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DISCRETE MATHEMATICAL STRUCTURES

QUESTION BANK

Module 1

1) Prove that the following are valid arguments

$p \to (q \to r)$
$p \vee \neg s$
q

$\therefore s \to r$

b) $p \to (q \wedge r)$
$r \to s$
$\neg (q \wedge r)$

$\therefore \neg p$

$p \to q$
$q \to (r \wedge s)$
$\neg r \vee (\neg r \vee u)$
$p \wedge t$

$\therefore u$

$u \to r$
$(r \to s) \to (p \vee t)$
$q \to (u \wedge s)$
$\neg t$

$\therefore q \to p$

$(p \wedge q)$
$p \to (r \wedge q)$
$r \to (s \vee t)$
$\neg s$

$\therefore t$

2. Prove that for any proportions p, q, r the compound proportions

$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology

3. Define a tautology and contradiction. Prove that the proposition, $[(p \to r) \wedge (q \to r)] \to (p \vee q) \to r$ is tautology

4. Apply principle of duality for the following

$(P \wedge Q) \vee (P \wedge R) \wedge T_0$

5. Simplify: $(p \vee q) \wedge \neg [(\neg p \vee q)]$

6. Prove the following logical equivalence using the laws of logic

i) $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)\} \Leftrightarrow \sim [(q \vee p)]$
ii) $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

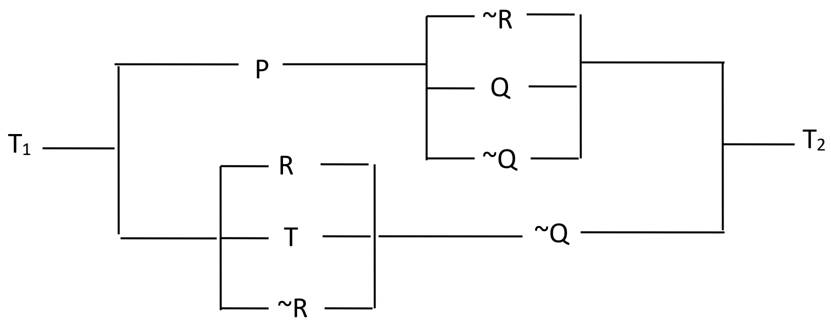
7. For the following statements consists of all non-zero integers. Determine the truth values of the following,
i) $\exists x \exists y\ [xy = 1]$
ii) $\exists x \forall y\ [xy = 1]$
iii) $\forall x \exists y\ [xy = 1]$

8. Define an open statement. Write down the negation of following statements: i) For all integer 'n' if n is not divisible by 2 then n is odd. ii) If k, m, n are any integers when (k - m) and (m - n) are odd then (k - n) is even.

9. Give i) a direct proof ii) an indirect proof iii) proof by contradiction statement : "If n is an odd integer, then n + 9 is an even integer".

10. The open statements p(x), q(x), r(x), and s(x) are given by p(x): x>=0 q(x): x2>=0 r(x): x2-3x-4 = 0 s(x): x2-3>0 Find the truth value of the following, i) $\exists x[p(x) \wedge q(x)]$ ii) $\forall x[p(x) \rightarrow q(x)]$ iii) $\forall x[q(x) \rightarrow s(x)]$ iv) $\forall x[r(x) \vee s(x)]$ v) $\exists x[p(x) \wedge r(x)]$ vi) $\forall x[r(x) \rightarrow p(x)]$

11. Simplify the following switch network



Module 2

Q1. Prove by Mathematical Induction , for every positive integer 4 divides $5^n + 2.3^{n-1} + 1$ (06 M)

Q2. Prove that $4^n < (n^2 – 7)$ for all positive integer n >= 6 (05 M)

Q3. Show that **$2^n > n^2$** for all positive integer **n** greater than **4**.

Q4.Prove that **$[1^2 + 2^2 + 3^2 + \text{- - - -} + n^2] = (n(n+1)(2n+1))/ 6$** using mathematical Induction (05 M)

Q5. Assume PASCAL language is case insensitive, an identifier consists of a single letter followed by upto seven symbols which may be letters or digits (26 letters , 10 digits). There are 36 reserved words. How many distinct Identifiers are possible in this version of PASCAL. (05 M)

Q6. Find the co-efficient of $a^2 b^3 c^2 d^5$ in the expression of $(a + 2b-3c + 2d + 5)^{16}$ (05 M)

Q7. Find the number of distinct terms in the expansion of $(W+X+Y+Z)^{12}$ (05 M)

Q8. By MI P.T

$1^2 + 3^2 + 5^2 + \text{-------------} + (2n-1)^2 = [n(2n + 1)(2n -1)]/ 3$ (05 M)

Q9. If **$U_1 = 3$ and $U_2 = 5$, $U_n = 3U_{n-1} – 2U_{n-2}$** where **n>=3**. Then prove that **$U_n = 2^n + 1$** for all positive integers n.

Q10. Find an explicit definition of the sequence defined recursively as follows

$a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$

Q11. Using mathematical induction prove that $3^n < (n+1)!$ for all positive integer $n \geq 4$.

Q12. If F_0, F_1, F_2 … are Fibonacci numbers, Prove that $\sum_{i=1}^{n} \frac{Fi-1}{2i} = 1 - \frac{Fn+2}{2n}$ for all positive integers of n.

If $F_{i's}$ are the Fibonacci numbers and $L_{i's}$ are the Lucas numbers, prove that $L_{n+4} - L_n = 5F_{n+2}$ for all positive integers n >= 0.

Q13. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations?

i) There is no restriction on the choice.
ii) Two particular persons will not attend separately.
iii) Two particular persons will not attend together.

Q14. i) How many arrangements are there for all letters in the word SOCIOLOGICAL
ii) In how many of these arrangements A and G are adjacent.
iii) In how many of these arrangements all the vowels are adjacent. (04 M)

Q15. In the expression of $(x+y+z)^7$ find the number of distinct terms (04 M)

Q16. Show that for positive integers n, t the co-efficient s of $X_1^{n1} X_2^{n2} X_3^{n3} \cdots X_t^{nt}$ in the expression of $(X1+X2+X3+ \ldots Xt)^n$ is $n! / (n_1!\, n_2!\, n_3! \ldots n_t!)$ where each ni is an integer with 0<=ni<= n, for all 1<=i<=t and $n_1+n_2+n_3 \text{----} n_t = n$. (04M)

Q17. How many ways 10 pens , 14 pencils and 15 books can be distributed among 3 boys. (04M)

Q18. Determine the co-efficient xyz^2 in the expansion of $(2x-y-z)^4$.

Q19. How many ways 10 roses, 14 sunflowers, 15 daffodils can be distributed among 3 girls.

Q20.A certain question paper contains two parts **A** and **B**, each containing **4** questions. How many different ways a student can answer **5** questions by selecting atleast **2** questions from each part?

Q20. How many numbers greater than **1000000** be formed by using digits **1,2,2,2,4,4,0**

Q21. Using mathematical induction show that 3 divides $n^3 - 7n + 3$ for all positive integer N.

Q22. Q30. For the Fibonacci sequence F0, F1, F2 … prove that $\mathrm{Fn} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1+\sqrt{5}}{2}\right)^n\right]$