, that are incident into v and this is denoted by $id(v)$

b) The Outgoing, or out degree of v is the number of edges in G that are incident from v and this is denoted by $od(v)$.

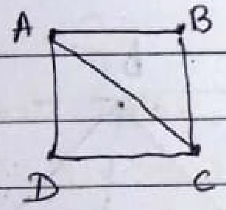
Theorem Prove that in a graph the number of verties of odd degree is even.

Eg:



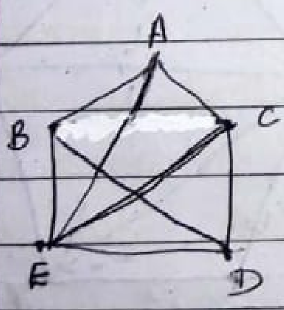
$deg(a) = 2$
$deg(b) = 2$
$deg(c) = 2$

Number of odd vertices are zero – even.



$deg(A) = 3$ — odd ~~de~~degree
$deg(B) = 2$
$deg(C) = 3$ — odd degree
$deg(D) = 2$ — 0

even



$deg(A) = 3$ — odd
$deg(B) = 3$ — odd
$deg(C) = 3$ — odd
$deg(D) = 3$ — odd

– even

3+2 =

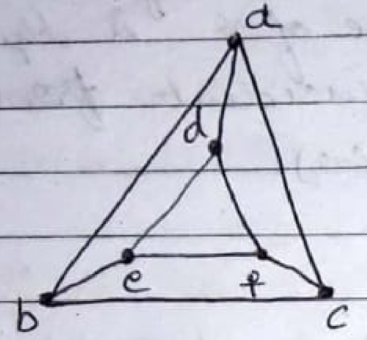
3

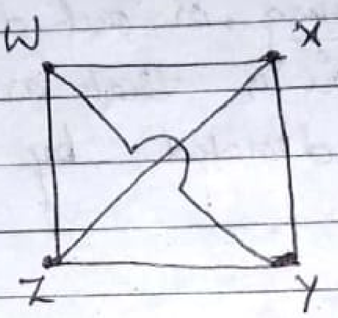
Note :- It is possible iff Number of odd degree Vertices are even.

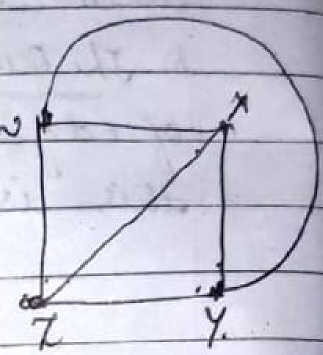
Planar Graphs

Defn A graph (or multigraph) G is called planar if G can be drawn in the plane with its edges intersecting only at vertices of G. Such a drawing of G is called an embedding of G in the plane.

Eg:1

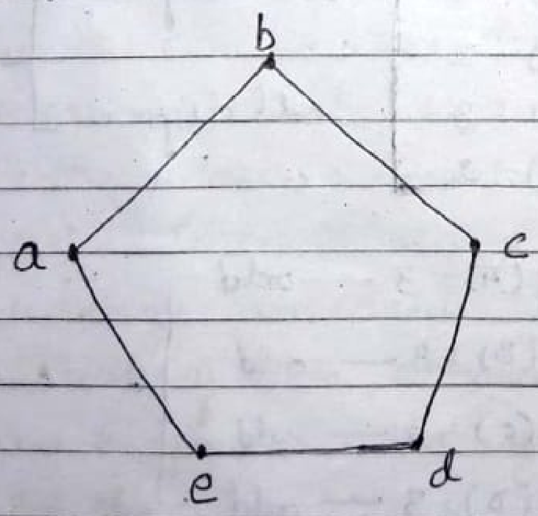


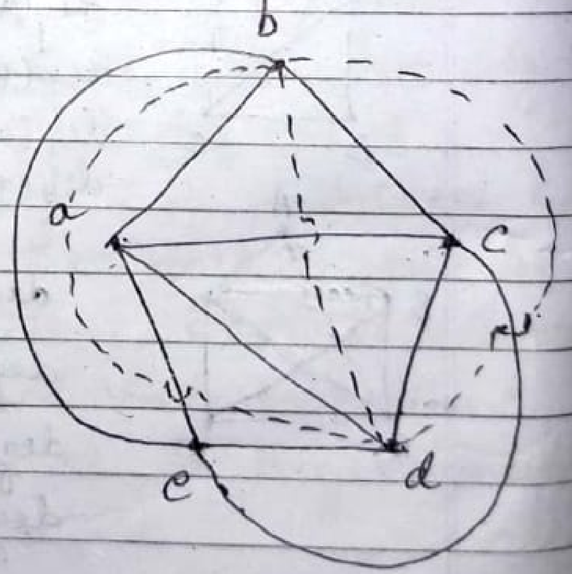




K_3 - 3 regular graph.
is a planar graph.

Eg2 K_5



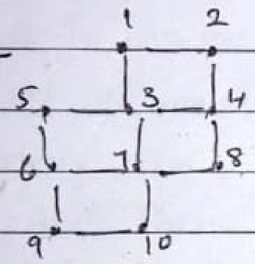


$\therefore K_5$ is nonplanar.

Defn. Bipartie Graph

A graph $G = \langle V, E \rangle$ is called bipartie if $V = V_1 \cup V_2$ with $V_1 \cap V_2 \neq \phi$ and every edge of G is of the form $\{a, b\}$ with $a \in V_1$ and $b \in V_2$. If each vertex in V_1 is joined with every vertex in V_2, we have a Complete bipartite graph. In this case, if $|V_1| = m$, $|V_2| = n$, the graph is denoted by $K_{m,n}$

Eg



G_1

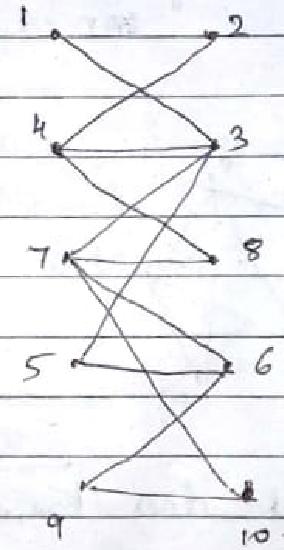
$f(1) = 1$

$f(2) = 2$

$\vdots$

$f(10) = 10$

$\cong$

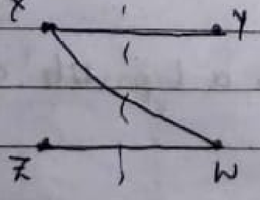


G_2.

Bipartie Graph.

G is bipartie graph if we can split V into components V_a and V_b where each $e \in E$ is of the form $\{v \in V_a, v \in V_b\}$

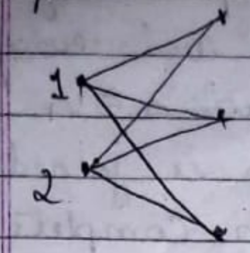
Subsets V_a, V_b Left right bi partite
2 partition.



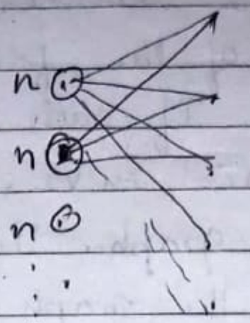
Discrete Maths 2 → TrevTutor.

Complete Bipartite Graph.

$K_{2,3}^{m,n}$



$K_{m,n}$



$3+3=6$

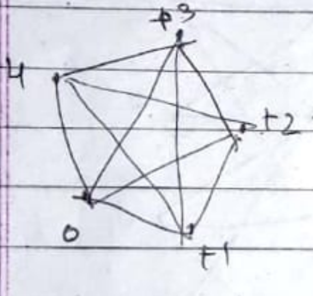
$2(3)=6$

$n+n+n \ldots n$ times n.

$m \cdot n =$ edges

How many edges are there in $K_{m,n}$.

mn edges



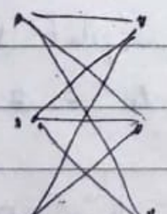
$\therefore K_m = \sum_{i=0}^{m-1} i$

What does $\overline{K_{m,n}}$ look like?

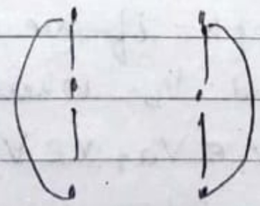
$K_{3,3}$

$\overline{E_G} = E_{K_M} - E_G$.

$K_{m,n}$



$\overline{K_{m,n}}$



The maximum no of edges in a bipartite graph with n vertices is $\left[\frac{n^2}{4}\right]$

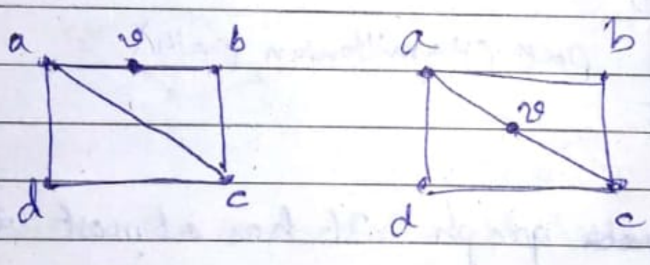
Eg $n = 6 = \frac{n^2}{4} = \frac{36}{4} = \underline{9}$

Defn A graph $G = \langle V, E \rangle$ is called bipartite if $V = V_1 \cup V_2$ with $V_1 \cap V_2 = \phi$ and every edge of G is of the form $\{a, b\}$ with $a \in V_1$ and $b \in V_2$. If each vertex in V_1 is joined with every vertex in V_2, we have a complete bipartite graph.

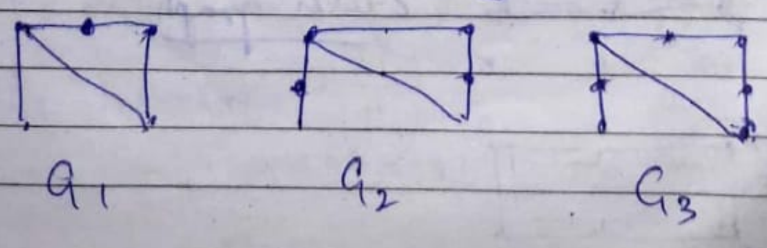
In this case if $|V_1| = m$ $|V_2| = n$ the graph is denoted by $K_{m,n}$.

Defn Let $G = \langle V, E \rangle$ be a loop free ~~ixxx~~ undirected graph, where $E \neq \phi$.

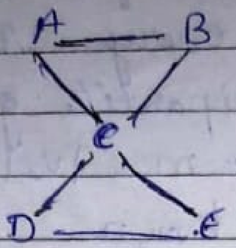
A elementary subdivision of G results when an edge $e = \{u, w\}$ is removed from G and then the edge $\{u, v\}$, $\{v, w\}$ are added to $G - e$, where $v \notin V$.

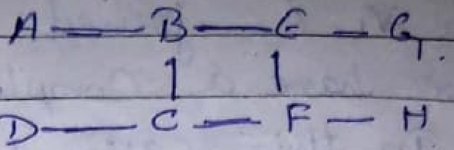


Defn Homeomorphic: The loop free undirected graph $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ are called homeomorphic if they are isomorphic or if they can both be obtained from the same loop free undirected graph by a sequence of elementary subdivisions.



Hamiltonian path : in a graph $G\langle V, E\rangle$, P is called hamiltonian path if it covers or visits every vertex exactly once.

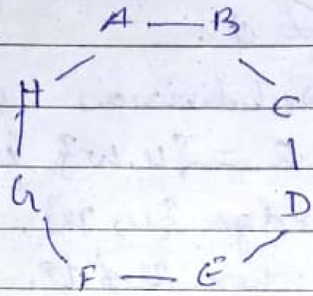




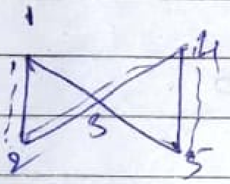
not hamiltonian graph.

A - B - C - D - E

~~Properties~~

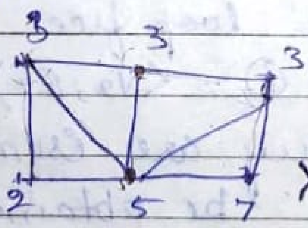


Hamiltonian Cycle.

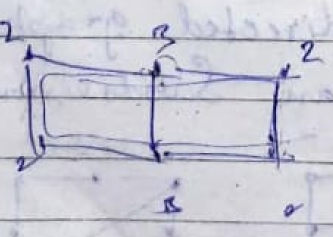


path (Hamiltonian path)

* * Note Euler ~~path~~ graph It has at most 2 odd degree vertices.



X not Euler graph.



Euler graph